Altuğ Özpineci

2 Midterms each 25%

1 Final 25%

HW 25%

1st Midterm Apr 1, Saturday

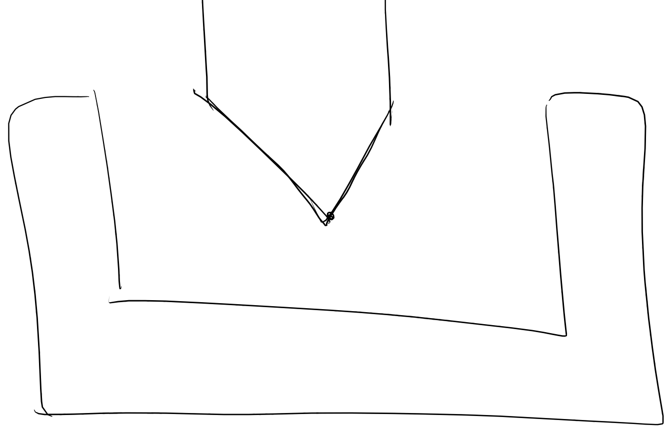
2nd Midterm May 6, Saturday

$13^{30} - 16^{30}$

Make-up : The monday after the finals

Quantum Mechanics

Stern-Gerlach experiment (1922)



$$U = -\vec{\mu}\cdot\vec{B}$$

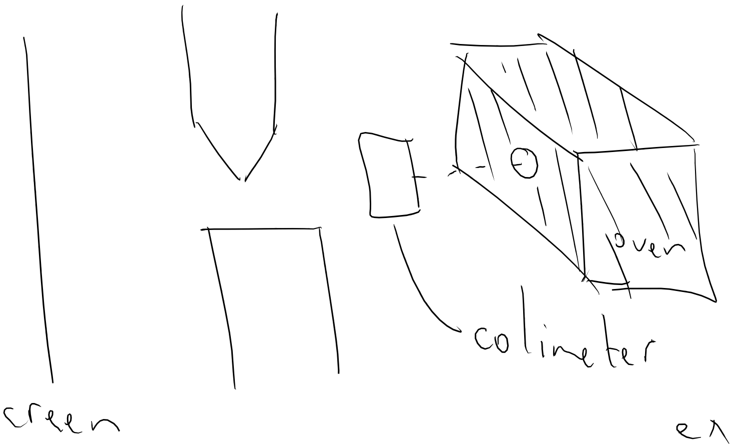
$$\vec{F} = -\vec{\nabla} U$$

$$= \vec{\nabla}(\vec{\mu}\cdot\vec{B})$$

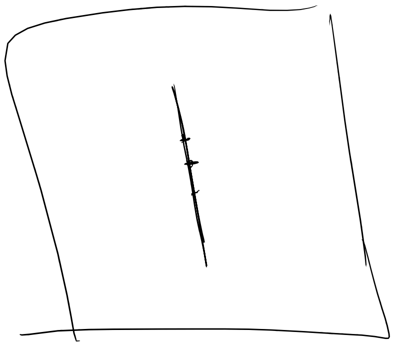
$$F_i = \mu_j \frac{\partial B_j}{\partial x_i}$$

$$F_i = \mu_j \frac{\partial B_i}{\partial x_j}$$

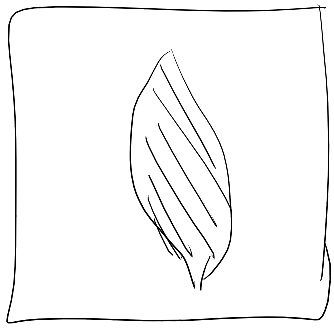
$$F_z = \mu_j \frac{\partial B_z}{\partial x_j}$$



