ORTA DOĞU TEKNİK ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ MÜDÜRLÜĞÜ

TEZ ŞABLONU ONAY FORMU
THESIS TEMPLATE CONFIRMATION FORM

1. Şablonda verilen yerleşim ve boşluklar değiştirilmemelidir.
2. **Jüri tarihi** Başlık Sayfası, İmza Sayfası, Abstract ve Öz'de ilgili yerlere yazılmalıdır.
3. İmza sayfasında jüri üyelerinin unvanları doğru olarak yazılmalıdır. Tüm imzalar **mavi pilot kalemle** atılmalıdır.
4. **Disiplinlerarası** programlarda görevlendirilen öğretim üyeleri için jüri üyeleri kısmında tam zamanlı olarak çalıştıkları anabilim dalı başkanlığının ismi yazılmalıdır. Örneğin: bir öğretim üyesi Biyoteknoloji programında görev yapıyor ve biyoloji bölümünde tam zamanlı çalışıyorsa, İmza sayfasına biyoloji bölümü yazılmalıdır. İstisnai olarak, disiplinler arası program başkanı ve tez danışmanı için disiplinlerarası program adı yazılmalıdır.
5. Tezin **son sayfasının sayfa** numarası Abstract ve Öz'de ilgili yerlere yazılmalıdır.
6. Bütün chapterlar, referanslar, ekler ve CV sağ sayfada başlamalıdır. Bunun için **kesmeler** kullanılmıştır. **Kesmelerin kayması** fazladan boş sayfaların oluşmasına sebep olabilir. Bu gibi durumlarda paragraf (¶) işaretine tıklayarak kesmeleri görünür hale getirin ve yerlerini **kontrol edin**.
7. Figürler ve tablolar kenar boşluklarına taşmamalıdır.
8. Şablonda yorum olarak eklenen uyarılar dikkatle okunmalı ve uygulanmalıdır.
9. Tez yazdırılmadan önce PDF olarak kaydedilmelidir. Şablonda yorum olarak eklenen uyarılar PDF dokümanında yer almamalıdır.
10. **Bu form aracılığıyla oluşturulan PDF dosyası arkalı-önlü baskı alınarak tek bir spiralli cilt haline getirilmelidir.**
11. Spiralli hale getirilen tez taslağınızdaki ilgili alanları imzalandıktan sonra, Tez Juri Atama Formu ile birlikte bölüm sekreterliğine teslim edilmelidir.
12. Tez taslaklarının kontrol işlemleri tamamlandığında, bu durum öğrencilere METU uzantılı öğrenci e-posta adresleri aracılığıyla duyurulacaktır.
13. Tez yazım süreci ile ilgili herhangi bir sıkıntı yaşarsanız, **Sıkça Sorulan Sorular (SSS)** sayfamızı ziyaret ederek yaşadığınız sıkıntıyla ilgili bir çözüm bulabilirsiniz.

1. Do not change the spacing and placement in the template.
2. Write **defense date** to the related places given on Title page, Approval page, Abstract and Öz.
3. Write the titles of the examining committee members correctly on Approval Page. **Blue ink** must be used for all signatures.
4. For faculty members working in **interdisciplinary programs**, the name of the department that they work full-time should be written on the Approval page. For example, if a faculty member staffs in the biotechnology program and works full-time in the biology department, the department of biology should be written on the approval page. Exceptionally, for the interdisciplinary program chair and your thesis supervisor, the interdisciplinary program name should be written.
5. Write **the page number of the last page** in the related places given on Abstract and Öz pages.
6. All chapters, references, appendices and CV must be started on the right page. **Section Breaks** were used for this. **Change in the placement** of section breaks can result in extra blank pages. In such cases, make the section breaks visible by clicking paragraph (¶) mark and **check their position.**
7. All figures and tables must be given inside the page. Nothing must appear in the margins.
8. All the warnings given on the comments section through the thesis template must be read and applied.
9. Save your thesis as pdf and Disable all the comments before taking the printout.
10. **Print two-sided the PDF file that you have created through this form and make a single spiral bound.**
11. Once you have signed the relevant fields in your thesis draft that you spiraled, submit it to the department secretary together with your Thesis Jury Assignment Form.
12. This will be announced to the students via their METU students e-mail addresses when the control of the thesis drafts has been completed.
13. If you have any problems with the thesis writing process, you may visit our **Frequently Asked Questions (FAQ)** page and find a solution to your problem.

☒ **Yukarıda bulunan tüm maddeleri okudum, anladım ve kabul ediyorum. / I have read, understand and accept all of the items above.**

Name : Hazel
Surname : Şentürk
E-Mail : e232285@metu.edu.tr
Date : 06.10.2021
Signature : ________________________

EXTENDING AN OUTRANKING MULTIPLE CRITERIA DECISION MAKING METHOD TO DIFFERENTIATE GAIN AND LOSS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

HAZEL ŞENTÜRK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

SEPTEMBER 2021

Approval of the thesis:

EXTENDING AN OUTRANKING MULTIPLE CRITERIA DECISION MAKING METHOD TO DIFFERENTIATE GAIN AND LOSS

submitted by **HAZEL ŞENTÜRK** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ____________

Prof. Dr. Esra Karasakal
Head of the Department, **Industrial Engineering** ____________

Prof. Dr. Esra Karasakal
Supervisor, **Industrial Engineering, METU** ____________

Prof. Dr. Orhan Karasakal
Co-Supervisor, **Industrial Engineering, Çankaya University** ____________

Examining Committee Members:

Prof. Dr. Yasemin Serin
Industrial Engineering, METU ____________

Prof. Dr. Esra Karasakal
Industrial Engineering, METU ____________

Prof. Dr. Gülser Köksal
Industrial Engineering, METU ____________

Assoc. Prof. Dr. Pelin Bayındır
Industrial Engineering, METU ____________

Assistant Prof. Özlem Karsu
Industrial Engineering, Bilkent University ____________

Date: 10.09.2021

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name Last name : Hazel Şentürk

Signature :

ABSTRACT

EXTENDING AN OUTRANKING MULTIPLE CRITERIA DECISION MAKING METHOD TO DIFFERENTIATE GAIN AND LOSS

Şentürk, Hazel
Master of Science, Industrial Engineering
Supervisor : Prof. Dr. Esra Karasakal
Co-Supervisor: Assoc. Prof. Dr. Orhan Karasakal

September 2021, 130 pages

In this study, the integration of Prospect Theory into ranking and sorting methods based on the dominance relations is studied. The well-known multi-criteria ranking method PROMETHEE and the well-known multi-criteria sorting method FlowSort are extended by using the prospect theory perspective. The proposed methods are used to rank and sort the alternatives in the case where the impact of losses is greater than gains for the same amount. When the results are compared with the PROMETHEE and FlowSort methods, the results show how the rankings and classes of the alternatives change according to the value of loss and gain that are determined by the decision-maker.

Keywords: Multi-Criteria Decision Making, Prospect Theory, PROMETHEE, FlowSort, Outranking Relations

ÖZ

BASKINLIK İLİŞKİSİ KULLANAN BİR ÇOK KRİTERLİ KARAR VERME YÖNTEMİNİN KAZANÇ VE KAYBI FARKLILAŞTIRARAK GENİŞLETİLMESİ

Şentürk, Hazel
Yüksek Lisans, Endüstri Mühendisliği
Tez Yöneticisi: Prof. Dr. Esra Karasakal
Ortak Tez Yöneticisi: Doç. Dr. Orhan Karasakal

Eylül 2021, 130 sayfa

Bu çalışmada Beklenti Teorisinin, baskınlık ilişkileri içeren sıralama ve sınıflandırma yöntemlerine entegrasyonu üzerine çalışılmıştır. En çok bilinen çok kriterli sıralama yöntemlerinden biri olan PROMETHEE yöntemi ve en çok bilinen çok kriterli sınıflandırma yöntemlerinden biri olan FlowSort yöntemleri beklenti teorisi bakış açısıyla değerlendirilmiştir. Geliştirilen yöntemler aynı miktardaki kaybın etkisinin kazançtan fazla olduğu durumlarda alternatifleri sıralama ve sınıflandırma için kullanılır. Sonuçlar PROMETHEE ve FlowSort yöntemleri ile karşılaştırıldığında, karar vericinin kayıp-kazanç arasında belirlediği değere göre alternatiflerin sıralamalarında ve sınıflarındaki değişimin etkisini gösterir.

Anahtar Kelimeler: Çok Kriterli Karar Verme, Beklenti Teorisi, PROMETHEE, FlowSort, Baskınlık İlişkileri

To my beloved family…

ACKNOWLEDGMENTS

I would like to express my deepest gratitude to my supervisor Prof. Dr. Esra Karasakal and my co-supervisor Assist. Prof. Dr. Orhan Karasakal for their continuous guidance, encouragements, criticism and support throughout my thesis study.

For and beyond this thesis study, I am indebted to my beloved family members, my mother Fatma Şentürk, my father Mustafa Şentürk, my sister Hilal Şentürk for their continuous love, belief, patience and altruism throughout my life.

I wish to thank Gökberk Tuncer who provide continuous encouragement and support throughout my study.

TABLE OF CONTENTS

ABSTRACT v
ÖZ vi
ACKNOWLEDGMENTS viii
TABLE OF CONTENTS ix
LIST OF TABLES xi
LIST OF FIGURES xv
LIST OF ABBREVIATIONS xvi
LIST OF SYMBOLS xviii
CHAPTERS
1 INTRODUCTION 1
2 LITERATURE REVIEW 3
2.1 RANKING METHODS BASED ON PROMETHEE 3
2.2 SORTING METHODS BASED ON PROMETHEE 4
2.3 PROSPECT THEORY 7
2.4 CONTRIBUTION TO LITERATURE 12
3 MULTICRITERIA RANKING PROBLEMS 15
3.1 BACKGROUND 15
3.1.1 PROMETHEE 15
3.1.2 THE PROSPECT THEORY 20
3.1.3 THE APPROACH TO CHOICE BEHAVIOR (Karasakal et al. (2019)) . 21
3.1.4 PT-PROMETHEE 25
3.2 THE PROPOSED METHOD 31

3.3 COMPUTATIONAL RESULTS 36
4 MULTICRITERIA SORTING PROBLEMS 59
4.1 BACKGROUND 59
4.2 THE PROPOSED METHOD 64
4.3 COMPUTATIONAL RESULTS 67
4.3.1 CPU 69
4.3.2 Auto MPG 71
4.3.3 Employee Selection (ESL) 73
4.3.4 Employee Rejection/Acceptance (ERA) 75
4.3.5 Lecturers Evaluation (LEV) 77
4.3.6 Car Evaluation (CEV) 79
4.3.7 Breast Cancer 82
4.3.8 Mammographic 85
4.3.9 General Remarks 87
4.4 ANALYSIS OF WEIGHTS 96
5 CONCLUSION AND FUTURE WORK 101
REFERENCES 105
APPENDICES
A. The Weight and the Threshold Values of each Criterion for Case 1 and 2 ... 123
B. Criterion Values of each Reference Alternative for each Data Set 124
C. Criterion Values of each Reference Alternative for each Data Set 125
D. Threshold Values of each Criterion for each Data Set 127
E. Criterion Values of each Reference Profile for each Data Set 129

LIST OF TABLES

TABLES

Table 3.1: Preference Functions of PROMETHEE, Brans et al. (1986) 17

Table 3.2: The additional preference functions proposed by Bozkurt (2007) 22

Table 3.3: The modified preference functions considering loss aversion, Lerche and Geldermann (2015). The red part illustrates the modified thresholds and preference functions. The original preference functions of PROMETHEE are shown with the black part.. 27

Table 3.4: The formulas of preference degree calculation to use in proposed methodology for leaving or entering flows.. 34

Table 3.5: The data taken Brans et al. (1986) 37

Table 3.6: Flow Values of PROMETHEE (Brans et al., 1986)............................. 38

Table 3.7: Flow Values of Proposed Method 39

Table 3.8: Criterion Values of Reference Alternatives.. 40

Table 3.9: Partial Ranking Results of PT-PROMETHEE by using each Reference Alternative.. 42

Table 3.10: Complete Ranking Results of PT-PROMETHEE by using each Reference Alternative 45

Table 3.11: Changes in Ranking among Comparison of Methods for THE World University Ranking 2019 Data Case I: The indifference threshold is 2 and the preference threshold is equal to the standard deviation 49

Table 3.12: Changes in Ranking among Comparison of Methods for THE World University Ranking 2019 Data Case II The indifference threshold is 5 and the preference threshold is equal to the standard deviation multiplied by 2................ 51

Table 3.13: Changes in Ranking among Comparison of Methods for THE World University Ranking 2020 Data Case I: The indifference threshold is 2 and the preference threshold is equal to the standard deviation. 53

Table 3.14: Changes in Ranking among Comparison of Methods for THE World University Ranking 2020 Data Case II The indifference threshold is 5 and the preference threshold is equal to the standard deviation multiplied by 2. 55
Table 4.1: The data sets with their number of alternatives, criteria, classes and source..........68
Table 4.2: The criteria of CPU data to evaluate the alternatives with their ranges and types..........69
Table 4.3: The total difference results between alternative classes comparing each run using CPU data..........70
Table 4.4: The criteria of Auto MPG data to evaluate the alternatives with their ranges and types 71
Table 4.5: The total difference results between alternative classes comparing each run using Auto MPG data..........72
Table 4.6: The criteria of ESL data to evaluate the alternatives with their ranges and types..........73
Table 4.7: The total difference results between alternative classes comparing each run using Employee Selection (ESL) data..........74
Table 4.8: The criteria of ERA data to evaluate the alternatives with their ranges and types..........75
Table 4.9: The total difference results between alternative classes comparing each run using Employee Rejection/Acceptance (ERA) data 76
Table 4.10: The criteria of LEV data to evaluate the alternatives with their ranges and types..........77
Table 4.11: The total difference results between alternative classes comparing each run using Lecturers Evaluation (LEV) data..........78
Table 4.12: The criteria of CEV data to evaluate the alternatives with their ranges and types..........80
Table 4.13: The difference of total results between alternative classes comparing each run using Car Evaluation (CEV) data 81

Table 4.14: The criteria of breast cancer data to evaluate the alternatives with their ranges and types 83
Table 4.15: The difference of total results between alternative classes comparing each run using Breast Cancer data 84
Table 4.16: The criteria of mammographic data to evaluate the alternatives with their ranges and types 85
Table 4.17: The difference of total results between alternative classes comparing each run using Mammographic data 86
Table 4.18: The total number of differences in case 2 analysis with 4 classes using ERA, ESL, LEV data sets 88
Table 4.19: Net flow and class values of the 8th alternative from CPU data to illustrate the first attitude. (The first attitude: The alternatives change the class when 2.25 is used as the loss aversion coefficient value comparing when 1.5 is used and again changes the class when 3 loss aversion coefficient value is used comparing when 2.25 is used) 90
Table 4.20: Net flow and class values of the 17th alternative from ESL data to illustrate the second attitude. (The second attitude: The alternatives stay in the same class whatever the loss aversion coefficient value is from 1.5 to 2.25.) 92
Table 4.21: Net flow and class values of 93.th alternative from CPU data to illustrate the third attitude. (The third attitude: The alternatives change the class once.) 94
Table 4.22: Weight values of experiments 97
Table 4.23: The results of weighting analysis experiments 98
Table A.1: The Weight and the Threshold Values of each Criterion for Case 1 .. 123
Table A.2: The Weight and the Threshold Values of each Criterion for Case 2 .. 123
Table B.1: Criterion Values of each Reference Alternative for THE World University Ranking 2019 Data Set 124
Table B.2: Criterion Values of each Reference Alternative for THE World University Ranking 2020 Data Set 124

Table C.1: The number of alternatives that are placed in the same rank in the comparisons using THE World University Ranking 2019 data sets 125
Table C.2: The number of alternatives that are placed in the same rank in the comparisons using THE World University Ranking 2020 data sets 126
Table D.1: Indifference Threshold (q) Values of each Criterion for each Data Set for both Cases 127
Table D.2: Preference Threshold (p) Values of each Criterion for each Data Set for Case I 127
Table D.3: Preference Threshold (p) Values of each Criterion for each Data Set for Case II 128
Table E.1: Criterion Values of each Reference Profile for CPU Data Set 129
Table E.2: Criterion Values of each Reference Profile for Auto MPG Data Set .. 129
Table E.3: Criterion Values of each Reference Profile for ESL Data Set 129
Table E.4: Criterion Values of each Reference Profile for ERA Data Set 129
Table E.5: Criterion Values of each Reference Profile for LEV Data Set 130
Table E.6:Criterion Values of each Reference Profile for CEV Data Set 130
Table E.7: Criterion Values of each Reference Profile for Breast Cancer Data Set 130
Table E.8: Criterion Values of each Reference Profile for Mammographic Data Set 130

LIST OF FIGURES

FIGURES

Figure 3.1: The S-shape value function of the Prospect Theory............................. 20
Figure 3.2: The piecewise linear value function of the Prospect Theory 21
Figure 3.3: Result of PROMETHEE I Method... 38
Figure 3.4: Result of PROMETHEE II Method 38
Figure 3.5: Partial Ranking of Proposed Methodology 39
Figure 3.6: Complete Ranking of Proposed Methodology 39
Figure 4.1: A flow and category representation with limiting profiles................... 60
Figure 4.2: A flow and category representation with central profiles 61
Figure 4.3: Net flow diagram of 8th alternative from CPU data 91
Figure 4.4: Net flow diagram of 17.th alternative from ESL data.......................... 93
Figure 4.5: Net flow diagram of 193.th alternative from CPU data 95

