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A simple growth model of schooling and public expenditure on education

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Abstract

This paper examines the long run effects of public expenditure on education on time allocation and the growth rate in a two-sector model of endogenous growth with physical and human capital. Individuals accumulate human capital by devoting time to school and public expenditure on education is an input in the human capital technology. The model shows that when the government modifies the proportion of resources assigned to education, the time spent in school may increase, fall or remain constant while the growth rate increases. This result depends on the intertemporal elasticity of substitution, and partially explains why shifts in school attainment of the labor force are not significantly related to the per capita GDP growth rate.

1. Introduction

Several empirical studies have tried to clarify how human capital affects the output and growth rate of an economy. Their results have been inconclusive. Mankiw, Romer and Weil (1992) found that schooling enrollment ratios, used as proxies for human capital, are significantly related to per capita GDP. Benhabib and Spiegel (1994) found that shifts in the average years of schooling are not significant in explaining GDP growth but the level of this variable is. Barro and Sala-i-Martin (1995) obtained that contemporaneous changes in schooling variables are not significantly related to the growth rate but their levels, observed at the start of the study period, are.[1]

This paper attempts to develop a theory to understand these troubling results. Benhabib and Spiegel (1994) pointed out that by simply treating human capital as another factor of production we may be misspecifying its

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1 Most growth regressions have been criticized because of the well-known problem of endogenous regressors that may generate bias in the parameters estimated. However, many use econometric techniques to correct this problem.

role since, in the spirit of Nelson and Phelps (1966), an educated labor force is better in creating, implementing and adopting new technologies and, hence, generating growth.[2]

This may explain why schooling attainment enters significantly in growth regressions, but it does not explain why changes in schooling do not. Barro and Sala-i-Martin (1995) answer this question by blaming measurement errors, including inaccuracies in the timing of the relationship between human capital and production. However, another possible explanation may be that the definition of human capital in most of these empirical studies is overly restrictive. In general, they only consider changes in schooling quantity and not changes in schooling quality. From the Barro and Lee (1996) data set, it seems clear that schooling varies considerably across countries and over time.

In this paper I propose that the weakness of the observed relationship between changes in school attainment and the long-run growth rate is due to the narrow definition of human capital used. More specifically, by allowing for public expenditure on education to interact with the amount of schooling, the direct relationship between changes in schooling attainment and growth rate may be eliminated. I construct a very simple endogenous growth model with physical and human capital where the latter is produced with time devoted to formal education as well as public expenditure on education. Glomm and Ravikumar (1992 and 1994a) have constructed similar models but they have not included decisions concerning time allocation and, therefore, they could not discuss how others factors interact with the schooling time in generating human capital. In the model here, public expenditure can be thought of as a measure of quality, which implies that, even though the time allocated to schooling remains constant, increases in schooling investment may take place because of improvements in the quality of schooling. The new technology I present is based on microeconomic and macroeconomic empirical evidence on the relevance of this factor in explaining the variation in the rate of return to education across individuals[3], and the long-run growth rate of an economy.[4]

In the model, all public expenditure is devoted to schooling, although the introduction of other types of public expenditure, as in Barro (1990) and Glomm and Ravikumar (1994a), would not modify the main

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2 This approach is developed in Romer (1990), where the stock of human capital determines the rate of growth.

3 Card and Krueger (1992) found that a decrease in the pupil/teacher ratio by five students and a 10% increase in teachers' pay are associated with 0.4% and 0.1% increase in the return to schooling, respectively.

4 The Barro and Sala-i-Martin (1995) regressions show that a 1.5% increase in public expenditure on education raises the growth rate by 0.3% per year.

conclusions. The government collects a part of the output generated in the economy through a proportional tax in order to provide free public educational services to households. Therefore, the tax rate can be understood as the size of government. Looking at the microeconomic empirical results of Card and Krueger (1992) one would expect that an increase in the size of government would lead to an increase in the time devoted to schooling. However, under the theoretical framework I develop, this behavior is true only when the intertemporal elasticity of substitution (IES) is large enough. The intuition behind this is as follows. If households are willing to shift consumption across time and the tax rate is set higher forever, individuals' optimal strategy will be to devote more time to the accumulation of human capital that will enable them to achieve higher earnings in the future and, as a result, greater consumption. Exactly the opposite behavior happens when the IES is low enough. By continuity, for an intermediate value of the IES, changes in the tax rate do not modify the allocation of time.

Hence, higher schooling quality may lead to an increase or a reduction in time spent on schooling, and even this variable may remain unaltered. Regarding the long-run growth rate, the relationship between the size of government and the growth rate presents a maximum for any IES value. This result is quite intuitive from Barro (1990). The negative *tax effect* reduces the growth rate and the *capital-labor ratio effect* operates to increase the interest rate and, therefore, the growth rate. When the tax rate is low, the second effect dominates and vice versa.

The main message of the paper is that by introducing public expenditure on education into the analysis the direct relationship between per capita GDP and shifts in school attainment may be eliminated. This result suggests that changes in quantity and quality should be considered when evaluating the importance of schooling for economic growth.