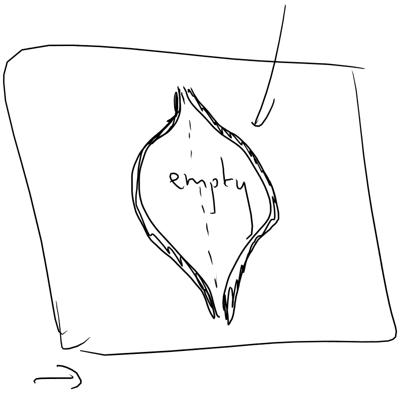
no B



expectation with B



observation

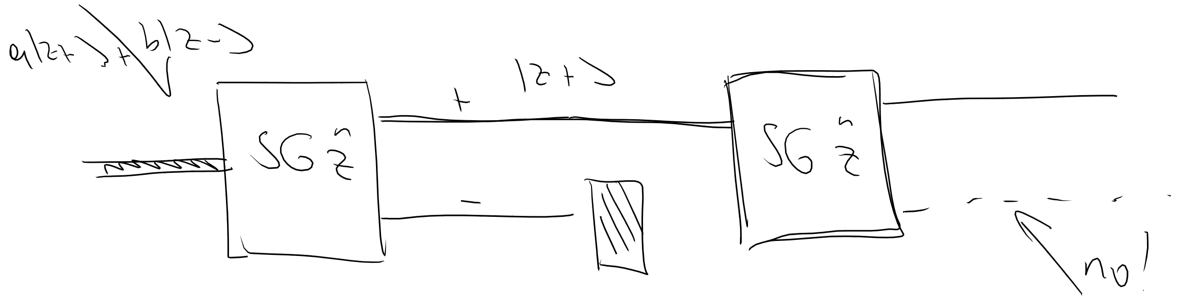


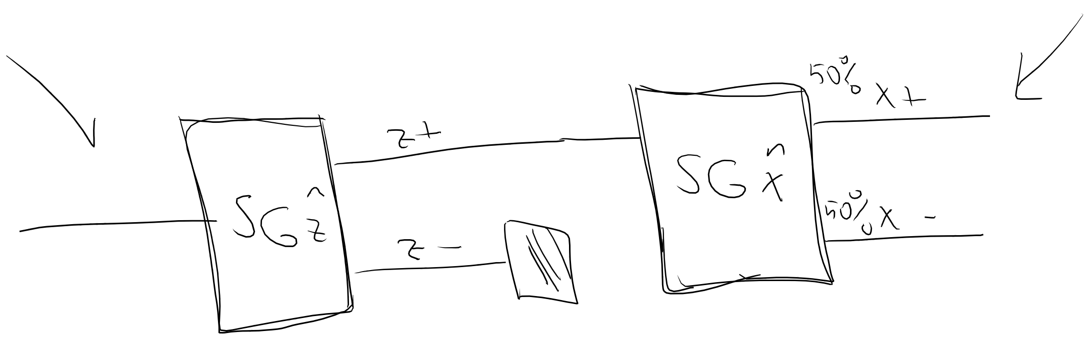
$$\vec{\mu} = \frac{e}{mc} \vec{S}$$

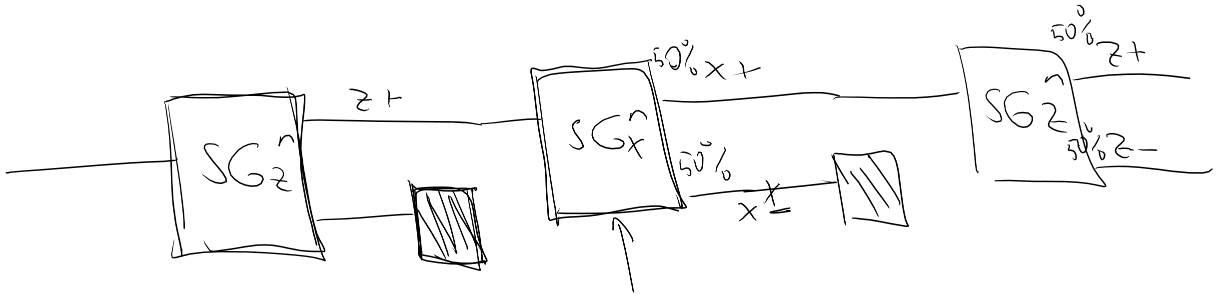
Ag 47 electrons

46 fill the spherical shell

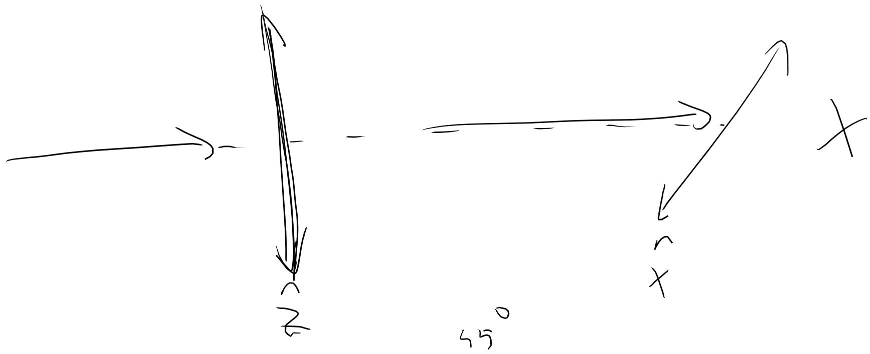
1 electron (5s) gives Ag its spin



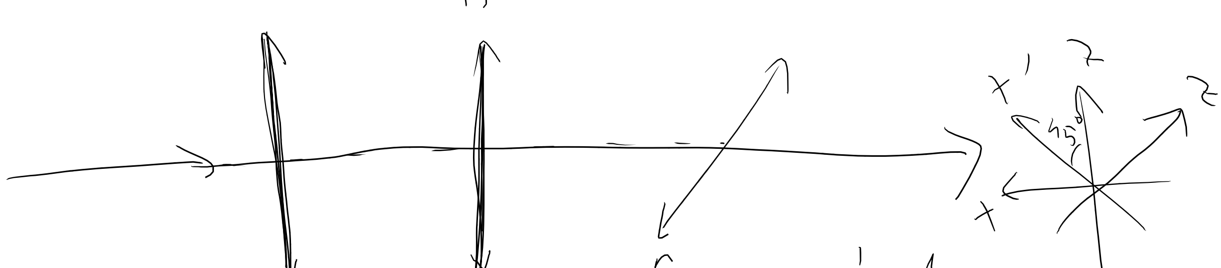




Optics – Polarizers 3D glasses



$$\hat z = \frac{1}{\sqrt{2}}\hat x' + \frac{1}{\sqrt{2}}\hat z'$$



$$\hat x' = \frac{1}{\sqrt{2}}\hat x + \frac{1}{\sqrt{2}}\hat z$$

$$\vec{E} = \vec{E}_0 \cos(ky - \omega t)$$

$$\vec{E}_0 = E_{0x}\hat{x} + E_{0z}\hat{z}$$

after 1^{st} filter

$$\vec{E}^{(1)} = E_{0z}\cos(ky-\omega t)\hat{z}$$

after 45° polarizer