LIST OF ABBREVIATIONS

ABBREVIATIONS

AHP : Analytic Hierarchy Process

ANP : Analytical Network Process

ave. : Average Value

CACH: Cache Memory in Kilobytes

CEV : Car Evaluation

CHMAX: Maximum Channels in Units

CHMIN: Minimum Channels in Units

DEA : Data Envelopment Analysis

DISSET : DISjunctive Sorting based on the Evidence Theory

DM : Decision Maker

EGN : Extended Grey Numbers

ELECTRE : ELimination Et Choice Translating Reality

ERA : Employee Rejection/Acceptance

ERVD : The Election based on Relative Value Distances

ESL : Employee Selection

FMEA : Failure Mode Effect Analysis

GAIA : Geometrical Analysis for Interactive Aid

GDSS : Group Decision Support Systems

HFPDM : Hesitant Fuzzy Prospect Decision Matrix

HFSs : Hesitant Fuzzy Sets

IVIFNs : Interval-Valued Intuitionistic Fuzzy Numbers

LEV : Lecturers Evaluation

MAUT/MAVT : Multi-Attribute Utility/Value Theory

MCDM : Multiple Criteria Decision Making

MMAX: Maximum Main Memory in Kilobytes

MMIN: Minimum Main Memory in Kilobytes

MYCT: Machine Cycle Time in Nanoseconds

PCE : Prospect Cross-Efficiency

PROMETHEE : Preference Ranking Organization METHod for Enrichment Evaluations

ref : Reference Point

SMAA : Stochastic Multicriteria Acceptability Analysis

SPA : Set Pair Analysis

st. dev. : Standard Deviation

THE : Times Higher Education

TODIM : An acronym in Portuguese of Interactive and Multicriteria Decision Making

TOPSIS : Technique for Order Preference by Similarity to Ideal Solution

TrIFNs : Trapezoidal Intuitionistic Fuzzy Numbers

VIKOR : VlseKriterijumska Optimizacija I Kompromisno Resenje

LIST OF SYMBOLS

SYMBOLS

{} : Curly bracket

$\in$: Is element of

$\sim$: Approximately

$\sum$: Summation

$\leqslant$: Less than and equal

$\geq$ *:* Greater than and equal

$>$: Greater than

$<$: Less than

iff : If and only if

$\Rightarrow$: If

e : Euler number 2.72

λ : Lambda (The loss aversion coefficient of the prospect theory)

p : The preference threshold

q : The indiffeence threshold

σ : Sigma (The gussion threshold)

p_L : The modified preference threshold

q_L : The modified indiffeence threshold

σ_L : The modified gaussian threshold

P : The preference relation

I : The indifference relation

J : The incomparable relations

d : Difference between two alternatives

ln : Logarithm

ε : Epsilon

& : And

$\forall$: For all

$\exists$: For any

$\Leftrightarrow$: If and only if

$\cup$: Union

$|h - h^{\sim}|$: Absolute value

+ : Addition

- : Subtraction

* : Multiplication

/ : Division

() : Parentheses

% : Percentage

CHAPTER 1

INTRODUCTION

Multiple Criteria Decision Making (MCDM) is a discipline that deals with problems consisting of multiple and generally conflicting criteria and evaluates the alternatives with respect to criteria. MCDM problems are mainly classified in three categories: choice, ranking and classification/sorting problems (Roy,1996). In choice problems, alternatives are evaluated to identify the best alternative. As for the ranking problems, the evaluation of the alternatives is done by ranking them in the order from the best to the worst.

Multiple criteria ranking problems are constructed on relative comparison of alternatives and define the order of alternatives according to their values on several attributes. It is possible to encounter ranking problems, for which various approaches have been proposed as solutions, such as project selection, supplier selection, decision of investment alternatives in organizations. Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE) is one of the well-known MCDM methods which is used for ranking alternatives based on pairwise preference.

Classification/sorting problems use exact judgements independent of the set of alternatives (Zopounidis and Doumpos, 2002) and are different considering the type of classes to assign the alternatives. The classes are nominal in classification problems, whereas in sorting problems, they are ordered from the best to the worst. Alternatives are assigned to the predefined ordered classes according to the criteria values in multi-criteria sorting problems. The sorting problems such as resource allocation, supplier evaluation, financial management are encountered in organizations.

This study aims to propose two new methodologies for ranking and sorting of alternatives. The ranking method proposed is based on PROMETHEE and Prospect Theory. The sorting method proposed is based on FlowSort and Prospect Theory. Prospect theory is a well-known approach based on the choice behavior of the decision maker (DM). Karasakal et al. (2019) is the main inspiration point of this study. This study redefines preference functions of PROMETHEE and FlowSort using the prospect theory perspective.

In both PROMETHEE and FlowSort, same threshold values are used in the preference functions for the calculation of entering and leaving flows. However, having a worse criterion value in pairwise comparison can cause to assign an alternative to a worse class considering choice behavior of the DM. Smaller threshold values in calculating entering flows than in calculating leaving flows in pairwise comparisons by inspiring from the prospect theory are determined in this study. In this study, PROMETHEE and FlowSort methods are redefined based on the prospect theory considering choice behavior of the DM.

This study is organized in five chapters. The literature review on the subject is presented in Chapter Two. Chapter Three covers the related background information on multicriteria ranking problems and the proposed method with computational results. Chapter Four explains the related background information on multicriteria sorting problems and the proposed method with computational results. Chapter Five summarizes the conclusion of the study.

CHAPTER 2

LITERATURE REVIEW

In this study, two novel MCDM methods based on prospect theory for ranking and sorting of alternatives are proposed. The proposed multi-criteria ranking method is based on PROMETHEE and the prospect theory. The proposed multi-criteria sorting method is based on FlowSort and the prospect theory. Literature review of related work is summarized in three parts respectively: the ranking methods based on PROMETHEE, the sorting methods based on PROMETHEE and the prospect theory.

2.1 RANKING METHODS BASED ON PROMETHEE

PROMETHEE, which is developed by Brans et al. (1986), is one of the well-known methods for MCDM problems. This method considers the outranking relations among alternatives based on the preference functions and the criteria weights to rank the alternatives partially or completely.

In PROMETHEE I, the alternatives are ranked according to the leaving and the entering flows. This method allows indifference, incomparability, preferability relation between alternatives by providing partial order. The net flow values are used in PROMETHEE II to ensure preferability or indifference relations among the alternatives by providing a complete order of the alternatives. In 2007, PROMETHEE III is introduced by Cavalcante and De Almeida (2007) for interval-based ranking. PROMETHEE IV (Brans et al., 1984) is proposed for partial and complete rankings of continuous solutions. As an extension of PROMETHEE II, PROMETHEE V is developed by Brans and Mareschal (1992) for the selection of alternatives with a set of segmentation constraints. In order to calculate the hardness degree of MCDM problems considering criterion weights, PROMETHEE VI is proposed by Brans and Mareschal (1995).

PROMETHEE group decision support system (GDSS) (Macharis et al., 1998) is developed by extending PROMETHEE II for group decision-making problems. The visual interactive module Geometrical Analysis for Interactive Aid (GAIA) is proposed for the purpose of graphical representation of complicated decision-making problems (Mareschal and Brans, 1988; Brans and Mareschal, 1994a).

PROMETHEE and GAIA methodologies have been implemented by Brans and Mareschal (1994b) on IBM compatible microcomputers. The resulting decision support system is then called as PROMCALC and GAIA.

In 2004, PROMETHEE method is extended by Figueira et al. (2004) in order to develop PROMETHEE TRI and PROMETHEE CLUSTER, which are used for sorting based problems and nominal classification respectively.

Interested readers may refer to Behzadian et al. (2010) to review PROMETHEE methods in detail.

2.2 SORTING METHODS BASED ON PROMETHEE

PAIRCLASS approach is proposed by Doumpos and Zopounidis (2004) by extending the PROMETHEE methodology for sorting problems. In PAIRCLASS approach, pairwise comparisons are applied between the alternatives to be sorted and reference alternatives which represent the classes. The preference function and weights are proposed using linear programming. The objective function is used to define the required parameters using the reference alternatives which have pre-specified classifications. Figueira et al. (2004) has proposed PROMETHEE CLUSTER and PROMETHEE TRI, which use central profiles for determining the class of alternatives. Pairwise comparisons are made between central profiles and alternatives to be classified, where the classes of alternatives are determined based on the deviation between them. The PROMSORT methodology, which uses limiting profiles and reference alternatives to determine classes, is proposed by Araz and Ozkarahan (2007). In order to sort the alternatives, pairwise comparisons are used and PROMETHEE I is used to construct relations. The alternatives that have a preference relation and no indifference or incomparability relations are assigned to the classes before the final assignment, which is made based on pairwise comparisons.

Another PROMETHEE based sorting method FlowSort is introduced by Nemery and Lamboray (2008) to assign alternatives to the completely ordered categories independently. In FlowSort method, alternatives are assigned to the predefined ordered categories. Determination of the categories in FlowSort method is done by either limiting or central profiles. Roy and Bouyssou (1993) have proposed using limiting profiles in Electre-Tri, whereas Doumpos and Zopounidis (2004) and Figueira et al. (2004) have proposed using central profiles. Limiting or central profiles are called reference profiles while determining the better profile that outranks the worst one. For each alternative, pairwise comparisons of PROMETHEE are made between the alternative to be sorted and all reference profiles. Categories of the alternatives are determined according to positive, negative or net flow values. Nemery and Lamboray (2008) have compared FlowSort and Electre-Tri methodologies in terms of consistency. The difference between the categories obtained by using positive and negative flow values of FlowSort is less than the difference between the categories obtained by using optimistic and pessimistic rules of Electre-Tri. Chapter 3 explains the FlowSort method in detail.

An exact algorithm for sorting problems has been developed by De Smet et al. (2012). The algorithm regroups the alternatives into completely ordered classes by considering preference degrees, which are calculated using the PROMETHEE methodology. Kadzinski and Ciomek (2016) have proposed a sorting methodology for preference modeling and robustness analysis of outranking based multi-criteria problems. The method proposed is implemented to an outranking methodology based on ELimination Et Choice Translating REality (ELECTRE) and

PROMETHEE. In order to determine the ordered clusters, Boujelben and De Smet (2016) has proposed a method, in which the valued preference model of the PROMETHEE methodology is used for comparisons between the alternatives and the central profiles of clusters, by using the k-means algorithm and DISjunctive Sorting based on the Evidence Theory (DISSET) method. Interested readers may refer to Boujelben (2016) for the literature review of PROMETHEE based sorting methods in detail. Wei et al. (2016), De Lima Silva et al. (2018) are suggested for detailed information on application of the PROMETHEE method in sorting problems.

DIS-CARD, which is a sorting method, is proposed by Kadziński and Słowiński (2012) to determine when the desired cardinality of classes is required. Assignment rules are represented by using ELECTRE-TRI-C, ELECTRE-TRI-nC and FlowSort with a mathematical model. The Interval-FlowSort method, which integrates the FlowSort and Interval Theory to use when the input data is specified by intervals, is proposed by Janssen and Nemery (2013). Lolli et al. (2015) has extended the FlowSort method by including Group Decision Support Systems (GDSS) and proposed FlowSort-GDSS to be used in the field of Failure Mode Effect Analysis (FMEA). The process of assigning the failure modes to the ordered priority classes including multi-DMs uses the FlowSort method.

The PCLUST model is proposed by Sarrazin and De Smet (2016) by extending PROMETHEE I for interval clustering using FlowSort. Following the proposal, a comparison between PCLUST model and P2CLUST model (De Smet (2013)) is done by Sarrazin et al. (2018) who argue that PCLUST model is better than P2CLUST model based on computational time.

Interested readers may refer to Hu (2016), de Lima Silva and de Almeida Filho (2020), Hu (2013) for recent MCDM methodologies and their comparisons with FlowSort methodology. Sepulveda et al. (2010), Sepulveda and Derpich (2014), Sepulveda and Derpich (2015), Collier and Lambert (2018) are being referred for applications of the FlowSort method.

Fuzzy-FlowSort Method is proposed by Campos et al. (2015). Previously, Integration of Fuzzy Theory and FlowSort Method has been mentioned by Nemery (2008) who explained the differences between the two studies. By integrating the SMAA method and the Fuzzy-FlowSort, Pelissari et al. (2019) has proposed SMAA-Fuzzy-FlowSort method which can deal with imperfect input data types such as interval or stochastic data or linguistic variables. In a case where there are no linguistic variables in input data, the method acts like Fuzzy-FlowSort and it can be used for robustness analysis of Fuzzy-FlowSort and FlowSort methods. Chen and Hu (2011) has emphasized that criteria in problems cannot be independent and propose a single-layer perceptron (SLP) method based on PROMETHEE considering the non-additive preference index. Comparison of the method with FlowSort methodology is explained in the study.

2.3 PROSPECT THEORY

The main focus of this study is the choice behavior of the DM. Due to nature of MCDM problems, the DM is involved in the problem to evaluate the alternatives. To represent the DM, Multi-Attribute Utility/Value Theory (MAUT/MAVT) is developed by Keeney and Raiffa (1976). In MAUT, the DM's preference is represented by using stable utility/value functions independent of any reference point. The additive difference choice model had been developed by Tversky (1969) for the purpose of representing the conditions, where consistent and predictable intransitivity occurs if $x > y$ and $y > z$ then $x > z$.

Kahneman and Tversky (1976) developed the prospect theory in order to analyze the decisions under risk. The prospect theory evaluates the alternatives with a difference function based on gains and losses relatively to a reference point. Outputs are considered as positive or negative deviations from the reference point. The effect of losses is higher than gains for the same amount. The marginal value function has s-shape and is concave for the gains, convex and steeper for the losses. The prospect theory is one of the well-known methods for behavioral decision-making under risk

(Barberis et al., 2001; Dong et al., 2015; Ravaja et al., 2016). In Kahneman and Tversky (1976)'s study, it is shown that the prospect theory dominates expected utility theory for decisions under risk. Interested readers may refer to Hoyer et al. (2002), Royne et al. (2012) to review application of prospect theory in detail.

The Cumulative Prospect Theory had been developed by Kahneman and Tversky (1992). The theory differs from the prospect theory as it uses cumulative decision weights instead of separate decision weights. Using different weighting functions for both gains and losses to evaluate any number of outcomes is also allowed in the cumulative prospect theory.

The prospect theory is extended to the MCDM problems by Korhonen et al. (1990) for the first time. Intransitive choice behavior of the DM's and rapid convergence of the reference direction are explained by using the additive utility difference model and the prospect theory.

It is shown by Salminen and Wallenius (1993) that the prospect theory outperforms the traditional value model in a deterministic environment for MCDM problems. The combination of different value functions is found consistent with prospect theory.

An interactive method based on prospect theory type value functions to solve the discrete deterministic MCDM problems is proposed by Salminen (1994). In order to eliminate the dominated alternatives, a piecewise linear value function is being used. The piecewise linear prospect theory of Salminen is compared by Lahdelma et al. (2003) with the convex cone method of Korhonen et al. (1984) considering the number of pairwise comparisons in methods and is described as more efficient than convex cone technique when the number of criteria is more than two. However, piecewise linear prospect theory can only be used if the DM's preference is modeled as a linear or piecewise linear value function.

The election based on Relative Value Distances (ERVD) method is proposed by Shyur et al. (2015). In this method, the shortest distance to the ideal point and the farthest distance to the nadir point are considered and used to evaluate the alternatives such as the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The ERVD differs from the the TOPSIS method by its reference points for each criterion and how it calculates the risk attitude of the DM by using value function of the cumulative prospect theory. The expected utility function in the ERVD is replaced with the S-shape value function of the prospect theory to determine the DM’s risk-averse and risk-seeking behaviors. The TOPSIS, VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), elimination et choix traduisant la realité (ELECTRE), the piecewise linear prospect theory method, and Analytic Hierarchy Process (AHP) are compared by Wu and Tiao (2018) using different number of alternatives and criteria with different utility functions. In conclusion of this comparison, the piecewise linear prospect theory and AHP are determined as better than the other methods in terms of rank consistency.

In 2016, Li et al. (2016) has proposed an MCDM method based on the prospect theory and the cloud model. The proposed method is appropriate, when the alternative values are uncertain or linguistic variables. The linguistic variables are converted to the cloud model, which are compared by using the prospect theory and dynamically by choosing each possible solution as a reference point. In this perspective, the cross-efficiency evaluation model in Data Envelopment Analysis (DEA) is proposed by Liu et al. (2018). The prospect values of each decision-making units are determined, and a Prospect Cross-Efficiency (PCE) model is proposed considering behaviors of the DM’s in the cross-efficiency calculation. Liu et al. (2014), Liu et al. (2017) propose MCDM methods based on prospect theory for interval numbers with a large number of criteria. Interested readers may examine Han et al. (2016) to investigate prospect theory applications in MCDM field in detail.

A well-known interactive outranking method TODIM (an acronym in Portuguese of Interactive and Multicriteria Decision Making) is proposed by Gomes and Lima (1992a and 1992b) considering the behavioral attitude of the DM based on the prospect theory. In TODIM, the DM selects the reference criterion and according to the reference criterion, relative importance of criteria is determined. Pairwise comparisons are made between alternatives to calculate partial and final measurement of dominance using preference functions of the prospect theory and the alternatives are ranked using the final measurement of dominance values. The TODIM methodology is generalized by Gomes and González (2012) based on the cumulative prospect theory. Lee and Shih (2015) have studied incremental analysis of TODIM for group decision making considering behavioral attitudes of the DMs (Lee and Shih (2015)). To overcome weight inconsistency, Llamazares (2018) has proposed a TODIM based method. Interested readers may refer to Gomes et al. (2009), Tseng et al. (2013), Adalı (2016), Soni et al. (2016), Sen et al. (2016), Alali and Tolga (2019) for TODIM applications in detail.

By combining the prospect theory with Stochastic Multicriteria Acceptability Analysis (SMAA-2) (Lahdelma and Salminen, 2001) for discrete and group MCDM problems, where DM's preferences cannot be defined exactly, Lahdelma and Salminen (2009) proposed the SMAA-P method. Based on the cumulative prospect theory and Set Pair Analysis (SPA), Wang et. Al (2009) and Hu and Yang (2011) developed a dynamic stochastic MCDM method under uncertainty considering risk attitude of the DM and lack of weight information in decision processes. The prospect theory is used to determine aspiration levels as initial reference points by Tan et al. (2014), who also proposed a method for stochastic MCDM problems with aspiration level based on the prospect stochastic dominance.

