PROBLEM-SOLVING PROCESSES OF MATHEMATICALLY GIFTED AND NON-GIFTED STUDENTS

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ABSTRACT

PROBLEM-SOLVING PROCESSES OF MATHEMATICALLY GIFTED AND NON-GIFTED STUDENTS

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This study aimed to investigate the use of problem-solving phases and strategies of seven mathematically gifted, seven successful, and six average students attending fifth-grade in different public and private schools. The participants were selected through purposeful sampling among those who volunteered to participate in the study in a city in Western Turkey. The study was designed as a qualitative case study. Data were collected through clinical task-based interviews that included six problems, researchers' field notes, and students' solution sheets. All sessions were audio-recorded. Participants' observation forms, solution sheets, and voice recordings were analyzed to determine the problem-solving phases suggested by Polya (2004) and strategies used as they attempted to solve each problem. The results showed that three groups of participants varied concerning the use and style of problem-solving phases and problem-solving strategies. Mathematically gifted students presented a higher number of attempts in applying problem-solving phases. They applied the phases sequentially while successful and average students applied them as not sequentially. Mathematically gifted students were also observed as using problem-solving strategies most and most efficiently. Adopting a different point of view and acting it out or simulation strategies were the most preferred and effectively used strategies by them while logical reasoning was the most used strategy by successful and average participants. However, logical reasoning was not utilized most effectively by the successful students. Mathematically gifted students were the most successful group in generalization and considering all conditions in a problem when using a problem-solving strategy.

Keywords: Problem-Solving Phases, Problem-Solving Strategies, Mathematically Gifted Students, Multiple Case Study, Clinical Task-Based Interview

ÖZ

MATEMATİKTE ÜSTÜN YETENEKLİ VE ÜSTÜN YETENEKLİ OLMAYAN ÖĞRENCİLERİN PROBLEM ÇÖZME SÜREÇLERİ

Sipahi, Yasemin
Yüksek Lisans, Matematik Eğitimi, Fen ve Matematik Bilimleri Eğitimi
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Çalışmanın amacı farklı özel ve devlet okullarında beşinci sınıfa devam eden yedi matematikte üstün yetenekli, yedi başarılı ve altı ortalama öğrencinin problem çözme stratejilerini ve aşamalarını incelemektir. Araştırmanın katılımcıları amaçlı örneklem yöntemiyle Türkiye’nin batısında yer alan bir şehirden gönüllü olarak katılmak isteyenler arasından seçilmiştir. Çalışma, nitel durum çalışması olarak tasarlanmıştır. Veriler, altı matematik problemini içeren kinik göreve dayalı görüşmeler, araştırmacı alan notları ve öğrencinin çözüm kağıtları aracılığıyla toplanmıştır. Tüm görüşmeler ses kaydına alınmıştır. Her bir problemi çözerken Polya (2004)’nın problem çözme aşamalarını ve problem çözme stratejilerini kullanım durumlarını belirlemek amacıyla katılımcıların gözlem formları, çözüm kağıtları ve ses kayıtları analiz edilmiştir. Çalışmanın sonuçları üç grubun da problem çözme aşamaları ve stratejilerinin kullanımı ve kullanım biçimi bakımından farklılaştığını göstermiştir. Matematikte üstün yetenekli öğrenciler problem çözme aşamalarını kulanma açısından en çok girişimde bulunan grup olmuştur. Başarılı ve ortalama öğrenciler problem çözme aşamalarını sıralı olmayan şekilde uygularken matematikte üstün yetenkli öğrenciler aşamaları sıralı bir şekilde uygulamışlardır. Aynı zamanda matematikte üstün yetenekli öğrenci grubu en çok problem çözme stratejisini kullanan ve bu stratejileri en verimli şekilde kullanan grup olmuştur. Farklı bir bakış açısını benimseme stratejisi ve canlandırma ya da simülasyon stratejisi matematikte üstün yetenekli öğrenciler tarafından en çok tercih edilen ve en verimli kullanılan strateji olurken akıl yürütme stratejisi en çok başarılı ve ortalama öğrenciler tarafından kullanılmıştır. Öte yandan akıl yürütme stratejisi başarılı öğrenciler tarafından en verimli şekilde kullanılan strateji olmamıştır. Matematikte üstün yetenekli öğrenciler problem çözme stratejisini kullanırken genelleme yapma ve problemdeki tüm durumlara dikkat ederek stratejiyi uygulamada en başarılı grup olmuştur.

Anahtar Kelimeler: Problem Çözme Aşamaları, Problem Çözme Stratejileri, Matematikte Üstün Yetenekli Öğrenciler, Çoklu Durum Araştırması, Klinik Görev Temelli Görüşmeler

To My Family

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Ministry of National Education	MoNE
Science and Art Center	SAC

CHAPTER 1

INTRODUCTION

Although until the 19th century, gifted children were not at the forefront (Borland, 2005), the root of giftedness as an extraordinary achievement has always been attention-grabbing for the humankind to discover (Renzulli, 2011; Ziegler & Heller, 2000). In the first studies about the fundamentals of giftedness, the term “gifted” is used as “genius” or “talented” in general terms of giftedness (Kaufman & Sternberg, 2008). These two concepts; giftedness and talent, were not thought independently of each other (Feldhusen, 2005), and were defined for an exceptional population of gifted students (Gagne, 2004). This concept has been defined from different viewpoints for many years. As a pioneer of research on giftedness research, Galton (1869) attributed the geniusness to inheritance (as cited in Kaufman & Sternberg, 2008) while Terman (1925) emphasized IQ scores for a definition of giftedness (as cited in Bergold et al., 2020; Chang, 1985; Kaufman & Sternberg, 2008; Simonton, & Song, 2009; Warne, 2019). On the other hand, Marland (1971) reported that there are more criteria that needed to be taken into consideration to define giftedness such as standardized test scores, teacher opinion, multiple abilities, and beware to define giftedness. In addition, Renzulli (2011) stated that giftedness should be examined under three traits: “above average intelligence”, “high levels of creativity” and “high levels of “task commitment” (p.81). Carroll (1993) associated giftedness with different components such as “verbal reasoning”, “number (speed)”, “speed of reasoning”, and “broad visualization” instead of a single criterion (p.494). Alongside, Gardner’s Theory of Multiple Intelligence (1983) emphasized different types of intelligence, each has its measurable abilities (Brualdi, 1996; Morgan, 1996). According to Kaufman and

Sternberg (2008), there is no single criterion to define giftedness since it is a very broad and variant concept. In the light of this information, some researchers defined giftedness with different characteristics. With this respect, intelligence is described in line with different components apart from one perspective. For that reason, giftedness can be defined with multiple disciplines and abilities, as in Marland's (1971) following definition: "professionally qualified people" who can show high-performance thanks to their high capability containing some form or combination of "general intellectual ability, specific academic aptitude, creative or productive thinking, leadership ability, visual and performing acts, psychomotor ability" (Marland, 1971, p.21). By considering all of these remarkable characteristics, many countries give importance to the education of gifted children (Köksal et al., 2017) since the information about how we enhance the learning is directly proportional to the understanding of how gifts and talents can be developed (Barfurth et al., 2009).

When giftedness was evaluated with different components that researchers expressed, some concepts became prominent like creativity (Carroll, 1976). Creativity was observed among gifted students as having creative thinking and generating creative products, especially while problems are solved. Gifted students as masters of dealing with "complex concepts" and "abstract materials" have a high level of thinking and verbal skills (Archambault et al., 1993) so they have distinctive characteristics such as being able to categorize problems more efficiently, being faster, and flexible in problem-solving comparing to their non-gifted peers (Shore and Kanevsky, 1993, as cited in Steiner, & Carr 2003). Therefore, gifted students differ from their non-gifted peers concerning these characteristics. It was expressed that gifted students "have the potential to be our future problem solvers." (Brody & Stanley, 2005, p. 27).

According to the studies related to strategies of gifted students, gifted students' preferences and problem-solving processes that gifted students overcome are different from non-gifted students (Bayazıt & Koçyiğit, 2017; Heinze, 2005; Hong & Aqui, 2004; Montague & Applegate, 2000; Overtoom-Corsmit & Span, 1986). Gifted students have a better skill about choosing and using the proper strategy to solve a problem (Steiner, 2006). Gifted students as the best problem solvers (Gorodetsky & Klavir, 2003) know more problem-solving strategies than their non-gifted peers (Benito, 1995; Steiner, 2006). In the study of Span and Overtoom-Corsmit (1986), averagely gifted students used the strategy of trial and error in problem-solving while highly gifted ones preferred different problem-solving strategies. Likewise, check and guess strategy such as trial and error was mainly observed among mainly non-gifted students by Bayazıt and Koçyiğit's (2017). In another study, Patisivian (2006) specifically expressed that gifted students mostly tended to use the following strategies: drawing a picture, making a table, and looking for a pattern. Similarly, Yıldız et al. (2012), Aydoğdu and Keşan (2016), and Bayazıt and Koçyiğit (2017) highlighted that making a drawing strategy is the most used strategy in problem solving among gifted students. On the other hand, other strategies such as intelligence guessing and testing, simplifying a problem and working backwards were also observed among gifted students (Aydoğdu & Keşan, 2016). In addition to these studies,in the study of Threlfall and Hargreaves (2008), gifted students have found as having similar performance regarding the use of strategies in problem solving and similar miscnpeptions when compared to older students of average ability.

As the term giftedness was started to be evaluated with different components (Chang, 1985), concerning the conception of giftedness, specific disciplines began to emerge. Krutetskii (1976) expressed that mathematically gifted students have a "mathematical cast of mind" (p.302). In other words, they can look at every object and situation in life from a mathematical perspective. From the point of domain-specific view, giftedness becomes prominent in specific areas (Subotnik et al., 2017), such as mathematics and physics, despite general perspectives in the field of giftedness. Gardner (1983) also named mathematical intelligence as "logical-mathematical intelligence" as other different areas of the field of intelligence (Morgan, 1996). The term mathematically gifted is described with students having ability and promising (Budak, 2012). It is thought that being special in mathematics and general giftedness are related to mathematical giftedness (Leikin et al., 2017). However, mathematically giftedness is related to being successful in school mathematics or general tasks. In addition to this, it refers to be able to have new mathematical ideas and be master in these ideas and be competent in problem-solving (Koshy et al., 2009). In the light of this information with problem-solving, in the study of Overtoom-Corsmit et al. (1990), gifted students solved the problems quickly compared to average ones. Similarly, Heinze (2005) stated that mathematically gifted students do not spend time on the problem when compared to non-gifted peers. Also, according to Ünal (2019), mathematically gifted students do not spend extra time understanding the problem. On the other hand, Sriraman (2003) reported that mathematically gifted students allocate remarkable time to understand the problem, identify the situation and devise a plan during a solving process. Another study conducted by Budak (2012) also highlighted that mathematically promising students devoted a long time thinking, reflecting, and planning during problem-solving. Moreover, Chang (1985) stated that the most important trait that distinguishes mathematically gifted students from their non-gifted peers is their thought processes in mathematics. Mathematically gifted students have extraordinary thinking about finding different ways to solve problems that they encountered previously (Chang, 1985; Greenes, 1981; Leikin, 2010). Wagner and Zimmermann (1986) evaluated mathematical giftedness as a measurable ability. This ability includes being able to distinguish the pattern, look at the other side of the problem, and present the problem in different ways. Similarly, Budak (2012) stated that mathematically gifted students can look for different ways to solve problems in the problem-solving process. Nevertheless, to the best of my knowledge, after searching for relevant studies with appropriate keywords (e.g. mathematical giftedness and problem-solving strategy) in available literature, there are no studies found to present which problem-solving strategies they prefer and to demonstrate how they use the problem-solving strategies in a detailed way.

In the literature presented above, there are studies related to problem-solving processes of gifted students (Akdeniz & Alpan, 2020; Archambault, 1993; Aydoğdu & Keşan, 2016; Overtoom-Corsmit et al., 1990; Pativisian, 2006; Span & Overtoom-Corsmit, 1986) and comparative studies with non-gifted students (Bayazıt & Koçyiğit, 2017; Gorodetsky & Klavir, 2003; Hong & Aqui, 2004; Montague & Applegate, 2000; Yıldız et al., 2012). Especially, comparative studies showed that gifted students' problem-solving processes for the recognition of strategy and phases were different from those of non-gifted students. On the other hand, there are three issues not answered in the previous studies and need further attention. First, few studies have been conducted to examine strategy use of gifted and non-gifted students comparatively, even though there are many studies related to problem-solving strategies of students in general. Moreover, those related to gifted students (Akdeniz & Alpan, 2020; Aydoğdu & Keşan, 2016; Pativisian, 2006) have been conducted only with gifted students. The studies particularly pointed out that gifted students gave importance to the problem-solving phases and the use of different types of problem-solving strategies. However, they do not provide satisfactory explanations about how gifted students might use problem-solving phases and strategies compared to their non-gifted peers. Second, in particular, when studies on the problem-solving processes are considered in the light of the problem-solving phases and strategies of mathematically gifted students, there is not a sufficient number of studies explaining which problem-solving phases and strategies mathematically gifted students would prefer and how they differ from their non-gifted peers. Mathematically gifted students were considered different from other ones regarding both qualitative and quantitative problem-solving processes (Budak, 2012; Chang, 1985; Greenes, 1981; Leikin, 2010). However, there were quite a few studies (Heinze, 2005; Sriraman, 2003; Threlfall & Hagreaves, 2008) comparing the problem-solving process of mathematically gifted and non-gifted students like successful and average students. These studies also drew attention to the fact that mathematically gifted students can use different problem-solving strategies. However, they did not express the use of these strategies in a detailed manner. Third, as explained above, the use of problem-solving phases followed by gifted students differs from study to study. Various studies (e.g., Budak, 2012; Heinze, 2005; Sriraman, 2003; Ünal, 2019) conducted with mathematically gifted students reported their use of the problem-solving process. Nevertheless, their results were not consistent with each other's concerning the use of phases at some points. For instance, Heinze (2005) and Ünal (2019) reported that mathematically gifted students spent a long time understanding the problem phase while Budak (2012) and Sriraman (2003) stated that they did not spend a long time for the same phase. Therefore, there is still no consistency concerning the use of the problem-solving phases among gifted students. This study may make inferences about these different results. In this sense, case studies can help us discover how giftedness should be approached with scientific data.

This study was planned to examine problem-solving phases and strategies used by mathematically gifted and non-gifted students and to reveal the possible reasons for similarities and differences by comparing them. Related studies (e.g., Bayazıt & Koçyiğit, 2017; Budak, 2012; Heinze, 2005; Threlfall & Hargreaves, 2008; Ünal, 2019) were generally conducted with primary and secondary gifted students, which may indicate that these studies gave the importance to work with early-age students. Also, students from middle schools were reached easier. Therefore, in particular, I aimed to investigate three groups of fifth-grade students' (i.e. mathematically gifted, successful, and average) problem-solving strategies and Polya's (2004) problem-solving phases in non-routine mathematical problems.

1.1 Research Questions

This study aimed to investigate the problem-solving process and strategy use of the fifth-grade mathematically gifted, successful, and average students enrolled in public and private middle schools**.** In this study, Polya's (2004) problem-solving phases and problem-solving strategies in mathematics problems, which was reported by Posamentier and Krulik (2009) in line with the fifth-grade level were analyzed and categorized. Furthermore, this study aimed to explore whether there are differences between mathematically gifted and successful and average students' processes and strategies in mathematics problems. Therefore, the following research questions guided the study.

1) How do mathematically gifted, successful, and average students use the problem-solving phases advocated by Polya (2004)?
2) Which problem-solving strategies do mathematically gifted, successful, and average students use while solving a non-routine problem?
3) How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?

1.2 Significance of the Study

There has been increased attention to gifted students and their education by authorities and stakeholders in recent years. Even though every country should accept that gifted students are among the most valuable sources for themselves (Johny, 2008; Sternberg, 2020), they are not understood precisely. That is, their needs are not exactly accommodated. For that manner, they might face wrong treatments (Baykoç & Aydemir, 2014).

Starting from the early ages of gifted children, their families are the first reference guiding and meeting their educational needs (Bildiren, 2018; Karakuş, 2010). On the other hand, their families do not necessarily have enough information about approaching them and guiding them when needed (Özdemir & Bostan, 2019). Counseling that families with gifted children most want to receive is concerned with is the child's characteristics (Ihlamur, 2017). Feeling inadequate to guide and understand their children is one of the most frequent problems for these parents (Girgin, 2019). Family and their gifted students do not speak in the same language or act in the same way when communicating. Gifted students having difficulties in sharing their ideas would have challenges in expressing their ideas and approaching and solving problems at home and school (Köksal et al., 2017). The parents cannot understand their gifted children's language or statements. It is concluded that parents need to be trained about approaching and understanding their children (Karakuş, 2010). They do not receive support, education, or training about approaching and communicating with their "gifted" kids by the relevant authorities or organizations. For example, Bildiren (2018) highlighted that the parents of gifted children were identified in Turkey do not receive any training regarding the characteristics of these children. This situation demonstrated that there is a problem for gifted students of not being understood from an early age.

Gifted students having difficulties in sharing their ideas would have challenges in expressing their ideas and approaching and solving problems at home and school (Köksal et al., 2017) even though they might be very good at, for example, problem-solving in mathematics. From the perspective of teachers and educators, a clear descriptive framework about attitudes towards gifted students and their education is not presented (McCoach & Siegle, 2007). Despite their talent, gifted ones are not understood by teachers, administrations, who have mostly inadequate information about the giftedness and their needs (Akgül, 2021; Marland, 1971). Çapan (2010) reported that prospective teachers think that they are not able to provide an atmosphere in education for gifted children. In another study, Archambault et al.(1993) reported that classroom teachers do not apply inquiry and thinking skills activities for gifted students more or less than non-gifted students. As seen in the US, critical thinking and creative problem solving are the areas in which gifted students do not have more chance than their non-gifted peers to focus. On the other hand, teachers who are unaware of the characteristics and teaching demands of “high ability” students have an unfavorable situation (Manning, 2006). That is, in a study by Koshy et al., (2009), teachers mostly complained that they were not adequately prepared and properly educated in mathematics for their mathematically promising students; therefore, they do not feel confident in helping them. Furthermore, researchers do not have enough information about how gifted children can use their exceptional knowledge and skills in different ways or how the teachers might help gifted students to solve more complicated tasks (Steiner & Carr 2003). In this context, knowledge about the needs of gifted students can also guide teachers concerning what they should do or not do for their education (Özdemir & Bostan, 2019).

Mathematically giftedness as having upper comprehension (Lubinski & Humphreys, 1990) is not an exact and standard definition accepted by the majority (Singer et al., 2017). As only a few studies conducted about mathematically gifted students’ problem-solving process (e.g., see Heinze, 2005; Sriraman, 2003) and mathematical giftedness is different from general giftedness (Leikin et al., 2013), more research is needed to understand if and how mathematically gifted students’ problem-solving processes are different from those of others. Also, researching students who are good at problem-solving could give information about their less capable peers (Montague, 1991). Furthermore, there is not much information about the reasons why several people can solve problems differently with “an aesthetic point of view” compared to the others (Tjoe, 2015, p.165). Pativisian (2006) also underlined the idea that the problem solving-process should be analyzed in a detailed way to understand how a mathematical problem is solved. Besides, longitudinal studies containing a wide range of practices can help to observe the mathematical ability of students in an efficient way (Koshy et al., 2009) since "the very nature of giftedness is still open to question" (Heller & Schofield, 2000, p.134).

In summary, the current study aims to contribute to the current literature by filling gaps in two areas. The first one is about understanding the giftedness and mathematical giftedness in various aspects and finding how to approach gifted and mathematically gifted students regarding problem-solving since these two groups of students do not necessarily have the same or similar characteristics. The second gap is about the lack of comprehensive studies related to understanding the similarities and differences between gifted and non-gifted students' problem-solving processes. There are not enough studies to enlighten which strategies and phases mathematically gifted students prefer during problem-solving processes. Thus, with the current study, I planned to contribute to filling these gaps by investigating the fifth-grade mathematically gifted, successful, and average students' problem-solving phases and strategies for the mathematical problems. The study will contribute to the related literature by presenting the similarities and differences of mathematically gifted, successful, and average students' problem-solving processes in a comprehensive manner.

1.3 Definition of Important Terms

Giftedness

"Students with gifts and talents perform - or have the capability to perform - at higher levels compared to others of the same age, experience, and environment in one or more domains" (NAGC, 2019, p. 1). In this study, as having the higher level, the giftedness was determined according to intelligence test administered in the Counseling and Research Centers in Turkey.

Mathematical Giftedness

According to Krutetskii (1976), mathematical giftedness is related to the ability to take, process, and maintain mathematical information with a "mathematical cast of mind" (p. 302). Having a mathematical cast of mind is about seeing the relationships in an environment from logical and mathematical perspectives. In this study, mathematically gifted students were identified according to two criteria. The first one was to be diagnosed as gifted as tested with intelligence scale. The second one was labeling by their classroom and mathematics teacher as gifted in mathematics and mathematical problem-solving.

Successful Students

In this study, successful students were defined as those who are not labeled as mathematically gifted or gifted students by stakeholders but are labeled by their classroom and mathematics teacher as successful students at mathematics and mathematical problem-solving.

Average Students

In this study, average students were defined as those who are not labeled as mathematically gifted, gifted, or successful but labeled by their classroom and mathematics teacher as average, below the successful students at mathematics and mathematical problem-solving.

Mathematical Problem

A mathematical problem as having given part, goal part, and feasible operations should also have arithmetic computation and algebraic operations in its solution process (Mayer & Hegarty, 1996). There are routine and non-routine problem types in mathematical problems. Routine problems have one direct solution while non-routine problems have different solving ways (Budak, 2012; Mayer & Hegarty, 1996). In this study, non-routine mathematical problems were used to obtain the data.

Problem Solving

Problem-solving is "an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine" (Cai & Lester, 2005, p. 221 as cited in Szabo & Andrews, 2018).

Problem Solving Phases

Polya (2004) suggested four phases to solve a problem conveniently: understanding the problem, devising a plan, carrying out the plan, and looking back.

Problem Solving Strategies

Mayer (1983) expressed that, "a problem-solving strategy is a technique that may not guarantee a solution, but serves as a guide in the problem-solving process." (as cited in Gick, 1986, p. 100). Nine problem-solving strategies reported by Posamentier and Krulik (2009) are investigated in this study; organizing data, intelligent guessing, and testing, solving a simpler equivalent problem, acting it out or simulation, working backward, finding a pattern, logical reasoning, making a drawing, adopting a different point of view. For example, consider the following problem; a question is presented like that: "Angelica has some 50 cents and dollars. The number of her dollars is twice the number of her 50 cents. In total, she has 12 dollars. So, how many 50 cents does she have?"

If a student wanted to use an intelligence guessing and testing strategy to solve this problem, s/he might give the numerical value to the number of dollars and cents to satisfy the condition of "number of her dollars is twice of her cents.". For instance, if s/he assigns "2" as the number of 50 cents, the number of dollars must be 4. So, the total money is equal to 6 dollars. However, this does not equal to 12 dollars, the expected correct answer. This time, s/he should continue trying another value for the number of 50 cents and dollars until reaching the right answer. For example, as the answer, 6, is half of the desired value, 12, s/he might "intelligently" guess that she should assign "$2 \times 2 = 4$" as the number of 50 cents.

Science and Art Center (SAC)

Science and Art Center is a government institution under the Ministry of National Education (MoNE) providing support education to the gifted students diagnosed as gifted from the field of general mental ability, visual arts, or musical talent fields to improve their abilities and enable them to use their capacities at the highest level (Bilsem, 2016). Gifted students attend these centers after their regular school time to work on projects about different disciplines to improve their problem-solving skills in the line with their talents. They take courses, do activities, and attend talent development workshops there (MoNE, 2019). To enter the SAC, students from first, second, and third-grade levels, observed and nominated as a candidate to be gifted by their class teachers, take a group exam conducted by the General Directorate of Special Education and Guidance Services. Then, students taking the exam successfully have a right to take the self-assessment exam with an intelligence test. After that, if students are successful in this exam, they can enter SAC as nominated in the general mental ability. Besides, the talent exam is held in the fields of painting and musical ability. Students are selected in this exam concerning the score determined by the commission. Finally, they are granted to enter the center to take the education programs that are regulated considering the students' learning capacity (Bilsem Online, 2020).

1.4 Motivation For the Study

As a student in my whole life, I have had many gifted friends who demonstrated that they see life differently from non-gifted ones. They used different words and had different behaviors when sharing an idea or way compared to others. Regarding the problem-solving process, they demonstrated high performance, especially with mathematical problems. On the other hand, the reality is not the same; they were not understood by their teachers as their non-gifted peers. Similarly, as a tutor and teacher, despite being aware of their high intelligence and differences, I do not feel well equipped about understanding gifted students. The reason is that we do not comprehend the gifted students' thinking process completely. They are curious, careful, sensitive students, and problem solvers about their environment. These traits make them different than others. What is more, they are talented in different areas such as musical, mathematical, and visual. Therefore, they become more systematic and complicated than others. On the other hand, these different types of areas bring different abilities in return. Moreover, their outstanding abilities, like being expertise in problem-solving, have become prominent in the literature. As problem-solving is considered crucial in many types of disciplines, gifted students with problem-solving abilities should become more crucial to study. In this context, investigating their problem-solving phases and strategies may help us understand them more in detail, especially at an early age. With this aim, I decided to analyze the fifth-grade students' problem-solving phases and strategies with which they used to explain their ways when solving.

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to investigate the fifth-grade mathematically gifted, successful, and average students' use of problem-solving strategy and problem-solving phases advocated by Polya (2004). This chapter will overview the related literature.

2.1 The Notion of a Mathematical Problem

Problem is related to not knowing the way firstly to achieve the goal (Duncker, 1945; Posamentier & Krulik, 2009). Grouws (1996) explained a problem from mathematical perspective by describing it as an issue that needs to be solved but the way to how to solve is not clear with current information at the first glance (as cited in Kayan & Çakıroğlu, 2008). Apart from this, according to Mayer and Hegarty (1996), for a problem to be a mathematical problem, mathematical methods such as arithmetic and algebraic method should be used in the solution process. This also indicated that a mathematical problem should be concerned with "a specific situation" (D'zurilla & Goldfried, 1971, p. 107). In conclusion, a mathematical problem, which does not have a prespecified solution, should have a mathematical procedure in the problem-solving process. Mathematical problems can be classified into routine and non-routine problems (Budak, 2012; Mayer & Hegarty, 1996). In routine mathematical problems, one can solve them in a direct and obvious way. For instance, (12+3) – (7-1) = ?. This type of problem has an exact solution for all solvers. In other respects, non-routine problems do not have a straightforward way to be solved. For instance, "Maria wants to find three consecutive even numbers whose sum is 60. What are these numbers?" (Posamentier & Krulik, 2009, p. 42). At first, a student might solve it to simplify the problem by using the solving simpler equivalent problem strategy. That is, s/he can select smaller three even numbers like 12+14+16, 16+18+20, and so on. Then, s/he can reach the right answer. Secondly, another student can use a logical reasoning strategy to solve it. S/he divides 60 to 3 and then, reorganizes 20+20+20 to provide them as consecutive numbers such as 18+20+22. So, the problem has different ways to be solved (Mayer & Hegarty, 1996). On the contrary to routine problems, non-routine problems serve as a student to solve the problems with different strategies. As Krulik and Rudnick (1989) stated that a problem should have unusual solutions for solvers. At the same time, there is consensus that upper-level mathematical abilities can come into view in favor of the non-routine problem-solving solving process (Szabo & Andrews, 2018). Budak (2012) also concluded in his study that mathematically promising students need to do extra non-routine problem-solving activities. As it turns out, students' problem-solving processes can be observed with the help of non-routine problems better. In this study, non-routine mathematical problems were used in light of this information.

2.2 Problem-Solving in Mathematics

Problem-solving is a cornerstone in every step of and an important part of mathematics and mathematics education (Posamentier, & Krulik, 2009). It is essential for both learning and doing mathematics (NCTM, 1998). The fundamental principle to study mathematics is problem-solving in which students can use their old knowledge for a new situation (NCGM, 1997). According to Szetala and Nicol (1992), problem-solving is a process in which solvers find the strategies to reach results by defining the given information. This process can be explained as a "goal-oriented process which requires the integrated use of a range of higher-order thinking skills, such as generating ideas, making interpretations and judgments, and using strategies to manage the complexity of situations" (Kirkwood, 2000, p.511). Similarly, Mayer & Hegarty (1996) stated that when a solver can comprehend how to use the given to reach the goal in a problem, problem-solving process begin to occur. Hence, it can be said that problem-solving is the best vehicle to observe students' thoughts and aspects about the problem-solving process and strategy use.

2.2.1 Problem-Solving Phases in Mathematics

Problem-solving is a particular activity "requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine." (Cai & Lester, 2005, p. 221). When these cognitive actions are considered, Kurtettski (1976, p.184) stated that the problem-solving process can be divided into three basic stages:

- Receiving information about the problem (related to an initial orientation towards its terms, an attempt to understand it),
- Processing (transforming) the obtained information to solve problems, and obtaining the desired results,
- Retaining information about the problem.

In the parallel line, Reif et al. (1976) mentioned three steps in the problem-solving process: problem analysis, construction, and check. In the phase of analysis, students describe the situation and goal of a problem clearly. In the construction phase, they comprehend the relations and use them to produce an efficient strategy. In the check phase, they control whether the goal has been achieved or not. Also, in this phase, they check the consistency of the solution and task. Also, Mason et al. (2010) expressed that there are three phases to approach a problem: Entry, Attack, and Review. The entry phase begins when students encounter a problem. Then, they try to understand the problem and determine what they will do about the solution. In the Attack phase, they try to solve the problem with their plan. Several plans can be adopted to solve the problem in this phase. Lastly, in the Review phase, they can check their works if they are satisfied with their plan or they do not have anything to do. Then, they extend their solutions. Garofalo and Lester's (1985) four components developed the problem-solving process. The four components are orientation, organization, execution, and verification. In orientation, students tried to understand the problem. In an organization, a plan is generated to solve a problem. In execution, students monitor their works and correspond to their plans. In verification, all three previous sections are evaluated. All of these phases have differences and similarities at some points. However, their roots came from Polya's (2004) research in *How to Solve It*. Hence, the present study used Polya's (2004) problem-solving phases. Polya (2004) pointed out that there are four phases in problem-solving to regulate changeable ideas and ways of thinking while solving a problem: Understanding the problem, Devising a plan, Carrying out the plan, and Looking back. These phases are explained in detail below (Polya, 2004):

Understanding the problem

In this phase, students should understand the situation of the problem. In other words, they should get the given, asked, and condition of the problem. They should also repeat or restate the problem, draw a figure related to the problem, and express what are verbally known and unknown. In this phase, the questions can be asked to help students to understand "*What is the unknown?, What are the data?, What is the condition?*" (Polya, 2004, p. 4).

Devising a plan

This section is about making a plan. Before solving the problem, the phase designs the solution containing "calculations, computations, or constructions" to find the unknown (Polya, 2004, p. 5). In this phase, strategies for solving the problem, for instance, logical reasoning or making a drawing, are designed using all the givens to conduct in the next phase. The question can be asked to encourage solvers to think: "Do you know a related problem?" (Polya, 2004, p. 5). In addition, previous knowledge and past experiences are important since the plan can be constructed according to the tested and well-work-out ideas from the previous works. If the ideas do not work as desired, the question “Could you restate the problem?” is asked to solvers to make them see the problem from a different viewpoint (Polya, 2004, p. 6).

Carrying out the plan

In this phase, the plan gives general instruction about the solution. The strategy planned in the previous phase is conducted here. If the strategy does not work, another strategy can be designed. This phase suggests that every detail of the plan should be checked and students should be sure about the steps of their plan. At this stage, the difference between looking and justification can come into view by asking the question “*Can you see clearly that the step is correct?”* (Polya, 2004, p. 8).

Looking back

This phase contains checking all the solution processes. Generally, students may finish the problem-solving process when they reach the unknown. Looking back to all the processes, results, and arguments can help not only the students develop their problem-solving skills but also they develop an understanding of the solution process. The following questions can help them: “Can you check the result or argument?”, “Can you derive the result differently?”, and “Can you use the result, or the method, for some other problem?” (Polya, 2004, p. 9).

2.2.2 Problem-Solving Strategies in Mathematics

Strategy is defined in Cambridge Dictionary (n.d.) with two meanings: plan as “being used to achieve something” and act as “the act of planning how to achieve something”. Similarly, a problem-solving strategy is a tool that is used to comprehend what sets of mathematical objects and relationships are (Schoenfeld, 2013). There are different types of problem-solving strategies (D’Zurilla & Goldfried, 1971) suggested for the solving process. Gick (1986) summarized the strategies concerning specific areas like geometry and algebra. In addition, asking questions, investigating the situations with diagrams, and applying trial and error are suggested to be used as problem-solving strategies (NCGM, 1997). From a comprehensive perspective, Posamentier and Krulik (2009) suggested nine problem-solving strategies used in mathematics for third to sixth-grade students: organizing data, intelligent guessing, and testing, solving a simpler equivalent problem, acting it out or simulation, working backwards, finding a pattern, logical reasoning, making a drawing, and adopting a different point of view. They have been used according to Posamentier and Krulik (2009) definitions:

- **Organizing data**

  Students analyze the given data in the problem. They can organize both visual and numerical data with a table or a list.
- **Intelligent guessing and testing**

  Students make reasonable guesses related to the problem’s logic instead of unreasonable guesses, then; they continue to intelligent guesses and test them in the context of the problem until they reach the right answer.
- **Solving a simpler equivalent problem**

  Students convert the original problem into a simpler form. Then, they reason on solution thanks to the simple form.
- **Acting it out or simulation**

  Students present the problem by using manipulative or other materials to comprehend the action of the problem.
- **Working backwards**

  Students approach problems reversely by thinking and making operations backward.
- **Finding a pattern**

  Students look for a geometric or numeric model or arrangement to solve the problem.

- **Logical reasoning**

  Students make logical deductions according to the given data and relations to solve the problem.

- **Making a drawing**

  Students represent the problem by drawing a picture or figure related to the problem situation to solve.

- **Adopting a different point of view**

  Students approach a problem from different aspects instead of its frequently used solution.

Posamentier and Krulik (2009) shared their suggestions about how these strategies can be used by teachers and students as a guide. They also presented non-routine mathematical problems to explain every strategy use. That's why these nine strategies were used as the expected strategies from the participants in this study.

If students want to be successful in problem-solving, they should have knowledge and ability of problem-solving strategies (Erbaş & Okur, 2012). Posamentier and Krulik (2009) stated that students generally approach a problem with only one strategy. However, gifted students are the ones who can approach the problem with various strategies compared to their non-gifted peers (Greenes, 1981; Krutetskii, 1976). According to the comparative studies with gifted and non-gifted students, the results showed that gifted ones find and use more efficient and different strategies in the problem-solving process (Benito, 1995). In addition, the use of some problem-solving strategies differs in gifted and non-gifted ones (Bayazıt, & Koçyiğit, 2017; Yıldız et al., 2012).

2.3 Gifted Students' Problem-Solving Processes

The term giftedness was defined with different traits since giftedness is not still described for a meeting of the minds (Johny, 2008; Singer et al., 2017; Ziegler, 2009). At first, Galton (1869) saw genius as an innate characteristic (as cited in Renzulli, 2011). After that, Terman (1925) pointed out intelligence test scores to define giftedness (Bergold et al., 2020; Chang, 1985; Kaufman & Sternberg, 2008; Simonton & Song, 2009; Warne, 2019). Apart from this, Marland's (1971) report showed that more than one criterion is effective to define giftedness, such as having a high score in the standardized test, opinion of teachers and stakeholders, sophisticated ability, and high attention. In the same way, Renzulli (2011) defined giftedness with the three-ring model, which contains "above average intelligence", "high levels of task commitment", and "high levels of creativity" (p. 81). In the three-ring model, these three criteria have an equal role in being. Above average ability refers to a measured intelligence test score above a certain point. Secondly, task commitment refers to endurance on a task for a long time. Thirdly, creativity refers to originality in thinking and approach (Renzulli, 2011). At the same time, Galton (1869) and Terman (1925) built a consensus on task commitment to determine giftedness (as cited in Renzulli, 2011). In this regard, a similar definition of gifted students is accepted in Science and Art Centers (SAC) in Turkey. In SAC, a gifted student is defined with the following qualities: as a faster learner compared to their non-gifted peers, be advanced in the fields related to creativity, art, and leadership, having the special academic ability, being able to comprehend abstract ideas, and act independently about their interests at the highest level (Bilsem, 2016). When these components are evaluated in a distinct perspective, extraordinary thinking in a task (Gagné, 2004), high intellectual capacity (Archambault et al., 1993), and problem-solving skills come to the forefront. At the same time, extraordinary thinking capacity and high intellectual capacity as outstanding features of gifted students affect problem-solving skills. In the literature, many definitions of giftedness were consubstantiated with problem-solving (Bayazıt & Koçyiğit, 2017). The literature also states that giftedness is mainly related to different and successful use of problem-solving processes and use of strategy (Benito, 1995). In various studies, different problem-solving phases were observed among gifted students such as orientation, execution and evaluation phases (Overtoom-Corsmit, 1986), control and division of the problem (Montague, 1991), understanding, plan, execution, and verification (Pativisan, 2006), thinking, reflecting, and planning (Budak, 2012). Specifically, gifted students in high school were found to be good at solving complex problems related to mathematics and science (Sowell et al., 1990). Their knowledge of strategy rises to prominence in comparison with their non-gifted peers (Steiner, 2006). Use of more strategies and different strategies were mainly observed in gifted students when compared to non-gifted students (Aydoğdu & Keşan, 2016; Bayazıt & Koçyiğit, 2017; Montague & Applegate, 2000; Yıldız et al. 2012). Moreover, gifted students can use different problem solving strategies by themselves (Aydoğdu & Keşan, 2016; Bayazıt & Koçyiğit, 2017; Pativisan, 2006; Yıldız et al. 2012).

