THE ROLE OF DYNAMIC GEOMETRY SOFTWARE ON MATHEMATICAL CREATIVITY OF PRE-SERVICE MATHEMATICS TEACHERS IN GEOMETRY TASKS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ZULAL MELEK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMETICS EDUCATION IN MATHEMATICS AND SCIENCE
EDUCATION

JULY 2021

Approval of the thesis:

THE ROLE OF DYNAMIC GEOMETRY SOFTWARE ON MATHEMATICAL CREATIVITY OF PRE-SERVICE MATHEMATICS TEACHERS IN GEOMETRY TASKS

submitted by **ZÜLAL MELEK** in partial fulfillment of the requirements for the degree of **Master of Science** i**n Mathematics Education in Mathematics and Science Education, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ____________

Prof. Dr. Erdinç Çakıroğlu
Head of the Department, **Mathematics and Science Education** ____________

Prof. Dr. Erdinç Çakıroğlu
Supervisor, **Mathematics and Science Education, METU** ____________

Examining Committee Members:

Assist. Prof. Dr. Betül Tekerek
Mathematics and Science Education, Kahramanmaraş
Sütçü İmam University ____________

Prof. Dr. Erdinç Çakıroğlu
Mathematics and Science Education, METU ____________

Assist. Prof. Dr. Betül Tekerek
Mathematics and Science Education, METU ____________

Date: 14.07.2021

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Zülal, Melek

Signature :

ABSTRACT

THE ROLE OF DYNAMIC GEOMETRY SOFTWARE ON MATHEMATICAL CREATIVITY OF PRE-SERVICE MATHEMATICS TEACHERS IN GEOMETRY TASKS

Melek, Zülal
Master of Science, Mathematics Education in Mathematics and Science Education
Supervisor: Prof. Dr. Erdinç Çakıroğlu

July 2021, 159 pages

The purpose of the study was to investigate the role of the dynamic geometry software on mathematical creativity of pre-service mathematics teachers in geometry tasks. The sample of the present study was five pre-service mathematics teachers in a public university in Ankara. The sample of the study was selected by the convenience sampling method from the pre-service mathematics teachers who took Exploring Geometry with Dynamic Geometry Applications course. The pre-service mathematics teachers asked to solve three different geometry tasks with more than one ways. The pre-service teachers asked to solve the task 1 in the paper-pencil environment and the task 2 in the GeoGebra environment. They were free to use any environment in the task 3. The source of the main data was participants' written works, GGB files, and video recordings. The data was analyzed by using written works, GGB files, and video recordings in terms of three dimensions of creativity: fluency, flexibility, and originality. Both mathematical creativity and the three dimensions of creativity were discussed by examining students' solutions in pencil-paper and dynamic geometry environments. Results reveal that GeoGebra set an environment for the pre-service teachers that enabled them to use their mathematical creativity in geometry tasks by supporting the different dimensions of creativity. While the fluency of pre-service teachers did not seem to be affected by the environment, it promotes the flexibility of pre-service teachers by allowing them to create alternative solution ways. Also, the combination of GeoGebra with the paper-pencil seemed to be more supportive of the pre-service teachers' flexibility. In short, GeoGebra has the potential to support mathematical creativity in solving geometry tasks because it allows individuals to provide more flexible and original solutions, especially when presented with a paper-pencil environment.

Keywords: Mathematical Creativity, Dynamic Geometry Software, GeoGebra, Paper-Pencil Environment, Pre-service Mathematics Teachers

ÖZ

DİNAMİK GEOMETRİ YAZILIMININ GEOMETRİ GÖREVLERİNDE ADAY MATEMATİK ÖĞRETMENLERİNİN MATEMATİKSEL YARATICILIĞI ÜZERİNDEKİ ROLÜ

Melek, Zülal
Yüksek Lisans, Matematik Eğitimi, Fen ve Matematik Bilimleri Eğitimi
Tez Yöneticisi: Prof. Dr. Erdinç Çakıroğlu

Temmuz 2021, 159 sayfa

Çalışmanın amacı, dinamik geometri yazılımının geometri görevlerinde matematik öğretmen adaylarının matematiksel yaratıcılığı üzerindeki rolünü incelemektir. Bu araştırmanın örneklemini, Ankara'da bir devlet üniversitesinde eğitim gören beş matematik öğretmen adayı oluşturmuştur. Araştırmanın örneklemi, Geometriyi Dinamik Geometri Uygulamaları ile Keşfetme dersini alan matematik öğretmen adaylarından uygun örnekleme yöntemi ile seçilmiştir. Matematik öğretmen adaylarından, üç farklı geometri görevini birden fazla yolla çözmeleri beklenmiştir. Öğretmen adaylarından 1. görevi kalem-kağıt ortamında ve 2. görevi GeoGebra ortamında çözmeleri istenmiştir. Öğretmen adayları 3. Görevde kullanacakları ortam konusunda özgür bırakılmışlardır. Bu çalışmada ana verileri katılımcıların yazılı çalışmaları, GGB dosyaları ve video kayıtları oluşturmuştur. Çalışmanın verileri öğrencilerin yazılı çalışmaları, GGB dosyaları ve video kayıtları yardımı ile matematiksel yaratıcılığın üç boyutu (akıcılık, esneklik ve özgünlük) dikkate alınarak analiz edilmiştir. Hem matematiksel yaratıcılık hem de yaratıcılığın üç boyutu öğrencilerin çözümlerini kalem-kağıt ve dinamik geometri ortamlarında inceleyerek ele alınmıştır. Araştırmanın sonuçları GeoGebra kullanımının matematiksel yaratıcılığın farklı boyutlarını destekleyerek, öğretmen adaylarına matematiksel yaratıcılıklarını kullanmalarını sağlayan bir ortam hazırladığını ortaya koymaktadır. Öğretmen adaylarının akıcılıkları ortamdan etkilenmemiş olsa da, GeoGebra kullanımı öğretmen adaylarının alternatif çözüm yolları oluşturmalarına olanak sağlayarak esnekliklerini artırmaktadır. Yanı sıra, GeoGebra ile kalem kağıt ortamının birlikte kullanımının öğretmen adaylarının esnekliğini desteklediği gözlemlenmiştir. Kısacası GeoGebra, özellikle kalem-kağıt ortamı ile sunulduğunda bireylerin daha esnek ve özgün çözümler sunmasına olanak sağladığı için geometri görevlerinin çözümünde matematiksel yaratıcılığı destekleme potansiyeline sahiptir.

Anahtar Kelimeler: Matematiksel Yaratıcılık, Dinamik Geometri Yazılımı, GeoGebra, Kalem-Kağıt Ortamı, Aday Matematik Öğretmenleri

To my mom

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my supervisor Prof. Dr. Erdinç Çakıroğlu, for his patience, support, and guidance throughout the lengthy research process. He always showed his kind understanding in every step of the study. He supported me with his valuable feedback, which improved me in terms of academically. It is a great honor for me to have a chance to study with him.

I would like to thank the thesis committee members Assist. Prof. Dr. Şerife Sevinç and Assist. Prof. Dr. Betül Tekerek for their suggestions and feedback to make my study more qualified.

I would like to express my sincere thanks to all pre-service mathematics teachers. This study would not be completed without their contribution and patience.

I am also thankful to my lovely friends Gülnur Akın, Nejla Öztürk, and Şeyma Nur Ertekin for their limitless support. They always encouraged me both in master thesis process and whenever I felt helpless and lonely. My sincere thanks to all close friends who made feel stronger myself.

Lastly, my endless thanks go to my family for their unconditional love and support throughout my life. My mother who is Yasemin Melek desired to reach the end of this process more than I did, and It is a great lucky to have such a mother because she always believe me more than I do. Furthermore, my special thanks go to my father Orhan Melek, my sisters Feyzanur and Zeynep, and my brother Mustafa. Their presence always gives me strength and makes feel me precious.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS

Creativity	Cr
Flexibility	Flx
Fluency	N
Originality	Or
Dynamic Geometry Software	DGS
GeoGebra	GGB
Paper-Pencil Environment	PP
Multiple Solution Task	MST
Ministry of Education	MoNE
Torrance Test of Creative Thinking	TTCT

CHAPTER 1

INTRODUCTION

Genius. Invention. Talent. And, of course, creativity. The highest levels of human capabilities might be described through these words. Humans feel that their performances are at the peak of their capabilities when they are involved in creative acts (Sawyer, 2011); even the ancient history of creativity is based on divine creators and mythic moments (Pope, 2005). The first booked use of nouns of creativity has been in recent 1875 (Pope, 2005); however, the research has begun recognizing that creativity is a practical and helpful way to learn and understand the world (Kaufman and Sternberg, 2010). In the mid-twentieth century, especially in the 1920s, the concept of creativity has born as an academic issue in the education and psychology environment (Pope, 2005). Creativity as an abstract concept has begun to be popular in the 1940s-1950s (Pope, 2005). Almost all the superior twentieth-century psychologists, who are Freud, Piaget, Roger, and Skinner, investigated the concept of creativity (Kaufman and Sternberg, 2010). Indeed, creativity is viewed as a mid-twentieth century product; it emerged as the modern response to the problems caused by accelerated social, technological, scientific, and military changes (Pope, 2005). In the same way, in today's world, which changes faster than ever before, humans still need to think in new and creative ways to handle the novel and difficult problems of the new world (Sternberg, 2007). Despite all, researchers still indicate no commonly accepted perspective on creativity (Haylock 1987; Plucker et al. 2004; Singer et al. 2017).

Guilford (1950) used divergent thinking as a base of creativity related to the four components: fluency, flexibility, originality, and elaboration. Torrance (1965) developed the Torrance Test of Creative Thinking to define and measure creativity based on identical four components. Torrance describes creativity as a process of being responsive to problems, gaps in knowledge, and disharmonies, which is needed to identify the issues and create solutions later testing the hypotheses. Additively, Sawyer (2011) discussed creativity as a social concept; and described it as creating novel, appropriate, proper, and valuable products by well-informed social group.

Another viewpoint is that National Advisory Committee on Creative and Cultural Education (NACCC, 1999) approaches creativity for educational purposes based on the nature of creative processes. Already, humanity has an expectation from education to prepare individuals for the new world that require having economic independence, constructive life, having tolerance, cultural diversity, and the ability to deal with the changing world. When humanity's expectation is considered, it is worthwhile to describe the creativity for educational purposes (NACCC, 1999). Therefore, the National Advisory Committee on Creative and Cultural Education (1999) has published a report about creativity; and Robinson and his colleagues touched on the importance of creativity to improve education for the twenty-first century standards. They described the four characteristics of the creative process; (i) thinking and acting imaginatively, (ii) having a purposeful activity to achieve an objective, (iii) creating something original, and (iv) valuable outcome related to the purpose. In this sense, their creativity definition is that:

> "Imaginative activity fashioned to produce outcomes that are both original and of value." (NACCC, 1999, p. 30)

In their definition (1999), and imagination is to make unfamiliar connections and notice relationships among ideas that have not been related; originality is to provide alternative ways to the expected and routine; a value is outcomes that could be judged purposes.

By contrast with some who think creativity is associated with "a lack of discipline in education" and thinks it is associated with only arts, they are of the opinion that creativity is in all human activities and all can have creative abilities or improve their creative abilities to use. Academic abilities do not mean that students have success or personal achievement in their academic life. However, creative abilities can improve academic capabilities, self-confidence, and self-esteem in related areas; students' creative skills in related areas could be developed by giving them the freedom to practice. In brief, they viewed creative education as enabling students to have original ideas and actions, and vital to all areas (e.g., arts, sciences, work-life, and all other areas of life) (NACCC, 1999).

While the others discussed whether students could be creative in all areas at the same time (Kaufman, Cole and Baer, 2009; Plucker and Zabelina, 2009; and Kattou, Christou and Pitta-Pantazi, 2015), and several studies supported the domain-specific creativity by assuming that if there is no reference in a domain in which it has occurred place, the creativity cannot be known (Plucker & Zabelina, 2009). In other words, a student who is creative in art or literature does not mean they are identified as creative in mathematics, and vice versa (Kattou, Christou, and Pitta-Pantazi, 2015).

One of the domains is mathematics, and many researchers studied mathematical creativity (Chamberlin & Moon, 2005; Ervynck, 1991; Sriraman, 2004). Contrary to what is believed, mathematics requires creative thinking to explore problems by its nature, not just mastering computational skills or replicating others (Mann, 2006). Chamberlin and Moon (2005) indicate that mathematical creativity may arise when a non-standard solution to a standard problem is aimed instead of using standard algorithms. In earlier, Ervynck (1991) depicted mathematical creativity as an ability to think and solve the problem by considering the deductive nature of a discipline; in that, it is an ability to create novel ideas through moving old ideas together with new ways. In addition, mathematical creativity is not handled only as an original work of professionals. It also refers to discovering something not already known by a person, even if it has been known by others (Sriraman, 2004). On this basis, Sriraman (2005) handled the mathematical creativity at the school level and described it as a process that requires unusual or insightful solution(s) to given problem(s) or the organization of new questions to examine the old problems from new angles. However, Ervynck (1991) indicated that students get used to solving a given problem by using certain algorithms; and students are often conditioned that mathematics is based totally on logic, certainty, proof, and clear explanations, but he claimed that “the creativity is none of these things” (Ervynck, 1991). Creativity requires understanding mathematics in a sophisticated way in a given context to create developments that extend a given theory. At the school level, such creative approaches are begun by promoting students to create extended mathematical investigations that are novel (Ervynck, 1991). These enable students "to begin to explore, to conjecture and test, to formulate and prove, in ways which give deeper meaning to mathematical processes" (Ervynck, 1991, p.53). An excellent mathematical mind already can think flexibly, manipulate and investigate any problems from various aspects (Dreyfus & Eisenberg, 1996). In a rapidly changing world, such flexible thinking is becoming necessary (Ervynck, 1991). Creativity requires adapting to the changing world and continuing advancements since these advancements in technology and science change humans' lives (Leikin, 2013). Therefore, each student should use the advantages of creativity in the school environment to "get a taste of mathematical creativity and realize their creative potential in mathematics” (Leikin, 2013, p.386). Thus, using technology in school mathematics is recommended to improve students' mathematical thinking, problem-solving, and creativity (Ersoy, 2003).

Already, the ability to use digital technologies is generally considered a vital skill for the twenty-first century as creativity is viewed as essential for twenty-first-century learning and teaching (Craft 2010); and the relationship between technology and creativity is handled as a key concern for twenty-first-century education (Page & Thorsteinsson, 2017). The effects of technology in today's world impact the human's way of life, think, work, create, and communicate, and technological devices offer a chance to imagine, make, and share in creative ways (Zhao, 2012). Creativity in the rapid technology change becomes much more essential; therefore, the discussion on education in a technology-rich environment becomes increasingly crucial (Henriksen et al., 2016). Technology can offer the necessary tools to encourage and strengthen students' creativity. Besides emphasizing the use of technology in education, integrating technology into mathematics education is also discussed, noting its positive impact on mathematical creativity (Idris & Nor, 2010). Using technology can enrich learning and provide greater flexibility; it is crucial to provide flexibility to improve students' creativity (NCTM, 2000).

In school mathematics, teachers might represent the mathematical ideas with physical manipulatives, which could be seen and handled; however, many mathematical ideas could not be represented by just using physical manipulative (Goldenberg, 2000). Association of Mathematics Teacher Educators (AMTE) (2006) recommended that technology be used when the representation is difficult or impossible without technology to facilitate mathematical discovery, understanding, and connections. NCTM (2008) asserts the calculators, interactive presentation devices, computer algebra systems, and interactive geometry software as the technological devices to ensure high-quality mathematics education.

There are two well-known types of technology in mathematical education: Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS). Dynamic Geometry Software is mainly used to teach and learn geometry and solve geometry tasks. DGSs might also describe as technological tools for doing dynamic geometry (Kokol-Voljc, 2007). The DGS encourages teachers and students to make reasoning abstractly and quantitatively and gives them the opportunity to make sense of the relationships among the district features of the problems or mathematical ideas. It is an environment that enables learners to construct dynamic figures that could not be created with paper-and-pencil (Contreras, 2013). To Contreras (2013), students' mathematical thinking abilities would be further when the DGS is used in this content. In that, the use of DGS promotes students to use proactive strategies since students are able to view the results and interpret them using instant feedbacks; and construct new understandings based on the relationships they observed (Hollebrands, 2007).

Moreover, students can access the world of mathematical theorems with the help of DGS; and develop mathematical explanations that provide deductive reasoning in mathematics (Jones, 2000). Especially, the DGS makes the geometrical configurations as sets of relations visible and explorable by providing convenience for creating drawings to illustrate geometry. The DGS-use provides a wide-ranging area for geometrical constructions and solutions. The DGS-use allows access to a wider range of possible geometrical activities; and offers deeper reflection and exploration than paper-pencil environment (Straesser, 2001).

One of the dynamic geometry software is the GeoGebra that is based on the relationships among points, lines, circles, and so on (Sangwin, 2007).

> "…GeoGebra, the brainchild of Markus Hohenwarter, which joins geometry, algebra and calculus. On the one hand, GeoGebra is a dynamic geometry system in which you work with points, vectors, segments, lines, and conic sections. On the other hand, equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards." (Sangwin, 2007, p. 1)

Moreover, the GeoGebra is an environment in which students can experience dynamic activities that are integrated with multiple representations of mathematical concepts (Haciomeroglu et al., 2009). In traditional classrooms, using the board to form the figure causes some difficulties such as not exact and unchangeable drawings; however, the GeoGebra allows changing and moving the figures, which encourages a better understanding of mathematics (Hohenwarter & Jones, 2007). Besides, it improves the students' problem-solving and reasoning skills by ensuring an interactive environment in which students can construct measure, manipulate, and analyze the concepts (Gittinger, 2012). It can be very useful for school mathematics since it provides faster and more flexible results (Sangwin, 2007).

Mathematical creativity is considered an important component of problem-solving (Plucker et al., 2004) and is defined as producing novel and useful solutions to problems (Chamberlin & Moon, 2005). Similarly, mathematical creativity at the school level is also related to problem-solving (Liljedahl & Sriraman, 2006; Leikin,

2009) and flexibility (Leikin, 2009); which might be encouraged by using GeoGebra in school mathematics (Gittinger, 2012 and Sangwing, 2007).

However, the mathematical curricula generally are arranged based on some techniques and procedures by viewing the notions to be learned; and the main component of curricula is mostly opposed to flexibility and fluency, which are typical of creativity. Yet, there have been much interference in introducing creativity into school mathematics and promoting teachers to create curricula that motivate creativity (Yerushalmy, 2009). In order to raise creativity in the classroom, students should handle challenging problems, realize the beauty of solutions to problems, learn to justify their solutions, and use different solution methods to be flexible (Sriraman, 2005). Namely, the GeoGebra prompts individuals to create their district strategies to receive creative feedback contributing to their creative reasoning because it does not provide the correct answers, unlike guidance from a teacher; it does not give clues on how to proceed. Creative feedback requires individuals to interpret and evaluate their solving strategies (Granberg & Olsson, 2015). On the other hand, the feedback is provided in multiple representations, direct manipulation of visual objects, systematic changes of parameters to enable students to identify and attend their phenomena; and provide an opportunity to begin conceptualization. This reveals the appreciation of individuals' creativity because individuals can experience conjectures before taking specific definitions or procedures (Yerushalmy, 2009). The technology supports the long-term guided inquiry prompt to flexible learning, and the long-term learning environment promotes the individual's creativity with the guided inquiry (Yerushalmy, 2009).

1.1 Purpose of Study and Research Question

The present study aims to investigate the role of dynamic geometry software on the mathematical creativity of pre-service mathematics teachers in multiple solution tasks. The pre-service teachers are expected to solve three different multiple solution tasks in many possible ways by using both GeoGebra and the paper-pencil environment. The pre-service teachers are observed in the paper-pencil environment for the first task and in the GeoGebra environment for the second task. In the third task, there is no restriction about the solving environment. The pre-services teachers' solutions were examined in terms of their mathematical creativity based on three dimensions of creativity: fluency, flexibility, and originality. Thus, it is also aimed to investigate the role of dynamic geometry software on the dimensions of creativity separately. The current study seeks to handle the following research questions:

1. How does the GeoGebra play a role in promoting the pre-service mathematics' teachers' mathematical creativity within the multiple solution tasks?
   1.1. What are the roles of both the paper-pencil and GeoGebra environments on pre-service teachers' mathematical creativity within multiple solution tasks, separately?
   1.2. What is the role of using paper-pencil and GeoGebra environment within multiple solution tasks on the three dimensions of mathematical creativity: fluency, flexibility, and originality, separately?

1.2 Significance of the Study

According to Sriraman et al. (2011), the concept of creativity should be a part of the curriculum beginning from early childhood because humans need creative abilities when they face life's several problems to solve them (Guilford, 1967). In Turkey, students are expected to increase their mathematical creativity examples by exploring their creativity after graduating from primary school (Mone, 2015). Already, the purpose of mathematics lessons is to teach the value of different points of view and meaningful learning (MoNE, 2009). Students' creativity might be supported by an environment in which students are conscious of differences in their problem-solving types and are allowed to create their ways; therefore, it is worth researching the creativity-supported environment (Mann, 2009).

Also, there is a view that creative students might be supported by creative teachers (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). Therefore, based on the current study's findings, it can be discussed how pre-service teachers' mathematical creativity can be supported during their education and how they can support their students' creative thinking abilities.

Moreover, the primary consideration of the current study is the lack of sufficient research on the role of dynamic geometry software on pre-service mathematics teachers' mathematical creativity. There are several studies on mathematical creativity at the school level (Leikin & Lev, 2007), problem-solving strategies of pre-service mathematics teachers in both the GeoGebra and paper-pencil environment (Koyuncu, 2013), students' thinking preferences in mathematical problem solving by comparing the paper-pencil and GeoGebra environments (Farihah, 2018), students' mathematical creative thinking and motivation within Problem Based Learning model via the GeoGebra (Selvy et al., 2020), and the learning process of analytic geometry concepts via the GeoGebra and its effect on the advancement of the creative thinking abilities of pre-service teachers (Yıldız et al., 2017). However, although Coelho and Cabrita (2015) investigated how the approach that combines the paper-pencil and Dynamic Geometry environment supports the development of geometric skills and understanding of geometric concepts as a part of the question of how technology used to encourage their creativity, the combination of the content area and data analysis framework makes a difference in the current study.

1.3 Definitions of the Important Terms

Mathematical creativity: It is defined as the process that results in unusual and/or insightful solution(s) to a given problem (Sriraman, 2004).

Fluency: It is defined as the number of appropriate solutions of individuals within a given time (Leikin, 2009).

Flexibility: It is based on different representations, properties, or divisions of mathematics; two solution strategies belong to different groups if they are formed with different representations, properties, or dimensions of mathematics (Leikin, 2009).

Originality: It is handled as level of insight and conventionality of the solution based on the learning background of individuals. It is determined as an original if the solution is unconventional or insightful (Leikin, 2009).

Dynamic Geometry Software: It is a type of technological tool to do dynamic geometry, improve to teach mathematics, support the development of geometrical concepts (Kokol-Voljc, 2007).

Multiple Solution Task: It is determined as a task which allow solving problem in different ways (Leikin & Levav-Waynberg, 2007).

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature related to the scope of the current study is reviewed. The chapter begins with a discussion of the history of creativity and the various views on the definition of creativity. The mathematical creativity and its relationship with problem-solving are presented; and a model for evaluating the mathematical creativity is discussed in detail. Next, the software used in the current study is detailed with its relationship with mathematical creativity. Finally, the earlier studies related to this study were summarized to make a clear picture.

2.1 Creativity: A Historical View

In the 1950s, Guilford challenged psychologists to focus more on creativity. Although creativity was already being studied before these years, Guilford stimulated the field and made creativity acceptable for research. It was a great starting point for talking about creativity in science research (Kaufman, 2016; Sawyer, 2011).

Guilford (1973) classifies thinking as convergent and divergent thinking and mentions creative thinking as *a subclass of thought in general*. Convergent thinking, frequently observed in schools, is considered as thinking tends to a single correct answer. Guilford (1973) defines divergent thinking as not bring high grades in school life, and that requires questioning, searching, and creating unconventional solutions, and he associates creativity with divergent thinking. Guilford (1973) asserted that at least ten primary characteristics underlay creative thinking, and some of those were flexibility, fluency, and elaboration, tolerance of ambiguity, originality, sensitivity, curiosity, independence, reflection, and action. However, much of his studies could be simplified to four main components: fluency, flexibility, originality, and elaboration (Kaufman, 2016). To Kaufman (2016), creativity already could not associate with only divergent thinking. Guilford's determination of characteristics for creative thinking guided researchers to study on measurement of creative thinking and develop creativity tests (Brown, 1989 and Kaufman, 2016).

The Torrance Tests of Creative Thinking (TTCT) is one of the most popular tests of creativity based on Guilford's divergent thinking perspective (Kaufman, 2016). Torrance (1965) defined creativity as:

> "The process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficulty; searching for solutions, making guesses, or formulating hypotheses about the deficiencies; testing and retesting these hypotheses and possibly modifying and retesting them, and finally communicating the results." (p.663)

He developed the Torrance Test of Creative Thinking (TTCT), which measures the four dimensions of creativity with both Verbal and Figural tests, which measure creativity with both Verbal and Figural tests. The Verbal subsets are Asking, Guessing Causes, Product Improvement, Unusual Uses, Unusual Questions, and Just Suppose, while the figural subsets were Picture Construction, Picture Completion, Lines/Circles (Kaufman, 2016). Indeed, the definition and test of Torrance were based on the four dimensions: *fluency, flexibility, originality, and elaboration* (Leikin, 2009, and Plucker & Makel, 2010). To Torrance (1974, as cited in Leikin, 2009), *fluency* refers to the continuity of common knowledge and *flexibility* related to producing divergent solutions to a problem. *Originality (novelty)* indicates unique thoughts and solutions. *Elaboration* refers to the ability to define, clarify, and generalize ideas. The originality (novelty) is more acknowledged than the other three components because Torrance defines creativity as a process of creating original and novel products (Leikin, 2009). According to Kaufman (2016), Guilford's and Torrance's definitions of creativity based on divergent thinking are the cornerstone of creativity. However, besides their intellection, there are some other definitions of creativity.

2.2 A Variety of Creativity Definitions

The common opinion of most researchers is that there is no common definition for creativity. (Sawyer, 2011 and Plucker et al., 2004). However, there are various views (Kaufman & Beghetto, 2009, Sawyer, 2011, and Plucker et al., 2004) on defining creativity, along with Guilford and Torrance's understanding of it.

Kaufman and Beghetto (2009) developed the Four C's of Creativity by adding the mini-c and Pro-c types to the already proposed little-c and big-C structure. To Kaufman and Beghetto (2009), the Big-C, also called eminent creativity, refers to the creativity of well-known people such as winners of the Pulitzer Prize in fiction. The little-c, also called everyday creativity, can be discovered in almost all people. They added two new concepts (mini-c and Pro-c) to their framework to emphasize the distinctions between Mini-C and Big-C creativity levels and to define creativity levels more clearly. Mini-c explains the creativity inherent in the learning process and is associated with children learning something for the first time. Beghetto and Kaufman (2007) define it as "*the novel and personally meaningful interpretation of experiences, actions, and events"* (p.73). Pro-c is the creativity displayed in the progression until a person with little-c arrives in Big-C and requires developmental and effortful improvement (Kaufman & Beghetto, 2009).

Sawyer (2011) considers the act of being creative as one of the highest levels of human performance and as essential to solving society's urgent problems and the world. Sawyer (2011) divides the definitions in creativity research into two main approaches: the individualist approaches and sociocultural approaches. Sawyer (2011), individualistic approaches define creativity as "*the new mental combination that is expressed in the World*" (p.209) while sociocultural approaches specify it as "*the generation of a product that is judged to be novel and also to be appropriate, useful, or valuable by a suitably knowledgeable social group"* (p.8).

Moreover, Plucker et al. (2004) consider that individuals or groups create novel and useful products for a social context with the interaction among aptitude, process, and environment. To Plucker et al. (2004), the interaction among aptitudes, cognitive processes, and environment emerges the creativity. However, generating novel and useful behaviors, ideas, or products, that are documentable works in the society, is referring the presence of creativity.

In brief, although there are various views on creativity, flexibility, fluency, originality, and elaboration were considered among the most important characteristics of it. It was seemed possible to evaluate creativity in both individual and social contexts.

2.3 Domain-Specific Creativity: Mathematical Creativity

To Milgram and Livne (2005), creativity was often visualized as part of giftedness, and it is not considered the distinction between the general and specific creative abilities and levels. However, Plucker (1998) and Baer (1998) pointed out domain-specific creativity by focusing on the nature of general vs. domain-specific creativity. They had a high opinion; examining creativity in one context differs from creativity in other contexts. In fact, Baer (1998) argues that creativity is domain-specific, but it would be varied within domains and even within specific tasks. To Baer (1998), the domains are defined as cognitive abilities such as visual, musical, mathematical, or linguistic abilities. Baer (1998) described the micro-domains such as story writing or poetry writing, which were considered task or content abilities.

Mathematics is also one of the domains where creativity has been addressed; contrary to what is believed, it requires much more than memorizing arithmetic facts and mastering algorithms (Mann, 2006). However, as in general creativity, it is extremely difficult to provide a widely accepted definition of mathematical creativity (Mann, 2006; Sriraman, 2005; Liljedahl & Sriraman, 2006). Still, there are various views (Ervynck, 1991 and Sriraman, 2004) on describing mathematical creativity, and the researchers commonly discussed the creativity in the school mathematics while describing mathematical creativity.

In mathematics, there is a belief that all is logical, precise, provable, and explainable; however, mathematical creativity is not associated with any of these (Ervynck, 1991). Indeed, an act of creativity needs at least one of the following objectives: (I) creating a useful concept, which means favorable for the future, (II) discovering the earlier unnoticed relations, (III) organizing a part of any theory in a way that makes it logically more straightforward. Essentially, Ervynck (1991) figures out mathematical creativity as the "create mathematical objects, together with discovering their mutual relationships." Yet, this view is different from algorithmic mathematical objects. Ervynck argues that mathematical creativity is the ability to solve problems or develop opinions, which takes into account the deductive nature of mathematics and the fitness of the concepts created concepts with the essence of mathematics. However, it needs an advanced understanding of mathematics to make creative developments that extend known theory. In school mathematics, instead of expecting them to reinvent mathematical concepts known for years, it is a creative enterprise for students to explore a novel context for them and expand it to conduct mathematical research. This allows students to explore deeper meanings in mathematical processes and provides the opportunity to think flexibly (Ervynck, 1991).

Likewise, Sriraman (2004) defines creativity as the process that results in novel and original work to a given problem. Sriraman considers the mathematical creativity separately at the professional and student level since each performs at their respective levels (2005). To Sriraman and Liljedahl (2006), the mathematical creativity among professional mathematicians is different from the mathematical creativity in the education curriculum. Mathematical creativity at the professional level could be defined as *"the ability to produce original work that significantly* *extends the body of knowledge"* (p.18). Professionals deal with the full of uncertain problems; however, most curricula do not offer students this open-ended view of mathematics (Sriraman, 2005). Already, at the school level, extraordinary creativity is not expected from students (Sriraman, 2005); however, students can be given a chance to deal with non-routine problems to observe mathematical creativity in the classroom (Sriraman, 2005). Liljedahl and Sriraman (2006) suggest that mathematical creativity at the school level is "the process that results in unusual and/or insightful solution(s) to a given problem or analogous problems.*"(*p.19) Besides, Liljedahl and Sriraman (2006) agree that mathematical creativity in school mathematics is associated with problem-posing or problem-solving.

2.3.1 Mathematical Creativity and Problem Solving

Many researchers have established a link between mathematical creativity and problem solving (Plucker et al., 2004; Chamberlin & Moon, 2005; Silver 1997; Liljedahl & Sriraman, 2006; Leikin, 2009). Mathematical creativity is considered an important component of problem-solving (Plucker et al., 2004) and is defined as producing novel and useful solutions to problems (Chamberlin & Moon, 2005). Similarly, mathematical creativity at the school level is also related to problem-solving (Liljedahl & Sriraman, 2006; Leikin, 2009).

Already, in a conversation about mathematical creativity (Liljedahl & Sriraman, 2006), Sriraman proposed the definition of creativity in school level mathematics as:

1. The process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or
2. The formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle (p.19)

Moreover, Silver (1997) associates mathematical creativity with the connection between problem-posing and problem-solving processes. Both the process and the product should be evaluated to comprehend how much creativity is evident in the problem-posing and problem-solving process. To observe creativity, a number of features that can be examined include the solution's novelty, the number of solutions produced, and the number of different solutions. Already, these are the features that are also evaluated in cognitive creativity tests (Torrance, 1965). Since the problem-solving process has the nature of observing these features, they serve both to observe creativity and develop students in a more creative disposition toward mathematics (Silver, 1997).

Besides, Leikin (2009) asserts that mathematical creativity could be evaluated through the multiple solution tasks based on Ervynck (1991), Silver (1997), and various researchers' both the definitions of creativity and the approaches of relations between mathematical creativity and problem-solving. Further, in this study, mathematical creativity was discussed in light of the model improved by Leikin (2009) that evaluates mathematical creativity in multiple solution tasks.

2.3.2 Leikin's View: Mathematical Creativity with MSTs

Leikin (2009) associates mathematical creativity with the problem-solving process and specifically focuses on using multiple solution tasks in exploring creativity.

To Leikin (2009), multiple solution tasks (MST) are tasks in which a student is directly required to solve a problem in different ways. Indeed, she (2009) specifies the solution spaces which enable researchers to investigate the several aspects of problem-solving performance in multiple solution tasks. An expert solution space means the complete set of solutions for a given problem in a specific time. In school mathematics, the expert solution space covers a conventional solution space. In the conventional solution spaces, problems are solved by curriculum recommendations and teachers' taught.

On the contrary, an unconventional solution space consists of undetermined solutions by the school curriculum. Moreover, an individual solution space is the individual's all solutions for a particular problem. The solution space in which individuals can present their solutions without help from others is defined as an available solution space. The solution space in which an individual presents her/his solutions with the help of others is defined as a potential solution space. Lastly, a collective solution space combines a group of individuals' solutions, and an expert solution space includes both individual and collective solution spaces. Leikin (2009) used the solution spaces to investigate individuals' use of creativity and evaluate whether a task assesses mathematical creativity.

Leikin (2009) developed a model based on solution spaces to evaluate individuals’ personal mathematical creativity. The model contains a scoring scheme of evaluating individuals’ creativity based on originality, fluency, and flexibility dimensions suggested by (Torrance, 1965). Since creativity can be evaluated for individuals, for groups, or tasks, this model made some discriminations based on the problem-solving modes (e.g., oral (interview), written (test), individual or group). However, the present study will use the part of the model which focused on written and individual solutions because of the scope of study that does not require any group work or interview.

In consideration of the model, Leikin (2009) discusses the three dimensions of creativity: fluency, flexibility, and originality in students' solutions to MSTs. Originality is evaluated in written settings with a small group of individuals based on the conventionality and the level of insight. She determines three different levels of originality, as creativity ranked from weakest to strongest: (i) an insight-based or unconventional solution, (ii) a model-based or partly unconventional solution (learned in a different context), and (iii) an algorithm-based or conventional (learned) solution. Overall originality of individual is found by gathering the individual's originality on the solutions in individual solution spaces. To Leikin (2009), fluency focuses on the solving process and the switches among different solutions and is the number of appropriate solutions in the individual solution space in a given time. Flexibility is evaluated according to the fact that two solutions will work in two different representations, properties (e.g., theorems, auxiliary, or definitions), or branches of mathematics. She determines three levels of flexibility, as creativity ranked from weakest to strongest, based on individual solution space: (i) the solution is almost identical with a previous solution, (ii) the solution belongs to one of the previously used groups but has a clear minor distinction, and (iii) the solution belongs to a group of solutions different from the solution(s) performed previously. The overall flexibility is found by gathering the individual's flexibility of the solutions in the individual solution space.

Finally, based on three dimensions of creativity, the mathematical creativity of an individual is evaluated, which represents the fluency, flexibility, and originality of an individual's solutions to the multiple solution tasks.

2.4 Dynamic Geometry Software: GeoGebra

In school mathematics, teachers illustrate the mathematical ideas with the help of physical manipulative; however, some mathematical ideas might not be represented by using just manipulative (Goldenberg, 2000). Association of Mathematics Teacher Educators (AMTE) (2006) suggested that technology should be used when the representation is difficult or impossible without technology to facilitate mathematical discovery, understanding, and connections. AMTE (2006) defined the technology as tools which are computers with appropriate software, Internet and digital sources, handheld computing tools, and emerging forms of corresponding devices and applications. One type of technological tool is Dynamic Geometry Software (DGS), a group of programs to do dynamic geometry. The DGS is improved to teach elementary mathematics, supports the development of geometrical concepts, particularly Euclidean geometry (Kokol-Voljc, 2007). The DGSs has three main characteristics such as (i) dynamic form of the paper-pencil environment with the dragging mode, (ii) a sequence of commands to form a new command, a macro, and (iii) Visualization of the movements of geometrical objects, a locus (Kokol-Voljc, 2007). Additionally, the DGS gives students the resources to make reasoning abstractly and quantitatively and give meaning to the relationships among the distinct features of the problem/mathematical ideas (Contreras, 2013). Also, it gives the opportunity to make the connection between real-world and mathematical ideas and create dynamic constructions that could not be represented with paper and pencil (e.g., measuring, dragging) (Contreras, 2013). It guides students to improve proactive strategies based on the relationships they observe since it is an environment where students able to view results and get instant feedbacks (Hollenbrands, 2003).

One of the Dynamic Geometry Software is the GeoGebra, which is based on the relationship between points, lines, circles, and so on (Sangwin, 2007). Sangwin (2007) describes the GeoGebra as *"the brainchild of Markus Hohenwarter, which joins geometry, algebra, and calculus"* (p.36). GeoGebra gives an environment to work with points, vectors, segments, lines, and conic sections, in which equations and coordinates could be entered directly. Also, functions could be defined algebraically and changed dynamically in the GeoGebra (Sangwin, 2007). Since there is no technical or financial limitation to using the GeoGebra, which means it freely available to download, and provides faster and flexible results, its use in school mathematics is considered beneficial (Sangwin, 2007).

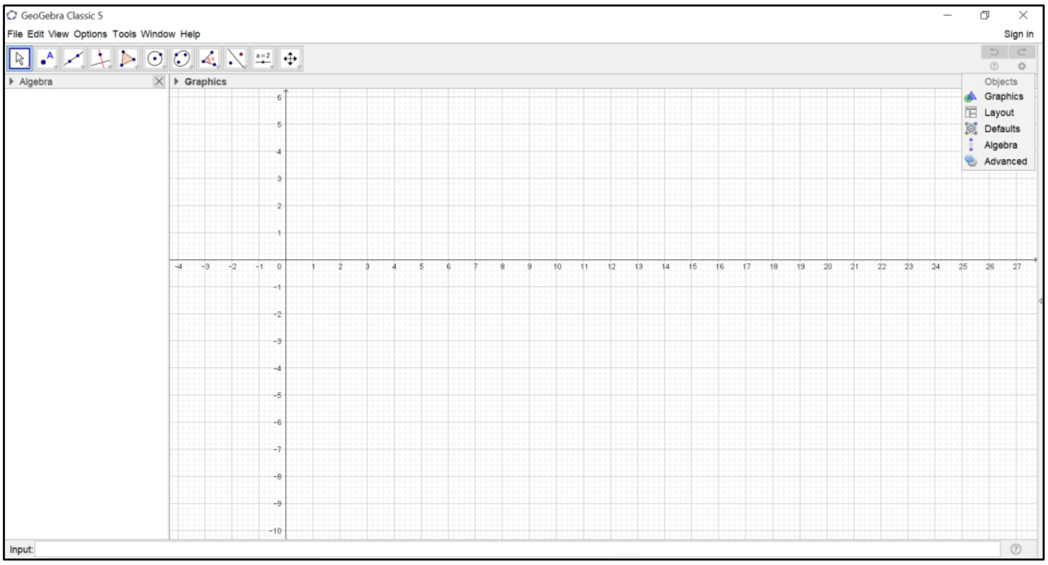


Figure 2. 1 GeoGebra window

The GeoGebra was developed to support mathematical understanding, which might be possible through the moving free objects that show how changes influence the other variables. In this way, students can comprehend the mathematical relations dynamically while solving a problem (Dikovic, 2009). GeoGebra supports discovery and experimental learning through its user-friendly multilingual interface, commands, and helps (Dikovic, 2009).

2.5 GeoGebra and Mathematical Creativity

Yerushalmy (2009) indicated that;

> "…technology can offer new opportunities to learners and teachers for whom traditional learning of mathematics does not provide opportunities for being creative" (p.102)

Since most mathematics curricula are developed based on rote learning, and the main components of curricula are mostly opposed to flexibility and fluency, there has been much interference in introducing creativity into school mathematics and promoting it for teachers to create curricula that motivate creativity (Yerushalmy, 2009). According to Sriraman (2005), to enhance creativity at the school level, students need to deal with challenging problems, discover the beauty of solutions, learn to justify their solutions, and produce different solution methods. To deal with solving challenging problems, students often utilize visualized mathematical representations, such as geometric figures, algebraic expressions, or graphs (Sedig & Sumner, 2006). Dynamic Geometry Software, referring to GeoGebra, allows students to construct, manipulate and explore these representations (Preiner, 2008). Also, the GeoGebra encourages students to get creative feedback that contributes to creative reasoning since it does not provide the correct answers and clues on how to proceed. Creative feedback requires individuals to interpret and evaluate their solving strategies (Granberg & Olsson, 2015). Indeed, the representations could also be constructed in the paper-pencil environment. However, the GeoGebra allows students to explore the representations dynamically and get immediate feedback that is difficult in paper-pencil environments. The dynamic construction, exploration, and instant feedback are crucial to concentrate more on cognitive aspects like problem-solving and reasoning (Heid & Edwards, 2001).

In this context, students' use of the GeoGebra in a way to explore the mathematical relationships has been shown that supports their problem solving (Berger, 2011), reasoning (Natsheh & Karsenty, 2014), and creative thinking abilities (Yerushalmy, 2009). This potential of GeoGebra to support students' creative thinking and problem-solving abilities might be used to design that expect to understand the relationship between the Dynamic Geometry Software and mathematical creativity.

2.6 Review of the Related Literature

In this part, studies related to mathematical creativity, the use of Dynamic Geometry Software on learning, teaching, and problem-solving activities, and differentiation between the Dynamic Geometry Software and the paper-pencil environment in terms of problem-solving and creative thinking abilities will be presented as a review of the literature.

Leikin and Lev (2007) investigated the multiple solution tasks as a tool to examine mathematical creativity at the school level. They studied with students from three ability groups, which are gifted, non-gifted, and regular, and asked them to solve problems in different ways, and there were six high school students in each group. Leikin and Lev (2007) discussed two tasks, conventional and non-conventional, to examine the multiple solution tasks as a tool for evaluating mathematical creativity. Their criteria for assessing mathematical creativity were novelty, flexibility, and fluency of students' solutions. According to their findings, gifted and regular students showed similar results in conventional tasks; however, Leikin and Lev (2007) observed significant differences in non-conventional tasks. That is, they hypothesized that non-conventional tasks are effective tools to investigate students' mathematical creativity at the school level.

Koyuncu (2013) investigated the plane geometry problem-solving strategies of pre-service mathematics teachers in both the GeoGebra and paper-pencil environment after pre-service teachers received instruction of the GeoGebra. He used the multiple case study design and studied with seven pre-service mathematics teachers in Turkey. After provided the GeoGebra training for pre-service teachers, the data were collected by classroom observations and interviews. The data analysis revealed that there were three types of pre-service teachers' solution: algebraic, geometric, and harmonic solutions. While they mainly created algebraic solutions in the paper-pencil environment and geometric solutions in the GeoGebra environment, which means mathematical problem-solving abilities might be affected from the different environments. The GeoGebra environment contributed to the mathematical understanding through its dynamic nature and enabled the pre-service teachers to save time and create different strategies. To Koyuncu (2013), the results of the study implied that the use of the GeoGebra environment in problem-solving activities might be beneficial from various aspects.