The rest of the paper is organized as follows. Section 2 discusses some of the empirical evidence on the relationship between the growth rate, changes in school attainment and public expenditure on education. Section 3 presents the model. Section 4 contains the results on the steady-state equilibrium. Section 5 analyzes how the time allocation and growth rate are affected when the size of government is modified. Section 6 summarizes the main conclusions. Lastly, an appendix contains the proofs of the results.

2. How public expenditure on education affects schooling time and growth

Most of the empirical studies on human capital, measured by school attainment and growth find that contemporaneous changes in human capital are not significantly related to the per capita GDP growth rate. Two examples are Benhabib and Spiegel (1994) and Barro and Sala-i-Martin (1995). In the first study, the authors use several human capital measures from several sources to confirm this result. They believe that human capital should not be treated as another factor in the production function of the economy but, in the spirit of Nelson and Phelps (1966), as a factor explaining total factor productivity growth. They show that human capital is significant under this treatment. The Barro and Sala-i-Martin (1995) analysis is more complete since they introduce a set of variables to evaluate the importance of human capital for growth. Apart from contemporaneous changes in schooling attainment, they consider schooling attainment, school-enrollment ratios, life expectancy, and the proportion of GDP devoted to public expenditure on education. Their results are similar to Benhabib and Spiegel's but their explanation of the puzzle consists of blaming measurement errors.

An interesting result of the second study is the finding of a significant role for public expenditure on education, that the authors interpret as measuring the quality of schooling. This empirical evidence is an indication that changes in schooling quality matter. The question is, are the two findings, no role for changes in school attainment, but a significant role for schooling quality, related? If so, how? My conjecture is that the interaction between changes in schooling quantity and quality might provide an explanation to the puzzle. One would expect that the government's decisions on educational expenditure will affect individual schooling decisions. Card and Krueger (1992) show that, at the microeconomic level, this is the case.

Buiter and Kletzer (1995) analyze the effect of public expenditure on education on private investment in human capital. They develop an overlapping generations model that excludes decisions on time allocation, in which human capital is accumulated with private and public expenditure. Agents invest in human capital when young and public expenditure is financed by taxes paid by middle-aged agents. They obtain that an increase in public expenditure may increase, reduce or hold constant private expenditure. Their results suggest that there is no clear-cut relationship between public funding of education and private spending on education.

To test empirically, in a rigorous way, whether this interaction is significant for growth is beyond the scope of this paper. Instead, I will look at data in a very simple way, by correlating per capita GDP growth rate, changes in average years of schooling and the proportion of GDP devoted to public expenditure on education from the well-known Barro and Lee (1994) data set.[5]

Tables 1, 2 and 3 show correlations between the per capita GDP growth rate and the human capital growth rate (measured by the average years of schooling growth rate); between the per capita GDP growth rate and the proportion of GDP devoted to public expenditure on education; and between human capital growth rate and this proportion, respectively. The data cover five-year sub-periods from 1960 to 1985. Correlations in Table 1, including those on the diagonal reflecting contemporaneous relationships, are either negative or positive and low. On the other hand, Table 2 shows higher and, in general, positive correlations between the growth rate and public expenditure on education, while Table 3 shows, in general, negative correlations between changes in human capital and public expenditure on education. This simple look at the data suggests that an increase in the proportion of GDP devoted to public expenditure on education might provoke an increase in per capita GDP growth rate and a decrease in the time spent in schooling. This would lead to a negative and weak relationship between contemporaneous changes in school attainment and growth.

Of course, this is a very simple way to look at the data and does not imply that these relationships are statistically significant. Here, I just try to get some intuition about what may be the reason behind these puzzling results. In the next section I develop a theoretical framework that captures the relationships in these three tables.

Table 1

Correlation Between Per Capita GDP Growth Rate (*grXX*) and Average Years of School Growth Rate (*schXX*) (*XX* Refers to Five-Year Sub-Periods)

	gr6065	*gr6570*	*gr7075*	*gr7580*	*gr8085*
Sch6065	0.0979	-0.0817	-0.0334	-0.0699	-0.0399
sch6570	-	0.0539	0.0735	-0.0013	-0.1749
sch7075	-	-	-0.0517	0.0849	-0.0304
sch7580	-	-	-	-0.1310	-0.1122
sch8085		-	-	-	-0.0732

Table 2

Correlation Between Per Capita GDP Growth Rate (*grXX*) and Public Expenditure on Education/GDP (*pexXX*) (*XX* Refers to Five-Year Sub-Periods)

	gr6065	*gr6570*	*gr7075*	*gr7580*	*gr8085*

5 The sample I consider is composed of 78 countries, those with no missing values, and cover five years sub-periods from 1960 to 1985. See Barro and Lee (1994) for a more detailed explanation of the data set.