2.4 Mathematically Gifted Students' Problem-Solving Processes

Mathematical giftedness has no universal definition accepted by all, and there is no consensus that this ability is learned or that people are born with it (Pitta-Pantazi et al., 2011; Rinn & Bishop, 2015; Singer et al., 2017). Nevertheless, there were many attempts to identify mathematical giftedness. In the domain of mathematical giftedness, one of the major comprehensive studies was carried out by Krutetskii (1976). He saw this ability as a developable trait with activity and instruction instead of seeing it as a constant ability. According to Krutetskii (1976), mathematical giftedness is related to seeing the world "through mathematical eyes" (p.302). In other words, mathematically gifted students can see the objects with the mathematical aspects, make spatial analyses, and consider quantitative relationships. They are prone to make the phenomena in their environment mathematical. In addition to this, mathematical giftedness is related to the ability of criticial thinking in mathematics (Chang, 1985) and high-level reasoning ability which is also related to mathematical ability (Ficici & Siegle, 2008). Krutetskii (1976) stated that these students approach terms in mathematical relationships with logical thinking and the stated mathematical abilities.

- The ability for the formalized perception of mathematical material, for grasping the formal structure of a problem,
- The ability for logical thought in the sphere of quantitative and spatial relationships, number and letter symbols; the ability to think in mathematical symbols,
- The ability for rapid and broad generalization of mathematical objects, relations, and operations,
- The ability to curtail the process of mathematical reasoning and the system of corresponding operations; the ability to think in curtailed structures,
- The flexibility of mental processes in mathematical activity,
- Striving for clarity, simplicity, economy, and rationality of solutions,
- The ability for the rapid and free reconstruction of the direction of a mental process, switching from a direct to a reverse train of thought (reversibility of the mental process in mathematical reasoning),
- Mathematical memory (generalized memory for mathematical relationships, type characteristics, schemes of arguments and proofs, methods of problem-solving, and principles of approach),
- A mathematical cast of mind (Krutetskii, 1976, pp. 350–351).

Besides, Wolfle (1986) presented the following list for the characteristics of mathematically gifted students derived from many research (e.g., Clark, 1983; Clendening & Davis, 1980; Gallagher, 1985; Wallace, 1983) and noted that they were observed in both gifted and mathematically gifted students.

- The ability to see a problem quickly and take the initiative to solve it,
- The ability to read rapidly and retain what is read, and can recall in detail,
- The ability to be reluctant to practice skills already mastered, finding such practice a waste of time,
- The ability to criticize constructively, sometimes argumentatively,
- The ability to be persistent in seeking task completion often sets very high personal goals and is a perfectionist,

- The ability to be keen and alert observers, note details and are quick to see similarities, differences, and anomalies,
- The ability to leap from concrete examples to abstractions, concepts, and syntheses quickly,
- The ability to be unwilling to accept statements without critical examination to find the "whys" and "hows",
- The ability to listen to only a part of the explanation and appear to lack concentration or interest, but know what is going on and usually know the answer,
- The ability to show rapid insights into cause-effect relationships,
- The ability to be often not willing to do busywork just to get a "grade.",
- The ability to be often perfectionist and show frustration with imperfection (Wolfle, 1986, pp. 82–83).

A study called Study of Mathematically Precocious Youth (SMPY) that was conducted at John Hopkins University utilized Scholastic Aptitude Test (SAT) scores to determine mathematically gifted students (Stanley & Benbow, 1982; Van Tassel-Baska, 2001). Lubinski and Benbow (2006) reported that the students with age less than 13-year-old and had an SAT score of 500 or more could understand all the courses at the high school level during a three-week summer program conducted within the SMPY. Students with a score of 700 or more in SAT were labeled as exceptional students in mathematics. Similarly, Lupkowski-Shoplik and Kuhnel (1995) stated that Elementary Student Talent Search (ESTS) program at the Carnegie Mellon University for mathematically talented elementary students was both a model and a service for the mathematically gifted in Pittsburg, the US. In this program, students took different courses like geometry, statistics, and probability, emphasizing problem-solving. They paid attention to share and discuss their problem-solving process and use different problem-solving strategies and solutions (Lupkowski-Shoplik & Kuhnel, 1995).

In parallel with these traits, “These students examine things thoroughly, observe relationships, recognize patterns, generalize results, and move rapidly from concrete to abstract thinking.” (Chang, 1985, p.77). Mathematically gifted students are able to generate their own problems, develop different thinking in problems, and look for particular relationships in solving process (Wolfle, 1986). In this sense, problem-solving skills come to forefront. The related studies stated that mathematically gifted students have different problem-solving processes with respect to the use of problem-solving phase compared to non-gifted students (Heinze, 2005; Sriraman, 2003). As the most remarkable characteristics of mathematically gifted students, being creative in problem-solving process drew attention (Ficici & Siegle, 2008). Likewise, being flexible in process of mathematical thinking was one of the qualifications in mathematically giftedness (Krutetskii, 1976). Grenees (1981) stated that mathematically gifted students differ from their non-gifted peers by finding and using unusual and alternative strategies in problem-solving process. According to the study of Sriraman (2003), mathematical giftedness and ability of problem-solving are interrelated. As gifted ones, talented students in mathematics have a remarkable mathematical memory about problem-solving strategies (Krutettskii, 1976; Leikin et al., 2013). In recent years, the Ministry of National Education (MoNE) (2018) has pointed out that the mathematics curriculum should give importance to people who value mathematics, who have high mathematical thinking and, who are good at problem-solving in mathematics. In the light of these traits, the studies that investigate the mathematically gifted students' problem-solving process have been appreciated. In the review of literature, it was ssen that there are some studies about the problem-solving processes of gifted students containing the use of strategy and phase in problem-solving. In the present study, the first part about the studies related to gifted students' problem-solving processes and the following part about the studies related to mathematically gifted students' problem-solving processes are handled in the following sections.

2.5 Studies of Gifted Students' Problem-Solving Processes

Span and Overtoom-Corsmit (1986) conducted a study with 14 highly gifted and 14 averagely gifted students from lower secondary education to comprehend the process of information in problem-solving. Students were chosen according to three standards which are “Dutch research project, the Raven Progressive Matrices, and a creativity test” (p. 276) and teachers’ views. Data were obtained with the interviews asking students seven mathematical problems. Mathematical problems were selected according to having three phases which are orientation, execution, and evaluation. The results showed that averagely gifted students did not pay attention exactly to problem situations while highly gifted students tried to analyze the aim and situation of the problem. The averagely gifted ones used only one strategy which is trial and error in problem-solving mostly when the highly gifted students utilized other useful and effective problem-solving strategies such as numbering and comparing systematic. In addition, a great majority of highly gifted ones could evaluate and discuss the strategy in the problem-solving process. In the problem-solving process, they tried to remember similar problems and paid attention to every phase of the problem-solving process as far as possible. This rate was low in averagely gifted students. Similar results were reached in Montague’s (1991) study conducted with three gifted and three learning-disabled gifted students from eight-grade. Students’ giftedness was identified with an IQ test, a standard scale of characteristics of giftedness, and the need for special program documentation. Data were obtained via mathematical word problems in clinical interviews. According to findings, gifted students were more knowledgeable with cognitive and metacognitive strategies in the problem-solving process. They used the strategies with high awareness and their past experiences in the process. Gifted students not only paraphrased the problems but also reread them in pieces during the solving process. Checking the process of problem-solving and regenerating the solution were observed among them. They also guided themselves by instructing and monitoring. On the other hand, learning-disabled gifted students had relatively little information about knowledge of strategies and they could not have control over the multiple strategies used in the problem-solving process. Parallel to Span and Overtoom-Corsmit (1986) and Montague (1991), Pativisan’s (2006) study was conducted with five Thai gifted students whose grades differ from eighth to tenth to investigate the problem-solving process. Students were selected with the criteria of "having similar scores on the second round of the entrance examination to Thai Mathematical Olympiad (TMO), project and do not participate in the training camp in the previous year" (p.36). Three non-routine mathematical problems containing number theory, counting, and geometry were asked to the students in the interview. Apart from Span and Overtoom-Corsmit (1986) and Montague (1991), Pativisan (2006) found different results. She stated that gifted students' problem-solving stages containing understanding, planning, executing, and verifying thinking were not arranged in a linear order. They could not pursue the stages from the first to the second or from the second to the third stage in the problem-solving process. In addition, they had logical analysis and systematic strategies in the problem-solving process such as "drawing pictures, making tables and looking for patterns" (p.62). They were open to apply alternative solutions in problem-solving by using their past experiences. They also tended to explain their ideas and reasoning in the solution process. In 2020, Akdeniz and Alpan conducted a study about the creative problem-solving (CPS) process of 151 gifted and talented students who were registered in the general mental ability field, musical ability field, and visual arts talent field in the Science and Art Center (SAC). Data which were obtained from the Creative Problem Solving Styles Inventory demonstrated that gifted and talented students gave importance to understand the problem, defined the situation of the problem, and produced many ideas about the solution of the problem. Gifted and talented students, especially registered in the general mental ability field, preferred to make a detailed plan, implement the plan and evaluate the plan in the problem-solving process. On the other hand, gifted and talented students, especially registered in the musical ability field tended to produce possible solutions in problem-solving. In Aydoğdu and Keşan's (2016) study, 27 ninth-grade gifted students' problem-solving strategies in geometry were investigated. Their strategies were examined according to Van Hiele's geometrical thinking level. Giftedness was determined according to being in the "0.42% slice (99.58% of the exam takers)" in the high school entrance test in 2013 (p. 49). Semi-structured interviews, open-ended problems, and Van Hiele Geometry Test were used to obtain the data. In the 2nd-level (informal deduction), gifted students utilized mostly simplifying the problem, making a drawing, and variable strategies while acting out the problem strategy was used the least. In the 3rd-level (deduction), making a drawing, using the known information, using the variable, and solving a simpler analogous problem were mostly used. On the other hand, they used the strategies of intelligent guessing and testing, summarizing the problem, and acting out the problem the least. In the top-level (rigor), students utilized simplifying the problem, making a drawing, using the known information, and use variable strategies. Acting out the problem strategy was the least preferred at this level.

The second part is about comparative studies among gifted and non-gifted students. Montague and Applegate (2000) investigated the problem perception and problem-solving strategies with seventh and eighth-grade learning disabled, average achieving, and gifted students. Learning disabled students had 85 points or more, average achieving students had between 85 and 115 points, and gifted ones got 130 points from the WISC-R test. All students were asked six 1-step, 2-step, and 3-step problems. Results showed that disabled and average students were not different from each other concerning spending time in problem-solving. They spent more time in problems compared to their gifted peers. Gifted students used more different strategies in the problem-solving process even if problems became more difficult. Also, they were faster than their non-gifted peers in problems. On the other hand, learning disabled and average students did not have any information about problem-solving strategies. Findings also indicated that in 1-step problems, and 2-step problems, there is no significant difference between learning disabled, average, and gifted groups in terms of the total number of problem-solving strategies. Similarly, Gorodetsky and Klavir's (2003) study was conducted with 121 seventh and eighth-grade gifted and eighth and ninth-grade average students to define sub-processes throughout problem-solving. Gifted ones were defined by the Ministry of National Education as the ones taking above 131 IQ. Questionnaire and verbal problems were used to obtain data. The results indicated that there are differences between average and gifted students' process of problem-solving concerning "selectivity". While gifted students worked on "Selective Combination" and "Selective Encoding" related to the interest for the problem in a comprehensive manner, average students concentrated on "Selective Retrieval" and "Selective Comparison" related to past experiences of them (p. 318). On the other hand, focusing on past experiences and similar problems to the previous ones were observed among mainly gifted students in Span and Overtoom-Corsmit (1986), Montague (1991), and Pativisan's (2006) studies. At the same time, Gorodetsky and Klavir's (2003) study also pointed out that the differences between gifted and average students should be attributed not only to selectivity in the same processes but also to different sub-processes. All in all, the pattern of the sub-processes of average students was different from the gifted students not only in quantitative but also in qualitative perspectives. Besides, Yıldız et al. (2012) examined the use of problem-solving strategies by comparing six gifted and six non-gifted eight-grade students. Gifted students were attending to Science and Art Center (SAC) in which only gifted students scoring above a certain threshold in intelligence tests can go. Five problems were asked to students throughout the clinical interview. Gifted students were able to use all the problem-solving strategies in mathematical problems when compared to their non-gifted peers. Also, they used more strategies than their non-gifted peers. They mostly used the strategies of accounting for all possibilities, making a drawing, and working backward. On other hand, non-gifted students mostly used the strategies of accounting for all possibilities, organizing data, and working backward. Besides, gifted students preferred finding a pattern strategy only once while non-gifted students used adopting a different point of view strategy only twice. The strategy of accounting for all possibilities was mostly used by both gifted and non-gifted groups while intelligence guessing and testing strategy was not used in any problem-solving process. Apart from all of them, results also pointed out that some strategies did not work for problem-solving. Although both groups of students used the strategies, gifted ones were not totally successful in ten strategies and non-gifted students were not successful in twelve strategies while solving the problem. Similarly, Beyazıt and Koçyiğit (2017) reported a study with 36 gifted and 36 non-gifted students from the seventh and eighth-grade levels to examine their non-routine problem-solving success. Gifted students took educational support from Science and Art Center (SAC) in which only some students, who were identified as gifted according to the WISC-R intelligence test, came. Data were collected with ten non-routine problems in a semi-structured interview. The results showed that not only gifted students were more successful than their non-gifted peers but also they used a greater number and variety of strategies than their non-gifted peers in the problem-solving process. Gifted students mainly used drawing a picture and making list strategy, whereas non-gifted ones preferred to use mainly check and guess and writing an algebraic equation or arithmetic expression strategies in the problem-solving process. Even though non-gifted students were able to use the same problem-solving strategies as their gifted peers, they were not good at analyzing information, identifying relationships, making inferences, and reaching more general ideas from the solution as gifted students. Besides, the study emphasized the idea that the use of some problem-solving strategies can help transition to a high-level problem-solving strategy for gifted students. In addition to the previous studies, another result indicated that gifted students could use some strategies for a problem in a coordinated way.

2.6 Studies of Mathematically Gifted Students' Problem-Solving Processes

Budak (2012) conducted a study in which four mathematically promising students' problem-solving abilities were examined throughout non-routine mathematics problems. Four mathematically promising students were chosen according to teachers' and administrators' views, and getting first, ninth, tenth, and fourteenth place on the mathematical contest made by the province was the goal. Data were gathered with three non-routine problems in the interview. The results showed that all mathematically promising students spent a long time understanding, reflecting, and making a plan concerning problems. They tried to find alternative ways for a solution when the first way did not work. Furthermore, Ünal (2019) conducted a thesis study with mathematically gifted students to investigate the problem-solving process of five mathematically gifted students. Two high schools, two eighth grade, and one seventh grade students who were from the Mathematics Olympiads Group arranged by the Ministry of National Education were asked eight non-routine mathematical problems in interview. Officially, they had been registered as being successful at National Mathematics Olympiad organized by TUBITAK in Turkey. The results indicated that all mathematically gifted students can apply all Polya's (2004) problem-solving stages with a high awareness for non-routine mathematical problems. In the phase of carrying out a plan, they especially paid attention to quantities, numbers, symbols, in short, all steps in a problem. On the other hand, they did not feel the need to check the correctness of their solutions since they thought that they could justify their problem-solving process. Although they had previous experiences with the problems they did not benefit from them in the solution process. In addition, they did not need to be given give extra time to understand and solve the problems. This result differs from the previous results in that they did not spend extra time in problem-solving as Budak (2012) indicated. This was because of the time difference between the paper-pencil test and the interview. Besides, in this study, the mathematically gifted students were able to express their solutions very clearly. They were aware of their computations, thinking ways, and steps in the problem-solving process.

In the second part, comparative studies between mathematically gifted and non-gifted students were explained there. Sriraman (2003) conducted a study exploring the problem-solving experiences of four ninth-grade mathematically gifted and five non-gifted students. Mathematical giftedness was identified concerning three criteria: higher than 124 IQ points, the Stanford Achievement Test (95 %), and teacher’s and counselor’s views. Data were gathered with five non-routine combinatorial problems from clinical interviews, students, and teachers’ notes. In the study, in the problem-solving process, gifted and non-gifted students differ from each other concerning “orientation”, “organization”, “execution”, and “reflection” steps (p. 156). Mathematically gifted ones showed high tenacity and maintained their motivation for the problems for a long time. They gave significant time to understand the problem, evaluate the assumptions, and make a plan for the problem. When they did not find a general conclusion about problems, they tried to continue by simplifying the problems and controlling the changeable situation in the problem. On the other hand, non-gifted ones were more superficial about the process. Nevertheless, before the study, the researcher encouraged the students to write the strategies and ideas about the problem. That’s why this may lead students to learn the process and procedure. Besides, Heinze (2005) conducted a study with mathematically and normal elementary students whose ages differ from six to ten to examine their thinking process. Mathematically gifted ones were identified according to their teachers' and parents’ views and intelligence test scores. Mathematical giftedness was determined in terms of solving the “indicative tasks developed by Käpnick (1998)” (p.175). Four problems were asked to obtain the data. Unlike Sriraman’s (2003) findings, the results in this study indicated that normal students spent more time on the “unsolvable” puzzle problems compared to the mathematically gifted students. The result also pointed out that mathematically gifted students work more systematically and logically than their non-gifted peers. Moreover, the study pointed out the idea that mathematically gifted students are prone to verbalize their problem-solving process better. They preferred to explain their ideas and strategies in problem-solving. This validates that mathematically gifted students’ problem-solving process is different than their non-gifted peers. In this line, Threlfall and Hargreaves (2008) conducted a larger-scale study with 475 nine-year-old mathematically gifted and 230 thirteen-year-old average students to explore thinking in mathematical problem-solving of younger gifted and older non-gifted students. Mathematically gifted ones who took World Class Test in 2002 were selected by the teacher with the criteria of being "in the top 10 percent ability range in mathematics" (p. 89). Older average students were selected by their teachers as average ability in mathematics. Data were gathered via ten everyday mathematics problems from World Class Tests (WCT). The results expressed that mathematically gifted students have the same approaches and answers to the problems as older average students. In the study, nine years old mathematically gifted ones showed similar performance with 13 years old average ability ones in the mathematical problems as part of responses to the questions, solving methods, and even making conceptual errors.

2.7 Summary of the Literature Review

Problem-solving is not only a major component of mathematics education but also is a way of thinking (Posamentier & Krulik, 2009) since it requires logical thinking and reasoning for students (Szetala & Nicol, 1992). Therefore, an activity containing problem-solving is the key to examine a student's thinking. In this regard, studies in the literature based on problem-solving were conducted to examine both gifted and non-gifted students' thinking. At first, a great majority of studies (Bayazıt & Koçyiğit, 2017; Gorodetsky and Klavir, 2003; Montague & Applegate, 2000; Pativisan, 2006; Yıldız et al., 2012) revealed that gifted students have remarkable specialty on problem-solving compared to their non-gifted peers. They were able to analyze the problem situation and their awareness about the process of problem-solving was very high. Also, they were successful at finding and applying unusual strategies for solutions. Furthermore, mathematically gifted students have different problem-solving processes concerning problem-solving phases and strategy use compared to non-gifted students (Heinze, 2005; Sriraman, 2003). Secondly, although gifted and non-gifted students used the same problem-solving strategies, gifted ones come into prominence on analyzing, interpreting, and reasoning. On the other hand, which strategy is preferred by mathematically gifted and non-gifted students and which phase is observed within the problem-solving process still need to be analyzed. Because some phases and strategy use still differs among them from study to study. Moreover, especially mathematically gifted students differ in thinking during problem-solving. The researcher suggested that further research could be conducted to examine mathematically gifted students' reasoning processes for different types of problems (Ünal, 2019; Yıldız et al., 2012). Also, literature shows me that there is a need to investigate problem-solving abilities and characteristics of mathematically promising students with non-routine problems compared to their non-gifted peers (Budak, 2012). Therefore, the next step in the literature should be about how mathematically gifted and non-gifted students differ in the problem-solving process. Their problem-solving strategies and phases should be examined in a detailed way with non-routine mathematical problems. Thus, the current study was designed to examine the problem-solving processes regarding problem-solving strategies and phases of mathematically gifted and non-gifted students (successful and average) via the case study. The data were obtained throughout the clinical task-based interview, researcher observation form, and students' solution sheets. Findings were be analyzed according to codes and themes by considering the explanations of Posamentier and Krulik (2009) and Polya (2004) who were the pioneers in the literature of problem-solving.

CHAPTER 3

METHODOLOGY

This chapter outlines the methodology of the current study including the research design, participant selection, instruments, method of data collection, data analysis, validity and reliability, and assumptions of this research.

3.1 Research Design

The present study investigated problem-solving phases and strategies of fifth-grade mathematically gifted, successful, and average students in mathematics while they were solving non-routine mathematical problems. The study adopted a multiple case study design (Merriam, 2009). The three groups of students, mathematically gifted students, successful, and average students in mathematics were considered to be the cases. A qualitative case study approach, specifically a multiple case study, examines real cases in their contexts and experiences (Stake, 2013). A case study design interested in "in-depth" investigation is different than "study of the isolated variable." (Yin, 2011, p.4). A multiple case study presents a wide range of evidence more comprehensively compared to a single study (Yin, 2011).

3.1.1 Pilot Study

A pilot study was conducted with participants to check the appropriateness of the main study. Before the data collection in the pilot study, approvals were obtained from the Middle East Technical University Human Subjects Ethics Committee (see Appendix G) and MoNE. Secondly, the written consent forms were collected from the parents via school administration. Finally, the data collection procedure for the pilot study started.

For the pilot study, participants were chosen from a public middle school from the Çankaya district in Ankara. Two girls and two boys from three different classes in the same school were selected according to teachers' views. They were labeled as successful students at mathematics and problem-solving in mathematics by their mathematics teachers. The eight mathematical problems were asked to four fifth-grade students in the pilot study. Clinical task-based interviews that were conducted with all of the students lasted approximately 100 minutes (equal to two lesson hours). Every interview was conducted in an empty class one by one. The responses and reactions were observed and gathered via interviews, observation forms, solution sheets, voice, and video recording by taking their permission. For the interview, the six mathematical problems were chosen from Posamentier and Krulik (2009) for the main study, and two mathematical problems were chosen from a preparedness test which was presented by Fİ Mathematics and by MoNE respectively.

According to the analysis of the pilot study data, certain revisions were made concerning the problems to be used in the main study. First of all, two problems from Fİ Mathematics and MoNE were determined that it would be better to not ask them**.** There were two reasons to make this decision. First, two questions were difficult for students to understand. It took more than 15 minutes to understand, so; observations were very time-consuming and inefficient for them concerning problem-solving phases and the use of strategies. Second, the same results were observed in these two problems when compared to the other six problems. As a result, it was decided that these two questions were removed and the remaining questions were considered adequate for the study. Secondly, the school atmosphere was very noisy since two students were affected by other students' voices although we were indoors and in an empty classroom. Moreover, a student was disrupted by an announcement of a fire drill in the school during the interview. Considering such unexpected issues, it was decided that the interviews should be conducted at the students' homes where they can feel more relaxed and secure. Thirdly, a survey was conducted after the interview ended. On the other hand, all observations were enough to examine students' problem-solving phase and strategy use in a detailed way. Therefore, the survey was omitted in the main study since students gave the same answers as the findings in the observation form and the survey. Lastly, two students got distracted by a video camera during the study since they looked at it for a long time and forgot to solve problems. Therefore, video recording was not used in the main study.

3.2 Participant Selection

The participants of this study were 20 fifth-grade students classified into three types of groups and selected by purposeful sampling for the main study. The first group of participants was seven mathematically gifted (M) students; three girls and four boys. They were selected from those attending SAC regularly, after school hours, to take various courses and do activities as gifted students. All of them were from different public and private middle schools in a city in Western Turkey, in Salihli. They have been taking various classes like Mathematics, Turkish, English, Social Studies, and Sciences in the same SAC. Apart from that, they also have been taking various courses in the center for years (see Table 3.1). They were selected as mathematically gifted participants of the study based on two criteria. First, they had been identified as gifted based on the intelligence scale. In this study, the mathematically gifted participants were nominated as gifted students with only the area of general mental ability from this scale in SAC (Bilsem Online, 2020). Second, their mathematics teacher in the SAC characterized these seven students as mathematically gifted compared to other gifted students attending the center. Based on her observations, she attributed the following characteristics to the selected students: "highly successful in problem-solving in mathematics", "making logical deductions", "having well-developed logical reasoning skills" "having different perspectives when comparing other gifted students", and "having ability to think of unusual ways for solving a problem". At the same time, the mathematics teacher reported these students at the end of the semester and throughout the study these students were accepted as mathematically gifted students of the study.

The second group of participants was seven successful (S) fifth-grade students; five girls and two boys. They were identified as the top problem solvers in mathematics by their mathematics teachers but not applied to or interested in the SAC and thus they have not been formally identified as gifted unlike the others in the aforementioned group. One of these students had been taking an out-of-school chess course for two years, and another one had been taking an out-of-school robotic coding course for a year. All of the students in this group were attending the same public middle school since this school was only one school to proper this condition. Five of them were from the same class, while two of them were from other classes in the school. They were selected according to their school teachers' remarks. Compared to other students in the school, their mathematics teachers and class teachers characterized them as "being highly successful in problem-solving in mathematics" and "having different perspectives in mathematics". Moreover, these students did not take any intelligence test or did not enter the SAC.

The third group of participants was six fifth-grade average (A) students in mathematics; a girl and five boys. They were not identified as gifted, mathematically gifted, or mathematically successful by their mathematics teachers. All of them were from the same class in a public middle school which was the same school with the successful participants in the study. They did not report any additional after-school courses that they were taking or any center for students that they were attending. Compared to other students in the school their mathematics teachers and class teachers characterized them as "average problem solvers in mathematics" and "having average success in mathematics."

Table 3. 1

Mathematically Gifted Students' (M) Information

Student	Student's School Type	Gender (Boy or Girl)	Attendance to SAC	The workshops attended in SAC
M1	Public	G	1 year (2 days per week)	Problem Solving and 3D (1 year)
M2	Public	B	1 year (4 days per week)	Robotic Coding (1 year)
M3	Public	B	1 year (4 days per week)	Technology Design and Archaeology (1 months)
M4	Private	B	1 year (4 days per week)	Stem, Archaeology, and Problem Solving (1 year)
M5	Private	G	2 years (A day per week)	Intelligence and Mind Game (1 year)
M6	Public	B	1 year (4 days per week)	Intelligence and Mind Game and Ancient Civilizations (6 months)
M7	Public	G	1years (4 days per week)	Problem Solving (2 years)

Note. M: Mathematically Gifted Student

3.3 Instruments

In a case study, "data have usually been derived from interviews, field observations, and documents." (Merriam, 2009, p. 203). Three instruments were used to obtain data in the present study: clinical task-based interviews, observation form, and students' solution sheets.

3.3.1 Clinical Task-Based Interview

A clinical task-based interview is a type of interview in which a solver discourse on his/her work during or after the problem-solving process. During the clinical task-based interview, an interviewer is in interaction with the solver on a task. It is used in studies related to mathematics education (Koichu & Harel, 2007). For clinical interviews, six mathematical problems were selected and adapted from Posamentier and Krulik (2009). The following four criteria were considered while selecting the problems:

1) The problems should be solved with at least more than one problem-solving strategy since the main focus to examine in the current study were students' strategy choices, uses, and the use of Polya's (2004) problem-solving phases.
2) The problems should be proper for students' level. The content of each problem must be appropriate to fifth-grade mathematics.
3) The mathematical contents of the problems should be as diverse (e.g., number and operation, algebra, and geometry) as possible.
4) The problems should be challenging for the students since these types of tasks can draw gifted students' attention more (Krutetskii, 1976).

Following the criteria above, problems were specified by the researcher and a mathematics teacher who has been working as a mathematics teacher for ten years in a public school. All problems were translated and adapted from English to Turkish by the researcher (see Appendix A for the Turkish translations as used in the study). Adaptation was done such as changing the name of the object, units of measurement, and meaning concerning the Turkish context. All problems were checked by a mathematics teacher who has been studying with gifted students in a SAC for three years, two mathematics teachers who have been studying with successful and average students in the middle schools for eight years, and the researcher concerning their suitability for fifth-grade gifted and non-gifted students. They gave suggestions about the words and the meaning of the problems. According to their feedback, problems were revised.

Table 3.2 shows all of the six problems used for the main study and the strategies that might be used by the students when solving them, as provided by Posamentier and Krulik (2009). During the clinical task-based interviews, prompting questions such as the following were used: “Can you tell me how you worked that out?” and “How did you decide?” (Hunting, 1997, pp. 153–154). Under some circumstances, further prompting questions were directed to understand which phase students could apply among Polya’s (2004) problem-solving phases: “What are the data?” and “Did you use all the data?”. Appendix B shows the list of the prompt questions by Hunting (1997) and Appendix C illustrates the list of phase questions suggested by Polya (2004). Table 3.2 demonstrates original problems and their expected problem-solving strategies while solving problems provided by Posamentier and Krulik (2009). The list of all the prompting questions by Hunting (1997) and phase questions suggested by Polya (2004) are presented in Appendix B and Appendix C, respectively.

Table 3. 2

Problems Used in the Task-based Interviews and Problem-Solving Strategies that Might Be Used for the Solutions

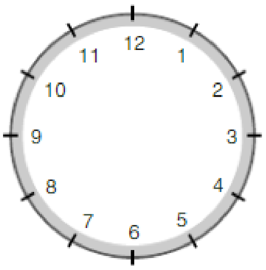
Problems	Expected Problem Solving Strategies
Problem 1 "Jean has 55 blocks to stack in a triangle display in the store window. She would like the top of the triangular display to have one block, the one below it to have two blocks, the one below that to have three blocks, and so on. Is it possible to make such a triangle with all 55 blocks, and if so, how many rows will the triangle have?" (Posamentier & Krulik, 2009, p. 9)	Making a drawing, finding a pattern, acting it out or simulation, organizing data, logical reasoning, solving a simpler equivalent problem
Problem 2 "Draw 2 straight lines across the face of a clock so that the sum of the numbers in each region is the same." (Posamentier & Krulik, 2009, p. 92)  	Logical reasoning, intelligent guessing and testing, adopting different point of view, acting it out or simulation

Table 3. 3 (continued)

Problem 3 "The Wolverines baseball team opened a new box of baseballs for today's game. They sent $\frac{1}{3}$ of their baseballs to be rubbed with special mud to take the gloss off. They gave 15 more baseballs to their star outfielder to autograph. The batboy took 20 baseballs for batting practice. They had only 15 baseballs left. How many baseballs were in the box at the start?" (Posamentier & Krulik, 2009, p. 66)	Working backwards, logical reasoning, making a drawing, acting it out or simulation
Problem 4 "Find the difference between the sum of all the even numbers less than 101 and the sum of all the odd numbers less than 101." (Posamentier & Krulik, 2009, p. 119)	Adopting a different point of view, logical reasoning, intelligence guessing and testing, finding a pattern, solving a simpler equivalent problem
Problem 5 "In my pocket, I have quarters and nickels. I have four more nickels than quarters. Altogether, I have $1.70 in my pocket. How many nickels and how many quarters do I have?" (Posamentier & Krulik, 2009, p. 34)	Acting it out simulation, intelligent guessing and testing, working backwards, adopting different point of view, logical reasoning
Problem 6 "There are 3 cacti growing in Jesse's yard. The Indian Fig cactus is 6 feet tall. The Golden Barrel cactus is 3 feet shorter than the Indian Fig cactus. The Saguaro cactus is 6 feet taller than the Golden Barrel cactus. How tall are the three cacti?" (Posamentier & Krulik, 2009, p. 104)	Intelligence guessing and testing, logical reasoning, adopting different point of view, making a drawing, organizing data, acting it out or simulation, solving simpler equivalent problem

3.3.2 Observation Form

Observation is a method to recognize "what people actually do, as opposed to what they think they do, or would like others to think they do." (Caldwell & Atwal, 2005, p. 42). An observation form was designed to guide the researcher to particularly observe. Polya's (2004) problem-solving phases and the strategies and behaviors were observed while students worked on the problems during the task-based interviews (see Appendix E). During the clinical interviews, I observed every student without intervention and took notes of their behaviors on the observation form (Gold, 1958). Therefore, I was an observer as a participant in the study. Appendix E presents the observation form in detail.

3.4 Method of Data Collection

In the pilot study, approvals were obtained from the Middle East Technical University Human Subjects Ethics Committee (see Appendix G) and MoNE (see Appendix H). Secondly, the written consent forms were collected from the parents via school administration. Finally, the data collection procedure for the pilot study started.

In the main study, the written consent forms were collected from the parents. Then, the data collection procedure for the main study started in the fall semester of 2020-2021 academic years. The study was conducted in the Salihli district from Manisa province. Data were collected via clinical task-based interviews and observation forms. Before the study, information about the study was given that the research would mainly focus on how they approach and solve the problems. Students were allowed to think about and solve the problems without time limitation since the aim of this study was about investigating students' problem-solving phase and strategy use instead of evaluating speed or success of problem-solving. They were encouraged to think aloud and express their feelings verbally during and after the study. Moreover, they were informed that they can leave the interviews whenever they want.

After the permission of clinical task-based interviews, interviews were conducted one by one to obtain data via six mathematical problems. Each face-to-face interview lasted approximately 90 minutes for every student at their own home in which they feel comfortable. The study was conducted in pandemic semester with its conditions. I and all participants were wearing masks. We talked at least two meters from each other during the study. I asked to all participants whether they have any semptom of coronavirus or not. After we were sure about that there was any semptom or sick about the coronavirus at their home, we started to the study. Before the interviews, I introduced math manipulatives so that they may use them in the problem-solving processes like calculation sticks, unit cubes, round counting pieces, and other sticks to students and explained how they can be used by the students for their mathematical understandings. Then, I gave these manipulatives to the students that they may use them in the problem-solving processes. Each participant was familiar with the manipulatives but they expressed that, in general, they did not use them in class. I especially explained that they do not have to use them but if they want, they can use all manipulatives whenever and wherever they want in the problem-solving process.

During the interviews, I was extremely careful about the point that students had to think and work on the problem by themselves without my influence. Therefore, I did not direct them verbally about their process in problem-solving. I listened to them and noted their behavior and the process during problem-solving on the observation forms. Apart from that, I only asked prompt questions to the students only in three cases. First of all, I asked questions like “Can you tell me what you are thinking?” when there was a long silence (Hunting, 1997, p. 153). Secondly, I asked the questions when I wanted students to express their process and problem-solving phases with their own words when they finished the process if they cannot explain themselves automatically or I cannot understand their written works clearly. In this situation, I generally asked a question like that: “Can you explain this process to me in your own words?” Thirdly, I asked questions when they finished solving the problem in the last phase of Polya (2004) to encourage them to find alternative problem-solving strategies: “Can you solve this problem differently?”

In the clinical interviews, I prepared the guiding questions list explained in Appendix B for these sections offered by (Hunting, 1997) such as:

- “Can you tell me how you worked that out?
- Can you say out loud what you are doing?
- How did you decide?
- Do you know a way to check whether you are right?
- Why?” (pp. 153–154).

And guiding questions were prepared to examine Polya’s (2004) problem-solving phases which are presented in Appendix C such as:

- “What is the condition?
- Do you know the related problem?
- Can you see clearly that the step is correct?
- Can you derive the result differently?” (pp. 4–9).

3.5 Data Analysis

To analyze the problem-solving strategies and phases of students, content analysis was used with a combination of voice recording, observation forms, and documents. Students’ answers were transcribed and data were analyzed in the light of the codes of problem-solving strategies which were organized according to Posamentier and Krulik’s (2009) explanations and problem-solving phase codes which were organized according to Polya’s (2004) explanations. After my coding, a second researcher who is a master's student in mathematics education checked the codes of the strategies, and phases of the participants. Then, with the agreement of the researchers, I made the analyses gingerly.

3.5.1 Analysis of Problem-Solving Phases

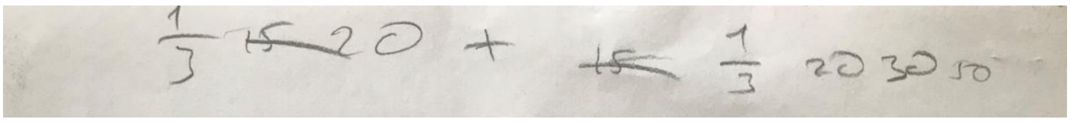
In the coding system, the problem solving phases were themes. The phase of understanding the problem, devising a plan, carrying out the plan, and looking back were the categories. The sub-phases for each category were the codes. The codings for the participants’ problem-solving phases and their sub-phases were carried out according to Polya’s (2004) four phases of problem-solving. In the following sections, the categories were explained.

- **Understanding the Problem**

Students try to be cognizant of the main parts which are given, asked, and presented with a condition in a problem. In this study, the “Understanding the problem” category was divided into three sub-phases and coded accordingly: repeat or restate the problem, draw a figure like an object, a notation, and a sign related to a problem to point out the data and unknown, and state verbally the unknown and data of the problem verbally. When students apply the sub-phase of repeating or restating the problem, they repeat or restate the statement of the problem verbally. In the sub-phase of drawing a figure like an object, a notation, and a sign, students draw them on paper when they tried to understand the problem. For example, Figure 3.1 shows a sample solution by a mathematically gifted student (M4) showing “draw a figure” in Problem-3 where he was trying to understand the problem first before going any further with the solution. He drew some notation related to the problem on the paper.

Figure 3.1

Part of a Mathematically Gifted Student’s (M4) Work on Problem-3 Showing



Similarly, statements such as the following belonged to one of the mathematically gifted students (M4), focusing on the given and unknown quantity asked for. The statements were also coded as the first phase of the problem-solving process as “state the unknown and data of problem”: “There are 15, 20, and 25 balls to take and the problem asks for a total number of balls” (M4, Problem-3). In this work, he wrote some notes and computations to understand the unknown and data in the problem.