$$\vec{E}^{(2)} = E_{0z}\cos(ky-\omega t)\frac{1}{\sqrt{2}}(\hat{x}' + \hat{z}')$$

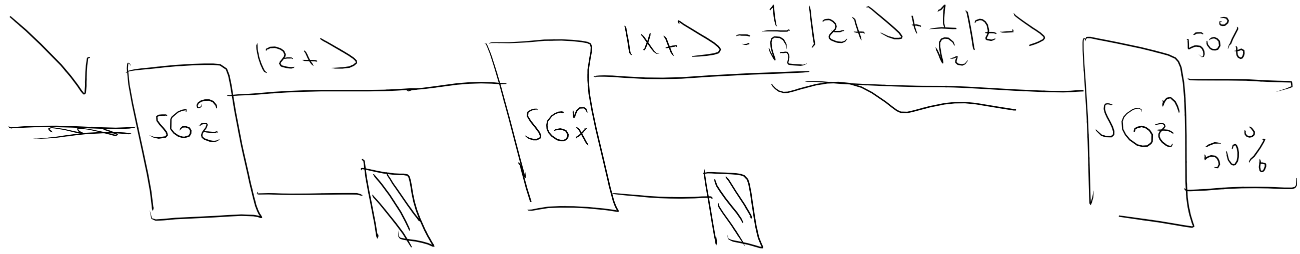
$$\vec{E}^{(2)} = \frac{1}{\sqrt{2}}E_{0z}\cos(ky-\omega t)\hat{x}'$$

$$\vec{E}^{(2)} = \frac{1}{\sqrt{2}}E_{0z}\cos(ky-\omega t)\frac{1}{\sqrt{2}}(\hat{x}+\hat{z})$$

$$|\text{initial state}\rangle \longrightarrow \begin{matrix}|z+\rangle \\ |z-\rangle\end{matrix}$$

$$\left.\begin{aligned}|z+\rangle &= \frac{1}{\sqrt{2}}|x+\rangle + \frac{1}{\sqrt{2}}|x-\rangle \\ |z-\rangle &= \frac{1}{\sqrt{2}}|x+\rangle - \frac{1}{\sqrt{2}}|x-\rangle\end{aligned}\right\}$$

$$|z+\rangle = \frac{1}{\sqrt{2}}|x+\rangle + \frac{1}{\sqrt{2}}|x-\rangle$$



$$\begin{pmatrix} |z+\rangle \\ |z-\rangle \end{pmatrix} \qquad \begin{pmatrix} |x+\rangle \\ |x-\rangle \end{pmatrix}$$