Pex6064	-0.0007	0.1301	0.2487	0.1394	0.2708
Pex6569	-	0.1187	0.1731	-0.0215	0.1698
Pex7074	-	-	0.0985	0.0106	0.1927
Pex7579	-	-	-	-0.1295	0.1990
Pex8084		-	-	-	0.1573

Table 3
Correlation Between Schooling Years Growth Rate (*schXX*)
and Public Expenditure on Education/GDP (*pexXX*)
(*XX* Refers to Five-Year Sub-Periods)

	Sch6065	*sch6570*	*sch7075*	*sch7580*	*sch8085*
Pex6064	-0.0131	0.3000	-0.1933	-0.0676	-0.1870
pex6569	-	0.2162	-0.2561	-0.1647	-0.2295
pex7074	-	-	-0.2164	-0.1479	-0.2440
pex7579	-	-	-	-0.0399	-0.0586
pex8084		-	-	-	-0.0601

3. A simple model of public expenditure on education

This section develops a very simple two-sector model of endogenous growth in which individuals accumulate human capital by devoting time to school and the public expenditure on education is an input in the human capital production function. I will assume, for simplicity, that this is not a market activity but rather it is carried out at home. The technology that I propose can be written as

$$\dot{H}_t = E_t^{\gamma}\left(H_t(1-U_t)\right)^{1-\gamma} - \delta H_t, \quad 0<\gamma<1 , \tag{1}$$

where $\dot{H}_t$ and H_t denote net investment in human capital and the level of human capital, respectively, δ is a positive depreciation rate, E_t is the public expenditure, and $1-U_t$ and U_t denote time allocated to education and work, respectively. Clearly, $1-U_t$ is a flow variable and H_t is a stock. The empirical studies use schooling attainment as a measure of H_t and enrollment ratios or shifts in the schooling attainment to measure the flow.

Notice that $1-U_t$ and the growth rate are directly related if the economy is in steady state. However, real economies are continuously suffering shocks that throw them out of their long-run equilibrium. The shocks herein refer to changes in the proportion of resources that the government decides to devote to education. The goal of this paper is to analyze how these changes affect the relationship between long-run growth and time allocation.

The goods sector consists of a continuum of competitive firms that I normalize to unity. These firms operate with a constant returns to scale Cobb-Douglas technology

$$Y_t = K_t^{\alpha}(H_t U_t)^{1-\alpha}, \quad 0<\alpha<1 \tag{2}$$

where Y_t denotes output and K_t is the level of physical capital. The government collects part of the total product generated in the economy through a proportional tax whose rate is τ_t . This specification amounts to the condition that the government's production function does not differ in form from that of the private sector. I will assume throughout the paper that τ_t is constant over time.[6] This policy parameter represents the size of the government since it is the proportion of total public expenditure with respect to total production in the economy.

From the profits maximization I get the usual conditions of equality between factor prices and marginal productivities

$$\begin{aligned} r_t &= \alpha K_t^{\alpha-1}(H_t U_t)^{1-\alpha} \\ w_t &= (1-\alpha) K_t^{\alpha}(H_t U_t)^{-\alpha} \end{aligned} \tag{3}$$

where r_t is the gross interest rate and w_t is the wage per efficiency unit. The aggregate good can be accumulated as physical capital, sold for consumption and used as public expenditure.

In each period the government spends all public revenues in such a way that the public budget is always balanced. This assumption is reasonable because, although in real economies governments may have budgetary deficits or surpluses, the budget should be balanced in the long run. Therefore, the government's budget constraint is

$$E_t = \tau K_{t\alpha}(H_t U_t)^{1-\alpha} \tag{4}$$

The economy is inhabited by a continuum of infinitely-lived identical households whose number is normalized to unity. There is no population growth, which implies that all variables are expressed in per capita terms. Each household derives utility from an aggregate consumption good, C_t , and maximizes its intertemporal utility discounted at a positive rate ρ

6 It could be argued that government's decisions on τ_t are endogenous since they try to reflect social preferences. However, the objective here is not to theorize on government behavior.

$$\int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}-1}{1-\sigma} dt \ . \tag{5}$$

Physical capital depreciates at a constant and positive rate, δ . Notice that I am assuming that both physical and human capital depreciates at exactly the same rate. This assumption is made to offer a clearer exposition and, as I will show later, it does not affect the main conclusions. Household's budget constraint can be expressed as

$$\dot{K}_t = \left(\left(1-\tau \right) r_t - \delta \right) K_t + \left(1-\tau \right) w_t H_t U_t - C_t \ \cdot \tag{6}$$

where $\dot{K}_t$ is the net investment in physical capital. The price of the aggregate good is taken as *numeraire.*

The representative household optimization problem (P) consists of choosing paths of consumption and time to be allocated to education and work that maximize the total discounted utility subject to (1) and (6), given K_0 and H_0 and the path of public expenditure, E_t .

This is a standard problem of dynamic optimization whose control variables are C_t and U_t and whose state variables are K_t and H_t . Assuming that homes have perfect foresight, they can advance interest rate, wage per efficiency unit of labor and public expenditure paths. Therefore, according to the maximum principle, the optimal trajectories of the representative household $\left\{ C_t, U_t, K_t, H_t \right\}_{t=0}^{\infty}$ satisfy the following first-order conditions[7]

$$C_t^{-\sigma} - \mu_{1t} = 0 \ , \tag{7}$$

$$\mu_{1t} (1-\tau) w_t H_t - \mu_{2t} (1-\gamma) E_t^{\gamma} H_t^{1-\gamma} (1-U_t)^{-\gamma} = 0 \ , \tag{8}$$

$$\dot{\mu}_{1t} = \mu_{1t} \rho - \mu_{1t} \left(\left(1-\tau \right) r_t - \delta \right) \ , \tag{9}$$

$$\dot{\mu}_{2t} = \mu_{2t} \rho - \mu_{1t} \left(1-\tau \right) w_t U_t - \mu_{2t} \left(\left(1-\gamma \right) E_t^{\gamma} H_t^{-\gamma} \left(1-U_t \right)^{1-\gamma} - \delta \right)$$

(10)

and the two usual tranversality conditions

$$\lim_{t \to \infty} e^{-\rho t} \mu_{1t} K_t = 0 \ , \tag{11}$$

[7] The fact that both the objective function and the two restrictions are concave guarantees that necessary conditions are also sufficient.

$$\lim_{t\to\infty} e^{-\rho t}\mu_{2t}H_t=0\ , \tag{12}$$

where $e^{-\rho t}\mu_{1t}$ and $e^{-\rho t}\mu_{2t}$ are the co-state variables or shadow prices associated with K_t and H_t respectively.

