IOP Conference Series: Materials Science and Engineering

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Determination of the 1st Buckling and Collapse Loads for Integrally Stiffened Panels by Artificial Neural Network and Design of Experiment Methodology

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Abstract. Buckling is a structural instability that load carrying capacity of a structural element may suddenly decrease. This sudden change in the load carrying capacity may cause catastrophic failures. Therefore, determination of the first buckling and collapse loads of structural elements is essential. FE analyses and structural testing are used to determine buckling characteristics of a structural element. However, in early design stages, FE analyses are time consuming and structural testing is costly. In this paper, artificial neural network tool is used to reduce computational effort to determine buckling loads of integrally stiffened structural panels in early design stages. Moreover, Latin Hypercube Sampling (LHS) methodology is used to reduce the number of required FE analyses to generate database that artificial neural network is based on. Mean errors and fit performance model results are compared to determine accuracy of the neural network results.

1. Introduction

Load carrying capacity of a structural element may suddenly decrease due to a structural instability called buckling. This sudden change in the load carrying capacity may cause catastrophic failures due to insufficient strength of entire structure. Therefore, determination of the first buckling and collapse loads of structural elements is essential. FE analyses and structural testing are used to determine buckling characteristics of a structural element. However, in early design stages, FE analyses are time consuming and structural testing is costly.

Any change in the structural geometry such as stringer height, thickness or spacing, affects buckling characteristics of structural elements. These changes result in increased computational time and cost in design. Response surface models, neural networks, cubic spline fits can be used to reduce computational time and to optimize the number of analyze points [1].

Artificial Neural Network is a computation tool that can classify and recognize patterns and provide accurate predictions for given inputs by learning from the previous data [2]. Furthermore, it is widely used in the industry for structural optimization purposes. As an example, artificial neural network was used on stiffened composite panels to reach optimum load carrying capacity and weight and reduce time consumed for analysis [3]. Furthermore, optimization of a compression member was achieved by use of artificial neural network. An analytical method was applied to determine load-displacement relation of different types of columns and the data generated was used to train neural network [4]. Also, design optimization of anisotropic laminated composites was achieved by developing artificial neural network and it was stated that use of the neural network leads to accurate enough solutions and decrease the time required for analysis in design process [5]. In another study thin-walled structures were optimized by developing a neural network. In this study, FE analysis results were used to train the neural network [6].

Artificial Neural Network was also used to create analysis tools for buckling and collapse loads of structural panels. In literature, post-buckling strength of a thin rectangular plate was obtained by creating a neural network as a function of initial imperfections [7]. Furthermore, post-buckling optimization was achieved by artificial neural network and utilized for stiffened panels [8]. In another study buckling load prediction of composite stiffened panels working under shear load was obtained by developing an artificial neural network [9].

Latin Hypercube Sampling (LHS) is a method of sampling that can be used to produce input values for estimation of expectation of functions of output variables [10]. LHS is commonly used design-of-experiment technique for design problems which consist of more than 2 variables. The selected design points are distributed so that they cover all design space but do not intersect each other. Therefore, LHS is used as a tool to reduce number of required FE analyses by covering the whole design space.

In this paper, an artificial neural network tool is used to reduce computational effort required to determine buckling loads of integrally stiffened structural panels in early design stages. Also, the panel mass is predicted in order not to design each trial panel in CAD (Computer-Aided Design) to determine the panel mass data. Moreover, the Latin Hypercube Sampling methodology is used to reduce the number of required FE analyses to generate the results database that artificial neural network is based on. Mean errors and fit performance model results are compared to determine accuracy of the neural network results. Finally, accuracy of artificial neural network based on the full design space solutions and based on design points selected by LHS are compared and represented for the first buckling, collapse load and mass predictions of integrally stiffened panels under compressive loads.

2. Design of Experiment

Design of experiment (DOE) is a general methodology to select the optimum number of parameters required to reach sufficiently accurate results in respectively short period of time. DOE methodologies analyze input data and clarify the outliers, then identify data points which are appropriate to current experiment and therefore improve the quality of the outputs which are taken from the experiment. Therefore, choosing appropriate design of experiment methodology to create a design space is essential in early design stages.