- **Devising a Plan**

  Students think and brainstorm on making a plan for the solution to a problem. In the solution of the problem, applying the phase of devising a plan occurs by using past experiences. Also, students try to be cognizant of using the data and conditions in the problem while devising a strategy in this phase. Therefore, in this study, the category was divided into two sub-phases: think of the same or similar problem and produce a strategy using all the given in the problem. For example, a mathematically gifted student was observed concerning thinking of the same or similar problem in problem-2. Mathematically gifted student (M5) stated: “I remember a similar problem to this problem. In SAC, I solve problems like it. So, I tried to remember what I did for the solution.” (M5, Problem-2) Furthermore, an average student (A3) was observed in problem-1 concerning production a strategy with using all the given in the problem. He stated: “If I put a triangle in the first place, two triangles in the second place, and three triangles in the third place, I should concentrate on this pattern. There are 55 triangles in total.” (A3-Problem-1) **[Then, he continued to carry out this plan.]**

- **Carrying out a Plan**

  Students patiently apply the plan that is made in the previous phase. Also, students conduct their plan by aware of every point in the problem’s context. They check their plan to see if it works. If the strategy does not work, another strategy should be conducted. Therefore, in this study, this category was divided into two sub-phases: checking all the steps of the plan and proving the correctness of the plan. To illustrate, checking all the steps of the plan category, a successful student (S4) in problem-6 checked her plan by looking at every step of the plan from beginning to end. To prove the correctness of the plan, she crosschecked problem 6.

- **Looking Back**

  Students check all the procedures in the final phase. They check the result and solution process to be sure if there is any incorrect point or not. Also, in this phase, the use of different strategies was observed in this study. Therefore, the question of "Can you solve the problem differently?" was asked to encourage students to think of another strategy different from the previous strategy if students did not share another strategy by themselves. As a result, the category was divided into two sub-phases: checking all conditions and steps of the problem and solving the problem with another strategy. For instance, concerning checking all the conditions and steps of the problem, a mathematically gifted student (M2) checked the problem's situation and his process for problem-1. In addition, concerning solving the problem with another strategy, an average student (A3) was observed to try to think on another strategy different from the previous one in problem-4.

3.5.2 Analysis of Problem-Solving Strategies

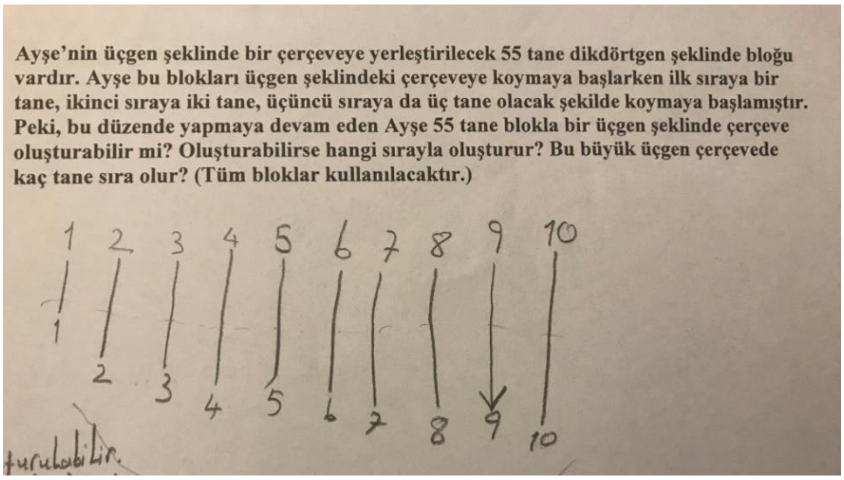
The problem-solving strategies were themes. Each strategy was the category. The explanations of each category were codes. Coding of the participants' problem-solving strategies was guided by Posamentier and Krulik's (2009) explanations as follows. The categories were explained in the following sections.

- **Organizing Data**

  Students organize visual or numerical data of a problem in a table or a list. For example, as presented in Figure 3.2, an average student (A5) used the problem-solving strategy of organizing data by constructing a table that presented orders and the number of triangles in problem-1. He organized the data of order in the triangle and showed them in a list with the number of orders.

Figure 3.2

Part of an Average Student's (A5) Work on Problem-1 Showing

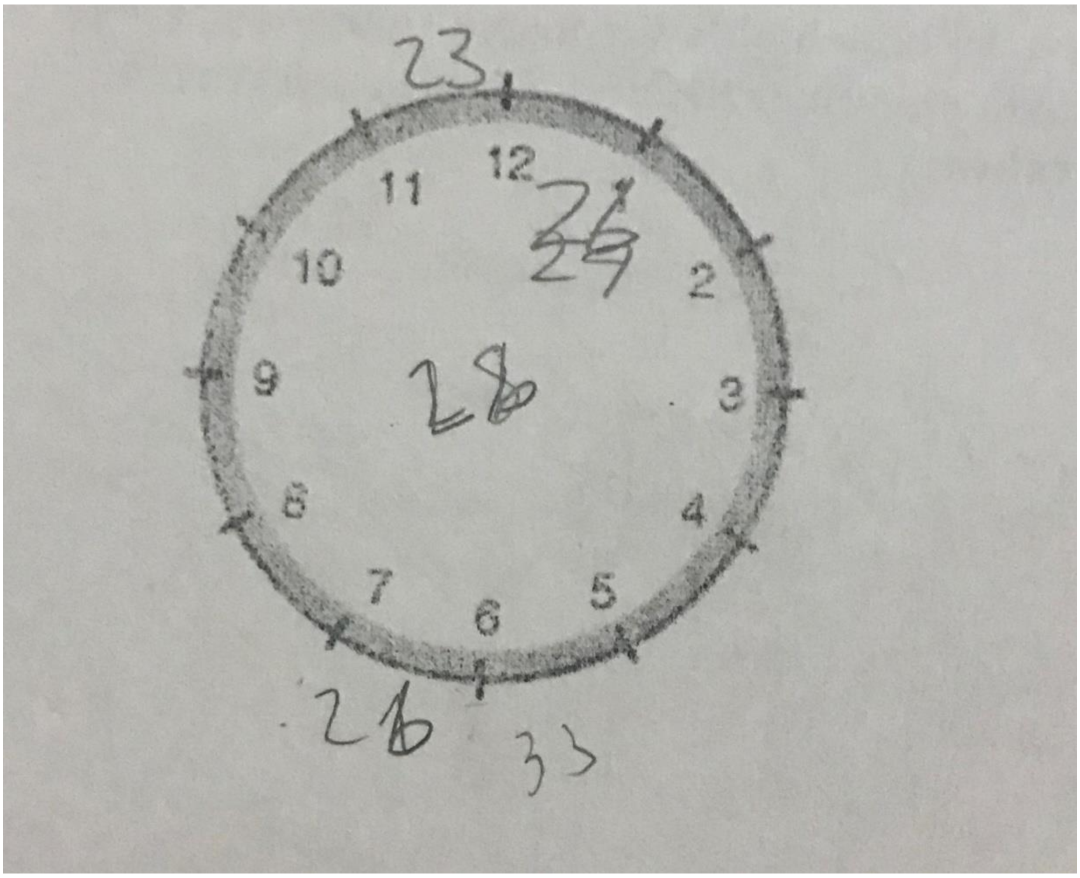


- **Intelligent Guessing and Testing**

  Students make an intelligent guess related to a problem, then, they test this guess in the context of the problem. Every guess should be based on former guesses. For example, in Figure 3.3, a successful student (S1) used the problem-solving strategy of intelligence guessing and testing by guessing an area in which the sum of all numbers is equal to each other and tested it in problem-2.

Figure 3.3

Part of a Successful Student's (S1) Work on Problem-2 Showing

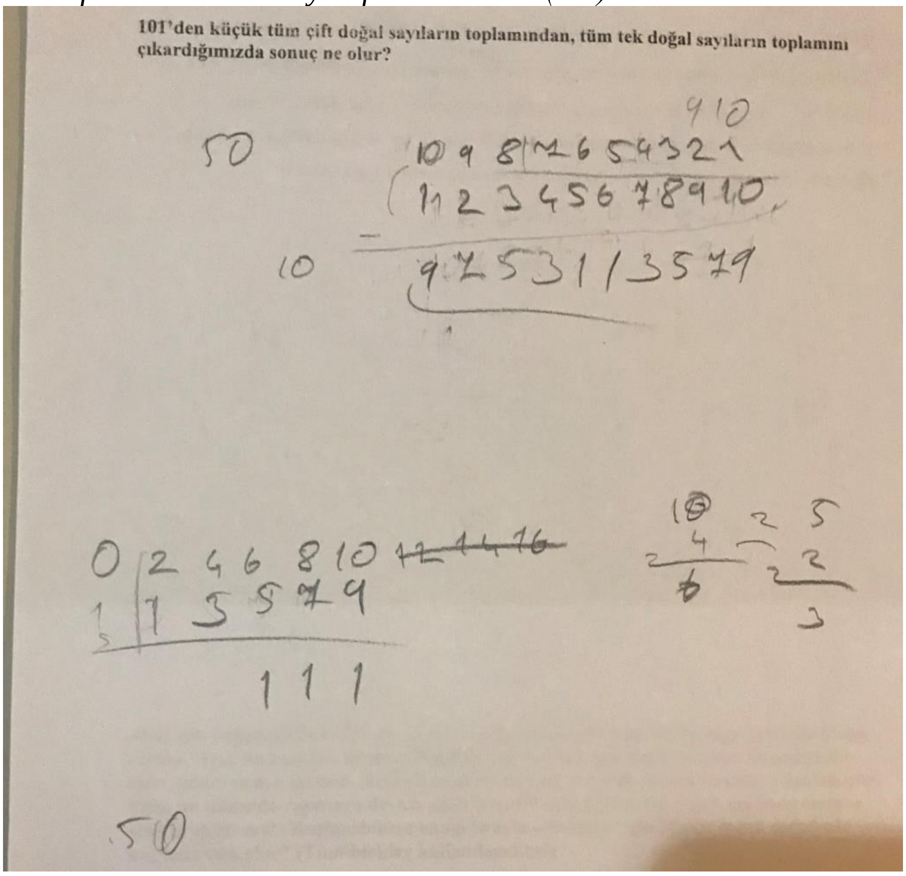


- **Solving a Simpler Equivalent Problem**

  Students change the original problem into a simpler form to see the solution of the original problem. For example, in Figure 3.4, a mathematically gifted student (M4) used the problem-solving strategy of solving a simpler equivalent problem by thinking the simpler version of problem-4. He thought the problem was if there are 10 numbers. Then, he applied this idea to the entire problem's situation.

Figure 3. 4

Part of a Mathematically Gifted Student's (M4) Work on Problem-4 Showing



- **Acting It Out or Simulation**

  Students act a role in a problem by using manipulatives/materials to animate the action in the problem. For example, in Figure 3.5, a mathematically gifted student (M6) used the problem-solving strategy of acting it out or simulation by using the sticks. He constructed a triangle concerning the pattern in problem-1.

Figure 3. 5

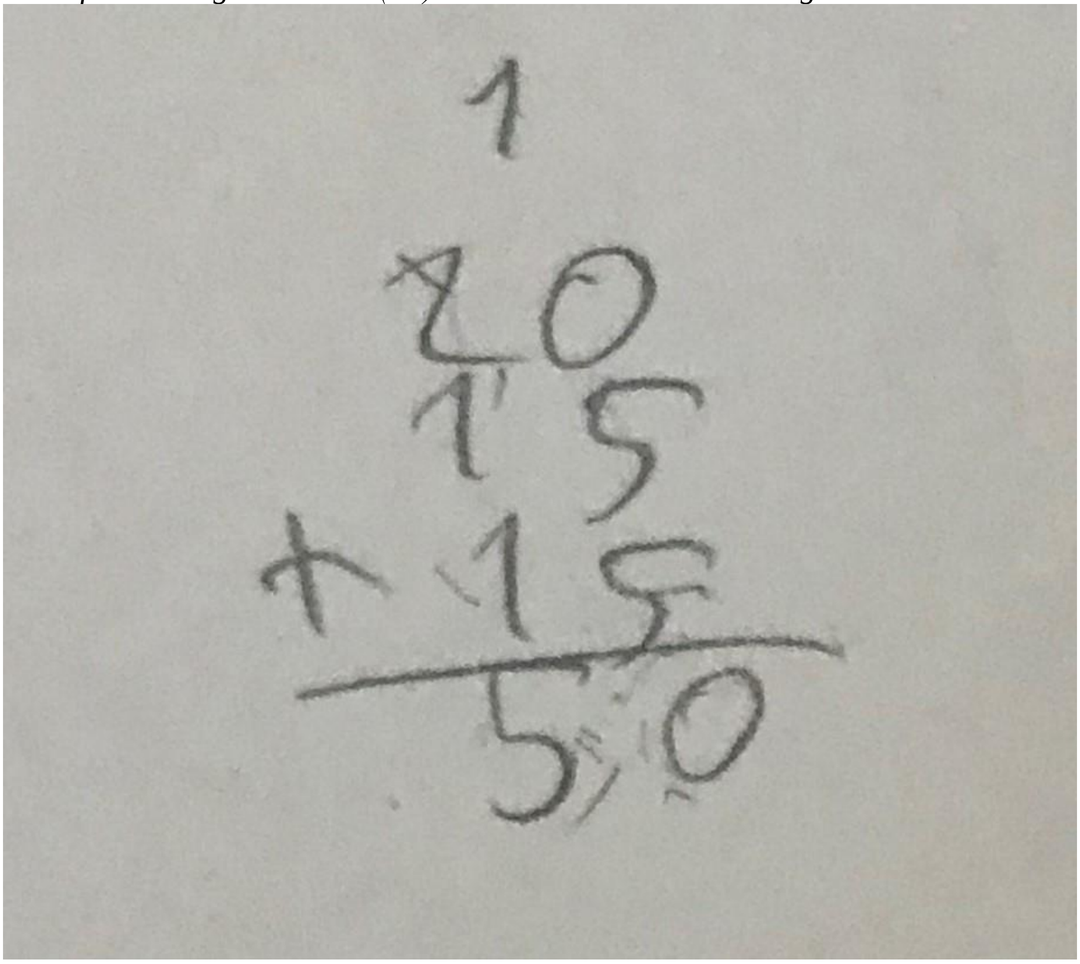
Part of a Mathematically Gifted Student's (M6) Work on Problem-1 Showing

- **Working Backwards**

  Students think reversely in the action. They follow the steps of the problem backward like making multiplication instead of division or making addition instead of subtraction. For example, in Figure 3.6, an average student (A1) used the problem-solving strategy of working backwards in problem-3. Firstly, he tried to find the sum of 15, 15, and 20. That is, he did addition instead of subtraction. Then, he tried to find the total number of balls.

Figure 3.6

Part of an Average Student's (A1) Work on Problem-3 Showing



- **Finding a Pattern**

  Students search for an order as a geometric or numeric pattern to solve a problem. For example, a successful student (S2) used the problem-solving strategy of finding a pattern. He found a pattern in problem-1 as the number of rectangles in the triangle is increasing like 1, 2, 3, 4, and so on.

- **Logical Reasoning**

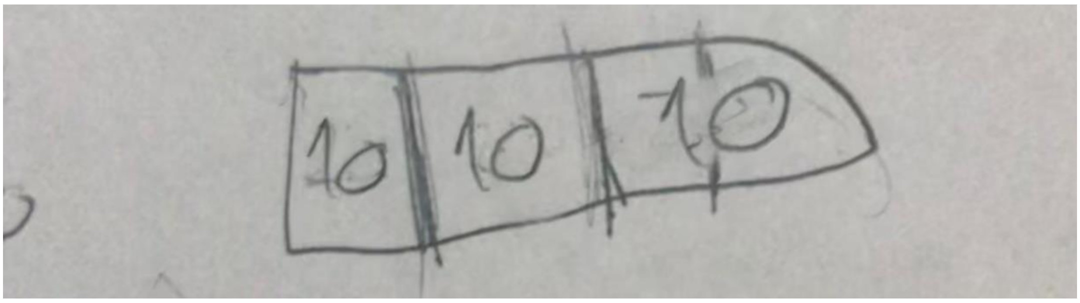
  Students make logical inferences from the context of a problem. These inferences do not mean that students express their idea whatever comes to their minds irrelevantly. That is, these inferences should contain “reading between the lines” in the context of the problem (p. 89). Thus, these inferences also should contain statements including “must be”. For instance, in problem-6, an average student (A3) made inferences about the heights of the people. He said that if Duru is smaller than Ada and Yasemin is taller than Duru, Duru must be the smallest one while Ada is the tallest one among them.

- **Making a Drawing**

  Students draw a picture, figure, or diagram with schematic representations to solve a problem. For instance, in problem-3, a successful student (S1) drew a figure that presents the number of balls. In this way, he divided all balls into equal parts. For instance, in Figure 3.7, a successful student (S1) drew a figure in problem-3 that presents the number of balls. In this way, he divided all the balls into equal parts.

Figure 3.7

Part of a Successful Student’s (S1) Work on Problem-3 Showing



- **Adopting a Different Point of View**

  Students consider a problem in a different perspective instead of its direct and known solution ways such as grouping the numbers differently, thinking the situation of a problem from different perspectives, etc. For example, a mathematically gifted student (M4) used adopting a different point of view strategy in problem-5. First, he took 50 pennies (Kuruş in Turkish) in 13.50 Turkish liras away since he tried to make his work easier. Then, he divided 13 Turkish liras into 2. Besides, he did not forget 50 pennies to add later that he took away at first.

3.6 Validity and Reliability

The first issue was validity in the present study. Instead of the term “validity”, the term “trustworthiness” is preferred to be used in qualitative studies since generalizability is not the case in qualitative research (Efron, & Ravid, 2013, p.70). Furthermore, trustworthiness was related to interval validity in a case study since both of them are interested in the degree to which findings are reliable (Merriam, 2009). To enhance the internal validity of the case study, triangulation was a helpful method by using various data sources (Merriam, 2009). Concerning triangulation of data sources, data were collected with multiple instruments in the current study: interviews, observation forms, and sheets. Moreover, Merriam (2009) stated that “triangulation using multiple sources of data means comparing and cross-checking data collected through observations at different times or in different places, or interview data collected from people with different perspectives or follow-up interviews with the same people.” (p. 216). In this sense, six mathematical problems were asked to all type of participants. In this way, different participants were observed in different places throughout the same tasks. Secondly, the triangulation method was applied in the analysis procedure as “peer examination” or “peer review” to enhance internal validity or trustworthiness (Merriam, 2009, p. 220). Merriam (2009) suggested that other people analyze or review the same data and findings are compared with the main researcher’s findings. In the current study, data were shared with a graduate student from mathematics education to be reviewed as an example of peer review.

In the validity of the instrument, content validity became prominent. It was interested in the definition of whole content and its elements adequately. This can be provided with expert opinion, literature review, and comparison with other instruments in other studies (Cohen et al., 2018). In the light of this information, in the study, teachers' opinions were taken as explained in section 3.3.1. Explanations of every phase and strategy were presented in detail in both the analysis and the result sections concerning related literature. Besides, the mathematical problems in the present study that students try to solve during the interview were presented by Posamentier and Krulik (2009)'s suggestions to be used by teachers. A pilot study also was conducted to ensure the instruments' validity.

The second issue was reliability. It refers to the "extent to which research findings can be replicated" (Merriam, 2009, p. 221). As it was said in the validity section, triangulation and peer examination are also suggested to improve the reliability of a qualitative study. Besides, the audit trail which explaining in detail how the data were obtained and how the codes were determined is a method to ensure the reliability of qualitative research (Merriam, 2009). In this regard, the participant selection criteria, mathematical problems selection criteria, data collection method, and coding system in analysis in the present study were explained in detail.

A student was chosen randomly from every three groups of students (15% of all data set) since "depending on the size of the data set, 10–25% of data units would be typical" (O'Connor & Joffe, 2020, p. 5). As a second coder, a graduate student from mathematics education who studied problem-solving strategy in her thesis analyzed this portion of the data set and coded them independently to ensure the reliability of the analysis. Percent agreement was calculated as 88% which can be accepted as "almost perfect" (McHugh, 2012, p. 279).

The last issue was external validity which refers to "which the findings of one study can be applied to other situations" (Merriam, 2009, p. 223). To enhance the external validity, “rich, thick description” was suggested (Merriam, 2009, p. 229). The present study provided thick descriptions with detailed participant descriptions and findings with participants’ quotes from the interviews (Merriam, 2009).

3.7 Assumptions

In the present study, it was assumed that all measurement tools were answered by the participants sincerely. Furthermore, all participants shared their own opinions without being affected by someone. Lastly, there was no linguistic problem that participants faced in the problem-solving process.

CHAPTER 4

RESULTS

In this chapter, the results of the study will be presented by answering the following questions:

1) How do mathematically gifted, successful, and average students use the problem-solving phases advocated by Polya (2004)?
2) Which problem-solving strategies do mathematically gifted, successful, and average students use while solving a non-routine problem?
3) How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?

In the light of the study, six mathematical problems that had been used in the study are introduced in Table 3.2.

The following section explained the first part of the results about the first research question.

4.1 Students' Use of Problem-Solving Phases

In Table 4.1, mathematically gifted students (M), successful students (S), and average students' (A) number of attempts to apply the problem-solving phases was introduced. All the main four phases were divided into sub-heads considering Polya's (2004) suggestions. Understanding the problem phase was divided into three sub-heads: repeating or restating the problem, drawing a figure to stress the data and unknown, and stating the unknown and data of the problem. The second phase, devising a plan, was divided into two sub-heads: thinking of the same/similar problem and producing a strategy using all the givens in the problem. The third one, carrying out the plan, was divided into checking all steps of the plan and proving the correctness of the plan. The last phase, looking back, had two sub-heads: checking all conditions and steps in the problem and solving the problem with another strategy. The sub-head, solve the problem with another strategy, was observed with the questions: “Can you solve this problem differently?” after finishing the all problem-solving process (see Data Analysis section). Table 4.1 presented the number of attempts of participants’ problem-solving phases uses while solving the problems. In this section, the results were presented as the participants’ number of attempts to use all problem-solving phases and sub-phases regarding the research question: How do mathematically gifted, successful, and average students use the problem-solving phases advocated by Polya (2004)? Appendix D also presents all participants’ problem-solving phases and sub-phases used for each problem in detail.

Table 4. 1

Participants' Number of Attempts to Use Problem-Solving Phases

Students	Problem-Solving Phases and Their Components								
	Understanding the problem			Devising a Plan		Carrying out the Plan		Looking Back	
	Repeat and/or Restate the problem	Draw a figure to stress the data and unknown	State the unknown and data of the problem	Think of the same or similar problem	Produce strategy with using all the givens in the problem	Check all the steps of the plan	Prove the correctness of the plan	Check all the conditions and steps in the problem	Solve the problem with another strategy
M	4	40	4	10	42	36	16	26	17
S	20	33	5	4	39	22	9	12	11
A	15	27	5	1	31	14	4	7	5

Note. M: Mathematically Gifted Students, S: Successful Students, A: Average Students

4.1.1 Understanding the Problem

When examined the understanding the problem phase, the following deduction could be shared that mathematically gifted students gave more time and attention to understand what condition, data, and the asked components are in a problem compared to successful and average students. They carefully read all the problems then, continue to apply the following phases. None of them continued to devise a planning phase without completely understanding the problem. These processes were not observed among successful students in the same way since when they read a problem, they proceeded to generate a strategy for solving it without understanding it exactly. Understanding some part of a problem was enough for them to continue. On the other hand, after a while, they were not sure about their solution process even if the solution is correct. For example, a conversation between the researcher and a successful student (S2) in problem-1 is:

> **[After a while, in problem 1, she asked me some points.]**
>
> S2: Which one is bigger: the triangle or the rectangle blocks?
>
> R: What does the problem say?
>
> S2: I did not understand the problem fully but I think that there is a pattern.
>
> S2: If the problem had given me that every order has bla bla blocks, I could have understood the problem better (S2, Problem-1).

As explained in these examples, a successful student (S2) did not give the importance to understand the condition, data, and the asked component as well as mathematically gifted ones. Although the condition of problem 1 (there is one block in the first place, there were two blocks in the second place, and three blocks in the third place) was explained in the problem context, he was confused like presented above.

A successful student (S5) in problem-6 stated: “I think that Ada is the tallest one, Duru is the shortest one but if the problem had given me how tall Ada was, I could have understood it.” (S5-Problem-6). Similarly, problem 6 (Duru is 20 cm shorter than Ada, Yasemin is 10 cm taller than Duru.) was explained in the problem contexts but the successful student (S5) could not comprehend exactly.

Average students were the most unsuccessful ones to understand problems among the three types of groups. Not only they could not understand the problem clearly but also they directly continued with the solution process without paying any attention to comprehending the problem. They also tended to change the situation of the problem as they desired it to be. For example, a conversation between researcher and A1 in problem-2 exemplifies that:

> A1: May I create my clock to solve this problem?
>
> R: Sure.
>
> A1: I want to change the situation of the problem. I mean, instead of two sticks, I want to combine two sticks as a stick. There is no difference in the solution.
>
> R: If you can solve this by paying attention to the condition of the problem, you can do it for sure.
>
> A1: Hmmm, I wish this clock was the digital watch. Then, it could be easier. If this situation will continue to be like that, I cannot understand and solve it (A1, Problem-2).

As explained in the example, the average student (A1) changed the situation but it caused him to be confused.

4.1.1.1 Repeating or Restating the Problem

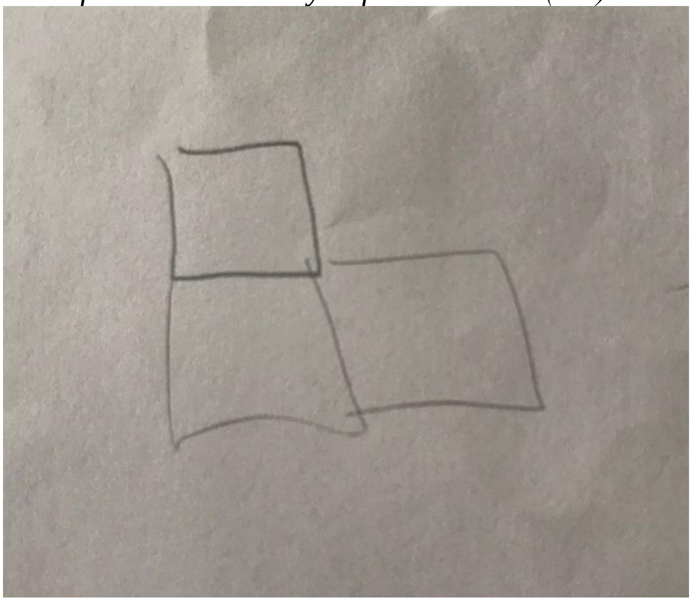
In Table 4.1, mathematically gifted students did not prefer to repeat or restate the problem very much since the lowest number of attempts was preferred in this group. When they preferred to do it, they repeated the problems instead of restating. They reread the problems slowly and stopped from time to time, then if they felt sure, they continued to read. When Table 4.1 was considered, contrary to mathematically gifted students, successful students were the group that preferred to apply repeat/restate the problem the most among all the groups. Similar to mathematically gifted students, successful students were observed that instead of restating, they repeated the problems to understand them. Following them, average students did it mostly after successful students. They repeated the problems to understand them before the solution. It was observed in both successful and average students that they reread the problems quickly and quietly when compared to mathematically gifted students. Therefore, they reread the problems more than mathematically gifted ones.

4.1.1.2 Drawing a Figure to Point Out the Data and Unknown

Except for two mathematically gifted students, all mathematically gifted students drew a figure about the data and the asked component in the problems. An example of drawing a figure in the understanding the problem phase was presented in Figure 4.1. A mathematically gifted student drew a small triangle to comprehend the context of problem-1. She drew a figure related to the triangle and its pattern in the problem when trying to understand it.

Figure 4. 1

Part of a Mathematically Gifted Student's (M7) Work on Problem-1 Showing



On the other hand, the number of attempts in drawing a figure to point out the data and unknown among successful students was less than mathematically gifted peers' attempts. For example, in problem-2 and problem-4, only 4 successful students could draw a figure. Average students had the lowest number in the attempt. For instance, in problem 2, only two average students could draw some figures about the problem. All in all, when Table 4.1 was examined, mathematically gifted had a priority about drawing related to the data and unknown of a problem in the paper when compared to their successful and average ones.

4.1.1.3 Stating the Unknown and Data of the Problem

In this code, in all three groups, low numbers drew attention. Mathematically gifted students stated the unknown and data of the problems verbally the least. With this regard, it was parallel to the sub-phase of repeating or restating the problem use. Similarly, successful and average students verbally stated the unknown and data in a problem the least compared to mathematically gifted ones.

Overall, the drawing a figure to stress the data and unknown was the most applied code among all types of participants. When considering the total number of attempts, successful students were at the highest rank in understanding the problem phase. Mathematically gifted students came in second place regarding the number of attempts because of the number of lowest applications of the phase of repeating or restating the problem and stating the unknown and data sub-phases. Average students came in third place with a very small difference from mathematically gifted students.

4.1.2 Devising a Plan

In the second phase, mathematically gifted and successful students paid attention to try to generate a plan for solving. Both groups of participants gave considerable time for planning in the problem-solving process. On the other hand, when mathematically gifted students tried to devise a plan, they did not go back to understanding the problem phase. However, successful students often turned back to problems' context when devising a plan. Moreover, mathematically gifted ones could estimate how long the plan takes time and whether the plan works or not. For example, a mathematically gifted student (M2) in problem-2 stated that:

> When I looked up at the clock, I understand that I can split it into areas and check if there is the same sum in every area. I can do it by making a guess. For example, I want to split the area like 12-1-2 and then check the results' correctness. On the other hand, this is so time-consuming for me. Therefore, I should find another shortcut to solve (M2, Problem-2).

As presented above, not only mathematically gifted student (M2) could devise a plan but also he could estimate how long this strategy would take.

Average students could try to devise a plan but they turned back to understanding the problem phase like successful students. Therefore, in some problems, they could not continue to devise a planning phase from understanding the problem phase in the problem-solving. Besides, they were not sure about their plan even if they spent considerable time. They were not successful about the foresightedness of the plan.

4.1.2.1 Thinking of Same or Similar Problem

By majority, mathematically gifted students thought of the same or similar problem except for problem-1 whether they solved it or not. For example, a mathematically gifted student (M5) stated that she solved a similar problem with her teacher in SAC. She tried to remember the solution process of that similar problem to solve problem-4. On the other hand, this number of attempts was not high compared to other sub-phases. The number of attempts of successful students in thinking of the same or similar problem less than half of the mathematically gifted students'. Except for problems-1 and 2, a successful student attempted to think of the same or similar problems to devise a plan. In the third place, for problem-4, there was only one average student who applied it.

4.1.2.2 Producing Strategy Using All Givens in the Problem

The most preferred sub-phase was to produce a strategy using all the givens in the problems for all the groups of participants. All the mathematically gifted students tried to generate a strategy for all the problems. They especially paid attention to use all the data and the given conditions in a problem when thinking about a strategy. For example, a mathematically gifted student (M5) in problem-2 stated: "Firstly, I can split the areas with sticks. Then, I should divide the place between 12 and 11 since the sum of the number is a large amount. Then, I can check the sum of all areas." (M5, Problem-2). Similarly, except for S7 for problem-4, all the successful students tried to apply this sub-head. Although the number of attempts of successful students was slightly smaller than mathematically gifted students' number of attempts, not all successful students used all the data when creating a strategy. For example, a conversation between the researcher and a successful student (S7) in problem-3 is:

> S7: If $\frac{1}{3}$ of the number of the ball is equal to 35, all of it should be equal to 105.
>
> R: Excuse me, what does “35” mean here?
>
> S7: In the problem, there are 15 and 20 balls; therefore, 35 balls are left (S7, Problem-3).

In this problem, the successful student (S7) did not consider the other “15” balls which were left. On the other hand, the number of attempts of average students was not as high as mathematically gifted and successful students since some average ones did not use all the conditions of a problem. They even got the situation of the problem wrong. For example, a conversation between the researcher and an average student (A6) in problem-4 is:

> A6: I think that I should think differently about this problem.
>
> R: What does it mean?
>
> A6. I mean, the problem says that I have to subtract the sum of all the odd natural numbers less than 101 from the sum of all even natural numbers less than 101. On the other hand, I want to do the operation with two numbers instead of all even and odd numbers less than 101. I use 100 and 99 since two of them are less than 101. Therefore, the answer to this problem should be 100-99=1 (A6, Problem-4).

To conclude, creating a strategy by using all the givens in the problem was the most used sub-phase in the phase of devising a plan. Besides, the mathematically gifted students were the group of participants who had the highest number of attempts in the phase of devising a plan. Also, they paid the highest attention to apply to the phase of devising a plan among three groups of participants.

4.1.3 Carrying Out the Plan

In this phase, mathematically gifted students had the highest number of attempts as seen in Table 4.1. They were trying to implement the plan which was devised in the previous phase patiently and carefully. They were all aware of what they were doing while carrying out the plan. If they were sure that their plan did not work, they turned back to the previous phase to devise another plan. Secondly, successful students did not have the highest number of attempts like their mathematically gifted peers. Besides, some successful students were not careful about the process in carrying out the plan. Some of them decided whether their plan would work very quickly compared to the mathematically gifted ones. They could carry out their plan as quickly as possible without being sure that the plan was completely conducted. Therefore, after a while some of them got confused about the plan's direction, so they reached irrelevant points and gave more than one answer about the problem. For example, a conversation between the researcher and a successful student (S5) in problem-4 occurred as:

> S5: I think that the answer is 1 since the subtraction of each even and odd number is equal to 1. That is, 100-99 = 1, 99-98 = 1, 97-96 = 1 and so on.
>
> **[After 1 minute, he shared her result.]**
>
> S5: The answer should be 100.
>
> R: If you finished your solving process, I want to ask you a question.
>
> S5: Yes, I finished although I cannot be sure. Now, you can ask.
>
> R: Can you solve this problem with a different strategy or way?
>
> S5: The even numbers were going to 0, the odd numbers were going to 1. The answer is -1 (S5, Problem-4).

As it was observed here that as time passed, his answers changed. Thirdly, average students came in last place concerning the number of attempts of the carrying out the plan phase. Some average students behaved like their successful peers.

Moreover, they mostly waited for a confirmation from me whether the plan would work or not. They devised a plan and carried out some part of the plan. After that, they asked me "Do you think I should continue?".

4.1.3.1 Check All the Steps of the Plan

The vast majority of students of mathematically gifted students check their plan's steps in the process. They did it by rereading and resolving the problems carefully. For example, a mathematically gifted student (M6) stated in the problem-3 that:

> Firstly, I summed 15, 20, and 25 since I wanted to find the left part and the question about the total number of balls was asked to me. Then, if $\frac{1}{3}$ of the total ball was left, a part of the ball was equal to 25. Two parts were equal to 50, and all the number of balls was 75. In this way, I checked the solution. (M6, Problem-3).

M1, M3, and M7 did not just prefer to check all the steps of the plan in problem-4. Among the successful students, the number of attempts to check was lower than mathematically gifted ones. They checked the problems like their mathematically gifted peers. On the other hand, some of them did not check their plans in some cases. For example, none of the successful students checked their plan in problem-4. On the other hand, average students had the lowest number of attempts regarding checking all the steps of the plan among all groups. The number of attempts of average ones was less than half of the mathematically gifted ones' number of attempts.

4.1.3.2 Prove the Correctness of the Plan

Contrary to the number of attempts regarding checking all the steps of the plan, the number of attempts of three groups of participants to prove the correctness of the plan was low. As illustrated in Table 4.1, the mathematically gifted students had a priority in this part among three groups of participants. Some of them preferred to prove the correctness of the plan. For instance, none of them did prove the correctness of the plan in problem-4. Successful students came in second place here. They tried to prove the correctness of plans in problems nearly less than half of the mathematically gifted students. They treated problem-4 like their mathematically gifted peers did. On the other hand, the average students' number of attempts to prove was equal to a quarter of the mathematically gifted students' number of attempts. This showed that in some problems, average students did not prove the correctness of their plan such as problems 1, 3, and 4. All participants proved the correctness of the plan by solving the problem again.

In conclusion, checking all steps of the plan was the most used sub-phase among groups in the carrying out the plan phase. Mathematically gifted students were the group of participants who mostly applied the carrying out the plan concerning its sub-phases. In the phase of carrying out the plan, the result showed that although students checked all steps of a plan, some of them did not prove the correctness of this plan. This was observed among all groups of participants. Mathematically gifted students stated that they crosscheck every point in a problem at any moment of a plan. Therefore, they did not feel compelled to prove the correctness again. For successful students, this behavior was observed not as much as mathematically gifted ones. However, average students did not apply to do it like their mathematically gifted and successful peers. Their number of attempts was quite low.

4.1.4 Looking Back

In the last phase, mathematically gifted students had the highest number of attempts. On the other side, the number of attempts of successful students to apply looking back phase was less than mathematically gifted students' number of attempts. The lowest number of attempts was in average students. They applied the looking back phase less than half of the number of attempts of successful and nearly one-third of mathematically gifted students.

4.1.4.1 Checking All the Conditions and Steps in the Problem

Mathematically gifted ones mostly checked all the conditions compared to the other two groups of participants, but this number was not high like the other sub-phases of the phases. They checked all the conditions after finishing solving the problem completely by taking the value they found and solving the problem again like checking all the steps of the plan and proving the correctness of the plan. This behavior was observed in both successful and average students as well. However, successful students checked all the conditions and steps in the problems in less time than half of the mathematically gifted ones. Some successful students did not pay attention to some parts of the problem. For example, a successful student (S6) in problem-6 stated: “I divided 450 to 3 since there are three students. Then, I added 10 to 150 cm to find the height of Ada. So, Yasemin should be 160 and Duru should be 140 cm.” (S6, Problem 6). In this problem, the condition was that sum of all three students’ height is equal to 480 cm. Average students were in the lowest level in checking all the conditions and steps in the problem. Their number of attempts was less than the number of the attempt of successful and nearly less than one-third of the number of mathematically gifted students’ attempts.