4. The steady state equilibrium

In this section I will construct a system of four differential equations on the variables C_t , U_t , K_t and H_t using the first-order conditions, the restrictions (1) and (6) and the maximum profit conditions, which allow me to characterize the balanced growth path (BGP). First of all, I define a competitive equilibrium for this economy.

Definition 1. The competitive equilibrium of the economy is a set of paths $\{C_t, U_t\}_{t=0}^{\infty}$, $\{H_t, K_t\}_{t=1}^{\infty}$ *and prices* $\{r_t, w_t\}_{t=0}^{\infty}$ *, given the policy parameter* $\{\tau\}_{t=0}^{\infty}$ *and* K_0, H_0 *, that solve the problem of the representative household (P), that satisfy the conditions of maximum profit of the firms (3) and the government budget constraint (4), and that clear product, physical capital and labor markets.*

Using (3), (4), (8) and (10) I get the growth rate of the human capital shadow price

$$\frac{\dot{\mu}_{2t}}{\mu_{2t}}=\rho+\delta-(1-\gamma)\tau^{\gamma}K_t^{\alpha\gamma}(H_tU_t)^{-\alpha\gamma}\left(\frac{U_t}{1-U_t}\right)^{\gamma}\ . \tag{13}$$

The consumption growth rate can be written by using (3), (7) and (9)

$$\frac{\dot{C}_t}{C_t}=(1-\tau)\frac{\alpha}{\sigma}K_t^{\alpha-1}(H_tU_t)^{1-\alpha}-\frac{\rho+\delta}{\sigma}\ . \tag{14}$$

I obtain the working time growth rate by fully differentiating (8) with respect to time and by taking into account (1), (3), (4), (6), (9) and (13)

$$\frac{\dot{U}_t}{U_t}=\frac{1}{B_t}[(1-\tau)\tau^{\gamma}K^{\alpha\gamma}(H_tU_t)^{-\alpha\gamma}\left(\frac{U_t}{1-U_t}\right)^{\gamma}-(1-\tau)\alpha K_t^{\alpha-1}(H_tU_t)^{1-\alpha}+$$
$$\alpha(1-\gamma)\left((1-\tau)K^{\alpha-1}(H_tU_t)^{1-\alpha}-\frac{C_t}{K_t}-\tau^{\gamma}K^{\alpha\gamma}(H_tU_t)^{-\alpha\gamma}U_t^{\gamma}(1-U_t)^{1-\gamma}\right)]$$

(15)

where $B_t \equiv \gamma \left(\frac{U_t}{1-U_t} \right)^{\gamma} \frac{1}{1-U_t} + \alpha (1-\gamma)$.

As I said above, the competitive equilibrium of this economy can also be represented by the system of four differential equations composed of expressions (1), (6), (14) and (15), after substituting (3) and (4), the initial values of the state variables and the two transversality conditions. To characterize the steady state simply, I redefine the variables in order for them to have zero growth rates in the steady state. To do this, I must investigate how the four variables behave when the economy is in the steady state equilibrium.

Definition 2. A steady state equilibrium is a competitive equilibrium set of paths $\{c_t, u_t, k_t, h_t\}_{t\geq 0}^{\infty}$, *given* τ , *such that* c_t, k_t, h_t and y_t *grow at a constant rate and* u *remains constant over time.*

From now on, the variables during the transition will be written in capitals and in the steady state in lowercase. With the last definition in mind, it is easy to check that c_t, k_t, h_t and y_t grow at the same rate. Since the two states and the consumption grow at the same rate, I can define the following intensive variables

$$W_t \equiv \frac{K_t}{H_t}, Z_t \equiv \frac{K_t}{H_t U_t}, X_t \equiv \frac{C_t}{H_t} . \tag{16}$$

Taking into account the definition of the steady state and these new variables, I get an implicit expression for u by equating the growth rates of c_t and h_t , and of μ_{1t} and μ_{2t}

$$\tau^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}} \left(\frac{u}{1-u} \right)^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}} \left(\frac{(1-\tau)\alpha}{1-\gamma} \right)^{\frac{\alpha\gamma}{1-\alpha(1-\gamma)}} \left(1-u-\frac{1-\gamma}{\sigma} \right) - \delta + \frac{\rho+\delta}{\sigma} = 0$$

. (17)

This latter expression implies that the intertemporal elasticity of substitution is a key parameter in determining the time allocation. The rest of the steady state values are functions of u such that proving the existence and the uniqueness of a solution to (17), the existence and uniqueness of the steady state equilibrium is guaranteed. The following proposition, whose proof is shown in the appendix, contains this result.