MCDM methods based on the prospect theory for fuzzy environment are proposed by Fan et al. (2013). The aspiration level of the DM is considered as a reference point and the difference between aspiration level and alternatives as gains and losses. TOPSIS method has been extended by Li and Chen (2014) based on prospect theory considering the risk psychology of the DM and ambiguous information under uncertainty for trapezoidal intuitionistic fuzzy numbers (TrIFNs). Another MCDM method which uses the prospect theory and the dempster-shafer theory is proposed by Li et al. (2014). The prospect theory represents the DM's risk attitude and the dempster-shafer theory reflects the uncertain weight information for the trapezoidal intuitionistic fuzzy environment. Wang et al. (2018) has developed a method for failure mode effect analysis (FMEA) using prospect theory and choquet integral. A hesitant fuzzy thermodynamic method based on the prospect theory for emergency decision making has been proposed by Ren et al. (2017). In the proposal, a negative exponential function is introduced into the prospect theory to convert the hesitant fuzzy decision matrix to the Hesitant Fuzzy Prospect Decision Matrix (HFPDM) based on expectation level. Interested readers may refer to Cunbin et al. (2016), Sun et al. (2017), Bai and Sarkis (2017) for fuzzy MCDM applications based on the prospect theory.

In 2012, the prospect theory and fuzzy numbers for uncertain MCDM problems based on TODIM are combined by Krohling and de Souza (2012). In 2013, Krohling et al. (2013) used the TODIM method with intuitionistic fuzzy information. New criteria interaction measures based on choquet integral used in TODIM method have also been developed by Gomes et al. (2013) that also extend the TODIM method based on the nonlinear cumulative prospect theory by using choquet integral. Passos and Gomes (2014) have extended the TODIM methodology for multicriteria classification problems and propose TODIM-FSE methodology. Gomes et al. (2014) and Araújo (2015) are suggested for TODIM-FSE applications.

Using the TODIM and fuzzy theory, Wei et al. (2015), Tan et al. (2015), Lourenzutti and Krohling (2015), Yu et al. (2016), Ren et al. (2016) and Jiang et al. (2016) have proposed new MCDM methodologies. Wang et al. (2017) extended the TODIM methodology with multi-hesitant fuzzy linguistic information for fuzzy environment to incorporate choquet integral for linguistic Z-numbers. Qin et al. (2017) used triangular intuitionistic fuzzy numbers in a TODIM based method. Interested readers may refer to Chen et al. (2015), Sang and Liu (2016), Ji et al. (2016), Zhang et al. (2018), Wang and Li (2018) and Qin et al. (2017) for applications of TODIM based methodologies in fuzzy environment in detail.

To determine the most preferred alternative with multiple reference points in the interval form for stochastic and intuitionistic fuzzy uncertainties, Hu et al. (2014) developed a new method based on the prospect theory. Yua et al. (2014), Zhang et al. (2017) have proposed a method based on the prospect theory by considering the stochastic hybrid MCDM problems including interval probability and unknown criteria weight. A stochastic MCDM method for the interval-valued intuitionistic fuzzy numbers (IVIFNs) based on the prospect theory has been proposed by Gao and Liu (2015). Thillaigovindan et al. (2016) used the prospect theory to determine the optimum criteria weights for fuzzy problems under risk. Yan and Liu (2016) and Zhou et al. (2017) proposed stochastic MCDM methods based on the prospect theory and the distance measures for extended grey numbers (EGN) which combines discrete and continuous grey numbers. Using the interval neutrosophic probability based on the regret theory, Wang et al. (2018) developed a new MCDM method that summarizes the similarities of the method and the prospect theory. A generalization of the TODIM method has been proposed by Lourenzutti and Krohling (2013) with intuitionistic fuzzy information. Zhang et al. (2017) developed a new method by combining SMAA and TODIM methodologies. Li et al. (2018) is being referred for applications of TODIM based stochastic MCDM methods in fuzzy environment. A different version of PROMETHEE II is proposed by Wang and Sun (2008) that use the prospect theory for trapezoidal fuzzy numbers where the prospect value function of trapezoidal fuzzy numbers is determined based on the DMs' risk attitudes. The preference function of PROMETHEE is redefined using the possible degree of prospect value function of trapezoidal fuzzy numbers. Peng et al. (2016) has extended TODIM and PROMETHEE II using the prospect theory for hesitant fuzzy sets (HFSs). The cumulative prospect theory and PROMETHEE has been combined by Liao et al. (2018) using the hesitant fuzzy linguistic thermodynamic method to select a green logistic provider.

2.4 CONTRIBUTION TO LITERATURE

This study aims at redefining preference functions of PROMETHEE and FlowSort from the prospect theory perspective. Bozkurt (2007) and later Karasakal et al. (2019) proposed to modify the preference functions of PROMETHEE based on choice behavior and develop two new preference function types in addition to the preference functions of PROMETHEE. The preference functions that are being proposed are used, when an equal amount of loss has a higher effect than an equal amount of gain.

The PT-PROMETHEE method has been proposed by Lerche and Geldermann (2015) by integrating reference dependency and loss aversion elements of the prospect theory into PROMETHEE. To integrate the loss aversion coefficient of the prospect theory, which suggests the steeper slope for losses than for gains, the preference functions of PROMETHEE are extended with smaller threshold values as mentioned by Bozkurt (2007) and an artificial reference alternative is used as a benchmark for the real alternatives. Interested readers may refer to Król et al. (2018) for PT-PROMETHEE application and comparison between PROMETHEE and PT-PROMETHEE in detail.

Determining the reference point is defined as the most important challenge for using the prospect theory by Markowitz (1952) and Barberis (2013). Baillon et al. (2019) emphasize how hard determining a reference point in an appropriate way is. Six reference point rules such as status quo or max-min rules are given and examined using various subjects in the study of Baillon et al. (2019) and the way to find the reference points can be determined differently in terms of the DMs' choice behavior. Clemen and Reilly (2013) have specified how people's behaviors can be inconsistent and can violate transitivity rules. Roy et al. (2014) has also mentioned the difficulty of determining parameters.

In this study, PROMETHEE and the prospect theory for ranking method, FlowSort and the prospect theory for sorting method are combined by using Özerol and Karasakal (2007)'s perspective. The relationship between the regret theory and PROMETHEE II has been studied by Özerol and Karasakal (2007) considering regret and rejoice in the decision process. The DM would feel regret if the chosen alternative is worse than another alternative or feel rejoice if the chosen alternative is better than another alternative in at least one criterion value. In this study, this situation is explained in gain or loss of the DM based on criterion values. Thus, for pairwise comparison of two alternatives considering all criteria, gain and loss degrees are possible. Gain and loss can occur just in the comparisons of real alternatives. Contrary to the related works in literature, an additional reference point is not suggested. In this study, the alternatives are compared with other alternatives. The proposed method is a generalization of PROMETHEE and FlowSort to use when the losses have higher impact than gains. Gains are qualified as positive flows; losses are defined as negative flows of PROMETHEE and FlowSort. The essential point of this study is that preference functions with steeper slopes are used for negative flow calculation since losses have a higher impact than gains for an equal amount. However, if the parameter value for having a steeper slope in negative flow calculations is used as 1, the method acts like PROMETHEE and FlowSort. The proposed methods are explained in Chapter Three and Chapter Four in detail.

CHAPTER 3

MULTICRITERIA RANKING PROBLEMS

3.1 BACKGROUND

3.1.1 PROMETHEE

PROMETHEE is a well-known outranking method that is developed by Brans et al. (1986) to analyze the multi-criteria problems simply, clearly and stably based on pairwise comparisons of the alternatives. Considering a set of m alternatives $A = \{a_1, \dots, a_m\}$ to be evaluated with respect to a set of n criteria $G = \{g_1, \dots, g_n\}$ to be maximized, PROMETHEE compares two alternatives with regard to each criterion according to a valued outranking relation belonging to the criterion.

Preference functions of PROMETHEE are shown in Table 3.1. The type of each preference function and the threshold values for each criterion are defined by the DM to determine the preference degree. The preference degree $P_j(a_1, a_2)$ represents the intensity of alternative a_1's preference with regard to alternative a_2 on criterion g_j for $j = 1, \dots, n$ and $a_1, a_2 \in A$. $P_j(a_1, a_2)$ is calculated based on the difference between value of alternatives for a specific criterion *j*.

$$P_j(a_1, a_2) = P[d_j(a_1, a_2)]$$

where $d_j(a_1, a_2)$ is the nondecreasing function of the difference between $g_j(a_1)$ and $g_j(a_2)$. $d_j(a_1, a_2)$ can be positive or negative. If $d_j(a_1, a_2)$ is negative or 0 than preference degree $P_j(a_1, a_2)$ is equal to 0. If $d_j(a_1, a_2)$ is positive than $P_j(a_1, a_2)$ takes a value between 0 and 1. $d_j(a_1, a_2)$ is calculated as follows:

$$d_j(a_1, a_2) = g_j(a_1) - g_j(a_2) \text{ for } j = 1, \dots, n \text{ and } a_1, a_2 \in A.$$

Preference degree $P_j(a_1, a_2)$ takes a value between the interval of 0 and 1 ($0 \leqslant P_j(a_1, a_2) \leqslant 1$) and it is interpreted as below:

- $P_j(a_1, a_2) = 0$ if there is no preference a_1 over a_2 on criterion *j* and a_1 and a_2 are indifference on criterion *j*;
- $P_j(a_1, a_2) \sim 0$ if there is weak preference a_1 over a_2 on criterion *j*;
- $P_j(a_1, a_2) \sim 1$ if there is strong preference a_1 over a_2 on criterion *j*;
- $P_j(a_1, a_2) = 1$ if there is strict preference a_1 over a_2 on criterion *j*.

Table 3.1: Preference Functions of PROMETHEE, Brans et al. (1986)

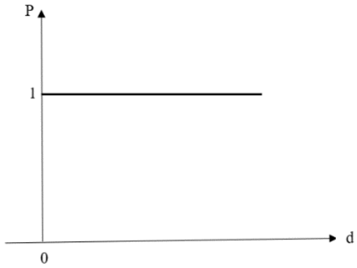
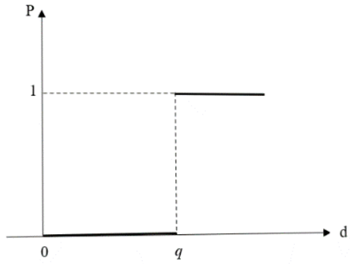
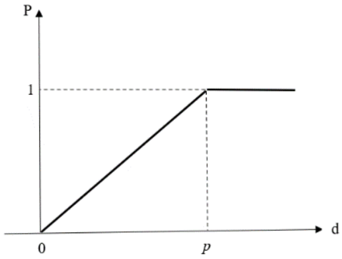
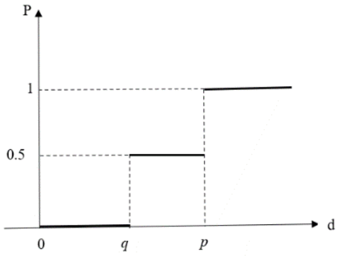
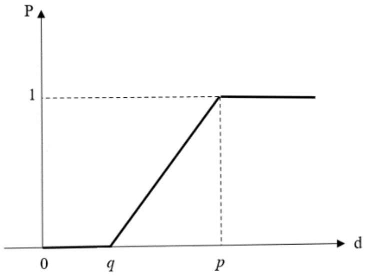
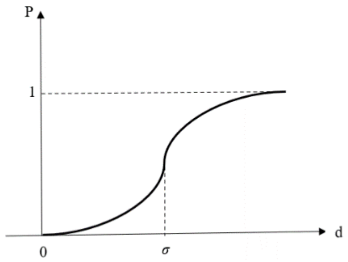
Usual Criterion	
U-Shape Criterion	
V-Shape Criterion	
Level Criterion	
V-Shape with Indifference	

Table 3.1 (continued)

Gaussian Criterion	

Considering each criterion g_j the weight value w_j is assigned such that $w_j > 0$ for $j = 1, \ldots, n$ and $\sum_{j=1}^{n} w_j = 1$. The weight values show the importance of the criteria according to the DM. The criteria weights can be determined by the DM or using the methods presented in Bozkurt (2007), Safari et al. (2012) and Jati and Dominic (2017)'s studies. Bozkurt (2007) used AHP, Matrix Multiplication Technique and ANP methodologies, Safari et al. (2012) and Jati and Dominic (2017) used the entropy method to determine the weight values. Using preference degrees and weight values, $\pi(a_1, a_2)$ values for each pair of alternatives are calculated as shown below:

$$\pi(a_1, a_2) = \sum_{j=1}^{n} w_j \, P_j(a_1, a_2)$$

$\pi(a_1, a_2)$ is called outranking degree and shows how much alternative a_1 is preferred to alternative a_2 considering all criteria.

- $0 \leqslant \pi(a_1, a_2) \leqslant 1$

The outranking degree takes a value in the range of 0 and 1 satisfying the following conditions. If $\pi(a_1, a_2)$ is close to 1, a_1 is preferred to a_2 strongly.

- $\pi(a_1, a_2) + \pi(a_2, a_1) \leqslant 1$

- $\pi(a_1, a_1) = 0$

Using the outranking degrees, $\phi^+(a_1)$ and $(\phi^-(a_1))$ are calculated. The summation is divided one less than the number of alternatives since $\pi(a_1, a_1)$ is equal to 0. The formulas are shown below:

$$\phi^{+}(a_1) = \frac{1}{m-1}\sum_{x=1}^{m}\pi(a_1, x),$$

$$\phi^{-}(a_1) = \frac{1}{m-1}\sum_{x=1}^{m}\pi(x, a_1)$$

$\phi^{+}(a_1)$ describes the positive outranking flow (the leaving flow) and $\phi^{-}(a_1)$ describes the negative outranking flow (the entering flow) of a_1. The positive flow of a_1 ($\phi^{+}(a_1)$) represents the outperforming degree of alternative a_1 to all other alternatives. Therewithal the negative outranking flow of a_1 ($\phi^{-}(a_1)$) represents the outperformed degree of a_1 to all other alternatives. The greater value of leaving flow ($\phi^{+}(a_1)$) and a smaller value of the entering flow ($\phi^{-}(a_1)$) comparing to other alternatives show that a_1 is preferred among the other alternatives.

Using the positive and the negative flow values, a partial ranking of the alternatives is provided by PROMETHEE I by allowing the preference (P), indifference (I) and incomparable (J) relations.

- $a_1 \, P \, a_2 \quad iff \quad \phi^{+}(a_1) \geq \phi^{+}(a_2)$ and $\phi^{-}(a_1) \leq \phi^{-}(a_2)$,
- $a_1 \, I \, a_2 \quad iff \quad \phi^{+}(a_1) = \phi^{+}(a_2)$ and $\phi^{-}(a_1) = \phi^{-}(a_2)$,
- $a_1 \, J \, a_2 \quad otherwise.$

The complete ranking of alternatives from the best to the worst is given in PROMETHEE II by considering the net flow of each alternative. Net flow value is calculated as follows:

$$\phi(a_1) = \phi^{+}(a_1) - \phi^{-}(a_1) \, ,$$

PROMETHEE II only allows the preference (P) and indifference (I) relations with the following rules:

- $a_1 \, P \, a_2 \quad iff \quad \phi(a_1) > \phi(a_2)$,
- $a_1 \, I \, a_2 \quad iff \quad \phi(a_1) = \phi(a_2)$.

3.1.2 THE PROSPECT THEORY

The Prospect Theory is proposed by Kahneman and Tversky (1976) to analyze the decisions under risk. According to the prospect theory, the outcomes are represented as positive or negative deviations (gains and losses) from a reference point. Reference points can be determined as aspiration level of the DM, status quo, minimal requirement of each criterion, a ghost alternative or one of the existing alternatives, etc. The Prospect Theory allows using different value functions based on the DM's preferences. Kahneman and Tversky (1976) recommend that the value function is commonly in a S-shape in which the concave above the reference point to represent the gains and the convex below the reference point to represent the losses. The preference functions are generally assumed to be steeper for the losses than for the gains since the losses have a higher effect than the gains for the same amount. The S-shape value function of the Prospect Theory is shown in Figure 3.1. The piecewise linear value function which is used widely in the literature is illustrated in Figure 3.2 as the extension of the S-shape value function.