4.1.4.2 Solving the Problem with Another Strategy

In this sub-phase, after finishing all the processes, all the participants were asked whether they could solve the problem with another different strategy or not. Mathematically gifted students showed more tendencies to try to think of a different problem-solving strategy. They tried to give considerable time to create a different strategy than the previous one(s) used in the solution. Moreover, the results indicated that they mostly thought of alternative strategies during the process instead of after the solution process. They did not prefer to use alternative strategies for problems that they solved relatively quickly or more easily. For example, problem-3 was the easiest one among other problems for most of the mathematically gifted students. Therefore, they did not think of another strategy. Most of them solved it with one strategy. On the other hand, when they faced more challenging problems, they tried to create alternative strategies both during and after the problem such as problem-5. The number of attempts to solve the problem with another strategy was lower in successful students. There was a notable number of students in the successful group who solved the problem with only one strategy. For example, there was one problem-solving strategy during the problem-solving process but any other strategy was not observed after finishing solving for S2 in problem-3, 4, 5, and 6; S3 in problem-3, 4, and 5; S4 in problem-4, 5, and 6; and S7, problem-3, 4, 5 and 6. In average students, the number of attempts was so low that it was less than half of successful students' and less than one-third of mathematically gifted students' number of attempts. Average students mostly tended to solve the problems with one strategy during the problem.

To conclude, checking all the conditions and steps in the problem was the most used sub-phase in the phase of looking back. Mathematically gifted students had the highest use of looking back phase. On the other hand, the number of attempts was not high like the other three phases. This is why, mathematically gifted students would rather check the condition and prefer to use and think of different strategies during the problem-solving process instead of after the problem-solving process. The number of attempts was lower in successful students. The number of the attempt of average students was the lowest among all the groups.

4.1.5 Summary of the Problem-Solving Phase Use

Concerning all the problem-solving phases, the phase of understanding the problem had the highest number of attempts to use among three groups of participants while the lowest number of attempts to use was in the phase of looking back. Besides, some similar behaviors were observed in carrying out the plan and looking back phases for all three groups of participants. The general results also showed that number of attempts to apply phases decreased from understanding the problem phase to the looking back phase for all the groups of participants.

From the groups' perspectives, mathematically gifted students had a priority over the use of phases even though they did not know the problem-solving phases before. Except understanding the problem phase, they generally tried to apply all the phases more compared to successful and average students. Moreover, they generally applied the respectively as firstly understanding the problem, secondly devising a plan, thirdly carrying out the plan, and lastly looking back respectively. They did not move on to the next phase without completing the previous phase. Successful students, on the other hand, had a priority on only understanding the problem phase. They came after their mathematically gifted peers in the number of applications of the other three phases. They did not apply the phases in order like mathematically gifted ones. Some of them directly started from devising a plan without applying the understanding the problem phase entirely. This situation caused students to give more than one answer to the problem as cited in carrying out the plan phase (e.g. S5 in problem-4). Lastly, average students came in last place for the numbers of every phase application. The effectiveness of use was the lowest. Making mistakes, not being sure about work on problems, and low motivation were more in number compared to mathematically gifted and successful students. Hence, these behaviors prevented them from applying problem-solving phases like the other two groups of participants.

After the process ended, when all the participants were asked to explain their works for problems' solutions, mathematically gifted students were the group who was able to express their ideas logically and clearly. Successful students were in second place. On the other hand, average students were the most unsuccessful group concerning explaining the problem-solving process explicitly.

4.2 Students' Use of Problem-Solving Strategies

In this section, the findings related to the second and the first part of the third question will be presented: “Which problem-solving strategies do mathematically gifted, successful, and average students use while solving a non-routine problem?” and “How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?”.

There were six mathematical problems presented to each group in this study which are introduced in Table 3.2. There were nine problem-solving strategies expected from participants to use when solving six mathematical problems. For each mathematical problem, the expected strategies used by the participants are listed in Table 3.2. Moreover, detailed use of problem-solving strategies by each participant in six mathematical problems is presented in Appendix F. In Table 4.2, the number of attempts to use problem-solving strategies of three groups of 20 participants, who were seven mathematically gifted (M), seven successful (S), and six average (A) fifth-grade students, were represented. Every strategy was split into two subsegments with prompt (P^{+}) and without prompt (P^{-}). P^{+}subsegment indicates that the student used this strategy after my prompt as a researcher such as asking in the looking back phase “Can you solve this problem with a different way or strategy?” when they finished solving the problem. Then, P^{-}subsegment indicates that a student tried to use the strategy without my prompt in the problem-solving process after finishing their problem-solving.

In the study, observations also showed that some strategies were used effectively by students to reach correct solutions. However, some students tried to use the same strategies but they were not successful to reach the correct solution. Therefore, in Table 4.2, the numbers inside the parenthesis, the case shows total attempts to reach correct answers in total answers using the strategy like # of total (successful and unsuccessful) attempt (# of successful attempt). Without the notation of parenthesis, the case shows the total unsuccessful number of attempts in using the strategy. For example, in Table 4.2, mathematically gifted students (M) had eight attempts to use the problem-solving strategy of making a drawing but none of them used it successfully to reach a correct solution in problems. On the other hand, the successful students (S) had ten attempts in total to use the problem-solving strategy of making a drawing but only three attempts were successful to reach the correct solution in the problems.

Table 4. 2

Participants' Number of Attempts to Use Problem-Solving Strategies

Students	Problem-Solving Strategies																	
	Organizing Data		Intelligent Guessing and Testing		Solving Simpler Equivalent Problem		Acting It Out or Simulation		Working Backwards		Finding a Pattern		Logical Reasoning		Making a Drawing		Adopting a Different Point of View	
	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-
M		1	2(2)	9(3)	2(2)	2(1)	9(3)	10(6)		8(4)	1	5(3)	1	13(6)	1	7	2	17(5)
	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-
S	1	1		11(5)		1(1)	6(1)	8(2)		7(2)		5(3)		21(2)	1	9(3)		12
	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-
A		2(2)	1	5(2)	1	2	5	2		5(1)		2(2)		15(5)	1	2		5(2)

Note. M: Mathematically Gifted Students, S: Successful Students, A: Average Students
Note. P^+: The solution that students solve with prompt of researcher, P^-: The solution that students solve without prompt of researcher, (a number): Number of solutions that students solve the problem correctly

4.2.1 Problem-Solving Strategies Used by the Different Groups of Students

All the participants tried to use all nine strategies for six problems. The majority of them used at least one expected strategy for the problems' solutions. The number and use of each problem-solving strategy for each participant are presented in detail in Appendix F. In addition to expected strategy use, for some groups of participants, there was unexpected strategy use in some problems.

When examined the Table 4.2, mathematically gifted students were the participants who used the most problem-solving strategy amongst all participants. Mathematically gifted groups also had considerable effort and motivation to implement a strategy for a solution. They found a strategy and tried to implement it until they were satisfied with the answer. They did not give up immediately without trying a strategy entirely. When they thought that the strategy did not help them to find the correct answer no longer, they did not hesitate to think of another strategy. Successful students were the second-highest problem-solving strategy users among all participants. By contrast with mathematically gifted students, some successful students insisted on practicing the same strategy for a problem instead of thinking of another strategy. For example, S3 only used logical reasoning in problem-4. She tried to solve the problem immediately. A conversation between researcher and S3 was:

> S3: There are 51 even numbers since we have to include "0". Then, there are also 50 odd numbers in 101. Therefore, I have to subtract 50 from 51. The result should be 1.
> R: Are you sure?
> S3: Yes, I am sure.
> R: Ok, Can you solve it with another strategy?
> S3: Of course, there are other strategies but I do not want to use them since I do not want to write all the numbers. In addition to that, these numbers are

> different from each other; so, I cannot find the sum of them (S3, Problem-4).

Similarly, S4 and S7 used only adopting a different point of view strategy in problem 4. S4 made all the number groups but she could not benefit from the strategy. The same case was observed for S7. She wrote all the numbers and tried to put them into groups. She reached the solution of “1” but she was not sure about it. Nevertheless, they did not attempt to find and use another strategy.

When examined in Table 4.2, average students were the participants who had the lowest number of attempts to find and use problem-solving strategies among all the participants in this study. Average students did not make considerable effort to implement a strategy for a solution. Furthermore, some average students lost their motivation when they could not solve the problems. For example, A6 tried to understand and solve problem-2 but when he could not reach the solution, he gave up. He did not even attempt to find another strategy. Therefore, some students could not generate a strategy for the problems among average students.

4.2.1.1 Problem-Solving Strategies Used by Mathematically Gifted Students

Mathematically gifted students were the group who used the most of the strategies in this study. They also generally tried to use more than one strategy in the problem-solving process except for some problems. In this group, some of them also preferred extra strategies apart from expected strategies for the problems. For example, M1 used adopting a different point of view strategy for problem-1. Similarly, she preferred making a drawing for problem-2. M5 also tried to use intelligence guessing and testing for problem-1. In addition, M3 and M7 utilized making a drawing strategy for problem-2. M3 also tried to use acting it out or simulation for problem-4. M3 benefitted from intelligence guessing and testing and solving simpler equivalent problem strategies for problem-3.

Table 4.2 showed that acting it out or simulation and adopting a different point of view were equally the most used strategies among mathematically gifted students. Following them, logical reasoning and intelligence guessing and testing strategies were the second and third most used strategies respectively. The problem-solving strategies of making a drawing and working backward were equally the fourth most preferred strategies by mathematically gifted students. The problem-solving strategy of finding a pattern was placed in fifth. The strategy of solving a simpler equivalent problem followed it with a small difference. Organizing data was the least used strategy among them. Only a student, M6, benefitted from it for problem-6.

Mathematically gifted students mainly tried to find and use the strategies without prompt of the researcher. They mostly used the problem-solving strategy of adopting a different point of view without prompt. The number of use without prompt was more than eight times of the number of use with the prompt. In second place, they tried to use logical reasoning without prompt. The number of use without prompt was equal to thirteen times of the number of use with prompt. In third place, the problem-solving strategy of acting it out or simulation was used with prompt and without prompt almost equally. Fourthly, intelligence guessing and testing were also preferred to solve the problem without prompt. As the fifth most used strategy, working backwards was preferred by them totally without prompt. Making drawing came in sixth place with a small difference. Finding a pattern was at seventh place concerning the use without prompt. As one of the least used strategies, the problem-solving strategy of solving the simpler equivalent problem was preferred as both with and without prompt in the same frequency. Lastly, organizing data was used without prompt only once.

Mathematically gifted students found at least one strategy for each problem. Apart from this, as the results pointed out above, all mathematically gifted students were not always successful to reach the correct solution by using the strategies. In this regard, they were able to use acting it out or simulation most effectively to reach the correct solutions. Secondly, logical reasoning was used to reach the correct solutions. In the third place, both intelligence guessing and testing and adopting a different point of view were utilized effectively in equal numbers. Mathematically gifted students used working backwards successfully in half shares. The problem-solving strategies of finding a pattern and solving simpler equivalent problem were effectively used the least to find the correct answer. They could not use making a drawing and efficiently organizing data to reach the correct answers.

4.2.1.2 Problem-Solving Strategies Used by Successful Students

In terms of the number of use of problem-solving strategies, successful students came in second place when compared to their mathematically gifted peers. Like mathematically gifted students, they tried to use more than one strategy except for some problems. However, the use of more strategies for the problems was higher in mathematically gifted students. Also, some of them preferred extra strategies apart from the expected strategies. There were unexpected strategies that only one student used. S1 used the problem-solving strategy of making a drawing for problem-3 and problem-solving strategy of acting it out or simulation for problem-4. In the group of successful students, there was only a student (S3) who could not use a strategy to solve problem-6.

Logical reasoning was mostly used when problems were solved among these groups. Compared to the mathematically gifted students, successful students used this strategy with quite a difference. Secondly, they tried to use acting it out or simulation. Thirdly, they preferred adopting a different point of view. Afterwards, the intelligence guessing and testing strategy was in fourth place with a small difference. Making a drawing was the sixth most used strategy. Working backwards and finding a pattern strategies were the sixth and the seventh most preferred strategies respectively. Similar to mathematically gifted ones, organizing data was one of the least used strategies in which S3 tried to use for problem-1 and S1 tried to utilize for problem-6. Lastly, solving the simpler equivalent problem strategy was used only once in problem-4 by S4.

Successful students like mathematically gifted students generally used the strategies without prompt. However, the number of strategies that successful students used without prompt was more than mathematically gifted students' number of strategies use. There were only three types of strategies that successful students preferred to use with prompt: acting it out or simulation, making a drawing, and organizing data. In acting it out or simulation strategy, the number of prompts was less than two times of the number of without prompts. In making a drawing, the number of prompts was nine times of without prompts number. Organizing data had an equal number in with and without prompt use.

Successful students, like their mathematically gifted peers, found at least one strategy for each problem and tried to solve them by using it except S3. S3 could not find any strategy to solve problem-6. Additionally, the results pointed out that like mathematically gifted students, all successful students were not always successful to reach the correct solution by using the strategies. Even, their number of attempts to find the right answer was less than mathematically gifted students' number of attempts. Successful students were able to use intelligence guessing and testing strategy most effectively to reach the correct solutions. Secondly, they could use making a drawing, finding a pattern, and acting it out or simulation equally and efficiently to find correct answers. Thirdly, the problem-solving strategies of logical reasoning and working backwards were utilized to reach the correct solutions. In the last place, the problem-solving strategy of solving a simpler equivalent problem was used efficiently once. They could not use adopting a different point of view and organizing data strategies effectively to reach correct the answers.

4.2.1.3 Problem-Solving Strategies Used by Average Students

In third place, average students showed the lowest number of attempts to use the strategies. In contrast to mathematically gifted and successful students, there was not much attempt of average students concerning the use of more than one strategy for the problems. Also, it was observed that apart from the expected strategies, the use of extra strategies was not preferred.

They mostly used logical reasoning but not as much as successful students. In second place, they preferred acting it out or simulation strategy like successful students. Intelligence guessing and testing strategy was used with a small difference in third place. Fourth, the problem-solving strategies of working backwards and adopting a different point of view were preferred equally. Fifth, the problem-solving strategies of solving the simpler equivalent problem and making a drawing were utilized by them in the same amount. Lastly, the problem-solving strategies of finding a pattern and organizing data were used the least while solving the problems. Finding a pattern strategy was used only twice as A1 and A3 tried to prefer it in problem-1. Also, the problem-solving strategy of organizing data was utilized only twice as A2 and A5 tried to use it in problem-2.

Average students tried to use the strategies without prompt of the researcher except acting it out simulation strategy. Their number of attempts was equal to successful students but less than half of the number of mathematically gifted students' attempts. They mostly used the problem-solving strategy of acting it out simulation with prompt. The number of the use with prompt was more than twice of the number of without prompt. In the problem-solving strategies of intelligence guessing and testing, solving a simpler equivalent problem, and making a drawing, they had an attempt to solve the problem with the prompt by using these strategies.

On the contrary to mathematically gifted and successful students, some average students did not generate a strategy to solve the problems. For instance, A1 could not find any strategy for problem-2, A2 could not find any strategy for problem-4, A5 could not find any strategy for problem-2 and problem-5, and A6 could not find any strategy for problem-2. Apart from this, average students used some strategies effectively. Firstly, they could use logical reasoning most effectively to reach the correct solutions. Secondly, they could use adopting a different point of view, finding a pattern, intelligence guessing and testing, and organizing data strategies efficiently and in the same frequency to reach the answers. Lastly, working backwards was used only once by A4 for problem-3 to reach the correct solution. They could not use making a drawing, solving a simpler equivalent problem, acting it out or simulation strategies in an efficient way to reach the correct answers.

4.2.2 Summary of Problem-Solving Strategy Use of Three Groups of Students

All in all, mathematically gifted ones had a priority among three groups of participants concerning the total number of strategies use. They used more than one strategy and applied them effectively. In addition, unexpected use of strategy was mostly observed in the group of mathematically gifted students. Acting it out or simulation, adopting a different point of view, and logical reasoning strategies were the most used three strategies among mathematically gifted students. Furthermore, they generally tried to use the strategies without prompt of the researcher. However, they were open to the prompt of me as a researcher and tried to find alternative ways to solve the problem after finishing the process in looking back phase. Similarly, acting it out or simulation was the most and effectively used strategy to reach the correct solutions. Besides, they reached the right answers by using the problem-solving strategies of logical reasoning, intelligence guessing and testing, and adopting a different point of view respectively. That is, the most used strategies also were the most effective strategies to reach the correct solutions for mathematically gifted ones.

Successful students were in second place in terms of the total number of strategies use, using more than one strategy, using the strategies effectively, and finding unexpected strategies for the problems. Logical reasoning was the most used strategy which was equal to one-fourth of their total number of strategies use. Then, acting it out or simulation strategy came as the most used ones among them. Additionally, they were in first place among three groups in finding and using strategies without prompt. This was probably because of the high number of using logical reasoning. They were not open to the strategy of thinking on with my prompt after finishing the problem-solving process in the phase of looking back as mathematically gifted ones. Successful students mostly used intelligence guessing and testing, making a drawing, finding a pattern, and acting it out or simulation respectively in an effective way to reach a correct solution for the problems. On the other hand, as the most benefitted strategy, logical reasoning was not used efficiently by them to solve the problems correctly. In other words, for successful students, the use of more strategies did not mean solving the problem more correctly.

Finally, average students were in last place in the total number of strategies use, using more than one strategy and using the strategies effectively in the problems. In the problem-solving strategies of logical reasoning, acting it out or simulation and intelligence guessing and testing, their performance mostly was parallel to successful students'. Similarly, the number of attempts to use strategies without prompt was equal to successful students' number of attempts to use strategies without prompt but average ones preferred to use some strategies with prompts more. On the other hand, they were not as successful to use all the strategies to reach the correct solutions as other groups of participants. They used logical reasoning and intelligence guessing and testing strategies effectively to reach the correct answers. This situation demonstrated that average students were able to find the right answers with a strategy that they used mostly. However, acting it out or simulation strategy which was the second most used strategy, were not adopted effectively by them to solve the problems correctly.

4.2.3 Problem-Solving Strategy Use Styles of Different Groups of Students

In this section, results were presented regarding the second part of the third research question as to the style of usage in problem-solving strategy "How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?". Results showed that although mathematically gifted, successful, and average students used the same strategies, there were differences as well as similarities in using and representing them in the problems. The cases, mentioned below, problems were chosen in line with one condition. This condition was that these three groups of students' use of the same strategies in the same problems. In this way, the similarities and differences of strategy use can be shown properly.

4.2.3.1 Intelligence Guessing and Testing

Problem-2 was the problem in which mathematically gifted, successful, and average students used intelligence guessing and testing strategy mostly. All mathematically gifted, six successful, and three average students preferred to use it in problem-2. Figure 4.2 presents a mathematically gifted student's (M3) work for problem-2 using intelligence guessing and testing strategy. M3 tried to separate big numbers from each other by drawing the lines between some numbers like 11 and 12 or 10 and 9. If he was not successful to make the sum of all areas the same, he tried to think 11 and 12 together but he did not prefer to take 10 next to 11 and 12. He continued to test his intelligence guesses until making all the areas equal.

Figure 4. 2

Part of a Mathematically Gifted Student's (M3) Work on Problem-2 Showing

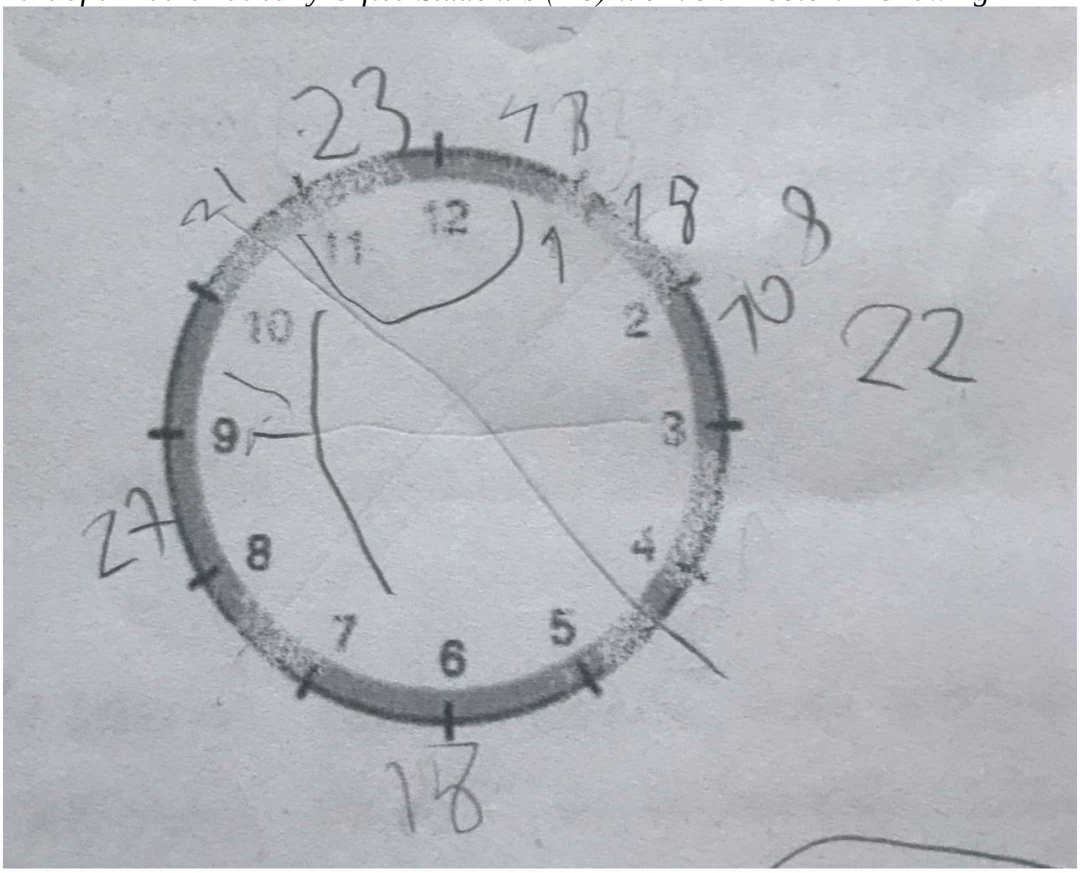


Figure 4.3 presents a successful students' (S2) work for problem-2 by using intelligence guessing and testing strategy. S2 tried to separate 11 and 12 from each other by using sticks and then he continued to check his guesses. Besides, he tried to combine small and big numbers with intelligence guesses and checks.

Figure 4. 3

Part of a Successful Student’s (S2) Work on Problem-2 Showing

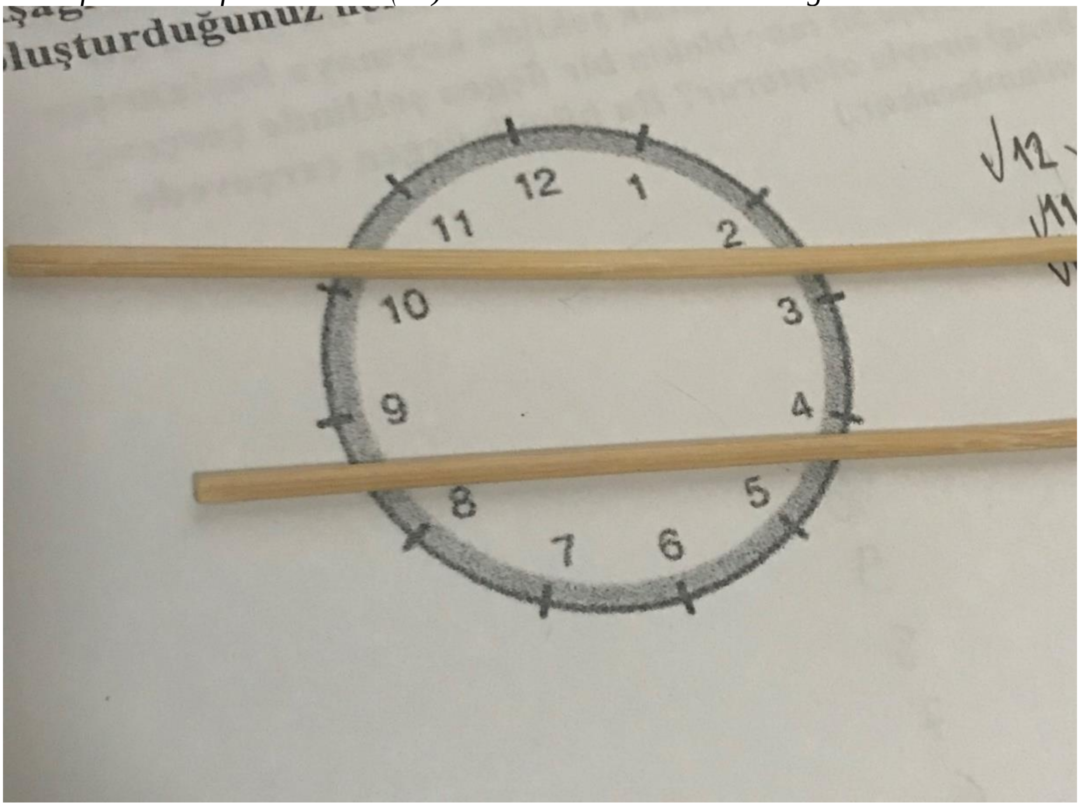
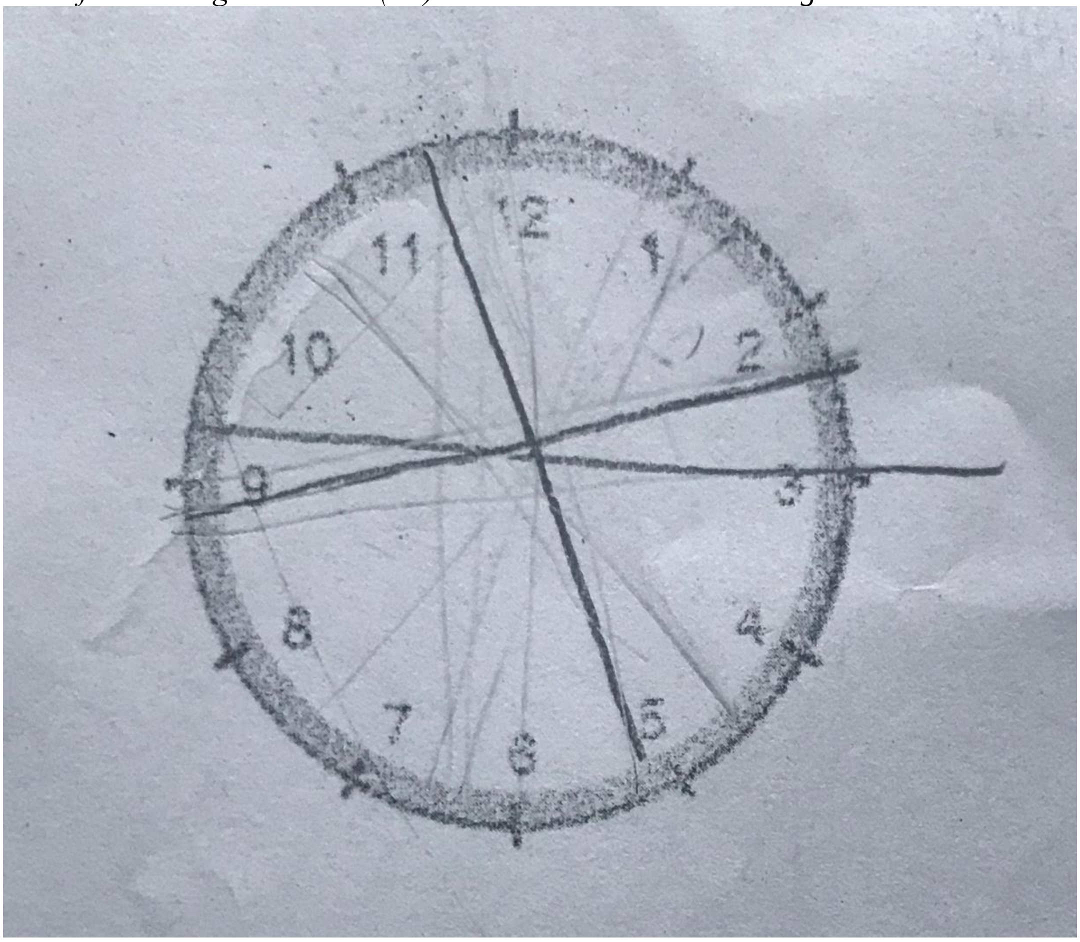


Figure 4.4 presents an average students’ (A2) work for problem-2 by using intelligence guessing and testing strategy. He tested lots of guesses about dividing the areas by drawing the line. Instead of focusing on separating big numbers from each other like other groups, they tried all numbers to divide. He continued to test his guesses by drawing the lines until the sum of each area is equal to each others. On the other hand, he could not reach the solution that he desired.

Figure 4. 4

Part of an Average Student's (A2) Work on Problem-2 Showing



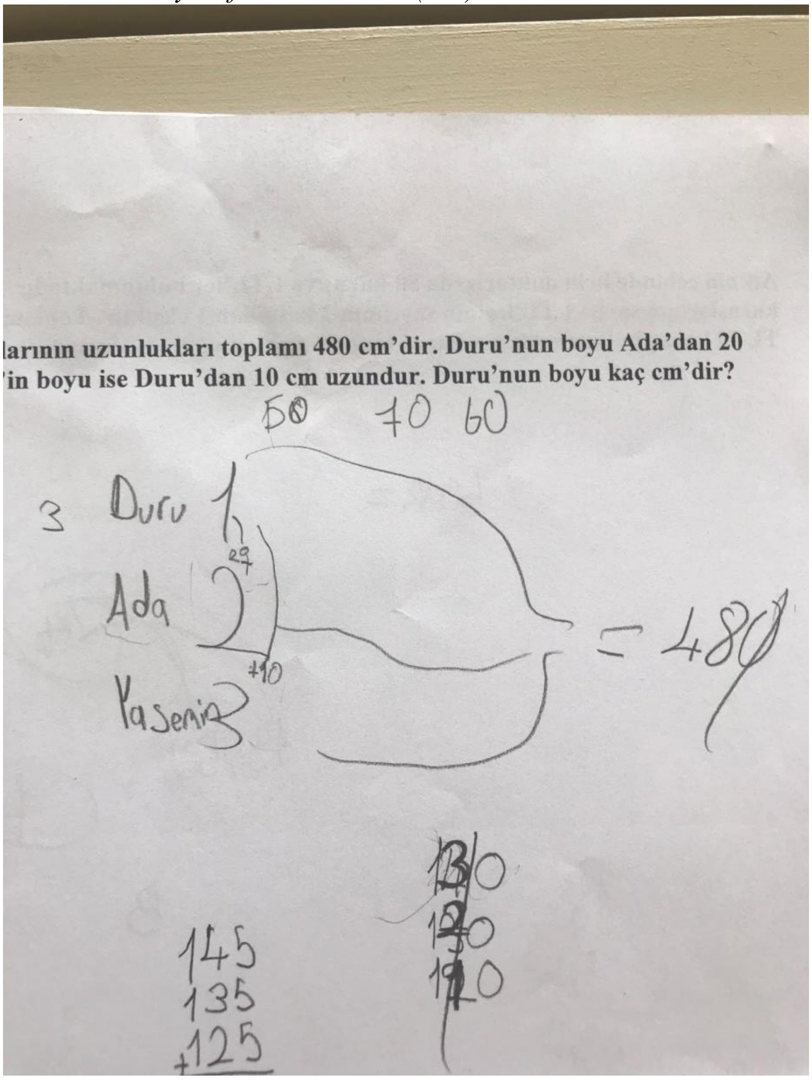
All in all, mathematically gifted and successful groups of participants (M3, S2) paid attention to big and small numbers in the clock. They consistently said that the numbers 12, 11, and 10 can be separated. That is, they guessed the place where the numbers were separated and how the clock could be divided. In addition, they tried to combine very small numbers like 1, 2, and 3 with big numbers 11, 10, and 9. In the light of these intelligence guesses that are related to the problem's condition, they tested all intelligence guesses in the clock. Therefore, there was no main difference concerning the use of intelligence guessing and testing strategy among these two groups of participants. On the other hand, average students make many relevant and irrelevant guesses. Therefore, they did more guesses than the other two groups of participants. Nevertheless, they were not successful to reach the correct solution. Mathematically gifted and successful students tried to make guesses and used the strategy more carefully. Therefore, they had more successful attempts to use this strategy compared to the average students.

4.2.3.2 Organizing Data

As one of the least used strategies among three groups of participants, the problem-solving strategy of organizing data was used in some problems like 1 and 6. M6 and S1 preferred to use the problem-solving strategy of organizing data in problem-6. Both of them put the height of every people in order. Figure 4.5 provides a mathematically gifted student's (M6) work on problem-6 using the problem-solving strategy of organizing data strategy. He organized three sets of data to see all the relationships. That is, he tried to understand completely the height differences between people in the problem. He made his inferences that Ada is the longest, Duru is the shortest one.

Figure 4. 5

Part of a Mathematically Gifted Student's (M6) Work on Problem-6 Showing

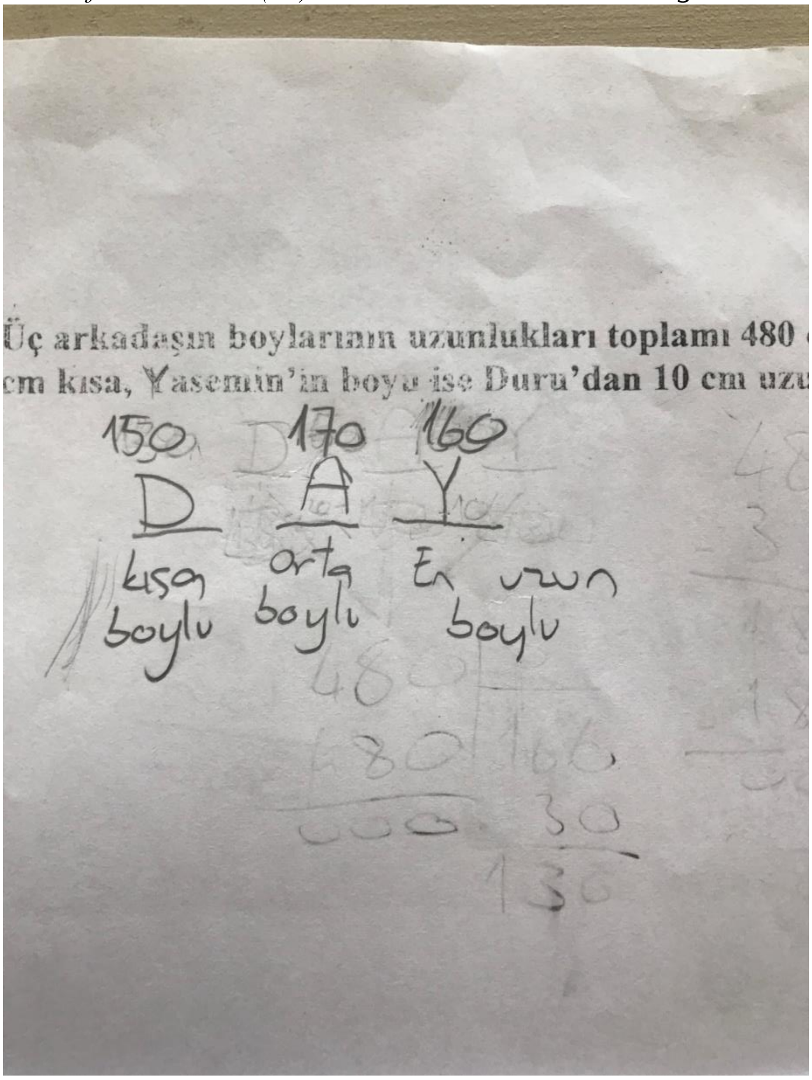


He also stated: "There are 10 cm differences among every person. Ada is the longest person, Duru is the shortest person. Yasemin is between them" (M6, Problem-6).

Figure 4.6 provides a successful student's (S1) use of organizing data strategy on problem-6. S1 organized the data according to the heights of them but she did not succeed to put them in the right place.

Figure 4. 6

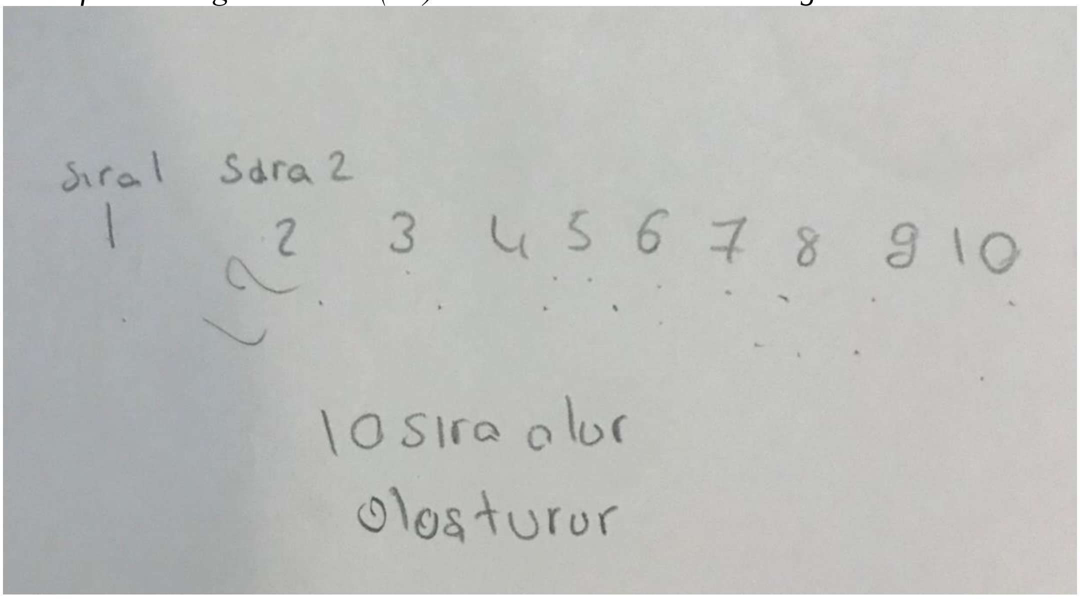
Part of a Successful Students' (S1) Work on Problem-6 Showing



Average students used the strategy of organizing data only in problem-1. There were no mathematically gifted and successful students who used this strategy in problem-1. Figure 4.7 provides the work of an average student (A2) on problem-1. He organized all the rows from 1 to 10 and constructed a list that showed the number and amount of rows. Then, he checked all the rows according to the pattern and the total number of rows in the problem. Finally, he could reach the correct solution.