Design space consists of several parameters. Those are variables of design space called factors, definitions of factors called levels, regression parameters and errors (Measurement errors etc.). In Equation 1, the design problem definition is given. Design matrix is $X^{n\times p}$ (factor), vector of regression parameters is $\beta^{p\times 1}$, vector of observations is $y^{n\times 1}$ and error vector is $\epsilon^{n\times 1}$ [11].

$$y = X\beta + \epsilon \quad (1)$$

Statistical error consists of two parts. The first one is the pure error and the second one is the error coming from the response surface model. Since FE analyses cannot be held without being input dependent, FE analyses errors are assumed as pure error. Furthermore, these errors can be negligible or can be calculated with uncertainty quantification. Hence, ϵ can be assumed as model error for FEA case and model is appropriate for experiment if model error is sufficiently low [12].

DOE methods create response surfaces to determine the effect of each factor on the response without replicating the experiment. There are several strategies to develop a design of experiment methodology. Typical experiment starts with an initial guess and guessed combinations of factors and their levels. The way to guess these factor combinations properly is nothing but design of experiment methodologies [12].

Classical and modern design of experiment approaches exist in the literature. Laboratory experiments are typical examples of classical DOE's, while computer-based experiments generally involve modern DOE techniques. There are two main differences between classical and modern DOE approaches [13]:

1. There is no random error in computer experiments while there is in laboratory experiments,
2. Probability density functions used in experiments are different in computer and laboratory experiments.

In the absence of repeatability, putting the sample points at the extremes of parameter space is the methodology for classical DOE approaches since this methodology is more reliable. Therefore, classical DOE techniques are suitable for experiment designs such as central composite design, full-factorial or fractional-factorial designs. Furthermore, possible design parameters are distributed uniformly between upper and lower bound in classical DOE methodologies. In the presence of repeatability which comes from deterministic computer simulations, modern DOE techniques were developed. In order to get the response trend precisely, space filling methods such as quasi-Monte Carlo sampling, orthogonal array sampling and Latin hypercube sampling are developed and employed to the modern DOE techniques. Besides, uniform and non-uniform probability distributions such as Gaussian or Weibull exist for design parameters in modern DOE methodologies [13].

In order to overcome, problems of modern DOE approaches such as non-uniform level of factor requirements or computational cost due to increase in simulation points, design of experiment problem should have a suitable sampling approach. The Latin Hypercube Sampling (LHS) ensures that the entire range of each input variable is completely covered without regard to which single variable or combination of variables might dominate the computer simulation response. This means that, a single sample will provide useful information when some input variables dominate certain responses while other input variables dominate other responses. By sampling over entire range, each variable has the opportunity to show up as important, if it indeed is important. Moreover, LHS is more efficient than simple random sampling in large range of conditions [14].

In this paper, full factorial approach is used to generate design space. Moreover, Latin Hypercube Sampling methodology is used to determine sub-spaces which are 30% 60% and 90% of the full factorial design space which will be compared by the accuracy later with full factorial design space.

3. Artificial Neural Network

A feed forward neural network with Levenberg-Marquardt backpropagation training algorithm which consists of layers which have artificial neurons which are interconnected to other neurons, is used in this study. A neuron and a neural network samples are shown in Figure 1.

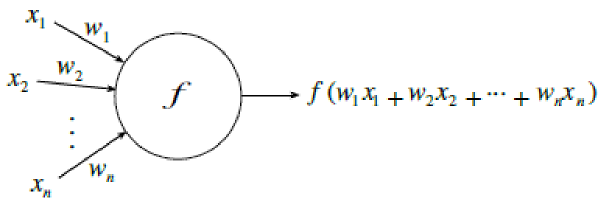
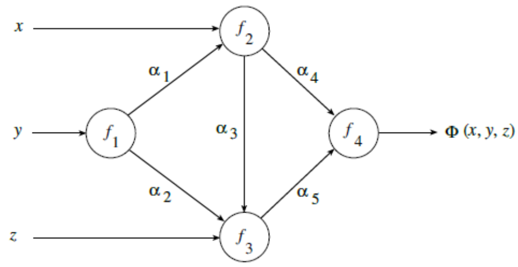

Figure 1. A Neuron (Left) and a Neural Network (Right) Models [15]

In this study, "Network/Data Manager Window" of the MATLAB tool has been used to develop the ANN. A neural network with 48 neurons is selected as layer architecture. Inputs are panel length, panel width, skin thickness, stringer thickness, stringer height and number of stringers whereas outputs are the first buckling load, collapse load and mass. In Figure 2, generic view of a neural network with 48 neurons (hidden layers) is shown.

