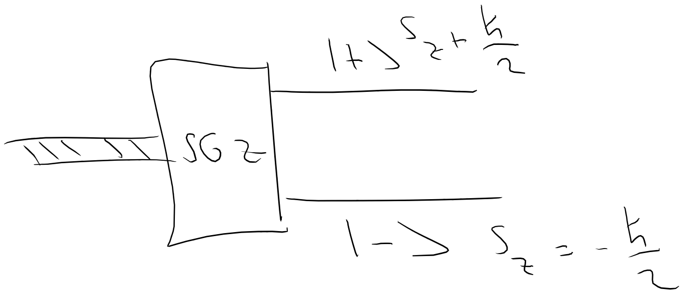
$$\{|\alpha_i\rangle\}$$

$$H|\alpha_i\rangle = \alpha_i|\alpha_i\rangle$$

$$\langle\alpha_i|\alpha_j\rangle = \delta_{ij}$$

$$H = \sum \alpha_i |\alpha_i\rangle\langle\alpha_i|$$



$$S_z = \frac{\hbar}{2}|+\rangle\langle+| + \left(-\frac{\hbar}{2}\right)|-\rangle\langle-|$$

$$S_z \doteq \begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{pmatrix}$$

$$\begin{pmatrix} |S_x+\rangle \\ |S_x-\rangle \end{pmatrix} \qquad \begin{pmatrix} |S_y+\rangle \\ |S_y-\rangle \end{pmatrix}$$

$$\Rightarrow |S_x+\rangle = \boxed{a|+\rangle + b|-\rangle} \qquad \boxed{\vec{r} = (1.2\,m)\,\hat{x}}$$

$$|S_x-\rangle = c|+\rangle + d|-\rangle$$

$$\boxed{|\gamma\rangle = \sum_i c_i|\alpha_i\rangle}$$

$$1 = \langle \gamma | \gamma \rangle = \sum_{i,j} c_i c_j^* \langle \alpha_j | \alpha_i \rangle$$

$$\underbrace{|\gamma\rangle, \quad c|\gamma\rangle}_{\text{correspond to the same physical system}} \qquad c \in \mathbb{C} \ \& \ c \neq 0$$

$$\langle \gamma | \gamma \rangle = 1$$

$$\left\{ |\gamma\rangle = \sum c_i |\alpha_i\rangle \right.$$

$$\langle \gamma | \gamma \rangle = \left(\sum_i c_i^* \langle \alpha_i | \right) \left(\sum_j c_j |\alpha_j\rangle \right)$$

$$= \sum_{i,j} c_i^* c_j \underbrace{\langle \alpha_i | \alpha_j \rangle}_{\delta_{ij}}$$

$$\boxed{1 = \sum_i |c_i|^2}$$

$|c_i|^2$: probability that a measurement of H will yield the value α_i

$$|\gamma\rangle^{\text{before}} = a_1 |\alpha_1\rangle + a_2 |\alpha_2\rangle$$

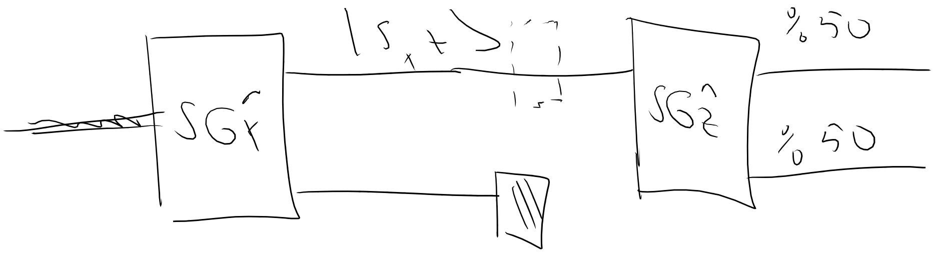
$$\Downarrow$$

$\boxed{\text{measurement of } H \text{ with outcome}}$ α_1

$$\Downarrow$$

$$|\gamma\rangle^{\text{after}} = |\alpha_1\rangle \qquad \text{collapse of the wave function.}$$