Bra-Ket notations

bra|ket $\langle\,|\,\rangle$

$|z+\rangle, |z-\rangle \ldots \quad |\alpha\rangle$: vectors

$$a|z+\rangle + b|z-\rangle$$

Hilbert spaces: infinite dimensional vector spaces

inner product

$$(|\alpha\rangle, |\beta\rangle) \longrightarrow \text{number}$$

$$\underbrace{(|\alpha\rangle, \cdot\,)}_{} : |\beta\rangle \longrightarrow \underbrace{(|\alpha\rangle, |\beta\rangle)}_{\text{number}}$$

$\xrightarrow{\text{bra}} \langle\alpha|$

$$\langle\alpha|(|\beta\rangle) = (|\alpha\rangle, |\beta\rangle) \equiv \langle\alpha|\beta\rangle$$

$$|\alpha\rangle \xrightarrow{\text{dual correspondence}} (|\alpha\rangle, \cdot) \equiv \langle\alpha|$$

$$|\alpha\rangle^* = |\alpha\rangle^\dagger = \langle\alpha|$$

$$(a|\alpha\rangle + b|\beta\rangle)^\dagger = a^*\langle\alpha| + b^*\langle\beta|$$

$$a|\alpha\rangle = |\alpha\rangle a \qquad a \in \mathbb{C}$$

$$a\langle\beta| = \langle\beta| a$$

$$(\langle\alpha|)(|\beta\rangle) = \langle\alpha|\beta\rangle$$

$$\underbrace{|\beta\rangle\langle\alpha|}_{\text{operator}} \neq \langle\alpha|\beta\rangle$$

number

$$\left.\begin{array}{l} |\alpha\rangle|\beta\rangle \\ \langle\beta|\langle\alpha| \end{array}\right\} \text{wrong (warning)}$$

$$|\alpha\rangle \otimes |\beta\rangle \rightarrow |\alpha\rangle|\beta\rangle \qquad \otimes : \text{\otimes}$$

$$\langle\beta| \otimes \langle\alpha| \rightarrow \langle\beta|\langle\alpha|$$

$$\underbrace{(|\alpha\rangle\langle\beta|)}\underbrace{(|\gamma\rangle)} = |\alpha\rangle\underbrace{\langle\beta|\gamma\rangle}_{\text{number}}$$

$$|\alpha\rangle\langle\beta|$$

$$\{ |z+\rangle, |z-\rangle \}$$

$$\{ |x+\rangle, |x-\rangle \}$$

$$\| |\alpha\rangle \| = \sqrt{\langle\alpha|\alpha\rangle}$$

$$\left.\begin{aligned} \langle z+|z+\rangle &= 1 \\ \langle z-|z-\rangle &= 1 \\ \langle z+|z-\rangle &= 0 \end{aligned}\right\}$$

$\{ |z+\rangle, |z-\rangle \}$ form an <u>ortho</u> <u>normal</u> basis

$$|\alpha\rangle = a|z+\rangle + b|z-\rangle$$

$$|\beta\rangle = c|z+\rangle + d|z-\rangle$$

$$\langle\beta|\alpha\rangle = \left(c^* \langle z+| + d^* \langle z-|\right)\left(a|z+\rangle + b|z-\rangle\right)$$

$$= c^* a \overbrace{\langle z+|z+\rangle}^{1} + c^* b \overbrace{\langle z+|z-\rangle}^{=0}$$

$$+ d^* a \underbrace{\langle z-|z+\rangle}_{=0} + d^* b \underbrace{\langle z-|z-\rangle}_{1}$$

$$\langle\beta|\alpha\rangle = \underline{c^* a + d^* b}$$

$$\langle\alpha|\alpha\rangle = |a|^2 + |b|^2$$

$$\| |\alpha\rangle \| = \sqrt{|a|^2 + |b|^2}$$