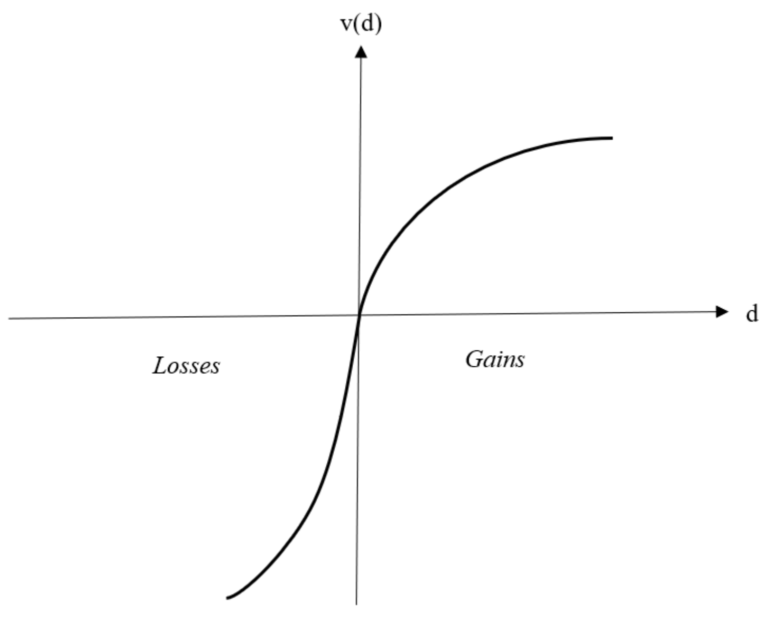


Figure 3.1: The S-shape value function of the Prospect Theory

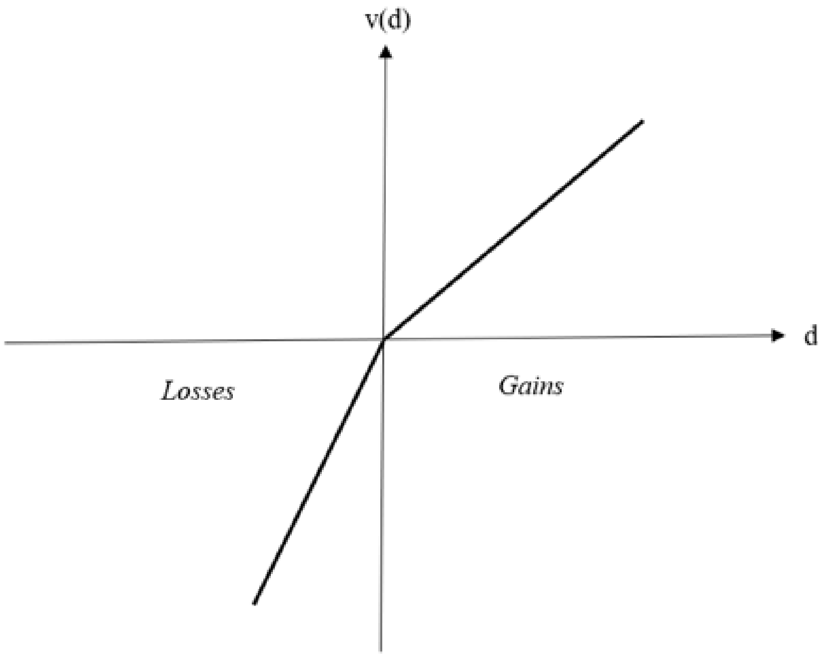


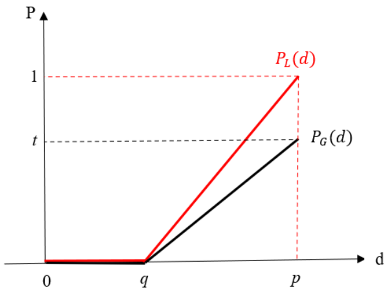
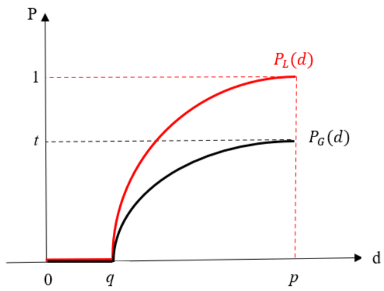
Figure 3.2: The piecewise linear value function of the Prospect Theory

Prospect theory is originally proposed for single criterion problems under uncertainty. Korhonen et al. (1990) has extended it and proposed for multi-criteria environment. Hybrid methods are proposed as the combination of prospect theory with different MCDM methods as explained in Chapter 2.

3.1.3 THE APPROACH TO CHOICE BEHAVIOR (Karasakal et al. (2019))

Karasakal et al. (2019) integrated the prospect theory into PROMETHEE II. Two different preference functions with a steeper slope for losses than for gains are determined considering the piecewise linear value function of the prospect theory to use in PROMETHEE as shown in Table 3.2.

Table 3.2: The additional preference functions proposed by Bozkurt (2007)

Linear Criterion (Prospect Theory)	
Exponential Criterion (Prospect Theory)	

In the study, the utility theory is mentioned in a way that it does not reflect the DM's choice behavior correctly, when losses have more impact than gains. To determine the DM's perspective about how important a loss is compared to a gain, the following question is asked to the DM: "*Considering the criterion under consideration, minimum how many units of gain can satisfy you upon one unit of loss?*" The answer represents the loss aversion coefficient value of the prospect theory that is used to determine slope of the functions. If the answer is one; the DM's choice behavior is consistent with the utility theory and preference functions of PROMETHEE are enough to calculate the outranking relations. However, if the answer is more than one; the utility theory is not enough to reflect the DM's satisfaction and the proposed functions can be used.

One of the preference functions is proposed as the extension of the preference function of the criteria with linear preference and indifference area whereas the other is proposed for the exponential criteria. Linear criterion preference function with the prospect theory perspective is appropriate when the marginal rate of substitution is constant. Exponential criterion preference function with the prospect theory perspective is concave and appropriate when the marginal rate of substitution is diminishing. If the small differences are more significant for the criterion value beyond the indifference area, the concave function is more suitable.

For the losses, preference degree $P_{jL}(a_1, a_2) = P_{jL}(g_j(a_1) - g_j(a_2))$ is calculated as follows. Alternative a_2 is assumed to be linear criterion with the prospect theory perspective:

- If the difference between criterion values of alternatives is less than or equal to the indifference threshold (q), preference degree is equal to zero as shown below:

$$g_j(a_1) - g_j(a_2) \leqslant q \;\Rightarrow\; P_{jL}(a_1, a_2) = 0$$

- If the difference is greater than the indifference threshold, the preference degree is calculated as follows:

$$g_j(a_1) - g_j(a_2) > q \;\Rightarrow\; P_{jL}(a_1, a_2) = \frac{((g_j(a_1) - g_j(a_2)) - q)}{(p - q)}$$

  where p is the preference threshold.

For the gains, preference degree $P_{jG}(a_1, a_2) = P_{jG}(g_j(a_1) - g_j(a_2))$ is calculated as follows. Alternative a_1 is assumed to be linear criterion with the prospect theory perspective:

- If the difference between criterion values of alternatives is less than or equal to the indifference threshold, preference degree is equal to zero as shown below:

$$g_j(a_1) - g_j(a_2) \leqslant q \;\Rightarrow\; P_{jG}(a_1, a_2) = 0$$

- If the difference is greater than the indifference threshold, the preference degree is calculated with the formula below:

$$g_j(a_1) - g_j(a_2) > q \;\Rightarrow\; P_{jG}(a_1, a_2) = t\,\frac{((g_j(a_1) - g_j(a_2)) - q)}{(p - q)}$$

  where t is $(gain/loss)^{-1}$ that is determined by the DM.

For the losses, preference degree $P_{jL}(a_1, a_2) = P_{jL}(g_j(a_1) - g_j(a_2))$ is calculated as follows. Alternative a_2 is assumed to be exponential criterion with the prospect theory perspective:

- If the difference between criterion values of alternatives is less than or equal to the indifference threshold, preference degree is equal to zero as shown below:

$$g_j(a_1) - g_j(a_2) \leqslant q \ \Rightarrow \ P_{jL}(a_1, a_2) = 0$$

- If the difference is greater than the indifference threshold, the preference degree is calculated as follows:

$$g_j(a_1) - g_j(a_2) > q \ \Rightarrow \ P_{jL}(a_1, a_2) = 1 - e^{-\lambda(g_j(a_1) - g_j(a_2) - q)}$$

where $\lambda = \frac{ln\varepsilon}{(p-q)}$ and ε is a small constant such as 0.01.

For the gains, preference degree $P_{jG}(a_1, a_2) = P_{jG}(g_j(a_1) - g_j(a_2))$ is calculated as follows. Alternative a_1 is assumed to be exponential criterion with the prospect theory perspective:

- If the difference between criterion values of alternatives is less than or equal to the indifference threshold, preference degree is equal to zero as shown below:

$$g_j(a_1) - g_j(a_2) \leqslant q \ \Rightarrow \ P_{jG}(a_1, a_2) = 0$$

- If the difference is greater than the indifference threshold, the preference degree is calculated using the following formula:

$$g_j(a_1) - g_j(a_2) > q \ \Rightarrow \ P_{jG}(a_1, a_2) = t - t(e^{-\lambda(g_j(a_1) - g_j(a_2) - q)})$$

where $\lambda = \frac{ln^{\varepsilon}/_{t}}{(p-q)}$ and $\varepsilon = 0.01$.

In the methodology, the preference degrees are calculated. If losses have a higher impact than gains considering the same amount, new preference functions are used. If gains and losses have equal impact, original preference functions of PROMETHEE are used. Outranking degrees are calculated using weight values and preference degrees as they are calculated in PROMETHEE as shown below.

$$\pi(a_1, a_2) = \sum_{j=1}^{n} w_j \, P_j(a_1, a_2)$$

Leaving and entering flows are determined as follows.

$$\phi^+(a_1) = \frac{1}{m-1} \sum_{x=1}^{m} \pi(a_1, x),$$

$$\phi^-(a_1) = \frac{1}{m-1} \sum_{x=1}^{m} \pi(x, a_1)$$

If losses have a higher impact than gains $\pi(a_1, x) = P_{jG}(a_1, x)$ and $\pi(x, a_1) = P_{jL}(x, a_1)$.

Net flow values are determined by the difference of leaving and entering flows as follows and alternatives are ranked by using PROMETHEE II procedure.

$$\phi(a_1) = \phi^+(a_1) - \phi^-(a_1)$$

3.1.4 PT-PROMETHEE

Lerche and Geldermann (2015) have proposed PT-PROMETHEE for deterministic problems by combining prospect theory and PROMETHEE. Reference dependency and loss aversion features of the prospect theory are integrated into PROMETHEE considering the piecewise linear value function of the prospect theory, where a reference alternative is introduced in the method. Pairwise comparisons are made between the reference alternative and the real alternatives in addition to the real alternatives themselves. In pairwise comparisons between the alternatives; the procedure is the same as it is in PROMETHEE. Gains and losses are only possible in pairwise comparisons with the reference point. The better criterion values of alternatives with respect to the reference alternative's criterion values represent gains and the worse values represent losses. The original preference functions of PROMETHEE are used for gains whereas the extended version of the preference functions of PROMETHEE by using the loss aversion coefficient of the prospect theory as suggested by Karasakal (2019) are used for losses. New preference functions with lower threshold values are shown in Table 3.3 by comparing the originals.

Table 3.3: The modified preference functions considering loss aversion, Lerche and Geldermann (2015). The red part illustrates the modified thresholds and preference functions. The original preference functions of PROMETHEE are shown with the black part

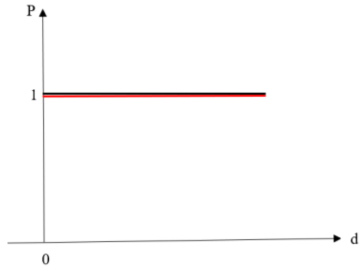
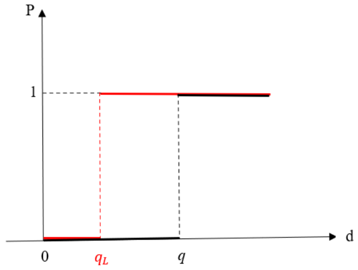
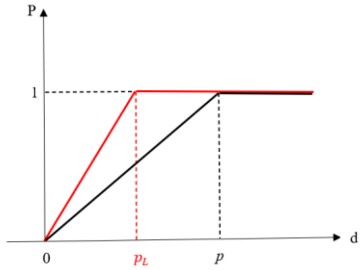
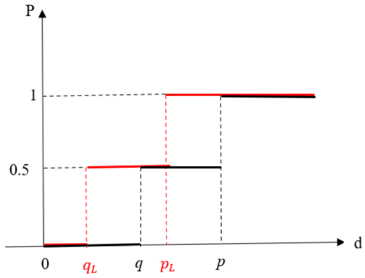
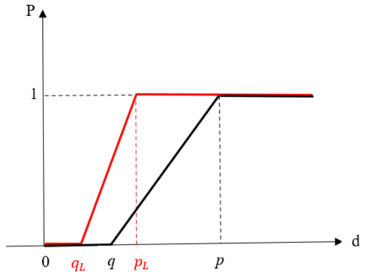
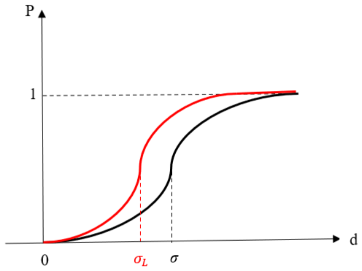
Usual Criterion	
U-Shape Criterion	
V-Shape Criterion	
Level Criterion	

Table 3.3 (continued)

V-Shape with Indifference	
Gaussian Criterion	

An artificial alternative is determined as the reference point by experts or the DM to illustrate the DM's expectation. The reference point can be used as a benchmark to show whether each alternative is a good choice or not for satisfying the expectation of the DM regarding the overall goal of the problem. However, setting the value of reference alternative properly is critical and the preference of DMs' can be inconsistent. Adding an additional and non-existing point can cause deviation from the reality. Defining the reference alternative in the right way is a challenge and extra load to the DM. Even though the information of the reference alternative is obtained properly, the alternative is not real. At this point, the real alternatives are compared with an artificial alternative where according to the not existing alternative the goodness or badness of the alternatives is determined. Additionally, a rise in the number of alternatives increases the number of pairwise comparisons, which requires more computational effort and computational time. The steps of method are explained as follows:

Type of preference functions, threshold values (p, q and σ), weights and loss aversion coefficients (λ) are determined for each criterion. Modified threshold values

(p_L, q_L and σ_L) are determined for losses by dividing the threshold values by λ, as suggested by Karasakal et al. (2019). Since losses have a higher impact than gains for the same amount, modified threshold values (p_L, q_L and σ_L) take lower values than the regular ones.

Pairwise comparisons between alternatives including the reference alternative are calculated. A potential loss or gain can be observed just in the pairwise comparison with the reference alternative. A loss occurs if a real alternative, a_1 has a lower value with respect to the reference alternative, a_r regarding any criterion j. Each pairwise comparison of a_r, that has a positive preference value, is a loss. The preference degree of a_r over alternative a_1 with respect to the criterion j is calculated by using the preference function for losses as shown below:

$$P_{Lj}(a_r, a_1) = P_{Lj}(g_j(a_r) - g_j(a_1))$$

If a real alternative a_1 has a better value than the reference alternative, a_r regarding any criterion j, a gain occurs. The regular preference function is used for all pairwise comparisons of any alternative, a_1 over reference alternative, a_r regarding any criterion j. The preference degree is calculated as follows:

$$P_j(a_1, a_r) = P_j(g_j(a_1) - g_j(a_r))$$

For the pairwise comparisons between real alternatives, regular preference functions are used. The preference degree of alternative a_1 over a_2 regarding any criterion j is obtained with the formula below:

$$P_j(a_1, a_2) = P_j(g_j(a_1) - g_j(a_2))$$

Outranking degrees are calculated. All pairwise comparisons of reference alternative, a_r over a real alternative, a_1 is a loss. The preference function for losses is used for the outranging degree calculation of a_r. Outranking degree of a_r over a_1 is obtained as follows:

$$\pi(a_r, a_1) = \sum_{j=1}^{n} w_j \, P_{Lj}(a_r, a_1)$$

For the preference degree of a real alternative, a_1 over the reference alternative, a_r or a real alternative, a_2, the regular preference function is used. The outranking degrees are obtained as shown below:

$$\pi(a_1, a_r) = \sum_{j=1}^{n} w_j \, P_j(a_1, a_r)$$

$$\pi(a_1, a_2) = \sum_{j=1}^{n} w_j \, P_j(a_1, a_2)$$

Leaving and entering flows of the reference alternative are calculated as follows to use in PROMETHEE I ranking:

$$\phi^+(a_r) = \frac{1}{m}\sum_{x=1}^{m} \pi(a_r, x)$$

$$\phi^-(a_r) = \frac{1}{m}\sum_{x=1}^{m} \pi(x, a_r)$$

The sum of outranking degrees is divided by m and not by $(m-1)$ as it is in the original PROMETHEE since the number of alternatives is increased by one with the reference alternative.

The leaving and entering flows of real alternatives are calculated as follows to use in PROMETHEE I ranking:

$$\phi^+(a_1) = \frac{1}{m}(\pi(a_1, a_r) + \sum_{x=1}^{m} \pi(a_1, x))$$

$$\phi^-(a_1) = \frac{1}{m}(\pi(a_r, a_1) + \sum_{x=1}^{m} \pi(a_1, x))$$

The net flow values are equal to the difference between leaving and entering flows as shown below to use for PROMETHEE II ranking:

$$\phi(a_r) = \phi^+(a_r) - \phi^-(a_r)$$

$$\phi(a_1) = \phi^+(a_1) - \phi^-(a_1)$$

3.2 THE PROPOSED METHOD

Combining the beneficial features of different methods and proposing a new hybrid methodology has been widely studied in the MCDM field. PROMETHEE and the prospect theory are two popular methods that are brought up with different MCDM methods.

The origin of our study can be defined as Karasakal et al. (2019)'s study. By combining the prospect theory and PROMETHEE based on the piecewise linear value function of the prospect theory, two new preference functions are proposed to be used in pairwise comparisons. If one unit of loss has higher impact than one unit of gain, the proposed preference functions are used. If gains and losses have equal importance, original preference functions of PROMETHEE are used. Modifying the preference functions of PROMETHEE using the loss aversion coefficient of the prospect theory, which gives a direction to Lerche and Geldermann (2015)'s study, is mentioned.

In this study, a methodology aiming to rank the alternatives in a discrete MCDM problem is proposed based on PROMETHEE. In the proposed method, pairwise comparisons are made just between the real alternatives. There is no additional effort to define an additional alternative. The method is a generalization of PROMETHEE and based on the piecewise linear value function of the prospect theory as in the study of Karasakal et al. (2019). If one unit loss has higher effect than one unit of gain as in the prospect theory, the proposed method reflects this attitude with higher entering flow values. If one unit of loss is equal to the one unit of gain, the loss aversion coefficient is equal to 1 ($\lambda = 1$) and the method acts the same as the original PROMETHEE. The method can be used for MCDM problems where PROMETHEE is applicable. Additionally, the proposed method is appropriate when the choice behavior of the DM may not be modelled using the MAUT due to the reason that the DM gives more importance to losses than gains. Interested readers may refer to Karasakal et al. (2019) for further information on problem types the proposed method can be used.

In our study, the prospect theory is integrated into PROMETHEE using Özerol and Karasakal (2007)'s perspective. Özerol and Karasakal (2007) mentioned the relation between the regret theory and PROMETHEE II considering the possible regret and rejoice in the decision process. Using similar consideration, a calculation is done to see if the current alternative was reference alternative, what the DM would feel. If the reference alternative has better value than the compared alternative, the DM would feel like she/he has gained. If the reference alternative has a worse value than the compared alternative, the DM would feel such that she/he has lost. Without an additional reference alternative, the existing alternatives are compared with all other alternatives as it is in PROMETHEE. Different than PROMETHEE, having a worse or a better alternative value on a criterion does not have the same impact, but has a higher effect as it is in the prospect theory.