$$|S_x +\rangle , \ |S_x -\rangle$$



$$|S_x +\rangle = a|+\rangle + b|-\rangle$$

$$|a|^2 = \frac{1}{2} \Longrightarrow a = \frac{1}{\sqrt{2}} e^{i\alpha}$$

$$|b|^2 = \frac{1}{2} \Longrightarrow b = \frac{1}{\sqrt{2}} e^{i\beta}$$

$$|S_x +\rangle = \frac{1}{\sqrt{2}} e^{i\alpha} |+\rangle + \frac{1}{\sqrt{2}} e^{i\beta} |-\rangle$$

$$|S_x +\rangle = \left[\cancel{e^{i\alpha}} \frac{1}{\sqrt{2}} \left(|+\rangle + e^{i\delta_1} |-\rangle \right) \right] \cancel{e^{-i\alpha}}$$

$$\delta_1 \equiv \beta - \alpha$$

$$|S_x +\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle + e^{i\delta_1} |-\rangle \right)$$

$$|S_x -\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle + e^{i\delta_2} |-\rangle \right)$$

$$0 = \langle S_x - | S_x + \rangle = \frac{1}{\sqrt{2}} \left(\langle +| + e^{-i\delta_2} \langle -| \right)$$

$$\times \frac{1}{\sqrt{2}} \left(|+\rangle + e^{i\delta_1} |-\rangle \right)$$

$$0 = \frac{1}{2}\left(1 + e^{i(\delta_1 - \delta_2)}\right)$$

$$e^{i(\delta_1-\delta_2)} = -1$$

$$\Rightarrow e^{i\delta_1} = -e^{i\delta_2}$$

$$\boxed{\begin{aligned} |S_x +\rangle &= \frac{1}{\sqrt{2}}\left(|+\rangle + e^{i\delta_1}|-\rangle\right) \\ |S_x -\rangle &= \frac{1}{\sqrt{2}}\left(|+\rangle - e^{i\delta_1}|-\rangle\right) \end{aligned}} \Leftarrow$$

$$S_x = \frac{\hbar}{2}|S_x+\rangle\langle S_x+| + \left(-\frac{\hbar}{2}\right)|S_x-\rangle\langle S_x-|$$

$$= \frac{\hbar}{2}\frac{1}{2}\left(|+\rangle + e^{i\delta_1}|-\rangle\right)\left(\langle +| + e^{-i\delta_1}\langle -|\right)$$

$$-\frac{\hbar}{2}\frac{1}{2}\left(|+\rangle - e^{i\delta_1}|-\rangle\right)\left(\langle +| - e^{-i\delta_1}\langle -|\right)$$

$$= +\frac{\hbar}{4}\cancel{\left(|+\rangle\langle +| - |+\rangle\langle +|\right)}$$

$$+\frac{\hbar}{4}\left(|+\rangle\langle -| e^{-i\delta_1} + |+\rangle\langle -| e^{-i\delta_1}\right)$$

$$+\frac{\hbar}{4}|-\rangle\langle +|\left(e^{i\delta_1} + e^{i\delta_1}\right)$$

$$+\frac{\hbar}{4}\cancel{|-\rangle\langle -|(1-1)}$$

$$S_x = \frac{\hbar}{2}|+\rangle\langle -|e^{-i\delta_1} + \frac{\hbar}{2}|-\rangle\langle +|e^{i\delta_1}$$

$$\boxed{S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\delta_1} \\ e^{+i\delta_1} & 0 \end{pmatrix}}$$

$$|+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$S_x|+\rangle = \frac{\hbar}{2}e^{i\delta_1}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} 0 & e^{-i\delta_1} \\ e^{i\delta_1} & \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$