Figure 4. 7

Part of an Average Student's (A2) Work on Problem-1 Showing



As a result, the mathematically gifted student (M6) organized three sets of data to understand the height differences between people in the problem. In this way, he was able to make logical inferences. After that, he moved on to another strategy to solve the problem. It can be said that he achieved what he was supposed to acquire by using the strategy of organizing data. Then, when he felt this strategy did not help him anymore, he continued with another strategy by bringing what they had acquired from. The successful student (S1) decided to use organizing data after her many attempts. She stated that Duru is the shortest and Yasemin is the longest one. On the other hand, when she found the height of every person in the problem, she could not put them in the right place in the list of the data. This is because she could not determine the heights right at first on the list. Therefore, the result showed that she did not use the strategy of organizing data effectively. In addition, mathematically gifted one used the strategy to proceed to another strategy but successful students used this strategy after using many strategies. Apart from this, the average student (A2) used the organizing data strategy efficiently. When Table

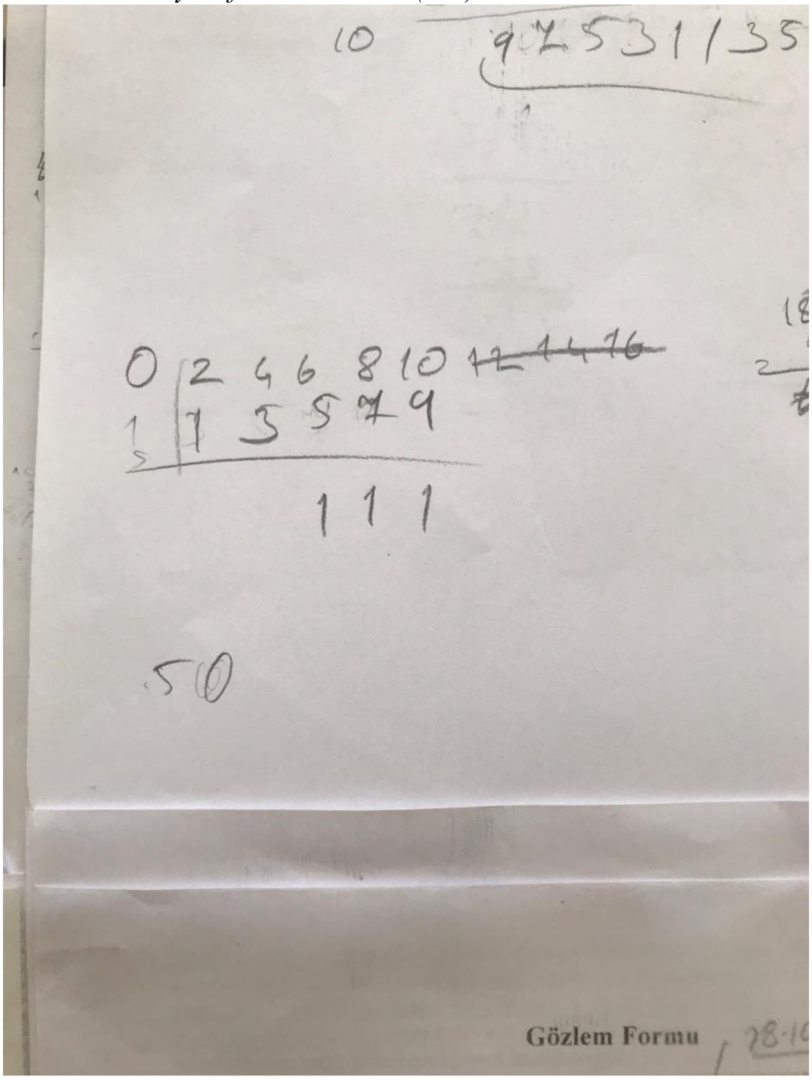
4.2 was examined, it can be seen that the problem-solving strategy of organizing data was effectively used mostly by average students. In problem-1, A2 put all the rows of data to the tenth place but he did not write other row numbers. He just wrote the first two order numbers. On the other hand, there was no common problem in which the use of strategies for all the groups of participants was compared at the same time.

4.2.3.3 Solving a Simpler Equivalent Problem

Like organizing data, as one of the least used strategies among three groups of participants, the problem-solving strategy of solving the simpler equivalent problem was preferred in some problems like problems 1, 3, 4, and 6. M4, S1 and, A6 tried to use solving the simpler equivalent problem in problem-4. All of them thought the problem-4 as a simple form. Figure 4.8 provides a mathematically gifted student's (M4) strategy use in problem-4. Firstly, he tested the condition with the first group of ten one-digit numbers, then a group of ten two-digit numbers. When he felt sure that the subtraction of every ten groups is the same (which equals "1"), he used it for all the numbers. In this way, he could reach the right answer.

Figure 4. 8

Part of a Mathematically Gifted Student's (M4) Work on Problem-4 Showing



A conversation between the researcher and M4 is:

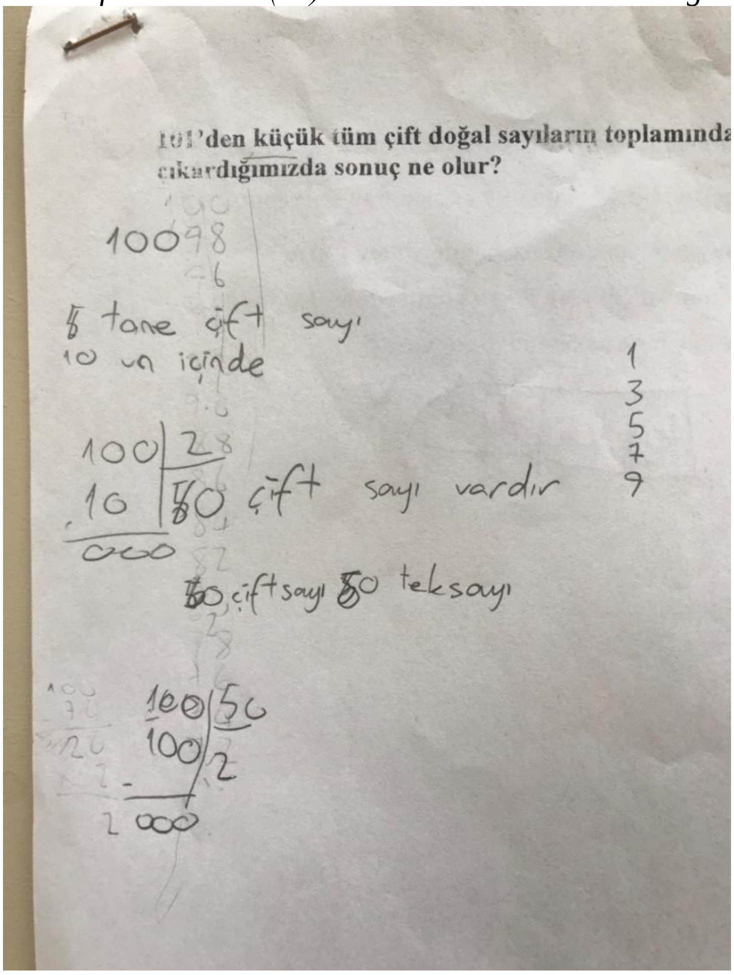
> M4: Firstly, I write the integers from 0 to 10. I want to think of just the first ten numbers.
>
> R: Why are you thinking like that?
>
> M4: Because firstly I want to test the small numbers. If I can see some relationships, I will try them on the rest of the numbers.
>
> **[After a while, he shared his ideas again.]**
>
> M4: I wrote all even numbers and all odd numbers up to 10. Then, I made subtraction respectively.
>
> R: How?

> M4: That is, I subtracted 9 from 10, 7 from 8, 6 from 7, and so on. Then, I find the result "1" at every turn. On the other hand, this will be the same on two-digit numbers.
> R: How?
> M4: For example, 20-19 is equal to 1, or 18-17 is also equal to 1. Every ten group of numbers' subtraction will be equal to 5. There are ten groups; so, the result should be 5 x 10: 50 (M4, Problem-4).

Figure 4.9 presents the strategy use of solving a simpler equivalent problem of a successful student (S1) on problem-4. She thought the situation simpler in the first step. That is, she tried to look at the first ten numbers. Then, she tried to generalize their results to the problem's main situation.

Figure 4. 9

Part of a S*uccessful Student's (S1) Work on Problem-4 Showing*



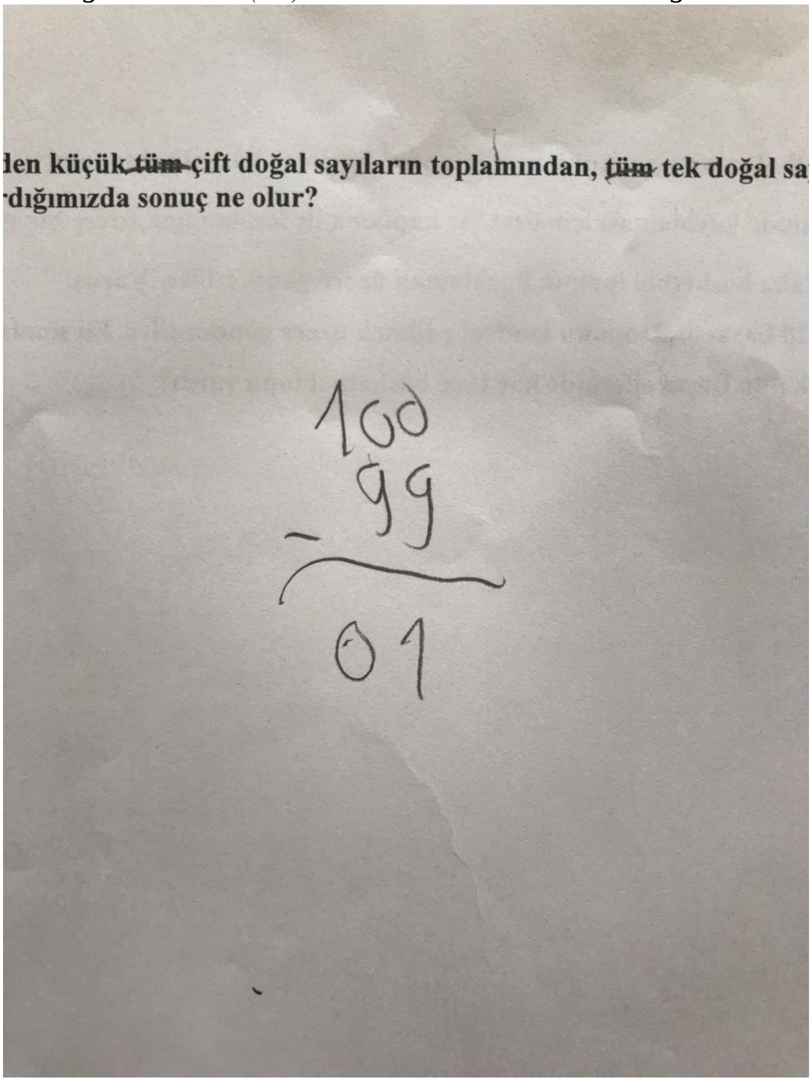
A conversation between researcher and S1:

> S1: Let's think that there are 6 even numbers between 0 and 10. Now, we think that there should be 60 even numbers in the first 100 numbers. This is the same for odd numbers but there are 5 odd numbers in the first ten numbers. Therefore, there should be 50 odd numbers in 100.
> R: Okey, what is the next step?
> S1: Now, I am going to subtract 50 from 60. The result is 10 (S1, Problem-4).

Figure 4.10 presents the strategy use of solving a simpler equivalent problem of an average student (A6). He made his operation as if there were only two numbers in the problem. That is, he tried to simplify the operation in the problem.

Figure 4. 10

Part of an Average Student's (A6) Work on Problem-4 Showing



A conversation between the researcher and an average student (A6) stated that:

> A6: Ok, we can delete all the numbers except for 100 and 99.
>
> R: Why?
> A6: Because we can subtract the biggest odd number from the biggest even number. Otherwise, the operation will be so time-consuming.
> R: So?
> A6: The result is 100-99: 1 (A6, Problem-4).

In conclusion, all three participants simplified the problem with small numbers in the first phase. Mathematically gifted one (M4) tested the condition with small numbers, then two-digit numbers. When he reached the desired result, he applied it to the whole problem. That is, he could make generalizations successfully. In this way, he was able to find the right answer. On the other hand, the successful student (S1) reduced the complicated nature of the problem without checking the small numbers in themselves. She directly concentrated on the sum of even and odd numbers instead of looking at the numbers one by one. The average student (A6) deleted all the numbers since he tried to simplify the operation when he should have simplified the condition of the problem. He ignored all the numbers except for 100 and 99. Therefore successful and average students were not able to use the strategy of solving the simpler equivalent problem effectively and reach the true answer as much as their mathematical gifted peers. Successful students had only an effective attempt to use the strategy while average students did not have any attempt to reach the solution.

4.2.3.4 Acting It Out or Simulation

As one of the most preferred strategies among three groups of participants, the problem-solving strategy of acting it out or simulation was preferred by the participants in almost all problems (see Appendix F). The number of participants used the strategy increased in problem-1. Six mathematically gifted, five successful, and two average students tried to use the strategy of acting it out or simulation strategy for problem-1. Figure 4.11 provides a mathematically gifted student's (M3) use of acting it out or simulation. He preferred to use calculation sticks from manipulatives to construct the triangle that was asked in problem-1. That is, he considered each rectangle as a whole. In the first row, there should be one rectangle. He put it as a 1 cm stick. Then, he put a 2 cm stick in the second row instead of two rectangles. Finally, he was able to reach the total sum of all rectangles which was equal to 55.

Figure 4. 11

Part of a Mathematically Gifted Student's (M3) Work on Problem-1 Showing

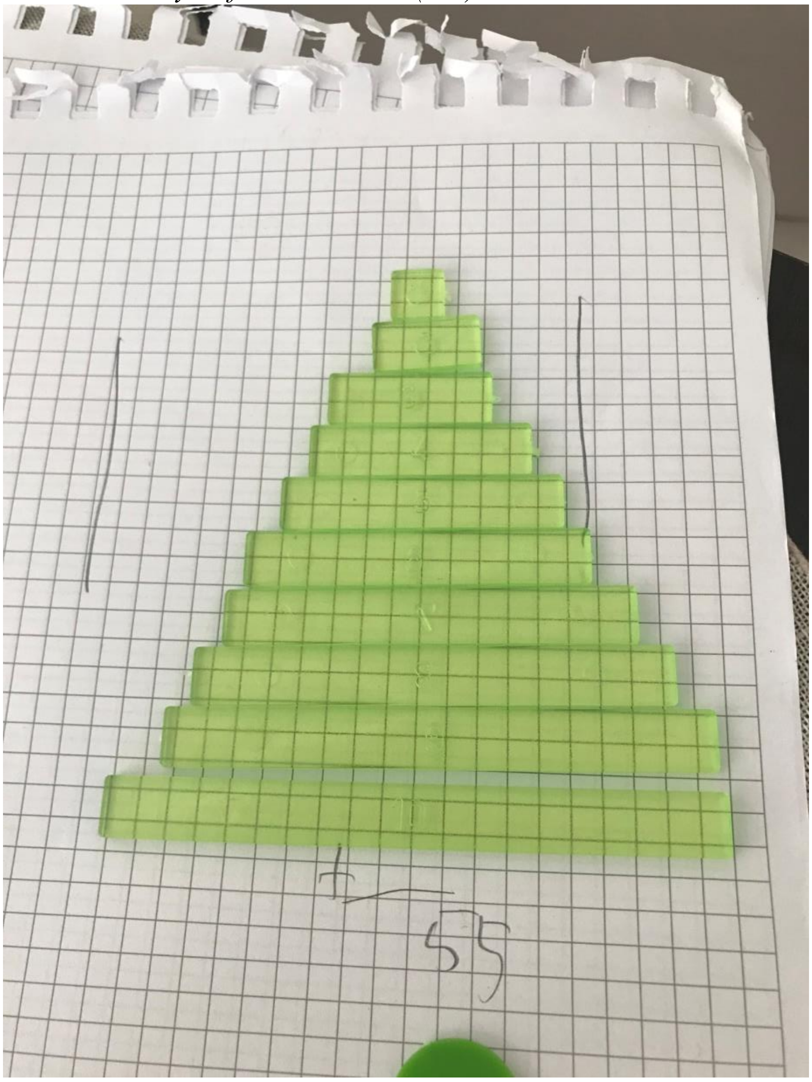


Figure 4.12 provides a mathematically gifted student's (M2) use of acting it out or simulation. In this work, he used the unit cubes as rectangles. He preferred to use unit cubes to construct the triangle for the problem-1. In the first row, he put a

rectangle as required in the problem. Then, he put two rectangles in the second row. He continued like this until he constructed the whole triangle.

Figure 4. 12

Part of a Mathematically Gifted Student's (M2) Work on Problem-1 Showing

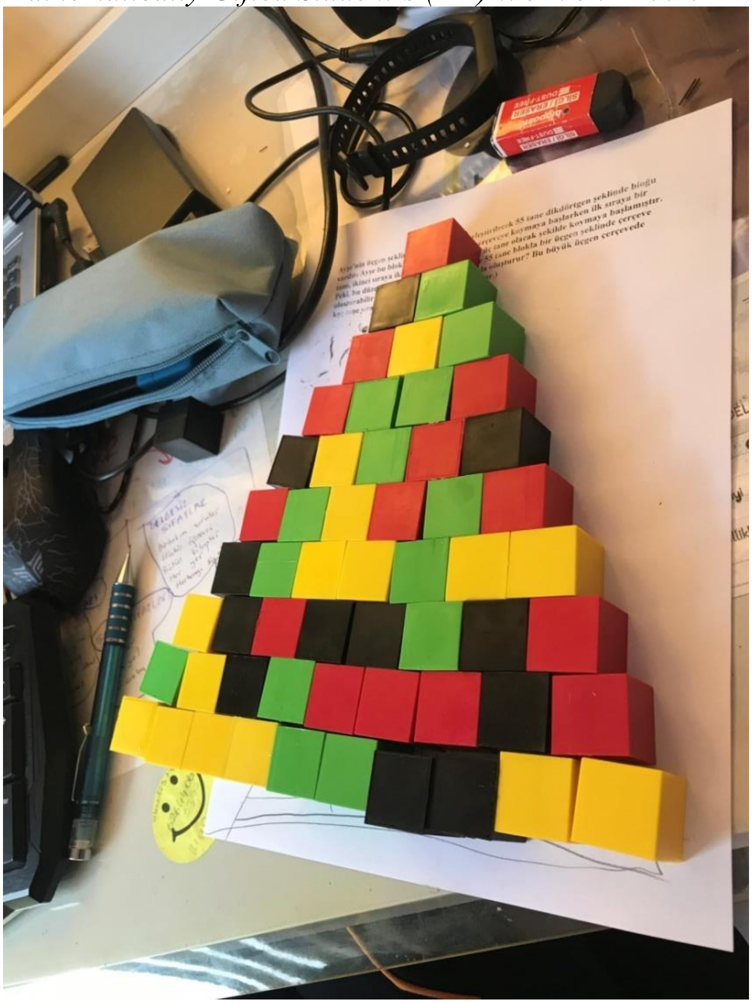


Figure 4.13 provides a successful students' (S1) use of acting it out or simulation in problem-1. She preferred to use unit cubes, calculation sticks, and round counting pieces to construct the triangle for the problem. On the other hand, she constituted

a small and a big triangle as one within the other. Also, she did not consider spaces inside the big triangle.

Figure 4. 13

Part of a Successful Students' (S1) Work on Problem-1 Showing

Figure 4.14 provides a successful student's (S5) use the strategy of acting it out or simulation in problem-1. He used calculation sticks to create the triangle for the problem. On the other hand, he focused on the frame's surface of the triangle

instead of the inside area. He covered three edges with 10 calculation sticks but he did not pay attention to the row of rectangles.

Figure 4. 14

Part of a Successful Student's (S5) Work on Problem-1 Showing

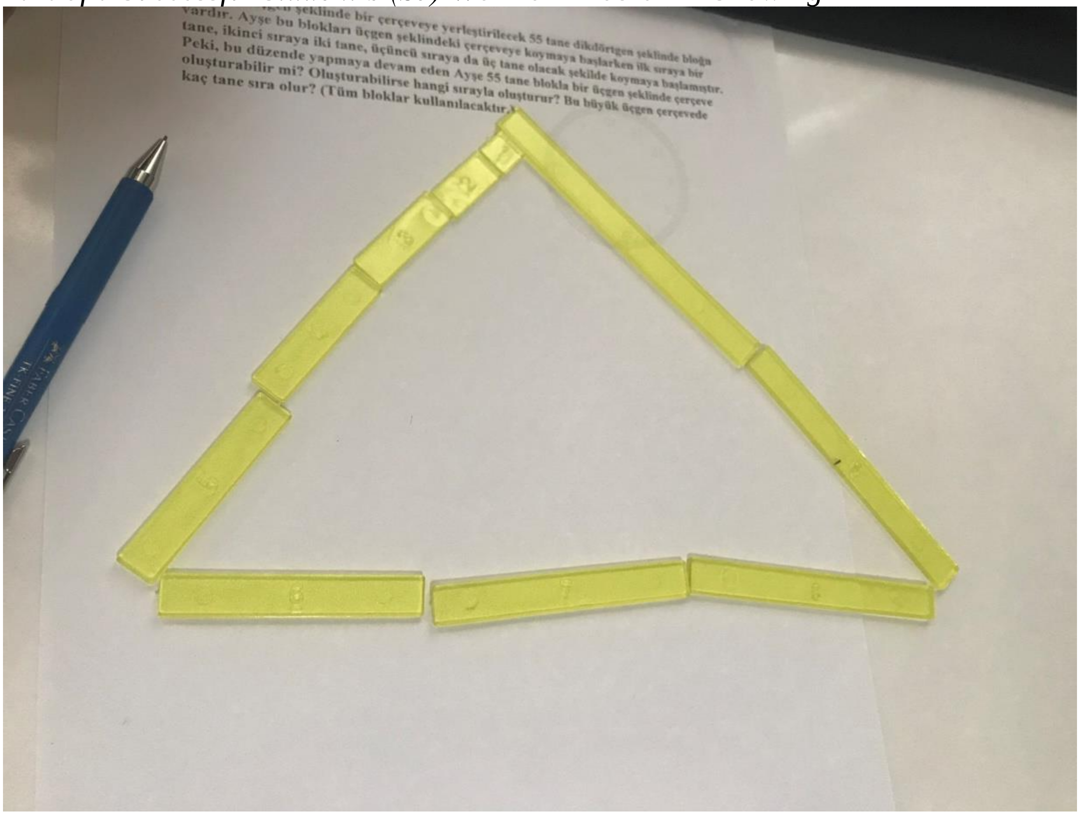


Figure 4.15 provides an average student's (A2) use of acting it out or simulation strategy in problem-1. He used calculation sticks to make a triangle. On the other hand, he mostly paid attention to the triangle's surface. He covered three edges with calculation sticks but he did not pay attention to the row of rectangles like S5.

Figure 4. 15

Part of an Average Student's (A2) Work on Problem-1 Showing

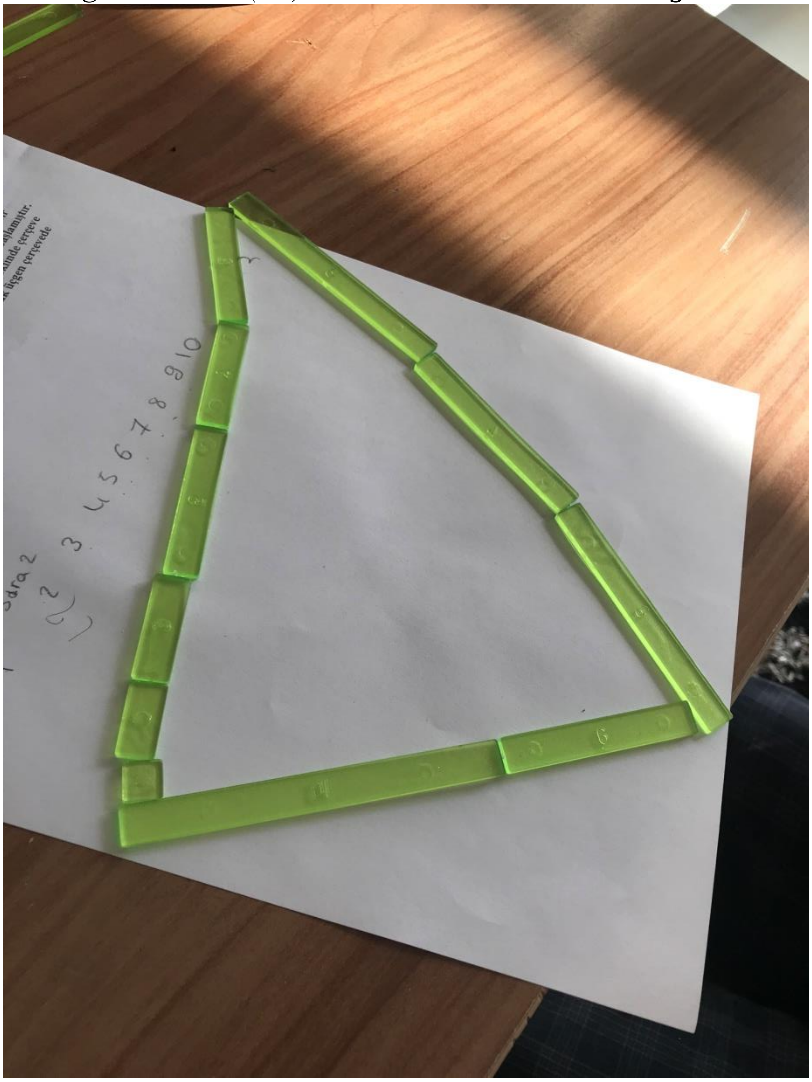
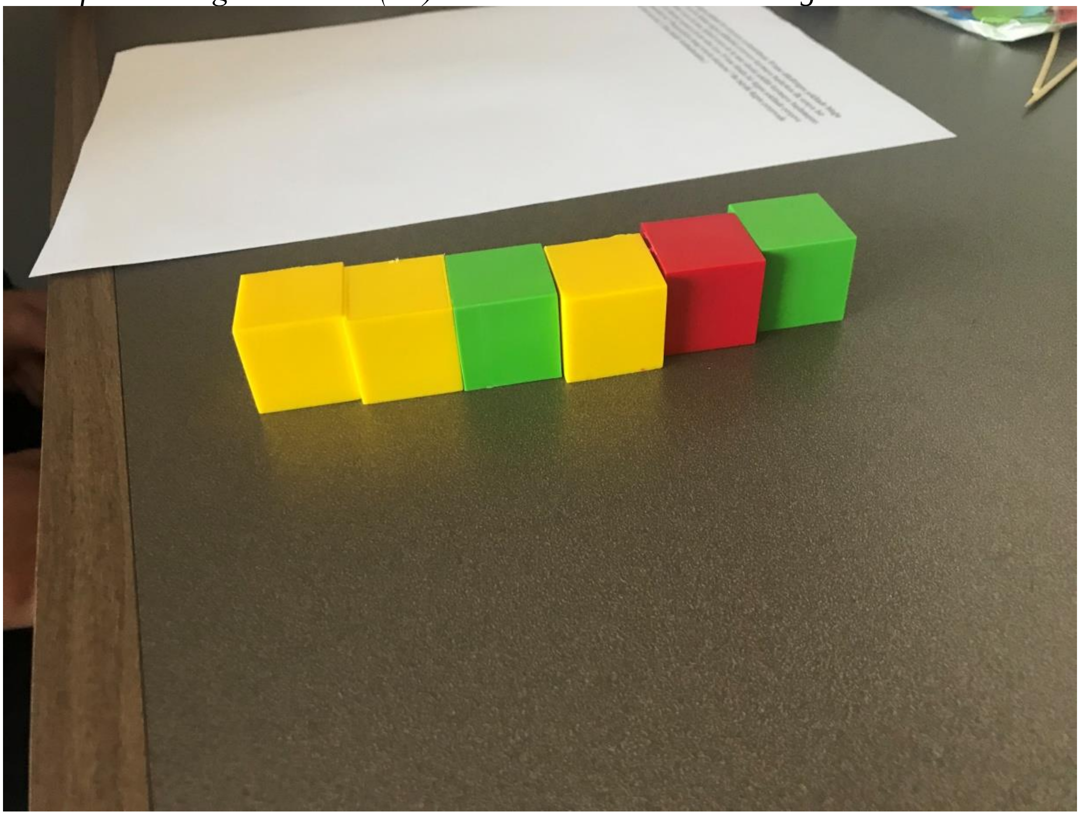


Figure 4.16 provides an average student's (A6) use of the strategy of acting it out or simulation in problem-1. He used unit cubes to form a triangle. He created the first row, the second row, and the third row. On the other hand, he did not consider whether the shape looked like a triangle or not.

Figure 4. 16

Part of an Average Student's (A6) Work on Problem-1 Showing



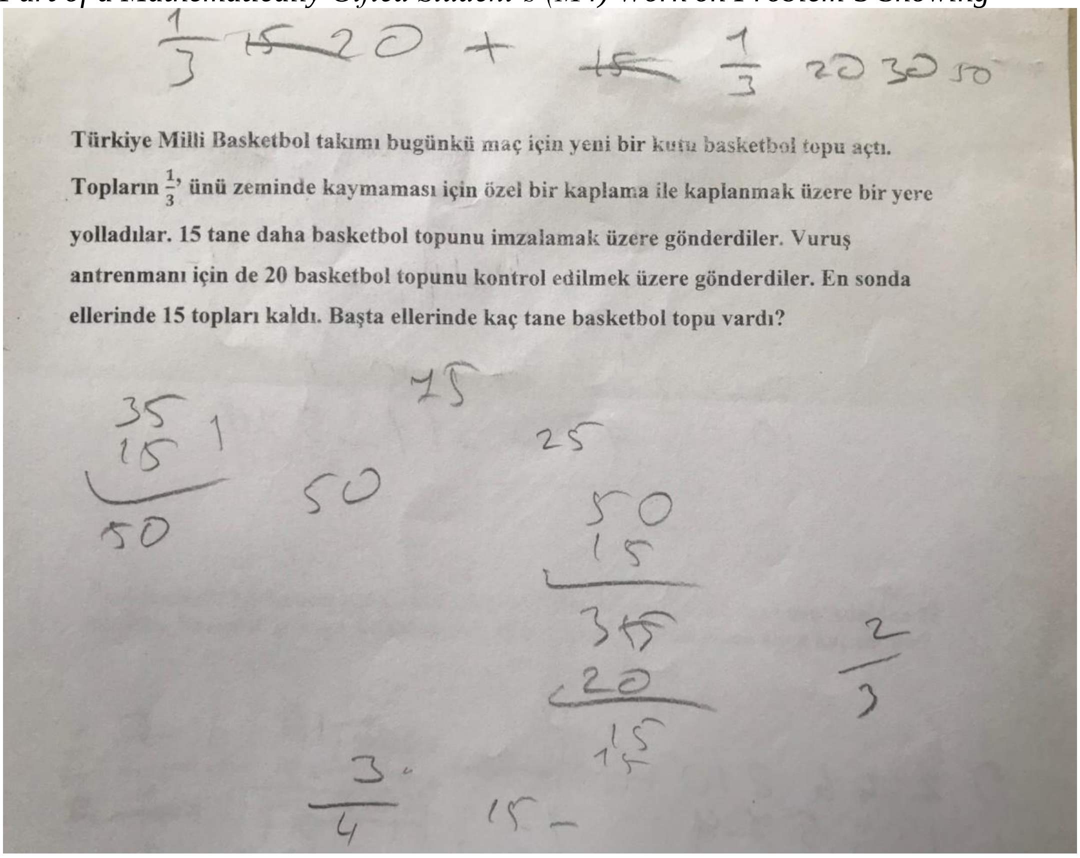
In conclusion, mathematically gifted students presented the triangle with both unit cubes and calculation sticks regarding all the conditions of the problem. Besides, they can differ in using manipulatives while applying the strategy of acting it out or simulation. While M3 preferred to use calculation sticks, M2 used unit cubes to present rectangles in the triangle. Both representation types of acting it out or simulation strategy were proper for problem-1. Before using the cubes, all of them asked me if they could use cubes instead of rectangles. I said that if their answers would satisfy the condition of the problem, they could use it. In the figure of unit cubes, students took the cubes according to the condition that there should be one cube in line 1, two cubes in line 2, three cubes in line 3, and so on. In the figure of calculation sticks, they took a 1 cm stick in the first line, a 2 cm long stick in the second line, a 3 cm long stick in the third line, and so on. M3 thought of the rectangle as rulers and he stated that the rulers can be divided into equal rectangles inside. In this way, they could create another type of triangle that also could satisfy the condition in the problem. That is, mathematically gifted students could make the transition between different materials like from rectangles to cubes and from rectangles to rulers. Secondly, S1 and S5 from the group of successful students tried to use all manipulatives types such as calculation sticks, unit cubes, round counting pieces. S1 constructed her triangle as seen in the figure she did not pay attention to the condition of the problem exactly. She constructed small and big intertwined triangles. The small triangle was suitable concerning the pattern in the problem although the manipulatives were not rectangles. On the other hand, the big triangle could not meet the condition of the problem. It looked like a frame but the small triangle did not fill the big frame. In addition, S5 used calculation sticks to form a triangle. However, he did not consider the inner area and rows. Both successful students did not state where the first place, second place, or another place are in the triangle. Concerning average students, A2 worked like S5. He just put the sticks by trying to attach them. Besides, in A6's work, there was no condition satisfying the problem. A6 used the cubes to form a triangle but he did not provide the shape of it. He just considered the pattern (1 rectangle - 2 rectangles - 3 rectangles and so on.). Therefore, both successful and average students did not use the strategy of acting it out or simulation as well as mathematically gifted students. When Table 4.2 was considered, successful students had more effective attempts than average students to use the strategy of acting it out or simulation. There was no student on average participants used the acting it out or simulation strategy effectively. Both groups of successful and average students did not apply all the conditions to the strategy like their mathematically gifted peers.

4.2.3.5 Working Backwards

The frequency of participants' use of working backwards increased in problem-3. Six mathematically gifted, five successful, and five average students used the strategy of working backwards for problem-3. Figure 4.17 presents a mathematically gifted student's (M4) work on problem-3. At first, he added 15, 15, and 25 to each other. That is, he began to work backward. Then, he found the rest of the ball as $\frac{2}{3}$ to make it equal to 50. Finally, he reached the answer which was equal to 75.

Figure 4. 17

Part of a Mathematically Gifted Student's (M4) Work on Problem-3 Showing



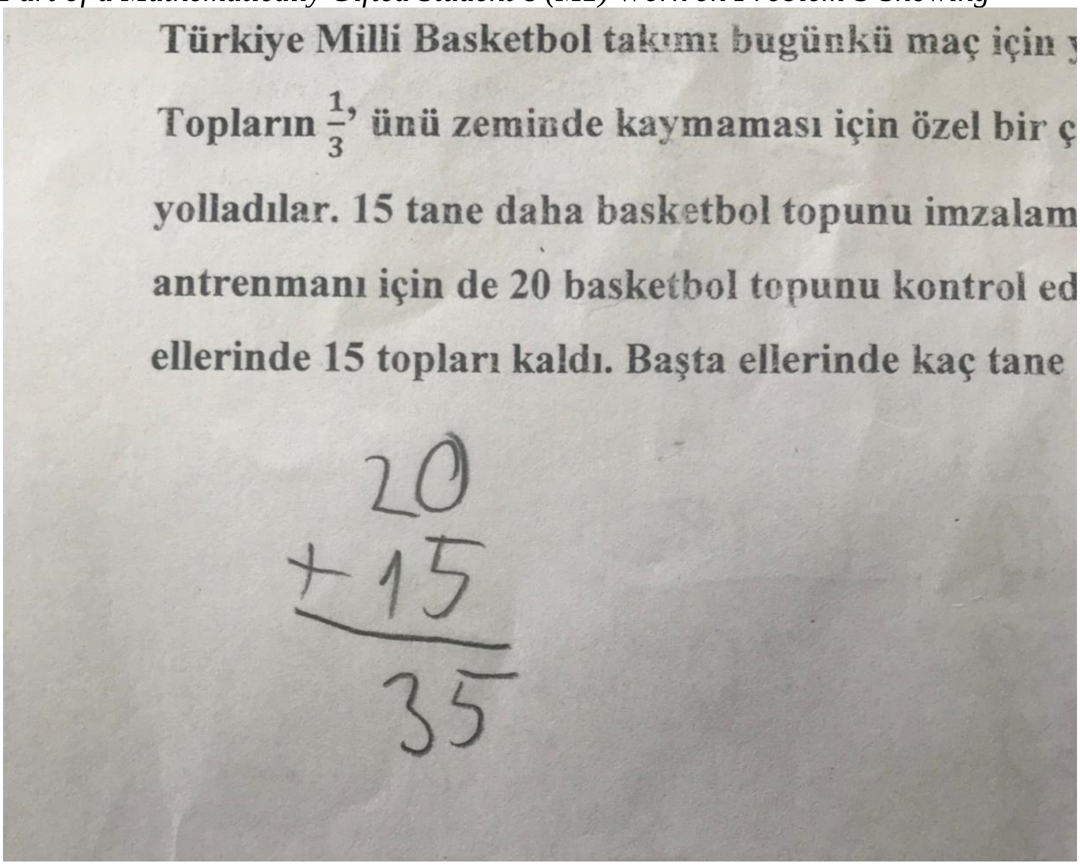
A conversation between the researcher and mathematically gifted student (M4) is:

> M4: I worked backward. So, the answer is 75.
> R: Can you explain how you reached it?
> M4: If $\frac{1}{3}$ of balls went, the rest of balls equal to $\frac{2}{3}$. That is, 15, 20, and 15 balls are equal to $\frac{2}{3}$ of total balls. Therefore, 75 is equal to the amount of the total ball (M4, Problem-3).