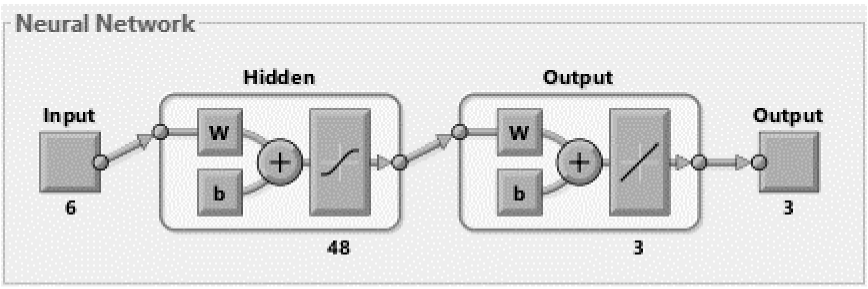

Figure 2. Generic View of Neural Network with 48 Neurons

Determination of the number of neurons in hidden layers is one of the most important aspects for neural network analysis. In order to obtain optimum number of neurons, trial and error approach is applied and errors for each neuron number is investigated for each output value. The mean error versus number of neurons for one of the output parameters is given in Figure 3 as an example.

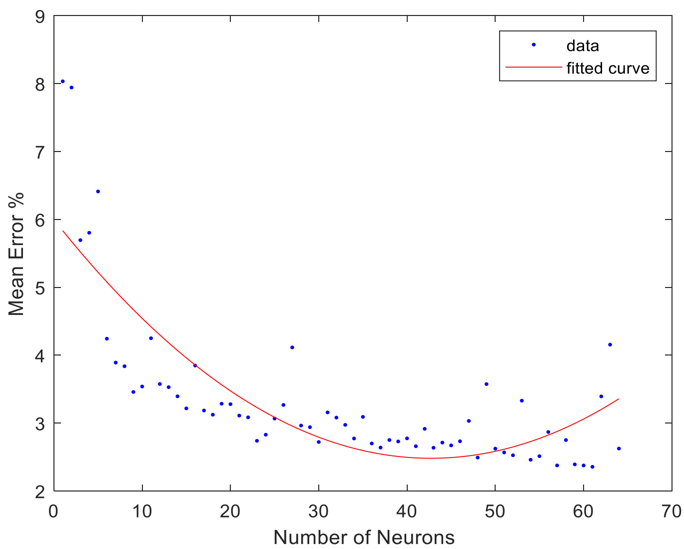

Figure 3. Mean Error vs Number of Neurons Study

In Figure 3, as the number of neurons increases mean error decreases. However, after 50 neurons, mean error starts to increase which means that optimum neuron number for the database given is around 40-50. In other words, the best fitted artificial neural network for the database given consists of 40-50 artificial neurons. Therefore, as shown in Figure 3, it is a wise choice to select number of neurons between 40 and 50. When the computational time of the neural network is investigated, it is decided that 48 neurons give both quick and accurate results. Finally, initialization and training parameters of the neural network are shown in Table 1.

Table 1. Training Parameters of the Neural Network

Number of Neurons	48
Training Ratio	0.90
Validation Ratio	0.05
Test Ratio	0.05
Maximum Failure	5000
Epochs	5000

4. Geometry and Database Descriptions

In order to create the database of structural analyses results required to train the neural network, a design space is determined and tested utilizing a FE analysis tool. Then, the first buckling loads, collapse loads and mass data are collected to generate the database. Geometric dimensions of the integrally stiffened panels are tabulated in Table 2.

