$$U(t_1, t_0)|\alpha, t_0\rangle = |\alpha, t_0; t_1\rangle$$

$$U^\dagger U = 1$$

$$U(t_2, t_1)\, U(t_1, t_0) = U(t_2, t_0)$$

$$U(t_1, t_0) = 1 - \frac{i}{\hbar} H(t_1 - t_0)$$ [if H is intepentent of time]

$$A|a_i\rangle = a_i|a_i\rangle$$

$$[H, A] = 0$$

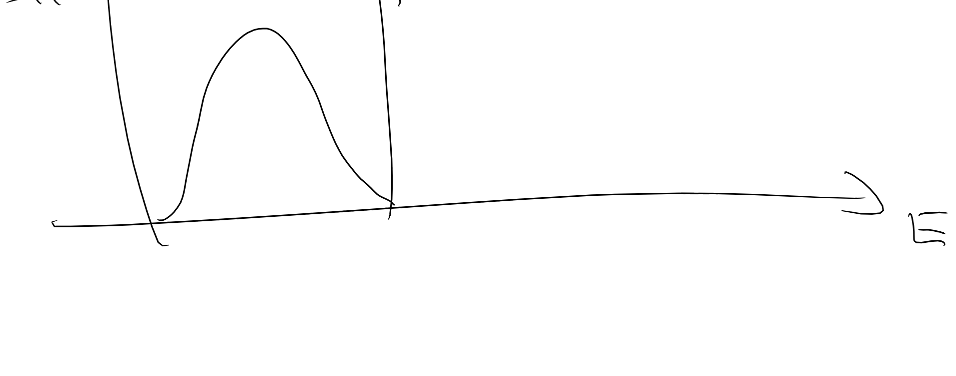
$$H|a_i\rangle = E_{a_i}|a_i\rangle$$

$$|\alpha, t_0\rangle = \sum c_i |a_i\rangle$$

$$|\alpha, t_0; t\rangle = \sum c_i e^{-\frac{i}{\hbar}E_{a_i}(t - t_0)} |a_i\rangle$$

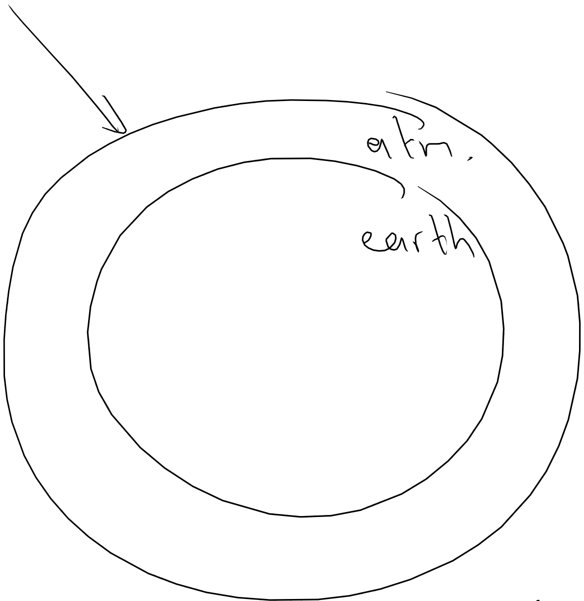
$$\langle B\rangle_\alpha^{(t)} = \sum c_i^* c_j e^{-\frac{i}{\hbar}(E_{a_i} - E_{a_j})} \langle a_i|B|a_j\rangle$$

Neutrino Oscillations



$\nu_e \quad \nu_\mu \quad \nu_\tau$

Solar neutrino problem



$\pi^\pm \to \mu^\pm \nu_\mu$

$\quad \hookrightarrow e \nu_e \nu_\mu$

$\pi^\pm \to e^\pm \nu_e \nu_\mu \bar{\nu}_\mu$

$$\frac{\# \nu_e}{\# \nu_\mu} \simeq \frac{1}{2} \text{ (exp)}$$

atmospheric neutrino problem.

$\simeq 1$ measured

$\{ |\nu_e\rangle, |\nu_\mu\rangle \}$ flavor basis

$\{ |\nu_1\rangle, |\nu_2\rangle \}$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$|\nu_e\rangle = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

$$\frac{(E^2 - \vec{p}^2 c^2)}{c^4} = m^2 \quad \sim 1 \text{ keV}$$

$$|\alpha, t=0\rangle = |\nu_e\rangle$$

$$|\langle \nu_e | \alpha, t\rangle|^2 = P(\nu_e \to \nu_e) \Leftarrow$$

$$|\langle \nu_\mu | \alpha, t\rangle|^2 = P(\nu_e \to \nu_\mu)$$

$$|\alpha, t\rangle = \cos\theta\, e^{-\frac{i}{\hbar}E_1 t}|\nu_1\rangle + \sin\theta\, e^{-\frac{i}{\hbar}E_2 t}|\nu_2\rangle$$

$$E_i = \sqrt{\vec{p}^2 c^2 + m_i^2 c^4} = pc\left(1 + \frac{m_i^2 c^2}{p^2}\right)^{1/2}$$

$$\simeq pc\left(1 + \frac{m_i^2 c^2}{2p^2}\right)$$

$$\boxed{E_i \simeq pc + \frac{m_i^2 c^3}{2p}}$$

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \alpha t\rangle|^2$$

$$= \left|\left(\cos\theta\langle\nu_1| + \sin\theta\langle\nu_2|\right)\left(\cos\theta\, e^{-\frac{i}{\hbar}E_1 t}|\nu_1\rangle + \sin\theta\, e^{-\frac{i}{\hbar}E_2 t}|\nu_2\rangle\right)\right|^2$$

$$= \left|\cos^2\theta\, e^{-\frac{i}{\hbar}E_1 t} + \sin^2\theta\, e^{-\frac{i}{\hbar}E_2 t}\right|^2$$

$$= \left|\cos^2\theta + \sin^2\theta\, e^{-\frac{i}{\hbar}(E_2 - E_1)t}\right|^2$$

$$= \left(\cos^2\theta + \sin^2\theta \cos\frac{\Delta E}{\hbar}t\right)^2$$

$$+ \left(\sin^2\theta \sin\frac{\Delta E}{\hbar}t\right)^2$$

$$P(\nu_e \to \nu_e) = \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \cos\left(\frac{\Delta E}{\hbar}t\right)$$

$$E_i \simeq pc + \frac{m_i^2 c^3}{2p}$$

$$\Delta E = E_2 - E_1 = (\Delta m^2)\frac{c^3}{2p}$$

$$t = \frac{L}{v} = \frac{L}{c}$$

$$\frac{\Delta E}{\hbar}t = \frac{\Delta m^2 c^3}{2p}\frac{L}{c} \simeq \frac{\Delta m^2 c^3 L}{2E/c}$$

$$p \simeq E$$

$$\boxed{P(\nu_e \to \nu_e) = 1 - B\left[\cos\left(\frac{\Delta m^2 c^3 L}{2E}\right) + 1\right]}$$

$$P(\nu_e \to \nu_\mu) = B\left[1 - \cos\left(\frac{\Delta m^2 c^3 L}{2E}\right)\right]$$

$$P(\nu_e \to \nu_e) = \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \cos\left(\frac{\Delta E}{\hbar} t\right)$$

$$\Delta E \, \Delta t \sim \hbar \qquad \Delta E \, \Delta t \sim \hbar$$

energy time
uncertainty relation