Similarly, Farihah (2018) analyzed the students' thinking preferences in mathematical problem solving by comparing the paper-pencil and GeoGebra environments based on their learning styles. She conducted a qualitative descriptive study with six eighth-grade students in Indonesia. The students had different learning styles; two visual, two auditory, and two kinesthetic students. The students were expected to solve the paper-pencil and the GeoGebra-based tasks, and Farihah (2018) compared the students' visual and non-visual thinking by investigating their solutions while using GeoGebra and paper-pencil. The result of her study showed that the GeoGebra environment influences the students' thinking styles based on the comparison between their solutions in the paper-pencil and GeoGebra environment, either visual, auditory, or kinesthetic. Moreover, the use of GeoGebra might encourage students to prefer using visual methods to non-visual methods and contribute to students' conceptual understanding by nurturing representation fluency as it supports them to create different solutions.

In addition to studies examining the mathematical thinking styles, Selvy et al. (2020) investigated students' mathematical creative thinking and motivation within a problem-based learning model via the GeoGebra. They studied with eleven senior high school students in Indonesia as a part of an experimental method with pre and post-test control group design. Their analysis displayed that students' mathematical creative thinking abilities and motivation were increased within the problem-based learning model via the GeoGebra, which means students' creative thinking abilities were higher than within using Problem Based Learning model without software. Thus, they (2020) claimed that GeoGebra is a valuable tool to contribute to improving mathematical creative thinking skills.

On the other hand, Coelho and Cabrita (2015) investigated how the approach that combines the paper-pencil and Dynamic Geometry environments supports the development of geometric skills and understanding of geometric concepts as a part of the question of how technology used to encourage their creativity. They studied with two groups of fifth-grade students within the qualitative case study. For the study based on reflection, rotation, and translation topics, they have developed an instructional exploratory task sequence. The students were required to solve eight different geometry tasks, and they solve some of them in paper-pencil, some of them in the GeoGebra, and some of them by using a combination of two environments. On the basis of the evaluation of creativity with respect to the solutions' originality, flexibility, and fluency, they (2015) claimed that the approach that combines the paper-pencil and Dynamic Geometry environment might increase the dimensions of creativity. Also, combining the Dynamic Geometry Environments with the paper-pencil might encourage students to produce more creative products and develop their knowledge and geometrical capability. This means that more technological approaches can promote more positive attitudes towards geometry in particular and mathematics in general.

Yıldız et al. (2017) investigated the learning process of analytic geometry concepts via the GeoGebra and its effect on the advancement of the creative thinking abilities of pre-service teachers. It was mixed-method research, and the Torrance Creative Thinking Test (TCTT) has been administered at the beginning and the end of the implementation. They studied with thirty pre-service teachers from Turkey and collected data through Torrance Creative Thinking Test Verbal A and Figural A forms. Also, they utilized semi-structured interviews and worksheets that the pre-service teachers produced in the GeoGebra environment. Based on the analyses of data, they asserted that the use of GeoGebra has positive effects on pre-service teachers' creative thinking abilities in all dimensions of creativity but one. To them, teaching through GeoGebra might have a positive effect on fluency, originality, and elaboration. Namely, the dynamic nature and ease of use of the software might help the three dimensions of creativity to be improved.

In conclusion, it is seen that the use of GeoGebra in teaching, learning, or problem solving promotes students' mathematical creativity and students' various abilities related to creativity: problem solving skill representational fluency, conceptual understanding, and dimensions of creativity. The combination of GeoGebra with the paper-pencil also encourages students to produce more creative products.

CHAPTER 3

METHODOLOGY

In this chapter, the methodology of current study will be showed in detail. The design of the study, participants of the study, data collection tools, role of reseacher, procedures of the study, and data analysis process will be presented. Then, quality of research, assumptions and limitations of the study, and ethics will be explained.

3.1 Research Design

In the present study, the researcher used the qualitative research design to understand the current situation deeply. To Creswell (2013), qualitative research aims to investigate the meaning individuals or groups attribute to problems and describe the issue in detail. It focuses on understanding a context in which participants address a point by getting contact with them directly (Creswell, 2013). Also, qualitative studies may focus on individual cases and aim to reply to what could be learned from a specific case (Denzin & Lincoln, 2000). This type of qualitative research is called a case study. A researcher analyzes “a bounded system” through multiple sources of information such as interviews, observations, or audio-visual materials (Creswell, 2013). In social and educational research, the cases can be people or programs. Each of them is similar to others and unique in many ways, and researchers are interested in both commonality and uniqueness of the cases to comprehend them (Stake, 1995). Hence, the multiple case study, one of the three types of case studies (intrinsic, instrumental, and multiple), is appropriate when the researcher needs to examine more than one case to examine coordination or phenomenon among single cases (Stake, 1995). The multiple case study research facilitates revealing the common characteristics or understanding a larger collection of cases (Denzin & Lincoln, 2000).

In the current study, the researcher needed to assemble a deep understanding of the pre-service teachers' solution strategies in multiple solutions tasks to answer the study's research questions, which would enable her to understand the role of dynamic geometry software on pre-service teachers' mathematical creativity abilities. Furthermore, to maximize the pre-service teachers' solutions, the researcher preferred to study more than one pre-service teacher. Therefore, the multiple case studies were selected as the appropriate design for this study, considering its features.

3.2 Participants

The participants of this study were senior students at Elementary Mathematics Education Program from the same university in Turkey. They were selected from the pre-service mathematics teachers who took the Exploring Geometry with Dynamic Geometry Applications course in the 2018-2019 Fall Semester the current study was conducted in the following 2018-2019 Spring Semester. The participants were senior students when taking the Exploring Geometry with Dynamic Geometry Applications course. This course helps pre-service mathematics teachers gain various perspectives in teaching and learning mathematics and geometry using GeoGebra. Within the this course, the pre-service mathematics teachers experience constructing different objects or figures, exploring mathematical properties, creating worksheets, tests, and presentation materials in the GeoGebra environment, and experience analyzing and solving various geometric problems. The reason for selecting pre-service mathematics teachers among those who took the Exploring Geometry with Dynamic Geometry Applications course was that they were assumed to have sufficient capability to use GeoGebra to create solutions for given geometry tasks in the GeoGebra. Moreover, prior to this study, they completed the Basic Computer course and could use computers at least at an average level.

One of the nonrandom sampling methods is convenience sampling. A convenience sample is conveniently available for a research study (Fraenkel et al., 2012). In the current study, eight pre-service mathematics teachers were selected by the convenience sampling method according to their availability for a three-time data collection period, and participation was voluntary. The pre-service teachers who were chosen had average course grades compared to other pre-service teachers in the Exploring Geometry with Dynamic Geometry Applications course. Then, the researcher chose one of the pre-service teachers for the pilot study and seven of them for the actual research. These students were selected considering not only their performance but also their willingness and communication with the researcher. One of the reasons for choosing them was that they could create available solutions for given tasks. In that, two of the pre-services were eliminated because they were not able to develop any solution for some of tasks, which After all, the study was conducted with five pre-service teachers, three of them were women and two of them were men. Additionally, the pre-service teachers were from a middle socioeconomic level, and they are from different cities in Turkey. However, all participants had been educated in public high schools in the same years. In other words, they had a geometry education with the same mathematics curriculum.

3.3 Data Collection Tools

In this study, the primary data sources were video recordings of each Multiple Solution Task's (MST) implementation process, written works, and GeoGebra (GGB) files of participants. Each participant solved three multiple solutions tasks and each task was implemented on different days. The researcher asked the participants to solve the given tasks in as many ways as possible and gave them approximately forty-five minutes for each task. Also, the researcher expected them to think aloud during the solution process to have a deep understanding of their way of thinking. The pre-service teachers created their solutions in a quiet environment where only they and the researcher were present under video recording.

Participants solved the tasks in the paper-pencil and GGB environments, and the researcher gathered their written works and GGB files at the end of each MSTs process. The written works and GGB files helped the researcher to determine and code participants' solutions. Also, each MSTs implementation process was recorded. Video recordings were chosen as data sources since they involved the participants' whole solution processes. They also enabled the researcher to determine the research findings explicitly and supported the researcher to review data many times later than the implementation process to code solution steps. Thus, video recordings improved the data and helped the written works and GGB files to complete the whole picture. Consequently, the findings of the study were enhanced with various data sources.

3.3.1 Multiple Solution Tasks

The data were collected through geometry tasks in this study, entitled Task 1, Task 2, and Task 3. These tasks' theme was geometry because GeoGebra mainly works with points, vectors, segments, lines, and conic sections that enable study on geometry tasks (Sangwin, 2007). Also, these tasks were classified as the Multiple Solutions Task because they would allow participants to solve them in different ways. Already, the Multiple Solutions Tasks require solving a problem in different ways (Leikin & Levav-Waynberg, 2007). Furthermore, multiple Solution Tasks also enable participants to solve the same tasks with other presentations, various properties of an object within a particular field, or different domains (Leikin, 2009). Therefore, they are considered tools for examining mathematical creativity (Leikin & Lev, 2007). Thus, the selected three tasks were appropriate for the study because they were suitable for solving in the GeoGebra environment and enabling participants to create different solutions.

The participants' grade levels were considered while choosing the multiple solution tasks. They were selected among the mathematics contents that had been covered by 9th-grade students so far. In addition, the assigned tasks were chosen from among the tasks used as multiple solution tasks in different studies (Leikin et al., 2011, Leikin & Lev, 2013, and Leikin, 2011). The Multiple Solution Tasks were in English, and they were translated from English to Turkish by the researcher. The expert in the field checked the translation of tasks to address the reliability and validity.

The multiple solutions geometry tasks and some detailed explanations about tasks were served below.

3.3.2 Multiple Solution Tasks: Task 1

The first task (Figure 3.1) was taken from Leikin et al. (2011). They used this task to compare the changes in geometric knowledge and creativity on regular and high-level students. Leikin et al. (2011) expected students to solve the task in as many ways as they could in sixty minutes. When the task was chosen, Leikin et al. (2011) considered that it is not conventional because some solutions require knowledge different from the school mathematics curriculum. Thus, Leikin et al. (2011) assumed that the task could be solved in more than one way.

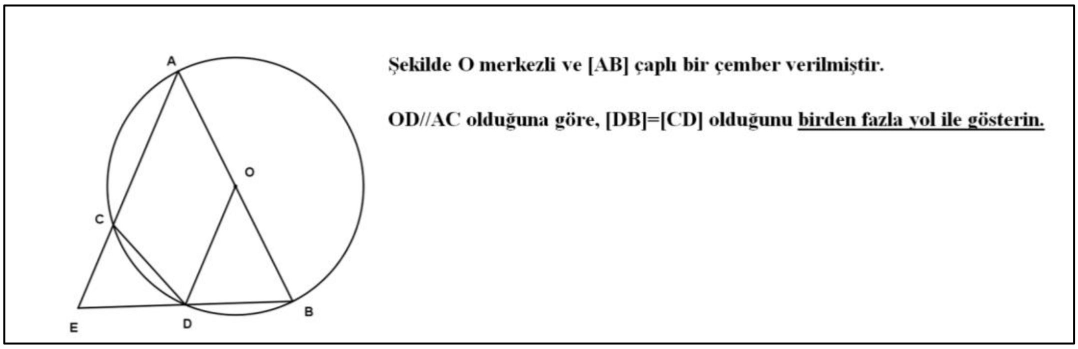


Figure 3. 1 The Multiple Solutions Task 1. Question in English: AB-diameter, O-center of the circle, and OD || AC. Show in as many ways as you can DB = CD (Leikin et al., 2011).

3.3.3 Multiple Solution Tasks: Task 2

The second task (Figure 3.2) was from Leikin and Lev (2013). They used the task to investigate the relationships between mathematical creativity, mathematical excellence, and general giftedness and the power of different types of multiple solutions tasks between other groups of students. This task was one of the five multiple solution tasks in their study. The 11^{th} and 12^{th}-grade students were expected to solve each task in as many ways as they can in an hour and a half. According to Leikin and Lev (2013), a different level of insight was embedded in these five tasks to evaluate mathematical creativity.

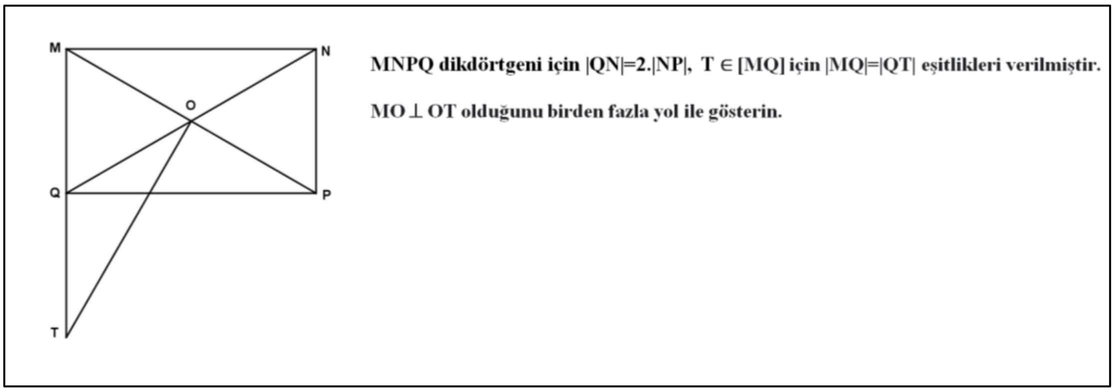


Figure 3. 2 The Multiple Solutions Task 2. Question in English: Given a rectangle MNPQ where QN = 2 NP. The diagonals meet a point O. T is on the line MQ so that MQ = QT. Show, in as many ways as you can, that MO is perpendicular to OT (Leikin & Lev, 2013).

3.3.4 Multiple Solution Tasks: Task 3

The third task (Figure 3.3) was from Leikin (2011). She used multiple solution tasks to investigate how teachers implemented multiple solution tasks (as new instructional tools) in classes and tried to connect these ways with the development of teachers' expertise. Her study consisted of two stages: the learning and implementation stages (a year for each stage). Twelve secondary school mathematics teachers were asked to integrate MSTs in their classes at the implementation stage (in the second year of the study). MSTs were thought of as a valuable instructional tool for teachers and providing a chance to understand more about mathematics.

Bir dik üçgende hipotenüse ait kenarortayın, hipotenüsün uzunluğunun yarısına eşit olduğunu birden fazla yol ile gösterin.

Figure 3. 3 The Multiple solutions Task 3. Question in English: Show, in as many ways as you can, that the median in a right triangle equals half of the hypotenuse (Leikin, 2011).

3.4 The Role of Researcher

The researcher has a role as a designer and practitioner of research and experimental periods. Also, she observed the participants' solution processes actively and analyzed the data through qualitative methods. The pre-service mathematics teachers were asked to solve the tasks in as many ways as they can in the paper-pencil environment and GeoGebra environment. They needed to solve each task independently, but if required, the researcher helped at a minimum level about the solution strategies or possible problems with GeoGebra. Also, since the researcher took the Exploring Geometry with Dynamic Geometry Applications course in her undergraduate education and is an active user of GeoGebra, she has sufficient experience solving geometry tasks in the GeoGebra environment.

3.5 Procedures

3.5.1 The Pilot Study

The pilot study was designed to investigate whether the instruments and planned process were appropriate for the actual research. The whole process is intended to implement in the actual research was assessed considering the feedbacks obtained from the pilot study. A student who took the Exploring Geometry with Dynamic Geometry Applications course in the 2018-2019 Fall Semester was invited by considering her performance on the course and being communicative to get valuable feedback about process and instruments voluntarily.

The researcher designed three different days to implement the three multiple solution tasks. In task 1 and task 2, she expected the participant to solve the given task in both the paper-pencil and GeoGebra environment. In task 3, she gave the freedom participant to create solutions in any environment or use two environments. On the first day of implementation, the researcher asked her to solve Task 1 in the paper-pencil environment. Then, the researcher guided her to create other solutions in the GGB environment by asking, "If I asked you to solve this question in the GGB environment, how would you create solutions?". On the second day of implementation, the researcher asked her to create her solutions for Task 2 on the GGB environment at first. Then, the researcher guided her to develop other solutions in the paper-pencil environment by asking, "If I asked you to solve this question in the paper-pencil environment, how would you create solutions?". On the last day, the researcher gave the freedom to choose any environment or use two of them to solve task 3. There was not a time limitation during her solution process of any task in that she could decide whether she created all possible solutions she can make. At the end of each day, the written works and GGB files were received, and the whole process was recorded with a video camera.

After the data collected, the researcher transcribed the video records and analyzed the written works and GGB files. The data were analyzed based on Leikin's framework (2009), which was created to evaluate the mathematical creativity in terms of fluency, flexibility, and originality in the multiple solution tasks. The participant developed more than one solution ways for all tasks in both the paper-pencil and GeoGebra environments. However, expecting to solve the same question in two environments caused that she tried to solve the task in the same ways in other environments instead of creating new solutions. For instance, after she solved task 1 in the paper-pencil environment, she transferred the ways formed in the paper-pencil environment to the GeoGebra environment instead of focusing on new strategies. Also, the researcher deduced that unlimited time is not convenient for the nature of the framework based on Leikin (2009), who suggested examining the mathematical creativity considering the solutions of a task at the given time. Finally, the researcher reviewed the whole process and revised a study plan in light of the feedback obtained from the pilot study.

According to the consequences of the pilot study, instead of unlimited time, all participants were asked to create as many solutions as they could within about forty-five minutes for the given task. Also, the participants were asked to develop solutions only in the paper-pencil environment for task 1. At the same time, they were expected to produce solutions for task 2 with at least one solution in the GeoGebra. Finally, for task 3, there was no restriction on the environment they will choose.

3.5.2 Data Collection

At first, the multiple solutions geometry tasks were translated into Turkish and adapted to pre-service mathematics teachers' understanding. Experts' opinions were taken to provide the validity and reliability of instruments before the pilot study would be designed. The experts were from mathematics educators from Middle East Technical University. After necessary permissions were obtained from the Middle East Technical University Human Subjects Ethics Committee, the pilot study was conducted. The pilot study was implemented with three MSTs for three days (27-28 February & 1 March 2019) in the spring semester of 2018-2019. Each task was executed on different days. Based on the feedback from the pilot study and an expert at METU, necessary revisions were done to be prepared for the actual research.

Then, five pre-service mathematics teachers enrolled in the elective course Exploring Geometry with Dynamic Geometry Applications in the 2018-2019 Fall Semester engaged in three multiple solutions tasks. Each task was implemented for each participant on different days during March 2019. The schedule was shown in Table 3.1 below.

The researcher applied the same procedure during the implementation of three multiple solutions tasks. Each participant solved three multiple solutions tasks (Task 1, Task 2, and Task 3), and each task was implemented on different days. The researcher asked the participants to solve the given tasks in as many ways as possible and gave them approximately forty-five minutes for each task. Also, the researcher expected them to think aloud during the solution process to have a deep understanding of their way of thinking. The pre-service teachers created their solutions in a quiet environment where only they and the researcher were present.

On the first day of each implementation, participants were asked to develop more than one solution for task 1 in the paper-pencil environment for about forty-five minutes and expected to think loudly during the solution process. For task 1, the participants were provided with enough worksheets to produce as many solutions as they wanted with the paper-and-pencil. On the second day of each implementation, participants were expected to create solutions with at least one solution in the GeoGebra about in forty-five minutes. They were asked to think aloud during the solution process. They were able to create other solutions in any environment that they want. Similarly, on the third day of each implementation, participants were asked to solve task 3 more than one way. However, there was no restriction about the solution environment; pre-services were free to use two environments. Likewise, they had forty-five minutes and were expected to think aloud during the solution process.

Table 3. 1. Time schedule of the present study

Date	Event	Duration
27-28 February & 1 March 2019	Pilot Study	There was no restriction about time
4-5-18 March 2019	Implementation of Task 1, Task 2 and Task 3 for Feyza	45 minutes for each task
5-8-15 March 2019	Implementation of Task 1, Task 2 and Task 3 for Ahmet	45 minutes for each task
7-11-14 March 2019	Implementation of Task 1, Task 2 and Task 3 for Umay	45 minutes for each task
7-12-13 March 2019	Implementation of Task 1, Task 2 and Task 3 for Beren	45 minutes for each task
16-23-26 March 2019	Implementation of Task 1, Task 2 and Task 3 for Kemal	45 minutes for each task

At the beginning of each implementation, the researcher gave the participants about ten minutes to read the task silently and understand what the task wants. Then, she asked the participants to share their ideas about possible solutions and to think aloud during the solution process. Finally, the researcher recorded the whole solution process and observed the pre-service teachers. She did not interfere in their solutions except for helping with any problem with using the GeoGebra software.

3.6 Analysis of Data

In this research, data were analyzed by the qualitative method to answer the intended questions of the study. The data analysis is the method of inferring the data (Merriam, 2009). The "coding" is a qualitative research method to figure out and name the categories and create a system for categorizing data into categories (Merriam, 2009).

In the current study, the pre-service mathematics teachers needed to solve three different multiple solutions tasks, which were shown previously, to investigate their mathematical creativity in as many ways as possible. The pre-service teachers were asked to solve task 1 in the paper-pencil environment, while they needed to create at least one solution in the GeoGebra for task 2. There was no limitation about the environment for task 3. This means there were pre-service teachers' written solutions for task 1, while there were both written and GeoGebra solutions for task 2 and task 3.

In the first step of data analysis, the researcher transcribed all solutions of pre-service teachers based on pre-service teachers' solutions in the paper-pencil and GeoGebra environment and video recordings. For example, in Figures 3.4 and 3.5, both a pre-service teacher's own solution and the pre-service teachers' solution transcribed by researcher were presented. The pre-service teachers thought aloud during implementation and made only verbal explanations for some steps of solutions. While the researcher transcribed their answers, she combined both the written and GeoGebra solutions and the verbal explanations in the video recordings.

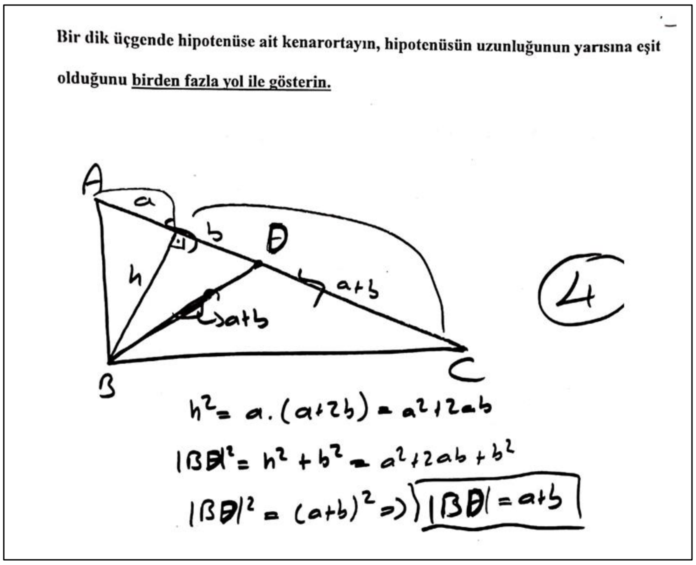


Figure 3. 4 Participant's solution fort ask 3 in the paper-pencil environment

1. Draw a line segment BF which is perpendicular AC and passing through the point B ($|AF| + |FD| = |AC|$)
2. Since the triangle ABC is a right triangle and BF is perpendicular to AC;
   $|BF|^2 = |AF| . (|FD| + |DC|)$ and $|BD|^2 = |BF|^2 + |FD|^2$
   $h^2 = a . (a + 2b)$ and $|BD|^2 = h^2 + b^2$
   $|BD|^2 = a^2 + 2ab + b^2$
   $|BD| = a + b$
3. $|AD| = |DC| = |BD|$; $|BD| = \frac{|AC|}{2}$

Figure 3. 5 The transcribed form of the solution given in Figure 3.4 by the researcher

Secondly, while the researcher transcribed the solutions, she determined all the mathematical concepts used in the answers for each task separately. Each solution way was divided into parts considering the mathematical concepts used by the pre-service teachers to observe the similarities and differences among the solution ways. The researcher applied this to all solutions created for each task, separately.

In Figures 3.6 and 3.7, some solutions formed in the paper-pencil and GeoGebra environment appear divided into parts. The different solution ways were coded so that it was clear which task they belong to. For example, the "T1.S1" code was used in the first solution type of task 1, while the "T2.S6" code was used in the sixth solution type of task 2.

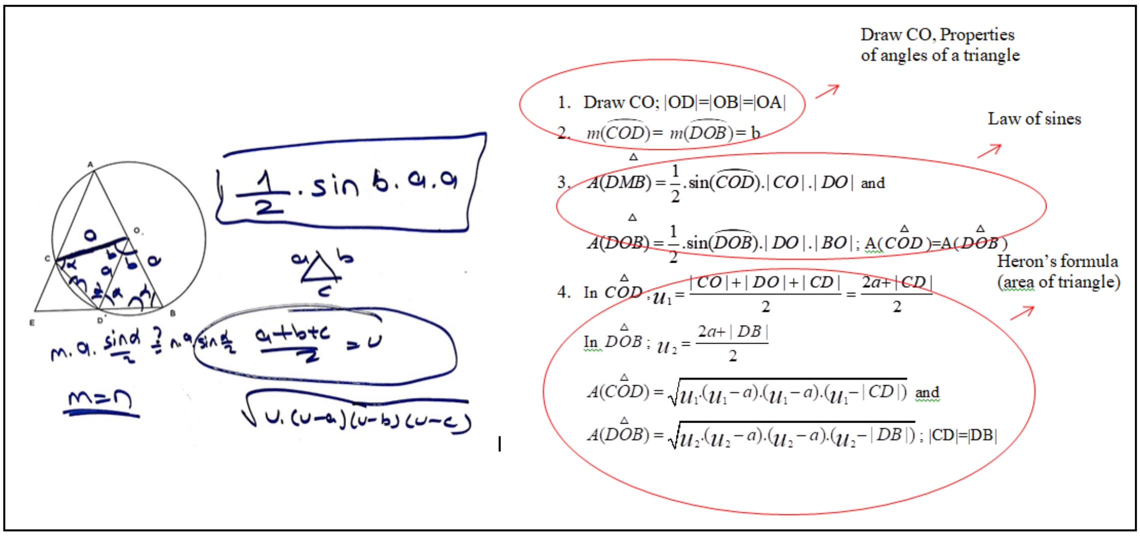


Figure 3. 6 The nineteenth solution type for task 1 (T1.S19) with the mathematical concepts used.

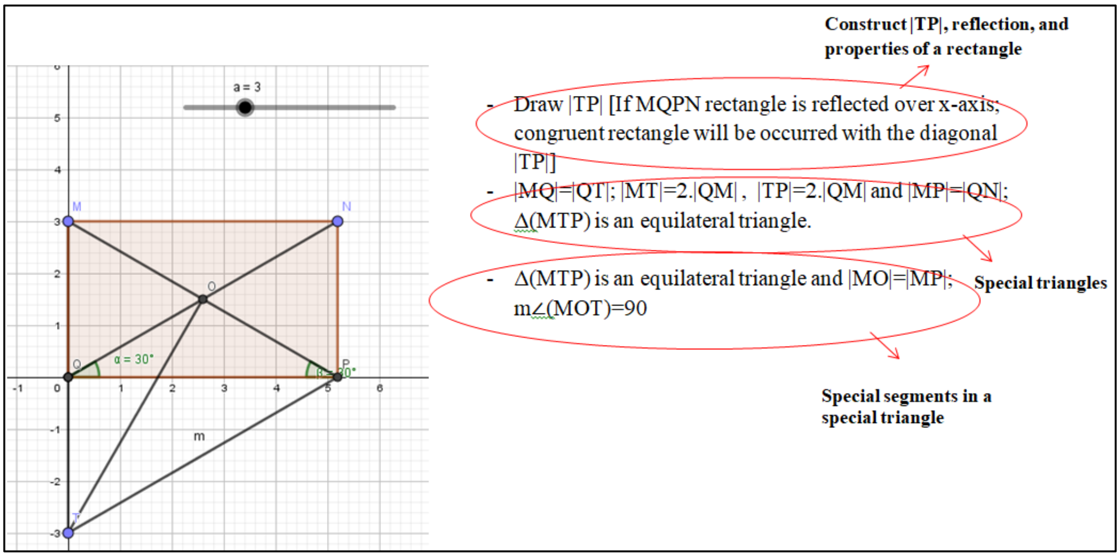


Figure 3. 7 The sixteenth solution type for task 2 (T2.S16) with the mathematical concepts used.

All the mathematical concepts used for Task 1 were as follows: *constructing auxiliary line segments and figures, types of angles (e.g., corresponding angles, alternate interior angles, and alternate angles), exterior angle theorem, congruent and similar triangles, corresponding angles, and sides, inscribed angle on a diameter, median, height, and angle bisector in a special triangle, inscribed angle theorem, central angle and arc relationship, angle properties of the triangle, congruent chords, median to the hypotenuse of a right triangle, law of cosine, law of sine, properties of the deltoid, Pythagorean theorem, Heron's formula, and making assumption way*. How some of these concepts were used in the solution ways can be seen in Figure 3.8.

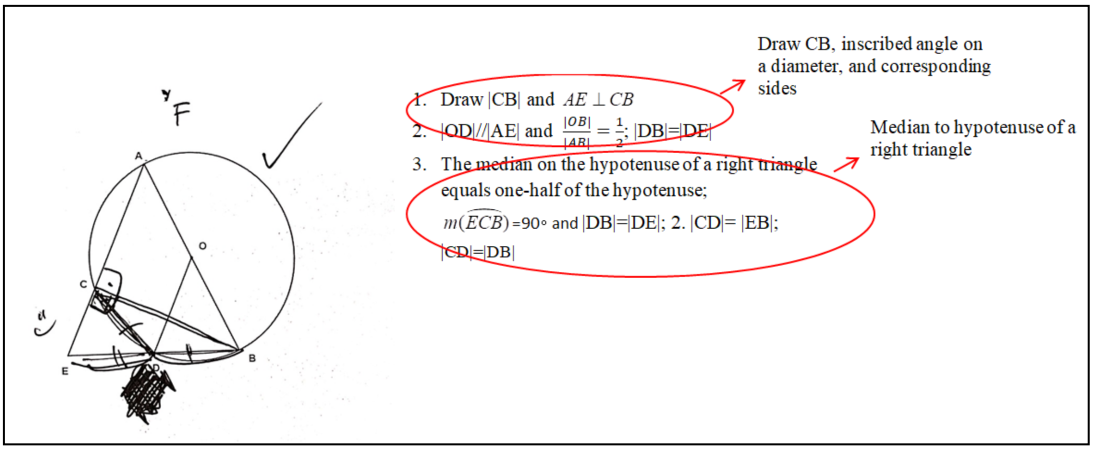


Figure 3. 8 The thirteenth solution type for task 1 (T1.S13) with the mathematical concepts used.

All the mathematical concepts used for Task 2 were as follows: *|QN| = 2. |NP|, |MQ| = |QT|, properties of a rectangle, median to the hypotenuse of a right triangle, special segments in special triangles, properties of special triangles, parallel and perpendicular lines, constructing auxiliary line segment TP, perpendicular bisector, reflection, inscribed angles on diameter, congruency and similarity, elements of a triangle, constructing a new geometric shape, constructing midpoint of |TO|, and algebraic solution ways including properties of angles of a triangle*. How some of these concepts were used in the solution ways can be seen in Figure 3.9.

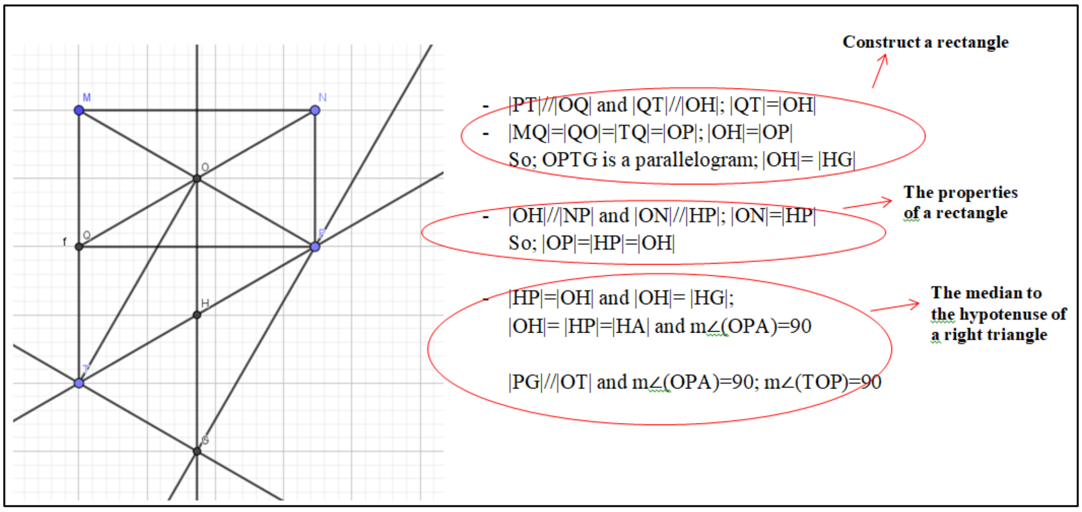


Figure 3. 9 The ninth solution type for task 2 (T2.S9) with the mathematical concepts used.

All the mathematical concepts used for Task 3 were as follows: *constructing a circle, congruency and similarity, properties of special triangles, properties of a circle, properties of a rectangle, properties of a parallelogram, corresponding angles, rotation, reflection, Pythagorean Theorem, and Euclid's Theorem.* How some of these concepts were used in the solution ways can be seen in Figure 3.10.

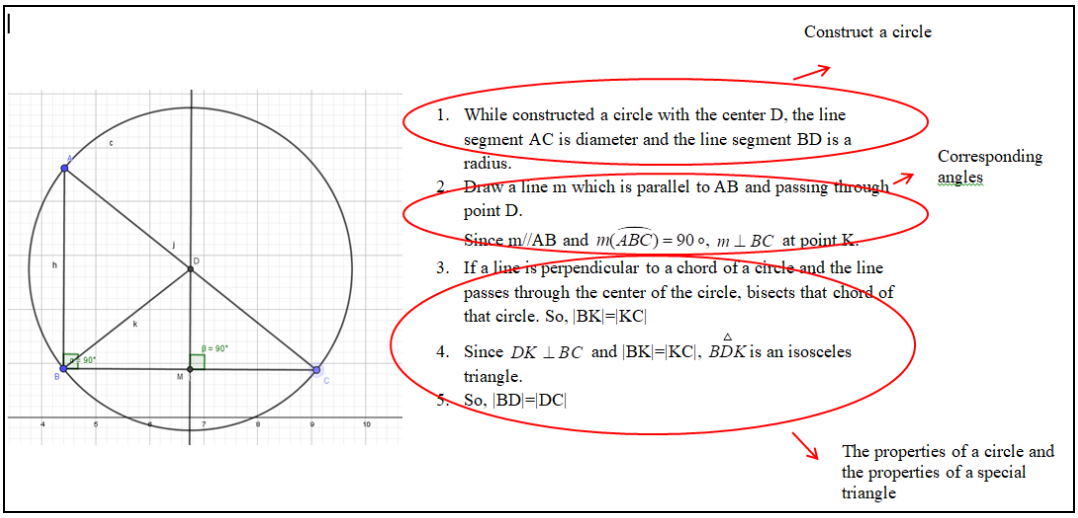


Figure 3. 10 The third solution type for task 3 (T3.S3) with the mathematical concepts used.

Lastly, the pre-service teachers' solutions were analyzed based on Leikin's framework (2009) to observe their mathematical creativity. To Leikin (2009), mathematical creativity is evaluated based on three dimensions: fluency, flexibility, and originality.

3.6.1 First Dimension of Creativity: Fluency

Leikin (2009) describes *fluency (N)* as the number of appropriate solutions of individuals within a given time. In the current study, the fluency of each pre-service teacher in each task was scored with the number of appropriate solutions. For example, if a participant created four different answers in task 1, the fluency was achieved four and coded as the "$N = 4$". Detailed coding for each pre-service teacher is presented in the findings section.

3.6.2 Second Dimension of Creativity: Flexibility

Leikin (2009) classifies the solution strategies created in the Multiple Solution Tasks to assess *the flexibility (Flx),* which is employed based on different representations, properties, or divisions of mathematics. Namely, two solution strategies belong to different groups if they are formed with different representations, properties, or dimensions of mathematics.

Leikin (2009) assesses the flexibility as follows:

- $\mathrm{Flx}_1 = 10$ for the first proper solution of each task.

Then, for each continuous solution way;

- $\mathrm{Flx}_i = 10$ if the solution way belongs to a different group of solutions from the previous solutions of the individual
- $\mathrm{Flx}_i = 1$ if the solution way belongs to one of the previous groups of solutions of the individual with the slight difference
- $\mathrm{Flx}_i = 0.1$ if the solution way is precisely the same as one of the previous groups of solutions of the individual

In the present study, the flexibility scores of each pre-service teacher were evaluated by considering the differentiation of the mathematical concepts they used within their own solutions. Therefore, detailed scoring of flexibility would be shown in the Findings Chapter for each pre-service teacher, separately.

3.6.3 Third Dimension of Creativity: Originality

Leikin (2009) handles *the originality (Or)* for individuals in a small group from similar learning backgrounds considering the level of insight and conventionality of the solution based on the learning background of individuals.

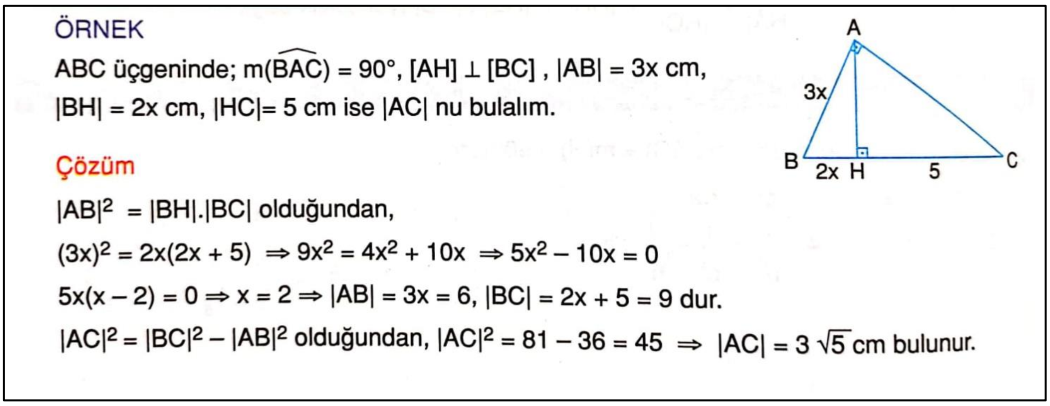
Leikin (2009) assesses the originality as follows:

- $Or_i = 10$ if the solution way is an insight-based or unconventional solution
- $Or_i = 1$ if the solution way is a model-based or partly-unconventional solution (seen in a different bunch)
- $Or_i = 0.1$ if the solution way is an algorithm-based or conventional solution (learned in school)

In the present study, the originality scores of each participant were mainly evaluated based on the conventionality of solutions. Namely, to assess the conventionality of pre-service teachers' answers, the Mathematics and Geometry Schoolbooks, which were used in the pre-services teachers' high school education period, were viewed focusing on the lectures and sample solutions. This enabled us to figure out how these solutions were created based on the taught in school. Based on Leikin’s framework (2009), the originality was assessed in this study as follows:

- $Or_i = 0.1$ if the solution is precisely created as taught in school and through memorized knowledge
- $Or_i = 1$ if some parts of the solution are created as taught in school
- $Or_i = 10$ if the solution is different from what was taught in school or rarely observed among the participants

Some examples of the mathematical concepts used in the answers with the High School Mathematics and Geometry Books are shown in the following (Figure 3.11, Figure 3.12, Figure 3.13, Figure 3.14, and Figure 15). The coincidences of these examples with the pre-service teachers' solutions and the scoring originality in the current study would be detailed in the Findings Chapter.


ÖRNEK

ABC üçgeninde; $m(\widehat{BAC}) = 90°$, $[AH] \perp [BC]$, $|AB| = 3x$ cm, $|BH| = 2x$ cm, $|HC| = 5$ cm ise $|AC|$ nu bulalım.

Çözüm

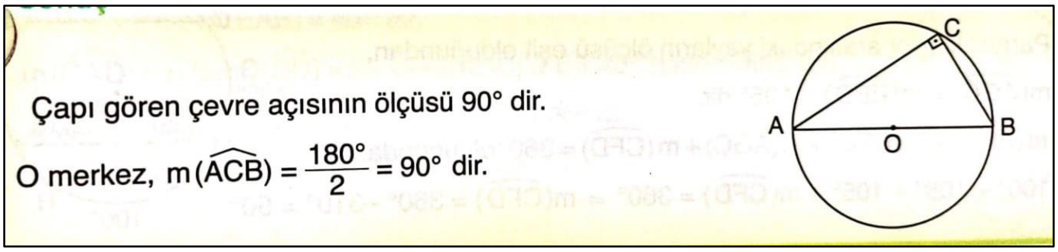
$|AB|^2 = |BH|.|BC|$ olduğundan,

$(3x)^2 = 2x(2x + 5) \Rightarrow 9x^2 = 4x^2 + 10x \Rightarrow 5x^2 - 10x = 0$

$5x(x - 2) = 0 \Rightarrow x = 2 \Rightarrow |AB| = 3x = 6$, $|BC| = 2x + 5 = 9$ dur.

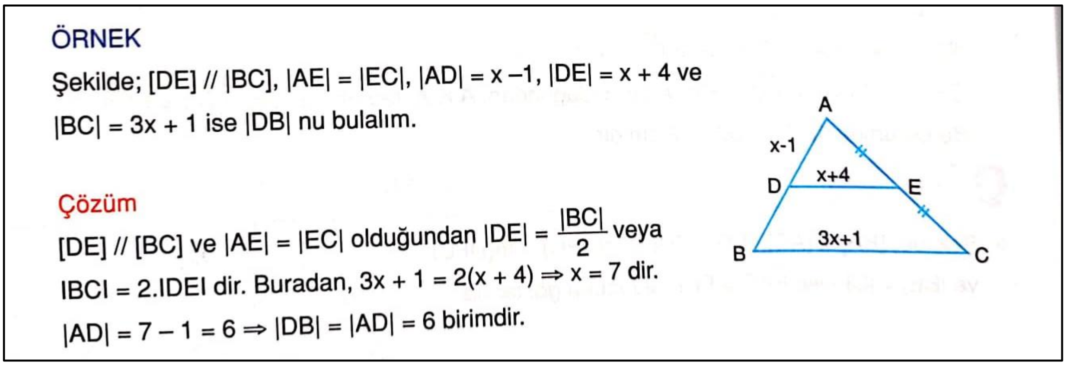
$|AC|^2 = |BC|^2 - |AB|^2$ olduğundan, $|AC|^2 = 81 - 36 = 45 \Rightarrow |AC| = 3\sqrt{5}$ cm bulunur.

Figure 3. 11 Example of using Euclid’s Theorem in the high school book (Peken & Aydın, 2009, p.178)


Çapı gören çevre açısının ölçüsü 90° dir.

O merkez, $m(\widehat{ACB}) = \frac{180°}{2} = 90°$ dir.