Table 2. Geometric Dimensions of Integrally Stiffened Panels

PANEL DIMENSIONS	
LENGTH (mm)	350, 400, 450
WIDTH (mm)	350, 400, 450
STRINGER HEIGHT (mm)	15, 20, 25
STRINGER THICKNESS (mm)	1.5, 1.75, 2.0
SKIN THICKNESS (mm)	1.5, 1.75, 2.0
NUMBER OF STRINGERS	3, 4, 5

All geometric dimensions in Table 2 are used as input columns for neural network analyses. In order to generate the database, 729 design points (all possible combinations of geometric variables – full factorial approach) are used; the respective geometries are developed and analyzed using the general static solver of ABAQUS. In the analyses, elastic and plastic material properties of Aluminum 7050 are used and nonlinear geometry option is activated. Boundary conditions of the panels are determined such as the panel is in a displacement-controlled test machine. In Figure 4, initial, buckled and collapsed views of an example integrally stiffened panel are shown respectively.

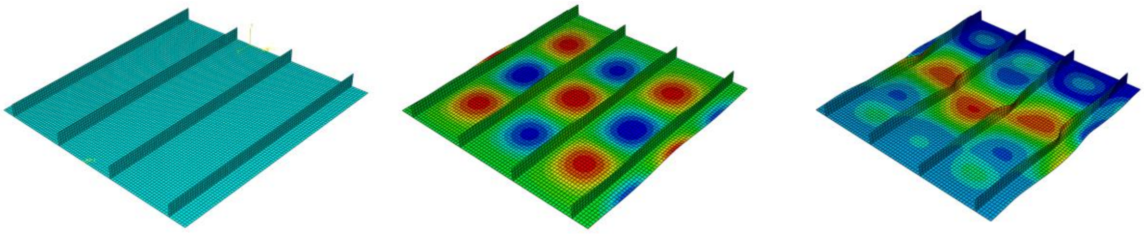

Figure 4. Initial, Buckled and Collapsed Views of an Example Integrally Stiffened Panel Respectively

5. Results

In order to validate the neural network, a database which is described in Section 4 is used. The neural network is trained by using 100% (full factorial approach), 90%, 60% and 30% of the design points which are selected utilizing LHS. The LHS approach creates the values of distribution function not by generating random numbers dispersed in chaotic way in the design interval but by assigning them certain fix values. The interval is divided into several layers of the same width, and the "x" values are assigned via the inverse transformation for the distribution function values corresponding to the center of each layer. By using reasonably high number of layers, the selected quantity x will have the proper probability distribution [17]. Then, the neural network results are compared with respective FE analyses results.

Fit performance model shown in Equation 2 is used to validate the neural network results [16]. In Equation 2, t indicates target value and d_m indicates neural network results. Target values are computed by FE analyses.

$$FIT = 100\left(1 - \left(\frac{\Sigma(t-dm)^2}{\Sigma\big(t-mean(t)\big)^2}\right)^{1/2}\right) \tag{2}$$

In Figures 5, 6 and 7, prediction fit values are compared with input data percentages for the collapse load, the buckling load and the mass predictions separately.