The essential point of this study is that different threshold values are used when calculating the leaving and entering flow values of alternatives. When the leaving flows are being calculated, the original preference functions of PROMETHEE are used. When the entering flows are being calculated, the preference functions are modified using the loss aversion coefficient of the prospect theory. To modify the preference functions, using smaller threshold values has been suggested by Karasakal et al. (2019) to obtain more sensitive results for the losing case. For obtaining the smaller threshold values, the original threshold values are divided by the loss aversion coefficient (λ) of the prospect theory. The modified threshold values of the preference functions for the losing case are calculated as shown below:

- $p_L = {}^{p}/_{\lambda}$ for preference threshold,
- $q_L = {}^{q}/_{\lambda}$ for indifference threshold,
- $\sigma_L = {}^{\sigma}/_{\lambda}$

The formulas of preference degree for leaving or entering flows are shown in Table 3.4. Leaving flows represent how much the current alternative outranks the other alternatives and how much the DM would gain if she/he chooses the current alternative, whereas entering flows represent how much the current alternative is outranked by the other alternatives and how much the DM would lose if she/he chooses the current alternative. $P_G(d)$ is used for leaving flow calculation and $P_L(d)$ is used for entering flow calculation.

Table 3.4: The formulas of preference degree calculation to use in proposed methodology for leaving or entering flows.

Usual Criterion	$P_G(d)\begin{cases}0 & d \leqslant 0\\ 1 & d > 0\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant 0\\ 1 & d > 0\end{cases}$
U-Shape Criterion	$P_G(d)\begin{cases}0 & d \leqslant q\\ 1 & d > q\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant \frac{q}{\lambda}\\ 1 & d > \frac{q}{\lambda}\end{cases}$
V-Shape Criterion	$P_G(d)\begin{cases}0 & d \leqslant 0\\ \frac{d}{p} & 0 < d \leq p\\ 1 & d > p\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant 0\\ \frac{d*\lambda}{p} & 0 \leq d \leq \frac{p}{\lambda}\\ 1 & d > \frac{p}{\lambda}\end{cases}$
Level Criterion	$P_G(d)\begin{cases}0 & d \leqslant q\\ 0.5 & q < d \leq p\\ 1 & d > p\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant \frac{q}{\lambda}\\ 0.5 & \frac{q}{\lambda} < d \leq \frac{p}{\lambda}\\ 1 & d > \frac{p}{\lambda}\end{cases}$
V-Shape with Indifference	$P_G(d)\begin{cases}0 & d \leqslant q\\ \frac{d-q}{p-q} & q < d \leq\\ 1 & d > p\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant \frac{q}{\lambda}\\ \frac{d*\lambda-q}{p-q} & \frac{q}{\lambda} < d \leq \frac{p}{\lambda}\\ 1 & d > \frac{p}{\lambda}\end{cases}$
Gaussian Criterion	$P_G(d)\begin{cases}0 & d \leqslant 0\\ 1-e^{-\frac{d^2}{2\sigma^2}} & d > 0\end{cases}$	$P_L(d)\begin{cases}0 & d \leqslant 0\\ 1-e^{-\frac{\lambda*d^2}{2\sigma^2}} & d > 0\end{cases}$

The step-by-step description of the proposed methodology is explained as follows in details:

1. The type of preference functions, the values of thresholds, weights and loss aversion coefficients for each criterion are determined. The preference functions can be determined based on the nature of criteria as mentioned in Abdullah et al. (2018)'s study. The loss aversion coefficient values can be defined based on the DM's expectation by using Hu et al. (2014)'s perspective. AHP, Matrix Multiplication Technique, ANP, entropy method can be used to determine the weight values.
2. $P_{Gj}(a_1, a_2)$ and $P_{Lj}(a_1, a_2)$ are calculated. Both represent the preference degree of a_1 over a_2 considering criterion j. $P_{Gj}(a_1, a_2)$ is used for the leaving flow calculation of alternative a_1 and $P_{Lj}(a_1, a_2)$ is used for entering flow calculation of alternative a_2. $P_{Gj}(a_1, a_2)$ is obtained by using preference functions with regular threshold values whereas $P_{Lj}(a_1, a_2)$ is calculated by using the preference functions with smaller threshold values. The formulas below show how preference indexes are obtained:

   $$P_{Gj}(a_1, a_2) = P_{Gj}(g_j(a_1) - g_j(a_2))$$
   $$P_{Lj}(a_1, a_2) = P_{Lj}(g_j(a_1) - g_j(a_2))$$

   The preference degrees show how much alternative a_1 has better or worse value comparing the alternative a_2 on criterion j. $P_{Gj}(a_1, a_2)$ and $P_{Lj}(a_1, a_2)$ both take a positive value in the range of 0 and 1. If the criterion value of alternative a_1 is a better value than the alternative a_2 both preference degrees take positive values, otherwise they are equal to zero. If the difference between the criterion values of alternative a_1 and alternative a_2 on criterion j is negative, the preferability of alternative a_1 over alternative a_2 on criterion j is equal to zero. If their values are not zero and the preferability of alternative a_1 is higher than the preferability of alternative of a_2 on criterion j, $P_{Lj}(a_1, a_2)$ takes a higher value than $P_{Gj}(a_1, a_2)$ when the loss aversion coefficient of criterion j is higher than 1.

3. The outranking degrees of a_1 over a_2 ($\pi_G(a_1, a_2)$ is to be used for the leaving flow of alternative a_1 and $\pi_L(a_1, a_2)$ is to be used for the entering flow of alternative a_2) are calculated as shown below:

$$\pi_G(a_1, a_2) = \left(\sum_{j=1}^{n} w_j \, P_{Gj}(a_1, a_2)\right)$$

$$\pi_L(a_1, a_2) = \left(\sum_{j=1}^{n} w_j \, P_{Lj}(a_1, a_2)\right)$$

4. The leaving and the entering flows of alternative a_1 are calculated as follows:

$$\phi^{+}(a_1) = \frac{1}{m-1} \sum_{x=1}^{m} \pi_G(a_1, x)$$

$$\phi^{-}(a_1) = \frac{1}{m-1} \sum_{x=1}^{m} \pi_L(x, a_1)$$

5. The net flow value is equal to the difference between leaving and entering flow values as shown below:

$$\phi(a_1) = \phi^{+}(a_1) - \phi^{-}(a_1)$$

6. After leaving, entering and net flow values are calculated, alternatives are either ranked according to leaving and entering flow values with PROMETHEE I or according to net flow values with PROMETHEE II same as in the original PROMETHEE procedure.

In the next section, case studies based on the proposed method and their results are explained in detail.

3.3 COMPUTATIONAL RESULTS

The proposed method is applied to the hydroelectric power station project selection problem, which is studied by Brans et al. (1986) and Times Higher Education (THE) world university ranking 2019 and 2020 data. The proposed method is a generalization of PROMETHEE including the loss aversion coefficient of prospect theory. The results show how the ranking is affected by adding the loss aversion coefficient to the problem when one unit of loss has a higher effect than one unit of gain.

For the hydroelectric power station project selection problem, the criterion values for each alternative, weight values, type of preference functions and threshold values are taken from the study of Brans et al. (1986). The weight values of each criterion are equal in the study, the other parameter values are as shown in Table 3.5. Criterion f-1 represents manpower, f-2 shows power, f-3 illustrates construction cost, f-4 shows maintenance cost, f-5 represents the number of villages to evacuate and f-6 shows the security level. Criterions f-1, f-3, f-4, f-5 should be maximized, f-2 and f-6 should be minimized. The hydroelectric power station projects are represented by the alternatives x-1, x-2, x-3, x-4, x-5 and x-6. As suggested by Kahneman and Tversky (1976), the loss aversion coefficient for each criterion is determined as 2.25.

Table 3.5: The data taken Brans et al. (1986)

Criterions	**Max/Min**	**Alternatives**						**Type of Criterion**	**Parameters**		
		x-1	**x-2**	**x-3**	**x-4**	**x-5**	**x-6**		**p**	**q**	**σ**
f-1	Min	80	65	83	40	52	94	U-shape	-	10	-
f-2	Max	90	58	60	80	72	96	V-shape	30	-	-
f-3	Min	6	2	4	10	6	7	V-shape with indifference	5	0.5	-
f-4	Min	5.4	9.7	7.2	7.5	2	3.6	Level	6	1	-
f-5	Min	8	1	4	7	3	5	Usual	-	-	-
f-6	Max	5	1	7	10	8	6	Gaussian	-	-	5

The flow values of PROMETHEE are given in Table 3.6 whereas the partial and complete rankings using PROMETHEE are given in Figure 3.3 and Figure 3.4, respectively.

Table 3.6: Flow Values of PROMETHEE (Brans et al., 1986)

Alternatives	Positive Flow	Negative Flow	Net Flow
x-1	0.220	0.366	-0.146
x-2	0.396	0.379	0.017
x-3	0.247	0.336	-0.090
x-4	0.329	0.349	-0.020
x-5	0.455	0.162	0.293
x-6	0.300	0.355	-0.055

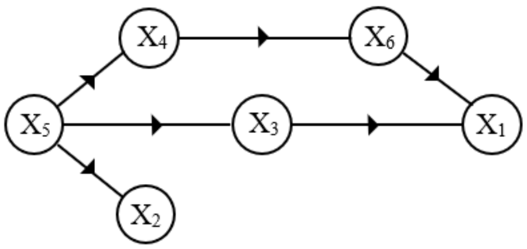


Figure 3.3: Result of PROMETHEE I Method

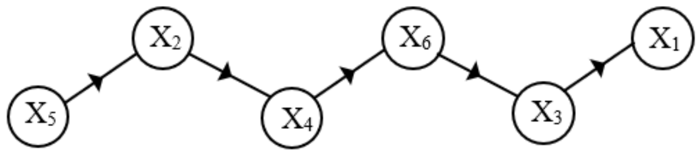


Figure 3.4: Result of PROMETHEE II Method

When the proposed procedure is applied, positive flows of alternatives remain at their values as in the original PROMETHEE. Negative flow values increase compared to the PROMETHEE and net flow values decrease.

The flow values of the proposed method are given in Table 3.7. The partial and complete rankings in case that the proposed method is used are shown in Figure 3.5 and Figure 3.6, respectively.

Table 3.7: Flow Values of Proposed Method

Alternatives	Positive Flow	Negative Flow	Net Flow
x-1	0.220	0.435	-0.215
x-2	0.396	0.467	-0.071
x-3	0.247	0.422	-0.176
x-4	0.329	0.442	-0.112
x-5	0.455	0.222	0.233
x-6	0.300	0.400	-0.100

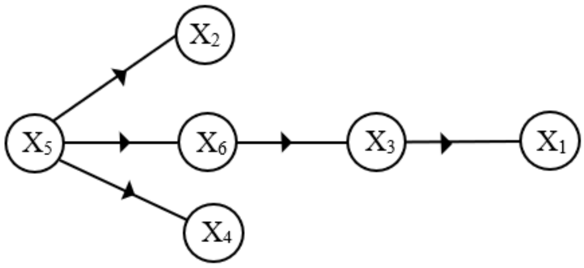


Figure 3.5: Partial Ranking of Proposed Methodology

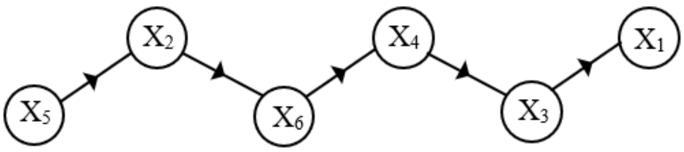


Figure 3.6: Complete Ranking of Proposed Methodology

In the partial ranking, x-5 is the best alternative. x-2 is incomparable with any other alternatives except x-5 when using both PROMETHEE and the proposed method. x-4 outranks x-6 in PROMETHEE. x-4 and x-6 are incomparable in the proposed method. x-3 and x-6 are incomparable in PROMETHEE. x-3 is outranked by x-6 in the proposed method.

In the complete ranking, x-6 is outranked by the x-4 in PROMETHEE. x-6 outranks x-4 based on the proposed method. Since the importance of losses increases, the ranking of alternatives is changed. Negative flow values increase while positive flow values remain the same in the proposed method compared to PROMETHEE. The small difference between negative flow values of alternatives in PROMETHEE becomes significant in the proposed methodology and the relations between alternatives change.

The proposed methodology and PT-PROMETHEE are compared as PT-PROMETHEE is the most similar method with the proposed methodology in literature. PT-PROMETHEE is applied to the hydroelectric power station project selection problem that is studied by Brans et al. (1986). Five different reference alternatives are introduced and the problem is solved with PT-PROMETHEE methodology by using each reference alternative. The loss aversion coefficient λ is taken 2.25 as suggested by Kahneman and Tversky (1976).

The first reference alternative is obtained by using average criterion values of real alternatives. By adding the standard deviation of criterion values of alternatives to the average values of alternatives, the second reference alternative is obtained. The outcome of subtracting the standard deviation of criterion values of alternatives from the average values of alternatives is the third reference alternative. The average value of the first and the third reference alternatives gives the fourth reference alternative. Finally, the fifth reference alternative is obtained by making the third reference alternative's values worse according to the criterions' maximization or minimization direction.

Reference alternative values to use in PT-PROMETHEE are shown in Table 3.8.

Table 3.8: Criterion Values of Reference Alternatives

Criterions	**Reference Alternatives**				
	r-1	**r-2**	**r-3**	**r-4**	**r-5**
f-1	69.0	48.6	89.4	79.2	90.0
f-2	76.0	91.5	60.5	68.2	60.0
f-3	5.8	3.1	8.5	7.2	9.0
f-4	5.9	3.1	8.7	7.3	8.8
f-5	4.7	2.1	7.2	6.0	7.3
f-6	6.2	9.2	3.1	4.6	3.0

The partial and the complete rankings are given in Table 3.9 and in Table 3.10, respectively for each reference point using PT-PROMETHEE methodology.

Table 3.9: Partial Ranking Results of PT-PROMETHEE by using each Reference Alternative

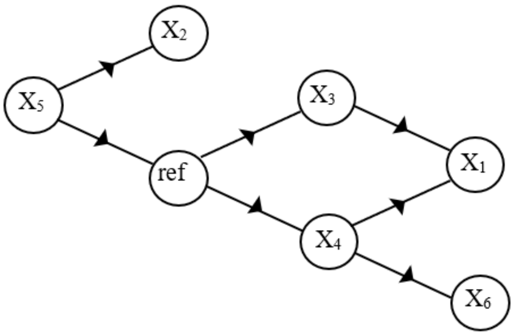
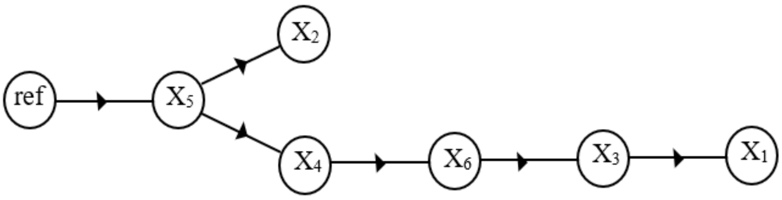
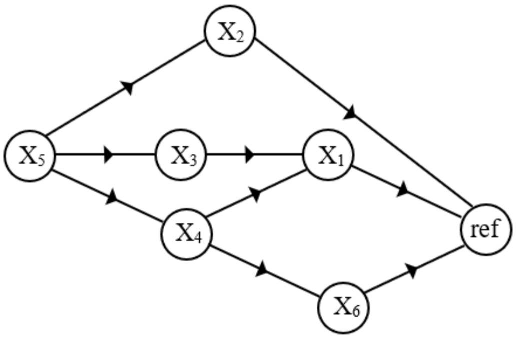
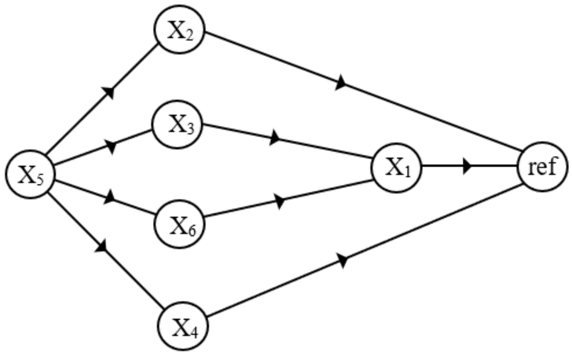
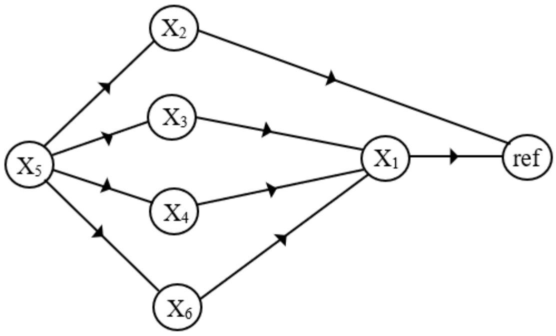
Reference Alternatives	**Partial Rankings of PT-PROMETHEE**
First Reference Alternative	
Second Reference Alternative	
Third Reference Alternative	

Table 3.9 (continued)

Fourth Reference Alternative	
Fifth Reference Alternative	

In the partial ranking using the first reference alternative, x-3 and x-4 are incomparable. However, x-4 outranks x-3 in the partial ranking using the second reference point. x-6 has an incomparability relation with both x-1 and x-3 when the first reference alternative is used. x-6 outranks both x-1 and x-3 when the second reference alternative is used. The first reference alternative is just outranked by x-5 and it has an incomparability relation with x-2 whereas the second reference point outranks all the real alternatives.