Figure 4.18 presents a mathematically gifted student's (M1) solution on problem-3 by using the strategy of working backwards strategy. First of all, she summed 20 and 15. On other hand, she did not prefer too much to make an operation on paper.

Figure 4. 18

Part of a Mathematically Gifted Student's (M1) Work on Problem-3 Showing



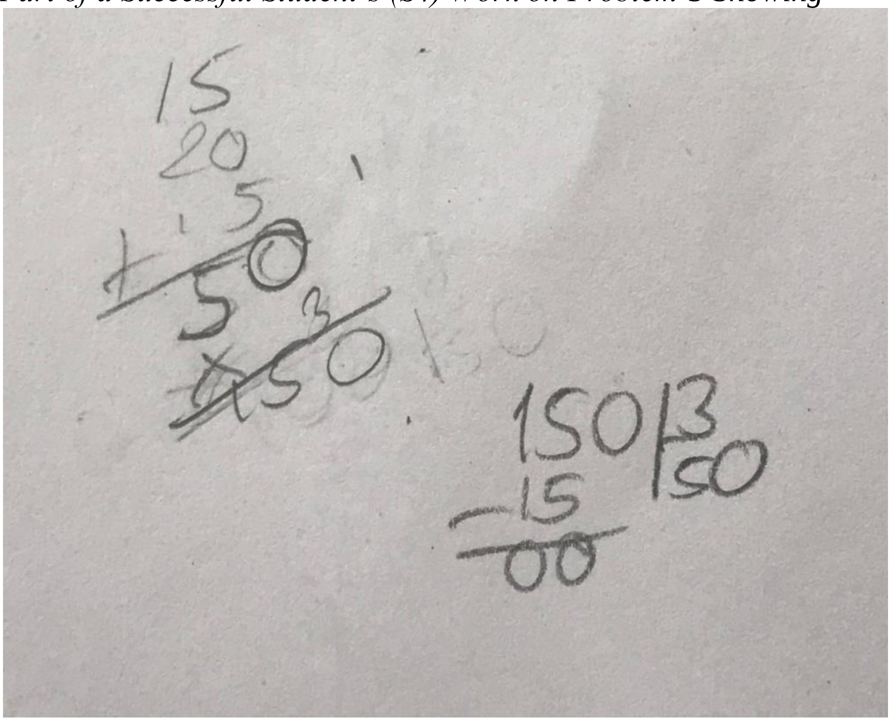
A conversation between the researcher and mathematically gifted students (M1) is:

M1: The answer is 150.
R: Can you explain how you found?
M1: Firstly, I summed up 15, 20, and 25. I find 50. Then, I multiply it by 3 to find total balls (M1, Problem-3).

Figure 4.19 provides a successful student's (S4) work when applying the strategy of working backwards. First of all, she summed 15, 20, and 15 and found 50. Then, she multiplied 50 with 3 since she thought that $\frac{1}{3}$ of the ball was equal to 50.

Figure 4.19

Part of a Successful Student's (S4) Work on Problem-3 Showing



Similar to M1, a successful student (S4) stated that:

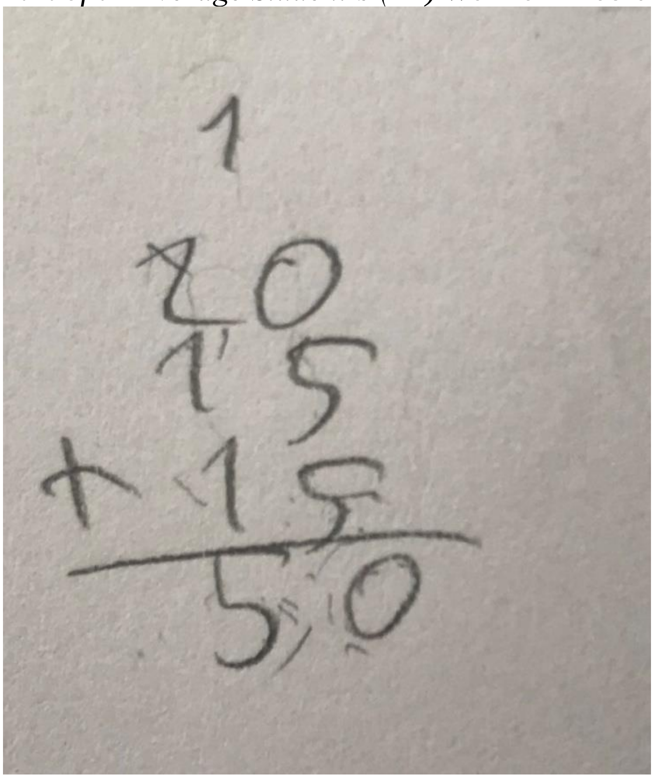
It can be 100.
[After a while, she changed her answer.]

> I found a different answer. Firstly, I summed up 15, 20, and 15, then I found 50. Then, I multiplied it by 3. The result should equal to 150. Even, I checked the result. If I divided 150 to 3, the answer would be 50. Therefore, I am sure about the result (S4, Problem-3).

Figure 4.20 presents the work of an average student's (A1) use the problem-solving strategy of working backwards in problem-3. He summed 15, 20, and 15. On the other hand, he was not sure about it. After that, he did not make any other operation and reached the result of 50.

Figure 4.20

Part of an Average Student's (A1) Work on Problem-3 Showing



A conversation between the researcher and an average student (A1) is:

> A1: I want to add 15, 20, and 15. Is this true?
> R: I do not know. It depends on you.

> A1: Okay, I want to continue. I add them and find 50. The result is 50.
> R: Why?
> A1: Because the question asked the number of total balls. Therefore, I add all the numbers (A1, Problem-3).

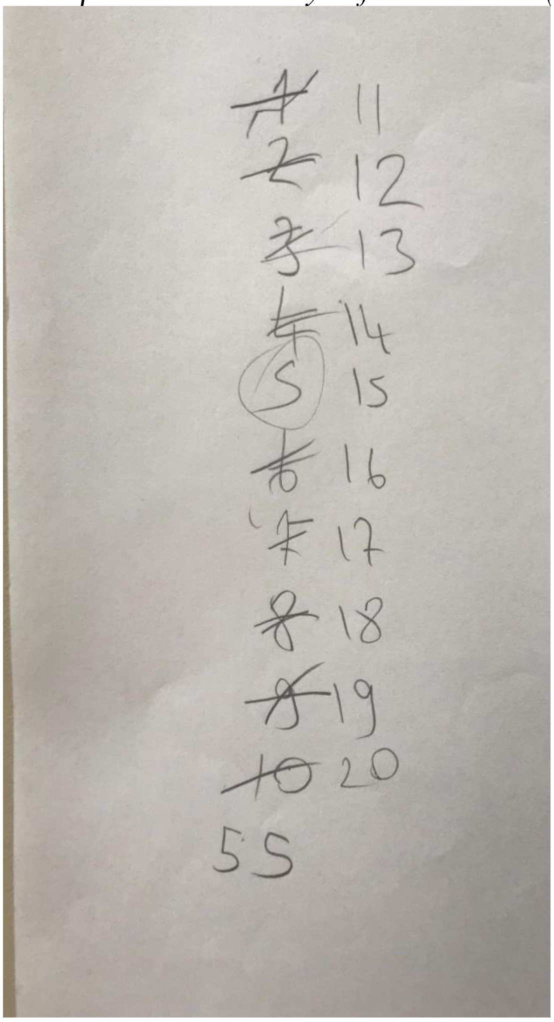
In conclusion, all the participants above worked backwards. The mathematically gifted student (M4) began to solve the problem by adding 15, 20, and 15. Then, he multiplied it by 3 and divided it 2 since $\frac{1}{3}$ of total balls were gone. The rest of the ball was equal to $\frac{2}{3}$ of the total ball. As a result, he reached the correct answer of 75. On the other hand, another mathematically gifted student (M1) found its result as 150 since she only multiplied 50 with 3. She thought $\frac{1}{3}$ as the rest of the balls. Therefore, she did not reach the correct answer. Successful student (S4) also approached the problem like M1 and she found 150 in the same way. Besides, the average student (A1) worked backward like their peers but he could not concentrate logically on the rest of the ball, relationships, and total ball. In total, mathematically gifted students had a priority to use the strategy of working backwards in an efficient way. Successful students tried to use the strategy effectively but not as much as mathematically gifted peers. Average students were the most unsuccessful ones concerning the effective use of the strategy of working backwards.

4.2.3.6 Finding a Pattern

The three groups' use of finding a pattern strategy was mainly observed in problem-1. Five mathematically gifted, five successful, and two average students used finding a pattern for problem-1. Figure 4.21 presents the work of a mathematically gifted student (M7) when using the strategy of finding a pattern in problem-1. Initially, she found a pattern between the numbers of rows like 1-2-3-4-5 and so on. Then, she summed all rows according to the pattern until reaching 55. Then, she found that there should be 10 rows.

Figure 4. 21

Part of a Mathematically Gifted Students' (M7) Work on Problem-1 Showing



M7 also stated that:

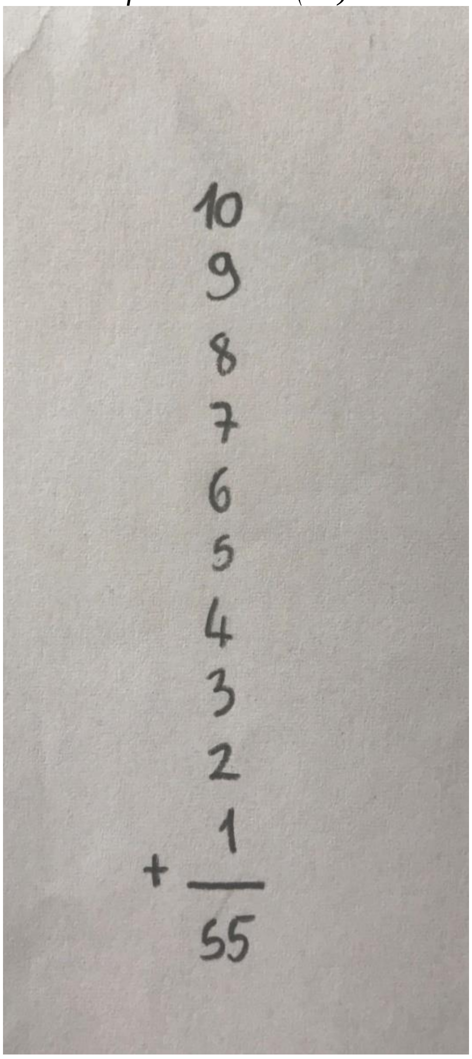
> The problem said that in the first place, there is a rectangle; in the second place, there are two rectangles and in the third place, there are three rectangles. In other words, there is a pattern that when the number of ordinal numbers increases, the number of rectangles increases in the same proportion. Therefore, I can find all the numbers according to this pattern until reaching 55 rectangles in total (M7, Problem-1).

Figure 4.22 presents the work of a successful student's (S5) work on problem-1 while using the problem-solving strategy of finding a pattern. He found a pattern as

1-2-3-4-5, and so on like M7. He summed all the rows until he reached 55 in the total number of rectangles.

Figure 4. 22

Part of a Successful Students' (S5) Work on Problem-1 Showing



In problem-1, an average student (A1) did not write his work on the paper. He thought the total number was the number of rows in the triangle. He also stated: "if there is a rectangle at first, two rectangles in second, three rectangles in third, this pattern should go to 55th order. Therefore, I have to count to 55" (A1, Problem-1).

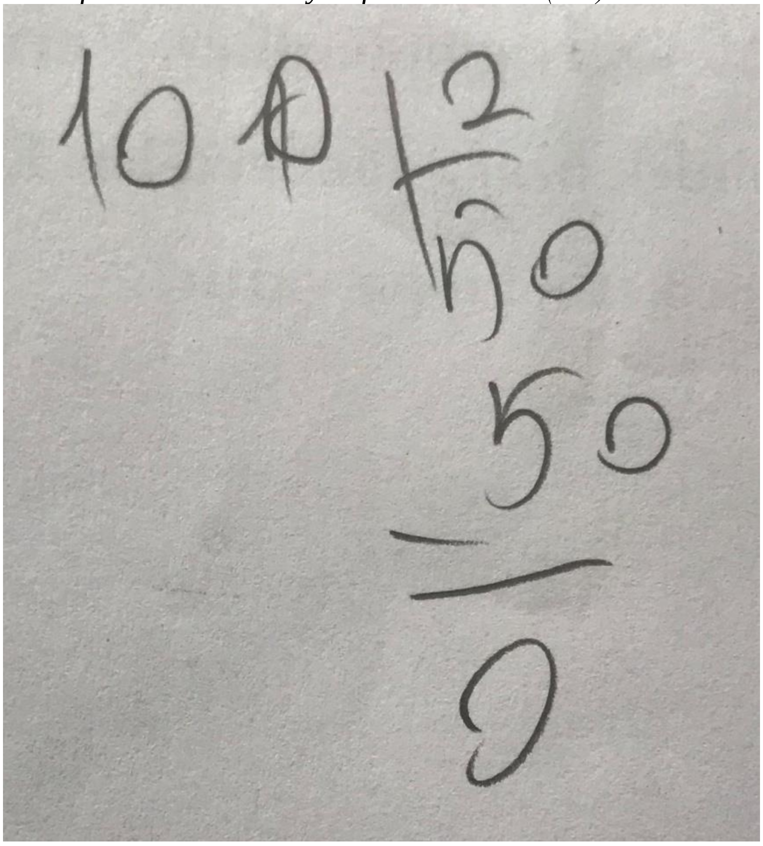
All in all, as seen in M7 and S5's works, mathematically gifted and successful students were able to comprehend the pattern and apply the strategy correctly in equal numbers (see Table 4.2). On the other hand, an average student (A1) misunderstood the pattern in the problem. He perceived 55 as the last number in the pattern instead of the total number of rectangles. Therefore, he could not use the strategy of finding a pattern effectively.

4.2.3.7 Logical Reasoning

Logical reasoning was one of the most preferred strategies among the three groups, especially for successful and average ones. It was used for all the problems with many attempts. The frequency increased in problem 2, 4, 5, and 6. In problem-4, five mathematically gifted, three successful, and three average students attempted to solve it with the strategy of logical reasoning as well. Figure 4.23 provides a mathematically gifted student's (M2) use of logical reasoning strategy in problem-4. Firstly, he thought that there were so many numbers. Therefore, he tried to find a shortcut. Then, he concentrated on the number of all the odd and even integers. He thought that there were 100 numbers in total and odd and even numbers were two groups. Then, he divided 100 into 2.

Figure 4.23

Part of a Mathematically Gifted Student's (M2) work on Problem-4 Showing

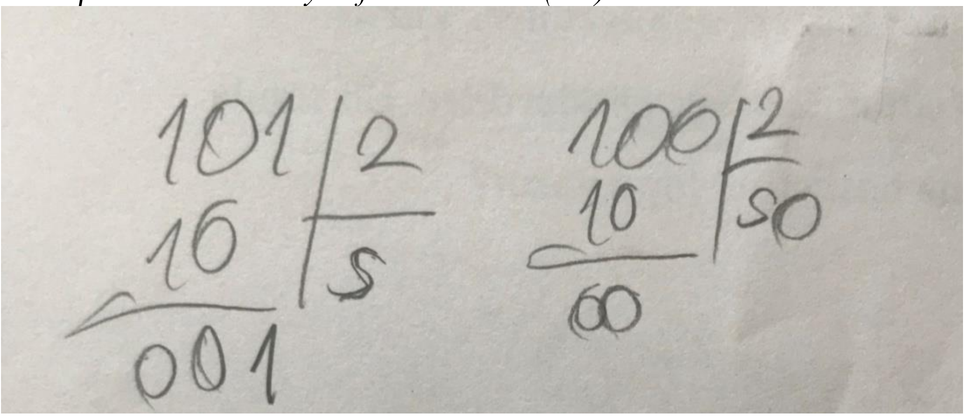


He also stated: "There must be 50 odd and 50 even numbers as two groups less than 101. Therefore, 101 should be divided to 2. The answer must be 0" (M2, Problem-4).

Figure 4.24 provides a mathematically gifted student's (M5) work of logical reasoning strategy use in problem-4. Firstly, she found the total numbers in 101. Then, she tried to concentrate specifically on odd and even numbers. In the first 50 numbers, she found total even and odd numbers. After that, she generalized the results to the whole problem.

Figure 4.24

Part of a Mathematically Gifted Student’s (M5) Work on *Problem-4 Showing*



She also stated that:

> The number of even and odd integers less than 101 should be equal. I should find how many even numbers are in the first 50 integers. There must be 26 even numbers including 0; so, there should be 24 odd numbers. Then, in the rest of the other group (second 50’s group), similarly, there should be 26 even and 24 odd numbers too. Therefore, there are 52 even numbers and 48 odd numbers. Oh, but I have to include 101. So, there are 49 odd numbers. The answer is 3 (52-49) (M5, Problem-4).

In problem-4, a successful student (S2) used logical reasoning. At first, he tried to sum all numbers but he hesitated to make this decision. Then, he changed his decision because the operation would be so time-consuming. Therefore, he concentrated on the number of integers in total. A conversation between the researcher and a successful student (S2) stated that:

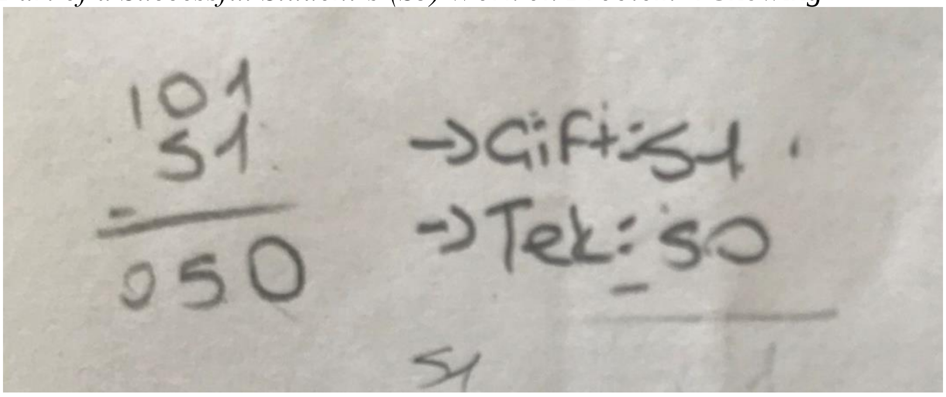
> S2: Do I have to write all the numbers one by one?
> R: You can solve however you want.
> S2: Hmmm. Maybe, I can multiply, but I am not sure.
> S2: May I include “101”?
> R: What problem says?
> S2: I am not sure but less than 101 means that I should not include 101.

> S2: I can divide 100 to 2, and then there must be 50 even and odd numbers. On the other hand, the sum of all numbers is not equal to 50 (S2, Problem-4).

Figure 4.25 presents the work of a successful student (S3) while using the strategy of logical reasoning. She concentrated on total numbers instead of the sum of odd and even integers and their subtractions.

Figure 4.25

Part of a Successful Student's (S3) Work on Problem-4 Showing

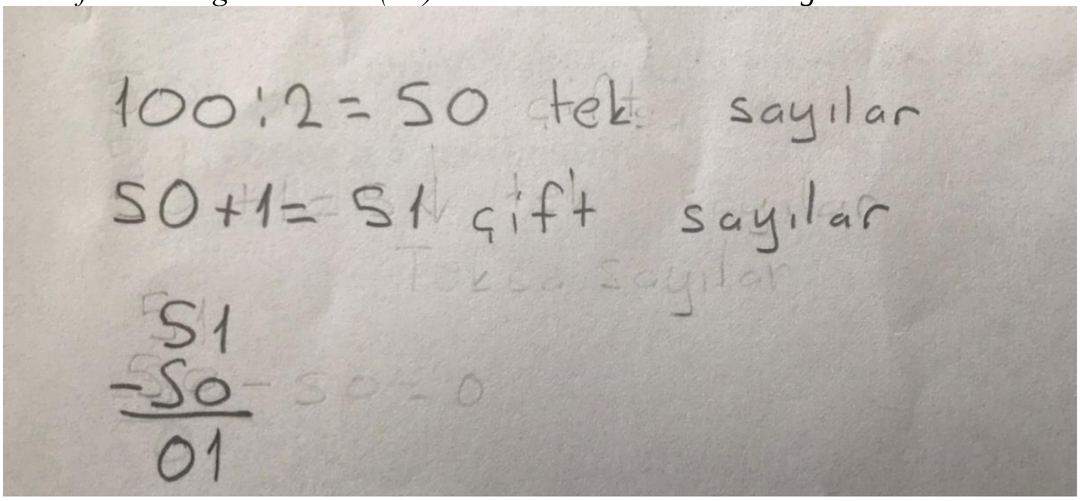


S3 also stated: "There are 51 even numbers with 0 and 50 odd numbers. Therefore, the answer must be 51-50=1" (S3, Problem-4).

Figure 4.26 presents an average student's (A4) use of logical reasoning strategy on problem-4. She just found the total number of even and odd integers in 101.

Figure 4.26

Part of an Average Student's (A4) Work on Problem-4 Showing



A conversation between the researcher and an average student (A4) stated that:

> A4: I am not sure about including 101. Is 101 less than 101?
> R: What do you think?
> A4: Hmm, firstly I include 101. There are 50 even and 50 odd numbers. The answer is 0.
> **[After a while, she changed her result.]**
> A4: I forget to take 0 to 50 even numbers. Now, there are 51 even and 50 odd numbers. Therefore, the answer is 1 (A4, Problem-4).

In conclusion, for problem 4, two mathematically gifted students (M2 and M5) could make inferences about even and odd numbers. On the other hand, they only concentrated on the number of integers instead of the sum of them which is asked in the problem. Similarly, S2 and S3 behaved like their mathematically gifted peers. Also, S2 was confused about taking 101 or not. It was observed in average students too. A4 was not sure about 101. He also only concentrated on the amount of numbers. From this perspective, three groups of participants showed similar mistakes using the strategy of logical reasoning.

In problem-6, four mathematically gifted, six successful, and five average students attempted to solve it with the logical reasoning strategy. A mathematically gifted student (M5) used the strategy of logical reasoning in problem-6. She concentrated on dividing 160 to 3 since she made an inference that if there were three people, total heights should be divided by 3. She also evaluated the height differences between people logically. She also stated in problem-6 that:

> The problem says that there are three friends. Therefore, I divided 480 by 3. Then, I found 160 cm. 160 cm must be a person's height and this person must be in the middle of all friends. In the problem, there must be 10 cm height differences between the three friends. Ada as the longest one must be 170 cm and Duru, as the shortest one, must be 150 cm. Hence, Yasemin, as in the middle place, must be 160 cm. (M5, Problem-6).

A successful student (S1) used logical reasoning in problem-6. She decided to begin with the longest person since she thought that the longest person can show her the height relationships between three people. She also stated that:

> I can solve this problem by concentrating on the longest person. Additionally, I can find the relationships between all the people.
> **[After a while, she continued.]**
> The longest person must be Yasemin. On the other hand, Duru must be the shortest one (S1, Problem-6).

On the other hand, although a successful student (S5) who also used logical reasoning in problem-6 could make the right inferences about the height of people, he did not pay attention to height differences and the total number of heights. He also stated: “I think that Ada must be longest and Duru must be the shortest person in the group but the problem should have given the height of Ada. In that way, the problem may have been so easy to solve” (S5, Problem-6).

An average student (A5) used logical reasoning in problem-6. He made his logical inferences about the height and relationships but he did not pay attention to the total number of heights. He even changed his results and reached the wrong relationships in the problem. He also stated that:

> I divided 480 by 3 since the problem says, three people. If Duru is 20 cm shorter than Ada, she must be 140 cm.
> **[After a while, he recognized that some points were wrong. He looked at his solution again.]**
> I think Duru must be 130 cm. A short time ago, I found 160 cm (480 : 3). Therefore, I added 20 cm to Ada and subtracted 20 cm from Duru. I added 10 cm to Yasemin because she is longer than 10 cm more. As a result, Yasemin must be 170 cm (A5, Problem-6).

As a result, in problem-6, the mathematically gifted student (M5) could comprehend the main points of problems and make inferences about the solutions. Not only she could pay attention to where she must begin, but also she generated logical conclusions about the data like equal height differences among people in the problem. In a controlled manner, she was able to reach all heights of people and could explain what she found at every step in the problem. A successful student (S1) could decide on where she should begin the problem. On the other hand, she did not reach the correct conclusion. Similarly, S7 could make logical thinking but she did not continue the next steps. S5 wanted the problem to give him more information about the data although he could make logical deductions about the data. Lastly, an average student (A5) was not able to make logical inferences about the longest and shortest person like their mathematically gifted and successful peers were able to make exact logical inferences. He made some operations wrong since he was not able to comprehend relationships the first time.

All in all, although all the three groups of participants had similar mistakes in problem-4 concerning the use of logical reasoning strategy, in problem-6, mathematically gifted students used the strategy of logical reasoning more effectively than successful and average students in total (see Table 4.2). Average students used the logical reasoning strategy more effectively than successful students in total. On the other hand, the effectiveness of mathematically gifted students' use of logical reasoning strategy differed regarding the type of the problem.

4.2.3.8 Making a Drawing

The use of making a drawing was high in problem-1. In problem-1, five mathematically gifted, three successful, and three average students attempted to solve with the strategy of logical reasoning. Figure 4.27 presents the work of a mathematically gifted student (M1) applying the strategy of making a drawing on problem-1. She drew the rectangles as small rectangles and tried to pay attention to the pattern in the problem. She also drew them as equal to each other. She began to draw from top to bottom.

Figure 4. 27

Part of a Mathematically Gifted Student's (M1) Work on Problem-1 Showing

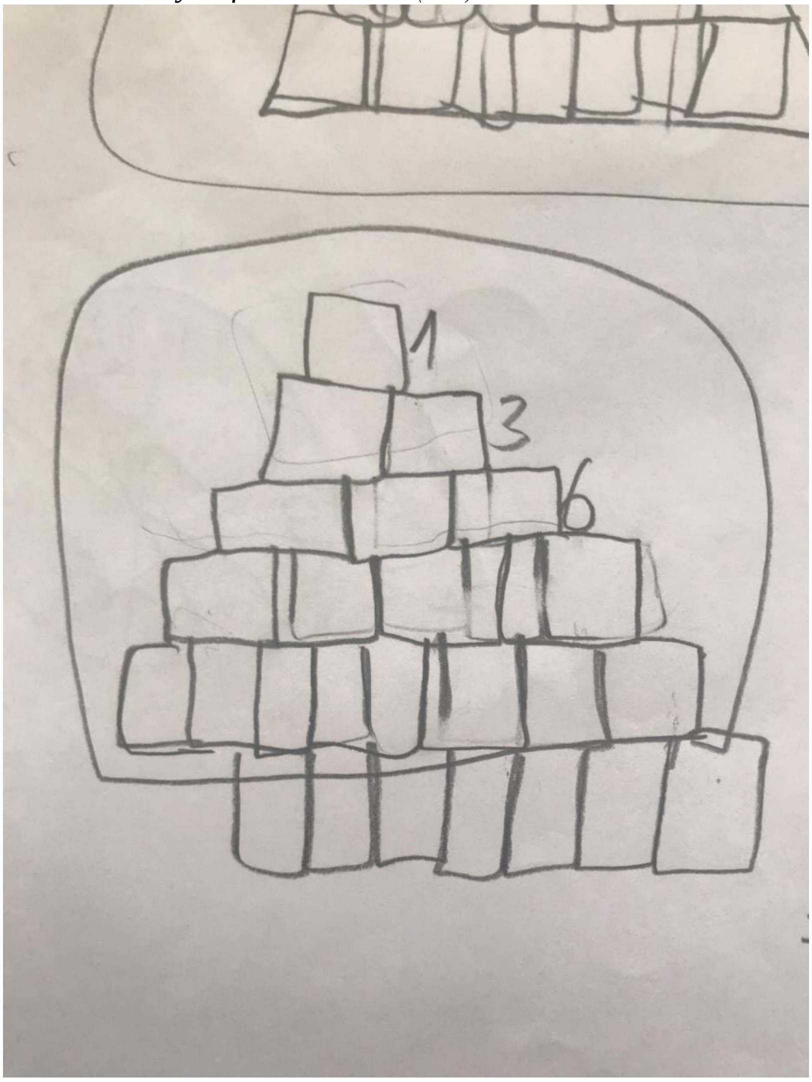


Figure 4.28 provides the work of a mathematically gifted student (M2) using the strategy of making a drawing for problem-1. Firstly, he drew a frame and then drew the rectangles inside. On the other hand, he did not pay attention to drawing rectangles equally. He began to draw from bottom to top.

Figure 4. 28

Part of a Mathematically Gifted Student's (M2) Work on Problem-1 Showing

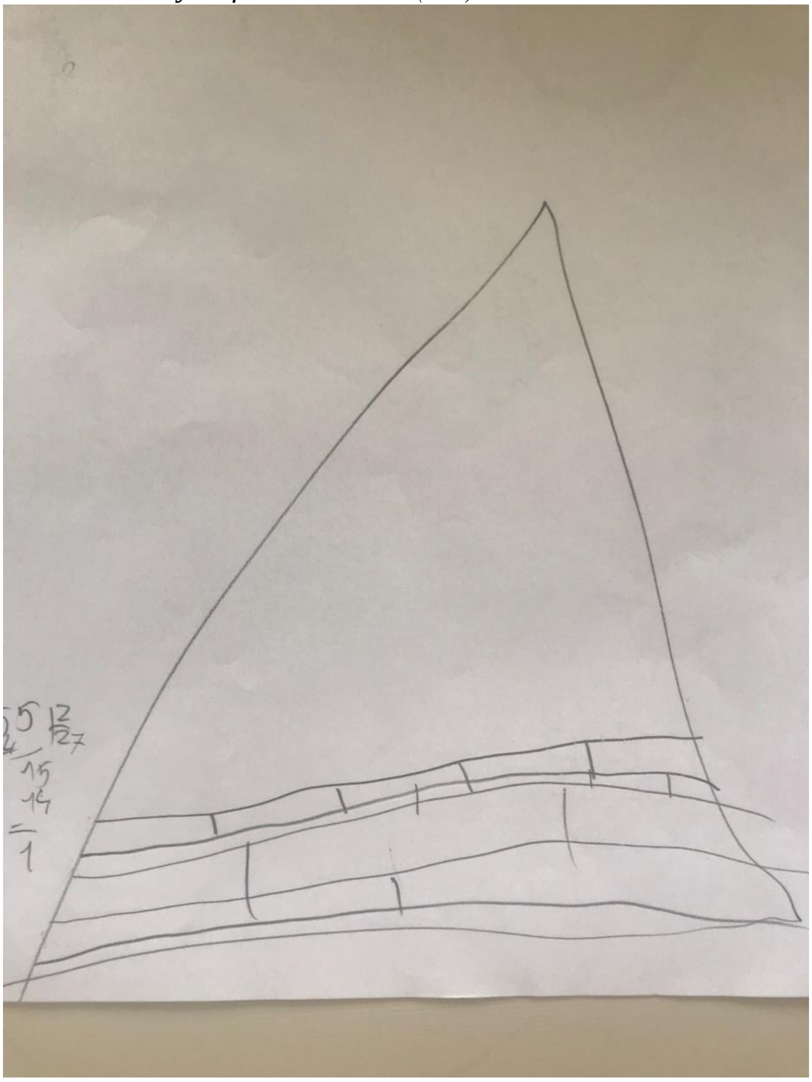


Figure 4.29 presents the work of a mathematically gifted student (M5) while she was using the strategy of making a drawing on problem-1. At first, she constructed the frame of the rectangle with rectangles, and then she drew the rectangles inside according to the frame of the problem. She drew the triangle from top to bottom.

Figure 4.29

Part of a Mathematically Gifted Student's (M5) Work on Problem-1 Showing

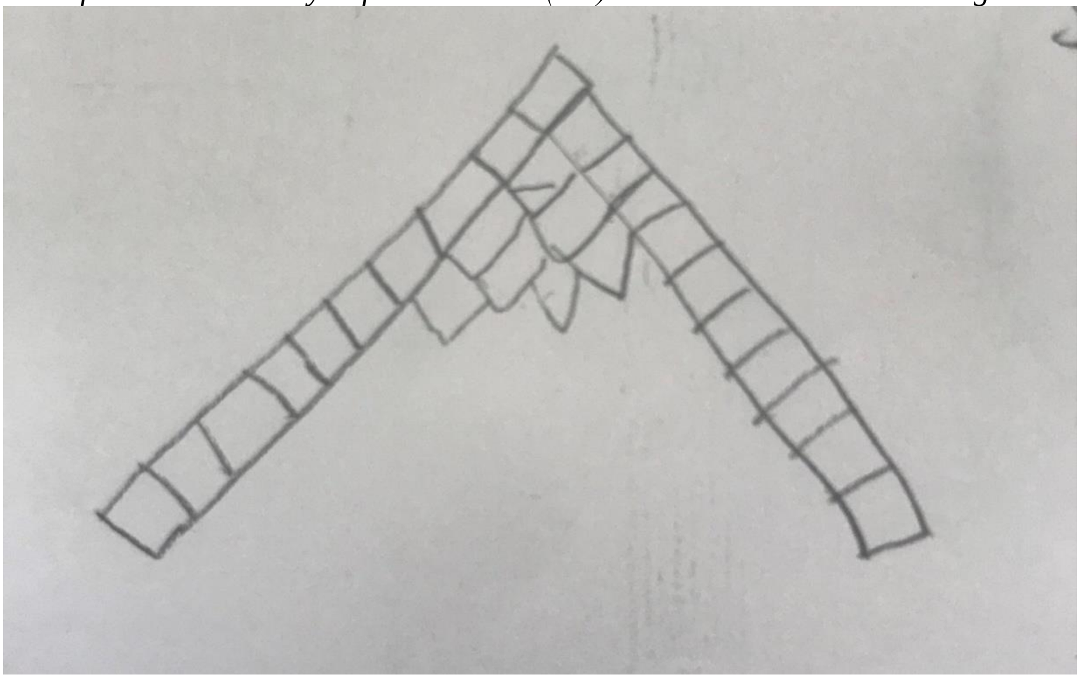


Figure 4.30 presents the work of a successful student (S1) while she was using the strategy of making a drawing on problem-1. At first, she constructed the frame of the rectangle with lines, and then she drew the rectangles inside according to the frame of the problem. On the other hand, she did not consider the equality of rectangles and formed the triangle from top to bottom.

Figure 4.30

Part of a Successful Student's (S1) Work on Problem-1 Showing

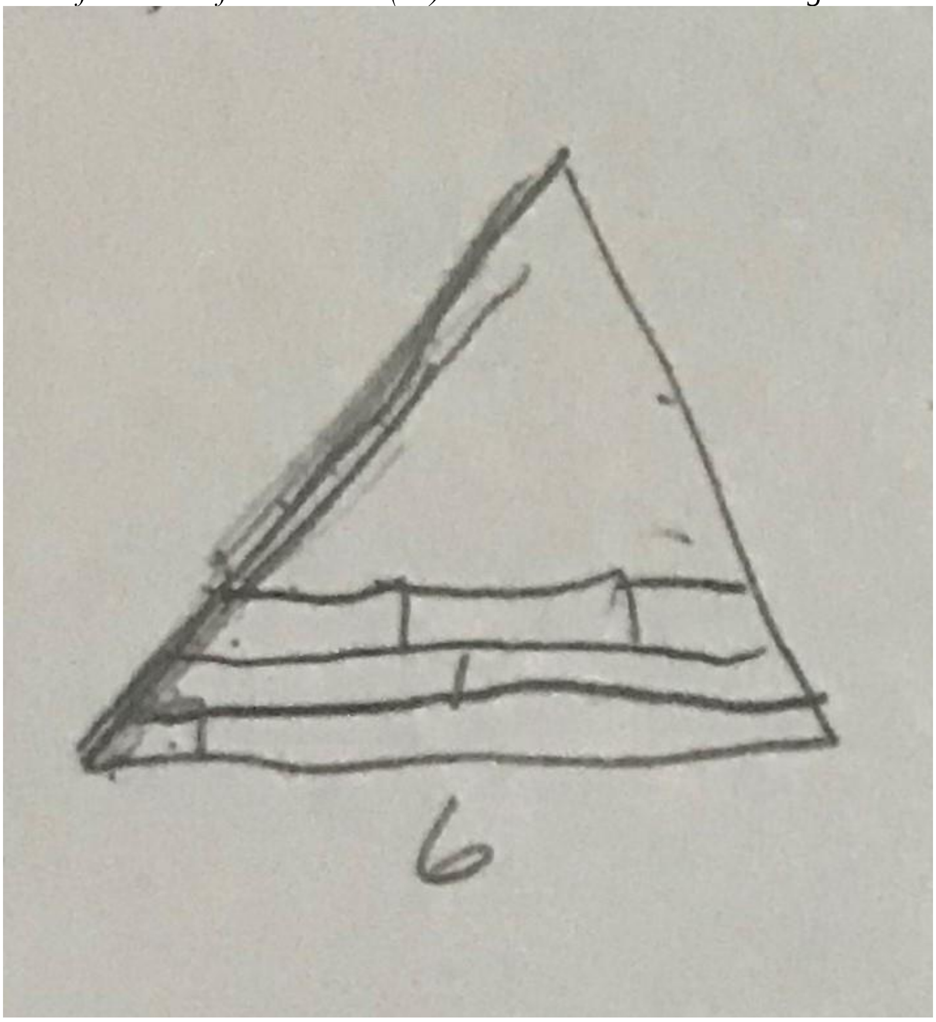


Figure 4.31 presents the work of a successful student (S3) while she was using the strategy of making a drawing on problem-1. At first, she drew a frame with lines, and then she drew the rectangles inside according to the frame of the problem. She drew the triangle from top to bottom and gave the number to each rectangle.