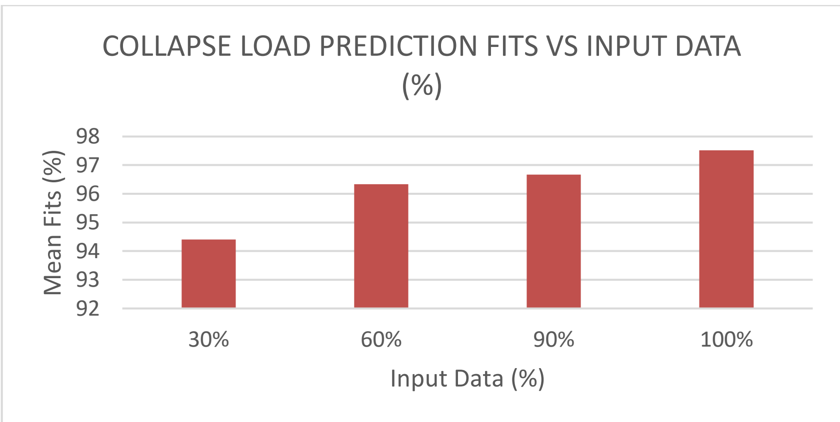

Figure 5. Collapse Load Prediction Fits versus Input Data Percentage

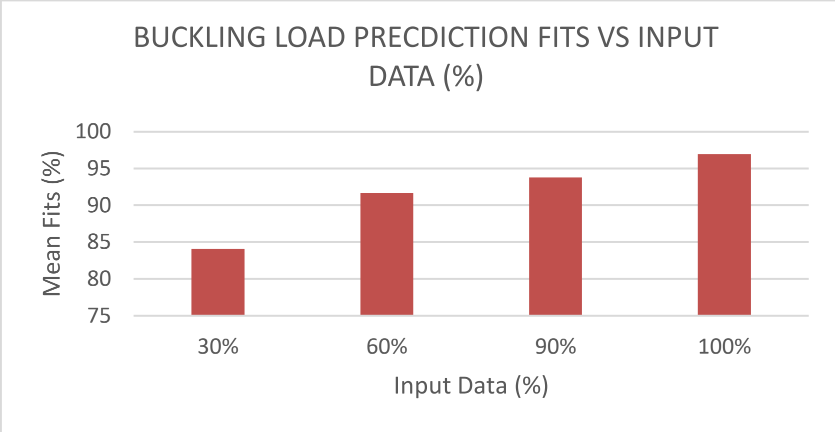

Figure 6. Buckling Load Prediction Fits versus Input Data Percentage

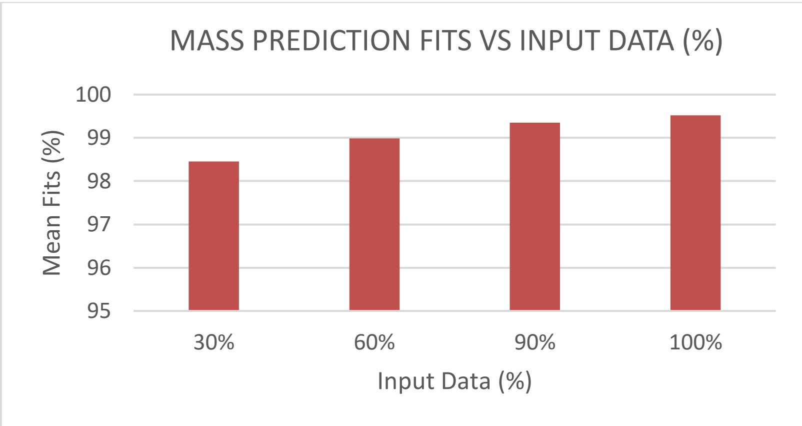

Figure 7. Mass Prediction Fits versus Input Data Percentage

As shown in Figures 5, 6, and 7, as the number of inputs creating the database increases, the error of neural network predictions decreases. Therefore, it can be stated that to obtain accurate results from a neural network, the training database should be as large as possible. In buckling predictions, accuracy drops significantly as the database shrinks. That can be explained by the fact that buckling load has a discontinuity trend since it is a structural instability and artificial neural network creates a non-parametric response surface model. Therefore, determination of the first buckling loads essentially requires a large database in order to give more accurate results. However, results also show that even 30% of the full database gives fit values higher than 80%. This proves the accuracy of the design point selections via Latin Hypercube Sampling. When collapse load and mass predictions are investigated, the neural network is quite accurate even when 30% of the full database is used. This could be explained by the fact that mass and collapse load have nearly stable responses to any change in the geometry.

Next, ten random samples from the database are selected to show the accuracy of the neural network predictions in a detailed sense. Example structural panel dimensions are given in Table 3.

Table 3. Sample Structural Panel Dimensions

		PANEL DIMENSIONS (mm)					
		WIDTH	LENGTH	STRINGER H.	SKIN T.	STRINGER T.	STRINGER #
EXAMPLE PANEL #	1	350	350	20	2	2	3
	2	350	350	25	2	2	3
	3	350	350	15	2	1.75	3
	4	400	350	20	1.5	1.5	3
	5	400	350	20	2	2	3
	6	400	350	25	2	2	3
	7	450	350	15	1.5	1.5	3
	8	450	350	20	1.5	1.5	3
	9	450	350	25	1.5	1.5	3
	10	450	350	15	1.5	2	3

The FE analysis results and neural network predictions of sample panels are compared in Table 4, 5, 6 and 7 for 100%, 90%, 60% and 30% of the design points.