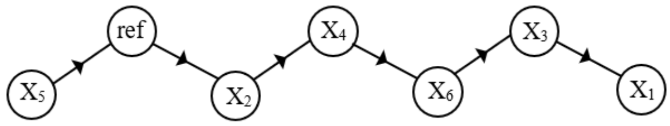
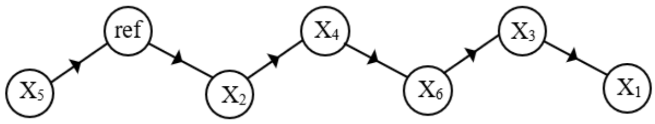
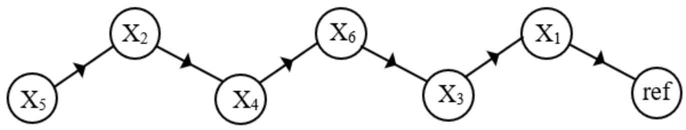
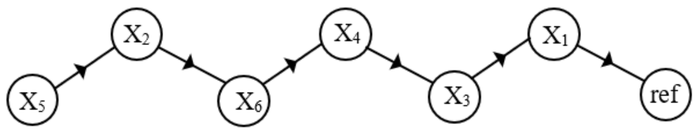
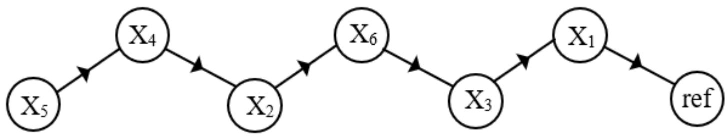
The partial ranking using the third reference alternative gives the same relations as the ranking using the first reference alternative between the real alternatives. The differences of outranking relation of alternatives comparing the results using the first and the second reference alternatives are the same as comparing the second and the third reference alternatives. The third reference point is outranked by all the existing alternatives.

In the partial ranking using the fourth reference point, x-4 is incomparable with all other alternatives except x-5 and the reference alternative. However, x-4 outranks both x-1 and x-6 in the ranking using the first reference alternative. x-4 outranks x-1, x-3 and x-6 when the second reference alternative is used. The fourth reference alternative is outranked by all the real alternatives.

In the partial order using the fifth reference alternative, x-4 outranks x-1. x-1 and x-4 are incomparable when the fourth reference point is used. The fifth reference alternative is outranked by all other alternatives.

The ranking of the real alternatives changes based on the reference alternatives values. An artificial alternative affects the relative importance of the real alternatives in PT-PROMETHEE.

Table 3.10: Complete Ranking Results of PT-PROMETHEE by using each Reference Alternative

Reference Alternatives	**Complete Rankings of PT-PROMETHEE**
First Reference Alternative	
Second Reference Alternative	
Third Reference Alternative	
Fourth Reference Alternative	
Fifth Reference Alternative	

In complete ranking results using the first, the second and the third reference points; the same relations between the real alternatives are obtained and the order of the reference alternatives is different.

A critical change is observed between the orders of x-4 and x-6, where x-4 outranks x-6 when the first, the second and the third reference points are used. However, x-4 is outranked by x-6 when the fourth reference alternative is used. This change between the relation of alternatives x-4 and x-6 is determined as rank reversal which can affect the DM’s choice and can lead to the selection of wrong alternative in a real-life case.

As a similar case, x-2 outranks x-4 in the complete rankings using the first, the second, the third and the fourth reference points. However, x-2 is outranked by x-4 when the fifth alternative is used. Namely, rank reversal is observed again.

When a new alternative is added to the model, the relative importance of the real alternatives changes. The comparison sets of rankings with different reference alternatives are not consistent even though all conditions such as parameter values, type of preference functions and weight values of each criterion are kept the same in all calculations. In our example with six alternatives and six criteria, only the rank of two alternatives is changed. However, if the number of alternatives or criteria is higher, a larger number of changes in the relative importance of alternatives is expected. Properly defining the reference alternative is a difficult process and brings an extra load to the DM. As the number of alternatives increases, the number of pairwise comparisons and computational time also increases to compare the real alternatives with an unreal reference alternative.

According to the reference alternatives’ values, order of the reference alternative is changed. Since the reference alternative determines the DM’s expectation, satisfactory levels of the real alternatives are changed. This change can cause wrong selection among real alternatives.

There is no artificial alternative in the proposed method. The DM should identify how many units of gain can satisfy one unit of loss. If the answer is more than one, PROMETHEE approach is modified to illustrate the difference between gain and loss. If the answer is one, the method acts the same as PROMETHEE.

In the next step, the proposed method is applied to the THE world university ranking 2019 data with 1258 alternatives and 2020 data with 1396 alternatives. THE ranks the universities based on five main criteria, which are teaching, research, citation, industry income and international outlook using the weighted sum method. The criterion values of alternatives and the weight values of criteria are taken as they identified by THE at their official website. Weight of 0.30 is used for teaching, research and citation criteria, 0.025 is used for industry income and 0.075 is used for international outlook. For each criterion linear preference function with indifference area is used since the preference degrees take different values between 0 and 1 according to the slope of the function. Therefore, more sensitive results can be obtained. There are two threshold values as indifference and preference thresholds in the linear preference function with indifference area. Since the preference functions of PROMETHEE are modified based on the threshold values, the linear preference function with indifference area is considered more appropriate to show the performance of the proposed method in comparison to the other methodologies used in experiments. However, the proposed method is appropriate to use with all of the preference functions of PROMETHEE. As suggested by Kahneman and Tversky (1976), coefficient value of 2.25 is used for loss aversion. By changing indifference and preference threshold values, two different cases are studied using both THE world university ranking 2019 and 2020 data. For the first case, indifference threshold values are taken 2 for each criterion and preference threshold values are determined as population standard deviation of alternative values whereas for the second case, indifference threshold values are taken 5 and preference threshold values are defined as twice of population standard deviation of alternative values. The weight and the threshold values of each criterion for case 1 and case 2 are shown in Appendix A.1 and A.2, respectively.

THE 2019 and 2020 data are examined using original PROMETHEE and PT-PROMETHEE methodologies for two cases. For PT-PROMETHEE methodology, the loss aversion coefficients are taken 2.25 for each criterion and three different reference points are determined for two cases. The first reference alternative is equal to the average value of alternatives and the second reference alternative is obtained by adding the standard deviation of alternatives to the average values. By subtracting the standard deviation of alternatives from average values, the third reference point is obtained. Criterion values of each reference point for THE World University Ranking 2019 and 2020 data sets are given in Appendix B.1 and B.2, respectively.

Difference between the rank of universities is calculated for each pair of ranking methods to analyze the divergence between methods. The results of using different indifference and preference threshold values are shown in Tables 3.11 and 3.12 for 2019 and in Table 3.13 and 3.14 for 2020 data.

Table 3.11: Changes in Ranking among Comparison of Methods for THE World University Ranking 2019 Data Case I: The indifference threshold is 2 and the preference threshold is equal to the standard deviation

Compared Methodologies	**Max change**	**Average change in ranking**	**Standard deviation of change**
Weighted Sum & PROMETHEE	216	44.30	41.05
Weighted Sum & PROPOSED	244	49.04	45.64
Weighted Sum & PT-PROMETHEE (ref=ave.)	216	44.35	41.05
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	216	44.27	41.01
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	216	44.31	41.05
PROMETHEE & PROPOSED	38	6.45	6.11
PROMETHEE & PT-PROMETHEE (ref=ave.)	2	0.72	0.51
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.93	0.39
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	2	0.12	0.34
PROPOSED & PT-PROMETHEE (ref=ave.)	38	6.55	6.10
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	37	6.53	6.13
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	38	6.47	6.09
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.30	0.51
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	4	0.67	0.56
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.88	0.47

The maximum change is equal to 244 for the first case with THE world university ranking 2019 data and the average change in the rankings is equal to 49.04. The standard deviation of changes is equal to 45.64 and the largest values for the three metrics are observed between the weighted sum and the proposed methodology. The second largest results belong to weighted sum comparisons with PROMETHEE and PT-PROMETHEE and the maximum change is equal to 216 here the average change in the rankings is 44.35 and the standard deviation is 41.05.

The third largest differences in all three metrics are observed in the proposed methodology comparisons with PROMETHEE and PT-PROMETHEE.

According to the comparisons between PROMETHEE and PT-PROMETHEE using three different reference values, the differences are not significant for all three measurements. The maximum number of changes is approximately 3 and both the average change and the standard deviation are less than 1.

The minimum number of changes is obtained as zero in all comparisons between each method pairs meaning that in all comparisons some alternatives take the same ranking value. The number of alternatives that are placed in the same ranking comparisons are given in Appendix C.1 and C.2 for THE World University Ranking 2019 and 2020 data sets, respectively. Note that the same alternatives do not stay in the same rank in each comparison.

Table 3.12: Changes in Ranking among Comparison of Methods for THE World University Ranking 2019 Data Case II The indifference threshold is 5 and the preference threshold is equal to the standard deviation multiplied by 2.

Compared Methodologies	**Max change**	**Average change in ranking**	**Standard deviation of change**
Weighted Sum & PROMETHEE	210	24.67	24.42
Weighted Sum & PROPOSED	223	33.72	31.53
Weighted Sum & PT-PROMETHEE (ref=ave.)	209	24.85	24.32
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	209	24.81	24.31
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	210	24.67	24.44
PROMETHEE & PROPOSED	91	12.36	12.98
PROMETHEE & PT-PROMETHEE (ref=ave.)	4	0.80	0.54
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.97	0.35
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.13	0.37
PROPOSED & PT-PROMETHEE (ref=ave.)	90	12.38	12.89
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	90	12.46	12.92
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	91	12.38	12.97
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.32	0.50
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.73	0.57
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	4	0.89	0.43

In the comparison between the weighted sum method and the proposed method, the largest changes are observed when using the second case with THE world university ranking 2019 data. The maximum change in ranking is equal to 223, the average change is equal to 33.72 and the standard deviation of change is equal to 31.53. The second largest changes are observed in the comparison of the weighted sum method with PROMETHEE and PT-PROMETHEE. In this case, the maximum changes are around 210, the average changes and the standard deviations are around 24.

PT-PROMETHEE comparisons using three different reference alternatives give similar difference values in itself and with PROMETHEE. The maximum changes are around 3, the average change and the standard deviation of changes are less than 1.

In comparing the cases with 2019 and 2020 data using the same indifference and preference thresholds, similar observations are obtained. The maximum values of all three metrics are obtained in comparisons between the weighted sum method and the proposed methodology. The larger values are observed in the analysis of THE world university ranking 2020 data comparing to the 2019 data for both cases due to the fact that the number of alternatives in 2020 data is more than the number of alternatives in 2019 data.

Table 3.13: Changes in Ranking among Comparison of Methods for THE World University Ranking 2020 Data Case I: The indifference threshold is 2 and the preference threshold is equal to the standard deviation.

Compared Methodologies	**Max change**	**Average change in ranking**	**Standard deviation of change**
Weighted Sum & PROMETHEE	281	51.41	48.00
Weighted Sum & PROPOSED	313	57.56	53.18
Weighted Sum & PT-PROMETHEE (ref=ave.)	282	51.48	47.97
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	282	51.39	47.96
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	281	51.42	48.01
PROMETHEE & PROPOSED	60	7.81	8.23
PROMETHEE & PT-PROMETHEE (ref=ave.)	3	0.74	0.51
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.93	0.39
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	2	0.11	0.33
PROPOSED & PT-PROMETHEE (ref=ave.)	59	7.94	8.19
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	59	7.92	8.20
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	60	7.82	8.22
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.29	0.49
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.69	0.55
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.89	0.47

The largest values are obtained in comparisons between the weighted sum method and the proposed methodology for the first case using THE world university ranking 2019 data. The maximum change is equal to 313, the average change is equal to 57.56 and the standard deviation is equal to 53.18. The second largest changes are observed in the weighted sum method compared with PROMETHEE and PT-PROMETHEE. The maximum number of changes are around 281, the averages are around 51 and the standard deviations are around 48.

In comparing the proposed method with PROMETHEE and PT-PROMETHEE, the maximum change is around 60, the average change and the standard deviation are around 7.80 and 8.20 respectively. In comparisons between PT-PROMETHEE itself and in between PT-PROMETHEE and PROMETHEE, the maximum changes are around 3. The average changes and the standard deviations are less than 1 and the minimum number of changes are observed as zero for all comparisons.

Table 3.14: Changes in Ranking among Comparison of Methods for THE World University Ranking 2020 Data Case II The indifference threshold is 5 and the preference threshold is equal to the standard deviation multiplied by 2.

Compared Methodologies	**Max change**	**Average change in ranking**	**Standard deviation of change**
Weighted Sum & PROMETHEE	221	28.07	28.74
Weighted Sum & PROPOSED	233	38.57	35.98
Weighted Sum & PT-PROMETHEE (ref=ave.)	219	28.28	28.60
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	221	28.24	28.59
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	221	28.07	28.74
PROMETHEE & PROPOSED	99	14.08	15.51
PROMETHEE & PT-PROMETHEE (ref=ave.)	3	0.79	0.51
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.97	0.34
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	2	0.12	0.34
PROPOSED & PT-PROMETHEE (ref=ave.)	97	14.13	15.39
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	98	14.23	15.44
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	98	14.09	15.51
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	3	0.31	0.51
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.75	0.54
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	3	0.93	0.42

By using THE world university ranking 2020 data for each metric, the largest comparison values are obtained between the weighted sum method and the proposed methodology for the second case. The maximum number of changes is equal to 233, the average change is equal to 38.57 and the standard deviation of change is equal to 35.98. The maximum number of changes in the comparisons of weighted sum method with PROMETHEE and PT-PROMETHEE is around 22 where the average changes are around 28.20 and the standard deviation of changes is around 28.60.

The following highest numbers are observed proposed method comparisons with PROMETHEE and PT-PROMETHEE. The maximum number of changes is around 98, the average values of the changes are around 14.10 and standard deviations of changes are around 15.40. In the comparison of PT-PROMETHEE using three different reference alternatives in itself and with PROMETHEE, the maximum number of changes is around 3. The average number of changes and the standard deviation of changes are under 1 and minimum change is observed as zero in all comparisons.

In all cases, the ranks of alternatives among each method are obtained differently for two different data sets. The largest changes are observed between weighted sum and remaining methods as the weighted sum is the most different method comparing the remaining methods used in this study. Pairwise comparisons are used in all methodologies except weighted sum method.

In all comparisons, the largest changes are observed in comparisons of the proposed method with the others. In PROMETHEE, preference functions are used with the same threshold values and their impacts are assumed to be equal for losses and gains whereas in PT-PROMETHEE, preference functions are considered with different threshold values for losses and gains since losing has a higher impact than gaining. However, the losing case is only valid in the comparisons with the reference alternative. PT-PROMETHEE algorithm causes a small difference in the entering flow of alternatives comparing PROMETHEE. As the number of alternatives increases, the impact of using loss aversion decreases. In our methodology, using preference functions with different threshold values is applied in all comparisons between each alternative pair which causes larger difference compared to PROMETHEE and PT-PROMETHEE. The entering flow values are affected by all other alternatives, as well as the reference alternative in PT-PROMETHEE.

In the comparison of PT-PROMETHEE method with itself when using different reference alternatives, the average change and the standard deviation values are calculated around zero which is negligible. Since the loss aversion coefficient affects only the comparisons with the reference alternative, there is no reasonable change in the results even though the reference alternative values are changed. As the number of alternatives increases, the effect of reference alternative decreases. Meanwhile the results are not significantly affected by the values of chosen reference alternatives. Additionally, the rank reversal had been detected in several cases when using THE world university ranking 2019 and 2020 data. Comparing the PT-PROMETHEE rankings with different reference alternative values, the outranking relation between the same alternatives show difference. When a new alternative is added to the instance, that new alternative is expected to take place in the sequence which has been obtained without itself. There could be better or worse alternatives to be compared with the new alternative. However, change in the outranking relations among the predefined alternatives causes an inconsistency in ranking. These results show that the outranking relations obtained by the use of PT-PROMETHEE are not consistent and depends on the value of the reference alternative.

CHAPTER 4

MULTICRITERIA SORTING PROBLEMS

4.1 BACKGROUND

FlowSort is a sorting method based on PROMETHEE, which has been proposed by Nemery and Lamboray (2008). In FlowSort method, a set of m alternatives $A = \{a_1, \dots, a_m\}$ are evaluated with respect to a set of n criteria $G = \{g_1, \dots, g_n\}$ to assign K categories which are predefined and completely ordered. The categories $C_1, C_2, \dots, C_K$ are such that $C_h > C_l$ with $h < l$ meaning that the category C_h is preferred to category C_l. The best and worst categories are defined as C_1 and C_K relatively. In FlowSort, K categories can be defined by using either limiting profiles as in Electre-Tri or central profiles as in the model proposed by Doumpos and Zopounidis (2004) and Figueira et al. (2004).

When the categories are defined by using limiting profiles, the limiting profiles set R contains $K + 1$ elements ($R = \{r_1, \dots, r_{K+1}\}$) since each category is determined by using an upper and a lower limiting profile. Limiting profiles are ordered from best to worst as in $r_1 >, \dots, > r_{K+1}$. Each profile is outperformed by the previous one. A category C_h is defined by an upper limiting profile r_h and a lower limiting profile r_{h+1}. The lower profile of better category C_{h-1} is described by r_h, and r_{h+1} describes the upper profile of worse category C_{h+1}. Performance of all alternatives in set A is assumed to be between the best limiting profile r_1 and the worst limiting profile r_{K+1}. The flow and category representation with limiting profiles are shown in Figure 4.1.