Figure 4.31

Part of a Successful Student's (S3) Work on Problem-1 Showing

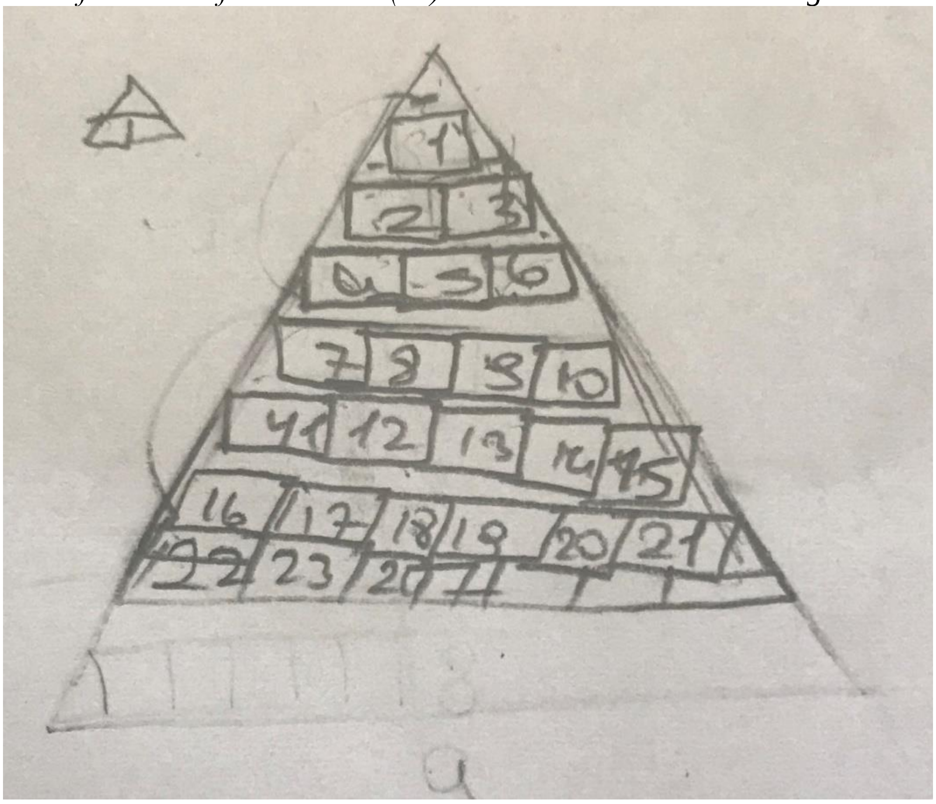


Figure 4.32 presents the work of a successful student (S5) while he was using the strategy of making a drawing in problem-1. First of all, he drew a frame of the triangle with lines, and then he wrote the numbers of each row around the frame of the triangle.

Figure 4.32

Part of a Successful Student's (S5) Work on Problem-1 Showing

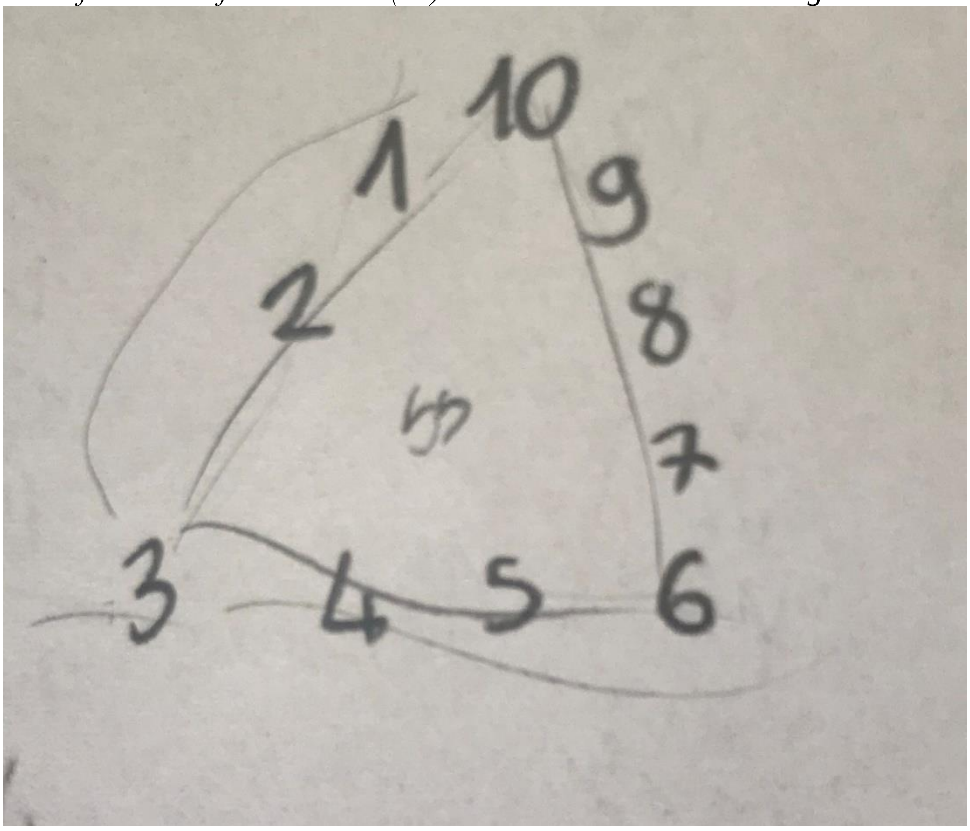


Figure 4.33 provides the work of a successful student (S6) using the strategy of making a drawing on problem-1. Firstly, she drew the frame of the rectangle with lines, and then she drew some rectangles until the 4th row. After that, she realized the pattern, and then she began to draw the rectangles as areas. She also wrote the numbers of all rows on these areas. On the other hand, her first triangle's frame was bigger than the final triangle's frame. Besides, she drew the triangle from top to bottom.

Figure 4.33

Part of a Successful Student's (S6) Work on Problem-1 Showing

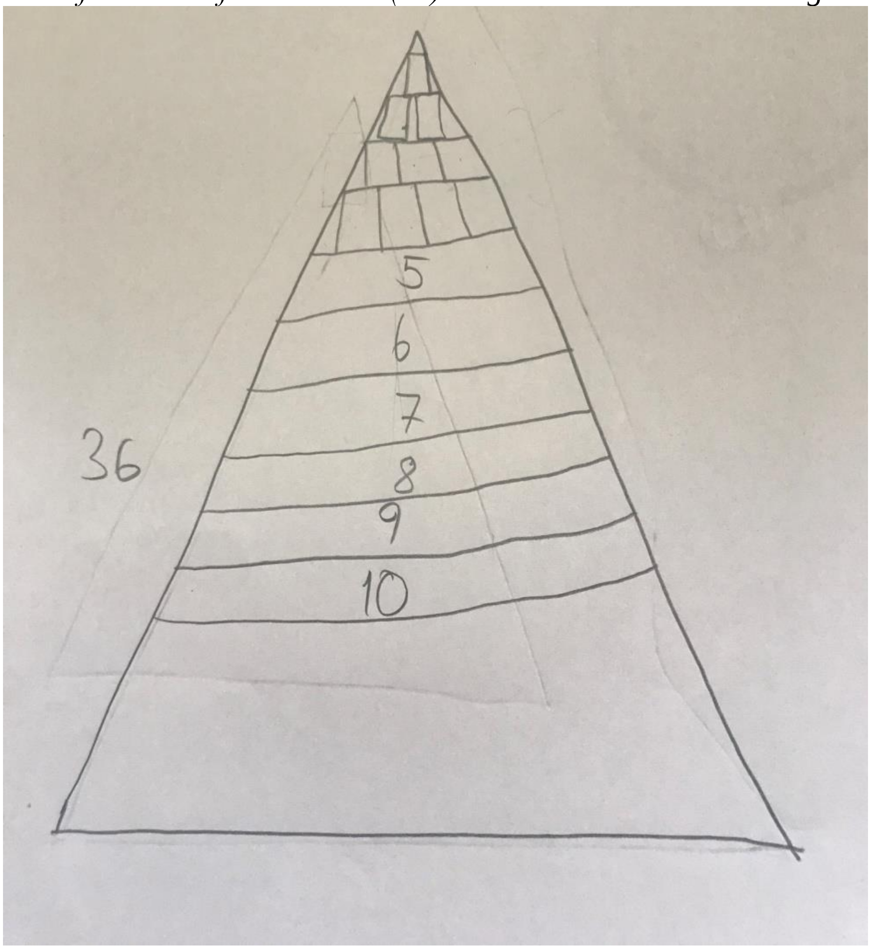
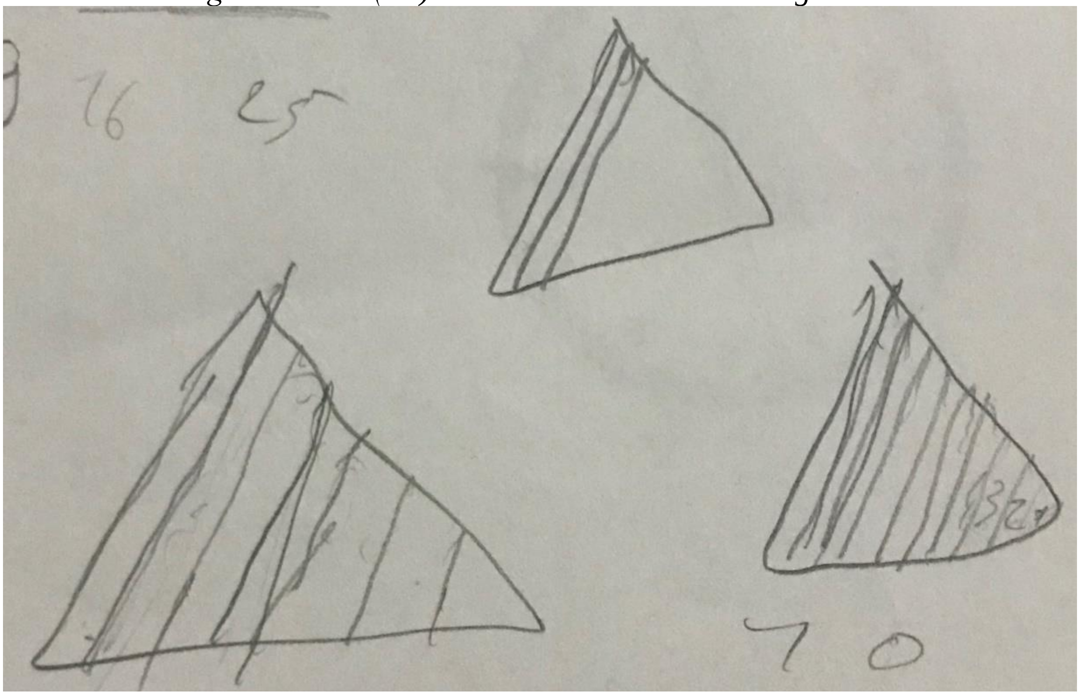


Figure 4.34 presents the work of an average student (A3) while he was using the strategy of making a drawing on problem-1. At first, he constructed three rectangles by drawing their frames with lines, and then he drew the rectangles inside. However, he did not consider the pattern and drew the rectangles as areas.

Figure 4.34

Part of an *Average Student's (A3) Work on Problem-1 Showing*



All in all, in two mathematically gifted students' (M1 and M5) works, the rectangles were placed in a triangle frame without a gap. Also, the shape of all the rectangles was equally drawn. The pattern, expressed in problem-1, was represented as drawing a rectangle in the first step, two rectangles in the second step, three rectangles in the third step, and so on. However, M2 drew the rectangles without paying attention to the equality of rectangles. Along the same line, the successful student (S1) drew the rectangles in a triangle with its pattern. On other hand, she could not pay attention to drawing all the rectangles equally like M2. Besides, S3 and S6 tried to draw the triangle by paying attention to patterns and rectangles. However, S5 mostly concentrated on numbers rather than triangles. Average student's (A3) work showed the same representation. He also drew the rectangles like a line in a triangle. On the contrary to mathematically gifted and successful peers, he could not show the rule/pattern in the problem. When all the works of the groups were considered, mathematically gifted was the first group to implement the strategy efficiently. On the other hand, none of the mathematically gifted groups used this strategy to reach the correct solutions. They used it to continue with another strategy or with a prompt. Although there were similarities between successful and mathematically gifted students' works, successful students mostly preferred to use the problem-solving strategy of making a drawing to reach correct solutions. Average students had the lowest use of the strategy of making a drawing concerning the effectiveness.

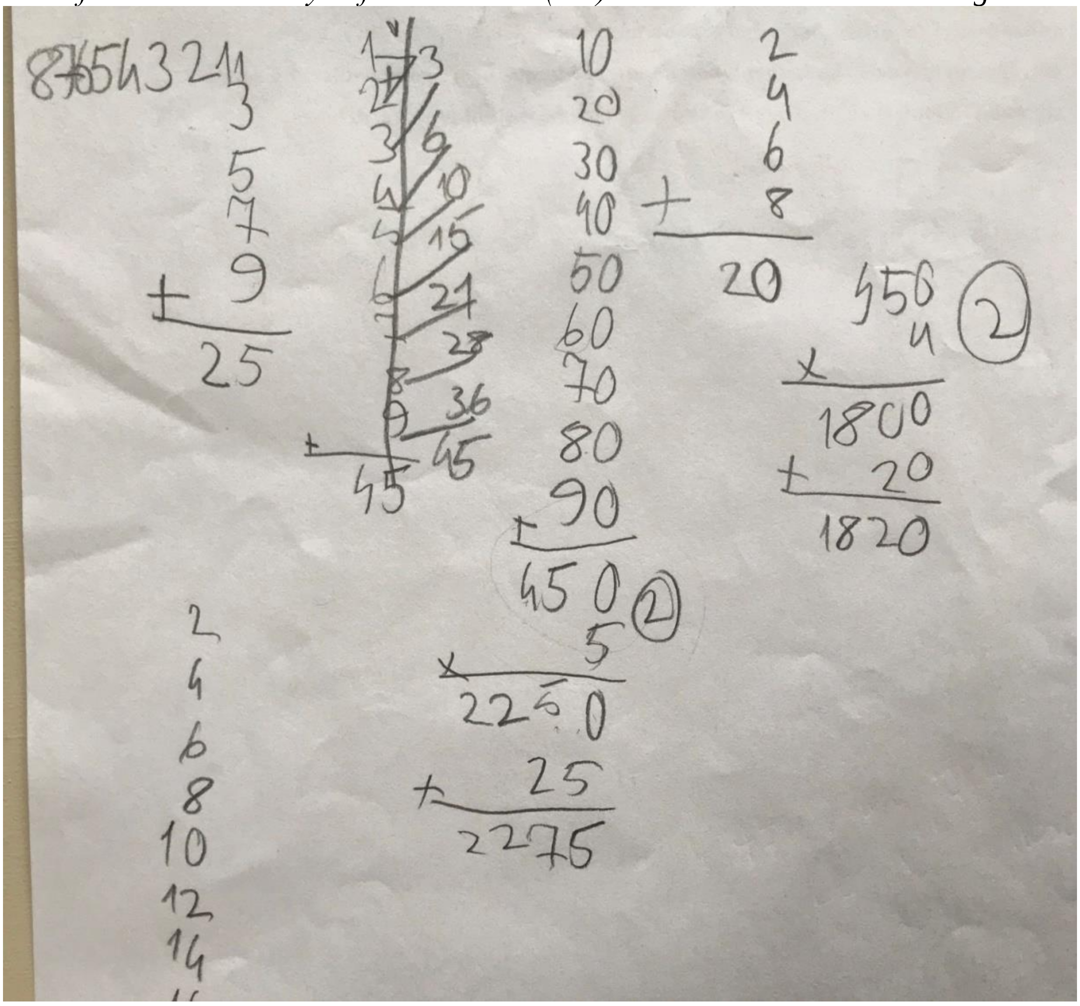
4.2.3.9 Adopting Different Point of View

Adopting a different point of view was one of the most preferred strategies especially among mathematically gifted and average groups. It was mainly preferred by the participants in problems 2, 4, 5, and 6. In problem-4, four mathematically gifted, five successful, and two average students attempted to solve it by the strategy of adopting a different point of view.

Figure 4.35 provides the work of a mathematically gifted student (M1) using the strategy of adopting a different point of view. She separated the numbers as one and two-digit numbers. Then, she also separated them according to odd and even numbers. She tried to make them as groups but operations were so time-consuming and complicated. Therefore, she wanted to continue later.

Figure 4.35

Part of a Mathematically Gifted Student's (M1) Work on Problem-4 Showing



She also stated that:

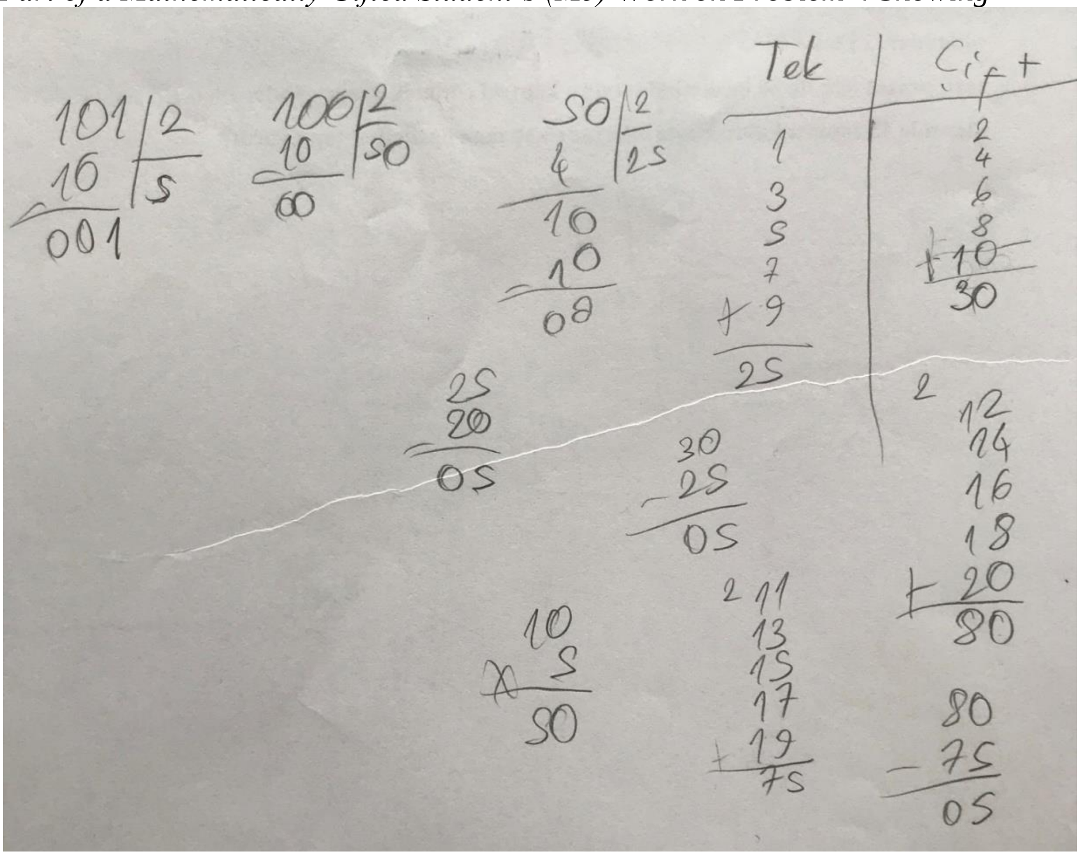
> I grouped the numbers as one-digit and two-digit numbers. Initially, I added all one-digit odd numbers and found 25. Then, I added two digits odd numbers but firstly, I made an operation with 20, 30, 40… 90, and 100 and found 450. Then, I multiplied 450 with 5 and it was equal to 2250. I added 25 to 2250. The sum of all odd numbers was 2275. Secondly, I added all one-digit even numbers and found 20. Then, I added two digits even numbers but firstly, I made an operation with 20, 30, 40… 90, and 100 and

> found 450. Then, I multiplied 450 with 4 since I did not have to take 0. I added 20 to 1820. However, the sum of all the even numbers was very small. I was confused about this result. Therefore, I want to stop here. If we have enough time, I may see this problem later (M1, Problem-4).

Figure 4.36 presents a mathematically gifted student's (M5) use of adopting a different point of view strategy on problem-4. First of all, she found the total numbers. After that, she found every sum of ten numbers. For example, she found the sum of even and odd numbers in ten groups often like 2 + 4+ 6 + 8 + 10 = 30. Then, she found 1 + 3 + 5 + 7 + 9 = 25. When she subtracted 30 from 25, she found 5. She realized that there are groups of ten. So, she multiplied 5 by 10 and found 50.

Figure 4.36

Part of a Mathematically Gifted Student's (M5) Work on Problem-4 Showing



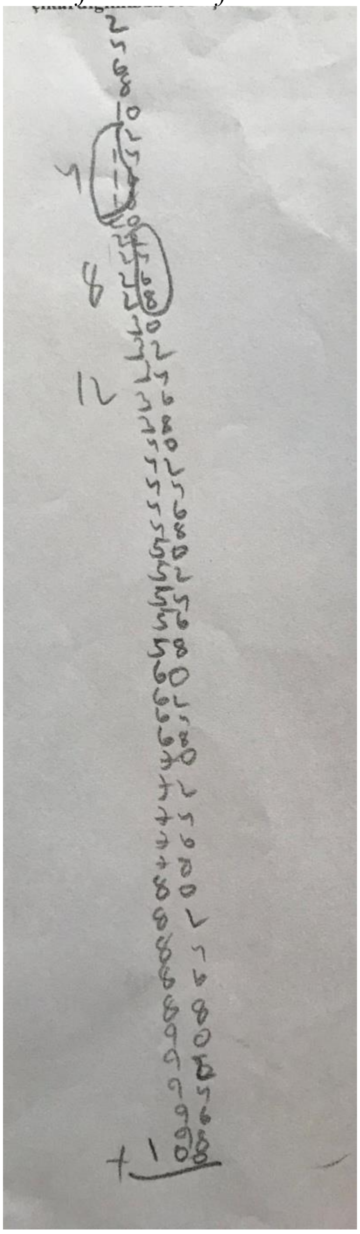
She also stated that:

> Firstly, I found that there are 100 numbers in total. Then, I added the first group of even and odd numbers. Then, I subtracted the odd sum from the even sum. Subtraction of every ten even and odd groups was equal to 5 at every turn. There were 10 groups, so; 5 x10: 50 was the result (M5, Problem-4).

Figure 4.37 provides a successful student's (S2) work implementing the strategy of adopting a different point of view in problem-4. Initially, he concentrated on the sum of even and odd numbers in the first group of ten. He realized that it may take a lot of operations. Hence, he tried to find all the one-digit numbers. Every sum of one-digit numbers was equal to 20. Then, he decided to add the sum of all tens digits of every group of ten. On the other hand, he was confused later.

Figure 4.37

Part of a Successful Student's (S2) Work on Problem-4 *Showing*



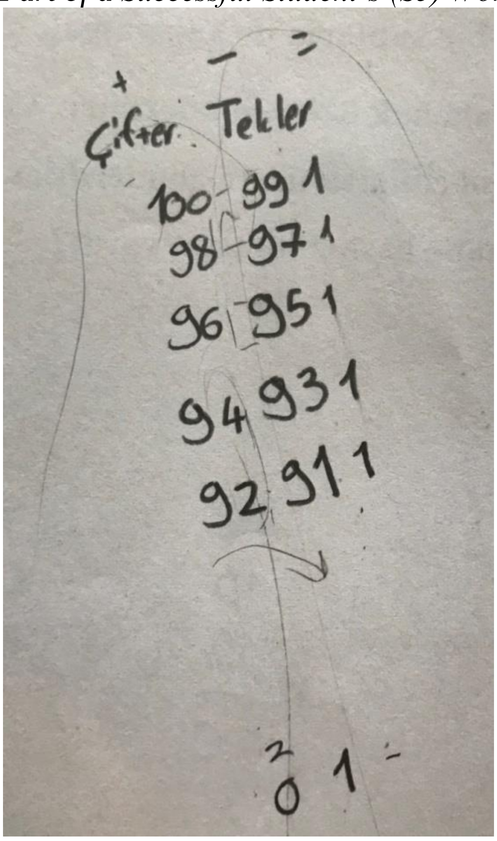
He stated that:

> The sum of the first group of ten even numbers was 20. However, I had to find more operations like that. Then, I added 12,14,16,18. I took ten parts of groups of them. For example, there were 9 of 20. That is 180. I added all numbers like that.
> **[After a while, he said he did not handle it.]**
> If we have time, I want to come back to this problem later (S2, Problem-4).

Figure 4.38 presents a successful student's (S5) work implementing the strategy of adopting a different point of view on problem-4. He subtracted from an odd number from an even number and found "1" every time. On the other hand, he did not comprehend total even and total odd numbers. Therefore, he reached the result "100".

Figure 4.38

Part of a Successful Student's (S5) Work on Problem-4 Showing

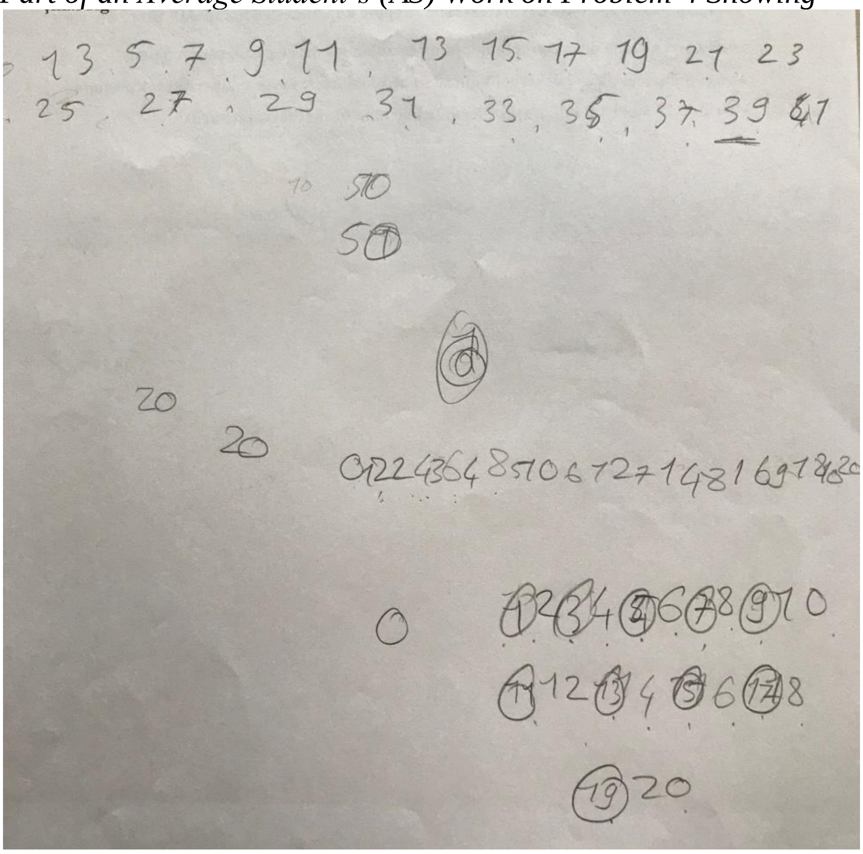


He stated: "I think the answer is 1 because I grouped all even and odd numbers separately. Then, I recognized that every subtraction of one odd and one even number was equal to 1. Therefore, the answer was 100" (S5, Problem-4).

Figure 4.39 provides the work of an average student (A3) using adopting a different point of view in problem-4. He concentrated on the quantity of numbers since he wrote all even and odd numbers until some point. He tried to make their groups according to their quantity. On the other hand, the problem asked subtraction.

Figure 4.39

Part of an Average Student's (A3) Work on Problem-4 Showing



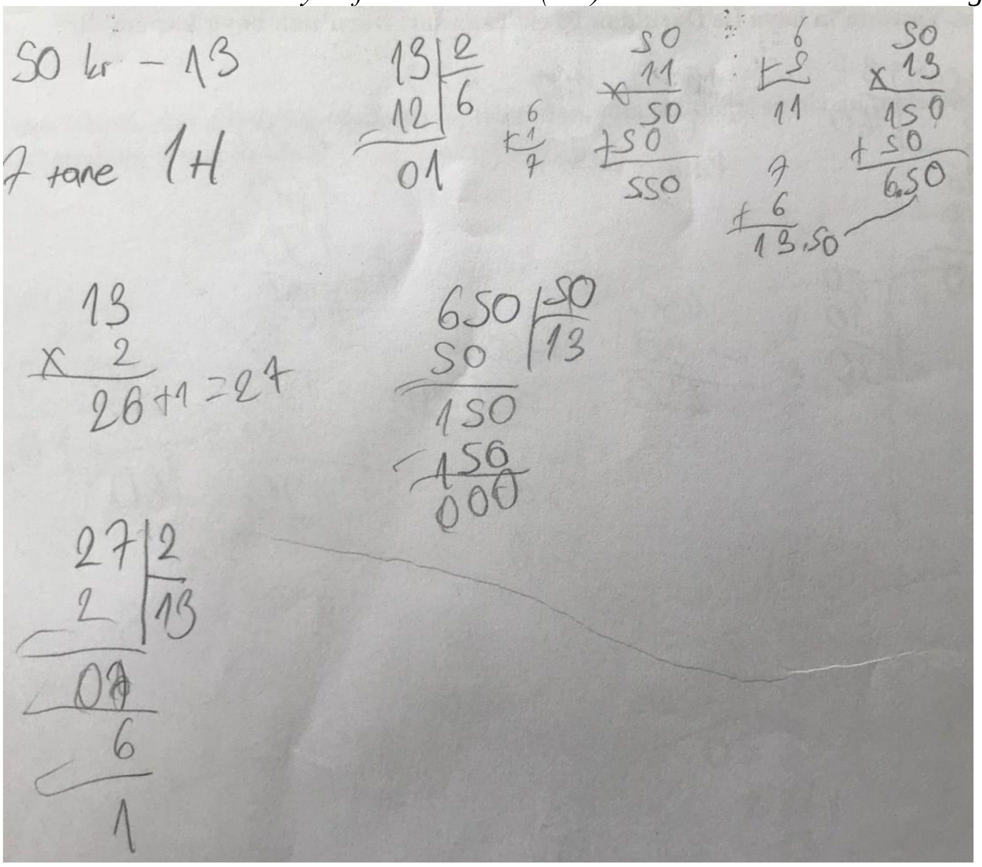
He also stated: “I wrote all even and odd numbers. For example, there were 10 even numbers and 10 odd numbers. If I counted until 50, I found 25 even and 25 odd numbers. Therefore, the answer was equal to 0” (A3, Problem-4).

All in all, mathematically gifted students (M1 and M5) differed from each other concerning using the strategy of adopting a different point of view. Although both mathematically gifted students focused on making the numbers as groups, they applied them differently. M1 grouped all even numbers in group and added them together. On the other hand, she recognized that her works would be so time-consuming. Another mathematically gifted student (M5) also grouped the numbers but she attempted to subtract all groups directly and she reached the right answer. Apart from this, a successful student (S2) tried to group numbers but he could not continue to apply it. Similarly, S5 grouped even and odd numbers. On the other hand, he could not pay attention to how many numbers were in total. In addition to this, when he thought about a solution, he changed the answer. Thirdly, the average student (A3) focused on only the quantity of numbers instead of subtraction of all the numbers even though he could make the numbers a group. As a result, he did the wrong generalization.

In problem-5, four mathematically gifted, a successful, and two average students attempted to solve it by the strategy of adopting a different point of view.
Figure 4.40 provides the work of a mathematically gifted student (M5) on problem-5 while she was using the strategy of adopting a different point of view strategy. At first, she approached the problem as if there were 13 TL (Turkish Liras) in total. In other words, she did not take 50 pennies which are like a surplus when made an operation. Then, she divided 13 to 2 since there was a ratio. Besides, she did not forget to add 50 pennies again.

Figure 4.40

Part of a *Mathematically Gifted Student's (M5) Work on Problem*-5 *Showing*



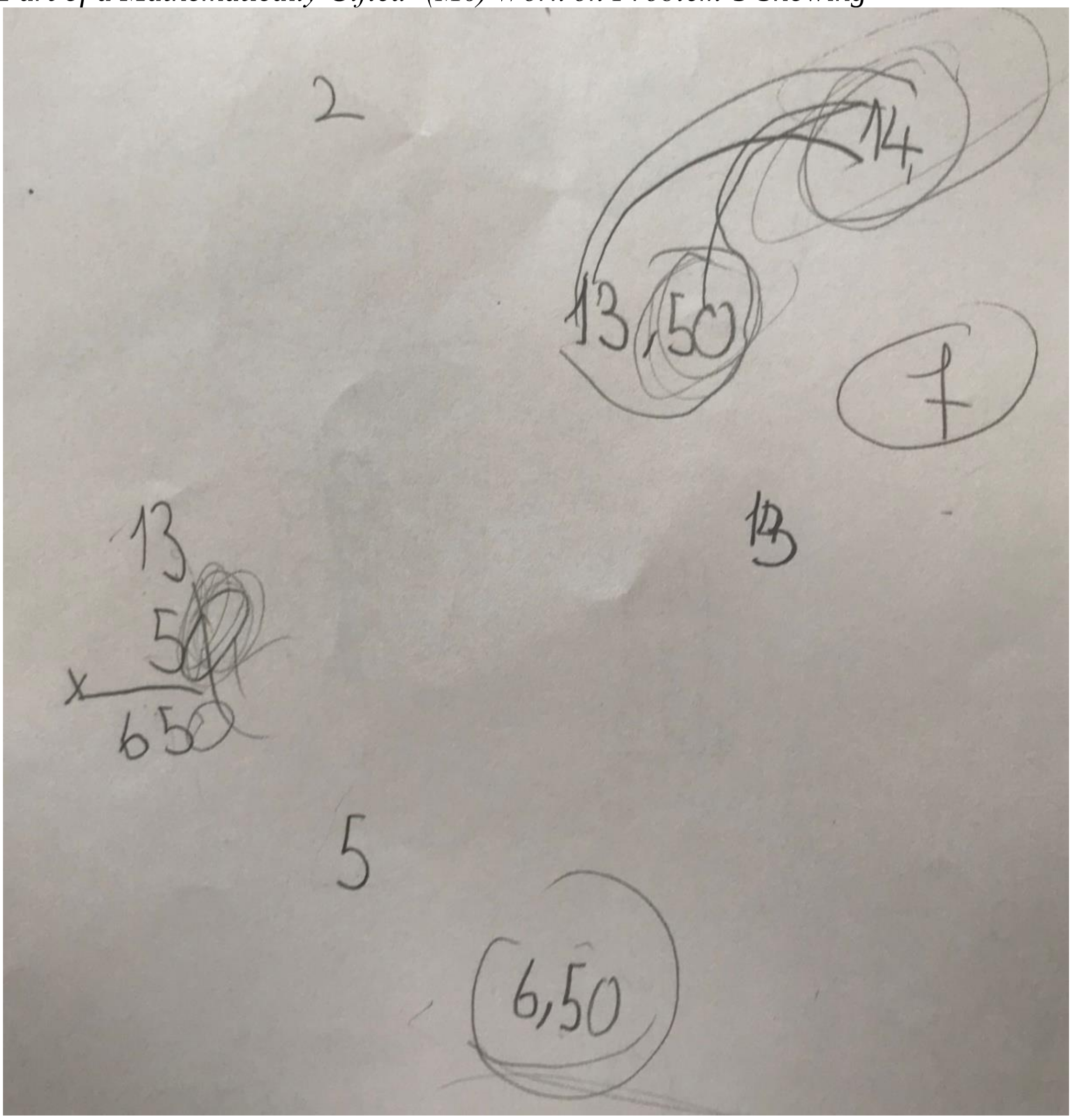
She also stated that:

> Firstly, I did not take "50 pennies" in 13.50 TL since this made my work easier. Then, I divided 13 into 2. I found 6 and I added "50 pennies" that I did not take into account in the first phase. Then, I found the number of 50 pennies was 13. Therefore, the number of "1 TL" was 7 since the number of "50 pennies" is one less than twice the number of "1 TL" in the problem. Even, I multiplied 13 and 50 and I found 650. That is, 6 TL and 50 pennies. Then, I added 6 and 7. I found 13. Finally, I added 50 pennies left. As a result, I could reach the right answer (M5, Problem-5).

Figure 4.41 provides mathematically gifted student's (M6) use the strategy of adopting a different point of view. At first, he considered total money as 14.00 TL instead of 13.50 TL. In other words, he added 50 pennies to 13.50 TL as one less. Then, he divided 14 by 2 because of the ratio in the problem-5.