Figure 3. 12 Example of the form of the inscribed angle on circle knowledge in the high school book (Şayakdokuyan, 2012, p.191)


ÖRNEK

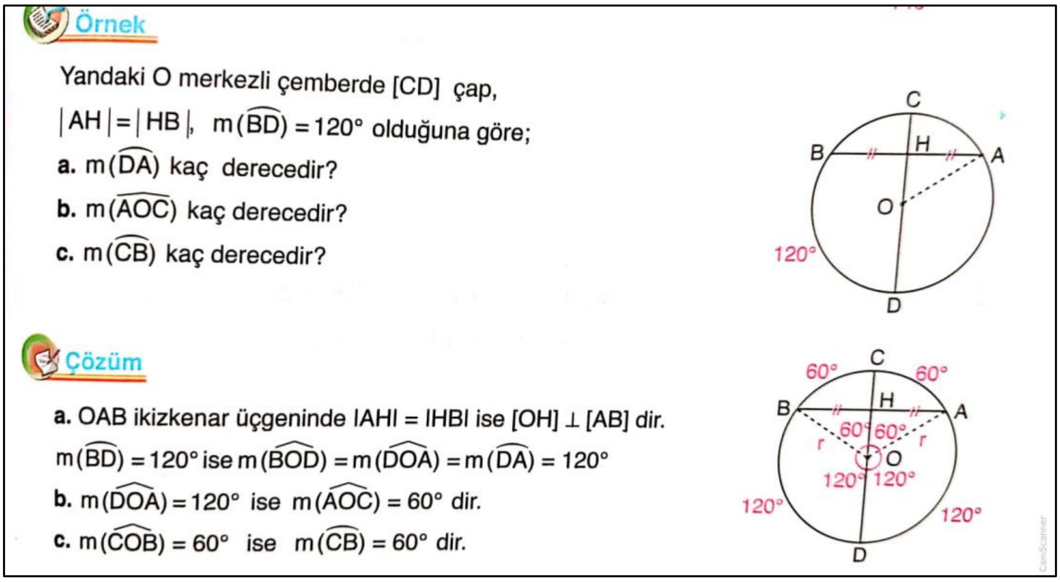
Şekilde; $[DE] // [BC]$, $|AE| = |EC|$, $|AD| = x - 1$, $|DE| = x + 4$ ve $|BC| = 3x + 1$ ise $|DB|$ nu bulalım.

Çözüm

$[DE] // [BC]$ ve $|AE| = |EC|$ olduğundan $|DE| = \frac{|BC|}{2}$ veya $|BC| = 2.|DE|$ dir. Buradan, $3x + 1 = 2(x + 4) \Rightarrow x = 7$ dir.

$|AD| = 7 - 1 = 6 \Rightarrow |DB| = |AD| = 6$ birimdir.

Figure 3. 13 Example of using triangles similarity in the high school book (Peken & Aydın, 2009, p.80)



Örnek

Yandaki O merkezli çemberde [CD] çap,

$|AH| = |HB|$, $m(\widehat{BD}) = 120°$ olduğuna göre;

a. $m(\widehat{DA})$ kaç derecedir?

b. $m(\widehat{AOC})$ kaç derecedir?

c. $m(\widehat{CB})$ kaç derecedir?

Çözüm

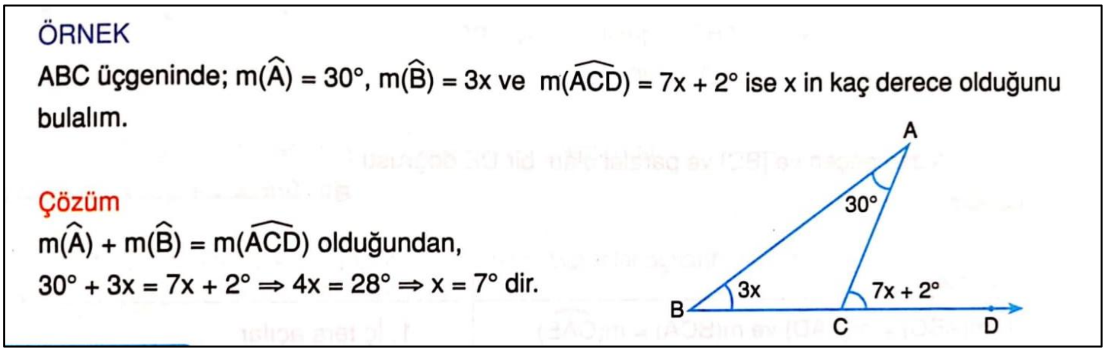
a. OAB ikizkenar üçgeninde $|AH| = |HB|$ ise $[OH] \perp [AB]$ dir.

$m(\widehat{BD}) = 120°$ ise $m(\widehat{BOD}) = m(\widehat{DOA}) = m(\widehat{DA}) = 120°$

b. $m(\widehat{DOA}) = 120°$ ise $m(\widehat{AOC}) = 60°$ dir.

c. $m(\widehat{COB}) = 60°$ ise $m(\widehat{CB}) = 60°$ dir.

Figure 3. 14 Example of using central angle and arc relationship in the high school book (Şayakdokuyan, 2012, p.189)



ÖRNEK

ABC üçgeninde; $m(\widehat{A}) = 30°$, $m(\widehat{B}) = 3x$ ve $m(\widehat{ACD}) = 7x + 2°$ ise x in kaç derece olduğunu bulalım.

Çözüm

$m(\widehat{A}) + m(\widehat{B}) = m(\widehat{ACD})$ olduğundan,

$30° + 3x = 7x + 2° \Rightarrow 4x = 28° \Rightarrow x = 7°$ dir.

Figure 3. 15 Example of using exterior angle theorem in the high school book (Peken & Aydın, 2009, p.55)

3.6.4 Evaluation of Creativity

Leikin (2009) discusses creativity in three phases as follows:

1. The creativity (Cr) of a specific solution is the multiplication of the solution's flexibility and originality score: $Cr_i = Flx_i \,.\, Or_i$
2. The total creativity on a multiple solutions task is the sum of creativity scores of each specific solution in the related task: $Cr = \sum_{i=1}^{n} Flx_i \,.\, Or_i$

3. The final creativity score (CR) of an individual in the related task is the multiplication of the total creativity score and the fluency score: $CR = n\ (\sum_{i=1}^{n} Flx_i \cdot Or_i)$

The third phase also includes the fluency scores because it is a fundamental division of creativity.

In this study, based on Leikin's framework (2009), each pre-service teacher's mathematical creativity was figured out for all three tasks separately. All solutions of pre-service teachers were analyzed in detail to have a deep understanding of the similarities and differences between the paper-pencil and GeoGebra environments in terms of fluency, flexibility, originality, and creativity. The analysis process would be presented in the Findings Part with more detailed justifications.

3.7 Quality of Research

In both qualitative and quantitative studies, the two main criteria are necessary to judge the quality of research: validity and reliability. Therefore, the investigation needs to give importance to these two main issues in each step of the study, such as collecting, analyzing, discussing data, and presenting the study (Merriam, 2009). In quantitative research, these issues are used as internal validity, external validity, and reliability. However, in qualitative research, the quality of study cannot be described directly by validity and reliability because the nature of quantitative and qualitative studies differs. To Lincoln and Guba (1985), in qualitative research, credibility, transferability, dependability, and confirmability substitute for internal validity, external validity, and reliability to ensure the quality of studies, respectively.

Credibility is the first issue to ensure the trustworthiness in qualitative research referring to internal validity. To Merriam and Tisdell (2016), there are five strategies to increase credibility: triangulation, member check, adequate engagement in data collection, researcher's position, and peer review. In the current study, triangulation, adequate engagement, and the researcher's position were used to maximize study credibility.

Triangulation was used to confirm credibility, which requires getting pieces of evidence from different sources (Creswell, 2007). Video recordings, pre-services' written works, and GGB files were used as multiple data sources, and these multiple data sources supported the analysis process of study. Also, To provide an adequate engagement strategy, the researcher observed five pre-service mathematics teachers three times each in addition to the pilot study (Merriam, 2009), which was sufficient time to observe the pre-service teacher's creativity in the given task. Also, it was enough for good observation since the pre-service teachers were familiar with the researcher and study environment before the current study. In qualitative research, the researcher's position is a significant issue for unprejudiced clarification of investigated phenomenon (Creswell, 2007). Therefore, it is needed to mention assumptions, biases, and dispositions of the study so that the reader understands the study better, ensuring credibility and consistency (Merriam, 2009). In this study, the role of the researcher was to determine the data collection instruments, observe the pre-service teachers during implementation, analyze the findings, and describe the discovery of the study. Also, the researcher did the pilot study before the actual research to diminish her biases.

To ensure trustworthiness in qualitative studies, transferability, is referring to external validity, is the second essential issue. According to Merriam and Tisdell (2016), while external validity interests the generalizability of current study findings to others, rich and thick representation is displayed to increase the transferability in the qualitative studies. In the recent research, to make the understanding and interpretation of readers clear, the participants, data collection period, data analysis period, and the findings were detailed, enabling them to transfer the results to similar situations.

In qualitative studies, the third issue of ensuring trustworthiness is dependability, referring to consistency or reliability. There are four approaches to increase the dependability of study: triangulation, peer review, researcher's position, and audit trail. The triangulation and researcher's position approaches were utilized to increase the credibility and dependability of the current study, as mentioned previously. Also, to ensure the audit trail, data collection, interpretation, and analysis period were described in detail.

Conformability, referring to objectivity, is the fourth one to ensure trustworthiness in qualitative research. The main concern of confirmability is to avoid and eliminate the researcher's subjective views and decisions (Fraenkel et al., 2012). Therefore, the whole process of research and the role of the researcher were discussed above to confirm the objectivity of this study.

3.8 Assumptions and Limitations

3.8.1 Assumptions

In the current study, the mathematical creativity of pre-service teachers was investigated in three multiple solution tasks concerning fluency, flexibility, and originality depending on Leikin's framework (2009), which means the mathematical creativity of pre-service teachers were addressed with specific dimensions in the limited number of multiple solution tasks. Still, it was assumed that the current study's findings would represent the mathematical creativity of pre-service teachers to a great extent.

All three tasks have previously been used as multiple solution tasks to observe mathematical creativity in various studies (Leikin et al., 2011, Leikin & Lev, 2013, and Leikin, 2011). Similarly, these tasks were used in the pilot study before implementing the actual study. It has been observed that these tasks are sufficient to create different solution types, which enable us to evaluate the mathematical creativity both in previous studies and in the pilot study. That is, the participant was able to create different solutions in a way that the mathematical creativity could be evaluated when these tasks were used in the pilot study. Thus, it was assumed that these tasks were sufficient to evaluate the pre-service teachers' mathematical creativity. Also, it was assumed that these tasks provided us with homogeneous environments to evaluate mathematical creativity that aroused in different environment (e.g. paper-pencil and GeoGebra) because all three tasks were geometry tasks convenient to evaluate mathematical creativity and they were used for common purposes in previous studies.

3.8.2 Limitations

In the present study, there were three critical limitations. The first one was participants and the selection of participants because they were not selected randomly. The participants were selected by purposive and convenient sampling procedure among the pre-service teachers who took the Exploring Geometry with Dynamic Geometry Applications course in the same university. Also, there were five cases, and each of the participants has unique characteristics, which means the findings might be invalid for other cases. These may be limitations because the sample of this research was not descriptive of all pre-service mathematics teachers in Turkey. Already, generalization of the findings of a particular case study with methodological justifications is the limitation of qualitative studies (Fraenkel et al., 2012). Fortunately, qualitative studies do not deal with generalizability (Merriam, 2009).

Another limitation may be the multiple solution tasks and the participants' solution processes. In this study, the instrument was limited to three geometry tasks. The participants were asked to solve three different geometry tasks in multiple solution ways, which might create some limitations for this study, such that the participants could make more or other solutions if the tasks were different. The qualification of multiple solutions tasks selected for this study in promoting mathematical creativity may be a limitation even though these tasks have been used to assess mathematical creativity in various studies (Leikin et al., 2011, Leikin & Lev, 2013, and Leikin, 2011). Moreover, the researcher asked them to think aloud during the solution process and recorded the whole process. Thinking loudly and being recorded in solving tasks may cause difficulty in focusing on solutions.

Lastly, the researcher carried out the whole process of this study like in most qualitative research. So, researcher bias may be another limitation of the study. The researcher made observations, transcribed the video recordings, and interpreted the findings of the study. She made an effort to be objective while observing and analyzing the video recordings and participants' solutions. Additionally, since the researcher's position has a significant role in removing the bias, the role of the researcher was clarified in detail at the role of the researcher part above.

3.9 Ethics

In the present study, the names and personal information of participants were kept confidential. Participants were coded by different names, so the researcher did not have any personal knowledge about the participants in the analysis process. The official permissions were received from the Human Research Ethics Committee to collect the data (see Appendix A). The researcher asked the pre-service teachers to participate in the research and obtained their permission orally.

CHAPTER 4

RESULTS

This chapter summarizes the findings of the study in three main sections and associated subsections. In the first section, the task 1 (T1) was analyzed concerning the fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of the pre-service mathematics teachers' solutions by using their solutions created in the paper-pencil environment and the GeoGebra environment. In the second section, the task 2 (T2) was analyzed concerning the fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of the pre-service mathematics teachers' solutions by using their solutions created in the paper-pencil environment and the GeoGebra environment. In the last section, the task 3 (T3) was analyzed concerning the fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of the pre-service mathematics teachers' solutions by using their solutions created in the paper-pencil environment and the GeoGebra environment.

4.1 Evaluation of Task 1

This section provides the solution types created by the pre-service mathematics teachers for task 1 with the mathematical ideas used in solutions and the analysis of task 1 with respect to fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of the solutions of each participant. The participants were asked to solve task 1 with more than one way in the paper-pencil environment within forty-five minutes.

Each pre-service teacher solved task 1 in more than one way and used various mathematical ideas in their solutions. All the solutions produced for task 1 were examined. Basic mathematical ideas in task 1 were the followings; drawing the line segments OC, AD, or CB, types of angles (e.g., corresponding angles, alternate interior angles, and alternate angles), exterior angle theorem, congruent triangles, corresponding sides or angles, inscribed angle on a diameter, median, height, and angle bisector in a special triangle, the relationship between inscribed angle and arc, the relationship between central angle and arc, properties of angles of a triangle, constructing new triangle, congruent chords with congruent angles or arcs, making an assumption, median to the hypotenuse of a right triangle, law of cosine, law of sine, the properties of a deltoid, Pythagorean Theorem, and Heron's formula.

All solutions were divided into parts based on the mathematical ideas used. There were twenty different solution types for task 1. Then, each solution type that was different from the others was identified with a different code to indicate which task the solution belongs. For instance, the first solution type of task 1 was shown as T1.S1 (the first solution type for task 1*)*, or the third solution type of task 1 was established as T1.S3 (the third solution type for task 1). There were twenty different solution types for task 1, and all solution types were shown in detail with the mathematical ideas used. Each figure in the subsequent pages (Figure 4.1 to Figure 4.20) summarizes the solution types and gives solutions of each participant provided these solutions.

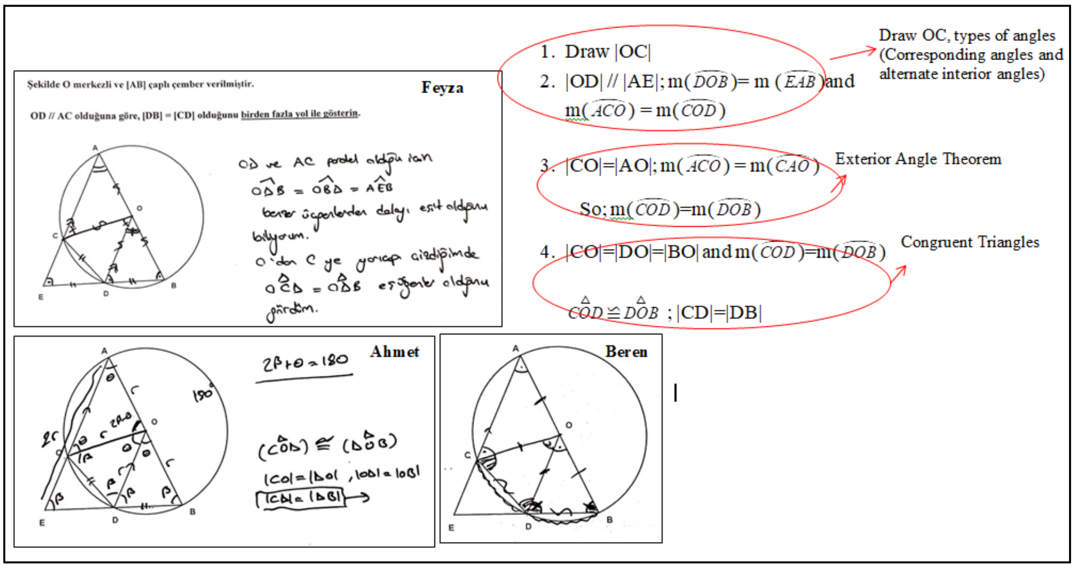


Figure 4. 1 T1.S1 (the first solution of task 1)

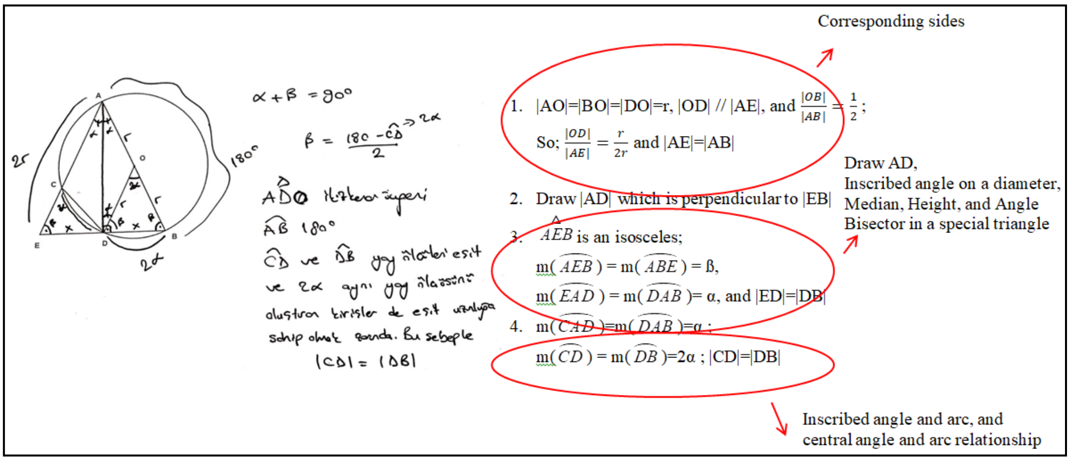


Figure 4. 2 T1.S2 (the second solution of task 1)

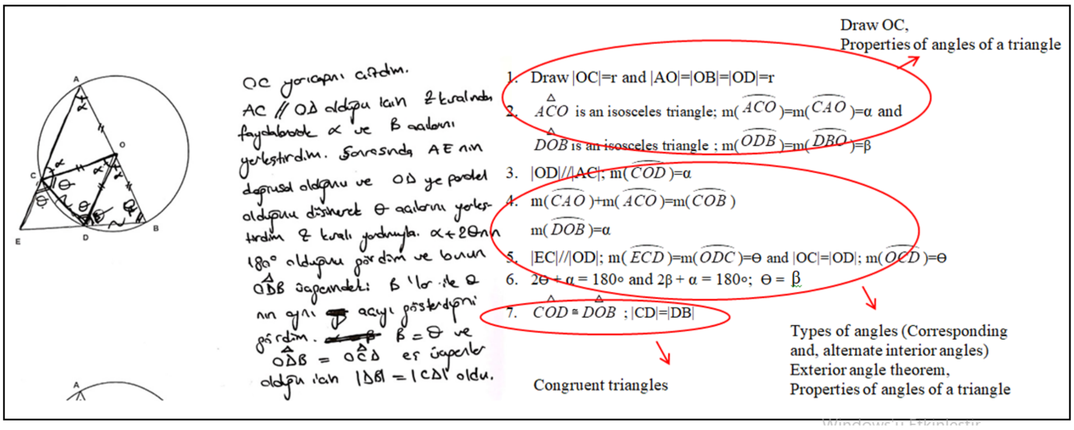


Figure 4. 3 T1.S3 (the third solution type of task 1)

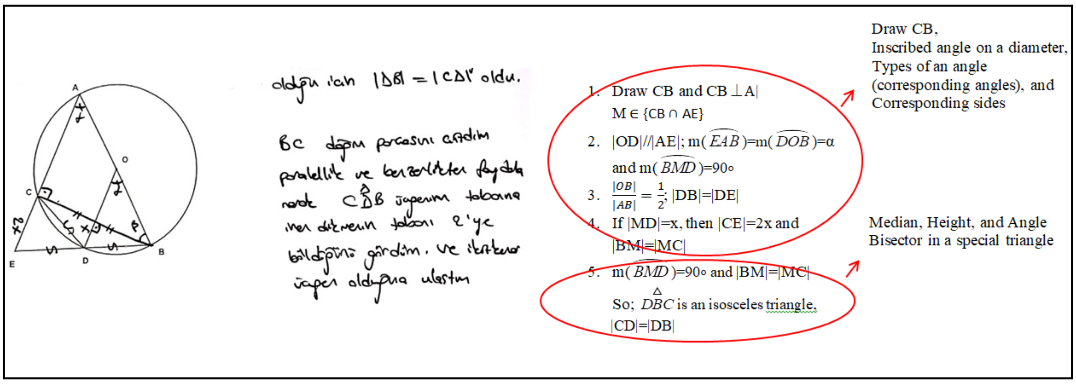


Figure 4. 4 T1.S4 (the fourth solution type of task 1)

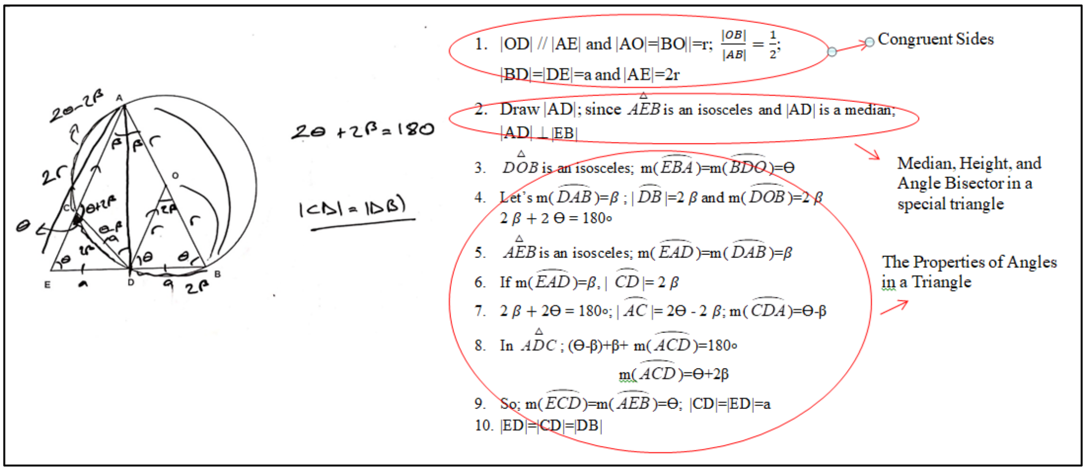


Figure 4. 5 T1.S5 (the fifth solution type of task 1)

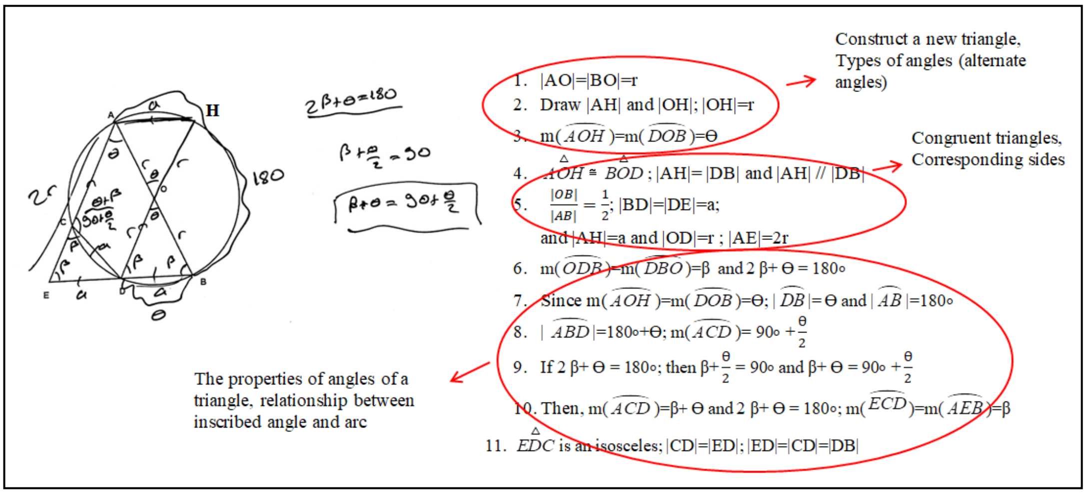


Figure 4. 6 T1.S6 (the sixth solution type for task 1)

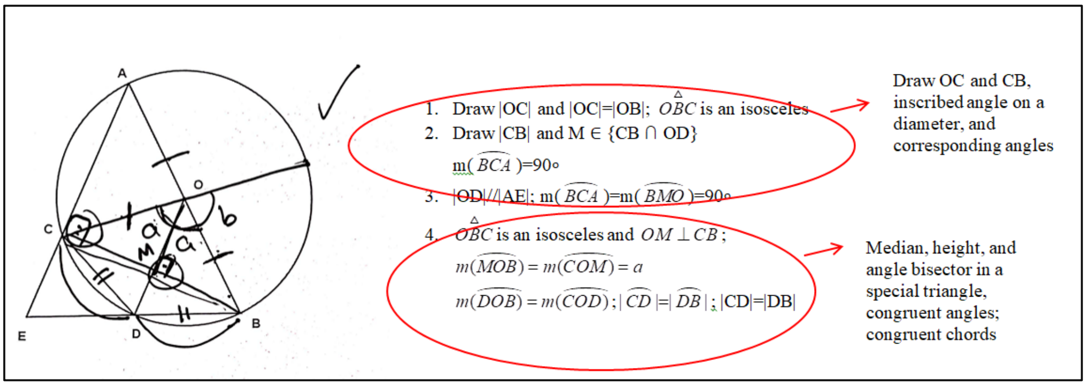


Figure 4. 7 T1.S7 (the seventh solution type of task 1)

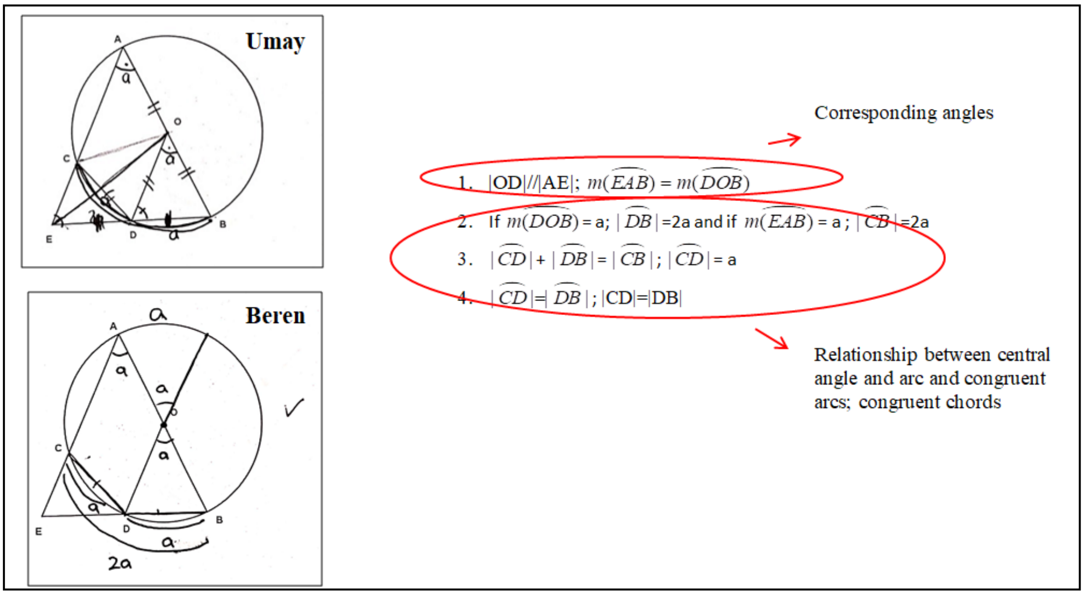


Figure 4. 8 T1.S8 (the eighth solution type of task 1)

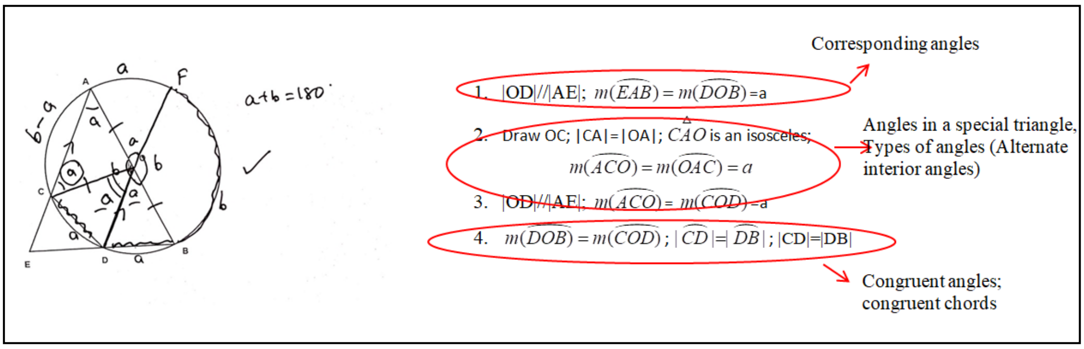


Figure 4. 9 T1.S9 (the ninth solution type of task 1)

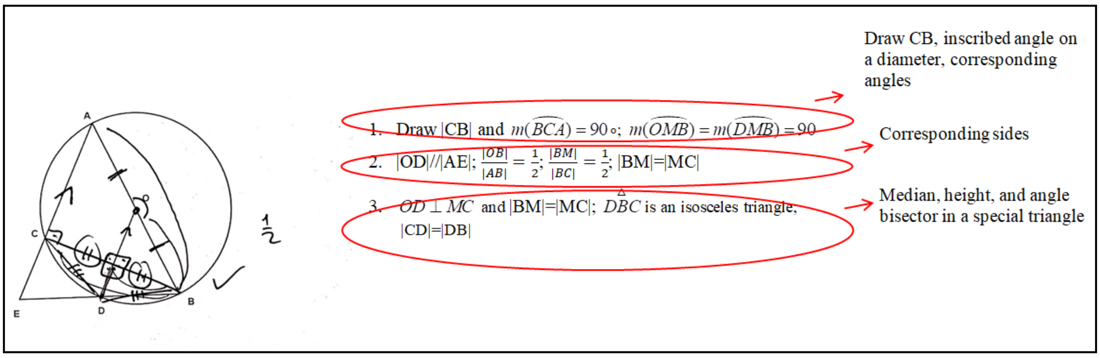


Figure 4. 10 T1.S10 (the tenth solution type of task 1)

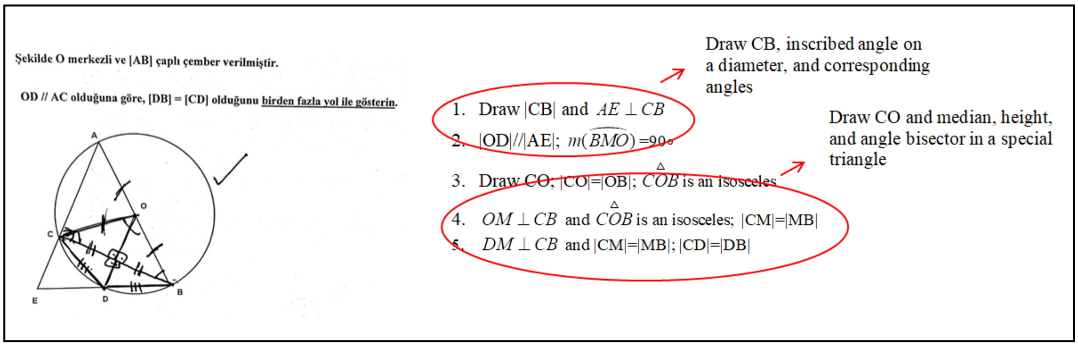


Figure 4. 11 T1.S11 (the eleventh solution type of task 1)

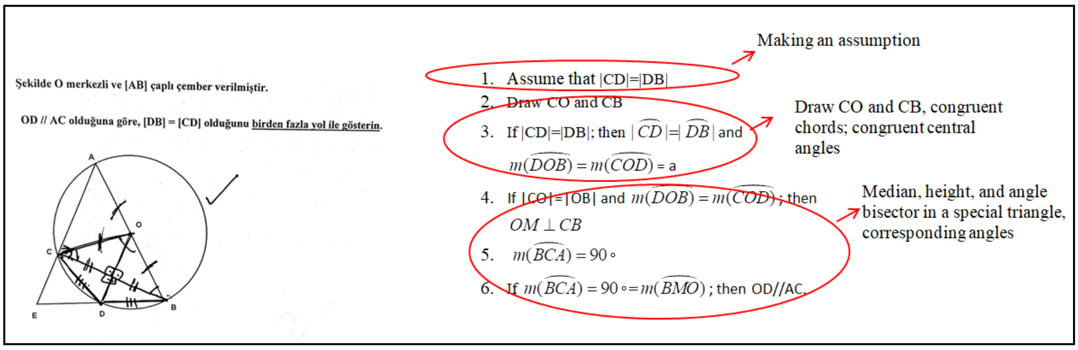


Figure 4. 12 T1.S12 (the twelfth solution type of task 1)

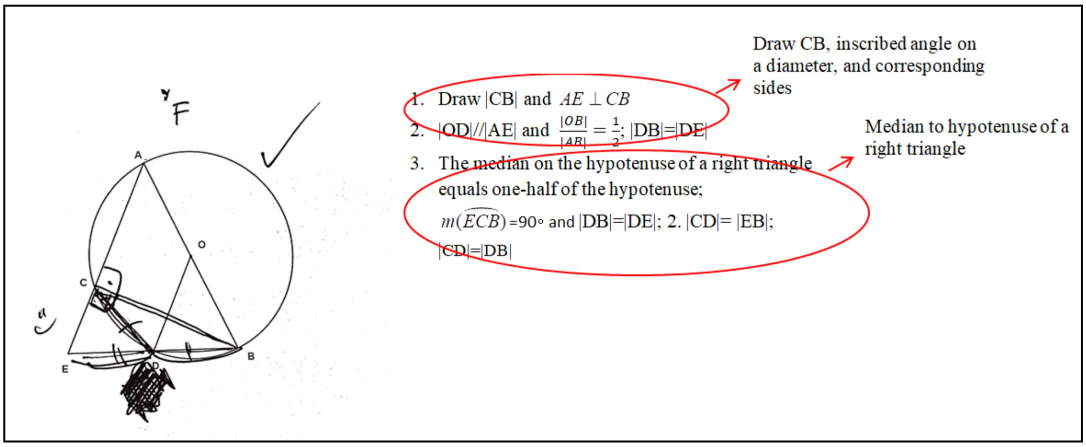


Figure 4. 13 T1.S13 (the thirteenth solution type of task 1)

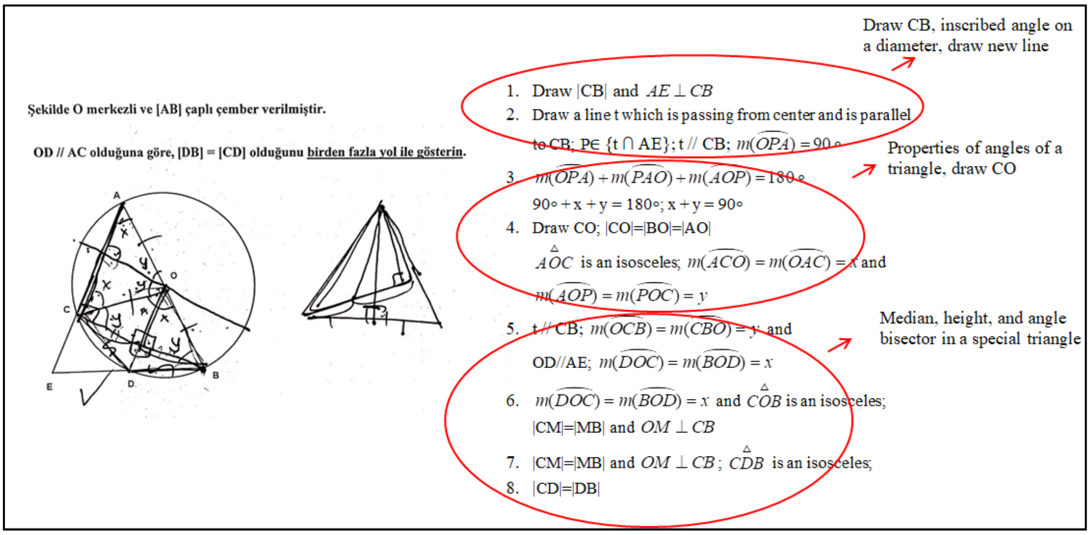


Figure 4. 14 T1.S14 (the fourteenth solution type of task 1)

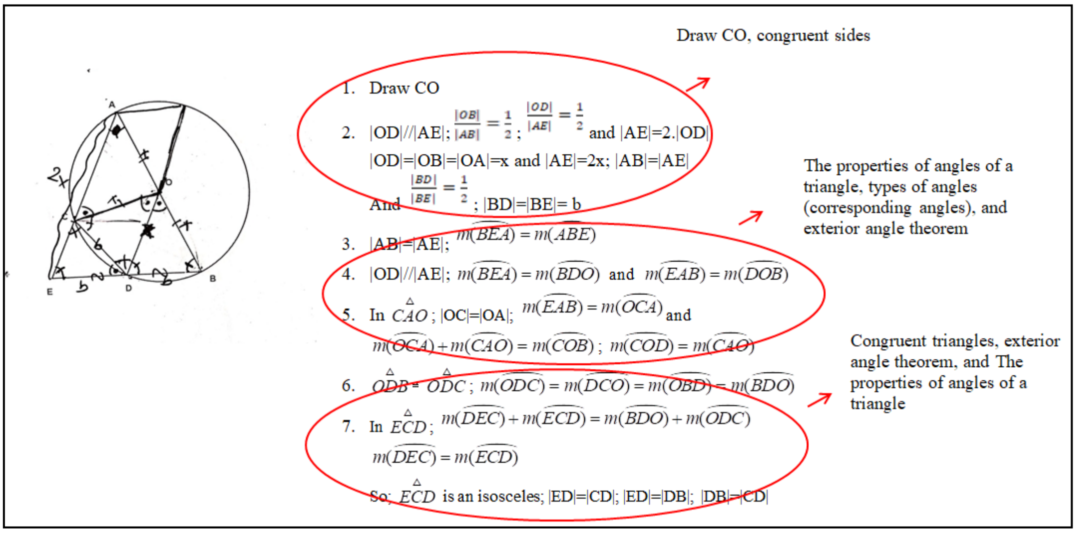


Figure 4. 15 T1.S15 (the fifteenth solution type of task 1)

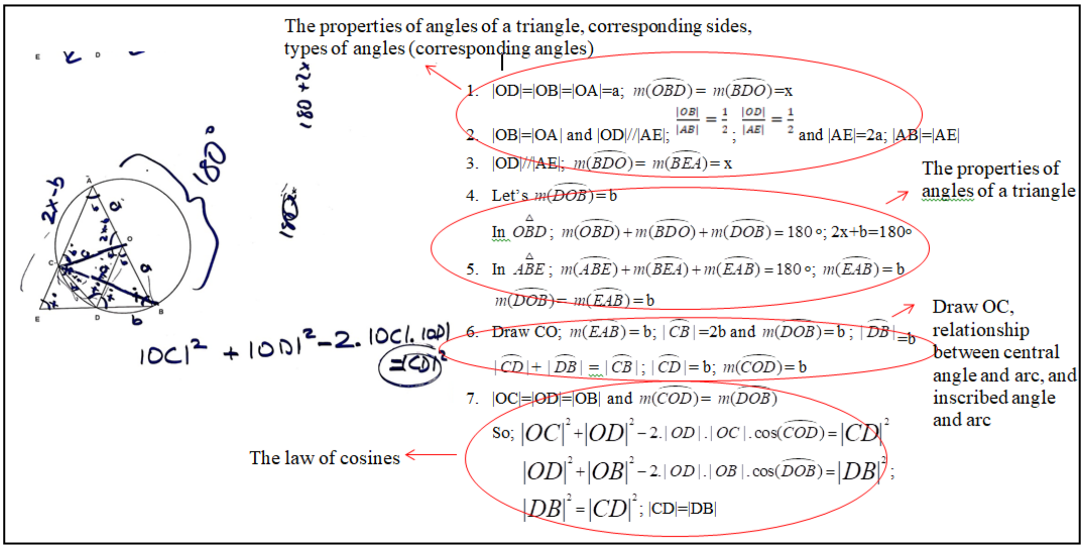


Figure 4. 16 T1.S16 (the sixteenth solution type of task 1)

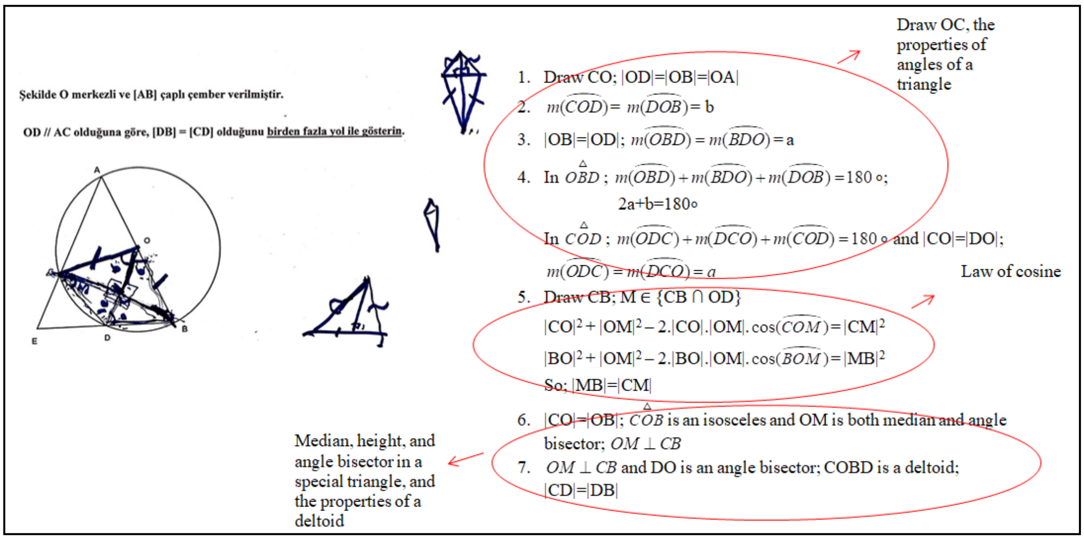


Figure 4. 17 T1.S17 (the seventeenth solution type of task 1)

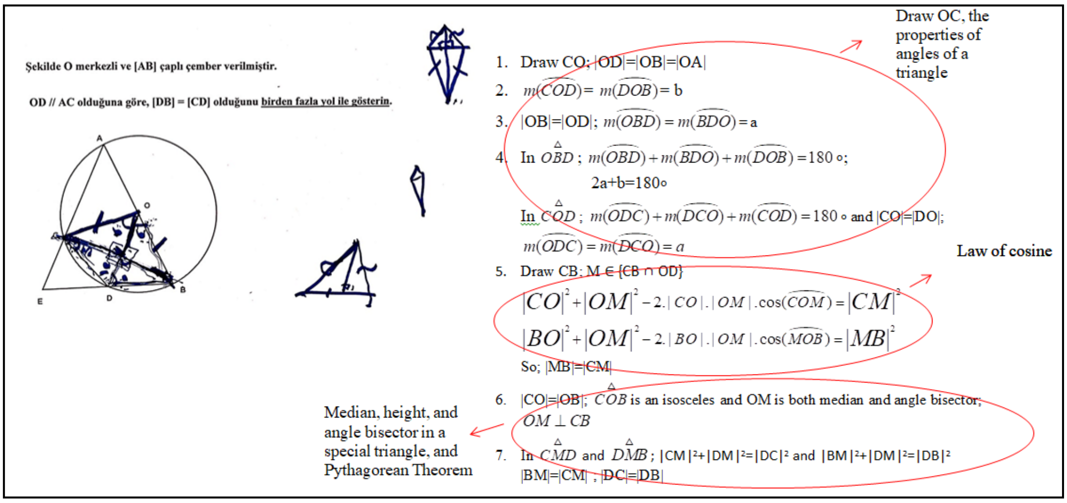


Figure 4. 18 T1.S18 (the eighteenth solution type of task 1)

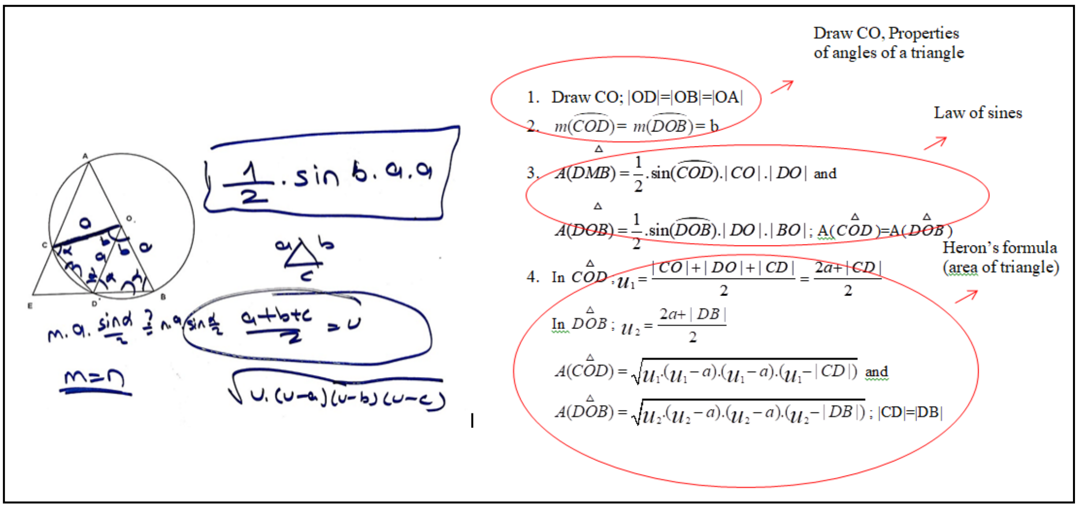


Figure 4. 19 T1.S19 (the nineteenth solution type of task 1)

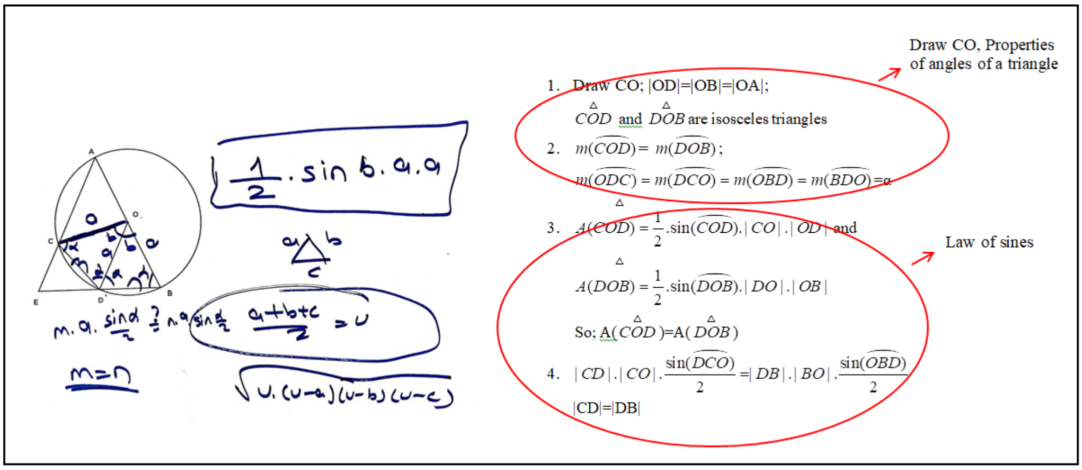


Figure 4. 20 T1.S20 (the twentieth solution type of task 1)

4.1.1 Evaluation of Fluency in Task 1

The fluencies (*N*) of the pre-service teachers were scored individually by viewing their solutions for task 1, based on the number of their appropriate solutions in the individual solution space. The pre-service teachers were allowed to solve task 1 with at least one solution in a paper-pencil environment. The fluency of each solution was assigned a unique code that displays the participant who came up with it. For example, the first participant, Feyza *(F)*, created four different solutions *(T1.S1, T1.S2, T1.S3, and T1.S4)* for task 1. To calculate her fluency score, each solution was scored 1 and shown as $N_{F.T1.S1}=1$, which denotes the fluency score of the first solution type of task 1, created by Feyza, is 1. Since 1 point was assigned for each solution; ($N_{F.T1.S1} = N_{F.T1.S2} = N_{F.T2.S3} = N_{F.T2.S4} = 1$), the total fluency of Feyza for the task 1 was $N_{F.T2} = 4$.