Table 4. Comparison of Analysis and Neural Network Results for 100% of the Input Data

		ANALYSIS RESULTS (N)			NEURAL NETWORK RESULTS (N)			ERRORS (%)			FIT VALUES (%)			
		F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	
EXAMPLE PANEL #	1	217981.0	99161.0	0.8	216011.6	98469.7	0.8	0.9	0.7	0.4	99.1	99.3	99.6	100% OF INPUT DATA
	2	229107.2	108370.0	0.8	230738.3	107863.8	0.8	0.7	0.5	1.1	99.3	99.5	98.9	
	3	163305.2	81181.0	0.8	160251.9	79567.4	0.8	1.9	2.0	0.8	98.1	98.0	99.2	
	4	127200.8	48556.0	0.7	126165.2	50527.0	0.7	0.8	4.1	0.3	99.2	95.9	99.7	
	5	210629.5	89451.0	0.9	208376.0	91645.8	0.9	1.1	2.5	0.2	98.9	97.5	99.8	
	6	215654.8	97623.0	1.0	214609.4	97916.9	1.0	0.5	0.3	0.7	99.5	99.7	99.3	
	7	113672.8	41475.0	0.8	115719.3	41432.4	0.8	1.8	0.1	0.9	98.2	99.9	99.1	
	8	124977.4	45062.0	0.8	126577.0	43519.0	0.8	1.3	3.4	0.2	98.7	96.6	99.8	
	9	132130.8	46793.0	0.8	132090.6	48362.6	0.8	0.0	3.4	0.8	100.0	96.6	99.2	
	10	125648.6	50911.0	0.8	126220.3	52041.5	0.8	0.5	2.2	1.6	99.5	97.8	98.4	

Table 5. Comparison of Analysis and Neural Network Results for 90% of Input Data

		ANALYSIS RESULTS (N)			NEURAL NETWORK RESULTS (N)			ERRORS (%)			FIT VALUES (%)			
		F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	
EXAMPLE PANEL #	1	217981.0	99161.0	0.8	218602.7	100926.5	0.8	0.3	1.8	0.0	99.4	98.1	100.0	90% OF INPUT DATA
	2	229107.2	108370.0	0.8	240976.8	121537.3	0.9	5.2	12.2	1.3	94.5	87.7	98.7	
	3	163305.2	81181.0	0.8	149974.3	76787.3	0.8	8.2	5.4	0.6	92.1	94.7	99.4	
	4	127200.8	48556.0	0.7	124356.8	44670.7	0.7	2.2	8.0	0.5	98.0	92.1	99.5	
	5	210629.5	89451.0	0.9	213132.0	79805.4	0.9	1.2	10.8	0.2	98.5	89.4	99.8	
	6	215654.8	97623.0	1.0	224335.1	98835.6	1.0	4.0	1.2	0.3	95.7	98.6	99.7	
	7	113672.8	41475.0	0.8	115186.1	39454.9	0.8	1.3	4.9	1.1	98.4	95.3	98.9	
	8	124977.4	45062.0	0.8	125664.8	46360.2	0.8	0.5	2.9	0.5	99.2	97.0	99.5	
	9	132130.8	46793.0	0.8	129550.8	45387.2	0.8	2.0	3.0	0.5	98.3	97.1	99.5	
	10	125648.6	50911.0	0.8	126192.3	44547.2	0.8	0.4	12.5	0.0	99.3	87.6	100.0	