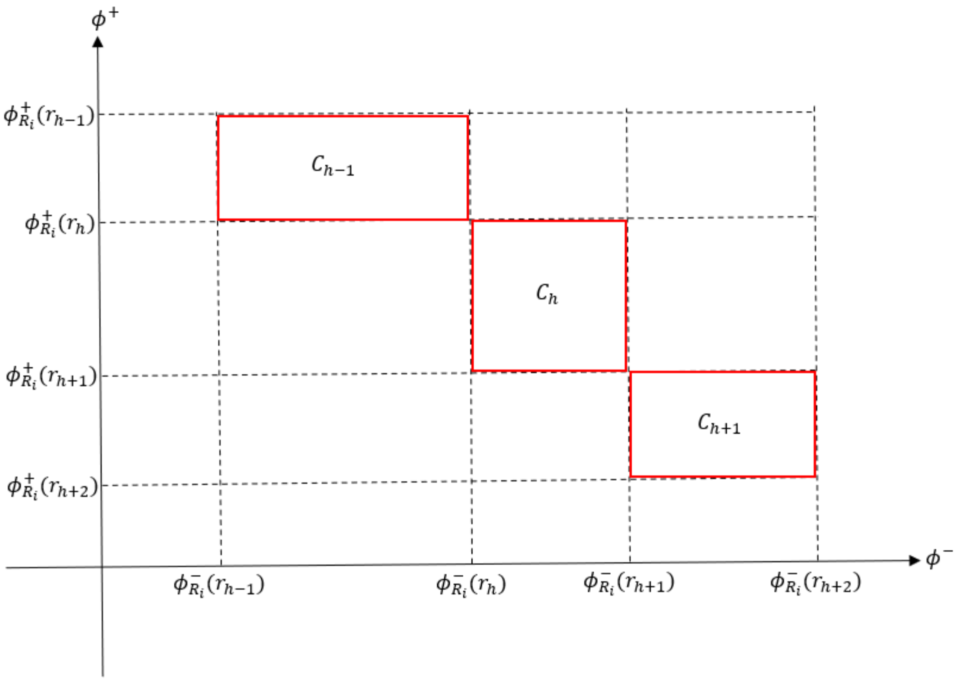


Figure 4.1: A flow and category representation with limiting profiles

When the categories are defined by using central profiles, the set of central profiles $R^{\sim}$ contains K elements ($R^{\sim} = \{r_1^{\sim}, \dots, r_K^{\sim}\}$). The central profiles are preordered and each profile outperforms the following ones after itself as in $r_1^{\sim} >, \dots, > r_K^{\sim}$. Each category $C_h^{\sim}$ is defined by a central profile or centroid $r_h^{\sim}$. The flow and category representation with central profiles are shown in Figure 4.2.

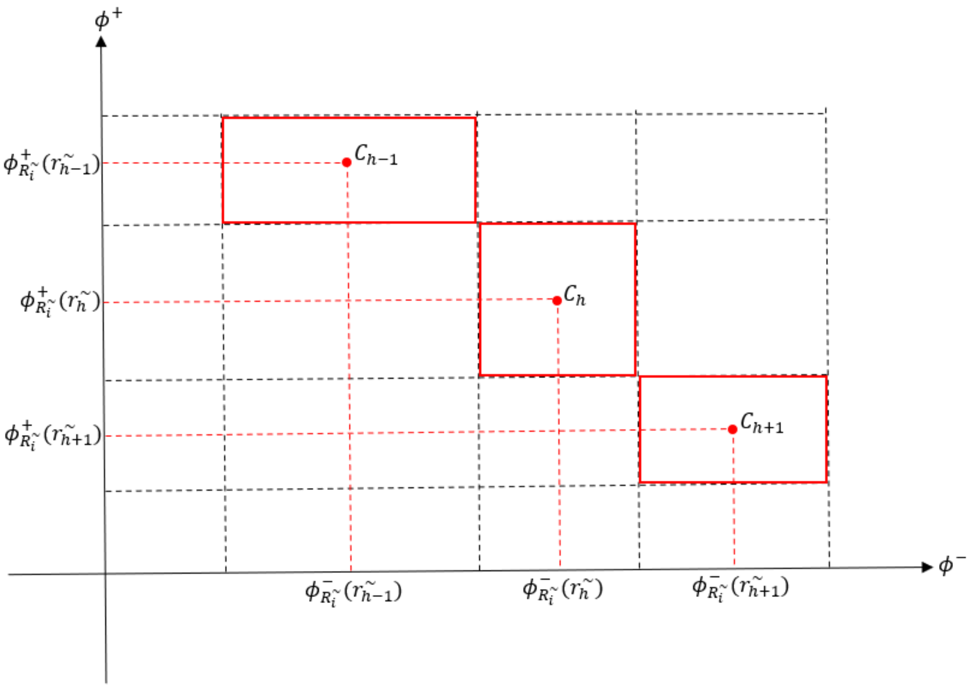


Figure 4.2: A flow and category representation with central profiles

Both limiting and the central profiles are illustrated by $R^* = \{r_1^*,\ r_2^*,\ \ldots\}$ and they are called reference profiles. The outranking relation between reference profiles can be defined as follows:

$$\forall h = 1, \ldots, K: r_h^* > r_{h+1}^* \Leftrightarrow \forall l = 1, \ldots, q: g_l(r_h^*) \geq g_l(r_{h+1}^*) \text{ and } \exists\, \mathrm{j}: g_j(r_h^*) > g_j(r_{h+1}^*)$$

Each alternative's performance in terms of outranking degrees of PROMETHEE is assumed to be between the worst and the best reference profiles. Pairwise comparisons are made between the alternative and the reference profiles for each alternative. An alternative a_i that is to be assigned to a category and reference profiles' set is determined as $R_i^* = R^* \cup \{a_i\}$. For each (x, y) pair in R_i^*, preference degree x over y on each criterion g_j is calculated, as in PROMETHEE, as follows:

$$P_j(x, y) = P[d_j(x, y)]$$

For each criterion g_j, the weight value w_j is determined with $w_j > 0$ for $j = 1, \dots, n$ and $\sum_{j=1}^{n} w_j = 1$. The weight values illustrate the relative importance of criteria. Using the weight values and preference degrees, the outranking degree of action x over action y is calculated as follows:

$$\pi(\mathrm{x}, y) = \sum_{j=1}^{n} w_j \, P_j(x, y)$$

Positive, negative and net flow values are calculated for each action by using outranking degrees as done in PROMETHEE.

$$\phi_{R_i^*}^{+}(x) = \frac{1}{m-1} \sum_{i=1}^{m} \pi(x, y),$$

$$\phi_{R_i^*}^{-}(x) = \frac{1}{m-1} \sum_{i=1}^{m} \pi(\mathrm{y}, \mathrm{x}),$$

$$\phi_{R_i^*}(x) = \phi^{+}(x) - \phi^{-}(x).$$

When the categories are defined by using limiting profiles, the assignment of an alternative a_i to a category C_h can be made based on two different rules according to the positive or negative flow values. Considering the set $R_i = R \cup \{a_i\}$, two assignment rules based on positive and negative flows are identified below.

$$C_{\phi^+}(a_i) = C_h \quad if \quad \phi_{R_i}^{+}(r_h) \geq \phi_{R_i}^{+}(a_i) > \; \phi_{R_i}^{+}(r_{h+1})$$

$$C_{\phi^-}(a_i) = C_h \quad if \quad \phi_{R_i}^{-}(r_h) < \phi_{R_i}^{-}(a_i) \leqslant \; \phi_{R_i}^{-}(r_{h+1})$$

The first rule is based on incoming flow values. The alternative a_i is assigned to category C_h if the positive flow of alternative a_i is between positive flows of limiting profiles r_h and r_{h+1}. The second rule is based on outgoing flows. The alternative a_i is assigned to category C_h if the negative flow of alternative a_i is between negative flows of limiting profiles r_h and r_{h+1}. Based on two different rules, two different assignments such as $C_{\phi^+}(a_i)$ and $C_{\phi^-}(a_i)$ are obtained. Assignment of an alternative based on the first and the second rule can be the same or different categories. This situation is illustrated as $C_{\phi^+}(a_i) \geq C_{\phi^-}(a_i)$ or $C_{\phi^+}(a_i) \leqslant C_{\phi^-}(a_i)$.

Based on the net flow values, alternative a_i is assigned to one unique category C_h using the following rule.

$$C_{\phi}(a_i) = C_h \quad if \quad \phi_{R_i}(r_h) \geq \phi_{R_i}(a_i) > \phi_{R_i}(r_{h+1})$$

The obtained categories based on positive and negative flow values are called the best and the worst categories. $C_{\mathrm{b}}(a_i)$ and $C_{\mathrm{w}}(a_i)$ represent the best and the worst categories of alternative a_i, respectively. The category of a_i, which is obtained by using net flow values, is always in between the best and the worst category of alternative a_i as given below.

$$C_{\mathrm{b}}(a_i) \geq C_{\phi}(a_i) \geq C_{\mathrm{w}}(a_i) \quad \forall ai \in A$$

When the categories are defined by using central profiles, the assignment of an alternative a_i to a category C_h can be made based on two different rules according to positive and negative flow values. Considering the set $R_i^{\sim} = R^{\sim} \cup \{a_i\}$, two assignment rules are specified below.

$$C_{\phi^+}^{\sim}(a_i) = C_h^{\sim} \quad if \quad \frac{\phi_{R_i^{\sim}}^{+}(r_h^{\sim}) + \phi_{R_i^{\sim}}^{+}(r_{h+1}^{\sim})}{2} < \phi_{R_i^{\sim}}^{+}(a_i) \leq \frac{\phi_{R_i^{\sim}}^{+}(r_h^{\sim}) + \phi_{R_i^{\sim}}^{+}(r_{h-1}^{\sim})}{2}$$

$$C_{\phi^-}^{\sim}(a_i) = C_h^{\sim} \quad if \quad \frac{\phi_{R_i^{\sim}}^{-}(r_h^{\sim}) + \phi_{R_i^{\sim}}^{-}(r_{h+1}^{\sim})}{2} \geq \phi_{R_i^{\sim}}^{-}(a_i) > \frac{\phi_{R_i^{\sim}}^{-}(r_h^{\sim}) + \phi_{R_i^{\sim}}^{-}(r_{h-1}^{\sim})}{2}$$

The arithmetic means of the central profiles are used in the rules. According to the rules, alternative a_i is assigned to the category $C_h^{\sim}$ if the positive or negative flow value of alternative a_i is between the arithmetic means of the central profiles $r_h^{\sim}$ and $r_{h+1}^{\sim}$ and the arithmetic mean of the central profiles $r_h^{\sim}$ and $r_{h-1}^{\sim}$. Positive flow values are used in the first rule whereas negative flow values are used in the second rule.

Considering the net flow values, alternative a_i is assigned to category C_h when the net flow value of alternative a_i is between the arithmetic mean of central profiles $r_h^{\sim}$ and $r_{h+1}^{\sim}$, and the arithmetic mean of central profiles $r_h^{\sim}$ and $r_{h-1}^{\sim}$ as shown below.

$$C_{\phi}^{\sim}(a_i) = C_h^{\sim} \quad if \quad \frac{\phi_{R_i^{\sim}}(r_h^{\sim}) + \phi_{R_i^{\sim}}(r_{h+1}^{\sim})}{2} < \phi_{R_i^{\sim}}(a_i) \leq \frac{\phi_{R_i^{\sim}}(r_h^{\sim}) + \phi_{R_i^{\sim}}(r_{h-1}^{\sim})}{2}$$

Alternative a_i can be assigned to the same or different categories according to the positive and negative flow-based assignment rules. The better category is described as the best category of alternative a_i whereas the worse category is defined as the worst category of alternative a_i. The category $C_{\tilde{\phi}}(a_i)$ based on the net flow value is between the best and the worst categories.

$$C_{\tilde{b}}(a_i) \geq C_{\phi}(a_i) \geq C_{\tilde{w}}(a_i) \quad \forall ai \in A$$

The central profile of a category C_h is assumed to be between two consecutive limiting profiles of category C_h for each criterion. The relation between limiting profiles and central profiles is established with the formula below.

$$g_j(r_h) \geq g_j(r_{\tilde{h}}) \geq g_j(r_{h+1})$$

The relation of categories that an alternative a_i is assigned by using limiting profiles or central profiles should be as follows. If $C_{\phi^+}(a_i) = C_h$, $C_{\phi^-}(a_i) = C_l$, $C_{\tilde{\phi}^+}(a_i) = C_{h^\sim}$ *and* $C_{\tilde{\phi}^-}(a_i) = C_{l^\sim}$, then it is expected that $|h - h^\sim| \leqslant 1$ and $|l - l^\sim| \leqslant 1$.

4.2 THE PROPOSED METHOD

FlowSort is a well-known sorting algorithm to assign alternatives to the predefined categories as explained in detail in Chapter 3. To the best of our knowledge, FlowSort and prospect theory have never been studied together. In this study, the FlowSort method is redefined with the gain and loss perspective of the prospect theory and a methodology aiming to sort the alternatives in a discrete MCDM problem is proposed. The method is a generalization of FlowSort. Same results are obtained from the proposed method and FlowSort when the loss aversion coefficient is 1 and the proposed method can be used in problems where FlowSort is applicable. Additionally, the proposed method is appropriate when the losses have a higher impact than the gains for the same amount.

Performance of the alternatives, in terms of positive or negative flow values in pairwise comparisons, affects the category assignment. In FlowSort, pairwise comparisons are made as made in the PROMETHEE methodology. For an alternative, having better or worse criterion values comparing the remaining alternatives has the same effect. On the contrary, in the proposed methodology, having a worse value on a criterion have a higher impact than having a better value for the same amount as in prospect theory. To reflect this property, threshold values of preference functions should be less for the calculation of negative flows than for the calculation of positive flows in pairwise comparisons. To obtain the smaller threshold values, the original threshold values are divided by the loss aversion coefficient (λ) of the prospect theory as suggested by Bozkurt (2007) as shown below. The original threshold values are used for the calculation of positive flows whereas the modified threshold values are used for the calculation of negative flows.

- $p_L = {}^{p}/_{\lambda}$ for preference threshold,
- $q_L = {}^{q}/_{\lambda}$ for indifference threshold,
- $\sigma_L = {}^{\sigma}/_{\lambda}$

In our method, the limiting profiles are used to specify predefined categories. However, the algorithm is also suitable to describe and apply them using the central profiles.

The steps of the proposed methodology are explained as follows:

1. The type of preference functions, the values of thresholds, weights and loss aversion coefficients for each criterion are determined. The set of limiting profiles $R = \{r_1, \dots, r_{K+1}\}$ such that $r_1 >, \dots, > r_{K+1}$, is introduced.
2. Considering $R_i = R \cup \{a_i\}$, where a_i is an alternative to be assigned to a category, for each pair (x, y) in R_i set, preference degrees $P_{Gj}(x, y)$ and $P_{Lj}(x, y)$ are calculated where both represent the preference degree of x over y considering criterion j. $P_{Gj}(x, y)$ is used to calculate the positive flow of x, whereas $P_{Lj}(x, y)$ is used to calculate the negative flow of y. Preference

functions with regular threshold values are used to obtain $P_{Gj}(x,y)$, and $P_{Lj}(x,y)$ is calculated by using the preference functions with smaller threshold values. The formulas for calculation of preference degrees are given below:

$$P_{Gj}(x,y) \ = P_{Gj}(g_j(x) - g_j(y))$$
$$P_{Lj}(x,y) = P_{Lj}(g_j(x) - g_j(y))$$

3. The outranking degrees of x over y are calculated with respect to the relative importance of criterions such that $w_j > 0$ for $j = 1, \dots, n$ and $\sum_{j=1}^{n} w_j = 1$. $\pi_G(x,y)$ is used for the calculation of the positive flow of x, whereas $\pi_L(x,y)$ is used for the calculation of the negative flow of y. The formulas are given below:

$$\pi_G(x,y) = \left(\sum\nolimits_{j=1}^{n} w_j \, P_{Gj}(x,y)\right)$$
$$\pi_L(x,y) = \left(\sum\nolimits_{j=1}^{n} w_j \, P_{Lj}(x,y)\right)$$

4. Positive, negative and net flow values for action x are calculated by using outranking degrees same as in PROMETHEE.

$$\phi^+(x) = \frac{1}{m-1}\sum\nolimits_{x=1}^{m} \pi_G(x,y),$$
$$\phi^-(x) = \frac{1}{m-1}\sum\nolimits_{x=1}^{m} \pi_L(y,x),$$
$$\phi(x) = \phi^+(x) - \phi^-(x).$$

5. Assignment of an alternative a_i to a category C_h is performed. Considering the positive, negative or net flow values, the category of alternative a_i is determined based on the rules shown below.
   - $C_{\phi^+}(a_i) = C_h \quad if \quad \phi^+_{R_i}(r_h) \geq \phi^+_{R_i}(a_i) > \ \phi^+_{R_i}(r_{h+1}),$
   - $C_{\phi^-}(a_i) = C_h \quad if \quad \phi^-_{R_i}(r_h) < \phi^-_{R_i}(a_i) \leqslant \ \phi^-_{R_i}(r_{h+1}),$
   - $C_{\phi}(a_i) = C_h \quad if \quad \phi_{R_i}(r_h) \geq \phi_{R_i}(a_i) > \ \phi_{R_i}(r_{h+1}).$

$C_{\phi^+}(a_i)$ and $C_{\phi^-}(a_i)$ can be the same or different categories such that $C_{\phi^+}(a_i) \geq C_{\phi^-}(a_i)$ or $C_{\phi^+}(a_i) \leqslant C_{\phi^-}(a_i)$. A unique category $C_{\phi}(a_i)$ is determined by using net flow values. $C_{\phi}(a_i)$ is always obtained between the worst and best categories which are defined by using positive and negative flow values.

In the next section, the results of case studies using the proposed methodology and FlowSort method are explained in detail.

4.3 COMPUTATIONAL RESULTS

For the experiments, five data sets from the UCI repository and three data sets from the WEKA machine learning framework are used. The source of the data set and the number of alternatives, criteria and classes for each data are specified in Table 4.1. FlowSort method and the proposed method with different loss aversion coefficient values are applied to the data sets and the difference between the class of alternatives are examined. In the experiments, 1.5, 2.25 and 3 are used as the loss aversion coefficient values. 2.25 is applied since it is suggested by Kahneman and Tversky (1976) whereas 1.5 and 3 are used since they have equal upper and lower distances to 2.25.

Two different cases are being considered using different preference threshold values and the cases are applied to the data sets with FlowSort and the proposed method. In the first case, preference thresholds p are equal to the differences between the maximum and the minimum values of alternatives for each criterion. In the second case, preference thresholds are equal to the differences between the maximum and the minimum values of alternatives multiplied by the loss aversion coefficient for each criterion. In both cases, indifference thresholds q are equal to 10% of the differences between the maximum and the minimum values of alternatives for each criterion. Indifference threshold values for both cases, preference threshold values for case I and case II are shown in Appendix D.1, D.2, and D.3, respectively.

FlowSort result is stable in the first case since preference threshold values are independent of the loss aversion coefficients. In the second case, both FlowSort and proposed method results may change based on loss aversion coefficient values since preference threshold values change dependently on loss aversion coefficient values.