Figure 4.41

Part of a Mathematically Gifted' (M6) Work on Problem-5 Showing



He also stated that:

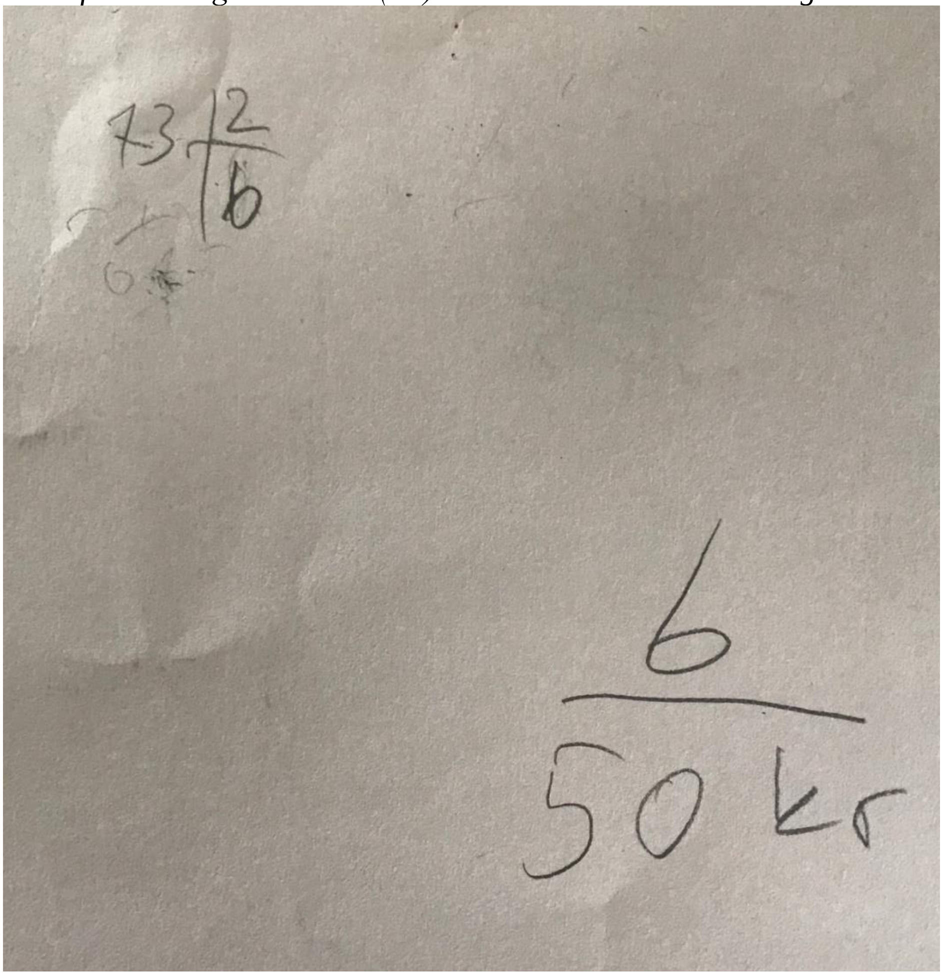
> First of all, I took 13.50 TL as 14 TL. That is, I added “one less” to 13.50 TL as “50 pennies”. Then, I divided 14 by 2. I found 7. After that, I subtracted 1 from 7 which is equal to 6. Therefore, there were 7 of 1 TL and 13 of 50 pennies (M6, Problem-5).

A successful student (S6) used the strategy of adopting a different point of view in problem-5 but she did not write it on the paper. She did not make an operation with 13.50, so she began the problem with 12.00 TL. On the other hand, she did not continue her plan. She stated: “I took 1.50 TL away from 13.50 TL. Then, I divided 12 to 2. Then, the problem says that “one less than twice”. I tried but I was confused a little bit” (S6, Problem-5).

Figure 4.42 provides the work of A1 while he was using the strategy of adopting a different point of view on problem-5. Initially, He took 50 pennies away from 13.50 TL. Then, he divided 13 into 2. On the other hand, he could not add 50 pennies to the final answer as he changed his decision.

Figure 4. 42

Part of an Average Student's (A1) Work on Problem-5 Showing



He also stated that:

> I did not take 50 pennies. That is, I divided 13 by 2. I knew I would add 50 pennies later.
> **[After a while, he continued.]**
> I did not have to add 50 pennies because in the problem "one less" was just equal to this operation (A1, Problem-5).

An average student (A4) also used the strategy of adopting a different point of view in problem-5 but he did not work on the paper. First of all, he changed the total amount of money to 12.00. Then, he divided 12 to 2. After a while, although he could continue in the right way to reach a solution, he could not handle his plan. He stated that:

> A4: I divided 13.5 to 2 but this operation did not work since I did not find an integer. So, I divided 12 into 2. I found 6. Then, I added 1.5 TL that I took away. 1.5 TL was equal to 2 of 50 pennies. Hence, I found 7. I mean, I had 7 TL. After, I multiplied 7 with 2 to find 50 pennies. I subtracted 1 from 14.
> **[After a while, she changed her result.]**
> A4: I took 1.50 TL from 14. Therefore, the number of 50 pennies was 13 and the number of 1TL was 17 (A4, Problem-5).

As a result, mathematically gifted students (M5 and M6) used the strategy of adopting a different point of view strategy differently. M5 took the amount of numbers away in the first step. Then, she transformed the problem into a different one. Also, she did not forget to add the amount later. Similarly, M6 converted the problem to another situation by adding the number to the given points in the problem. As a result, not only two mathematically gifted students used the strategy effectively but also they used it in different ways. Successful student (S6) thought in parallel with M5 but she could not continue because of her confusion. The average student (A1) was aware of the condition in the problem and what he had to do. However, he was not able to proceed. Similarly, even though A4 could adopt the problem at a different point, he could not continue his plan effectively.

All in all, mathematically gifted students differed concerning using the strategy of adopting a different point of view. In addition to this, mathematically gifted ones had a more sophisticated solution in this strategy and they could handle it in comparison to successful and average students. Successful students and average students' approaches with the strategy of adopting a different point of view were similar to each other. On the other hand, when Table 4.2 was examined, average students used the strategy of adopting a different point of view more effectively than successful students.

4.2.4 Summary of Use Styles of Problem Solving Strategy of Different Groups of Students

There were nine expected problem-solving strategies throughout all the six non-routine mathematical problems in this study. Mathematically gifted, successful, and average students found and used the same strategies during the solving process but there were differences in terms of use and style.

Mathematically gifted students gave more energy and showed more patience to problems and their solutions compared to successful and average students. They were able to continue to the endpoint of a strategy in which they took what they needed. The group of mathematically gifted applied the strategies by considering all points in the problem. They were able to check and pursue the steps of their strategy. In addition, strategy use differed and was not the same at all. That is, all mathematically gifted students did not apply one strategy in the same way. In some strategies like acting it out or simulation, making a drawing, and adopting a different point of view, they showed different works in the same problem. However, not all mathematically gifted students were successful to apply every strategy like working backwards strategy effectively for the problems. Secondly, successful students were not as perseverant as their mathematically gifted peers. They stopped the strategy process when a challenge occurred and then they changed their strategy immediately. Therefore, they frequently had to come back to the previous strategy. They were not successful at applying the strategies by considering all the points in the problem as the group of mathematically gifted. They were not able to check and pursue the steps of the strategies as mathematically gifted ones. Lastly, average students were the most unsuccessful groups among all participants to apply strategies efficiently. They did not pay attention to all the points and conditions when applying strategies. They generally concentrated on numbers and operations instead of the conditions in the problem. They failed to check and pursue the steps of their strategies. Strategies generally stayed as thoughts. They were not confident about continuing to apply strategies in some cases (see Figure 4.20 work of A1 in problem-3). All in all, mathematically gifted students mainly used the strategies effectively if not all. The same situation was observed among successful students, but the number of effective strategies use was less than mathematically gifted ones. Successful and average students had similar mistakes in the use of strategy. They were not as good at generalizations and applying the strategy considering all conditions of the problem in the problem-solving process as mathematically gifted students. The lowest number in effective strategy use belonged to average students.

CHAPTER 5

DISCUSSION, IMPLICATIONS, AND RECOMMENDATIONS

In this chapter, the purpose and results of the study are restated with the related literature. Then, implications and further recommendations are mentioned.

5.1 Discussion

The purpose of this study was to investigate the use of problem-solving phases and problem-solving strategies of seven mathematically gifted, seven successful, and six average students from fifth-grade for the six mathematical problems. Along this line, the following research questions will be discussed in this section.

- How do mathematically gifted, successful, and average students use the problem-solving phases advocated by Polya (2004)?
- Which problem-solving strategies do mathematically gifted, successful and average students use while solving a non-routine problem?
- How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?

In this study, the data had been gathered via clinical task-based interviews, which were done with 20 students and observation. In the interview, six mathematical problems were asked. Every participant was especially informed that the study was not conducted to get correct answers. The aim of the study was explained as observing which problem-solving phase and strategy they use and how they use them during the problem-solving process. Therefore, the participants were also informed that there was no time limitation for solving the problems and if they wanted, they were free to leave them. They were also encouraged to think aloud to express their ideas verbally when working on the problems. All interviews and participants' works were recorded.

5.1.1 Comparison of Three Groups of Students' Problem-Solving Phases Use

Regarding the research question of "How mathematically gifted, successful, and average students use problem-solving phases advocated by Polya (2004)?", the findings revealed that mathematically gifted students differed from successful and average students as they attempted to follow problem-solving phases more than the other two groups (see Table 4.1). They were also more systematic in following these phases than the successful and average students. That is, the phase of understanding the problem was the first step which was followed by the phase of devising a plan. The phases of carrying out a plan and looking back came respectively after devising a plan. In some cases of successful and average students, they applied the first and the second phases but then, they turned back to the first phase, the phase of understanding the problem. However, the mathematically gifted students systematically pursued the order, from the first to the second, the third, and the fourth phases. They just turned back to the first phase, which is devising a plan, while they were carrying out the plan to change their strategy when it did not work. As a result, mathematically gifted students' utilization of the problem-solving phase was sequential while successful and average students' utilization in the problem-solving phase was not sequential. In parallel to this result, Gorodetsky and Klavir (2003) pointed out that gifted and non-gifted students have a different pattern of process and sub-process in problem-solving. Heinze (2005) also stated that gifted students' works were more systematic than non-gifted students. On the other hand, the result was different in Pativisan's (2006) study conducted with five gifted Thai students whose grades varied from eight to ten. According to her study, problem-solving stages were observed as understanding, planning, executing, and verifying among gifted students. On the other hand, they did not apply the stages in a linear order. The present study's results also highlighted that mathematically gifted students did not continue with the next phase without completing the previous phase in the process. The literature emphasized similar results that gifted groups applied every phase of problem-solving more carefully (Akdeniz & Alpan, 2020; Montague, 1991; Span and Overtoom-Corsmit, 1986).

During the phase of understanding the problem, the present study pointed out that mathematically gifted students repeated the problem and stated the unknown and the data in the problem less than successful students who repeated the problem the most. That's why, mathematically gifted students showed the lowest number of attempts in applying the phase of understanding the problem (see Table 4.1). However, in this study, having the highest number of finding the correct solution meant that mathematically gifted students gave remarkable time and effort to understand the problem. Sriraman (2003) also found similar results among four ninth-grade mathematically gifted and five non-gifted students, and Budak (2012) observed similar findings among four mathematically promising students. Moreover, even though mathematically gifted students had the lowest number of attempts in the phase of understanding the problem, they were not only cognizant of their works but also expressed that in the most logical way. The results of Pativisian's (2006) study about gifted students supported the present study's findings that gifted students were good at explaining their work. Similarly, Budak (2012) and Heinze (2005) stated that mathematically gifted students were good at explaining their problem-solving processes.

During the phase of devising a plan, mathematically gifted students were the participants who tried to think the same or a similar problem during problem-solving processes the most, but the number of attempts was lower in comparison to other sub-processes. The same result was observed in Overtoom-Corsmit's (1986) and Montague's (1991) studies conducted with gifted students. It was understood that gifted ones were more prone to remembering past experiences and using them in new situations. However, Gorodetsky and Klavir (2003) pointed out contradictory findings that non-gifted ones thought on past experiences more when gifted ones were interested in new situations. Similarly, Ünal (2019) also emphasized that mathematically gifted students did not prefer to use their past knowledge even though they were aware of them. Secondly, using strategies by giving high awareness and remarkable time was more common among gifted students (Bayazıt & Koçyiğit, 2017; Montague, 1991; Yıldız et al., 2012). Moreover, Budak's (2012) study showed that all the mathematically promising students gave remarkable time to understand and make a plan in his study. Results of the present study validated this conclusion. Production of strategy by using all the given components in the problem had the highest number of attempts in the phase of devising a plan among mathematically gifted, successful, and average students in the present study. Nevertheless, mathematically gifted students had the top number in the attempt of producing strategy with remarkable attention and patience.

During the phase of carrying out the plan, checking all the steps of the plan was high among mathematically gifted students in the present study. In the same way, in the study of Span and Overtoom-Corsmit (1986), many highly gifted students checked their strategy more compared to averagely gifted students. Besides, according to Akdeniz & Alphan's (2020) study conducted with 151 gifted and talented students, especially gifted students from the mental ability field had a priority about making and evaluating their plan during the problem-solving process. Apart from this, proving the correctness of the plan did not have the high number of attempts among all participants but mathematically gifted students had the highest number of attempts concerning proving the plan in the problem-solving process. In the literature, awareness about what they did and what they were going to do during the problem-solving process was high both in gifted students (Montague, 1991; Overtoom-Corsmit, 1986; Pativisan, 2006) and in mathematically gifted students (Heinze, 2005; Ünal, 2019). Hence, some mathematically gifted students may not have needed to prove their strategy in the present study.

During the phase of looking back, the large majority of mathematically gifted students gave importance to looking back at all the conditions and findings in the present study. However, Ünal's (2019) findings stated that mathematically gifted students did not check their solutions at the last phase since they were able to be aware of every step during the problem-solving process and they just checked every moment of the plan. From this perspective, the same behavior was observed in carrying out the plan phase too. This could explain why some mathematically gifted students did not need to check all their works in the present study. The number of attempts of checking all the conditions and steps in the problem for successful and average students was less than mathematically gifted students' number of attempts. Also, when successful and average students tried to check the process, they mostly concentrated on operational control rather than the whole process control. Besides, mathematically gifted students were more eager to find other strategies for the possible solution of the problem after they finished the process. These findings are similar to the mathematical problem-solving behaviors reported by Pativisian (2006) for five gifted students in Thai and to the creative problem-solving behaviors reported by Akdeniz and Alpan (2020) for gifted students in music in Turkey.

5.1.2 Comparison of Three Groups of Students' Problem-Solving Strategy Use and Styles

In the light of the research question "Which problem-solving strategies do mathematically gifted, successful, and average students use while solving a non-routine problem?", the present study indicated that adopting a different point of view and acting it out or simulation were the most used strategies among mathematically gifted students while logical reasoning and acting it out or simulation were the most used strategies among successful and average students (see Table 4.2). In contrast to this result, gifted students used the acting it out strategy the least in Aydoğdu and Keşan’s (2016) study. Moreover, in the present study, organizing data, solving a simpler equivalent problem, and finding a pattern strategies were the three least used strategies among three groups of participants. The study of Yıldız et al. (2012) did not confirm this result since non-gifted students mostly preferred the organizing data. According to Bayazıt and Koçyiğit’s (2017) findings, gifted students preferred to use making a list more when non-gifted peers used guess and check strategy more. When the present study and related literature were examined, the possible reason for these differences may be related to the type of problems, grade level, and type of gifted students since the problems which were used in the studies were different from each other, and the present study was conducted with fifth-grade mathematically gifted students which were not participants of Aydoğdu and Keşan’s (2016); Bayazıt and Koçyiğit’s (2017); and Yıldız et al.’s (2012) studies.

In the light of the research question “How do mathematically gifted, successful, and average students use problem-solving strategies while solving a non-routine problem?”, in the present study, the mathematically gifted group had the highest number of attempts in using more problem-solving strategies. Many research studies related to giftedness problem-solving process (Akdeniz & Alpan, 2020; Bayazıt & Koçyiğit, 2017; Budak, 2012; Overtoom-Corsmit, 1986; Pativisan, 2006; Yıldız et al. 2012) built a consensus about this mastery. On the contrary, Montague and Applegate (2000) added that gifted, average, and learning disabled students did not differ concerning the use of the total number of strategies in some problems like 1-step and the 2-step problem. When the present study and literature are evaluated together, problem types may be a factor to affect the number of problem-solving strategies use.

The present study stated that mathematically gifted students were the most successful group to use strategies effectively to reach the correct solutions manner compared to successful and average groups. Using strategies effective was noticed among gifted students more in the study of Bayazıt and Koçyiğit (2017), Montague (1991), and Span and Overtoom-Corsmit (1986). In the present study, even though mathematically gifted ones had the highest number of attempts to find a problem-solving strategy in the problem-solving process, some of them were not successful to use some strategies effectively to reach the correct solutions. Similarly, some successful and average students were able to find the expected strategy for the problems' solutions but they could not solve the problems correctly by using the strategies. Okur (2008) supported this result from his study with ninth-grade students that finding a suitable strategy may not guarantee the correct solution. Yıldız et al. (2012) also stated that gifted students could not use all the problem-solving strategies successfully in their study.

In the present study, there were similarities between successful and average students' styles of problem-solving strategy use compared to mathematically gifted students. Their mistakes were similar to each other. They could not concentrate on the conditions of the problem and the situation as their mathematically gifted peers while using a problem-solving strategy. On the other hand, mathematically gifted students used the strategies completely until the strategy did not work for the problem. They were more successful in generalization in the strategy use. Besides, they checked the effectiveness of their strategy use and consider all conditions of the problem when thinking and applying a problem-solving strategy. In this sense, Gorodetsky and Klavir (2003) stated that gifted students were different from non-gifted students not only quantitatively but also qualitatively. Similarly, according to Montague's (1991) study, gifted students were more knowledgeable about the strategies. At the same time, Yim et al. (2008) conducted a study with two gifted students in mathematics stated that they were able to comprehend the clear points of a problem. That is, they were good at where they should begin in the problem-solving. It showed that this ability can help them use a strategy effectively and make a logical generalization in the problems in the present study.

To sum up, observation of problem-solving can give the proper information about the similarities and differences of different types of problem solvers' characteristics. In the present study, mathematically gifted students' process concerning problem-solving phase application and effective use of problem-solving strategies were generally confirmed with the related literature. However, some sub-points under the Polya's (2004) problem-solving phases and use of problem-solving types may differ among gifted and mathematically gifted students. Besides, effective use of problem-solving strategies was not observed only among mathematically gifted students in the present study. There were some cases in which successful or average students were the most successful group in using some problem-solving strategies efficiently to reach the correct solutions to the problems (see Table 4.2). Furthermore, it could be deducted that a successful problem-solving process should be related to not only selecting the true problem-solving strategy but also being able to use the problem-solving strategy effectively for both gifted and non-gifted students. In addition to this, the literature and this study pointed out that not only gifted and non-gifted students use different problem-solving strategies but also mathematically gifted and gifted students may differ in problem-solving strategy use considering the least and the most used strategies.

5.2 Implications for Teachers

Some implications for the teachers who work with mathematically gifted, successful, and average students on their problem-solving process are presented in this section.

The result of the present study revealed that an effective problem-solving process was not only related to the use of problem-solving strategy but also related to the use of the problem-solving phase for three groups of participants since mathematically gifted students had the highest number of attempts in both problem-solving strategy use and use of problem-solving phase. Therefore, they were the most successful participants in reaching the correct solution which teachers should pay attention to. Teachers can create problem-solving activities in which students can be observed concerning both the problem-solving phase and strategies at the same time like as in the present study. In this way, the reasons for the non-effective and effective solution process may be observed comprehensively. Additionally, teachers should teach their students about the utilization of the problem-solving phase and problem-solving strategies with these activities.

Although all the three groups of participants used the same strategy in some particular problems in the current study, mathematically gifted students were more determined than successful students. Average students were the poorest problem solvers in the study since they were not sure about their strategies and work as their mathematically gifted and successful peers. They could not feel sure about solving problems and asked the researcher whether they should continue or not. They changed mostly their answers. Therefore, it may be the reason that they could not attempt to find different strategies. Considering all these results, teachers should give importance to problem-solving activities more in the classrooms. They can also encourage poor problem solvers to solve the problem with more than one strategy. In addition, the teachers should encourage them to solve/approach the problems from different perspectives and employing different strategies. Also, they should introduce the problems that might allow the use of different strategies.

5.3 Limitation of the Current Study and Recommendations for Further Research

There are some recommendations for further studies on problem-solving phases and problem-solving strategies of mathematically gifted, successful, and average students on their problem-solving processes.

In this study, the problem-solving phases and problem-solving strategies of seven mathematically gifted, seven successful, and six average students from fifth-grade level were investigated. The results of the present study and the studies from the literature were conducted with various grade levels showed that problem-solving strategies used by students were not parallel to each other considering the most and the least used strategies. The present study was conducted with only fifth-grade students and limited to the participants of the study. There should be more studies to clarify whether the grade level may differ the results or not. Furthermore, the present study was conducted with mathematically gifted, average, and successful students. On the other hand, studies (Aydoğdu and Keşan, 2016; Beyazıt and Koçyiğit, 2017; Pativisan, 2006; Span and Overtoom-Corsmit, 1986; Yıldız et al., 2012) related to problem-solving strategy use in literature were conducted among only gifted or gifted and non-gifted students. There is no study to investigate mathematically gifted students' problem-solving strategy use and use styles in detail in the literature. When considered the present study and these studies from the literature, it can be said that mathematically gifted students and gifted students could use the same strategies but they differed with respect to the least and the most used strategies. However, these results did not come from the same study and there is no detail about the use style. Therefore, there should be more studies to investigate the use and use style of problem-solving strategies of mathematically gifted and gifted students.

In the present study, students were selected according to the teachers' opinions. On the other hand, some mathematically gifted, successful, and average students' problem-solving processes showed similarities. This showed that teachers' opinion was not always a reliable criterion to select three types of participants separately in this study. Besides, there was no certainty that some successful students were possible gifted students although successful students did not take any intelligence scale. When taking into account all of them, there were two recommendations Firstly, teachers should be educated about the giftedness and gifted student' characteristics. Secondly, a specific scale was developed to define mathematical giftedness. Further studies can be interested in these two recommendations.

In the present study, the group of mathematically gifted has been taking different courses in SAC. They also attended different middle schools while successful and average students attended the same middle school (see Table 3.1). The results indicated that the use of the same or similar problem sub-phase was applied by the mathematically gifted group the most among the three groups of participants (see Table 4.1). These students indicated that they solved the problem before in SAC or their classes in the schools. Therefore, there is no certainty about whether in the present study, the courses or their different teachers also influenced their problem-solving process or not. Future studies can be done considering this factor. Participants can be selected in line with the factors that they take the same courses at the same time or non of them take any courses and they attend the same schools. In this way, the effect of courses or schools/teachers on the problem-solving process can be eliminated.

In the present study, in some problems, some mathematically gifted students differed in the same strategy use style like adopting a different point of view and acting it out or simulation strategies. Besides, in some problems, a strategy could not be used effectively by the mathematically gifted students while the same strategy was productively used by them in another problem. Apart from this, some strategies became prominent for the groups. The problem-solving strategy of adopting a different point of view was mostly used by mathematically gifted students. On the other hand, logical reasoning strategy was mostly observed among both successful and average students. Apart from this, organizing data had the lowest attempt for all the groups. However, there is no certainty about whether this result was related to the content of the problem or not since the mathematical content of the six problems used in the current study was limited to geometry, numbers and operation, and algebra. In the literature, results of some studies (Aydoğdu & Keşan, 2016; Bayazıt & Koçyiğit, 2017; Yıldız et al., 2012) were not parallel to the present study concerning problem-solving strategy use since there were more different problems having different contents. Therefore, other mathematical concepts e.g. probability and measurement with different types of problems can be studied further in problem-solving-related studies. In this way, the reasons for the students' differences in the problem-solving process can be examined comprehensively and it can contribute to the literature of mathematically gifted students.

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APPENDICES

A. MATHEMATICAL PROBLEMS USED IN THE STUDY

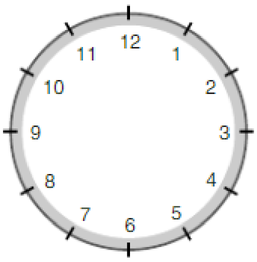
Posamentier and Krulik (2009) suggested mathematical problems with different use of problem-solving strategies. The adapted and translated (to Turkish) six mathematical problems which were used in the study are presented below.

Problem 1

Ayşe'nin üçgen şeklinde bir çerçeveye yerleştirilecek 55 tane dikdörtgen şeklinde bloğu vardır. Ayşe bu blokları üçgen şeklindeki çerçeveye koymaya başlarken ilk sıraya bir tane, ikinci sıraya iki tane, üçüncü sıraya da üç tane olacak şekilde koymaya başlamıştır. Peki, bu düzende yapmaya devam eden Ayşe 55 tane blokla bir üçgen şeklinde çerçeve oluşturabilir mi? Oluşturabilirse hangi sırayla oluşturur? Bu büyük üçgen çerçevede kaç tane sıra olur? (Tüm bloklar kullanılacaktır.)

Problem 2

Aşağıda gördüğünüz saatte sayıların üstüne gelmeyecek şekilde öyle düz iki çizgi çizin ki oluşturduğunuz her bölgedeki sayıların toplamı birbirinin aynısı olsun.



Problem 3

Türkiye Milli Basketbol takımı bugünkü maç için yeni bir kutu basketbol topu açtı. Topların $\frac{1}{3}$' ünü zeminde kaymaması için özel bir çamurla ovalanmak üzere bir yere yolladılar. 15 tane daha basketbol topunu imzalamak üzere gönderdiler. Vuruş antrenmanı için de 20 basketbol topunu kontrol edilmek üzere gönderdiler. En sonda ellerinde 15 topları kaldı. Başta ellerinde kaç tane basketbol topu vardı?

Problem 4

101'den küçük tüm çift doğal sayıların toplamından, tüm tek doğal sayıların toplamını çıkardığımızda sonuç ne olur?

Problem 5

Ali'nin cebinde belli miktarlarda 50 kuruş ve 1 TL'ler bulunmaktadır. Cebindeki 50 kuruşlarının sayısı 1 TL'lerinin sayısının 2 katından 1 eksiktir. Toplamda cebinde 13 TL 50 kuruş bulunan Ali'nin kaçar tane 50 kuruşu ve 1 TL'si vardır?

Problem 6

Üç arkadaşın boylarının uzunlukları toplamı 480 cm'dir. Duru'nun boyu Ada'dan 20 cm kısa, Yasemin'in boyu ise Duru'dan 10 cm uzundur. Duru'nun boyu kaç cm'dir?

B. FOLLOW-UP QUESTIONS USED DURING THE INTERVIEW

Hunting (1997, pp. 153–154) suggested some follow-up questions for the researcher during a clinical interview which is also used in the current study.

- **Can you tell me what you are thinking?**

This question is useful after about 10 seconds of silence where it is not certain that productive mental activity is taking place.

- **Can you say out loud what you are doing?**

When a student seems to be engaged in thought, after giving a short time, the interviewer may interrupt. Indicators of activity include inaudible utterances, scratch work on paper, motor activity such as tapping, eye, and other body movements.

- **Can you tell me how you worked that out? How did you know? How did you decide?**

A student may respond with an answer to a problem without any apparent clue as to the way the answer was obtained. These questions are intended to convey to the student that you are interested in how the result was determined. As such it is designed to encourage a verbal explanation.

- **Was that just a lucky guess?**

  If the student makes a response but does not give an explanation, then this question often has the effect of putting the student at ease and relieving tension. Sometimes in an effort to obtain information students will respond with the first thing that comes into their head. Students are generally happy to admit guessing.

- **The other day another student told me…**

  If there are grounds for supposing that the student isn't confident about the solution offered, or the interviewer wants to test the strength of a conviction, an alternative solution from a neutral and anonymous third party may be proposed for consideration. The advantage of attributing the alternative solution to a third party is that the student could feel it in his or her best interests to agree with a view emanating from the interviewer, just because the source of that view has power and status in the situation.

- **Do you know what _ means?**

Success on a task may depend on knowledge of a particular term used in presentation of the problem. Potentially problematic vocabulary can be nullified by clarifying the meaning of the term. Teachers being teachers have an uncontrollable urge to teach. Should a teacher explain a point during an interview? The answer to this question rests on whether the teacher primarily intends to assess the status of the student's mathematical knowledge. It is not wrong to provide a student information. In fact, there are benefits in seeing how far the student is able to progress on the basis of some assistance. It may be that the information provided allows the student to incorporate other knowledge previously untapped. It is worth bearing in mind that the interview itself is a learning experience for the student. The extent to which the teacher digresses into a didactic frame during an interview will dictate how much progress will be made through the interview given the time available. We generally discourage teachers from digressing during formal training.

- **Do you know a way to check whether you are right?**

Problem solutions, particularly those involving basic arithmetic operations, can be checked by means of estimation, rounding, or the appropriate inverse operation. Encouraging checking provides another window into a student's depth of understanding.

- **Why?**

In response to an explanation a student may make an assertion. Asking why is a sensible way of encouraging further explanation.

- **Pretend you are the teacher. Could you explain what you think to a younger child? How would you explain?**

  Encouraging children to formulate viewpoints or design settings for younger children provides an opportunity to capture their understanding of a situation or problem.

C. QUESTIONS TO EXAMINE PROBLEM-SOLVING PHASES

Questions that should be asked to examine problem -solving phases use of participants according to Polya's (2004, pp. 4–9) four phases model in the problem-solving process:

- **Understanding the Problem**
  What is the unknown?
  What are the data?
  What is the condition?
  Is it possible to satisfy the condition?

- **Devising a Plan**
  Could you restate the problem?
  Did you use all the data?
  Did you use the whole condition?

- **Carrying out a Plan**
  Can you see clearly that the step is correct?
  Can you also prove that the step is correct?

- **Looking Back**
  Can you check the result?
  Can you check the argument?
  Can you derive the result differently?
  Can you see it at a glance?

D. PROBLEM-SOLVING PHASES USE OF PARTICIPANTS

The table 4.3 which is below presents the every participants' use of problem-solving phases in all problems in the interview.

Tablo 4.3

Participants' Use of Problem-Solving Phases

P.	Problem Solving Phases								
	Understanding the problem			Devising a Plan		Carrying out the Plan		Looking Back	
	Repeat/Restate the problem	Draw a figure to point out the data and unknown	State the unknown & data of the problem	Think of the same/similar problem	Produce strategy with using all givens in the problem	Check all steps of the plan	Prove the correctness of the plan	Check all conditions and steps in the problem	Solve the problem with another strategy
1	S1,S3,A3	M1,M2,M3, M4,M5,M6, M7,S1,S2,S3,S4,S5,S6,S7,A1,A2,A4,A3,A5,A6	M6,S5		M1,M2,M3,M4, M5,M6,M7,S1,S2,S3,S4,S5,S6,S7, A1,A2,A3,A4,A5, A6	M1,M2,M3,M4,M5,M6,M7, S1,S2,S3,S4,S5,S7,A2,A3,	M1,M4,M5,S3,S7	M1,M2,M3, M5,M6,M7, S2,S3,S4,S7	M1,M4,M5, M6, M7,S2,S5,S7
2	S3,S2,S7, A3,A6	M1,M2,M3, M4,M5,M6, M7,S2,S4,S5,S7,A2,A3		M1,M5	M1,M2,M3,M4, M5,M6,M7,S1,S2,S3,S4,S5,S6,S7, A2,A3,A4	M1,M2,M3,M4,M5,M6,M7, S1,S3,S4,S5,A3,A4	M4,M5,M6, M7,S3,S5,A4	M2,M3,M4, M5,S1,S3,S5,A4	S1
3	M7,S1,S3, S4,S5,S7, A2,A3,A4, A6	M1,M2,M3, M4,M5,M7, S1,S2,S4,S5, S6,S7,A1,A2,A3,A4	M4,M6,S4,S5,A4	M4,M7,S4	M1,M2,M3,M4, M5,M6,M7,S1,S2,S3,S4,S5,S6,S7, A1,A2,A3,A4,A5, A6	M1,M2,M4,M5,M6,M7,S1,S3,S4,S5,S6,S7, A2,A4	M4,M5,M6,S1,S6	M4,M5,M6, M7,S6,A4	M4,M6,S5,S6,A4,A6

Table 4.3 (continued)

4	M3,M4,S1, S3,S7,A3, A4,A6	M1,M2,M3, M4,M5,M6, M7,S1,S4,S5 ,S7,A3,A4,A 5,A6	M6,S5,A 4,A5	M4,M5,S3, A6	M1,M2,M3,M4, M5,M6,M7,S1,S2 ,S3,S4,S6,A1,A3, A4,A5,A6	M2,M4,M5,M 6,A3,A4,A5		M4,M5,M6, A3,A4,A5	M6,S6,A3
5	M4,S1,S3, S5,S6,A3, A4	M1,M3,M4, M5,M6,M7, S1,S2,S3,S4, S5,S7,A1,A2 ,A3,A4,A6		M4,S4	M1,M2,M3,M4, M5,M6,M7,S1,S2 ,S3,S4,S5,S6,S7, A1,A2,A3,A4,A6	M4,M2,M3,M 5,M6,M7,S4,S 7,A3,A4	M4,M5,S5,A 3,A4	M2,M4,M5, M6,S5,A4	M3,M4,M5, M6,M7,S1,S 3,S5,A2
6	S1,S3,S4, A3,A4,A5	M1,M2,M3, M4,M5,M6, M7,S1,S2,S4 ,S5,S6,S7,A1 ,A2,A3,A4,A 5,A6	M6,S5,A 4,A6	M4,M5, M7,S6	M1,M2,M3,M4, M5,M6,M7,S1,S2 ,S4,S5,S6,S7,A1, A2,A3,A4,A5,A6	M1,M2,M3,M 4,M5,M6,S1,S 4,S5,S6,A1,A3 ,A4	M1,M4,M5, M6,S4,S5,A4	M1,M2,M4, M5,M6,S1,S 4,S5,A4	M1,M3,M4, M5, S6,A2

Note. P: The problems, M: Mathematically gifted student, S: Successful student, A: Average student

E. OBSERVATION FORM

Observer: Class: Student: Getting started with the problem:	Time: Date: School/Institution/Teacher:
Was there any question of the student about the problem?	How did the student approach the problem?
Did the student understand the problem?	Was there any point the student cannot understand the problem?
Did the student explain the problem in her/his word?	What did the student think about the solution?
Comments	
During the problem solving:	
Did the student have any questions about the problem?	Did the student have a plan for the problem? If yes, how did she/he make this plan?
Which type of plan did the student follow?	Did the student explain her/his plan?
Why did the student follow the plan? Did she/he use this plan before?	Did the student solve a similar problem before?
Comments	
At the end of the problem solving:	
Was there any question of the student?	Did the student use the plan that she/he thought in the devising a plan phase?

Was the student sure whether the solution was correct or not? If yes, how did she/he can be sure?	Did the student control the result of the problem? If yes, how did she/he control it?
Did the student solve the problem differently?	Were there any challenging points during the problem-solving process for the student?
Were there any hinter points during the problem-solving process for the student?	

F. PROBLEM-SOLVING STRATEGY USE OF PARTICIPANTS

The table 4.4 below presents all type of participants' use of problem-solving strategies within all problems in the interview.

Table 4.4

Participants' Use of Problem Solving Strategies

P.	Organizing Data		Intelligence Guessing and Testing		Solving Simpler E. Prob.		Acting It Out or Simulation		Working Backwards		Finding a Pattern		Logical Reasoning		Making a Drawing		Adopting Diff. Point of View	
1	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-
		S3,A2*,A5*	M5*			M4	M7,S7	M2*,M3*,M4,M5*,M6*,S1,S2*,S3*,S5,A2,A6				M1*,M3,M4*,M6,M7*,S1,S2,S5*,S6*,S7*,A1*,A3*		A3,A6*	M5	M1,M2,M7,S1*,S2*,S3,S4*,S5,S6,S7,A3,A4	M1	
2	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-	P^+	P^-
				M1,M2,M3*,M4,M5,M6,M7*,S1*,S2,S3,S5*,S6,S7,A2,A3,A4*			S1	M6*						M2*,M4*,S2,S3*,S4,S5,S6,A3		M1,M3,M7		M1,M3,M4,M5*,M6,S2,S3,S4,S7

Table 4.4 (continued)

3	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	$P^{\cdot}$	P^{-}
			M6*	S1	M6*		S5,A4,A6	M3,S1		M1,M2,M4*,M5*,M6*,M7*,S2,S3*,S4,S5,S6*,A1,A2,A3,A4*,A5				M3,S4,S7,A6*	S6,A4	S1,S4		
4	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	$P^{\cdot}$	P^{-}
						M4*,S1,A6	M3	S1			M3			M2,M3,M5,M6,M7,S2,S3,S6,A1,A3,A4			M3	M1,M4,M5*,M6,S2,S4,S5,S6,S7,A3,A5*

Table 4.4 (continued)

5	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	$P^{\cdot}$	P^{-}
			A1	S5*,S6,A3,A6*			M4, M5*, M6*, M7,S1,S3, S5*, A1,A2	M1,M2*,M3,S2		M1,M3,S1,S3			M4	M1,S1,S2,S4,S5,S7,A1,A2,A3*				M4,M5*,M6*,M7,S6,A1,A4
6	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	P^{+}	P^{-}	$P^{\cdot}$	P^{-}
	S1	M6	M1	M6*,S1*, S5*	M6*,A2	A3	M1*, M3, M6, A6	S1,						M1*,M2*,M3*,M5*,S1,S2*,S4,S5,S6, S7,A2,A3*, A4*,A5,A6		M4		M3,M4*,M5,M7, S1,S6,A1*

Note. M: Mathematically gifted student, S: Successful student, A: Average student

Note. P.: Problems, P^{+}: The solution that students solve with prompt of researcher, P^{-}: The solution that students solve without prompt of researcher, *: The right solution

G. APPROVAL OF THE UNIVERSITY HUMAN SUBJECT ETHICS COMMITTEE

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21 Ocak 2020

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Ayhan Kürşat ERBAŞ

Danışmanlığını yaptığınız **Yasemin SİPAHİ'nin "Üstün Yetenekli Çocukların Problem Çözme Sırasında Kullandıkları Stratejilerin ve Çözüm Süreçlerinin İncelenmesi"** başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **024-ODTU-2020** protokol numarası ile onaylanmıştır.

Saygılarımızla bilgilerinize sunarız.

Prof.Dr. Mine MISIRLISOY
Başkan

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Üye

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