The second participant, Ahmet *(A)*, created three different solutions *(T1.S1, T1.S5, and T1.S6)* for task 1 in the paper-pencil environment. For each solution 1 point was assigned ($N_{A.T1.S1} = N_{A.T1.S5} = N_{A.T1.S6} = 1$). Therefore, the total fluency of Ahmet for task 1 was $N_{A.T1} = 3$.

The third participant, Umay *(U)*, created eight different solutions *(T1.S7, T1.S8, T1.S9, T1.S10, T1.S11, T1.S12, T1.S13, and T1.S14)* for task 1 in the paper-pencil environment. For each solution 1 point was assigned ($N_{U.T1.S7} = N_{U.T1.S8} = N_{U.T1.S9} = N_{U.T1.S10} = N_{U.T1.S11} = N_{U.T1.S12} = N_{U.T1.S13} = N_{U.T1.S14} = 1$). Therefore, the total fluency of Umay for the task 1 was 8 ($N_{U.T1}=8$).

The fourth participant, Beren *(B)*, created three different solutions *(T1.S1, T2.S14, T1.S8, and T1.S15)* for task 1 in the paper-pencil environment. For each solution 1 point was assigned ($N_{B.T1.S1} = N_{B.T1.S8} = N_{B.T1.S15} =1$). *T*herefore, the total fluency of Beren for task 1 was $N_{B.T1} = 3$.

The fifth participant, Kemal *(K)*, created five different solutions *(T1.S16, T1.S17, T1.S18, T1.S19, and T1.S20)* for task 1. For each solution 1 point was assigned ($N_{K.T1.S16} = N_{K.T1.S17} = N_{K.T1.S18} = N_{K.T1.S19} = N_{K.T1.S20} =1$). *T*herefore, the total fluency of Kemal for task 1 was $N_{K.T2} = 5$.

Table 4.1 summarizes the fluency scores of each participant for task 1.

Table 4. 1 Fluency scores of each participant for task 1

Participants	Fluency Scores
Feyza	4
Ahmet	3
Umay	8
Beren	3
Kemal	5

4.1.2 Evaluation of Flexibility in Task 1

As stated in the previous section, there were twenty different solution types for task 1. While each of them was different solution types, some of them included common mathematical ideas within. The flexibility *(Flx)* of each solution was evaluated individually based on the differentiation of mathematical ideas used in the solutions. The flexibility of each solution was assigned a unique code that displays the participant who provided the solution. For example, the first pre-service teacher, Feyza *(F)*, created four different solutions *(T1.S1, T1.S2, T1.S3, and T2.S4)* for task 1. Her first solution's flexibility, the first solution type (T1.S1), was displayed as $Flx_{F.T1.S1}$. Also, the total flexibility of Feyza for task 1 was shown as $Flx_{F.T1}$.

Feyza *(F)* created four different solutions for task 1 *(T1.S1, T1.S2, T1.S3, and T2.S4).* The first solution type (T1.S1) was the first appropriate solution of Feyza for task 1, which means its flexibility was scored 10. It was shown as $Flx_{F.T1.S1}=10$. The second solution of Feyza (T1.S2) was also scored 10 ($Flx_{F.T1.S2}=10$). As detailed in the solution types, the mathematical ideas used in the second solution type were utterly different from those used in the first solution type. The third solution of Feyza, which was the third solution type (T1.S3), was scored 0.1 ($Flx_{F.T1.S3}=0.1$). She began the third solution by drawing OC, then completing it with the congruent triangles ($\Delta(COD) \cong \Delta(DOB)$) as in the first solution way. Besides, the mathematical concepts used in the third type's remaining steps were also nearly the same as those used in the first solution type. The ideas used for both solution types were drawing OC, types of angles, exterior angle theorem, and congruent triangle. So, the third solution type was scored 0.1 due to the overlap between the first and third solution types. The last solution way of Feyza was the fourth solution type (T1.S4), which was scored 1 ($Flx_{F.T1.S4}=1$). Any of her previous solutions alone did not contain the mathematical concepts used in the fourth solution type. However, she brought together the mathematical concepts used in her earlier solutions within the fourth solution type. The fourth solution type was scored 1 for not completely repeating any previous solution, even though the mathematical ideas were used before in her different solution types. Therefore, the total flexibility of Feyza for task 1 was 21.1 ($Flx_{F.T1}=21.1$).

Ahmet *(A)* created three different solutions for task 1 *(T1.S1, T1.S5, and T1.S6).* The first solution type (T1.S1) was the first appropriate solution of Ahmet for task 1, which means its flexibility was scored 10 ($Flx_{A.T1.S1}=10$). The second solution of Ahmet *(T1.S5)* was also scored 10 ($Flx_{A.T1.S5}=10$). When the first and fifth solution types were examined, it was apparent that there was no common concept between the mathematical concepts used in the two solutions. For this reason, the flexibility of the fifth solution type was scored 10. However, in the sixth solution type, Ahmet used the ratio of corresponding sides (|BO|=|OA|) again, which was related to OD and AC's parallelism. He reused both the angle properties of a triangle and the relationship between angles and arcs as like in the fifth solution. Although he did not completely repeat the fifth solution type, the sixth solution type's flexibility was scored 1 ($Flx_{A.T1.S6}=1$) because the mathematical ideas used were the same as the previous solution type. Therefore, the total flexibility of Ahmet for task 1 was 21 ($Flx_{A.T1}=21$).

Umay *(U)* created eight different solutions for task 1 *(T1.S7, T1.S8, T1.S9, T1.S10, T1.S11, T1.S12, T1.S13, and T1.S14).* The seventh solution type *(T1.S7)* was the first appropriate solution of Umay for the task 1, which means its flexibility was scored 10, which was shown as $Flx_{U.T1.S7}=10$. The second solution of Umay was the eighth solution type *(T1.S8).* In the eighth solution type, Umay displayed some corresponding angles using the parallelism of the line segment OD and the line segment AC, as she did in the previous solution. In the last step of the solution, she concluded that the CD and BD chords should also have equal lengths since the lengths of the CD and BD arcs are identical, used in her previous solution. Although she did not wholly repeat the fifth solution type, the eighth solution type's flexibility was scored 1 ($Flx_{U.T1.S8}=1$) because the mathematical concept used was the same as the previous solution type. Then, she produced the ninth solution type *(T1.S9).* The first step (corresponding angles (m∠(EAB = m∠(DOB)) and last steps of the ninth solution type were precisely the same as the eighth solution type. The ninth solution type was scored 1 ($Flx_{U.T1.S9}=1$) because the mathematical concepts used in between the first and last steps were not used in her previous solutions. Another appropriate solution was the tenth solution type (T1.S10). The tenth solution type was parallel with the seventh type of solution. Only one of the mathematical concepts she used in the tenth solution type was different from the ideas used in the seventh solution type. She used the ratio of corresponding sides (|BO|=|OA|) again, which was related to the parallelism between OD and AC, for the first time in the tenth solution type. Despite commonly used mathematical concepts in both seventh and ninth solution types, it was scored 1 ($Flx_{U.T1.S10}=1$) because the tenth solution type was not the same as the previous solution types. Moreover, all mathematical concepts used in the eleventh solution type (T1.S11) were used with a different order in the seventh solution type, which means she repeated the previous solution way in this solution type. The eleventh solution type was scored 0.1 ($Flx_{U.T1.S11}=0.1$) because of the lack of any different mathematical concepts. Then, she formed the twelfth solution type (T1.S12), which was scored 1 ($Flx_{U.T1.S12}=1$). In the twelfth solution type, Umay attempted to demonstrate that the information given (OD//AC) is accurate by assuming that the desired situation (|DB|=|CD|) is correct. The ideas she used in the twelfth solution type were not precisely the same as any solution. However, they have previously used ideas except for "making an assumption." Similarly, the mathematical concepts used in the thirteenth solution type (drawing CB, inscribed angle on a diameter, corresponding sides) were previously used except for "median to the hypotenuse of a right triangle." Yet, it was not precisely the same as previous solutions. So, the thirteenth solution type (T1.S13) was scored 1 ($Flx_{U.T1.S13}=1$). Her last solution was the fourteenth solution type (T1.S14), which was scored 1 ($Flx_{U.T1.S14}=1$). The mathematical ideas she used in the fourteenth solution type (e.g., drawing CB, inscribed angle on a diameter, drawing CO; and median, height, and angle bisector in a special triangle) were among the concepts she frequently used in the previous solutions. In addition to these concepts, Umay used the angle properties of a triangle and drawing a new line segment was passing from a center and parallel to CB. Thus, the fourteenth solution type was not utterly the same with any previous solution. Therefore, the total flexibility of Umay for task 1 was 16.1 ($Flx_{U.T1}=16.1$).

Beren *(B)* created three different solutions for task 1 *(T1.S1, T1.S8, and T1.S15).* The first solution type *(T1.S1)* was the first appropriate solution of Beren for task 1, which means its flexibility was scored 10 ($Flx_{B.T1.S1}=10$). The second solution of

Beren was the eighth solution type *(T1.S8).* In the eighth solution type, she displayed some corresponding angles using the parallelism of the line segment OD and the line segment AC, as she did in the previous solution. Although the continuing steps of solution types were different, the mathematical ideas that the two solutions were based on were identical. The eighth solution type was scored 1 ($Flx_{B.T1.S8}=1$), as it did not completely differ from her previous solution. Lastly, she created the fifteenth solution type (T1.S15), which was also scored 1 ($Flx_{B.T1.S15}=1$). The fifteenth solution type included all the concepts used in the first solution type and some mathematical concepts together with them, such as congruent sides based on OD//AC and |BO| = |OA| and the properties of angles of a triangle. However, the fifteenth solution type was not completely the same as the first solution type. Therefore, the total flexibility of Beren for task 1 was 12 ($Flx_{B.T1}=12$).

Kemal *(K)* created five different solutions for task 1 *(T1.S16, T1.S17, T1.S18, T1.S19, and T1.S20).* The sixteenth solution type *(T1.S16)* was the first appropriate solution of Kemal for task 1, which means the flexibility of it was scored 10 ($Flx_{K.T1.S16}=10$). The second solution of Kemal was the seventeenth solution type *(T1.S17).* Some of the concepts that formed the seventeenth solution type were used in the sixteenth solution type. In addition to these, Kemal used the median, height, and angle bisector in a special triangle and the properties of a deltoid. Despite its similarities with the previous solution type, the seventeenth solution type was scored 1 ($Flx_{K.T1.S17}=1$) due to the different mathematical concepts. Moreover, until the last step of the eighteenth solution type, Kemal used the same mathematical concepts he used in the seventeenth solution type. The only change he made to the eighteenth solution type was to use the Pythagorean Theorem instead of using a deltoid's properties. It was scored 1 ($Flx_{K.T1.S18}=1$) because of using a different mathematical concept in the last step of the solution. In the nineteenth solution type (T1.S19), as in the seventeenth and eighteenth solution type, he drew the line segment OC and used the properties of angles of the triangle generated. In that solution type, unlike the previous ones, he used the Law of Sine and Heron's Area Formula instead of the Law of Cosine. Yet, using the Law of Sine instead of the Law of Cosine does not mean that the nineteenth solution type was completely different. So, the flexibility of the nineteenth solution type was scored 1 ($Flx_{K.T1.S19}=1$). Lastly, he created the twentieth solution type (T1.S20), which was scored 0.1 ($Flx_{K.T1.S20}=0.1$) because it was part of the nineteenth solution type. He completed the twentieth solution type without using Heron's Area Formula, which he used in the nineteenth solution. Without making changes in the other steps, he repeated the previous solution way in this solution type. Therefore, the total flexibility score of Kemal was 13.1 ($Flx_{K.T1}=13.1$).

The flexibility score of each participant in the task 1 are shown in Table 4.2.

Table 4. 2 Flexibility score of each participants for task 1

Participants	Flexibility Scores
Feyza	21.1
Ahmet	21
Umay	16.1
Beren	12
Kemal	13.1

4.1.3 Evaluation of Originality in Task 1

There were twenty different solution types in task 1. While some of the solutions were insight-based or unconventional solutions, some were model-based or partly-unconventional solutions. Some of them were algorithm-based or conventional solutions. The originality *(Or)* of each solution type was scored depend on its conventionality and insight level. Each participant's total originality score on task 1 was the sum of each solution's originality score. Moreover, each solution's originality was assigned a unique code that displays whom the solution belongs to.

For instance, the second participant, Ahmet *(A),* produced three different solutions *(T1.S1, T1.S5, and T1.S6)* for task 1. The originalities of these solutions were displayed as $Or_{A.T1.S1}$ (the originality of the first solution type that Ahmet produced for task 1), $Or_{A.T1.S5}$ (the originality of the fifth solution type that Ahmet created for task 1*)* $Or_{A.T1.S6}$ (the originality of the sixth solution type that Ahmet produced for the task 1). Hence, the total originality of Ahmet for task 1 was shown as $Or_{A.T1}$ (the originality of Ahmet for task 1).

Feyza *(F)* created four different solutions *(T1.S1, T1.S2, T1.S3, and T2.S4)* for task 1. The first solution type *(T1.S1)* was her first appropriate solution. In the first solution type, Feyza firstly made the information hidden in task 1 apparent. For instance, she displayed the corresponding angles m∠(DOB) = m∠(EAB), alternate interior angles m∠(ACO) = m∠(COD) within the figure, and congruent triangles Δ(COD) ≘Δ (DOB). Also, she used the exterior angle theorem, which was based on the algebraic way. In the first solution type, she did not adjust the given figure's structure, or there were no insight-based steps within the solution. For these reasons, the first solution type was evaluated as an algorithm-based conventional solution, and its originality was scored 0.1 ($Or_{F.T1.S1}=0.1$). Similarly, she created the second solution type (T1.S2), which was scored 0.1 ($Or_{F.T1.S2}=0.1$). At the beginning of the second solution type, she used the ratio of the corresponding sides (|BO|=|OA|), which was related to OD and AC's parallelism. Using corresponding sides is the way which was commonly taught and recommended in mathematics classes to solve geometry tasks. The other ideas (inscribed angle on a diameter, median, height, and angle bisector in a special triangle, and the relationship between angles on a circle and arcs) used in the rest of the second solution type were used the same way as they were taught. She did not make any explanation about the bases of these concepts. For example, she said that "Since triangle AEB is isosceles; the length of median, height, and angle bisector has to be equal". Such discourses contained no evidence to evaluate the second solution type as unconventional or partly-unconventional. Thus, the second solution type was a conventional solution way, which means its originality score was 0.1 ($Or_{F.T1.S2}=0.1$). In the third solution type, she firstly displayed the corresponding angles (m∠(ACO) = m∠(CAO) = α and m∠(ODB) = m∠(DBO) = β) with the help of isosceles triangles $\Delta(ACO)$ and $\Delta(DOB)$. Then, she formed two algebraic equations (2ϴ + α = 180∘ and 2β + α = 180∘) with the use of corresponding angles, alternate interior angles, and exterior angle theorem; and concluded the solution with congruent triangles $\Delta(COD) \cong \Delta(DOB)$. As seen in the third solution type, Feyza used the algebraic concept to form a congruent triangle and did not change the figure's given representation. Thus, the originality of the third solution type was scored 0.1 ($Or_{F.T1.S3}=0.1$). Lastly, in the fourth solution type, she used the mathematical concepts as taught in the classes. For example, she claimed that "the angle at the circumference is 90°" without giving any reason. Also, she said, as in the second solution type, "since triangle AEB is isosceles; the length of median, height, and angle bisector has to be equal." Since she did not show any reason for her discourses, the fourth solution type was evaluated as conventional and scored 0.1 ($Or_{F.T1.S4}=0.1$). It was also noted that all of the mathematical concepts used in Feyza's solutions were commonly used by the other participants. So, none of them was evaluated as partly-conventional or unconventional because they were not rarely used concepts among the participants. Therefore, the total originality score of Feyza for the task 1 was 0.4 ($Or_{F.T1}=0.4$).

Ahmet *(A)* created three different solutions *(T1.S1, T1.S5, and T1.S6)* for task 1. The first solution type *(T1.S1)* was his first appropriate solution. In the first solution type, Ahmet followed the same steps with Feyza. Firstly, he made the information hidden in task 1 apparent. For instance, he displayed the corresponding angles m∠(DOB) = m∠(EAB), alternate interior angles m∠(ACO) = m∠(COD) within the figure, and congruent triangles COD and DOB. Also, he used "the exterior angle theorem" which was based on algebraic operations. In the first solution type, he did not adjust the structure of the given figure, or there was no insight-based step within the solution. For these reasons, the first solution type was evaluated as an algorithm-based conventional solution, and its originality was scored 0.1 ($Or_{A.T1.S1}=0.1$). Secondly, Ahmet created the fifth solution type (T1.S5) for task 1. He used the ratio of corresponding sides (|BO|=|OA|), which was related to OD and AC's parallelism. "Using corresponding sides" is the way which was commonly taught and recommended in mathematics classes to solve geometry tasks. In the same way, he used the property of median, height, and angle bisector in a special triangle as it was taught. Namely, he asserted, "Since triangle AEB is isosceles; the length of median, height, and angle bisector has to be equal." It was also obviously seen that the last step of the fifth solution type was based on algebraic operations. Thus, the fifth solution type was classified in the algorithm-based conventional solution category and scored 0.1 ($Or_{A.T1.S5}=0.1$). The last solution of Ahmet was the sixth solution type (T1.S6). In the sixth solution way, he used similar mathematical concepts he used in his previous solutions, such as types of angles, congruent triangles, corresponding sides, and the properties of angles of a triangle, which were common within the curriculum. Also, the last step of the sixth solution way was based on algebraic operations. Hence, the sixth solution type was an algorithm-based conventional solution and was scored 0.1 ($Or_{A.T1.S6}=0.1$). Therefore, the total originality score of Ahmet for task 1 was 0.3 ($Or_{A.T1}=0.3$).

Umay *(U)* created seven different solutions *(T1.S7, T1.S8, T1.S9, T1.S10, T1.S11, T1.S12, T1.S13, and T1.S14)* for task 1. The seventh solution type *(T1.S7)* was her first appropriate solution. In the seventh solution type, Umay formed an inscribed angle on the diameter ($\angle$(BCA)) by drawing the line segment CB and defined the angle as 90◦ because she claimed: "the inscribed angle on a diameter has to be 90◦." Also, she used the median, height, and angle bisector property in a special triangle as it was taught. Namely, she claimed, "Since the triangle OBC is an isosceles and $\overleftrightarrow{OM} \perp \overleftrightarrow{CB}$, the measure of an angle $\angle$(DOB) and $\angle$(COD) are equal." The last mathematical concept he used was that if two central angles in a circle are congruent, their intercepted arcs and chords are congruent. While using these concepts, Umay did not explain the reasons or make a solution that requires an arrangement in the given figure's representation. She even used the concepts taught in the lesson. So, the seventh solution type was scored 0.1 ($Or_{U.T1.S7}=0.1$). Similarly, the eighth, ninth, tenth, eleventh, thirteenth, and fourteenth solution types were scored 0.1 ($Or_{U.T1.S8} = Or_{U.T1.S9} = Or_{U.T1.S10} = Or_{U.T1.S11} = Or_{U.T1.S13} = Or_{U.T1.S14} = 0.1$). Namely, Umay used these solution types taught and recommended in the mathematics curriculum for solving geometry tasks; and were frequently used by the other participants. For example, Umay used the being equal of median, height, and angle bisector in a special triangle, which was covered in the mathematics curriculum and commonly used among the participants, within three solution types *(T1.10, T1.S11, and T1.S14)*. Besides, she displayed the congruent sides, congruent angles, and alternate interior angles *(T1.S8, T1.S9, T1.S10, T1.S11, and T1.S13)*, which were based on the parallelism between the line OD and AC. Showing congruent figures also was one of the mathematical concepts included in the mathematics curriculum. On the other hand, the concept of median to the hypotenuse of a right triangle used in the thirteenth solution type was not used by other participants. Still, this concept was also one of the curriculum concepts and frequently recommended to solve geometry tasks. Within the rest of the concepts used in these six solution types, there were no mathematical concepts used apart from those taught in the curriculum. Even the fourteenth solution type included a step based on algebraic operations. Hence, these six solution types were evaluated as conventional solutions, while the fourteenth solution type was an algorithm based conventional solution. On the other hand, Umay made an assumption in the twelfth solution type (T1.S12), she assumed that |DB|=|CD|. She showed that her assumption (|DB|=|CD|) supplied the OD//AC given by using the mathematical concepts she used in her other solutions. Although she used the similar mathematical concepts (drawing CO and CB, relationship between congruent chords and intersected angles, median, height, and angle bisector in a special triangle, and corresponding angles) with her previous solutions, "making assumption" was not used for the task 1 by any participant. Also, making assumption was not in the scope of the mathematical curriculum to solve the given geometry curriculum. Thus, the originality of the twelfth solution type was scored 10 ($Or_{U.T1.S12}=10$). Therefore, the total originality score of Umay was scored 10.7 for the task 1 ($Or_{U.T1}=10.7$).

Beren *(B)* created three different solutions *(T1.S1, T1.S8, and T1.S15)* for task 1. The first solution type *(T1.S1)* was her first appropriate solution. In the first solution type, Beren followed the same steps with Feyza and Ahmet. Firstly, she made the information hidden in task 1 apparent. For instance, Beren displayed the corresponding angles ∠(DOB) = m ∠(EAB) alternate interior angles m∠(ACO) = m∠(COD) within the figure, and congruent triangles Δ(COD) and Δ(DOB). Also, she used the exterior angle theorem, which was based on an algebraic way. In the first solution type, she did not adjust the given figure's structure, or there were no insight-based steps within the solution. For these reasons, the first solution type was evaluated as an algorithm-based conventional solution, and its originality was scored 0.1 ($Or_{B.T1.S1}=0.1$). Then, Umay created the eighth solution type (T1.S8) as her second solution. In the second solution type, Beren followed the same steps with Ahmet. She displayed the corresponding angles that arose from the parallelism between OD and AC. Then, she stated that the arcs intersected with the CD and DB had an equal length, which means "if two arcs are congruent, then the corresponding chords are congruent." The eighth solution type did not include any mathematical concepts except for the taught in the curriculum or rarely used among the participants. Thus, the originality of the eighth solution type was 0.1 ($Or_{B.T1.S8}=0.1$). Lastly, she created the fifteenth solution type (T1.S15). In the fifteenth solution type, while she used the mathematical concepts such as congruent sides (e.g., |AB|=|AE| and |BD|=|BE|), corresponding angles ∠(BEA) = m ∠(BDO), and congruent triangles Δ(ODB) and Δ(ODC), she made algebraic operations by using the exterior angle theorem and the properties of the angles of a triangle. As in her previous solutions, the fifteenth solution type included commonly taught mathematical ideas based on algebraic operations. Hence, the originality of the fifteenth solution type was score 0.1 ($Or_{B.T1.S15}=0.1$). Therefore, the total originality score of Beren for task 1 was 0.3 ($Or_{B.T1}=0.3$)

Kemal *(K) formed* five different solutions *(T1.S16, T1.S17, T1.S18, T1.S19, and T1.S20)* for task 1. When the five solutions developed by Kemal were viewed, it was apparent that his all solution ways were dependent on algebraic operations and formulas. He used mathematical formulas such as the law of cosine, the law of sine, the Heron's formula, and the Pythagorean Theorem. Besides, he also used the mathematical concepts commonly were within the curriculum, such as the properties of angles of a triangle, corresponding sides based on parallelism, the relationship between arcs and angles, and median, height, and angle bisector in a special triangle. Thus, sixteenth, eighteenth, nineteenth, and twentieth solution types were evaluated as algorithm-based conventional solutions, and their originality score were 0.1 ($Or_{K.T1.S16}= Or_{K.T1.S17}= Or_{K.T1.S19}= Or_{K.T1.S20}=0.1$).

Table 4. 3 Originality score of each participant for task 1

Participants	Originality Scores
Feyza	0.4
Ahmet	0.3
Umay	10.7
Beren	0.3
Kemal	1.4

However, the seventeenth solution type's originality was scored 1 ($Or_{K.T1.S17}=1$), which means it was evaluated as partly-conventional because Kemal demonstrated the deltoid COBD in the last step of the seventeenth solution type. The properties of a deltoid were not used by any participant except for Kemal. Even the participants did not make a solution using a geometric figure apart from the circle and triangle. Hence, using the properties of a deltoid was one of the rarely used mathematical concepts, and the originality of the seventeenth solution was scored 1

($Or_{K.T1.S17}=1$). Therefore, the total originality score of the Kemal for task 1 was 1.4 ($Or_{K.T1}=1.4$).

The originality scores of each participant for task 1 are shown in table 4.3.

4.1.4 Evaluation of Creativity in Task 1

There were twenty different solution types for task 1, which were evaluated concerning their fluency, flexibility, and originality, respectively. The creativity of each solution (Cr) and each participant's total creativity (CR) for task 1 were scored depending on the scores of fluency, flexibility, and originality of the solutions. A specific *creativity score of each solution* (Cr_i) was the product of the solution's flexibility and originality score ($Cr_i = Flx_i . Or_i$). The *total creativity score of all solutions* on task 1 was the sum of the creativity scores of each solution ($Cr = \sum_1^n (Flx_i . Or_i)$). Therefore, each participant's final creativity score was calculated as the product of the fluency score and the total creativity score (CR = n ($\sum_{i=1}^n Flx_i . Or_i$)).

Feyza *(F)* produced four different solutions in task 1 *(T1.S1, T1.S2, T1.S3, and T2.S4)*. The first solution type's flexibility score was $Flx_{F.T1.S1} = 10$, and the originality score of the first solution type was $Or_{F.T1.S1} = 0.1$. The creativity score of her solution ($Cr_{F.T1.S1}$), which was the first solution type of task 1, was scored by multiplying its flexibility and originality score ($Cr_{F.T1.S1} = Flx_{F.T1.S1} \times Or_{F.T1.S1}$); which means the creativity score of the first solution type for the task 1 was 1 ($Cr_{F.T1.S1} = 1$). Similarly, the creativity score of the second, third, and fourth solution types of task 1 ($Cr_{F.T1.S2}$, $Cr_{F.T1.S3}$, *and* $Cr_{F.T1.S4}$) was scored as the product of their flexibility and originality scores. The creativity of the second solution type was scored 1 ($Cr_{F.T1.S2} = 1$), the creativity of the third solution type was scored 0.01 ($Cr_{F.T1.S3}= 0.01$,) and the creativity of the fourth solution type was scored 0.1($Cr_{F.T1.S4}= 0.1$). Thus, the total creativity of Feyza's solutions for task 1 was scored 2.11 ($Cr_{F.T1} =2.11$), which was scored as the sum of the creativity scores on each solution. Therefore, the final creativity score of Feyza for task 1 was the product of her fluency score *($N_{F.T1}$=4)*, and her total creativity score *($Cr_{F.T1}$ = 2.11)*, was scored 8.44 *($CR_{F.T1}$ = 8.44).* The scoring creativity of Feyza is shown in the Table 4.4.

Table 4. 4 Scoring the creativity of Feyza for task 1.

	Fluency	Flexibility	Originality	Creativity
T1.S1	1	10	0.1	1
T1.S2	1	10	0.1	1
T1.S3	1	0.1	0.1	0.01
T1.S4	1	1	0.1	0.1
Total	4	21.1	0.4	**2.11**
Final creativity of Feyza for task 1				**8.44**

Ahmet *(A)* produced three different solutions for task 1 *(T1.S1, T1.S5, and T1.S6)*. The first solution type's flexibility score was *$Flx_{A.T1.S1}$ = 10*, and the originality score of the first solution type was *$Or_{A.T1.S1}$ = 0.1*. The creativity score of his solution (*$Cr_{A.T1.S1}$)*, which was the first solution type of task 1, was scored by multiplying its flexibility and originality score *($Cr_{A.T1.S1}$ = $Flx_{A.T1.S1}$ x $Or_{A.T1.S1}$);* which means the creativity score of the first solution type for task 1 was scored 1 *($Cr_{A.T1.S1}$ = 1).* Similarly, the creativity score of the fifth and sixth solution types (*$Cr_{A.T1.S5}$ and $Cr_{A.T1.S6}$)* were scored as the product of their flexibility and originality scores. The fifth solution type's creativity was scored 1 (*$Cr_{A.T1.S5}$ = 1),* and the creativity of the sixth solution type was scored 0.1(*$Cr_{A.T1.S6}$= 0.1).* Thus, the total creativity of Ahmet's solutions for task 1 was scored 2.1 *($Cr_{A.T1}$ =2.1)*, which was scored as the sum of the creativity scores of each solution. Therefore, the final creativity score of Ahmet for task 1, was the product of his fluency score *($N_{A.T1}$=3)*, and his total creativity score *($Cr_{A.T1}$ = 2.1)* was scored 6.3 *($CR_{A.T1}$ = 6.3).* (seen in the table 4.5).

Table 4. 5 Scoring the creativity of Ahmet for task 1.

	Fluency	Flexibility	Originality	Creativity
T1.S1	1	10	0.1	1
T1.S5	1	10	0.1	1
T1.S6	1	1	0.1	0.1
Total	3	21	0.3	**2.1**
Final creativity of Ahmet for the task 1				**6.3**

Umay *(U)* produced eight different solutions for task 1 *(T1.S7, T1.S8, T1.S9, T1.S10, T1.S11, T1.S12, T1.S13, and T1.S14)*. The seventh solution type's flexibility score was $Flx_{U.T1.S7} = 10$, and the originality score of the seventh solution type was $Or_{U.T1.S7} = 0.1$. The creativity score of her solution ($Cr_{U.T1.S7}$), which was the seventh solution type of task 1, was scored by multiplying its flexibility and originality score ($Cr_{U.T1.S7} = Flx_{U.T1.S7}$ x $Or_{U.T1.S7}$); which means the creativity score of the seventh solution type for the task 1 was scored 1 ($Cr_{U.T1.S7} = 1$). Similarly, the creativity scores of the eighth, ninth, tenth, eleventh, twelfth, thirteenth, and fourteenth solution types ($Cr_{U.T1.S8}$, $Cr_{U.T1.S9}$, $Cr_{U.T1.S10}$, $Cr_{U.T1.S11}$, $Cr_{U.T1.S12}$, $Cr_{U.T1.S13}$, *and* $Cr_{U.T1.S14}$) were scored as the product of their flexibility and originality scores. The creativity of eighth solution type was scored 0.1 ($Cr_{U.T1.S8} = 0.1$), the creativity of ninth solution type was scored 0.1 ($Cr_{U.T1.S9} = 0.1$), the creativity of tenth solution type was scored 0.1 ($Cr_{U.T1.S10} = 0.1$), the creativity of eleventh solution type was scored 0.01 ($Cr_{U.T1.S11} = 0.01$), the creativity of twelfth solution type was scored 10 ($Cr_{U.T1.S12} = 10$), the creativity of thirteenth solution type was scored 0.1 ($Cr_{U.T1.S13} = 0.1$), and the creativity of fourteenth solution type was scored 0.1 ($Cr_{U.T1.S14} = 0.1$). Thus, the total creativity of Umay's solutions for task 1 was scored 11.51 ($Cr_{U.T1} =11.51$), which was scored as the sum of the creativity scores of each solution. Therefore, the final creativity score of Umay for task 1, was the product of her fluency score ($N_{U.T1}=8$), and her total creativity score ($Cr_{U.T1} = 11.51$) was scored 92.08 ($CR_{U.T1} = 92.08$) (seen in Table 4.6)

Table 4. 6 Scoring the creativity of Umay for task 1.

	Fluency	**Flexibility**	**Originality**	**Creativity**
T1.S7	1	10	0.1	1
T1.S8	1	1	0.1	0.1
T1.S9	1	1	0.1	0.1
T1.S10	1	1	0.1	0.1
T1.S11	1	0.1	0.1	0.01
T1.S12	1	1	10	10
T1.S13	1	1	0.1	0.1
T1.S14	1	1	0.1	0.1
Total	8	16.1	10.7	**11.51**
Final creativity of Umay for task 1				**92.08**

Beren *(B)* produced three different solutions for task 1 *(T1.S1, T1.S8, and T1.S15)*. The first solution type's flexibility score was $Flx_{B.T1.S1} = 10$, and the originality score of the first solution type was $Or_{B.T1.S1} = 0.1$. The creativity score of her solution ($Cr_{B.T1.S1}$), which was the first solution type of task 1, was scored by multiplying its flexibility and originality score ($Cr_{B.T1.S1} = Flx_{B.T1.S1} \times Or_{B.T1.S1}$); which means the creativity score of the first solution type for the task 1 was scored 1 ($Cr_{B.T1.S1} = 1$). Similarly, the creativity score of the eighth and fifteenth solution types ($Cr_{B.T1.S8}$ *and* $Cr_{B.T1.S15}$) were scored as the product of their flexibility and originality scores. The creativity of the eighth solution type was scored 0.1 ($Cr_{B.T1.S8} = 0.1$), and the creativity of the fifteenth solution type was scored 0.1($Cr_{B.T1.S15}= 0.1$). Thus, the total creativity of Beren's solutions for task 1 was scored 1.2 ($Cr_{B.T1} =1.2$), which was scored as the sum of the creativity scores of each solution. Therefore, the final creativity score of Beren for task 1 was the product of his fluency score ($N_{B.T1}=3$), and his total creativity score ($Cr_{B.T1} = 1.2$) was scored 3.6 ($CR_{B.T1} = 3.6$) (seen in Table 4.7).

Table 4. 7. Scoring the creativity of Beren for task 1.

	Fluency	Flexibility	Originality	Creativity
T1.S1	1	10	0.1	1
T1.S8	1	1	0.1	0.1
T1.S15	1	1	0.1	0.1
Total	3	12	0.3	**1.2**
Final creativity of Beren for task 1				**3.6**

Kemal *(K)* produced five different solutions for task 1 *(T1.S16, T1.S17, T1.S18, T1.S19, and T1.S20)*. The sixteenth solution type's flexibility score was $Flx_{K.T1.S16} = 10$, and the originality score of the sixteenth solution type was $Or_{K.T1.S16} = 0.1$. The creativity score of his solution ($Cr_{K.T1.S16}$), which was the sixteenth solution type of task 1, was scored by multiplying its flexibility and originality score ($Cr_{K.T1.S16} = Flx_{K.T1.S16} \times Or_{K.T1.S16}$); which means the creativity score of the sixteenth solution type for the task 1 was 1 ($Cr_{K.T1.S16} = 1$). Similarly, the creativity score of the seventeenth, eighteenth, nineteenth, and twentieth solution types of task 1 ($Cr_{K.T1.17}$, $Cr_{K.T1.S18}$, $Cr_{K.T1.S19}$, *and* $Cr_{K.T1.S20}$) were scored as the product of their flexibility and originality scores. The creativity of the seventeenth solution type was scored 1 ($Cr_{K.T1.17} =1$), the creativity of the eighteenth solution type was scored 0.1 ($Cr_{K.T1.18} =0.1$), the creativity of the nineteenth solution type was scored 0.1 ($Cr_{K.T1.19} =0.1$), and the creativity of the twentieth solution type was scored 0.01 ($Cr_{K.T1.20} =0.01$). Thus, the total creativity of Kemal's solutions for task 1 was scored 2.21 ($Cr_{B.T1} =2.21$), which was scored as the sum of the creativity scores on each solution. Therefore, the final creativity score of Kemal for task 1 was the product of his fluency score ($N_{K.T1}=5$), and her total creativity score ($Cr_{K.T1} = 2.21$) was scored 11.05 ($CR_{K.T1} = 11.05$) (seen in Table 4.8).

Table 4. 8. Scoring the creativity of Kemal for task 1.

	Fluency	**Flexibility**	**Originality**	**Creativity**
T1.S16	1	10	0.1	1
T1.S17	1	1	1	1
T1.S18	1	1	0.1	0.1
T1.S19	1	1	0.1	0.1
T1.S20	1	0.1	0.1	0.01
Total	5	13.1	1.4	**2.21**
Final creativity of Kemal for task 1				**11.05**

4.2 Evaluation of Task 2

This section provides the solution types created by the pre-service mathematics teachers for task 2 with the mathematical ideas used in solutions and the analysis of task 2 considering the fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of solutions of each participant. The participants were asked to solve task 2 in more than one way, with at least one solution being in the GeoGebra environment within approximately forty-five minutes.

Each pre-service teacher solved task 2 in one or more than one way and used various mathematical ideas in their solutions. When all the solutions produced for task 2 were examined, the basic mathematical ideas developed for task 2 were identified as the followings; |QN| = 2. |NP|, the properties of a rectangle, |MQ| = |QT|, the median to the hypotenuse of a right triangle, the special segments in special triangles, the properties of special triangles, the parallel and perpendicular lines, constructing |TP|, the perpendicular bisector, the reflection, the inscribed angles on diameter, the congruence, the elements of a triangle, constructing a new geometric shape, constructing the midpoint of |TO|, and algebraic solution ways including properties of angles of a triangle.

All solutions were grouped based on the type of mathematical ideas used. There were seventeen different solution types for task 2. Then, each solution type, different from the others, was assigned a unique code to indicate which task the solution belongs to. For instance, the first solution type of task 2 was shown as *T2.S1 (the first solution type for task 2)*, or the fourth solution type of task 2 was established as *T2.S4 (the fourth solution type for task 1).* There were seventeen different solution types for task 2, and all solution types were shown in detail with the mathematical ideas used. Each figure in the subsequent pages (Figure 4.21 to Figure 4.37) summarizes the solution types and gives solutions of each participant provided these solutions.