Proposition 1. If the steady state equilibrium exists, it is unique. A sufficient condition for the existence of a balanced growth path is

$$(\tau\gamma)^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}}\left((1-\gamma)\alpha\right)^{\frac{\alpha\gamma}{1-\alpha(1-\gamma)}}(1-\tau)^{\frac{(1-\gamma)(1-\alpha)}{1-\alpha(1-\gamma)}}\left(\frac{\sigma-1}{\sigma}\right)-\delta+\frac{\rho+\delta}{\sigma}=0 \cdot \tag{18}$$

The growth rate of the economy can be written as a function of u as follows

$$\theta=\frac{\alpha}{\sigma}(1-\tau)^{\frac{\alpha\gamma}{1-\alpha(1-\gamma)}}\left(\frac{1-\gamma}{\alpha}\right)^{\frac{1-\alpha}{1-\alpha(1-\gamma)}}\tau^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}}\left(\frac{u}{1-u}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}}-\frac{\rho+\delta}{\sigma}=0 \; . \tag{19}$$

I need an extra condition to ensure the existence of sustained growth; that is, to ensure that the growth rate is positive in the long run. As shown in the proof of Proposition 1, the fulfillment of both transversality conditions requires $u>\gamma$. Since θ is an increasing function in u , a sufficient condition for θ to be positive can easily be reached by substituting γ for u in (19) and by imposing this new expression to be positive. The last result on the balanced growth path refers to stability. As usual, to prove local stability I construct a system of three differential equations on W_t , X_t and U_t and linearize it around the steady state. Proposition 2, whose proof appears in the appendix, contains this result.

Proposition 2. The BGP is locally stable.

5. Long-run effects of changes in the tax rate on time allocation and the growth rate

In this section I carry out an exercise of comparative statics to determine how different fiscal policies affect the growth rate and individuals' time allocation decisions. I start by studying the relationship between time spent working and the policy parameter. Applying the implicit function theorem to expression (17) I obtain

$$\frac{\mathrm{du}}{\mathrm{d\tau}}=\frac{\frac{\gamma(1-\alpha-\tau)u(1-u)}{\left(1-\alpha(1-\tau)\right)\tau(1-\tau)}\left(1-u-\frac{1-\gamma}{\sigma}\right)}{(1-u)\left(u-\frac{\gamma(1-\alpha)}{1-\alpha(1-\gamma)}\right)+\frac{(1-\alpha)(1-\gamma)}{\sigma\left(1-\alpha(1-\gamma)\right)}} \; . \tag{20}$$

Note that the denominator of (20) is positive since $u>\gamma$. Therefore, the sign of (20) will depend on the tax rate and the intertemporal elasticity of substitution values. For small enough values of the IES, the working time reaches a maximum at $\tau^{¿}=1-\alpha$, while for large enough values of IES this variable reaches a minimum at $\tau^{¿}$. For intermediate values of

IES the time allocation is not affected by changes in τ. The relationship is summarized in the following proposition.

Proposition 3. The behavior of the steady state value of working time when τ *changes is as follows*

Case 1: if $-\delta+\dfrac{\delta+\rho}{\sigma}<0$ *(small enough IES) then* $1-u-\dfrac{1-\gamma}{\sigma}>0$ *and* u *reaches a maximum at* $\tau^{¿}=1-\alpha$.

Case 2: if $-\delta+\dfrac{\delta+\rho}{\sigma}>0$ *(large enough IES) then* $1-u-\dfrac{1-\gamma}{\sigma}<0$ *and* u *reaches a minimum at* $\tau^{¿}=1-\alpha$.

Case 3: if $-\delta+\dfrac{\delta+\rho}{\sigma}=0$ *(for some intermediate value of IES) then* $1-u-\dfrac{1-\gamma}{\sigma}=0$ *and* u *is a constant function of* τ.

This causality would only consist of the second case if human capital does not depreciate. Hence, the existence of depreciation is a key assumption, which deserves to be justified by means of a short review of the empirical evidence on this parameter.

Haley (1976) estimated simultaneously all parameters of a life-cycle model of human capital accumulation from the observations of the age-earnings profile. The depreciation rate turned out to be significant for all educational groups (0.019, 0.041, 0.043, 0.035 and 0.027 for individuals with less than 8, 8, 9-11,12 and 13-15 years of schooling, respectively), except for the two groups with higher education (16 or more years of school). Heckman's (1976) objective was to estimate an extended Ben-Porath model that included a term for the value of leisure in order to test whether this term would improve the fit of the earnings functions. He obtained a positive and significant depreciation rate of 0.037 for the whole sample and of 0.047, 0.037, and 0.07 when the sample was broken into three educational groups of 12, 13-16 and 16 years of schooling, respectively. Heckman, Lochner and Taber (1998) assumed a zero depreciation rate in order to carry out their estimation of a human capital technology because of the lack of any peak in the life-cycle wage-age profile reported in some works (e.g. Mincer, 1974; Meghir and Whitehouse, 1996). However, other authors have shown the opposite result (e.g. Murphy and Welch (1992); Mulligan (1998)). Looking at the empirical evidence, the assumption of human capital depreciation seems reasonable.