Table 6. Comparison of Analysis and Neural Network Results for 60% of Input Data

	ANALYSIS RESULTS (N)			NEURAL NETWORK RESULTS (N)			ERRORS (%)			FIT VALUES (%)			
EXAMPLE PANEL #	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	60% OF INPUT DATA
1	217981.0	99161.0	0.8	219395.3	101609.9	0.8	0.6	2.5	0.5	99.2	97.3	99.6	
2	229107.2	108370.0	0.8	225407.4	103066.8	0.8	1.6	4.9	0.7	98.5	95.4	99.1	
3	163305.2	81181.0	0.8	157919.7	84002.8	0.8	3.3	3.5	1.3	96.8	96.2	98.5	
4	127200.8	48556.0	0.7	133067.5	53176.1	0.7	4.6	9.5	1.8	95.3	90.2	98.0	
5	210629.5	89451.0	0.9	222533.7	97969.6	0.9	5.7	9.5	1.1	94.2	90.2	98.7	
6	215654.8	97623.0	1.0	226138.0	100957.9	1.0	4.9	3.4	2.0	95.0	96.3	97.8	
7	113672.8	41475.0	0.8	119391.4	38085.8	0.8	5.0	8.2	1.0	94.9	92.1	98.8	
8	124977.4	45062.0	0.8	133338.9	46475.9	0.8	6.7	3.1	0.0	93.2	96.6	99.8	
9	132130.8	46793.0	0.8	138049.8	52206.1	0.8	4.5	11.6	0.1	95.4	88.1	99.9	
10	125648.6	50911.0	0.8	130186.5	51524.3	0.8	3.6	1.2	0.0	96.3	98.5	99.9	

Table 7. Comparison of Analysis and Neural Network Results for 30% of Input Data

	ANALYSIS RESULTS (N)			NEURAL NETWORK RESULTS (N)			ERRORS (%)			FIT VALUES (%)			
EXAMPLE PANEL #	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	F_COLLAPSE	F_BUCKLE	MASS	30% OF INPUT DATA
1	217981.0	99161.0	0.8	200429.5	75127.8	0.8	8.1	24.2	2.6	91.6	75.7	97.6	
2	229107.2	108370.0	0.8	241288.0	84072.8	0.8	5.3	22.4	0.4	95.1	77.5	99.4	
3	163305.2	81181.0	0.8	139575.2	61345.9	0.8	14.5	24.4	0.5	85.1	75.5	99.7	
4	127200.8	48556.0	0.7	132331.5	52765.6	0.7	4.0	8.7	3.1	96.4	91.5	97.2	
5	210629.5	89451.0	0.9	204519.4	73205.1	0.9	2.9	18.2	0.1	96.7	81.7	99.8	
6	215654.8	97623.0	1.0	211537.5	79375.1	1.0	1.9	18.7	0.6	97.7	81.2	99.7	
7	113672.8	41475.0	0.8	126740.6	35880.5	0.8	11.5	13.5	1.0	88.9	86.4	98.7	
8	124977.4	45062.0	0.8	126522.4	44919.9	0.8	1.2	0.3	2.8	99.1	99.5	96.9	
9	132130.8	46793.0	0.8	132330.3	65258.8	0.8	0.2	39.5	1.2	99.8	60.7	98.5	
10	125648.6	50911.0	0.8	139751.1	54046.6	0.8	11.2	6.2	1.1	89.2	94.0	98.7	

As shown in Table 4, 5, 6 and 7, prediction accuracy increases as the database enlarges. Also, there is a significant decrease in the accuracy of buckling load as the database shrinks. However, even 30% of the full database gives accurate results for the collapse load and the mass predictions.

6. Conclusions

In this study, an artificial neural network is trained to determine the first buckling load, the collapse load and the mass of an integrally stiffened panel in early design stages. Latin Hypercube Sampling methodology is employed to examine whether time spent for FE analyses used to create the neural network training database can be reduced by choosing design points that generate the database in an efficient way. To generate the database integrally stiffened panels are designed and analyzed by a commercial FE analysis tool. The database includes the first buckling load, the collapse load and the mass data with respect to six geometric variables which are the panel length, the panel width, the stringer height, the stringer thickness, the skin thickness and the number of stringers. It was shown in this study that the total time spent for generating a database can significantly be reduced by employing Latin Hypercube Sampling methodology. Accuracy of the neural network predictions are determined by fit performance model. According to the results, the accuracy is high in prediction of collapse load and mass, due to the fact that those parameters have more continuous and predictable curves. However, the buckling load prediction accuracy decreases as the database shrinks due to nature of the buckling phenomenon itself. Therefore, determination of the buckling load essentially requires higher number of inputs.

Overall, the artificial neural network combined with the design of experiment methodologies is a useful tool to predict load carrying capacities of structural elements in early design stages. Effective selection of design points can reduce the computational effort. Moreover, it can be used as a conceptual design tool to design integrally stiffened structural panels that has an optimum weight and optimum load carrying capacity at once.

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