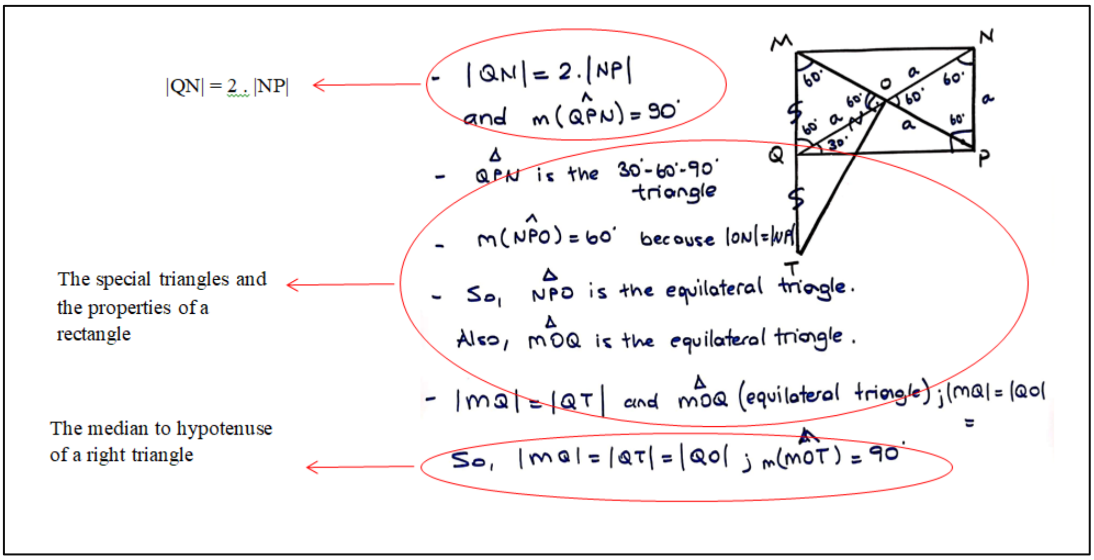


Figure 4. 21 T2.S1 (the first solution type of task 2)

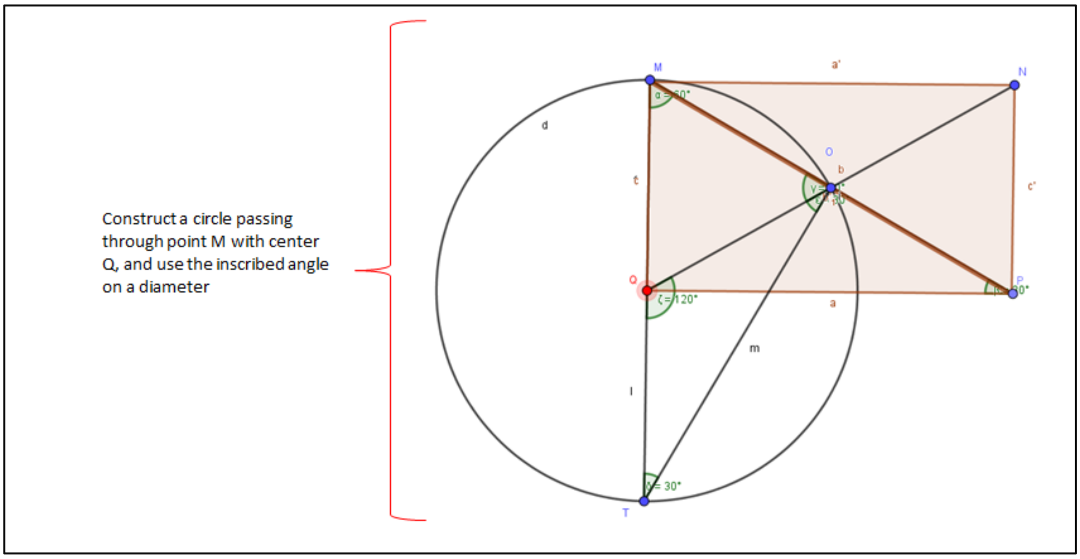


Figure 4. 22 T2.S2 (the second solution type of task 2)

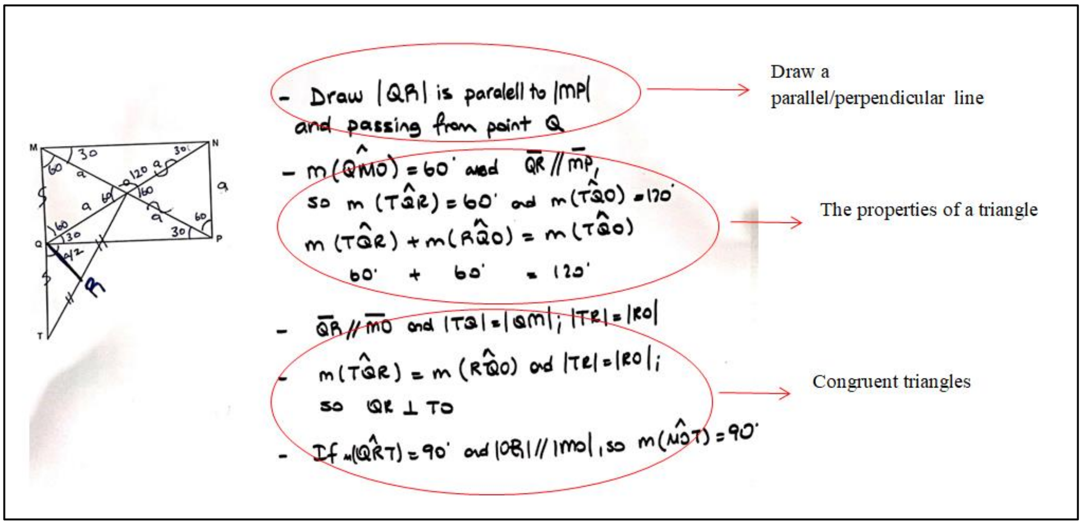


Figure 4. 23 T2.S3 (the third solution type of task 2)

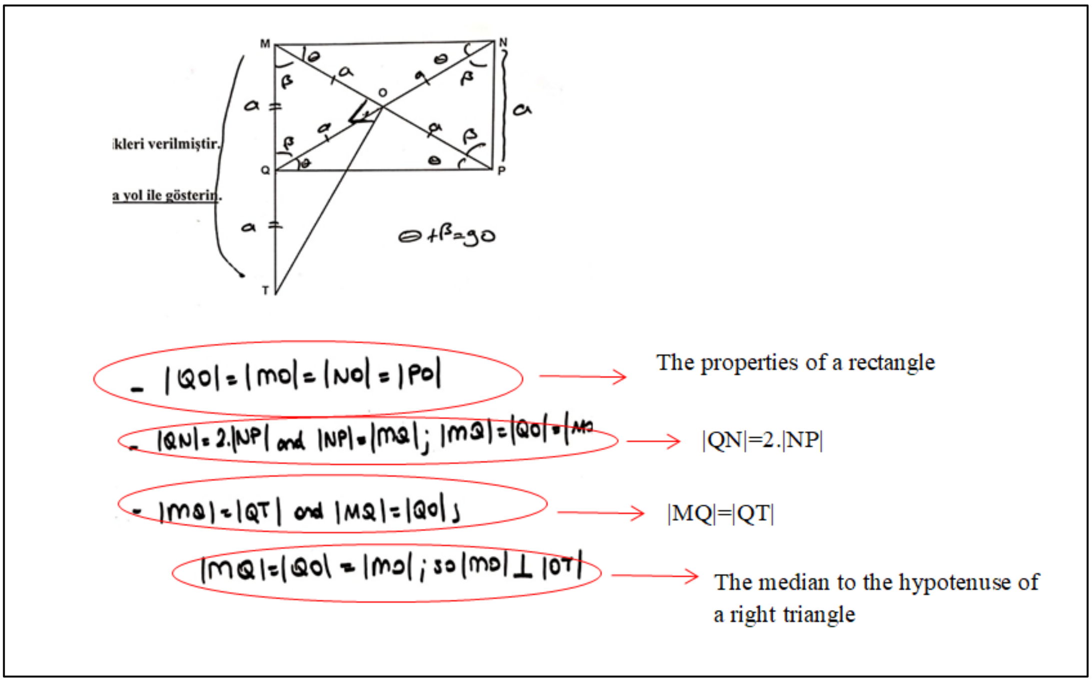


Figure 4. 24 T2.S4 (the fourth solution type of task 2)

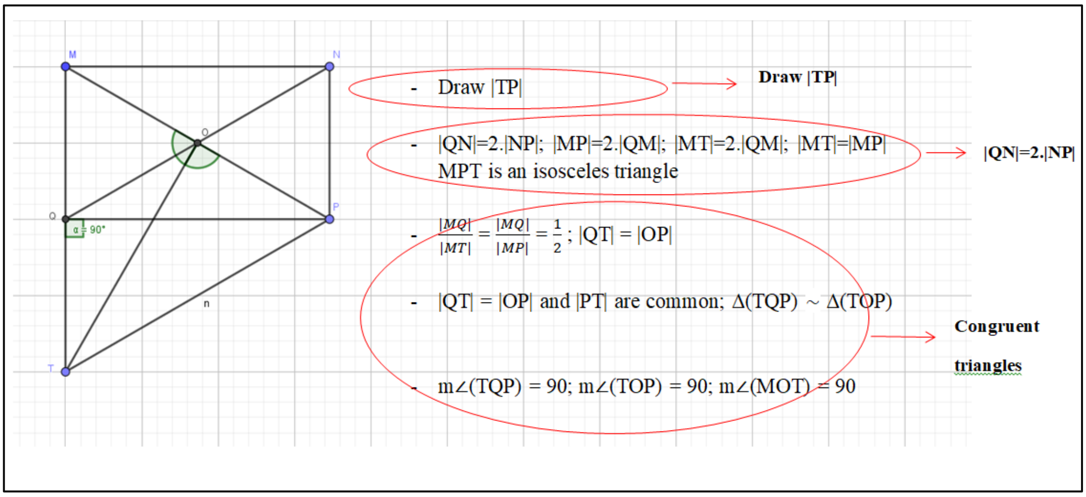


Figure 4. 25 T2.S5 (the fifth solution type of task 2)

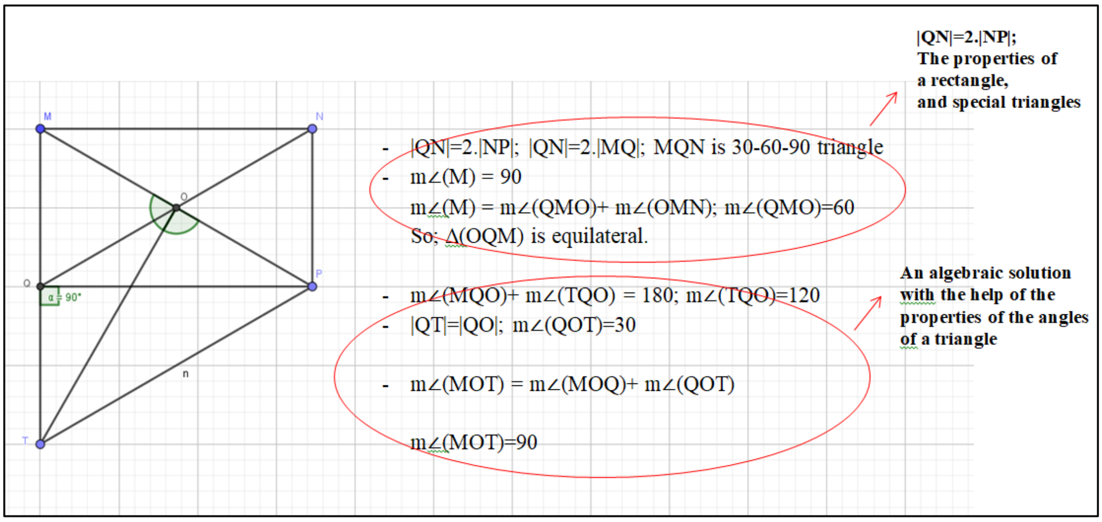


Figure 4. 26 T2.S6 (the sixth solution type of task 2)

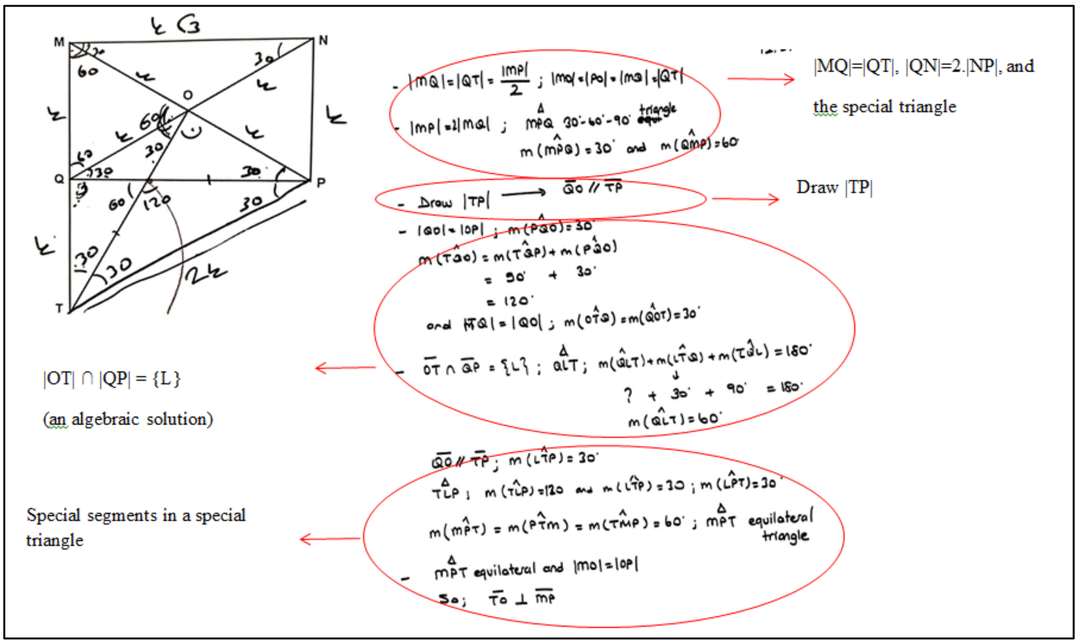


Figure 4. 27 T2.S7 (the seventh solution type of task 2)

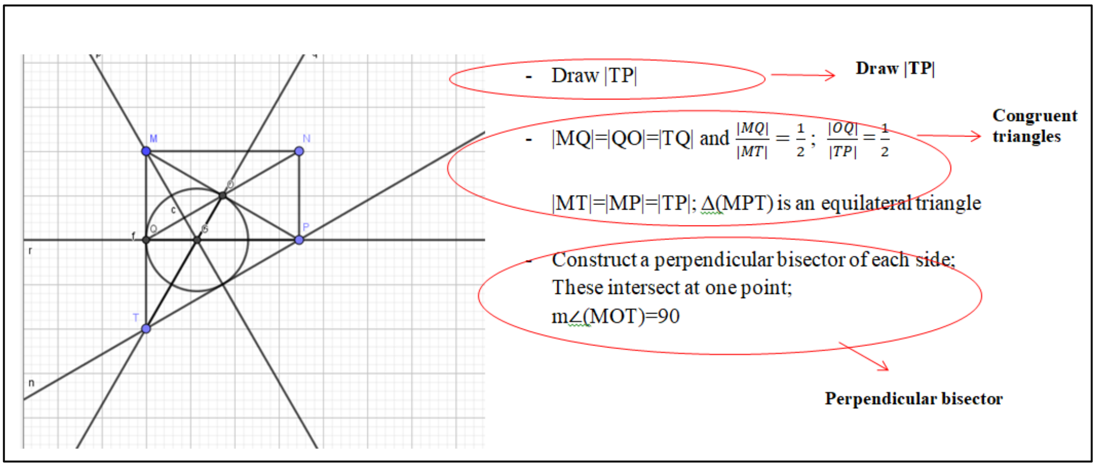


Figure 4. 28 T2.S8 (the eighth solution type of task 2)

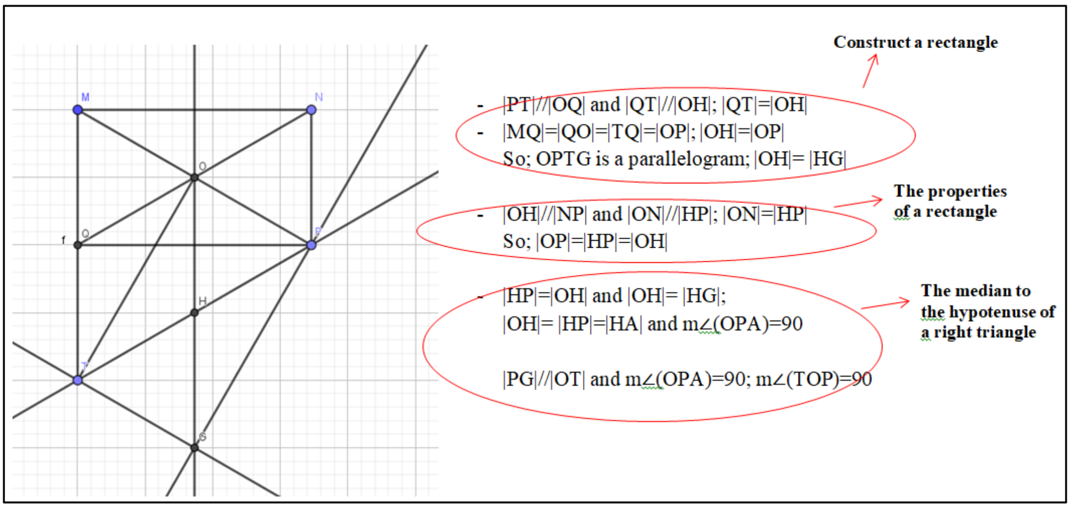


Figure 4. 29 T2.S9 (the ninth solution type of task 2)

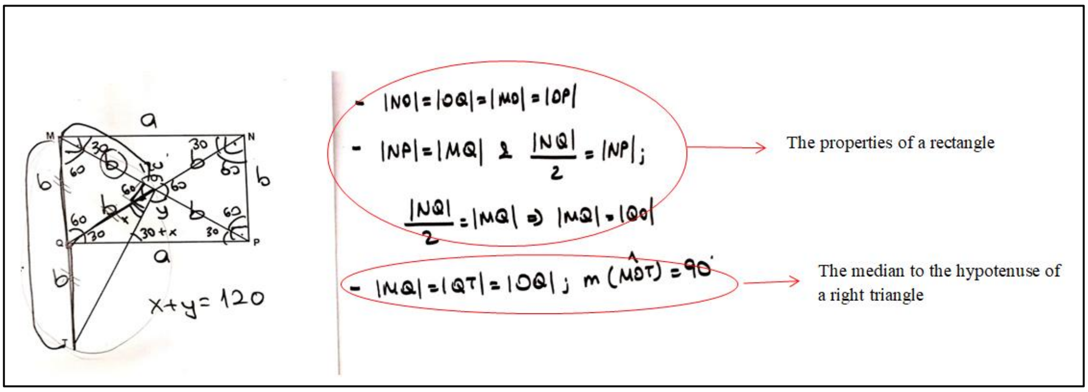


Figure 4. 30 T2.S10 (the tenth solution type of task 2)

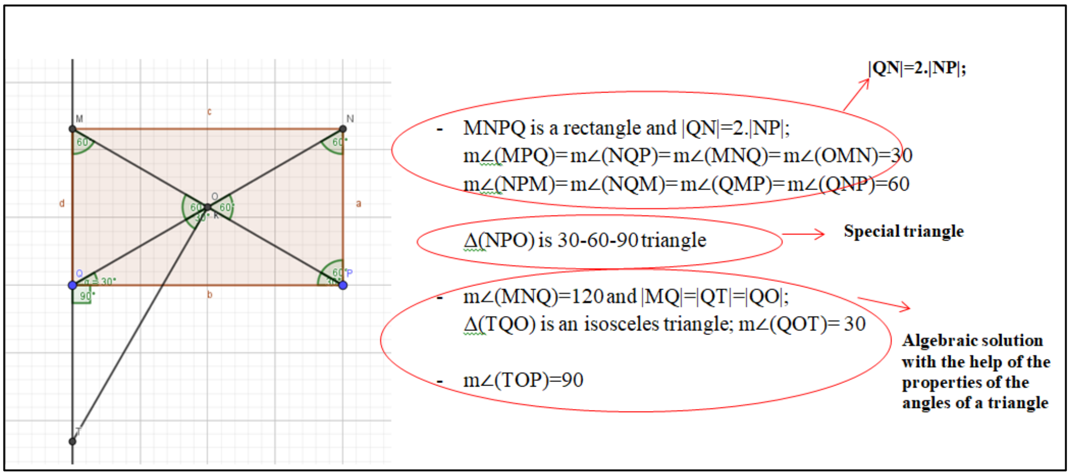


Figure 4. 31 T2.S11 (the eleventh solution type of task 2)

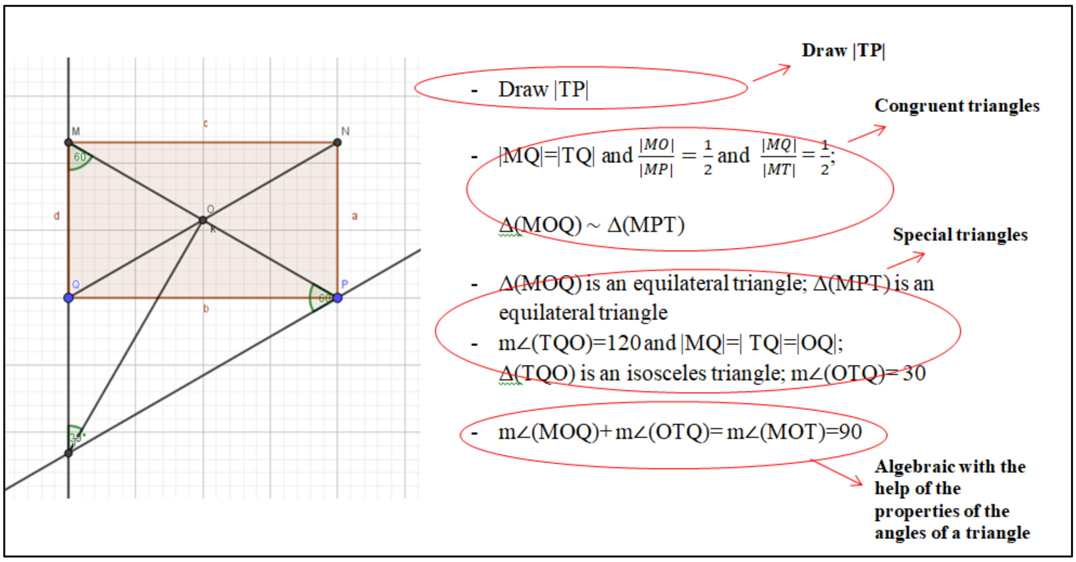


Figure 4. 32 T2.S12 (the twelfth solution type of task 2)

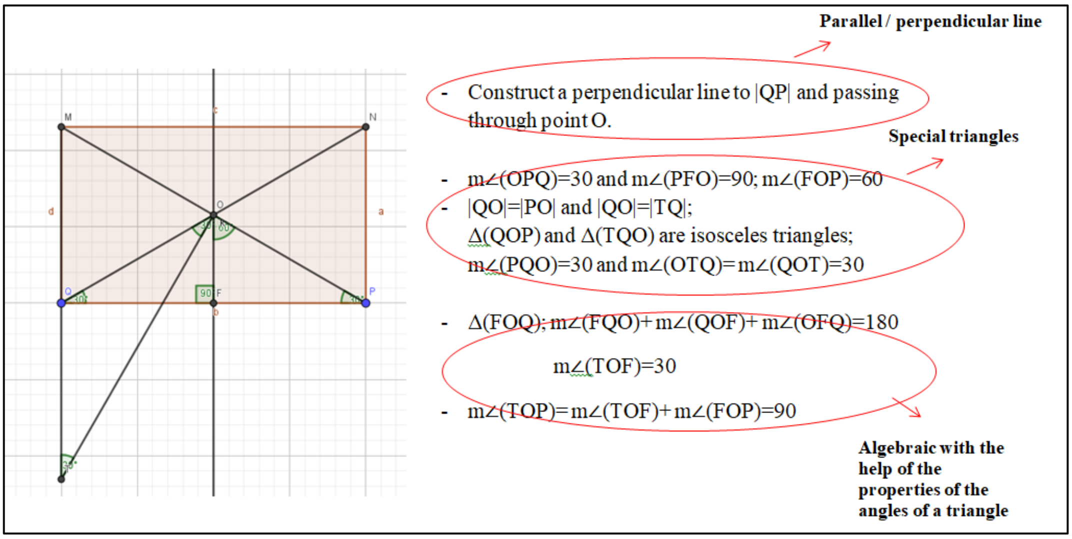


Figure 4. 33 T2.S13 (the thirteenth solution type of task 2)

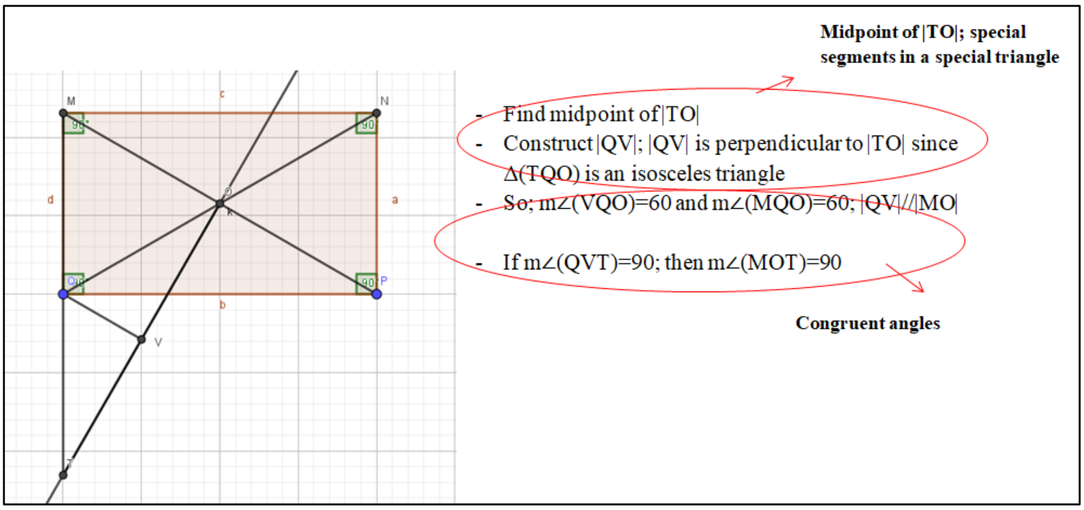


Figure 4. 34 T2.S14 (the fourteenth solution type of task 2)

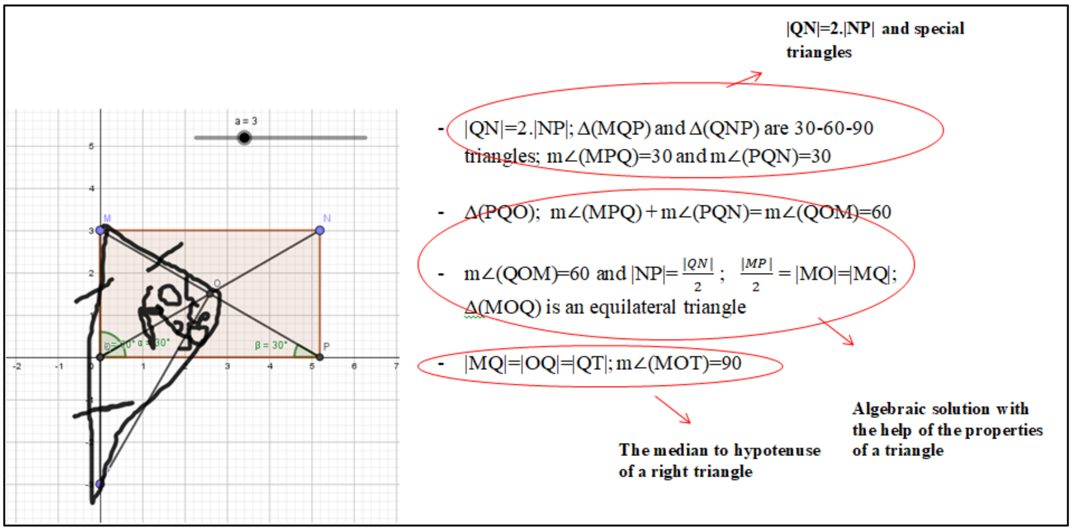


Figure 4. 35 T2.S15 (the fifteenth solution type of task 2)

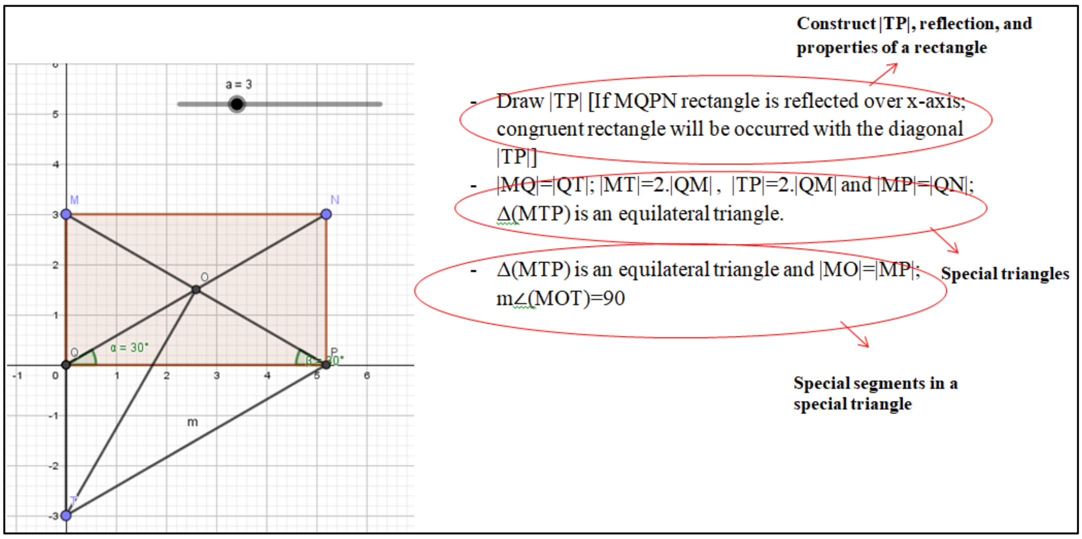


Figure 4. 36 T2.S16 (the sixteenth solution type of task 2)

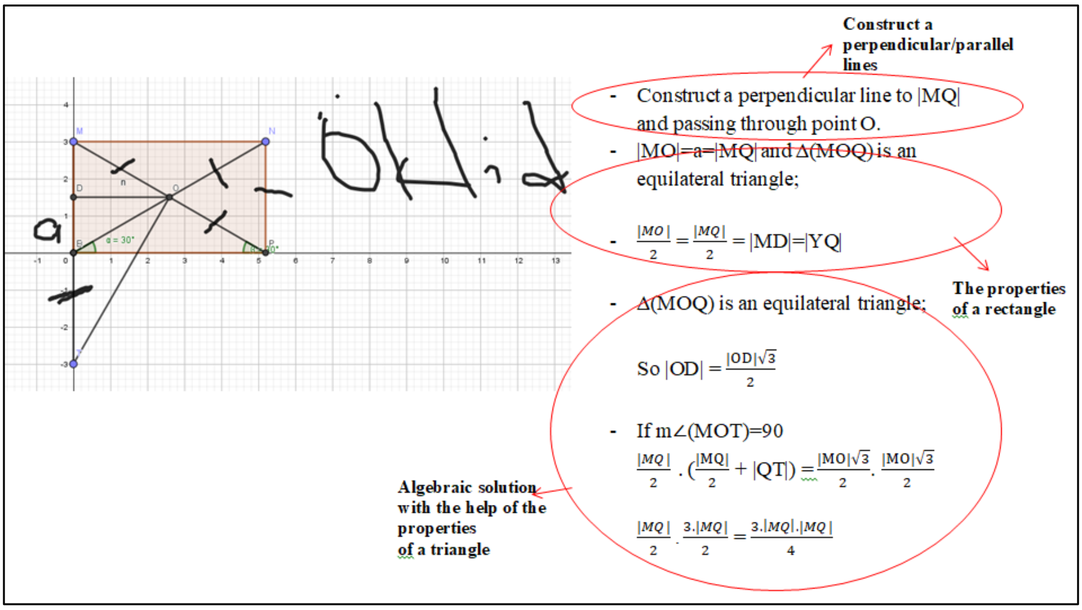


Figure 4. 37 T2.S17 (the seventeenth solution type of task 2)

4.2.1 Evaluation of Fluency in Task 2

The fluencies (*N*) of the pre-service teachers were scored individually by viewing their solutions for task 2, based on the number of their appropriate solutions in the individual solution space. Since the pre-service were allowed to solve task 2 with at least one solution being in the GeoGebra environment, there were solutions provided in both paper-and-pencil environment and dynamic geometry environment. The fluencies of each solution were also displayed, considering the environment separately. Each solution's fluency was assigned a unique code that displays to whom the solution belongs and in which environment it was solved. For example, the first participant, Feyza *(F)*, created three different solutions *(T2.S1, T2.S2, and T2.S3)* for task 1, two of them were in the paper-pencil environment, and one of them was in the GeoGebra environment. Feyza created the first solution type in the paper-pencil environment, and the fluency of the first solution type was shown as $N_{F.T2.S1.PP.}$ Feyza created the second solution type in the GeoGebra environment, and the fluency of this solution was displayed as $N_{F.T2.S2.GGB}$. For each solution, 1 point was assigned ($N_{F.T2.S1.PP} = N_{F.T2.S2.GGB} = N_{F.T2.S3.PP} = 1$). Therefore, the total fluency of Feyza for task 2 was $N_{F.T2} = 3$, while the total fluency of Feyza for task 2 in the paper-pencil environment was $N_{F.T2.PP} = 2$, and the total fluency of Feyza for task 2 in the GeoGebra environment was $N_{F.T2.GGB} = 1$.

The second participant, Ahmet *(A)*, created seven different solutions *(T2.S2, T2.S4, T2.S5, T2.S6, T2.S7, T2.S8, and T2.S9)* for task 2, two of them were in the paper-and-pencil environment, and five of them were in the GeoGebra environment. For each solution, 1 point was assigned ($N_{A.T2.S2.GGB} = N_{A.T2.S4.PP} = N_{A.T2.S5.GGB} = N_{A.T2.S6.GGB} = N_{A.T2.S7.PP} = N_{A.T2.S8.GGB} = N_{A.T2.S9.GGB} = 1$). Therefore, the total fluency of Ahmet for task 2 was $N_{A.T2} = 7$, while the total fluency of Ahmet for task 2 in the paper-pencil environment was $N_{A.T2.PP} = 2$ and the total fluency of Ahmet for task 2 in the GeoGebra environment was $N_{A.T2.GGB} = 5$.

The third participant, Umay *(U)*, created one solution *(T2.S10)* for task 2 in the paper-pencil environment. For her solution 1 point was assigned ($N_{U.T2.S10.PP} =1$). Therefore, the total fluency of Umay for task 2 was $N_{U.T2} = 1$, while the total fluency of Umay for task 2 in the paper-pencil environment was $N_{U.T2.PP} = 1$, and the total fluency of Umay for task 2 in the GeoGebra environment was $N_{U.T2.GGB} =0$.

The fourth participant, Beren *(B)*, created five different solutions *(T2.S13, T2.S14, T2.S15, T2.S16 and T2.S2)* for task 2; all of them were in the GeoGebra environment. For each solution 1 point was assigned ($N_{B.T2.S13.GGB} = N_{B.T2.S14.GGB} = N_{B.T2.S15.GGB} = N_{B.T2.S16.GGB} = N_{B.T2.S12.GGB} = 1$). Therefore, the total fluency of Beren for task 2 was $N_{B.T2} = 5$ while the total fluency of Beren for task 2 in the GeoGebra environment was $N_{B.T2.GGB} = 5$.

The fifth participant, Kemal *(K)*, created three different solutions *(T2.S17, T2.S18, and T2.S19)* for task 2, and all of them were in the GeoGebra environment. For each solution 1 point was assigned ($N_{K.T2.S17.GGB} = N_{K.T2.S18.GGB} = N_{K.T2.S19.GGB} = 1$). Therefore, the total fluency of Kemal for task 2 was $N_{K.T2} = 3$, while the total fluency of Kemal for task 2 in the GeoGebra environment was $N_{K.T2.GGB} = 3$.

Fluency scores of each participant is shown in Table 4.9.

Table 4. 9 Fluency scores of each participant for task 2

Participants	Fluency Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	1	2	3
Ahmet	5	2	7
Umay	0	1	1
Beren	5	0	5
Kemal	3	0	3

4.2.2 Evaluation of Flexibility in Task 2

As stated in the previous section, there were seventeen different solution types for task 2, from all participants. While each of them was different solution types, some of them included common mathematical ideas within. The flexibility *(Flx)* scores were evaluated based on the differentiation of mathematical ideas used in the solutions. There were solutions solved in both paper-and-pencil and dynamic geometry environments since the pre-service were allowed to solve task 2 with at least one solution being in a dynamic geometry environment. The flexibility of each solution was assigned a unique code that displays to whom the solution belongs and in which environment it was solved. For example, the first pre-service teacher, Feyza *(F)*, created three different solutions *(T2.S1, T2.S2, and T2.S3)* for task 2. The two of them *(T2.S1 and T2.S3)* were in the paper-pencil environment, and the flexibilities of these solutions were displayed as $Flx_{F.T2.S1.PP}$ and $Flx_{F.T2.S3.PP}$ (the flexibility of the first and third solution types that Feyza produced in the paper-pencil environment for task 2). One of the solutions was in the GeoGebra environment, and the flexibility of this solution was displayed as $Flx_{F.T2.S2.GGB}$ (the originality of the second solution type that Feyza produced in the GeoGebara environment for task 2). Hence, the total flexibility of Feyza for task 2 was shown as $Flx_{F.T2}$.

Feyza (F) created three different solutions for task 2 *(T2.S1, T2.S2, and T2.S3).* The first solution type (T2.S1) was the first appropriate solution of Feyza for task 2, which means its flexibility was scored 10. Since the first solution was in the paper-pencil environment, it is shown as $Flx_{F.T2.S1.PP}=10$. Feyza created her second solution (T2.S2) in the GeoGebra environment, and it was also scored 10 ($Flx_{F.T2.S2.GGB}=10$). Constructing a circle passing through point M, with center Q, and using the inscribed angle on diameter in the second solution had no common concept with the first solution type, which means the geometric concept used in the second solution was totally different from the previous solution. Moreover, Feyza created the third solution (T2.S3) in the paper-pencil environment. The flexibility of this solution was also scored 10 and shown as $Flx_{F.T2.S3.PP}=10$; because the geometric concepts used in the third solution type were totally different from the first and the second solution type. Feyza, respectively, constructed a parallel line and used a triangle and congruence properties in the third solution way (seen in the table). She did not use these ideas in previous solutions. Therefore, the total flexibility of Feyza for task 2 was $Flx_{F.T2}=30$, while the total flexibility of Feyza for task 2 in the paper-pencil environment was $Flx_{F.T2.PP}=20$, and the total flexibility of Feyza for task 2 in the GeoGebra environment was $Flx_{F.T2.GGB}=10$.

Ahmet (A) created seven different solutions *(T2.S4, T2.S2, T2.S5, T2.S6, T2.S7, T2.S8, and T2.S9)* for task 2. The fourth solution type *(T2.S4)* was the first appropriate solution of Ahmet for task 2, which means its flexibility was scored 10. Since the fourth solution way was in the paper-pencil environment, it is shown as $Flx_{A.T2.S4.PP}=10$. Ahmet created her second solution *(T2.S2)* in the GeoGebra environment, and it was also scored 10 ($Flx_{A.T2.S2.GGB}=10$). Constructing a circle passing through point M with center Q and using the inscribed angle on diameter in the second solution way had no common concept with Ahmet’s first solution, which means the geometric concept used in the second solution way was totally different from the previous solution. The fifth solution type *(T2.S5)* was Ahmet's third appropriate solution for task 2, which was created in the GeoGebra environment. This solution's flexibility was scored 1 ($Flx_{A.T2.S5.GGB}=1$) due to Ahmet used one of the mathematical concepts that shaped the fifth solution way in the previous solution. In the fourth solution type, Ahmet noticed the median to the hypotenuse of a right triangle using |QN|=2.|NP|. Likewise, he presented an isosceles triangle by using |QN|=2.|NP| in the fifth solution type and completed the solution with the help of congruent triangles $(\Delta(TQP) \sim \Delta(TOP))$. It showed that |QN|=2.|NP| is a common concept in the fourth and fifth solution type to improve them. Hence, the flexibility of the fifth solution type was scored 1 ($Flx_{A.T2.S5.GGB}=1$). The sixth solution type *(T2.S6)* was Ahmet's fourth appropriate solution created in the GeoGebra environment. This solution's flexibility was scored 1 ($Flx_{A.T2.S6.GGB}=1$) due to Ahmet used one of the mathematical concepts that shaped the sixth solution way in the previous solution. In the fifth solution type, Ahmet formed an isosceles triangle with the help of |QN|=2.|NP| and the properties of a rectangle, then he finalized his solution with an algebraic solution based on the properties of angles of a triangle. In the same way, he began the fourth solution type by using |QN|=2.|NP| and a rectangle's properties. Although their final steps were different, two solution types were founded on common mathematical concepts. Hence, the flexibility of the sixth solution type was scored 1 ($Flx_{A.T2.S6.GGB}=1$). Ahmet's appropriate fifth solution was the seventh solution type *(T2.S7)*, created in the paper-pencil environment. The flexibility of this solution was scored 1 ($Flx_{A.T2.S7.PP}=1$). Ahmet formed a solution in the seventh solution type, which was based |MQ|=|QT|, |QN|=2.|NP|, special triangles, and drawing |TP|, these were commonly used mathematical concepts in his previous solutions, e.g., T2.S4, T2.S5, and T2.S6. Even though his solution became distinct with algebraic operations and special segments in a special triangle, the seventh solution type's flexibility was scored 1 ($Flx_{A.T2.S7.PP}=1$) because he formed his solution depending on commonly used mathematical concepts in his former solutions. Ahmet's sixth appropriate solution was the eighth solution type *(T2.S8)* produced in the GeoGebra environment. This solution's flexibility was also scored 1 ($Flx_{A.T2.S7.PP}=1$) because the eighth solution type had common mathematical concepts with the fifth solution type. He based both solutions on the special triangle formed by drawing |TP|, then used congruent triangles within both solutions. While the eighth solution type was finalized depends on perpendicular bisector, the eighth solution type's flexibility was scored 1 (FlxA.T2.S8.GGB=1) due to its correspondence with the fifth solution type. Lastly, his seventh solution was the ninth solution type of task 2 (T2.S9) produced in the GGB environment. The flexibility of this solution way was also scored 1 ($Flx_{A.T2.S9.GGB}=1$). He made a distinct beginning by constructing a new rectangle. Yet, he repeated some parts of the fourth solution type by using the properties of a rectangle and median to the hypotenuse of a right triangle in the ninth solution type. Hence, the flexibility of the ninth solution type was scored 1 ($Flx_{A.T2.S9.GGB}=1$). Therefore, the total flexibility of Ahmet for task 2 was $Flx_{A.T2}=25$, while the total flexibility of Ahmet for task 2 in the paper-pencil environment was $Flx_{A.T2.PP}=11$, and the total flexibility of Ahmet for task 2 in the GeoGebra environment was $Flx_{A.T2.GGB}=14$.

Umay (U) created one solution *(T2.S10)* for task 2 in the paper-pencil environment. The tenth solution type was the first and only appropriate solution of Umay. For this reason, the flexibility of it was scored 10 ($Flx_{U.T2.S10.PP}=10$). Therefore, the total flexibility of Umay for task 2 was $Flx_{U.T2} = 10$, while the total flexibility of Umay for task 2 in the paper-pencil environment was $Flx_{U.T2.PP} = 10$, and the total flexibility of Umay for task 2 in the GeoGebra environment was $Flx_{U.T2.GGB} =0$.

Beren *(B)* generated five different solutions *(T2.S11, T2.S12, T2.S13, T2.S14, and T2.S2)* for task 2, and all of them were formed in the GeoGebra environment. The eleventh solution type was the first appropriate solution of Beren for task 2, which means its flexibility was scored 10 ($Flx_{B.T2.S11.GGB}=10$). The twelfth solution type *(T2.S11)* and the thirteenth solution type *(T2.S13)* were the second and third appropriate solution of Beren in task 2, respectively. They were scored 1 ($Flx_{B.T2.S12.GGB}=Flx_{B.T2.S13.GGB}=1$) owing to their resemblance with eleventh solution type. Even though the starting points of the three solutions were distinct, the finalized solutions depend on special triangles and an algebraic solution with the help of the properties of the angles of a triangle. Lastly, the fourth and fifth appropriate solutions of Beren were the fourteenth *(T2.S14)* and second solution type *(T2.S2)*, respectively. They were both scored 10 (($Flx_{B.T2.S14.GGB}=Flx_{B.T2.S2.GGB}=10$) because the mathematical concepts used in these solutions were totally different from her previous solutions. Especially in the fourteenth solution type, he found a midpoint of |TO|, made use of special segments in special triangle TQO, and obtained a congruent angle with the angle of MOT in the final part of the solution. However, in the second solution type, he preferred to construct a circle passing through point M with center Q and use the inscribed angle on a diameter. The two solution types *(T2.S14 and T2.S2)* had no common mathematical concepts with her previous solutions. Therefore, the total flexibility of Beren for task 2 was $Flx_{B.T2} = 32$, while the total flexibility of Beren for task 2 in the GeoGebra environment was $Flx_{B.T2.GGB} = 32$, and the total flexibility of Beren for task 2 in the paper-pencil environment was $Flx_{B.T2.PP} = 0$.

Kemal *(K)* generated three different solutions *(T2.S15, T2.S16, and T2.S17)* for task 2, and all of them were formed in the GeoGebra environment. The fifteenth solution type *(T2.S15)* was the first appropriate solution of Kemal for task 2, which means the flexibility of it was scored 10 ($Flx_{K.T2.S15.GGB}=10$). His second appropriate solution was the sixteenth solution way, which was scored as 1 ($Flx_{K.T2.S16.GGB}=1$) because Kemal used some of the mathematical concepts that shaped the fifteenth solution way in his previous solution. For example, although their starting steps were different, he used the special triangles and special line segments in a special triangle in both solution types *(T2.S14 and T2.S15)*. The third appropriate solution of Kemal was the seventeenth solution type for task 2, which was scored as 1 ($Flx_{K.T2.S17.GGB}=1$). He created the seventeenth solution by gathering the mathematical concepts in his previous solutions. The seventeenth solution type was especially started by constructing a perpendicular line to |MQ| and passing through point O. It then was maintained by using the properties of a rectangle and algebraic solution with the help of the properties of a triangle MOQ. While the properties of the rectangle also were used in the sixteenth solution type, the algebraic solution with the help of the properties of a triangle MOQ was used in the sixteenth solution type. So, the flexibility of the seventeenth solution way was scored 1. Therefore, the total flexibility of Kemal for task 2 was $Flx_{K.T2} = 12$, while the total flexibility of Kemal for task 2 in the GeoGebra environment was $Flx_{K.T2.GGB} = 12$, and the total flexibility of Kemal for task 2 in the paper-and-pencil environment was $Flx_{K.T2.PP} = 0$.