I cannot exactly establish what small and large enough IES means because its threshold value depends on δ and ρ. What I can say is that all cases can occur for empirically plausible values of this parameter.[8] The time allocation condition (8) implies that the value of devoting a unit of time to work (wage per time unit) must be equal to the value of devoting a unit of time to school (marginal product of time in the educational sector valued in terms of relative shadow prices)[9]. A marginal increase in τ reduces the value of time in the good sector because of the *tax effect* and leads to a drop in the capital-labor ratio. However, it has an ambiguous effect on the value of time in the human capital sector because although more resources are being devoted to education, the relative shadow price, $\frac{\mu_{2t}}{\mu_{1t}}$, is falling. The response of working time will depend on the magnitude of the reaction of the relative shadow price and, in turn, this will depend on the IES value.

A low IES means that individuals are less willing to shift consumption across time. Therefore, they will prefer to accumulate physical capital to produce human capital since this latter activity implies a reduction in the current labor income and, hence, it forces them to postpone today's consumption. In this case individuals will devote more time to work and less to education. As τ rises, the value of devoting an additional unit of time to work becomes lower and eventually u reaches a peak (case 1). The opposite behavior happens if the IES is high enough (case 2). For an intermediate value of the IES both effects are exactly offset and changes in τ do not affect the allocation of time (case 3).

The same analysis can be carried out with the growth rate. Using (19) and (20) I obtain

$$\frac{d\theta}{d\tau}=(1-\tau)\frac{\alpha}{\sigma}z^{\alpha-1}\frac{\dfrac{\gamma(1-\alpha-\tau)u(1-u)}{\left(1-\alpha(1-\tau)\right)\tau(1-\tau)}}{(1-u)\left(u-\dfrac{\gamma(1-\alpha)}{1-\alpha(1-\gamma)}\right)+\dfrac{(1-\alpha)\gamma(1-\gamma)}{\left(1-\alpha(1-\gamma)\right)}}\ .$$

(21)

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8 For a discussion on intertemporal elasticity of substitution estimates see Auerbach, Kotlikoff and Skinner (1983).

9 After some manipulations this condition becomes

$$(1-\tau)(1-\alpha)\left(\frac{k_t}{h_t}\right)^{\alpha(1-\gamma)}=\frac{\mu_{2t}}{\mu_{1t}}(1-\gamma)\tau^{\gamma}\left(\frac{u}{1-u}\right)^{\gamma}\ .$$

This expression may help the reader to follow the explanation in the text.

The growth rate reaches a maximum at $\tau^{¿}=1-\alpha$ for any value of the IES. That implies that a proportion of output that is equal to the labor share must be devoted to education in order to achieve the highest growth rate. Of course, there is no reason for the government to maximize the long-run growth rate or any other variable *per se*. For a benevolent government, the appropriate objective would be to maximize the utility attained by the representative household. However, this paper is not concerned with analyzing optimal public expenditure on education.

As in Barro (1990), the effects of the government on growth involve two channels: an increment in τ reduces θ (this is the *tax effect)*, but also increases the marginal productivity of physical capital, which increases θ . The second force dominates when the government is small, and the first one when it is large enough. The size of the government that maximizes the long-run growth rate corresponds to the natural condition for productive efficiency: the social cost of producing a unit of public expenditure is one and the social benefit is $\frac{\partial Y_t}{\partial \tau}=\frac{1-\alpha}{\tau}$.

Figures 1a and 1b illustrate the results. They have been constructed by using expressions (17) and (19) and by setting the parameter values at $\alpha=0.33$, $\gamma=0.65$, $\delta=0.05$ and $\rho=0.04$; σ values have been selected to show the three possible relationships between time allocation and public expenditure on education-output ratio. Note that the lower the IES, the higher the growth rate for any tax rate, because this implies that less time is devoted to schooling for any tax rate.

Remember that I am assuming that both types of capital depreciate at the same rate. It is easy to verify that similar results would be obtained if this condition is removed. The derivation of the comparative statics of the model with different depreciation rates is shown in the appendix.

This exercise has shown that, if a more general production function of human capital is assumed, faster growth is not necessarily linked to devoting more time to school. Movements in other variables that affect the growth rate, in this case the public expenditure on education, may also affect the allocation of time generating the result.

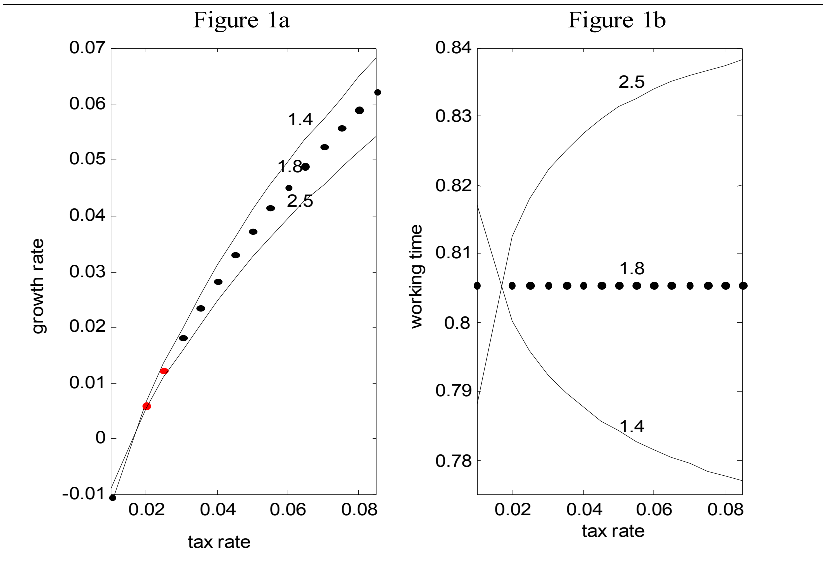

Fig. 1. $\alpha = 0.33$, $\gamma = 0.65$, $\delta = 0.05$, $\rho = 0.04$, $\sigma = \{1.4, 1.8, 2.5\}$

6. Summary and conclusions

A puzzling result from growth accounting exercises is that contemporaneous changes in schooling attainments of the labor force are not significantly related to the growth rate. A possible explanation of this puzzle is the existence of other types of investment in human capital whose interaction with the schooling time causes this result. I explored this possibility by developing a very simple two-sector model of endogenous growth in which individuals accumulate human capital by devoting time to school and where public expenditure on education is an input in human capital production. Changes in the proportion of resources devoted to improve the quality of schooling, as measured in the model by the size of government, affect individuals' time allocation as well as the growth rate of output.

While the relationship between the growth rate and size of the government always presents a maximum, the extent of schooling may reach a maximum, a minimum or be a constant function in the tax rate, and this behavior crucially depends on the intertemporal elasticity of substitution. When the IES is small enough, individuals are less willing to shift consumption across time. So they will prefer to save in order to produce human capital and, hence, they will work more and will devote less time to education. A high enough IES brings about the opposite behavior. For an intermediate value of the IES the relative value of the time in both activities does not change and neither does the allocation of time. Therefore, this simple model is able to give an explanation to the empirical facts summarized in Section 2.

Appendix

Proof of Proposition 1

To prove the existence of a BGP the only thing I need to prove is that (17) has a solution. Note that in the steady state the fulfillment of both transversality conditions requires

$$z^{\alpha\gamma}\tau^{\gamma}\left(\frac{u}{1-u}\right)^{\gamma}(\gamma-u)<0 \ . \tag{22}$$

Hence, all values of u have to be strictly higher than γ. Next I define the following auxiliary function which allows me to prove the existence of a single solution of (17)

$$F(\hat{u})=\tau^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}}\left(\frac{\hat{u}}{1-\hat{u}}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha(1-\gamma)}}\left(\frac{(1-\tau)\alpha}{1-\gamma}\right)^{\frac{\alpha\gamma}{1-\alpha(1-\gamma)}}\left(1-\hat{u}-\frac{1-\gamma}{\sigma}\right)-\delta+\frac{\rho+\delta}{\sigma} \ . \tag{23}$$

It easy to check that $F(\hat{u})$ is strictly decreasing for all possible equilibrium values of u. Note that the relevant range is $(\gamma, 1)$ since the transversality conditions have to be satisfied. Then, if u exists it is unique. To prove existence I have to look for two values of $\hat{u}$ such that $F(\hat{u})$ changes sign. Note that for values near one, $F(\hat{u})$ goes to minus infinity. Setting $\hat{u}=\gamma$ in the auxiliary function and imposing this expression to be positive, I obtain (18).

Proof of Proposition 2

First, I construct the dynamic system composed of three differential equations on W_t, X_t and U_t

$$\frac{\dot{W}_t}{W_t}=(1-\tau)W_t^{\alpha-1}U_t^{1-\alpha}-\frac{X_t}{W_t}-\tau^{\gamma}W_t^{\alpha\gamma}U_t^{-\alpha\gamma}U_t^{\gamma}(1-U_t)^{1-\gamma}\equiv\Phi(W_t,X_t,U_t) \ , \tag{24}$$

$$\frac{\dot{X}_t}{X_t}=\frac{\sigma-\alpha}{\sigma}(1-\tau)W_t^{\alpha-1}U_t^{1-\alpha}+\frac{X_t}{W_t}+\frac{\dot{W}_t}{W_t}+\delta-\frac{\delta+\rho}{\sigma}\equiv\Omega(W_t,U_t) \ , \tag{25}$$

$$\frac{\dot{U}_t}{U_t}=\frac{1}{B_t}[(1-\tau)\tau^{\gamma}W_t^{\alpha\gamma}U_t^{-\alpha\gamma}\left(\frac{U_t}{1-U_t}\right)^{\gamma}-$$
$$(1-\tau)\alpha W_t^{\alpha-1}U_t^{1-\alpha}+\alpha(1-\gamma)\frac{\dot{W}_t}{W_t}]\equiv\Psi(W_t,X_t,U_t)$$

(26)