The flexibility scores of each participant for task 2 are shown in Table 4.10.

Table 4. 10. Flexibility scores of each participant for task 2

Participants	Flexibility Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	10	20	30
Ahmet	14	11	25
Umay	0	10	10
Beren	32	0	32
Kemal	12	0	12

4.2.3 Evaluation of Originality in Task 2

There were seventeen different solution types in task 2. While some of the solutions were insight-based or unconventional, others were model-based or partly unconventional solutions and some of them were algorithm-based or conventional solutions. The originality *(Or)* of each solution type was scored depend on its conventionality and insight level. Each participant's total originality score on task 2 was the sum of each solution's originality score. There were solutions solved in both paper-pencil and dynamic geometry environments since the participants were allowed to solve task 2 with at least one solution being in a dynamic geometry environment. The originalities of the solutions were also evaluated, considering the environment separately. Each solution's originality was assigned a unique code that displays to whom the solution belongs and in which environment it was created. For instance, the first participant, Feyza *(F)*, produced three different solutions *(T2.S1, T2.S2, and T2.S3)* for task 2. Two of them *(T2.S1 and T2.S3)* were in the paper-pencil environment, and the originalities of these solutions were displayed as $Or_{F.T2.S1.PP}$ and $Or_{F.T2.S3.PP}$ (the originality of the first and third solution types that Feyza produced in the paper-pencil environment for task 2). One of the solutions was in the GeoGebra environment, and the originality of this solution was displayed as $\mathrm{Or}_{F.T2.S2.GGB}$ (the originality of the second solution type that Feyza produced in the GeoGebra environment for task 2). Hence, the total originality of Feyza for task 2 was shown as $\mathrm{Or}_{F.T2}$.

Feyza first solution type *(T2.S1)* was her first appropriate solution, formed in the paper-pencil environment. In the first solution type, Feyza firstly made both the information given with task 2 (e.g., |QN|=2. |NP|) and the information hidden in task 2 (e.g., the equilateral triangles NPO and MOQ) apparent, which did not require adjusting the representation of the given figure. Also, she used the concept of "the median to the hypotenuse of a right triangle" as usually taught in mathematics classes without comprehending and explaining its structure. She claimed that "a triangle is a right triangle if the segment drawn to the base in a triangle divides the base into two equal parts and its length is equal to these parts." However, it did not explain the main structure of the concept of the median to the hypotenuse in a right triangle. For these reasons, the first solution type was evaluated as a conventional solution, and its originality was scored 0.1 ($Or_{F.T2.S1.PP}=0.1$). The second solution of Feyza was the second solution type *(T2.S2)*, which is created in the GeoGebra environment. Feyza constructed a circle passing through point M with center Q based on insight to examine a figure's structure and displayed the inscribed angle on a diameter. It required comprehending the given figure's structure rather than using the concept of "the median to the hypotenuse of a right triangle" directly. Hence, the second solution type was evaluated as an unconventional solution, and its originality was scored 10 ($Or_{F.T2.S2.GGB}$*=10).* The third solution type *(T2.S3)* was the last solution of Feyza, which was created in the paper-pencil environment. She formed two congruent triangles (QTR and MTO) by constructing the line |MP| parallel to |MP|. Forming congruent triangles with the help of parallel/perpendicular lines were commonly taught and used in mathematics classes. It did not require an understanding of the figure's construction in this way. Also, she based the step in using "the properties of the triangle" on algebraic operations. So, the third solution type (T2.S3) was evaluated as an algorithm based conventional solution; and its originality was scored 0.1 ($Or_{F.T2.S3.PP}$*=0.1).* Therefore, the total originality of Feyza for task 2 was $Or_{F.T2}$*=10.2,* while the total originality of Feyza for task 2 in the paper-pencil environment was $Or_{F.T2.PP}$*=0.2,* and the total originality of Feyza for task 2 in the GeoGebra environment were $Or_{F.T2.GGB}$*=10*

Ahmet *(A)* produced seven different solutions *(T2.S4, T2.S2, T2.S5, T2.S6, T2.S7, T2.S8, and T2.S9)* for task 2. The fourth solution type (T2.S4) was his first solution, formed in the paper-pencil environment. He made the given knowledge with the task *(|QN|=2.|NP| and |MQ|=|QT|)* apparent without making changes in the structure of a figure as commonly used in the mathematics classes. Then, he displayed "the median to the hypotenuse of a right triangle" as taught in mathematics class, without comprehending and explaining its structure. In the same way as Feyza, he claimed that "a triangle is a right triangle if the segment drawn to the base in a triangle divides the base into two equal parts and its length is equal to these parts," which did not explain the main structure of the concept of "the median to the hypotenuse of a right triangle." Thus, the fourth solution type was a conventional solution, and its originality score was 0.1 ($Or_{A.T2.S4.PP}$*=0.1).* The second solution of Ahmet was the second solution type *(T2.S2),* which is created in the GeoGebra environment. Ahmet constructed a circle passing through point M with center Q and displayed the inscribed angle on a diameter. It required comprehending the given figure's structure rather than using the concept of "the median to the hypotenuse of a right triangle" directly. Hence, the second solution type was evaluated as an unconventional solution, and its originality was scored 10 ($Or_{F.T2.S2.GGB}=10$). Thirdly, he created the fifth solution type in the GeoGebra environment. He formed two congruent triangles (TOP and TQP) by constructing the line |TP|. Forming congruent triangles with the help of constructing a new line is commonly learned and used in mathematics classes and did not require an understanding of the figure's construction in this way. Thus, the fourth solution type *(T2.S4)* was evaluated as a conventional solution; and its originality was scored 0.1 ($Or_{F.T2.S5.GGB}=0.1$). The fourth solution of Ahmet was the sixth solution type of task 2 (T2.S6), which was formed in the GeoGebra environment. He used the equation of |QN|=2·|NP|, the properties of a rectangle, and special triangles' properties based on an algorithm. Obviously, the whole solution was created on algebraic operations, including the last step of the solution. For this reason, the sixth solution type was an algorithm-based conventional solution, which means the originality of the sixth solution type was 0.1 ($Or_{F.T2.S6.GGB}=0.1$). The fifth solution of Ahmet was the seventh solution type (T2.S7), which was formed in the paper-pencil environment. He first made the given |MQ|=|QT|, |QN|=2·|NP|, and special triangles within the figure appeared. Then, he created an algorithm based solution with the triangle formed by constructing |TP|. The seventh solution type was based on an algorithm, which means the seventh solution type's originality score was 0.1 ($Or_{F.T2.S7.PP}=0.1$). The last two solutions of Ahmet were the eighth and ninth solution types produced in the GeoGebra environment. In the eighth solution type, Ahmet showed an equilateral triangle (MPT) with the line segment TP and the congruent triangles. He asserted that "if the triangle MPT is an equilateral triangle, a perpendicular bisector of each side intersect in a single point." and verified his view by constructing a perpendicular bisector of each side. The eighth solution type required some insight and was rarely used among the participants. For these reasons, its originality was scored 1 ($Or_{F.T2.S8.GGB}=1$). In the ninth solution type, Ahmet formed a new parallelogram OPGT, a rectangle, associating it with the rectangle MNPQ. Then, he displayed the median to the hypotenuse of a right triangle on the new rectangle. In this solution, he reshaped the given figure, and it was rarely used in the participants. Hence, the originality of the ninth solution way was scored 1 ($Or_{F.T2.S9.GGB}=1$). Therefore, the total originality score of Ahmet for task 2 was $Or_{A.T2}=12.4$, while the total originality of Ahmet for task 2 in the paper-pencil environment was $Or_{A.T2.PP}=0.2$, and the total originality of Ahmet for task 2 in the GeoGebra environment was $Or_{A.T2.GGB}=12.2$.

Umay (U) produced one solution (T2.S10) for task 2, which was in the paper-and-pencil environment. In the tenth solution type, she made the equal lengths and the measure of angles apparent in the figure by using the properties of a rectangle. Then, she used the concept of "the median to the hypotenuse of a right triangle" as taught in mathematics classes without explaining its structure. She, as previous participants did, claimed that "a triangle is a right triangle if the segment drawn to the base in a triangle divides the base into two equal parts and its length is equal to these parts, which did not explain the main structure of the concept of "the median to the hypotenuse of a right triangle." Thus, the tenth solution way was a conventional solution, and the originality was score 0.1 ($Or_{U.T2.S10.PP}=1$). Therefore, the total originality score of Umay for task 2 was $Or_{U.T2}=0.1$, while the total originality of her for task 2 in the paper-pencil environment also was $Or_{U.T2.PP}=0.1$. Since she could not create any solution in the GeoGebra environment, her originality score in the GeoGebra environment was $Or_{U.T2.GGB}=0$.

Beren *(B)* created five different solutions *(T2.S11, T2.S12, T2.S13, T2.S14, and T2.S2)* for task 2. Her first solution was the eleventh solution type *(T2.S11)* of task 2, formed in the GeoGebra environment. Beren firstly made the given knowledge within the figure *(e.g.,* $|QN|=2\cdot|NP|$ *and special triangle NPO)* apparent. Then, she showed the answer with the help of an algebraic solution based on the properties of a triangle's angles. The eleventh solution type was algorithm-based, and the originality was scored 0.1 ($Or_{B.T2.S11.GGB}=0.1$). Her second solution was the twelfth solution type (T2.S12), formed in the GeoGebra environment. She developed two congruent triangles (MOQ and MPT) by constructing the line |TP|. Forming a congruent triangle by constructing a new line is commonly learned and used in mathematics classes and did not require an understanding of the figure's construction in this way. Also, in the progressing part of the solution, she used algebraic operations to complete her solution. For these reasons, the twelfth solution type *(T2.S12)* was evaluated as an algorithm based conventional solution; and its originality was scored 0.1 ($Or_{F.T2.S12.GGB}=0.1$). The thirteenth solution type was her third solution in task 2, created in the GeoGebra environment. Beren constructed the perpendicular line to |QP| and passing through point O and revealed two isosceles triangles QOP and TQO. Then, she showed the answer with the help of an algebraic solution based on the properties of the angles of a triangle. The way of constructing perpendicular lines was commonly learned and used in mathematics classes and did not require an understanding of the figure's construction in this way. Also, she based the last step in using "the properties of the triangle" on algebraic operations. Hence, the thirteenth solution type was evaluated as algorithm-based, and the originality was scored 0.1 ($Or_{F.T2.S13.GGB}=0.1$). The fourteenth solution type *(T2.S14)* was the fourth solution of Beren, created in the GeoGebra environment. Beren found the midpoint of |TO| that is the point V, and constructed |QV| is perpendicular to |TO| since TQO is an isosceles triangle (which is known from the previous solution). Then, she showed that the measure of angle MOT is 90° with the help of congruent angles. Constructing a perpendicular line with the help of the midpoint of sides was commonly learned in mathematics classes. Also, using congruence angles or congruence triangles was commonly used among the participants. Thus, the fourteenth solution type was a conventional solution, and the originality was score 0.1 ($Or_{F.T2.S14.GGB}=0.1$). Lastly, she created the second solution type (T2.S2) as her last solution in task 2. As explained in the solution of Feyza and Ahmet, the second solution type was evaluated as an unconventional solution and was scored 10 ($Or_{F.T2.S2.GGB}=10$). Therefore, the total originality score of Beren for task 2 was $Or_{B.T2}=10.4$, while the total originality in the GeoGebra environment also was $Or_{B.T2.GGB}=10.4$. Since she did not create any solution in the paper-pencil environment, her originality score in the paper-pencil environment was $Or_{B.T2.PP}=0$.

Kemal (K) produced three different solutions (*T2.S15, T2.S16, and T2.S17)* for task 2. His first solution was the fifteenth solution type *(T2.S15)* of task 2, formed in the GeoGebra environment. Firstly, he showed the knowledge given and hidden within the figure (|QN|=2.|NP| and special triangles). Then, Kemal displayed that the MOQ is an equilateral triangle with an algebraic solution based on a triangle's properties. Then, he used the concept of “the median to the hypotenuse of a right triangle” as taught in mathematics class, without comprehending and explaining its structure. He, as previous participants did, claimed that "a triangle is a right triangle if the segment drawn to the base in a triangle divides the base into two equal parts and its length is equal to these parts, which did not explain the main structure of the concept of “the median to the hypotenuse of a right triangle.” The fifteenth solution type was also evaluated as an algorithm-based conventional solution because there was an algebraic step. He used the concepts learned in the mathematic classes. Thus, the originality of the fifteenth solution way was scored 0.1 ($Or_{K.T2.S15.GGB}=0.1$). The second solution of Kemal was the sixteenth solution type of task 2, which was created in the GeoGebra environment. In the sixteenth solution type, Kemal constructed |TP|, and verbally claimed that “if the rectangle MNPQ is reflected over the x-axis, congruent rectangle would be occurred with the diagonal |TP| since the line segments |NP| and |OT| are parallel and congruent.”. Then, he displayed the triangle MTP is an equilateral triangle, which means |TO| is perpendicular to |MP| because the line segments |MO| and |OP| has an equal length. The verbal claim required reshaping the given figure and based some insight. Also, the concept of “special segments in a special triangle” was rarely used among the participants. For these reasons, the originality of the sixteenth solution type was scored 1 ($Or_{K.T2.S16.GGB}=1$). The last solution of Kemal was the seventeenth solution type of task 2, which was created in the GeoGebra environment. Kemal firstly constructed a perpendicular line segment to |MQ|, passing through point O. Then, he claimed that “If the measure of angle MOT is 90◦, I should be able to apply the Euclid’s Theorem.” While constructing a perpendicular line was commonly learned and used in the mathematics classes, applying Euclid’s Theorem was an algorithm-based. Thus, the seventeenth solution type was an algorithm based conventional solution, and the originality was scored 0.1 (*$Or_{K.T2.S17.GGB}$=1)*. Therefore, the total originality score of Kemal for task 2 was *$Or_{K.T2}$=1.2*, while the total originality in the GeoGebra environment was *$Or_{K.T2.GGB}$=1.2*. Since he did not create any solution in the paper-pencil environment, his originality score in the paper-pencil environment was *$Or_{K.T2.PP}$=0*.

The originality scores of each participant for task 2 are shown in Table 4.11.

Table 4. 11 The originality scores of each participant for task 2

Participants	Originality Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	10	0.2	10.2
Ahmet	12.2	0.2	12.2
Umay	0	0.1	0.1
Beren	10.4	0	10.4
Kemal	1.2	0	1.2

4.2.4 Evaluation of Creativity in Task 2

There were seventeen different solution types for task 2, which were evaluated concerning their fluency, flexibility, and originality, respectively. The creativity of each solution (Cr) and each participant's total creativity (CR) for task 2 were scored depending on the scores of fluency, flexibility, and originality of the solutions. A specific solution (Cri) creativity score was the product of the solution's flexibility and originality score *($Cr_i = Flx_i \cdot Or_i$)*. The total creativity score on task 2 was the sum of the creativity scores on each solution *($Cr = \sum_{i=1}^{n} Flx_i \cdot Or_i$)*. Besides, the total creativity score was separately evaluated, considering the environment in which they were created because not all solutions were produced in the GeoGebra environment. Therefore, the participant's final creativity score was scored as the product of the fluency score and the total creativity score (CR = n ($\sum_{i=1}^{n} Flx_i \cdot$ Or_i)).

Feyza *(F)* produced three different solutions for task 2 *(T2.S1, T2.S2, and T2.S3)*. The flexibility score of the first solution type was $Flx_{F.T2.S1.PP} = 10$, and the originality score of the first solution type was $Or_{F.T2.S1.PP} = 0.1$. The creativity score of her solution ($Cr_{F.T2.S1.PP}$), which was the first solution type of task 2, was scored by multiplying its flexibility and originality score ($Cr_{F.T2.S1.PP} = Flx_{F.T2.S1.PP} \times Or_{F.T2.S1.PP}$); which means the creativity score of the first solution type for the task 2 was 1 ($Cr_{F.T2.S1.PP} = 1$). Similarly, the creativity score of the second and third solution type of task 2 ($Cr_{F.T2.S2.GGB}$ *and* $Cr_{F.T2.S3.PP}$) were scored as the product of their flexibility and originality scores. The creativity of the second solution type was scored 100 ($Cr_{F.T2.S2.GGB} = 100$), and the creativity of the third solution type was scored 0.1 ($Cr_{F.T2.S3.PP} = 0.1$). Thus, the total creativity of Feyza for task 2, which was the sum of the creativity scores on each solution, was scored 101.1 ($Cr_{F.T2} = 101.1$). While her solutions' total creativity were formed in the paper-pencil environment was scored as 1.1 ($Cr_{F.T2.PP} = 1.1$), the total creativity of her solution was created in the GeoGebra environment scored 100 ($Cr_{F.T2.GGB} = 100$). Therefore, the final creativity score of Feyza for task 2 was the product of her fluency score ($N_{F.T2}=3$), and her total creativity score ($Cr_{F.T2} = 101.1$) was scored 303.3 ($CR_{F.T2} = 303.3$) (seen in Table 4.12).

Table 4. 12. Scoring the creativity of Feyza for task 2

	Fluency	**Flexibility**	**Originality**	**Creativity**
T2.S1	1	10	0.1	1
T2.S2	1	10	10	100
T2.S3	1	10	0.1	1
Total	3	30	10.2	**102**
Final creativity of Feyza for task 2				**306**

Note. Solution was created in the GeoGebra environment was highlighted.

Ahmet *(A)* created seven different solutions for task 2 *(T2.S4, T2.S2, T2.S5, T2.S6, T2.S7, T2.S8, and T2.S9)*. The flexibility score of the first solution type was $Flx_{A.T2.S4.PP} = 10$, and the originality score of the first solution type was $Or_{A.T2.S4.PP} = 0.1$. The creativity score of his solution ($Cr_{A.T2.S4.PP}$), which was the fourth solution type of task 2, was scored by multiplying its flexibility and originality score ($Cr_{A.T2.S4.PP} = Flx_{A.T2.S4.PP} \times Or_{A.T2.S4.PP}$); which means the creativity score of the fourth solution type for the task 2 was 1 ($Cr_{A.T2.S4.PP} = 1$). In the same way, the creativity score of subsequent solutions of Ahmet ($Cr_{A.T2.S2.GGB}$, $Cr_{A.T2.S5.GGB}$, $Cr_{A.T2.S6.GGB}$, $Cr_{A.T2.S7.PP}$, $Cr_{A.T2.S8.GGB}$, *and* $Cr_{A.T2.S9.GGB}$) were scored as the product of their flexibility and originality scores. Namely, the creativity of the second solution type was scored 100 ($Cr_{A.T2.S2.GGB} = 100$), the creativity of the fifth solution type was scored 0.1 ($Cr_{A.T2.S5.GGB}= 0.1$), the creativity of the sixth solution type was scored 0.1 ($Cr_{A.T2.S6.GGB}= 0.1$), the creativity of the seventh solution type was scored 0.1 ($Cr_{A.T2.S7.PP}= 0.1$), the creativity of the eighth solution type was scored 1 ($Cr_{A.T2.S8.GGB}= 1$), and the creativity of the ninth solution type was scored 1 ($Cr_{A.T2.S9.GGB}= 0.1$).

Table 4. 13. Scoring the creativity of Ahmet for task 2

	Fluency	**Flexibility**	**Originality**	**Creativity**
T2.S4	1	10	0.1	1
T2.S2	1	10	10	100
T2.S5	1	1	0.1	0.1
T2.S6	1	1	0.1	0.1
T2.S7	1	1	0.1	0.1
T2.S8	1	1	1	1
T2.S9	1	1	1	1
Total	7	25	12.4	103.3
Final creativity of Ahmet for task 2				**723.1**

Note. Solutions were created in the GeoGebra environment was highlighted.

Thus, the total creativity of Ahmet for task 2, which was the sum of the creativity scores on each solution, was scored 103.3 ($Cr_{A.T2}$ = 103.3). While the total creativity of Ahmet's solutions were formed in the paper-pencil environment was scored 1.1 ($Cr_{A.T2.PP}$ = 1.1), the total creativity of his solutions was created in the GeoGebra environment was scored 102.2 ($Cr_{A.T2.GGB}$ = 102.2). Therefore, the final creativity score of Ahmet for task 2 was the product of her fluency score *($N_{A.T2}$=7)* his total creativity score ($Cr_{A.T2}$ = 103.3) was scored 303.3 ($CR_{A.T2}$ = 723.1) (seen in Table 4.13).

Umay *(U)* produced one solution for task 2 *(T2.10)*. The flexibility score of the tenth solution type was $Flx_{U.T2.S10.PP} = 10$, and the originality score of the tenth solution type was $Or_{U.T2.S10.PP} = 0.1$. The creativity of the tenth solution type ($Cr_{U.T2.S10.PP}$) was scored by multiplying its flexibility and originality score ($Cr_{U.T2.S10.PP} = Flx_{U.T2.S10.PP} \, x \, Or_{U.T2.S10.PP}$, which means the tenth solution type's creativity score for task 2, was 1 ($Cr_{U.T2.S10.PP} = 1$). Since Umay formed just one solution for task 2, her total creativity score for task 2 was 1 ($Cr_{U.T2} =1$). While the total creativity of her solution was formed in the paper-pencil environment was scored 1 ($Cr_{U.T2.PP}$ = 1), the total creativity in the GeoGebra environment was scored 0 ($Cr_{U.T2.GGB}$ = 0). Therefore, the final creativity score of Umay for task 2 was the product of her fluency score ($N_{U.T2}=1$), and her total creativity score ($Cr_{U.T2} = 1$), was scored 1 ($CR_{U.T2} = 1$) (seen in Table 4.14).

Table 4. 14. Scoring the creativity of Umay for task 2

	Fluency	Flexibility	Originality	Creativity
S7	1	10	0.1	1
Total	1	10	0.1	1
Final creativity of Umay for task 2				**1**

Beren *(B)* formed five different solutions for task 2 *(T2.S11, T2.S12, T2.S13, T2.S14, and T2.S2).* The flexibility score of the eleventh solution type was $Flx_{B.T2.S11.GGB} = 10$, and the originality score of the eleventh solution type was $Or_{B.T2.S11.GGB} = 0.1$. The creativity score of her solution ($Cr_{B.T2.S11.GGB}$), which was the eleventh solution type of the task 2, was scored by multiplying its flexibility and originality score ($Cr_{B.T2.S11.GGB} = Flx_{B.T2.S11.GGB}$ x $Or_{B.T2.S11.GGB}$); which means the creativity score of the eleventh solution type for the task 2 was 1 ($Cr_{B.T2.S11.GGB} =1$). Similarly, the creativity score of subsequent solutions of Beren ($Cr_{B.T2.S12.GGB}$, $Cr_{B.T2.S13.GGB}$, $Cr_{B.T2.S14.GGB}$, *and* $Cr_{b.T2.S2.GGB}$) was scored as the product of their flexibility and originality scores Especially, the creativity of the twelfth solution type was scored 0.1 ($Cr_{B.T2.S12.GGB}= 0.1$), the creativity of the thirteenth solution type was scored 0.1 ($Cr_{B.T2.S13.GGB}= 0.1$), the creativity of the fourteenth solution type was scored 1 ($Cr_{B.T2.S14.GGB}= 1$), *and* the creativity of the second solution type was scored 100 ($Cr_{B.T2.S2.GGB} = 100$). Thus, the total creativity of Beren for task 2 was the sum of the creativity scores on each solution, which was scored 102.2 ($Cr_{B.T2}$ = 102.2). While the total creativity in the paper-pencil environment was scored 0 ($Cr_{B.T2.PP}$ = 0), the total creativity of her solutions were created in the GeoGebra environment was scored 102.2 ($Cr_{B.T2.GGB}$ = 102.2). Therefore, the final creativity score of Beren for task 2 was the product of her fluency score ($N_{B.T2}=5$), and her total creativity score ($Cr_{B.T2}$ = 102.2) was scored 511 ($CR_{B.T2}$ = 511).

Table 4. 15. Scoring the creativity of Beren for task 2.

	Fluency	Flexibility	Originality	Creativity
T2.S11	1	10	0.1	1
T2.S12	1	1	0.1	0.1
T2.S13	1	1	0.1	0.1
T2.S14	1	10	0.1	1
T2.S2	1	10	10	100
Total	5	32	10.4	102.2
Final creativity of Beren for task 2				**511**

Note. Solutions were created in the GeoGebra environment was highlighted.

Kemal *(K)* formed three different solutions for task 2 *(T2.S15, T2.S16, and T2.S17).* The flexibility score of the fifteenth solution type was $Flx_{K.T2.S15.GGB} = 10$, and the originality score of the fifteenth solution type was $Or_{K.T2.S15.GGB} = 0.1$. The creativity score of his solution ($Cr_{K.T2.S15.GGB}$) was scored by multiplying its flexibility and originality score ($Cr_{K.T2.S15.GGB} = Flx_{K.T2.S15.GGB}$ x $Or_{K.T2.S15.GGB}$); which means the creativity score of the seventeenth solution type for the task 2 was 1 ($Cr_{K.T2.S15.GGB} =1$). Similarly, the creativity score of subsequent solutions of Kemal ($Cr_{K.T2.S16.GGB}$ *and* $Cr_{K.T2.S17.GGB}$) was scored as the product of their flexibility and originality scores. The creativity of the sixteenth solution type was scored 1 ($Cr_{K.T2.S16.GGB}= 1$), and the creativity of the seventeenth solution type was scored 0.1 ($Cr_{K.T2.S17.GGB}= 0.1$). Thus, the total creativity of Kemal for task 2 was the sum of the creativity scores on each solution, which was scored 2.1 ($Cr_{K.T2}$ = 2.1). While the total creativity in the paper-pencil environment was scored 0 ($Cr_{K.T2.PP}$ = 0), his solutions' total creativity was created in the GeoGebra environment was scored 2.1 ($Cr_{K.T2.GGB}$ = 2.1). Therefore, the final creativity score of Kemal for task 2 was the product of his fluency score ($N_{K.}T2=3$), and his total creativity score ($Cr_{K.T2}$ = 2.1) was scored 6.3 ($CR_{B.T2}$ = 6.3).

Table 4. 16. Scoring the creativity of Kemal for task 2.

	Fluency	**Flexibility**	**Originality**	**Creativity**
T2.S15	1	10	0.1	1
T2.S15	1	1	1	1
T2.S17	1	1	0.1	0.1
Total	3	12	1.2	2.1
Final creativity of Kemal for task 2				**6.3**

Note. Solutions were created in the GeoGebra environment was highlighted.

4.3 Evaluation of Task 3

This section provides the solution types created by the pre-service mathematics teachers for task 3 with the mathematical ideas used in solutions and the analysis of task 3 considering the fluency (N), flexibility (Flx), originality (Or), and creativity (Cr) of solutions of each participant. The participants were asked to solve task 2 in more than one way, with at least one solution being in the GeoGebra environment within approximately forty-five minutes. The participants were free to create their solutions in the paper-pencil or GeoGebra environments or used both of them.

Each pre-service teacher solved task 3 more than one way and used various mathematical ideas in their solutions. When all the solutions produced for task 3 were examined, it was decided that the basic mathematical ideas in task 3 were the followings; constructing a circle, congruency, the properties of special triangles, corresponding angles, the properties of a circle, the properties of a rectangle, rotation, reflection, Pythagorean Theorem, Euclid's Theorem, and the properties of a parallelogram.

All solutions were divided into the parts which were determined concerning the mathematical ideas used. There were thirteen different solution types for task 3. Then, each solution type, different from the other, was specified with a unique code to indicate which task the solution belongs to. For instance, the first solution type of task 3 was shown as *T3.S1 (the first solution type for task 3),* or the third solution type of task 3 was established as *T3.S3 (the third solution type for task 3).* There were thirteen different solution types for task 3, and all solution types were shown in detail with the mathematical concepts used (Table 4.38 to 4.50).

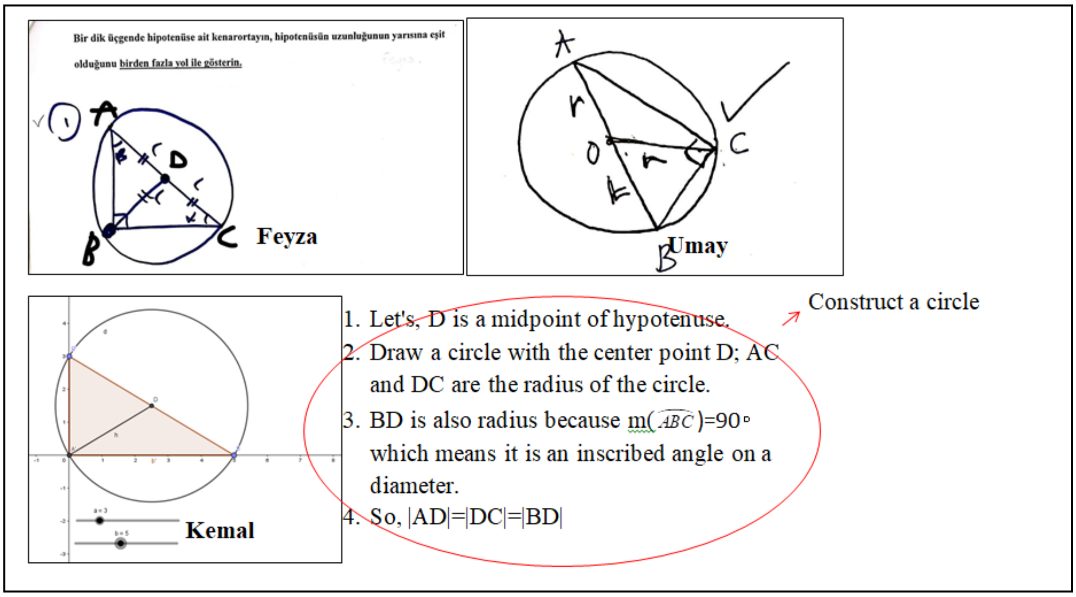


Figure 4. 38 T3.S1 (the first solution type of task 3)

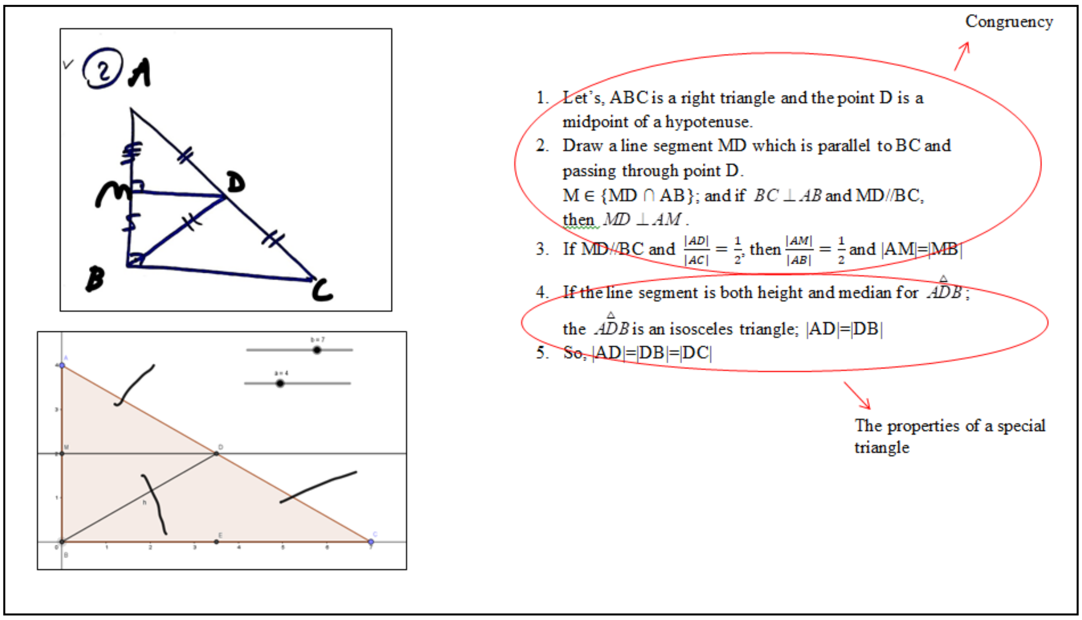


Figure 4. 39 T3.S2 (the second solution type of task 3)

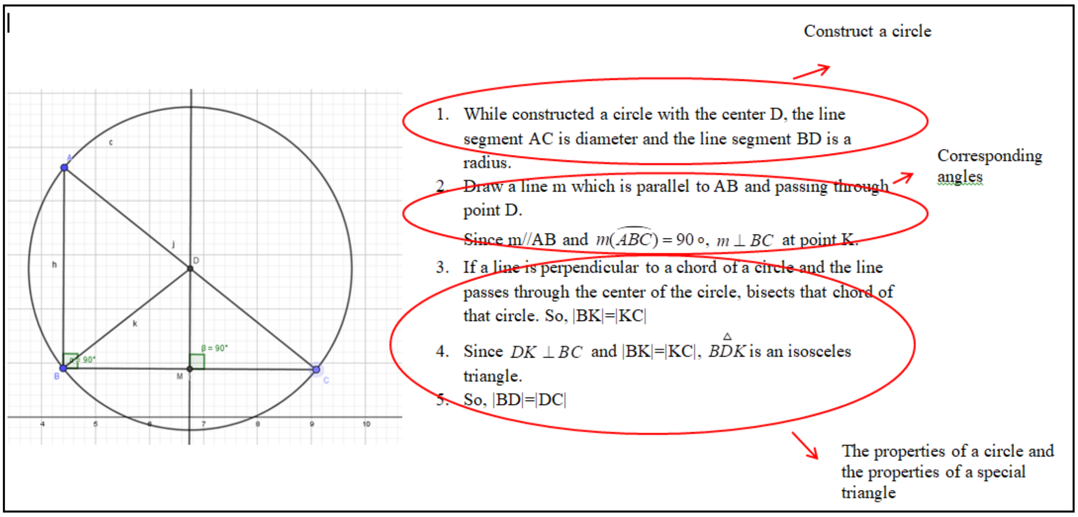


Figure 4. 40 T3.S3 (the third solution type of task 3)

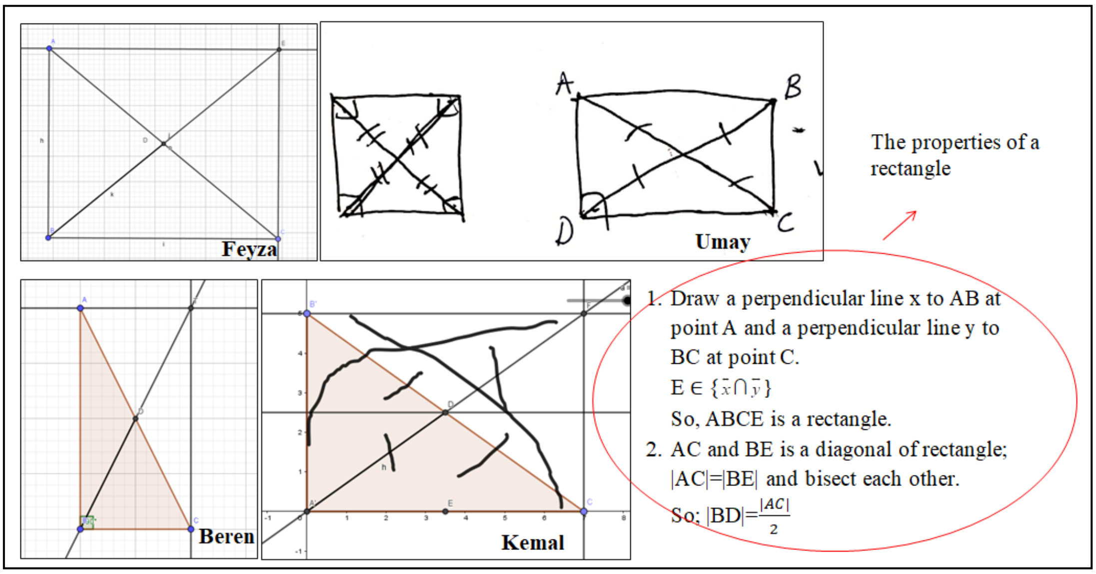


Figure 4. 41 T3.S4 (the fourth solution type of task 3)

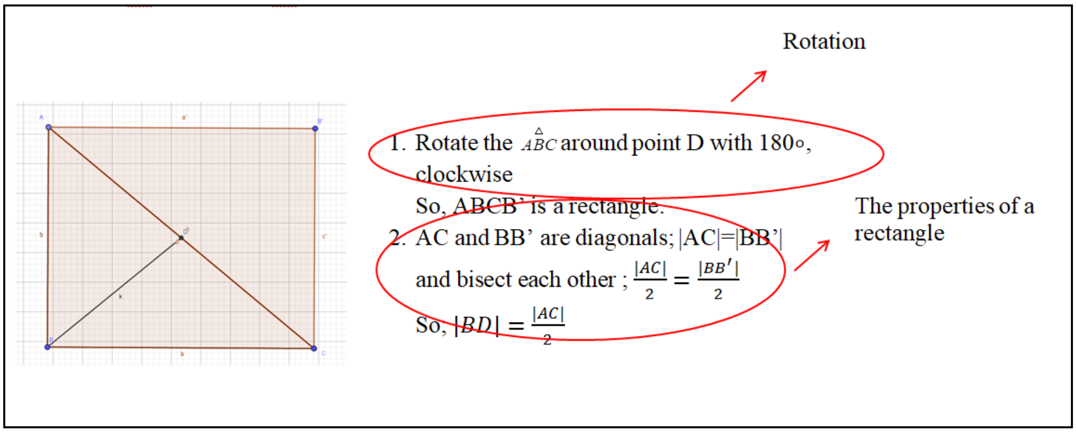


Figure 4. 42 T3.S5 (the fifth solution type of task 3)

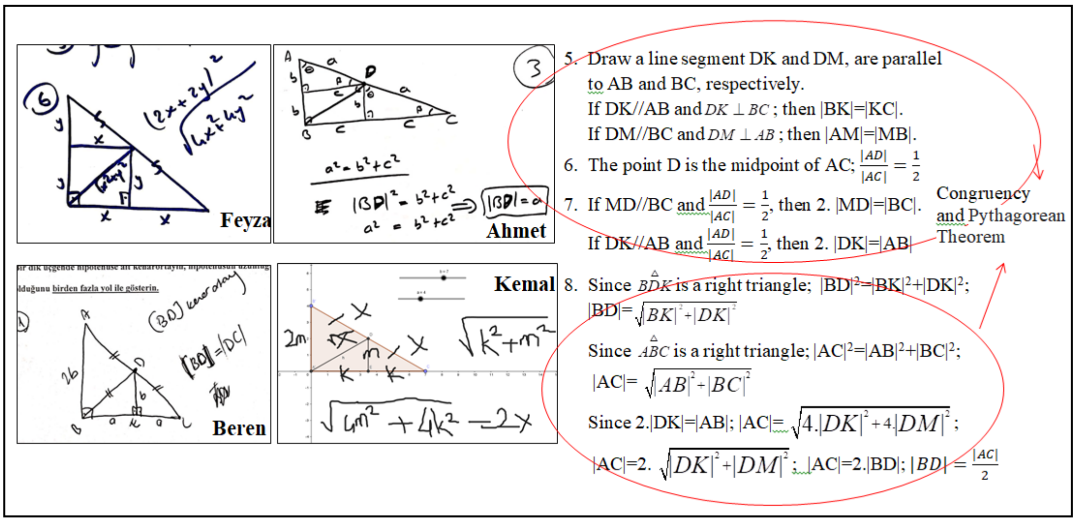


Figure 4. 43 T3.S6 (the sixth solution type of task 3)

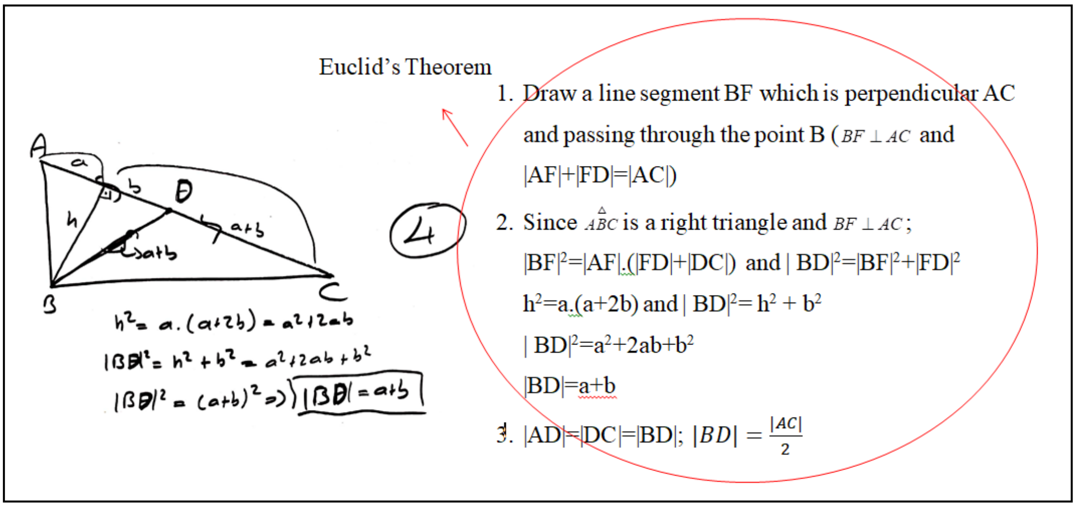


Figure 4. 44 T3.S7 (the seventh solution type of task 3)

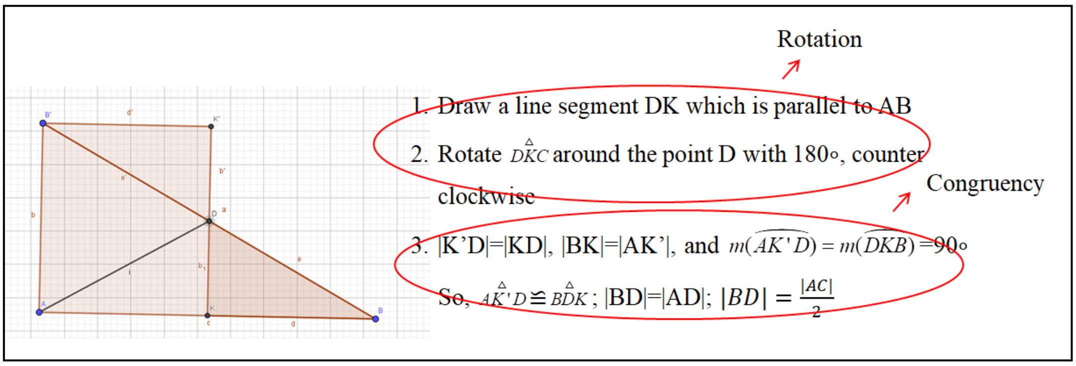


Figure 4. 45 T3.S8 (the eighth solution type of task 3)