Next, as usual, I linearize the system around the steady state. It is easy to check that the components of the Jacobian matrix associated with the system linearized around w,x,u may be written as

$$J_{SS}=\begin{pmatrix}\Phi_w^{'} & \Phi_x^{'} & -\Phi_w^{'}-\Phi_x^{'}+a\\ \Phi_w^{'}+\Phi_x^{'}+b & 0 & -\Phi_w^{'}-\Phi_x^{'}+a-b\\ c+d\,\Phi_w^{'} & d\,\Phi_x^{'} & -c+e+d\left(-\Phi_w^{'}-\Phi_x^{'}+a\right)\end{pmatrix}, \quad (27)$$

where $\Phi_w^{'}$ and $\Phi_x^{'}$ are partial derivatives of Φ respect to W_t and X_t evaluated at the steady state, respectively, and a,b,c,d and e are positive expressions

$$a\equiv\alpha\gamma\tau^{\gamma}z^{\alpha\gamma}u^{\gamma-1}(1-u)^{-\gamma}(u-\gamma)\ , \quad (28)$$

$$b\equiv(1-\alpha)(1-\tau)\frac{\sigma-\alpha}{\sigma}z^{\alpha-1}\ , \quad (29)$$

$$c\equiv\frac{1}{B}\left(\alpha\gamma(1-\gamma)\tau^{\gamma}z^{\alpha\gamma}\left(\frac{u}{1-u}\right)^{\gamma}+(1-\tau)\alpha(1-\alpha)z^{\alpha-1}\right), \quad (30)$$

$$d\equiv\frac{\alpha(1-\gamma)}{B}\ , \quad (31)$$

$$e\equiv\frac{(1-\gamma)}{B}\gamma z^{\alpha\gamma}\tau^{\gamma}\left(\frac{u}{1-u}\right)^{\gamma}\frac{1}{u(1-u)}\ . \quad (32)$$

From the Routh-Hurwitz theorem, if the determinant of the Jacobian matrix is negative, the system is stable since this result ensures the existence of at least one negative eigenvalue. Unfortunately, the complexity of this system prevents me from obtaining more concrete results on the transition. In this case the determinant is

$$Det(J_{SS})=\Phi_x^{'}\left[ca-e\left(\Phi_w^{'}+\Phi_x^{'}+b\right)\right]<0\ . \quad (33)$$

Comparative statics when the depreciation rates differ

By equating the versions of equations (9) and (13) when both depreciation rates are different, and (1) and the version of (14), once introduced (4) and taken into account (16), I obtain two expressions that allow me to solve for the steady state values of u and z in this case

$$\left(1-\tau\right)\alpha z^{\alpha-1}-\delta_h+\delta_k-\left(1-\gamma\right)\tau^{\gamma}z^{\alpha\gamma}\left(\frac{u}{1-u}\right)^{\gamma}=0\ , \tag{34}$$

$$\tau^{\gamma}z^{\alpha\gamma}u^{\gamma}\left(1-u\right)^{1-\gamma}-\left(1-\tau\right)\frac{\alpha}{\sigma}z^{\alpha-1}-\delta_h+\frac{\delta_k+\rho}{\sigma}=0\ , \tag{35}$$

where δ_h and δ_k denote human and physical capital depreciation rates, respectively. By using the implicit function theorem, I check that $\frac{du}{d\tau}=0$ if $\tau=1-\alpha$ and $\frac{dz}{d\tau}<0$. Substituting one expression into the other and differentiating with respect to τ , I obtain

$$\frac{du}{d\tau}=\frac{\gamma\left(\frac{1}{\tau}+\frac{\alpha}{z}\frac{dz}{d\tau}\right)\left(1-u-\frac{1-\gamma}{\sigma}\right)}{\frac{\gamma}{u\left(1-u\right)}\left(\left(1-u\right)\left(u-\gamma\right)+\frac{1-\gamma}{\sigma}\right)}\ . \tag{36}$$

Taking into account the version of (14) in this case, the growth rate can be written as

$$\theta=\left(1-\tau\right)\frac{\alpha}{\sigma}z^{\alpha-1}-\frac{\rho+\delta_k}{\sigma}\ . \tag{37}$$

Differentiating respect to τ I get

$$\frac{d\theta}{d\tau}=\frac{\alpha}{\sigma}z^{\alpha-1}\left(-1-\left(1-\tau\right)\frac{1-\alpha}{\alpha}\frac{dz}{d\tau}\frac{\alpha}{z}\right)\ . \tag{38}$$

The sign of $\frac{d\theta}{d\tau}$ can be analyzed by inspecting $\frac{du}{d\tau}$. $\frac{d\theta}{d\tau}$ equals zero if τ equals $1-\alpha$, $\frac{d\theta}{d\tau}>0$ if $\tau<1-\alpha$ and $\frac{d\theta}{d\tau}<0$ if $\tau>1-\alpha$.

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Özet

Öğrenime ve kamu eğitim harcamalarına ilişkin basit bir büyüme modeli

Bu makale, eğitime yapılan kamu harcamalarının fiziksel ve beşerî sermayeli iki sektörlü bir içsel büyüme modelinde zaman tahsisine ve büyümeye etkisini incelemektedir. Bireyler öğrenime vakit ayırarak beşerî sermayelerini artırmaktadır ve devletin eğitim harcamaları beşerî sermaye teknolojisinde girdi olmaktadır. Modelde devlet, kaynaklarından eğitime ayırdığı payı değiştirdiğinde büyüme oranı artmaktadır; buna mukabil dönemler arası ikame elastikiyetine göre öğrenime ayrılan vakit artabilmekte, azalabilmekte veya sabit kalabilmektedir. Bu bulgu, iş gücü eğitim düzeyinin fert başına GSYİH artış oranıyla anlamlı bir ilişki göstermemesini kısmen izah etmektedir.