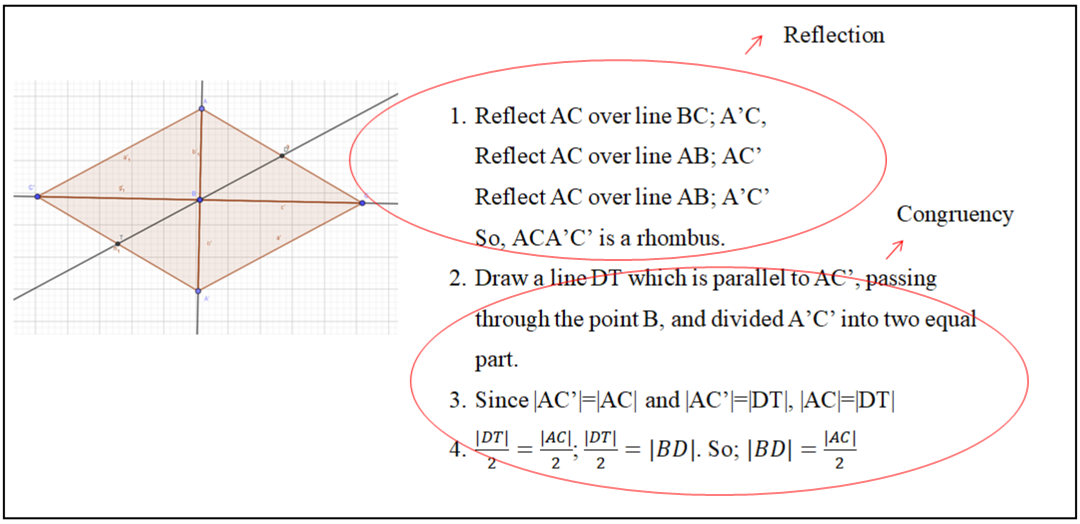


Figure 4. 46 T3.S9 (the ninth solution type of task 3)

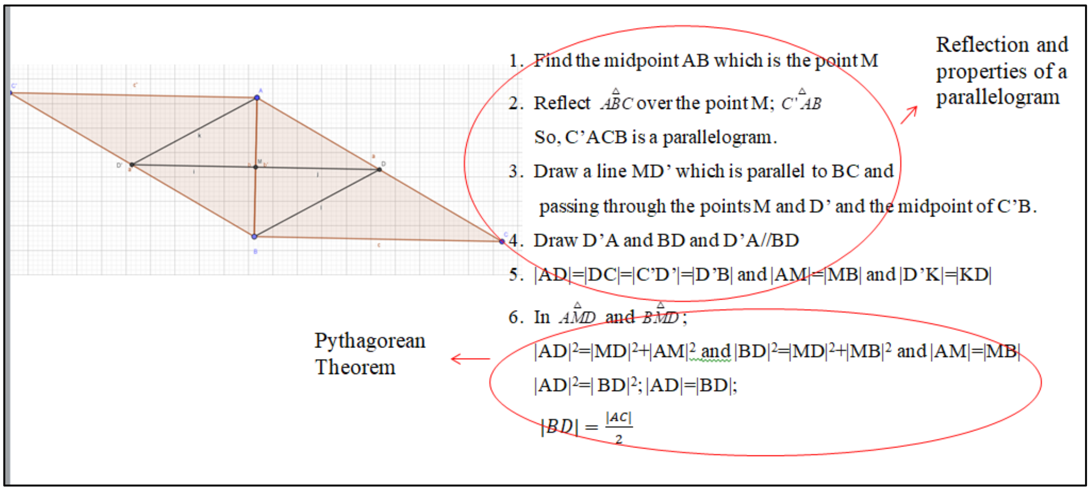


Figure 4. 47 T3.S10 (the tenth solution type of task 3)

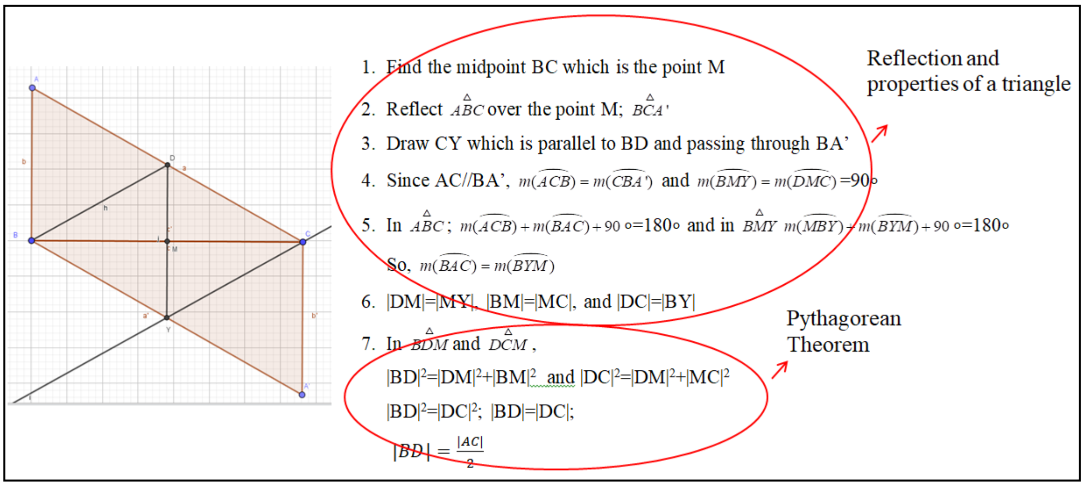


Figure 4. 48 T3.S11 (the eleventh solution type of task 3)

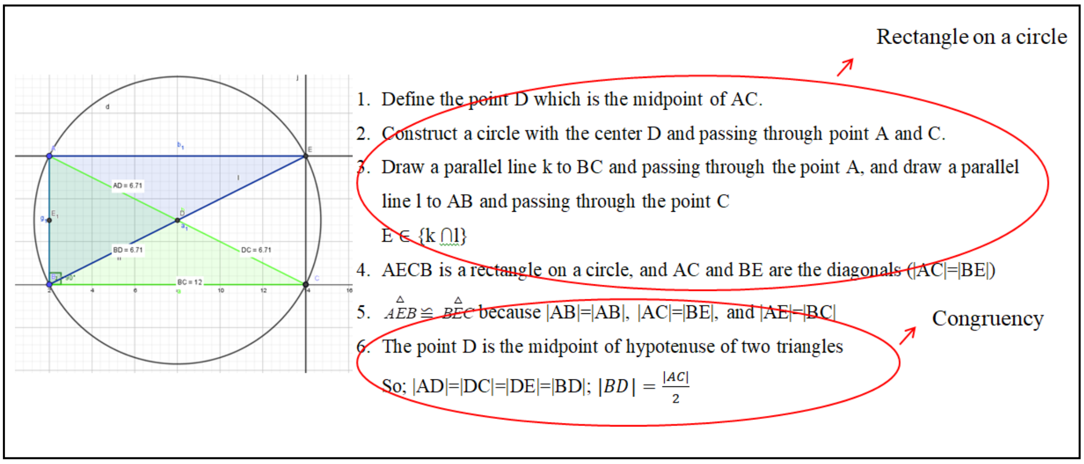


Figure 4. 49 T3.S12 (the twelfth solution type of task 3)

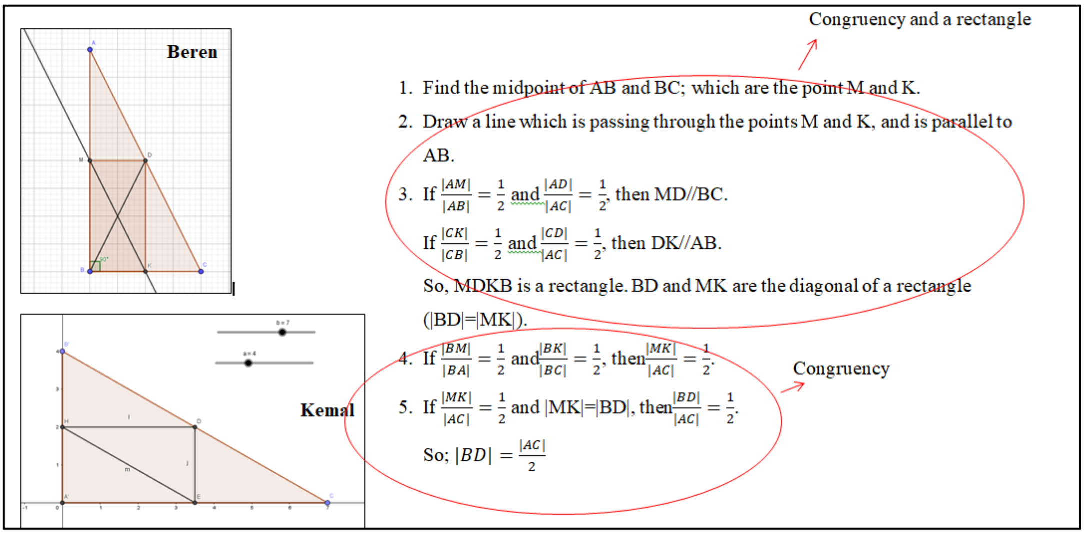


Figure 4. 50 T3.S13 (the twelfth solution type of task 3)

4.3.1 Evaluation of Fluency in Task 3

The fluencies (*N*) of the pre-service teachers were scored individually by viewing their solutions for task 3, based on the number of their solutions in the individual solution space. Since the pre-service were free to choose an environment to solve task 3, there were solutions in both the paper-and-pencil environment and GeoGebra environment. The fluencies of the solutions were also evaluated, considering the environment separately. Moreover, each solution's fluency was assigned a unique code that displays to whom the solution belongs and in which environment it was solved. For example, the participant, Feyza *(F)*, created six different solutions *(T3.S1, T3.S2, T3.S3, T3.S4, T3.S5, and T3.S6)* for task 2, three of them were in the paper-and-pencil environment, and three of them were in the GeoGebra environment. Feyza created her first solution for task 3 in the paper-pencil environment, and the fluency of this solution was displayed as $N_{F.T3.S1.PP.}$ She created her third solution for task 3 in the GeoGebra environment, and the fluency of this solution was displayed as $N_{F.T3.S3.GGB}$. For each solution 1 point was assigned ($N_{F.T3.S1.PP} = N_{F.T3.S2.PP} = N_{F.T3.S3.GGB} = N_{F.T3.S4.GGB} = N_{F.T3.S5.GGB} = N_{F.T3.S6.PP} = 1$). Therefore, the total fluency of Feyza for task 3 was $N_{F.T3} = 6$, while the total fluency of Feyza for task 3 in the paper-pencil environment was $N_{F.T3.PP} = 3$, and the total fluency of Feyza for task 3 in the GeoGebra environment was $N_{F.T3.GGB} = 3$.

The second participant, Ahmet *(A)*, created eight different solutions *(T3.S5, T3.S6, T3.S7, T3.S8, T3.S8, T3.S9, T3.S10, and T2.S11)* for task 3; two of them were in the paper-pencil environment. Six of them were in the GeoGebra environment (T3.S8 was created twice by Ahmet with small change). For each solution 1 point was assigned ($N_{A.T3.S5.GGB} = N_{A.T3.S6.PP} = N_{A.T3.S7.PP} = N_{A.T3.S8.GGB} = N_{A.T2.S8.GGB} = N_{A.T2.S9.GGB} = N_{A.T2.S10.GGB} = N_{A.T2.S11.GGB} = 1$). Therefore, the total fluency of Ahmet for the task 3 was $N_{A.T3} = 8$, while the total fluency of Ahmet for the task 3 in the paper pencil environment was $N_{A.T3.PP} = 2$ and the total fluency of Ahmet for the task 3 in the GeoGebra environment was $N_{A.T3.GGB} = 6$.

The third participant, Umay *(U)*, created three different solutions *(T3.S1, T3.S4, and T3.S12)* for task 3; two were in the paper-pencil environment, and one of them was in the GeoGebra environment. For each solution 1 point was assigned ($N_{U.T3.S1.PP} = N_{U.T3.S4.PP} = N_{U.T3.S12.GGB} =1$). Therefore, the total fluency of Umay for task 3 was $N_{U.T3} = 3$, while the total fluency of Umay for task 3 in the paper-pencil environment was $N_{U.T3.PP} = 2$, and the total fluency of Umay for task 3 in the GeoGebra environment was $N_{U.T3.GGB} = 1$.

The fourth participant, Beren *(B)*, created three different solutions *(T3.S4, T3.S6, and T3.S13)* for task 3; one of them was in the GeoGebra environment, and two of them were in the GeoGebra environment. For each solution 1 point was assigned ($N_{B.T3.S6.PP} = N_{B.T3.S4.GGB} = N_{B.T3.S13.GGB} = 1$). Therefore, the total fluency of Umay for task 3 was $N_{B.T3} = 3$, while the total fluency of Beren for task 3 in the paper-pencil environment was $N_{B.T3.PP} = 1$, and the total fluency of Beren for task 3 in the GeoGebra environment was $N_{B.T3.GGB} = 2$.

The fifth pre-service teacher, Kemal *(K)*, created five different solutions *(T3.S1, T3.S2, T3.S4, T3.S6, and T3.S13)* for task 3 and all of them were in the GeoGebra environment. For each solution 1 point was assigned ($N_{K.T3.S1.GGB} = N_{K.T3.S2.GGB} = N_{K.T3.S4.GGB} = N_{K.T3.S6.GGB} = N_{K.T3.S13.GGB} = 1$). Therefore, the total fluency of Kemal for the task 3 was $N_{K.T3} = 5$ while the total fluency of Kemal for the task 3 in the GeoGebra environment was $N_{K.T3.GGB} = 5$.

The fluency scores of each participant for task 3 are shown in Table 4.17.

Table 4. 17 Fluency scores of each participant for task 3

Participants	Fluency Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	3	3	6
Ahmet	6	2	8
Umay	1	2	3
Beren	2	1	3
Kemal	5	0	5

4.3.2 Evaluation of Flexibility in Task 3

As stated in the previous section, there were thirteen different solution types for task 3. While each of them was different solution types, some of them included common mathematical ideas within. The flexibility *(Flx)* of each solution was evaluated individually based on the differentiation of mathematical ideas used. There were solutions solved in both the paper-pencil environment and GeoGebra environment since the pre-service were free to solve the task in both environments. The flexibility of each solution was assigned a unique code, which displays to whom the solution belongs and in which environment it was solved. For example, the third participant, Umay *(U)*, created three different solutions *(T3.S1, T3.S4, and T3.S12)*. The two of them *(T3.S1 and T3.S4)* were in the paper-pencil environment, and the flexibilities of these solutions were displayed as $Flx_{U.T3.S1.PP}$ and $Flx_{U.T3.S4.PP}$ (the flexibility of the first and fourth solution types that Umay produced in the paper-pencil environment for task 3). One of the solutions was in the GeoGebra environment, and the flexibility of this solution was demonstrated as $\mathrm{Flx}_{U.T3.S12.GGB}$ (the flexibility of the twelfth solution types that Umay produced in the GeoGebra environment for task 3). Hence, the total flexibility of Umay for task 2 was shown as $\mathrm{Flx}_{U.T3}$.

Feyza (F) created six different solutions for task 3 *(T3.S1, T3.S2, T3.S3, T3.S4, T3.S5, and T3.S6).* The first solution type (T3.S1) was the first appropriate solution of Feyza for task 3, which means its flexibility was scored 10. Since Feyza created the first solution type in the paper-pencil environment, it is shown as $Flx_{F.T3.S1.PP}=10$. In the first solution type, Feyza claimed that "If I draw a circle with the center D and diameter AC, and $m\angle(ABC)=90°$, then B is on the circle. Thus, BD is a radius and half of the hypotenuse". However, in the second solution type (T3.S2), she used congruency and the special triangle properties, which means the mathematical concepts used were different from the previous solution. Thus, the flexibility of the second solution type was scored 10. Since it was created in the paper-pencil environment, it was shown as $Flx_{F.T3.S2.PP}=10$. In the third solution type (T3.S3), Feyza solved task 3 using her previous solutions' mathematical concepts. For example, she constructed a circle with the center O and diameter AB. Then, she constructed the parallel line to AB and showed corresponding angles based on the parallelism between the line and AB. While "constructing a circle" was used in the first solution type, "parallelism between the lines" was used in the second solution type. Also, in the last step of both the second and third solution type, she showed the isosceles triangles (ADB and BDK) to complete her solution, respectively. Thus, the third solution type's flexibility was scored as 1 because Feyza used the mathematical concepts that shaped the third solution way in her previous solutions. When it was created in the GeoGebra environment, it was shown as $Flx_{F.T3.S3.GGB}=1$. In the fourth solution type, Feyza constructed a rectangle. She firstly drew a perpendicular line x to AB at point A and a perpendicular line y to BC at point C. Point E was the intersection point of the line x and y, and ABCE was a rectangle. Then, she defined the line segments AB and BE as the diagonal of a rectangle, and they bisected each other. She asserted that "If one of the diagonal were defined as the hypotenuse of a right triangle, then the median of hypotenuse would be the half of the hypotenuse." Until the fourth solution type, Feyza did not use the properties of the rectangle. The fourth solution type was considered completely different, as it did not contain common mathematical concepts with her previous solutions. So, its flexibility was scored 10, and since it was formed on the GeoGebra environment, it was shown as $Flx_{F.T3.S4.GGB}=10$. In fact, in the fifth solution type, Feyza again used the properties of a rectangle. However, in the fifth solution type, the method she used to construct the rectangle was different from the previous one. These two solution types were evaluated as different because the construction method is different and includes a new mathematical concept called rotation. Thus, the fifth solution type's flexibility was scored 1 since it was not completely different from her previous solutions, and it was shown as $Flx_{F.T3.S5.GGB}=1$. The last solution of Feyza was the sixth solution type of task 3. Feyza drew line segments DK and DM, which were parallel to AB and BC, respectively. Then, she displayed the congruent lines (e.g., |BK|=|KC|,

|AM|=|MB|, 2. |MD|=|BC|, and 2. |DK|=|AB|) depending on the parallelism between the lines (e.g., DK//AB and DM//BC). In the last step of the solution type, she used the Pythagorean Theorem in the right triangle ABC. Although she did not use the Pythagorean Theorem in her previous solutions, she formed a congruency by drawing the parallel line segment to AB in the second solution type, which means the first part of the sixth solution type was almost the same as the second solution type. However, since the Pythagorean Theorem was not used in the previous solutions, the sixth solution type's flexibility was scored as 1, and it was shown as $Flx_{F.T3.S6.PP}=1$. Therefore, the total flexibility of Feyza for task 3 was $Flx_{F.T3}=33$, while the total flexibility of Feyza for task 3 in the paper-pencil environment was $Flx_{F.T3.PP}=21$, and the total flexibility of Feyza for task 3 in the GeoGebra environment were $Flx_{F.T3.GGB}=12$.

Ahmet (A) created eight different solutions *(T3.S5, T3.S6, T3.S7, T3.S8, T3.S8, T3.S9, T3.10, and T3.S11)* for task 3. The fifth solution type *(T3.S5)* was the first appropriate solution of Ahmet for task 3, which means its flexibility was scored 10. Since the fifth solution type was in the GeoGebra environment, it is shown as $Flx_{A.T3.S5.GGB}=10$. Then, he solved the task with the sixth solution type (T3.S6) in the paper-pencil environment. In the fifth solution type, Ahmet rotated the triangle ABC around point D with 180∘, clockwise to construct a rectangle, then he used the properties of a rectangle. However, in the sixth solution type, he drew a line segments DK and DM, which were parallel to AB and BC, respectively. Then, she displayed the congruent lines (e.g. |BK|=|KC|, |AM|=|MB|, 2. |MD|=|BC|, and 2. |DK|=|AB|) depending on parallelism between the lines (e.g., DK//AB and DM//BC). In the last step, she used the Pythagorean Theorem in the right triangles BDK and ABC. Obviously, there was no common mathematical idea used in the two solution types. Thus, the flexibility of the sixth solution type was scored as 10 and shown as $Flx_{A.T3.S6.PP}=10$. Similarly, the flexibility of the seventh solution type (T3.S7) was scored as 10. Namely, in the seventh solution type, Ahmet drew a line segment BF which was perpendicular to AC, and passing through the point B. He then used the Euclid’s theorem to display the equality of the line segments AD, DC, and BD. The mathematical concepts used in the seventh solution type completely different from the mathematical concepts used in his previous solutions. Since it was formed in the paper-pencil environment, it was shown as $Flx_{A.T3.S7.PP}=10$. In the eighth solution type, Ahmet constructed a parallel line segment to AB, and triangle DKC was shaped. Then, he rotated the triangle DKC around the point D with 180°, counterclockwise. He displayed $|BD| = \frac{|AC|}{2}$ with the help of the congruency between triangle AK'D and BDK. The mathematical concepts used in the eighth solution type (e.g. rotation and congruency) were used in his fifth and sixth solution types, respectively. Thus, the flexibility of the eighth solution way was 1, and shown as $Flx_{A.T3.S8.GGB}=1$. The solution that Ahmet created from now on was again the eighth solution type. In this solution, he drew a line segment DM which was parallel to BC, and formed the triangle DMB. Then, he rotated it around point D with 180°, clockwise. Again, he displayed $|BD| = \frac{|AC|}{2}$ with the help of the congruency between triangles BMD and DM'C. It is clear that Ahmet repeated his previous solution in this solution. Thus, the flexibility of the eighth solution type, which was formed for the second time, was scored as 0.1 and shown as $Flx_{A.T3.S8.GGB}=0.1$. Also, he created the ninth solution type (T3.S9) in the GeoGebra environment. Ahmet, again, used the congruency between the lines in the ninth solution type. However, he constructed a rhombus by using reflection over the line. Although congruency was used in his previous solutions, there was no solution that was exactly the same as the ninth solution. Thus, the flexibility of the ninth solution type was scored as 1, and shown as $Flx_{A.T3.S9.GGB}=1$. Then, he created the tenth solution type (T3.S10) for task 3 in the GeoGebra environment. She formed a parallelogram C'ACB by reflecting the triangle ABC over the point M which was the midpoint of AB. Then, she used the Pythagorean Theorem in the triangles AMD and BMD which were constructed by using the properties of a parallelogram. The mathematical concepts used in the tenth solution type (e.g. reflection and Pythagorean Theorem) were previously used in the ninth solution type and the sixth solution type, respectively. Thus, the flexibility of the tenth solution type was score 1, and shown as $Flx_{A.T3.S10.GGB}=1$. The last solution of Ahmet was the eleventh solution type (T3.S11), which was formed in the GeoGebra environment. In the eleventh solution type, he used the properties of the triangle as different from the ninth solution type. So, the eleventh solution type's flexibility was 1 because it was not completely the same as the previous one. It was shown as $Flx_{A.T3.S11.GGB}=1$. Therefore, the total flexibility of Ahmet for task 3 was $Flx_{A.T3}=34.1$ while the total flexibility of Ahmet for task 3 in the paper-and-pencil environment was $Flx_{A.T3.PP}=20$ and the total flexibility of Ahmet for task 3 in GGB environment was $Flx_{A.T3.GGB}=14.1$.

Umay (U) generated three different solutions (T3.S1, T3.S4, and T3.S12) for task 3. She formed two of her solution in the paper-pencil environment and one of them in the GeoGebra environment. The first solution type was her first solution for task 3, and its flexibility was scored 10 ($Flx_{U.T3.S1.PP}=10$). Secondly, she created the fourth solution type (T3.S4) in the paper-pencil environment. She drew the rectangle ABCD and drew the line segment AC, divided the rectangle into two equal right triangles. She displayed the $|BO| = \frac{|AC|}{2}$ by using the properties of a rectangle. However, the first solution type based on the circle, which means the second solution of Umay was entirely different from the previous one. Thus, the flexibility of the fourth solution type scored 10 ($Flx_{U.T3.S4.PP}=10$). The last solution of Umay for task 3 was the twelfth solution type *(T3.S12)*. In the twelfth solution type, Umay first defined the midpoint of AC, point D, and constructed a circle with the center D and passing through the points A and C. Then, she drew a parallel line k to BC and passing through point A, and drew a parallel line l to AB and passing through point C. Point E was the intersection of the line k and l and formed the rectangle ABCD. Using the properties of a rectangle, she displayed two congruent triangles (AEB and BEC), and completed the solution.

Umay also used the rectangle properties in the fourth solution type, which means the twelfth solution type was not different from her previous solution. Thus, the twelfth solution type's flexibility was scored 1 and shown as ($Flx_{U.T3.S12.GGB}=1$). Therefore, the total flexibility of Umay for task 3 was $Flx_{U.T3}=21$, while the total flexibility of Umay for task 3 in the paper-and-pencil environment was $Flx_{U.T3.PP}=20$ and the total flexibility of Umay for task 3 in GeoGebra environment was $Flx_{U.T3.GGB}=1$.

Beren (B) formed three different solutions (T3.S6, T3.S4, and T3.S13) for task 3. She formed two of her solution in the GeoGebra environment and one of them in the paper-pencil environment. The sixth solution type was her first solution for task 3, and its flexibility was scored 10 ($Flx_{B.T3.S6.PP}=10$). Secondly, she created the fourth solution type (T3.S4) in the GeoGebra environment. As mentioned in previous paragraphs, the fourth solution type was based on the properties of a rectangle. However, in the sixth solution type, Beren drew the line segments DK and DM, which were parallel to AB and BC, respectively. Then, she displayed the congruent lines (e.g., |BK|=|KC|, |AM|=|MB|, 2. |MD|=|BC|, and 2. |DK|=|AB|) depending on the parallelism between the lines (e.g., DK//AB and DM//BC). In the last step of the solution type, she used the Pythagorean Theorem in the right triangle ABC. Thus, the fourth solution type was different from the previous one, and its flexibility was scored 10 ($Flx_{B.T3.S4.GGB}=10$). The last solution of Beren was the thirteenth solution type (T3.S13), which was created in the GeoGebra environment. In the thirteenth solution type, Beren defined M and K, which were the midpoints of AB and BC, respectively. Then, she constructed a rectangle MDKB, within the right triangle ABC, by using the congruency between the lines. In the last step of the solution, she again used congruency to complete the solution. The thirteenth solution type was not completely the same as her previous solutions. Yet, it included the common concepts (e.g. the properties of a rectangle and congruency) with her previous solutions. Thus, its flexibility was scored 1 and shown as ($Flx_{B.T3.S12.GGB}=1$). Therefore, the total flexibility of Beren for task 3 was $Flx_{B.T3}=21$, while the total flexibility of Beren for task 3 in the paper-pencil environment was $Flx_{B.T3.PP}=10$ and the total flexibility of Beren for task 3 in the GeoGebra environment were $Flx_{B.T3.GGB}=11$.

Kemal *(K)* generated five different solutions *(T3.S1, T3.S6, T3.S13, T3.S4, and T3.S2)* for task 3, and all of them were formed in the GeoGebra environment. The first solution type *(T3.S1)* was the first appropriate solution of Kemal for task 3, which means the flexibility of it was scored as 10 ($Flx_{K.T3.S1.GGB}=10$). The second appropriate solution of Kemal was the sixth solution type (T3.S6). While the first solution type based on constructing a circle, Kemal used the congruency and the Pythagorean Theorem in the sixth solution type. Namely, the sixth solution type was different from his previous solution, and its flexibility was scored 10 ($Flx_{K.T3.S6.GGB}=10$). Thirdly, Kemal formed the thirteenth solution type (T3.S13) in the GeoGebra environment. In the thirteenth solution type, Kemal defined M and K, which were the midpoints of AB and BC, respectively. Then, she constructed a rectangle MDKB, within the right triangle ABC, by using the congruency between the lines. In the last step of the solution, she again used congruency to complete the solution. Similarly, he also drew the line segments DK and DM in the sixth solution way as in the thirteenth. Yet, he used a rectangle's properties in the thirteenth solution type while he used the Pythagorean Theorem in the sixth solution type. Hence, the thirteenth solution's flexibility was score 1 ($Flx_{K.T3.S13.GGB}=1$) because there was a common mathematical concept used before. Moreover, he created the fourth solution type (T3.S4) in the GeoGebra environment. The fourth solution type was dependent on the properties of a rectangle. The concept of a rectangle was used before in his solution ways, which means the fourteenth solution type's flexibility was 1 ($Flx_{K.T3.S4.GGB}=1$) because it was not entirely the same as the previous solutions. The last solution of Kemal was the second solution type (T3.S2). In the second solution type, he constructed a line segment MD (the point M was the midpoint of hypotenuse of a right triangle ABC) parallel to the line segment BC. Then, she displayed the congruent lines with the help of MD//BC. The sixth solution type included what has been created so far in the second solution type. However, in the other part of the second solution type, Kemal used a special triangle's properties ADB, which was an isosceles triangle. Since this step was different from the sixth solution type, the second solution type's flexibility was scored 1 ($Flx_{K.T3.S2.GGB}=1$). Therefore, the total flexibility of Kemal for task 3 was $Flx_{K.T3} = 23$, while the total flexibility of Kemal for task 3 in the GeoGebra environment was $Flx_{K.T3.GGB} = 23$, and the total flexibility of Kemal for task 3 in the paper-pencil environment was $Flx_{K.T3.PP} = 0$. Table X presents the flexibility scores of each participant for task 3.

The flexibility scores of each participant for task 3 are shown in Table 4.19.

Table 4. 18 Flexibility scores of each participant for task 3

Participants	Flexibility Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	12	21	33
Ahmet	14.1	20	34.1
Umay	1	20	21
Beren	11	10	21
Kemal	23	0	23

4.3.3 Evaluation of Originality in Task 3

There were thirteen different solution types in task 3. While some of the solutions were insight-based or unconventional solutions, some were model-based or partly unconventional solutions. Some of them were algorithm-based or conventional solutions. The originality *(Or)* of each solution type was scored depend on its conventionality and insight level. Each participant's total originality score on task 3 was the sum of each solution's originality score. Also, there were solutions in both the paper-and-pencil and dynamic geometry environments since the participants were free to choose both. The originalities of the solutions were also evaluated, considering the environment separately. Each solution's originality was assigned a unique code that displays to whom the solution belongs and in which environment was created. For example, the third participant, Umay (U), created three different solutions *(T3.S1, T3.S4, and T3.S12).* Two of the solutions *(T3.S1 and T3.S4)* were in the paper-and-pencil environment, and the originalities of these solutions were displayed as $Or_{U.T.S1.PP}$ *(the originality of the first solution type that Umay produced in the paper-pencil environment for the task 3)* and $Or_{U.T3.S4.PP}$ (*the originality of the fourth solution type that Umay produced in the paper-pencil environment for task 3).* One of the solutions was in the GeoGebra environment, and the originality of this solution was displayed as $Or_{U.T3.S12.GGB}$. Hence, the total originality of Umay for task 3 was shown as $Or_{U.T3}$ *(the originality of Umay for task 3).*

Feyza (F) created six different solutions (T3.S1, T3.S2, T3.S3, T3.S4, T3.S5, and T3.S6) for task 3. Her first appropriate solution was the first solution type of task 3, created in the paper-pencil environment. In the first solution type, she claimed that "If I draw a circle with the center D and diameter AC, and $m\angle$ (ABC) = 90∘; then B is on the circle. Thus, BD is a radius and half of the hypotenuse". The first solution type was not based on algebraic operations and required comprehending the task's structure. Also, the first solution type was not recommended in the mathematics classes to solve the geometry tasks. Thus, the originality of the first solution type was scored 10. Since it was formed in the paper-pencil environment, it was shown as $Or_{F.T3.S1.PP}=10$. Feyza drew a line segment MD in the second solution type, which was parallel to BC to form a congruency among lines. Then, she used the special triangles' properties such that "in an isosceles triangle, the height drawn to the base is the median." The two concepts used in the second solution type were taught in the mathematics class in the scope of the geometry curriculum, and the concept of the congruency frequently used by the participants. So, it was evaluated as the conventional solution and scored 0.1 ($Or_{F.T3.S2.PP}$). In the third solution type, Feyza constructed a circle with the center O and diameter AC. She defined the point D as the midpoint of the hypotenuse. Then, she constructed the line m, which was parallel to AB and passing through the point D. Since the line m and the line segment AB was parallel and $m\angle$ (ABC) = 90∘, the line m was perpendicular to the line segment BC. The line segment BC was also the chord of a circle, and she stated that "if a line is a perpendicular a chord and the line pass through the center of a circle, the line bisects that chord of a circle." Since $DK \perp BC$ and |BK|=|KC|, the triangle BDC is isosceles. The mathematical concepts used in this solution type were "circle, corresponding angles, properties of a circle, and special triangles." She used these concepts as taught in the mathematics classes and did not require an understanding of the structure of task 3. So, the third solution type was the conventional solution and scored 0.1 ($Or_{F.T3.S3.GGB}=0.1$). In the fourth solution type, she constructed a rectangle ABCE with perpendicular lines to AB and BC. The diagonals of the rectangle (AC and BE) were equal and bisected each other (|AD|=|BD|=|CD|=|ED|). In the triangle ABC, the line AC was the hypotenuse, and the line segment BD was the median. Thus, she showed the median in the right triangle is half of the hypotenuse. In the fourth solution type, she changed the given representation, and it required some insight to construct a new geometric shape on the given one. Hence, it was a partly conventional solution type and scored 1 ($Or_{F.T3.S4.GGB}=1$). In the same way, in the fifth solution type (T3.S5), she constructed a rectangle. However, in the fifth solution type, she constructed the rectangle by rotating the right triangle around point D with 180°, which required some insight to form the new geometric shape by rotating the given. Also, "constructing the rectangle by rotating" used by only two participants. Thus, the fifth solution type was score 1, and shown as ($Or_{F.T3.S5.GGB}=1$). The last solution of Feyza was the sixth solution type (T3.S6), which was formed in the paper-pencil environment. In the sixth solution type, Feyza used the Pythagorean Theorem based on algebraic operations after formed the congruency as in the second solution type (T3.S2). Since a significant part of the solution type included algebraic operations, the sixth solution type was evaluated as an algorithm-based conventional solution. Thus, the originality of the sixth solution type was scored 0.1 ($Or_{F.T3.S6.GGB}=0.1$). Therefore, the total originality of Feyza for task 3 was $Or_{F.T3}=12.3$, while the total originality of Feyza for task 3 in the paper-pencil environment was $Or_{F.T3.PP}=10.2$ and the total originality of Feyza for task 3 in the GeoGebra environment was $Or_{F.T3.GGB}=2.1$.

Ahmet (A) created eight different solutions *(T3.S5, T3.S6, T3.S7, T3.S8, T3.S8, T3.S9, T3.S10, and T3.S11)* for task 3. His first appropriate solution was the fifth solution type of task 3 (T3.S5), created in the GeoGebra environment. In the fifth solution type (T3.S5), he constructed a rectangle by rotating the right triangle around the point D with 180◦, which required some insight to form the new geometric shape by rotating the given. Also, "constructing the rectangle by rotating" was used by only two participants. Thus, the fifth solution type was evaluated as the partly-conventional solution and score 1 ($Or_{A.T3.S5.GGB}=1$). Then, he formed the sixth solution type (T3.S6) for task 3 in the paper-pencil environment. Ahmet used the Pythagorean Theorem based on the algebraic operation after formed the congruency. The significant part of the sixth solution type included algebraic operations. The sixth solution type was created by four of the participants (Feyza, Ahmet, Beren, and Kemal), which means it was commonly used. So, the sixth solution type was an algorithm based conventional solution and scored 0.1 ($Or_{A.T3.S6.PP}=0.1$). The seventh solution type (T3.S7) was the other solution of Ahmet, which was formed in the paper-pencil environment. He drew a line segment, which was perpendicular to the hypotenuse and passing through point B, in a way to apply Euclid's Theorem. Then, he used Euclid's Theorem based on algebraic operations, which was commonly used in the mathematics classes to solve the geometry tasks and was algorithm-based. So, the seventh solution type was evaluated as an algorithm based conventional solution and scored 0.1 ($Or_{A.T3.S7.PP}=0.1$). After that, he created the eight solution type (T3.S8) twice for task 3. For the first time, he drew a line segment DK parallel to AB and formed the triangle DKC. Then he displayed the congruent triangle to DKC by rotating the triangle DKC around point D with 180 ∘. However, the second time, he drew a line segment DM, which was parallel to BC, and formed the triangle DMB. Then, he created the congruent triangle to DMB by rotating the triangle DMB around point D with 180 ∘. In the last step of both solutions, he used the triangles' congruency, and he showed the intended. The two solutions were based on the congruency between the triangles, and that was commonly used in mathematics classes. It did not require understanding the structure of the figure. Thus, the eight solution type was a conventional solution and scored 0.1 ($Or_{A.T3.S8.GGB}=0.1$). Since Ahmet produced the eighth solution type twice, he got 0.1 twice for the originality score, which means the originality score from the eight solution type was 0.2. In the ninth solution type (T3.S9), Ahmet formed the new figure, a rhombus, by reflecting the lines. Then, he completed the solution with the help of a rhombus's properties and the congruency among the lines. Although the ninth solution type did not require comprehending the structure, it was a solution type produced by just one participant and was not based on algebraic operations. Hence, the ninth solution type was the partly conventional solution, and the originality score was 1 ($Or_{A.T3.S9.GGB}=1$). In both tenth (T3.S10) and eleventh solution (T3.S11) types, Ahmet formed a parallelogram by reflecting the triangle ABC over the point M that was the midpoint of AB. Then, he completed the solutions with the algebraic operations based on the Pythagorean Theorem. In the eleventh solution type, he also used the triangles' properties (the sum of the interior angle of a triangle is 180∘) depending on algebraic operations. Hence, both solution types were based on conventional solutions because the significant part of the solutions included algebraic operations. The originality score of the tenth and eleventh solutions were 0.1 ($Or_{A.T3.S10.GGB}=0.1$ *and* $Or_{A.T3.S11.GGB}=0.1$). Therefore, the total originality of Ahmet for task 3 was $Or_{A.T3}=2.6$, while the total originality of Ahmet for task 3 in the paper-pencil environment was $Or_{A.T3.PP}=0.2$, and the total originality of Ahmet for task 3 in the GeoGebra environment was $Or_{F.T3.GGB}=2.4$.

Umay (U) created three different solutions *(T3.S1, T3.S10, and T3.S12)* for task 3. Her first appropriate solution was the first solution type of task 3 (T3.S1), which was created in the paper-pencil environment. In the first solution type, she drew a circle with the center D and defined its diameter as AC. Then, she determined the point B, which was on the circle. The angle ABC was the inscribed angle on a diameter, which means $m\angle(ABC)=90^{\circ}$. In ABC triangle, she drew the line segment OC, the radius of a circle, and the median to hypotenuse (|AC|=2r and |OC|=r). She stated that “the median to the hypotenuse in a right triangle was the half of the hypotenuse." The first solution type were not based on the algebraic operations and required comprehending the structure of the task. Also, the first solution type was not recommended in the mathematics classes to solve the geometry tasks. Thus, the originality of the first solution type was scored as 10. Since it was formed in the paper-and-pencil environment, it was shown as $Or_{U.T3.S1.PP}=10$. Secondly, Umay created the fourth solution type (T3.S4) in the paper-and-pencil environment. Firstly, she described a rectangle ABCD and drew the line segment AC and BD, which were the rectangle's diagonals. The rectangle diagonals were equal in length and bisect each other (|AO|=|BO|=|CO|=|DO|). She concluded that "in the triangle ABC, the line AC was the hypotenuse, and the line segment BO was the median. Thus, she showed that the median in the right triangle is half of the hypotenuse. In the fourth solution type, she changed the representation of the given, and it required some insight to construct a new geometric shape on the given one. Hence, it was a partly conventional solution type and scored as 1 ($Or_{U.T3.S4.PP}=1$). Lastly, she created the twelfth solution type (T3.S12) for task 3 in the GeoGebra environment. In the twelfth solution type, she constructed the rectangle on the circle with the other participants' constructions. Then, she described two congruent triangles within the rectangle by using the properties of a rectangle. The twelfth solution type required adjusting the representation of the given figure and some insight. Thus, it was a partly conventional solution and scored as 1 ($Or_{U.T3.S12.GGB}=1$). Therefore, the total originality of Umay for task 3 was $Or_{U.T3}=12$, while the total originality of Umay for task 3 in the paper-and-pencil environment was $Or_{U.T3.PP}=11$, and the total originality of Umay for task 3 in the GeoGebra environment was $Or_{U.T3.GGB}=1$.

Beren (B) formed three different solutions *(T3.S6, T3.S4, and T3.S13)* for task 3. Her first appropriate solution was the sixth solution type of task 3 (T3.S6), created in the paper-and-pencil environment. She used the Pythagorean Theorem based on algebraic operations after forming the congruency. The significant part of the sixth solution type included algebraic operations. The sixth solution type was created by four of the participants (Feyza, Ahmet, Beren, and Kemal), which means it was commonly used. So, the sixth solution type was an algorithm-based conventional solution and scored 0.1 ($Or_{U.T3.S6.PP}=0.1$). Secondly, she created the fourth solution type (T3.S4) for task 3 in the GeoGebra environment. In the fourth solution type, she constructed a rectangle ABCE with perpendicular lines to AB and BC. The rectangle (AC and BE) was equal and bisected (|AD|=|BD|=|CD|=|ED|). In the triangle ABC, the line AC was the hypotenuse, and the line segment BD was the median. Thus, she showed that the median in the right triangle is half of the hypotenuse. In the fourth solution type, she changed the given representation, and it required some insight to construct a new geometric shape on the given one. Hence, it was a partly conventional solution type and scored 1 ($Or_{U.T3.S4.GGB}=1$). Lastly, she created the thirteenth solution type (T3.S13) for task 3 in the GeoGebra environment. In the thirteenth solution type, Beren defined M and K, which were the midpoints of AB and BC, respectively. Then, she constructed a rectangle MDKB, within the right triangle ABC, by using the congruency between the lines. In the last step of the solution, she again used congruency to complete the solution. She created the thirteenth solution type based on congruency, which was commonly used in mathematics classes. Also, it did not require understanding the structure of the figure. So, the thirteenth solution type was the conventional solution and scored 0.1 ($Or_{U.T3.S13.GGB}=.1$). Therefore, the total originality of Beren for task 3 was $Or_{B.T3}=1.2$, while the total originality of Beren for task 3 in the paper-pencil environment was $Or_{B.T3.PP}=0.1$, and the total originality of Beren for task 3 in the GeoGebra environment was $Or_{B.T3.GGB}=1.1$.

Kemal (K) formed five different solutions *(T3.S1, T3.S6, T3.S13, T3.S4, and T3.S2)* for the task 3. Firstly, he created the first solution type (T3.S1) for the task 3 in the GeoGebra environment. He defined the midpoint of the hypotenuse, D, and constructed a circle with the center D and passing through point C. He stated that "since the point B on the circle and $m\angle(ABC)=90^{\circ}$, the line segment AC was the diameter of a circle". Thus, he stated that the line segments AD, CD, and BD were the circle radius, and BD was half of the hypotenuse. The first solution type was not based on algebraic operations and required comprehending the task's structure.

Also, the first solution type was not typically recommended in the mathematics classes to solve the geometry tasks. Hence, the first solution type's originality was scored as 10 and shown as ($Or_{K.T3.S1.GGB}=10$). Secondly, he created the sixth solution type (T3.S6) in the GeoGebra environment. He used the Pythagorean Theorem based on algebraic operations after formed the congruency. The significant part of the sixth solution type included algebraic operations. The sixth solution type was created by four of the participants (Feyza, Ahmet, Beren, and Kemal), which means it was commonly used. So, the sixth solution type was an algorithm based conventional solution and scored 0.1 ($Or_{K.T3.S6.PP}=0.1$). Then, he formed the thirteenth solution type (T3.S13) in the GeoGebra environment. In the thirteenth solution type, Kemal defined M and K, which were the midpoints of AB and BC, respectively. Then, he constructed a rectangle MDKB, within the right triangle ABC, by using the congruency between the lines. In the last step of the solution, he again used congruency to complete the solution. He created the thirteenth solution type based on congruency, which was commonly used in mathematics classes. Also, it did not require understanding the structure of the figure. So, the thirteenth solution type was the conventional solution and scored 0.1 ($Or_{U.T3.S13.GGB}=0.1$). Moreover, he produced the fourth solution type (T3.S4) for task 3 in the GeoGebra environment. In the fourth solution type, he constructed a rectangle ABCE with the help of perpendicular lines to AB and BC. The line segments (AC and BE) were equal and bisected (|AD|=|BD|=|CD|=|ED|). In the triangle ABC, the line AC was the hypotenuse, and the line segment BD was the median. Thus, he showed that the median in the right triangle is the half of hypotenuse. In the fourth solution type, he changed the representation of given, and it required some insight to construct a new geometric shape on the given one. Hence, it was partly-conventional solution type and scored 1 ($Or_{K.T3.S4.GGB}=1$). Lastly, she created the second solution type (T3.S2) in the GeoGebra environment. In the second solution type, Kemal drew a line segment MD which was parallel to BC to form a congruency among lines. Then, he used the properties of the special triangles such that “in an isosceles triangle, the height drawn to the base is the median”. The two concepts used in the second solution type were taught in the mathematics class in the scope of the geometry curriculum, and the concept of the congruency frequently used by the participants. So, it was evaluated as the conventional solution, and scored 0.1 ($Or_{K.T3.S2.GGB}$). Therefore, the total originality of Kemal for task 3 was $Or_{K.T3}=11.3$ while the total originality of Kemal for task 3 in the paper-and-pencil environment was $Or_{K.T3.PP}=0$ and the total originality of Kemal for task 3 in GeoGebra environment was $Or_{K.T3.GGB}=11.3$.

The originality scores of each participant for task 3 are shown in Table 4.19.

Table 4. 19 Originality scores of each participant for task 3

Participants	Originality Scores		
	GeoGebra	Paper-and-Pencil	Total
Feyza	2.1	10.2	12.3
Ahmet	2.4	0.2	2.6
Umay	1	11	12
Beren	1.1	0.1	1.2
Kemal	11.3	0	11.3

4.3.4 Evaluation of Creativity in Task 3

There were thirteen different solution types for task 3, which were evaluated concerning their fluency, flexibility, and originality, respectively. The creativity of each solution (Cr) and each participant's total creativity (CR) for task 3 were scored depending on the scores of fluency, flexibility, and originality of the solutions. A specific solution (Cri) creativity score was the product of the solution's flexibility and originality score ($Cr_i = Flx_i \cdot Or_i$). The total creativity score on task 3 was the sum of the creativity scores on each solution ($Cr = \sum_{i=1}^{n} Flx_i \cdot Or_i$). Also, the total creativity score was separately evaluated, considering the environment in which they were created because not all solutions were produced in the GeoGebra environment. Therefore, the participant's final creativity score was scored as the product of the fluency score and the total creativity score (CR = n ($\sum_{i=1}^{n} Flx_i \cdot$ *Or_i*)).

Feyza *(F)* produced six different solutions for task 3 *(T3.S1, T3.S2, T3.S3, T3.S4, T3.S5, and T2.S6)*. The flexibility score of the first solution type was $Flx_{F.T3.S1.PP} = 10$, and the originality score of the first solution type was $Or_{F.T3.S1.PP} = 10$. The creativity score of her solution ($Cr_{F.T3.S1.PP}$), which was the first solution type of task 3, was scored by multiplying its flexibility and originality score ($Cr_{F.T3.S1.PP} = Flx_{F.T3.S1.PP} \times Or_{F.T3.S1.PP}$); which means the creativity score of the first solution type for the task 3 was 10 ($Cr_{F.T3.S1.PP} = 100$). Similarly, the creativity scores of her other solutions for task 3 (e.g., $Cr_{F.T3.S2.PP}$, $Cr_{F.T3.S3.GGB}$, $Cr_{F.T3.S4.GGB}$, $Cr_{F.T3.S5.GGB}$, *and* $Cr_{F.T3.S6.PP}$) were scored as the product of their flexibility and originality scores. The second solution type's creativity was scored 1 ($Cr_{F.T3.S2.PP}=1$), and the creativity of the third solution type was scored 0.1 ($Cr_{F.T3.S3.GGB}=0.1$).

Table 4. 20. Scoring the creativity of Feyza for task 3.

	Fluency	Flexibility	Originality	Creativity
T3.S1	1	10	10	100
T3.S2	1	10	0.1	1
T3.S3	1	1	0.1	0.1
T3.S4	1	10	1	10
T3.S5	1	1	1	1
T3.S6	1	1	0.1	0.1
Total	6	33	10.2	**112.2**
Final creativity of Feyza for task 3				**673.2**

Note. Solution was created in the GeoGebra environment was highlighted.

Then, the creativity of fourth solution type was 10 ($Cr_{F.T3.S4.GGB}=10$), the creativity of fifth solution type was 1 ($Cr_{F.T3.S5.GGB}=1$), and the creativity of the sixth solution type was 0.1 (*$Cr_{F.T3.S6.PP}$=0.1).* Thus, the total creativity of Feyza for task 3, which was the sum of the creativity scores on each solution, was scored 112.2 *($Cr_{F.T3}$ = 112.2).* While her solutions' total creativity was formed in the paper-pencil environment was scored 101.1 ($Cr_{F.T3.PP}$ = 101.1), her total creativity was created in the GeoGebra environment was scored as 11.1 ($Cr_{F.T3.GGB}$ = 11.1). Therefore, the final creativity score of Feyza for task 3 was the product of her fluency score *($N_{F.T3}$=6)*, and her total creativity score *($Cr_{F.T3}$ = 112.2)* was scored 673.2 *($CR_{F.T3}$ = 673.2)* (seen in Table 4.20).

Ahmet *(A)* produced eight different solutions for task 3 *(T3.S5, T3.S6, T3.S7, T3.S8, T3.S8, T3.S9, T3.S10, and T3.S11)*. The flexibility score of the fifth solution type was $Flx_{A.T3.S5.GGB} = 10$, and the originality score of the fifth solution type was $Or_{F.T3.S5.GGB} = 1$. The creativity score of his solution (*$Cr_{F.T3.S5.GGB}$)*, which was the fifth solution type of task 3, was scored by multiplying its flexibility and originality score *($Cr_{F.T3.S5.GGB} = Flx_{F.T3.S5.GGB}$ x $Or_{F.T3.S5.GGB}$);* which means the creativity score of the fifth solution type for the task 3 was 1 *($Cr_{F.T3.S5.GGB}$ = 1).* In the same way, the creativity scores of his other solutions for the task 3 (e.g. *$Cr_{F.T3.S6.PP}$, $Cr_{F.T3.S7.PP}$, $Cr_{F.T3.S8.GGB}$, $Cr_{F.T3.S8.GGB}$, $Cr_{F.T3.S9.GGB}$, $Cr_{F.T3.S10.GGB}$, and $Cr_{F.T3.S11.GGB}$)* were scored as the product of their flexibility and originality scores. The creativity of the sixth solution type was scored 1 (*$Cr_{F.T3.S6.PP}$=1* and the creativity of the seventh solution type were scored 1 (*$Cr_{F.T3.S7.PP}$=1).* Ahmet created the eighth solution type twice, and the creativity of each was scored separately. In the first time, the creativity score of the eight solution type was 0.1 (*$Cr_{F.T3.S8.GGB}$=0.1)* since the flexibility score of the eighth solution type was 1 and the originality score of the eighth solution type was 1. However, the eighth solution type's creativity score for the second time was 0.01 (*$Cr_{F.T3.S8.GGB}$=0.01)* because the flexibility score of the eighth solution for the second time was 0.1. Then, the creativity score of the ninth solution type was 1 (*$Cr_{F.T3.S9.GGB}$=1),* the creativity score of the tenth solution type was 0.1 (*$Cr_{F.T3.S10.GGB}$=0.1),* and the creativity score of the eleventh solution type was 0.1 (*$Cr_{F.T3.S11.GGB}$=0.1).* Thus, the total creativity of Ahmet for task 3, which was the sum of the creativity scores on each solution, was scored 13.4 *($Cr_{A.T3}$ = 13.4).*

While his solutions' total creativity was formed in the paper-pencil environment was scored 2 (CrA.T3.PP = 2), the total creativity of his solution was created in the GeoGebra environment scored 11.4 ($Cr_{A.T3.GGB}$ = 11.4). Therefore, the final creativity score of Ahmet for task 3, was the product of her fluency score *($N_{A.T3}$=8)*, and her total creativity score *($Cr_{A.T3}$ = 13.4)*, was scored 107.2 *($CR_{A.T3}$ = 673.2)* (seen in Table 4.21).

Table 4. 21. Scoring the creativity of Ahmet for task 3.

	Fluency	**Flexibility**	**Originality**	**Creativity**
T3.S5	1	10	1	10
T3.S6	1	10	0.1	1
T3.S7	1	10	0.1	1
T3.S8	1	1	0.1	0.1
T3.S8	1	0.1	1	0.1
T3.S9	1	1	0.1	0.1
T3.S10	1	1	0.1	0.1
T3.S11	1	1	1	1
Total	8	34.1	3.5	**13.4**
Final creativity of Ahmet for task 3				**107.2**

Note. Solution was created in the GeoGebra environment was highlighted.

Umay *(U)* formed three different solutions for task 3 *(T3.S1, T3.S4, and T12)*. The flexibility score of the first solution type was $Flx_{U.T3.S1.PP} = 10$, and the originality score of the first solution type was $Or_{U.T3.S1.PP} = 10$. The creativity score of her solution ($Cr_{U.T3.S1.PP}$), which was the first solution type of task 3, was scored by multiplying its flexibility and originality score ($Cr_{U.T3.S1.PP} = Flx_{U.T3.S1.PP}$ x $Or_{U.T3.S1.PP}$); which means the creativity score of the first solution type for task 3 was 100 ($Cr_{U.T3.S1.PP} = 100$). Similarly, the creativity scores of her other solutions for task 3 (e.g., $Cr_{U.T3.S4.PP}$ *and* $Cr_{U.T3.S12.GGB}$) were scored as the product of their flexibility and originality scores. The fourth solution type's creativity was scored 10

($Cr_{U.T3.S4.PP}$=*10)*, and the creativity of the twelfth solution type was scored 1 ($Cr_{U.T3.S12.GGB}$=*1)*. Thus, the total creativity of Umay for task 3, which was the sum of the creativity scores on each solution, was scored 111 *($Cr_{U.T3}$ = 111)*. While her solutions' total creativity was formed in the paper-pencil environment was scored 110 (CrU.T3.PP = 110), the total creativity of her solution was created in the GeoGebra environment scored 1 ($Cr_{U.T3.GGB}$ = 1). Therefore, the final creativity score of Umay for task 3 was the product of her fluency score *($N_{U.T3}$=3)*, and her total creativity score *($Cr_{U.T3}$ = 111)* was scored 333 *($CR_{U.T3}$ = 333)* (seen in Table 4.22).

Table 4. 22. Scoring the creativity of Umay for task 3.

	Fluency	**Flexibility**	**Originality**	**Creativity**
T3.S1	1	10	10	100
T3.S4	1	10	1	10
T3.S12	1	1	1	1
Total	3	21	12	**111**
Final creativity of Umay for task 3				**333**

Note. Solution was created in the GeoGebra environment was highlighted.

Beren *(B)* created three different solutions for task 3 *(T3.S6, T3.S4, and T13)*. The flexibility score of the sixth solution type was *$Flx_{B.T3.S6.PP}$ = 10*, and the originality score of the sixth solution type was *$Or_{B.T3.S6.PP}$ = 0.1*. The creativity score of her solution ($Cr_{B.T3.S6.PP}$*)*, which was the sixth solution type of task 3, was scored by multiplying its flexibility and originality score *($Cr_{B.T3.S6.PP}$ = $Flx_{B.T3.S6.PP}$ x $Or_{B.T3.S6.PP}$);* which means the creativity score of the sixth solution type for the task 3 was 1 *($Cr_{B.T3.S6.PP}$ = 100)*. Similarly, the creativity scores of her other solutions for task 3 (e.g., *$Cr_{B.T3.S4.GGB}$ and $Cr_{B.T3.S13.GGB}$)* were scored as the product of their flexibility and originality scores. The creativity of the fourth solution type was scored 10 ($Cr_{B.T3.S4.GGB}$=*10)* and the creativity of thirteenth solution type was scored 0.1 ($Cr_{B.T3.S13.GGB}$=*0.1)*. Thus, the total creativity of Beren for task 3, which was the sum of the creativity scores on each solution, was scored 11.1 *($Cr_{B.T3}$ = 11.1).* While her solutions' total creativity were formed in the paper-pencil environment was scored 1 ($Cr_{B.T3.PP}$ = 1), the total creativity of her solutions were created in the GeoGebra environment scored 10.1 ($Cr_{B.T3.GGB}$ = 10.1). Therefore, the final creativity score of Beren for task 3 was the product of her fluency score *($N_{B.T3}$=3),* and her total creativity score *($Cr_{B.T3}$ = 11.1)* was scored 33.3 *($CR_{B.T3}$ = 33.3)* (seen in Table 4.23).

Table 4. 23. Scoring the creativity of Beren for task 3

	Fluency	**Flexibility**	**Originality**	**Creativity**
T3.S6	1	10	0.1	1
T3.S4	1	10	1	10
T3.S13	1	1	0.1	0.1
Total	3	21	1.2	**11.1**
Final creativity of Beren for task 3				**33.3**

Note. Solution was created in the GeoGebra environment was highlighted.

Kemal *(K)* produced five different solutions for task 3 *(T3.S1, T3.S6, T3.S13, T3.S4, and T3.S2).* The flexibility score of the first solution type was $Flx_{K.T3.S1.GGB}$ *= 10,* and the originality score of the first solution type was $Or_{K.T3.S1.GGB}$ *= 10*. The creativity score of his solution ($Cr_{K.T3.S1.GGB}$*),* which was the first solution type of task 3, was scored by multiplying its flexibility and originality score *($Cr_{K.T3.S1.GGB}$ = $Flx_{K.T3.S1.GGB}$ x $Or_{K.T3.S1.GGB}$);* which means the creativity score of the first solution type for the task 3 was 100 *($Cr_{K.T3.S1.GGB}$ = 100).* Similarly, the creativity scores of his other solutions for task 3 (e.g., $Cr_{K.T3.S6.GGB}$, $Cr_{K.T3.S13.GGB}$, $Cr_{K.T3.S4.GGB}$, *and* $Cr_{K.T3.S2.GGB}$*)* were scored as the product of their flexibility and originality scores. The creativity of the sixth solution type was scored 1 ($Cr_{K.T3.S6.GGB}$*=1),* the creativity of the thirteenth solution type was scored 0.1 ($Cr_{K.T3.S13.GGB}$*=0.1),* the creativity of the fourth solution type was 1 ($Cr_{K.T3.S4.GGB}$*=1),* and the creativity of the second solution type was scored 0.1 ($Cr_{K.T3.S2.GGB}$*=0.1).* Thus, the total creativity of Kemal for task 3, which was the sum of the creativity scores on each solution, was scored 102.2 *($Cr_{K.T3}$ = 102.2).* While the total creativity of his solutions was formed in the paper-pencil environment was scored 0 (CrK.T3.PP = 0), his solutions were created in the GeoGebra environment scored 102.2 ($Cr_{K.T3.GGB}$ = 102.2). Therefore, the final creativity score of Kemal for task 3 was the product of his fluency score *($N_{K.T3}$=5),* and his total creativity score *($Cr_{K.T3}$ = 102.2)* was scored 511 *($CR_{K.T3}$ = 511)* (seen in Table 4.24).

Table 4. 24. Scoring the creativity of Kemal for task 3

	Fluency	**Flexibility**	**Originality**	**Creativity**
T3.S1	1	10	10	100
T3.S6	1	10	0.1	1
T3.S13	1	1	0.1	0.1
T3.S4	1	1	1	1
T4.S2	1	1	0.1	0.1
Total	5	23	11.3	**102.2**
Final creativity of Kemal for task 3				**511**

Note. Solution was created in the GeoGebra environment was highlighted.

CHAPTER 5

DISCUSSION AND IMPLICATIONS

The main goal of this study was to investigate the role of Dynamic Geometry Software in promoting the mathematical creativity of pre-service teachers in geometry tasks. For that purpose, the researcher examined the pre-service teachers' solutions in both paper-pencil and GeoGebra environments in three dimensions of mathematical creativity: Fluency, Flexibility, and Originality. This chapter discusses the main findings of this study under the heading of these three dimensions and the overall creativity in relation to the related literature. Finally, recommendations and implications are presented.

5.1 Fluency

For all geometry tasks, all pre-service teachers created at least one solution (see Table 5.1). Namely, two pre-service teachers, Feyza and Ahmet, formed the most number of solutions in the environment where they could use the GeoGebra (e.g., Task 3). Furthermore, while Kemal produced the same number of solutions in the paper-pencil environment and the GeoGebra included environment (e.g., task 1 and task 3), he had approximately the same number of solutions for task 2. On the other hand, while Umay constructed a few solutions for tasks 2 and 3 that GeoGebra was involved in, its fluency was considerably high for task 1 that required solve in the paper-pencil environment.

Table5. 1 Pre-service teachers' fluency scores for all tasks

	Fluency Scores						
Participants	TASK 1	TASK 2			TASK 3		
		GGB	PP	Total	GGB	PP	Total
Feyza	4	1	2	3	3	3	6
Ahmet	3	5	2	7	6	2	8
Umay	8	0	1	1	1	2	3
Beren	3	5	0	5	2	1	3
Kemal	5	3	0	3	5	0	5

The most and least number of solutions of pre-service teachers were investigated considering the environments they produced their solutions, and a noticeable pattern was not observed, which implies that the number of valid answers to be produced was not affected by the environment. It has been observed in different studies that the pre-service teachers or students create at least one solution in both GeoGebra and paper-pencil environments (Koyuncu et al., 2015 and Farihah, 2018). In the present study, the pre-service teachers' creating at least one solution for all tasks might connect with the multiple solution tasks' structure. Currently, the MSTs were defined as assignments that required students to solve the problem in different ways and explore mathematical creativity (Leikin, 2009). Even, Leikin (2009) implies that the MSTs already tend to increase the students' fluency. Besides, the fact that pre-service teachers can create at least one solution for each task might also be related to their mathematical background. Leikin et al. (2011) emphasize the fluency is firmly based upon knowledge.

On the other hand, there are available examples of studies that claimed that the Dynamic Geometry Software might support the students' fluency (Farihah, 2018, Levav-Waynberg & Leikin, 2012, and Yıldız et al., 2017). According to Farihah (2018), the Dynamic Geometry Software might promote the students' conceptual understanding by assisting their fluency. Yıldız et al. (2017) conclude that teaching in the GeoGebra environment positively affects fluency based on creating a large number of solutions.

5.2 Flexibility

When the flexibility scores of the pre-service teachers were investigated (see Table 5.2), the highest flexibility of all, except Beren, was obtained for task 3. The highest flexibility score of Beren was formed for task 2. For all pre-service teachers, the environments where the highest flexibility scores were observed were the ones in which the GeoGebra was involved. The use of Dynamic Geometry Software in solving geometry tasks might support the differentiation of pre-service teachers' solutions and mathematical concepts they used. In other words, the Dynamic Geometry Environment enables the pre-service teachers to create alternative solutions for the same geometry task. This finding is consistent with the other research studies comparing the two environments (Coşkun, 2011; Iranzo & Fortuny, 2011; Levav-Waynberg & Leikin, 2012; and Koyuncu et al., 2015). Unlike the current study, the previous studies did not directly examine the concept of fluency as a dimension of mathematical creativity within the framework of creativity; they address the participants' abilities might be related to mathematical creativity such as creating alternative solution ways (Coşkun, 2011; Iranzo & Fortuny, 2011; and Koyuncu et al., 2015). On the other hand, when the solutions of pre-service teachers in the GeoGebra and paper-pencil environment were examined separately, we cannot deduce that the flexibility of solutions produced in the GeoGebra environment is more than those created in the paper-pencil environment. For example, in both task 2 and task 3, the flexibility score of Feyza in the paper-pencil environment was higher than in the GeoGebra environment. In comparison, the flexibility score of Kemal in the GeoGebra environment was higher than in the paper-pencil environment. Indeed, the combination of the GeoGebra with the paper-pencil environment for the geometry tasks might promote flexibility rather than using GeoGebra alone. There are previous studies that support this by claiming that even though Dynamic Geometry Software supports enhancing some dimension of creativity, it is more advantageous to present the Dynamic Geometry Software together with the paper-pencil environment to improve students' mathematical creative thinking abilities (Coelho & Cabrita, 2015 and Selvy et al., 2020).

Table5. 2 Pre-service teachers' flexibility scores for all tasks

	Flexibility Scores						
Participants	TASK 1	TASK 2			TASK 3		
		GGB	PP	Total	GGB	PP	Total
Feyza	21.1	10	20	30	12	21	33
Ahmet	21	14	11	25	14.1	20	34.1
Umay	16.1	0	10	10	1	20	21
Beren	12	32	0	32	11	10	21
Kemal	13.1	12	0	12	23	0	23

5.3 Originality

When the overall originality scores of the pre-service teachers were examined (as seen in table 5.3), the highest originality score of all pre-service teachers was observed in task 2 or task 3 that were solved in GeoGebra included environment. Namely, the originality of solutions created in the paper-pencil environment was weaker than the solutions formed in the GeoGebra environment. It has also been observed in the studies of Coelho and Cabrita (2015) that when GeoGebra was used to solve the questions, students created more detailed and original solutions. They imply that the pencil-paper environment prevents students from creating alternative ways and limits their ability to produce original solutions by attracting them into technical procedures.

Table5. 3 Pre-service teachers' originality scores for all tasks

	Originality Scores						
Participants	TASK 1	TASK 2			TASK 3		
		GGB	PP	Total	GGB	PP	Total
Feyza	0.4	10	0.2	10.2	2.1	10.2	12.2
Ahmet	0.3	12.2	0.2	12.4	2.4	0.2	2.6
Umay	10.7	0	0.1	0.1	1	11	12
Beren	0.3	10.4	0	10.4	1.1	0.1	1.2
Kemal	1.4	1.2	0	1.2	11.3	0	11.3

The lowest originality scores are viewed in algorithm-based or commonly taught solutions, and the paper-pencil environment seems preferable to create these solutions. It is an environment where the pre-service teachers are more accustomed to producing algorithm-based solutions and use taught mathematical concepts more frequently. While pre-service teachers tend to create algorithm-based solutions in the pencil-paper environment, they use geometric solutions in the GeoGebra environment (Iranzo & Fortuny, 2011; Koyuncu et al., 2014). Even, Koyuncu et al. (2015) imply that students feel more comfortable setting up and solving equations than searching geometric relationships. Indeed, the emphasis in Turkish schools has been on algebra than geometry (MoNE, 2009). For these reasons, the originality of pre-service teachers in the paper-pencil environment was lower, which might arise from their mathematical background.

Besides, creating a solution in the GeoGebra environment might require the pre-service teachers to have more conceptual understanding than the paper-pencil environment. In the paper-pencil environment, they might tend to solve as taught in the school. For instance, while a square is constructed in the paper-pencil environment based on estimation, it requires the use of perpendicular lines in the GeoGebra environment. While students reconstruct the figures given in the GeoGebra environment, they can easily notice the figures' sub-figurations than in the paper-pencil environment. (Koyuncu et al., 2015). This might be due to the dynamic nature of the GeoGebra environment. For example, it was observed that "dragging or measuring" help students to comprehend the structure of the problem and promoted students to explore the possible answers (Christou et al., 2015). Even it is implied that the GeoGebra environment allows students to explore the relationships by providing them visualization aids (Saha et al., 2010). Another claim is that construction in the GeoGebra environment takes longer than the paper-pencil environment, allowing students to observe a better and deeper understanding of the task (Koyuncu et al., 2015). The fact that the GeoGebra environment supports the conceptual understanding of pre-service teachers might have enabled them to create original solutions to the given task.

5.4 Creativity

While the creativity scores of the pre-service teachers were evaluated individually for each task (as seen in Table 5.4), no significant patterns were observed in favor of any of the environments. For example, while the creativity scores of Feyza in the GeoGebra environment was higher for task 2; her creativity score in the paper-pencil environment was higher for task 3. Either, the creativity score of Kemal in the GeoGebra environment was higher for both the task 2 and 3. In comparison, the creativity score of Umay in the paper-pencil environment was higher for both tasks. It is thought that this differentiation might arise from the background of the pre-service teachers or the multiple solution tasks used. Selvy et al. (2020) imply that the students' creative thinking abilities vary and suggest that students might not be accustomed to solving tasks that require creative thinking ability. Also, according to them, the improvement of the mathematical creative thinking ability needs time and continuous practice with non-routine problems.

Table5. 4 Pre-service teachers' creativity scores for all tasks

	Final Creativity Scores						
Participants	TASK 1	TASK 2			TASK 3		
	(Final Creativity)	GGB (Total)	PP (Total)	Final Creativity	GGB (Total)	PP (Total)	Final Creativity
Feyza	8.44	100	2	306	11.1	101.1	673.2
Ahmet	6.3	102.2	1.1	723.1	11.4	2	107.2
Umay	92.08	0	1	1	1	110	333
Beren	3.6	102.2	0	511	10.1	1	33.3
Kemal	11.05	2.1	0	6.3	102.2	0	511

On the other hand, the highest overall creativity scores were observed in task 2 and task 3 (Table 5.4), which included the GeoGebra environment. It means the mathematical creativity in the environment in which students used only the paper-pencil was weaker than in the environment in which GeoGebra was included. This might be because the paper-pencil environment alone could not support the two dimensions of creativity: flexibility and originality, as much as the GeoGebra included environment, as stated in the previous sections. Namely, the GeoGebra environment supports the flexibility of the pre-service teachers by providing them to create different solutions for one task (Coşkun, 2011, Iranzo & Fortuny, 2011, and Koyuncu et al., 2015). Also, it encourages the pre-service teachers to create a more original and detailed solution by moving away from technical procedures (Coelho & Cabrita, 2015) and promoting their conceptual understanding (Christou et al., 2015, Koyuncu et al., 2015, and Saha et al., 2010). Currently, previous studies (Coelho & Cabrita, 2015, Saha et al., 2010, and Selvy et al., 2020) argues that the GeoGebra environment also improves the students' creative thinking abilities when it presented together with the paper-pencil in addition to supporting the different dimension of the mathematical creativity. On these bases, it was observed that the GeoGebra environment has the potential to support mathematical creativity in solving geometry tasks, especially when presented with the paper-pencil environment. The GeoGebra environment provides students with an environment where they can comprehend the task, think critically to create solutions, problem-based learning, and promote critical, creative, and innovative thinking skills (Kim & Md-Ali, 2017).

On the other hand, when it is considering that the improvement of mathematical creativity needs time (Kim & Md-Ali, 2017 and Selvy et al., 2010) and the GeoGebra has the potential to promote mathematical creativity, it was thought that teaching mathematics through the GeoGebra also might improve the mathematical creativity. In the previous studies (Kim & Md-Ali, 2017, Selvy et al., 2020, Yıldız et al., 2017), it has been concluded that teaching through the GeoGebra has a positive effect on creative thinking ability.

All in all, the GeoGebra set an environment for the pre-service teachers that enabled them to use their mathematical creativity in geometry tasks by supporting the different dimensions of creativity. While the fluency of pre-service teachers did not seem to be affected by the environment, the differentiation of fluency arises from the nature of multiple solutions tasks or their mathematical background. The use of GeoGebra supports the differentiation of pre-service teachers' solutions and mathematical concepts they used in solving geometry tasks, which means it promotes the flexibility of pre-service teachers by allowing them to create alternative solution ways. More importantly, the combination of GeoGebra with the paper-pencil seemed to be more supportive of the pre-service teachers' flexibility. The GeoGebra environment, on the other hand, allows the pre-service teachers to explore the relationships in the geometry task through the nature of software, while it required spending more time. While this promotes the conceptual understanding of pre-service teachers, it enables them to create profound and original solutions. In short, GeoGebra has the potential to support mathematical creativity in solving geometry tasks because it allows individuals to provide more flexible and original solutions, especially when presented with a paper-pencil environment.

5.5 Recommendations, Implications, and Further Research

The Dynamic Geometry Software has the potential to support the mathematical creativity of students. The software provides an environment in which students can improve flexible and original solution ways. Therefore, the teachers can use the dynamic geometry software in mathematics classes to support the students' mathematical creativity.

The school mathematics curricula should be reviewed in a way so that would support the improvement of students' creative thinking abilities. For example, the school mathematics textbooks could include multiple solution tasks which could be solved in the dynamic geometry environment. Moreover, both in dynamic geometry and paper-pencil environments, pre-service teachers tend to create algorithm-based solutions. The reason might be related to their prior learning, usually based on setting up algebraic equations. Therefore, supporting problem-solving with the dynamic geometry software might enable developing different and deep solution ways, promoting their mathematical creativity.

The teacher education curricula could be enriched with greater emphasis on an integrated learning environment with Dynamic Geometry Software in order to support students' mathematical creativity. Also, this can improve the pre-service teachers' creative thinking abilities, which they can reflect on to their students in the future.

In this study, the researcher investigated the role of pencil-paper and GeoGebra environments on the pre-service teachers' mathematical creativity, however, did not evaluate the differences that the different tasks used could create. It was assumed that the multiple solution tasks used were homogeneous environments in evaluating the pre-service teachers' mathematical creativity and the reasons were clarified as one of the limitations of this study. That is, it was assumed that the different multiple solution tasks would no effect when assessing mathematical creativity in different environments. However, in the future research, the role of using different multiple solution tasks on the pre-service teachers' mathematical creativity can be examined. Also, based on the results of current study, the role of environments (e.g. paper-pencil and GeoGebra) on the pre-service teachers' mathematical creativity in each task can be investigated separately.

In the present study, the researcher examined the mathematical creativity of five particular cases at a specific university to figure out the role of dynamic geometry software on their mathematical creativity. Future studies might be preferred to study different cases at various universities and with diverse topics in the mathematics curriculum. Also, it might be efficient to work with students from high or middle school to have more significant results about improving students' creativity.

In this study, the researcher analyzed the role of Dynamic Geometry Software on creativity with the help of geometry tasks. Future researchers might study the role of dynamic geometry software on mathematical creativity using different mathematical topics such as algebra.

REFERENCES

Association of Mathematics Teacher Educators (2006). Preparing teachers to use technology to enhance the learning of mathematics – A position of the Association of Mathematics Teacher Educators. Retrieved Septembern25, 2012, from http://www.amte.net/publications.

Association of Mathematics Teacher Educators (2006). Preparing teachers to use technology to enhance the learning of mathematics – A position of the Association of Mathematics Teacher Educators. Retrieved Septembern25, 2012, from http://www.amte.net/publications.

Baer, J. (1998). The case for domain specificity of creativity. *Creativity research journal*, *11*(2), 173-177.

Beghetto, R. A., & Kaufman, J. C. (2007). Toward a broader conception of creativity: A case for" mini-c" creativity. *Psychology of Aesthetics, Creativity, and the Arts*, *1*(2), 73.

Berger, M. (2011). Using mathematical discourse to understand students' activities when using GeoGebra. *Proceedings of PME 35*, *2*, 137-144.

Brown, R. T. (1989). Creativity. In *Handbook of creativity* (pp. 3-32). Springer, Boston, MA.

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education*, *17*(1), 37-47.

Christou, C., Mousoulides, N., Pittalis, M., & Pitta-Pantazi, D. (2005). Problem solving and problem posing in a dynamic geometry environment. *The Mathematics Enthusiast*, *2*(2), 125-143.

Coelho, A., & Cabrita, I. (2015). A Creative Approach to Isometries Integrating Geogebra and Italc with 'Paper And Pencil'Environments. *Journal of the European Teacher Education Network*, *10*, 71-85.

Contreras, J. N. (2013). Fostering Mathematical Creativity through Problem Posing and Modeling using Dynamic Geometry: Viviani's Problem in the Classroom. *Journal of Mathematics Education at Teachers College ,4*, Fall-Winter, 66-72.

Coskun, S. (2011). A Multiple Case Study Investigating The Effects Of Technology On Students' Visual And Nonvisual Thinking Preferences Comparing Paper-pencil And Dynamic Software Based Strategies Of Algebra Word Problems.

Craft, A. (2010). Creativity and education futures: Learning in a digital age. Stafordshire: Trentham Books.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches.* Thousand Oaks, CA: Sage

Creswell, J. W. (2013). *Qualitative inquiry and research design : choosing among five approaches.* Los Angeles : SAGE Publications, 2013. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip&db=cat06966a&AN=metu.b1651586&site=eds-live&authtype=ip,uid

Denzin, N. K., & Lincoln, Y. S. (2000). *Handbook of qualitative research*. Thousand Oaks, Calif. : Sage Publications, 2000. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip&db=cat06966a&AN=metu.b1146339&site=eds-live&authtype=ip,uid

Dikoviç, L. (2009). Applications GeoGebra into teaching some topics of mathematics at the college level. Computer Science & Information Systems, 6(2), 191-203

Dreyfus, T., & Eisenberg, T. (2012). On different facets of mathematical thinking. In *The nature of mathematical thinking* (pp. 269-300). Routledge.

Ersoy, Y. (2003). Teknoloji destekli matematik eğitimi - 1: Gelişmeler, politikalar ve stratejiler. İlköğretim Online, 2(1), 18-27.

Ervynck, G. (1991). Mathematical Creativity. In D. Tall (Ed.), *Advanced Mathematical Thinking* (pp.42-53).Dordrecht, Netherlands: Kluwer.

Farihah, U. (2018, April). Students' thinking preferences in solving mathematics problems based on learning styles: A comparison of paper-pencil and geogebra. In *Journal of Physics: Conference Series* (Vol. 1008, No. 1, p. 012079). IOP Publishing.

Fraenkel, J. R., Wallen, N. E., & Hyun H.H. (2012). *How to design and evaluate research in education*. New York: McGraw-Hill.

Gittinger, J. D. (2012). A Laboratory Guide for Elementary Geometry using GeoGebra. *North American GeoGebra Journal*, *1*(1).

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *The Journal of Mathematical Behavior*, *37*, 48-62.

Guilford, J. P. (1950). Creativity. *American Psychologist*, 5(9), 444–454

Guilford, J. P. (1967). Creativity: Yesterday, today and tomorrow. *The Journal of Creative Behavior*, *1*(1), 3-14.

Guilford, J. P. (1973). Characteristics of Creativity.

Haciomeroglu, E. S., Bu, L., Schoen, R. C., & Hohenwarter, M. (2009). Learning to develop mathematics lessons with GeoGebra. *MSOR Connections, 9*(2), 24-26.

Haylock, D. W. (1987). A framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics*, 18(1), 59–74.

Heid, M. K., & Edwards, M. T. (2001). Computer algebra systems: revolution or retrofit for today's mathematics classrooms?. *Theory into practice*, *40*(2), 128-136.

Henriksen, D., Mishra, P., & Fisser, P. (2016). Infusing creativity and technology in 21st century education: A systemic view for change. *Journal of Educational Technology & Society*, *19*(3), 27-37.

Hohenwarter, M. & Jones, K. (2007). Ways of linking geometry and algebra:The case of GeoGebra. Proceedings of the British Society for Research into Learning Mathematics, 27(3), 126-131.

Hollebrands, K. (2003). High school students‘ understandings of geometrictransformations in the context of a technological environment. Journal of Mathematical Behavior, 22(1), 55-72.

Hollebrands, K. F. (2007). The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for Research in Mathematics Education*, 38(2), 164–192.

Idris, N., & Nor, N. M. (2010). Mathematical creativity: usage of technology. *Procedia-Social and Behavioral Sciences*, *2*(2), 1963-1967.

Iranzo, N., & Fortuny, J. M. (2011). Influence of GeoGebra on problem solving strategies. In *Model-Centered Learning* (pp. 91-103). Brill Sense.

Jones K. (2000). Providing a foundation for deductive reasoning: students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics*, 44(1-3), 55-85.

Kattou, M., Christou, C., & Pitta-Pantazi, D. (2015, February). Mathematical creativity or general creativity?. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1016-1023).

Kaufman, J. C. (2016). *Creativity 101*. Springer publishing company.

Kaufman, J. C., & Beghetto, R. A. (2009). Beyond big and little: The four c model of creativity. *Review of general psychology*, *13*(1), 1-12.

Kaufman, J. C., & Sternberg, R. J. (Eds.). (2010). *The Cambridge handbook of creativity*. Cambridge University Press.

Kaufman, J. C., Cole, J. C., & Baer, J. (2009). The construct of creativity: Structural model for self-reported creativity ratings. *The Journal of Creative Behavior*, *43*(2), 119-134.

Kim, K. M., & Md-Ali, R. (2017). GeoGebra: towards realizing 21st century learning in mathematics education. *Malaysian Journal of Learning and Instruction*, 93-115.

Kokol-Voljc, V. (2007). Use of mathematical software in pre-service teacher training: The case of dgs. *Proceedings of the British Society for Research into Learning Mathematics*, *27*(3), 55-60.

Koyuncu, I., Akyuz, D., & Cakiroglu, E. (2015). Investigating plane geometry problem-solving strategies of prospective mathematics teachers in technology and paper-and-pencil environments. *International Journal of Science and Mathematics Education*, *13*(4), 837-862.

Koyuncu, İ. (2013). *Investigating the use of technology on pre-service elementary mathematics teachers' plane geometry problem solving strategies* (Master's thesis, Middle East Technical University).

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students*. (Ch.9, pp. 129–145). Rotterdam: Sense Publisher.

Leikin, R. (2011). Multiple-solution tasks: from a teacher education course to teacher practice. *ZDM*, *43*(6-7), 993-1006.

Leikin, R. (2013). Evaluating mathematical creativity: The interplay between multiplicity and insight1. *Psychological Test and Assessment Modeling*, *55*(4), 385.

Leikin, R., & Lev, M. (2007, July). Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In *Proceedings of the 31st international conference for the psychology of mathematics education* (Vol. 3, pp. 161-168).

Leikin, R., & Lev, M. (2013). Mathematical creativity in generally gifted and mathematically excelling adolescents: What makes the difference?. *Zdm*, *45*(2), 183-197.

Leikin, R., Levav-Waynberg, A., & Guberman, R. (2011). Employing multiple solution tasks for the development of mathematical creativity: Two comparative studies. In *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education (CERME7)* (pp. 1094-1103).

Levav-Waynberg, A., & Leikin, R. (2012). The role of multiple solution tasks in developing knowledge and creativity in geometry. *The Journal of Mathematical Behavior*, *31*(1), 73-90.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For the learning of mathematics*, *26*(1), 17-19.

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic inquiry. Sage Publications.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, 30, 236–262.

Mann, E. L. (2009). The search for mathematical creativity: Identifying creative potential in middle school students. *Creativity Research Journal*, *21*(4), 338-348.

Merriam, S. B. & Tisdell, E. J. (2016). Qualitative research: A guide to design and implementation. (4th ed.). John Wiley & Sons Inc.

Merriam, S. B. (2009). *Qualitative Research: A Guide to Design and Implementation* (2. b.). San Francisco: John Wiley & Sons Inc.

Milgram, R. M., & Livne, N. L. (2005). Creativity as a general and a domain-specific ability: The domain of mathematics as an exemplar. In *Creativity Across Domains* (pp. 205-222). Psychology Press.

Milli Eğitim Bakanlığı [Ministry of Education] (2015). Ġlkokul matematik dersi 1, 2, 3 ve 4. sınıflar öğretim programı [Elementary mathematics curriculum for 1,2,3 and 4 graders]. Ankara, Turkey.

Ministry of National Education [MoNE] (2009). İlköğretim Matematik Dersi 6-8. Sınıflar Öğretim Programı. Ankara: Talim ve Terbiye Kurulu

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: some definitions and characteristics. *Procedia-Social and Behavioral Sciences*, *31*, 285-291.

National Advisory Committee on Creative and Cultural Education. (1999). *All our futures: Creativity, culture and education* (p. 62). Suffolk: DfEE publications.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

Natsheh, I., & Karsenty, R. (2014). Exploring the potential role of visual reasoning tasks among inexperienced solvers. *ZDM*, *46*(1), 109-122.

Page, T., & Thorsteinsson, G. (2017). Teaching Creativity across the Curriculum through Design Education?. *Journal of Educational Technology*, *14*(1), 7-19.

Peken, M. & Aydın, İ. (2009). *Ortaöğretim Geometri 1*. A. Baysal (Ed.). Ankara, Farklı Yayıncılık.

Plucker, J. A. (1998). Beware of simple conclusions: The case for content generality of creativity. *Creativity Research Journal*, *11*(2), 179-182.

Plucker, J. A., & Makel, M. C. (2010). Assessment of creativity.

Plucker, J. A., Beghetto, R. A., & Dow, G. T. (2004). Why isn't creativity more important to educational psychologists? Potentials, pitfalls, and future directions in creativity research. *Educational psychologist*, 39(2), 83–96.

Plucker, J., & Zabelina, D. (2009). Creativity and interdisciplinarity: One creativity or many creativities?. *Zdm*, *41*(1), 5-11.

Pope, R. (2005). *Creativity: Theory, history, practice*. Routledge.

Preiner, J. (2008). Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra (Unpublished Doctoral Dissertation). University of Salzburg, Salzburg, Austria.

Saha, R. A., Ayub, A. F. M., & Tarmizi, R. A. (2010). The effects of GeoGebra on mathematics achievement: enlightening coordinate geometry learning. *Procedia-Social and Behavioral Sciences*, *8*, 686-693.

Sangwin, C. (2007). A brief review of GeoGebra: dynamic mathematics. *MSor Connections*, *7*(2), 36.

Sawyer, R. K. (2011). *Explaining creativity: The science of human innovation*. Oxford university press.

Sedig, K., & Sumner, M. (2006). Characterizing interaction with visual mathematical representations. *International Journal of Computers for Mathematical Learning*, *11*(1), 1-55.

Selvy, Y., Ikhsan, M., & Johar, R. (2020, February). Improving students' mathematical creative thinking and motivation through GeoGebra assisted problem based learning. In *Journal of Physics: Conference Series* (Vol. 1460, No. 1, p. 012004). IOP Publishing.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *Zdm*, *29*(3), 75-80.

Singer F.M., Sheffield, L.J. & Leikin, R. (2017). Advancements in research on creativity and giftedness in mathematics education: introduction to the special issue. *ZDM Mathematics Education*, 49(1): 5. doi:10.1007/s11858-017-0836-x

Sriraman, B. (2004). The characteristics of mathematical creativity. *Mathematics Educator*, *14*(1), 19-34.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education*, 17, 20–36.

Sriraman, B., Yaftian, N., & Lee, K. H. (2011). Mathematical creativity and mathematics education: A derivative of existing research. In B. Sriraman & K. H. Lee (Eds.), The elements of creativity and giftedness in mathematics (pp. 119-130). Rotterdam, the Netherlands: Sense Publishers.

Stake, R. E. (1995). The art of case study research. Thousand Oaks : Sage Publications, c1995. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip&db=cat06966a&AN=metu.b1292995&site=eds-live&authtype=ip,uid

Sternberg, R. (2007). Creativity as a habit. In Tan, A-G. (Ed.), *Creativity: a handbook for teachers* (pp. 3-25). London: World Scientific Publishing Limited.

Straesser. R. (2001). Cabri-Geometre: Does dynamic geometry software (DGS) change geometry and its teaching and learning? *International Journal of Computers for Mathematical Learning*, 319-333. Netherlands: Kluwer Academic Publishers.

Şayakdokuyan, A. (2012). *Ortaöğretim Geometri Ders Kitabı 11. Sınıf*. A. Göztok (Ed). Ankara, Ekoyay Eğitim Yayıncılık

Torrance, E. P. (1965). Scientific views of creativity and factors affecting its growth. *Daedalus*, 663-681.

Yerushalmy, M. (2009). Educational technology and curricular design: Promoting mathematical creativity for all students. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 101–113). Rotterdam: Sense Publishers.

Yildiz, A., Baltaci, S., & Demir, B. K. (2017). Reflection on the Analytic Geometry Courses: The Geogebra Software and Its Effect on Creative Thinking. *Universal Journal of Educational Research*, *5*(4), 620-630.

Zhao, Y. (2012). World class learners: Educating creative and entrepreneurial students. Thousand Oaks, CA: Corwin Press.

APPENDIX

A. METU HUMAN SUBJECTS ETHICS COMMITTEE APPROVAL

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
ueam@metu.edu.tr
www.ueam.metu.edu.tr

Sayı: 28620816 / 672

19 ARALIK 2018

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Erdinç ÇAKIROĞLU

Danışmanlığını yaptığınız Zülal MELEK'in "**Dinamik geometri yazılımlarının, matematik öğretmen adaylarının geometri görevlerinde matematiksel yaratıcılıklarındaki rolü**" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2018-EGT-196** protokol numarası ile araştırma yapması onaylanmıştır.

Saygılarımla bilgilerinize sunarım.

Prof. Dr. Tülin GENÇÖZ
Başkan

Prof. Dr. Ayhan SOL
Üye

Prof. Dr. Ayhan Gürbüz DEMİR (4.)
Üye

Prof.Dr. Yaşar KONDAKÇI
Üye
İZİNLİ

Doç. Dr. Emre SELÇUK
Üye

Doç. Dr. Pınar KAYGAN
Üye

Dr. Öğr. Üyesi Ali Emre TURGUT
Üye