The reference profiles are determined between the maximum and the minimum values of alternatives for each criterion. The maximum values are used as the best reference profile of the criteria whereas the minimum values are used as the worst reference profile of the criteria. The intermediate reference profiles are determined such that each reference profile is equidistant from the other. If there are three reference profiles, the middle reference profiles are equal to the median of the range of the alternative values and weight values of each criterion are taken as equal.

The total difference of alternatives' classes in the comparisons of each run, which are obtained using different loss aversion coefficient values in FlowSort and proposed methodology, are explained for both case 1 and case 2. The results show how the classes of alternatives are affected by adding the loss aversion coefficient of prospect theory to the problem. The data sets and the results for both cases are explained as follows:

Table 4.1: The data sets with their number of alternatives, criteria, classes and source

Data Set	Number of Alternatives	Number of Criteria	Number of Classes	Source
CPU	209	6	2	UCI
Auto MPG	392	7	2	UCI
Employee Selection (ESL)	488	4	2	WEKA
Employee Rejection/Acceptance (ERA)	1000	4	2	WEKA
Lecturers Evaluation (LEV)	1000	4	2	WEKA
Car Evaluation	1728	6	4	UCI
Breast Cancer	278	6	2	UCI
Mammographic	351	4	2	UCI

4.3.1 CPU

The CPU data is taken from the UCI repository. The data includes 209 alternatives to be sorted the CPU time performances into 2 classes based on 6 criteria. The classes are specified as below:

- Acceptable – Class-1
- Unacceptable – Class-2

Criterion values of each reference profile for CPU data set are shown in Appendix E.1. The criteria of CPU data to evaluate the alternatives with their ranges and types are shown in Table 4.2.

Table 4.2: The criteria of CPU data to evaluate the alternatives with their ranges and types

Criteria	Range	Type
MYCT: machine cycle time in nanoseconds	17-1500	Lower the better
MMIN: minimum main memory in kilobytes	64-32000	Higher the better
MMAX: maximum main memory in kilobytes	64-64000	Higher the better
CACH: cache memory in kilobytes	0-256	Higher the better
CHMIN: minimum channels in units	0-52	Higher the better
CHMAX: maximum channels in units	0-176	Higher the better

The total difference results between alternative classes that compare each run by using CPU data are reported in Table 4.3 for case 1 and case 2.

Table 4.3: The total difference results between alternative classes comparing each run using CPU data

Compared Methods	Number of Total Differences	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	0	1
FlowSort-Proposed Method (λ=2.25)	3	0
FlowSort-Proposed Method (λ=3)	1	0
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	3	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	1	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	2	0

In case 1, the highest total number of alternatives placed in different classes comparing the results is 3. This number is observed in two comparisons. The first is from the comparisons of FlowSort and the proposed method using the loss aversion coefficient of 2.25 and the second is obtained from the comparisons of the proposed method using the loss aversion coefficient of 1.5 and 2.25. The total number of differences from the comparison of the proposed method using the loss aversion coefficient equivalent to 2.25 and 3 is equal to 2. The differences of the comparisons between FlowSort and proposed method using loss aversion coefficient of 3, and between proposed method runs using loss aversion coefficient as 1.15 and 3 are equal to 1. There is no change in the classes of alternatives obtained with FlowSort and the proposed method with loss aversion coefficient of 1.5.

In case 1, the alternatives that change classes are the same in the comparisons. The highest change is observed when the loss aversion coefficient is used as 2.25.

In case 2, the difference is observed as 1 in the comparison of FlowSort and the proposed method using the loss aversion coefficient as 1.5. In the remaining comparisons, no changes are observed. Class of an alternative can change if the superiority relationship between the net flows of alternative and the reference profile that has net flow value close to the alternative changes. Alternative is assigned to the same class, if the decrease in the net flow of the alternative is small compared to the decrease in the net flow of the closest reference profile and not sufficient for the alternative to be placed in a different class.

4.3.2 Auto MPG

The auto MPG data from the UCI repository includes 392 alternatives to be sorted into 2 classes based on 7 criteria based on the city-cycle fuel consumption in miles per gallon. The classes are specified as below:

- Acceptable – Class-1
- Unacceptable – Class-2

Criterion values of each reference profile for Auto MPG data set are shown in Appendix E.2. The criteria of Auto MPG data to evaluate the alternatives with their ranges and types are given in Table 4.4.

Table 4.4: The criteria of Auto MPG data to evaluate the alternatives with their ranges and types

Criteria	**Range**	**Type**
Cylinders	3-8	Lower the better
Displacement	1010-9800	Lower the better
Horsepower	46-230	Lower the better
Weight	1613-5140	Lower the better
Acceleration	80-950	Lower the better
Model year	70-82	Lower the better
Origin	More fuel consuming car (1), Medium fuel consuming car (2), Low fuel consuming car (3)	Higher the better

The total number of alternatives placed in different classes comparing each run using Auto MPG data are shown in Table 4.5 for case 1 and case 2.

Table 4.5: The total difference results between alternative classes comparing each run using Auto MPG data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	14	1
FlowSort-Proposed Method (λ=2.25)	27	1
FlowSort-Proposed Method (λ=3)	25	1
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	13	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	11	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	2	0

In case 1, higher changes are observed in FlowSort comparisons. The maximum number of changes is observed as 27 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 2.25. The next higher change is equal to 25 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 3. Finally, in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 1.5, the difference is 14.

The highest difference within proposed method comparisons is 13 which is observed when the loss aversion coefficients are 1.5 and 2.25. When the loss aversion coefficients are 1.5 and 3, the difference is equal to 11 between the proposed method comparisons. The difference is 2 between the proposed method comparisons when loss aversion coefficients are 2.25 and 3.

In case 1, the alternatives that change classes are the same in the comparisons. The highest change is observed when the loss aversion coefficient is taken as 2.25.

In case 2, the difference is observed as 1 in FlowSort comparisons with the proposed method using any of the three-loss aversion coefficient values. The class-changing alternative is the same in all three comparisons. In proposed method comparisons, no changes are observed. Alternatives are assigned to the same classes if the decreases in the net flows of the alternatives are small compared to the decreases in the net flows of the closest reference profiles and are not sufficient for the alternatives to be placed in different classes.

4.3.3 Employee Selection (ESL)

The ESL data is taken from the WEKA machine learning framework. The data includes 488 profiles of applicants for certain industrial jobs to be sorted into 2 classes based on 4 criteria. The criteria values are determined by expert psychologists based on the psychometric test results and interviews with the candidates. All criteria types are considered as “higher the better”.

The classes are specified as below:

- Acceptable – Class-1
- Unacceptable – Class-2

Criterion values of each reference profile for ESL data set are shown in Appendix E.3. The criteria of ESL data to evaluate the alternatives with their ranges and types are presented in Table 4.6.

Table 4.6: The criteria of ESL data to evaluate the alternatives with their ranges and types

Criteria	Range	Type
Criterion-1	0-9	Higher the better
Criterion-2	0-9	Higher the better
Criterion-3	2-8	Higher the better
Criterion-4	2-8	Higher the better

The results for total differences between alternative classes comparing each run using ESL data are reported in Table 4.7 for case 1 and case 2.

Table 4.7: The total difference results between alternative classes comparing each run using Employee Selection (ESL) data

Compared Methods	Number of Total Differences	
	Case 1	Case 2
FlowSort-Proposed Method (λ=1.5)	1	0
FlowSort-Proposed Method (λ=2.25)	18	0
FlowSort-Proposed Method (λ=3)	0	0
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	17	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	1	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	18	0

In case 1, higher changes are observed within the proposed method comparisons using the loss aversion coefficient of 2.25. The number of changes is 18 in proposed method comparisons where loss aversion coefficient is 2.25 with FlowSort and 3 with proposed method. The difference is 17 in the comparison of the proposed method when the loss aversion coefficients are 2.25 and 1.5. The change is equal to 1 when the proposed method with the loss aversion coefficient of 1.5 is compared with either FlowSort or the proposed method with the loss aversion coefficient of 3. There is no change in the comparison of FlowSort and the proposed method with the loss aversion coefficient of 3.

In case 1, the alternatives that change classes are the same in the comparisons. The highest change is observed when the loss aversion coefficient is taken as 2.25.

In case 2, no class change is observed in comparisons regarding the high difference of flow values of the reference profiles and the alternative since superiority relationships between the alternatives and the reference profiles do not change.

4.3.4 Employee Rejection/Acceptance (ERA)

The ERA data from the WEKA machine learning framework originates from an academic decision-making experiment. The data set includes 1000 profiles of applicants to be accepted or rejected based on 4 criteria. The criteria values are attributes of a candidate such as an experience, verbal skills, etc., All criteria types are considered as "higher the better".

The classes are specified as below:

- Acceptable – Class-1
- Unacceptable – Class-2

Criterion values of each reference profile for ERA data set are shown in Appendix E.4. The criteria of ERA data to evaluate the alternatives with their ranges and types are shown in Table 4.8.

Table 4.8: The criteria of ERA data to evaluate the alternatives with their ranges and types

Criteria	Range	Type
Criterion-1	0-14	Higher the better
Criterion-2	0-14	Higher the better
Criterion-3	0-13	Higher the better
Criterion-4	0-14	Higher the better

The total difference results between alternative classes comparing each run using ERA data are given in Table 4.9 for case 1 and case 2.

Table 4.9: The total difference results between alternative classes comparing each run using Employee Rejection/Acceptance (ERA) data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	119	0
FlowSort-Proposed Method (λ=2.25)	143	0
FlowSort-Proposed Method (λ=3)	143	0
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	24	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	24	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	0	0

In case 1, higher class changes are observed in FlowSort comparisons. The highest difference is 143 and obtained from both comparisons between FlowSort and the proposed method with loss aversion coefficient of 2.25, and between FlowSort and the proposed method with loss aversion coefficient of 3. The next higher difference is 119 in the comparison of FlowSort and the proposed method with loss aversion coefficient of 1.5.

In the proposed method comparisons between loss aversion coefficients of 1.5 and 2.25, and between loss aversion coefficients of 1.5 and 3, the differences are equal to 24. There is no change in the proposed method comparison when the loss aversion coefficients are 2.25 and 3.

In case 1, the alternatives that change classes are the same in the comparisons. The numbers of changes increase when the loss aversion value increases from 1.5 to 2.25. The changes stay the same when the loss aversion coefficient increase from 2.25 to 3.

In case 2, no class change is observed in comparisons regarding the high difference of flow values of the reference profiles and the alternative since superiority relationships between the alternatives and the reference profiles do not change.

4.3.5 Lecturers Evaluation (LEV)

The LEV data is taken from the WEKA machine learning framework. The data set includes 1000 anonymous lecturer evaluations taken at the end of MBA courses. The criteria values are the score of the lecturers according to four attributes such that oral skills, contribution to their professional/general knowledge. All criteria types are considered as “higher the better”.

The classes are specified as below:

- Acceptable – Class-1
- Unacceptable – Class-2

Criterion values of each reference profile for LEV data set are shown in Appendix E.5. The criteria of LEV data to evaluate the alternatives with their ranges and types are shown in Table 4.10.

Table 4.10: The criteria of LEV data to evaluate the alternatives with their ranges and types

Criteria	Range	Type
Criterion-1	0-4	Higher the better
Criterion-2	0-4	Higher the better
Criterion-3	0-4	Higher the better
Criterion-4	0-4	Higher the better

The total difference results between alternative classes comparing each run using LEV data are given in Table 4.11 for case 1 and case 2.

Table 4.11: The total difference results between alternative classes comparing each run using Lecturers Evaluation (LEV) data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	118	0
FlowSort-Proposed Method (λ=2.25)	177	0
FlowSort-Proposed Method (λ=3)	134	0
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	59	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	16	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	43	0

In case 1, higher changes are observed in FlowSort comparisons. The highest difference is 177 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 2.25. The number of class changes is 134 between FlowSort and the proposed method with loss aversion coefficient of 3. The difference from the comparison between FlowSort and the proposed method with loss aversion coefficient of 1.5 is equal to 118.

The highest difference in the proposed method comparisons themselves is 59 when the loss aversion coefficients are 1.5 and 2.25. The difference is equal to 43 between the proposed method comparisons when the loss aversion coefficients are 2.25 and 3. The difference between the proposed method comparisons with the loss aversion coefficients of 1.5 and 3 is 16.

In case 1, the alternatives that change classes are the same in the comparisons. The highest change is observed when the loss aversion coefficient is used as 2.25.

In case 2, no class change is observed in comparisons regarding the high difference of flow values of the reference profiles and the alternative. Alternatives are assigned to the same classes if the decreases in the net flows of the alternatives are small compared to the decreases in the net flows of the closest reference profiles and are not sufficient for the alternatives to be placed in different classes.

4.3.6 Car Evaluation (CEV)

The CEV data is retrieved from the UCI Machine Learning Repository. The data set includes 1728 cars to be sorted into 4 classes based on 6 criteria. The classes are specified as below:

- Very good – Class-1
- Good – Class-2
- Acceptable – Class-3
- Unacceptable – Class-4

Criterion values of each reference profile for CEV data set are shown in Appendix E.6. The criteria of CEV data to evaluate the alternatives with their ranges and types are shown in Table 4.12.

Table 4.12: The criteria of CEV data to evaluate the alternatives with their ranges and types

Criteria	**Range**	**Type**
Price	Very high (1), High (2), Medium (3), Low (4)	Higher the better
Maintenance cost	Very high (1), High (2), Medium (3), Low (4)	Higher the better
Number of doors	2 doors (1), 3 doors (2), 4 doors (3), More than 4 doors (4)	Higher the better
Number of person that can be carried	2 persons (1), 4 persons (2), More than 4 persons (3)	Higher the better
Luggage boot capacity	Small (1), Medium (2), Big (3)	Higher the better
Safety	Low (1), Medium (2), High (3)	Higher the better

The difference of total results between alternative classes comparing each run using CEV data are reported in Table 4.13 for case 1 and case 2.

Table 4.13: The difference of total results between alternative classes comparing each run using Car Evaluation (CEV) data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	153	21
FlowSort-Proposed Method (λ=2.25)	346	21
FlowSort-Proposed Method (λ=3)	394	108
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	193	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	241	69
Proposed Method(λ=2.25)-Proposed Method (λ=3)	48	69

In case 1, The highest change is observed as 394 in the comparison between FlowSort and the proposed method with loss aversion coefficient of 3. The difference in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 2.25 is equal to 346. The difference is equal to 153 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 1.5.

The number of changes is equal to 193 in the comparison between proposed method runs when the loss aversion coefficients are 1.5 and 2.25. The difference is obtained as 241 when the proposed method result using the loss aversion coefficient as 1.5 and 3 are compared. The difference is equal to 48 in the comparison of the proposed method results using the loss aversion coefficient as 2.25 and 3.

In case 1, the alternatives that change classes are the same within the comparisons. The number of changes increase when the loss aversion value increases. The highest change is observed when the loss aversion coefficient is taken as 3.

In case 2, the total highest class-change is observed as 108 in the comparison between FlowSort and the proposed method with loss aversion coefficient of 3. The next higher differences are equal to 69 when the proposed method with the loss aversion coefficient of 1.5 is compared with the proposed method results with the loss aversion coefficients of 2.25 and 3. The numbers of changes are equal to 21 when the FlowSort is compared with the proposed method results with the loss aversion coefficients of 1.5 and 2.25. There are no changes in the comparison of proposed method results when the loss aversion coefficients are 1.5 and 2.25.

In case 2, 21 alternatives that change classes in the comparison of FlowSort and the proposed method are the same when the loss aversion coefficient is 1.5, 2.25 and 3. Other than 21 alternatives, 87 alternatives change the classes in the comparison of FlowSort and the proposed method when the loss aversion coefficient is 3. 69 alternatives of 87 alternatives are the same alternatives in the comparisons of proposed method when the loss aversion coefficients are 3, 1.5 and 2.25.

4.3.7 Breast Cancer

The breast cancer data set from the UCI repository is obtained from the University Medical Center, Institute of Oncology, Ljubljana, Yugoslavia. The patients are sorted into 2 classes as no-recurrence-events and recurrence-events according to 7 attributes. Breast cost information is excluded since the attribute cannot include the evaluation.

The classes are specified as below:

- No-recurrence-events – Class-1
- Recurrence-events – Class-2

Criterion values of each reference profile for Breast Cancer data set are shown in Appendix E.7. The criteria of breast cancer data to evaluate the alternatives with their ranges and types are shown in Table 4.14.

Table 4.14: The criteria of breast cancer data to evaluate the alternatives with their ranges and types

Criteria	**Range**	**Type**
Menopause	Premeno (1), Ge40 (2), Lt40 (3)	Lower the better
Tumor Size	0-4 (1), 5-9 (2), 10-14 (3), 15-19 (4), 20-24 (5), 25-29 (6), 30-34 (7), 35-39 (8), 40-44 (9), 45-49 (10), 50-54 (11)	Lower the better
Inv-nodes	0-2 (1), 3-5 (2), 6-8 (3), 9-11 (4), 12-14 (5), 15-17 (6), 24-16 (7)	Lower the better
Node-caps	No (1), Yes (2)	Lower the better
Deg-malig	1-3	Lower the better
Irradiant	No (1), Yes (2)	Lower the better

The difference of total results between alternative classes comparing each run using breast cancer data are given in Table 4.15 for case 1 and case 2.

Table 4.15: The difference of total results between alternative classes comparing each run using Breast Cancer data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	15	0
FlowSort-Proposed Method (λ=2.25)	24	0
FlowSort-Proposed Method (λ=3)	20	0
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	9	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	5	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	4	0

In case 1, the higher changes are observed in FlowSort comparisons. The highest difference is 24 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 2.25. The difference is between FlowSort and the proposed method with loss aversion coefficient of 3 is equal to 20. The difference in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 1.5 is equal to 15.

The highest difference in proposed method comparisons is 9 when the loss aversion coefficients are 1.5 and 2.25. The difference is equal to 5 in the proposed method comparisons when the loss aversion coefficients are 1.5 and 3. There are 4 changes between the proposed method comparisons with loss aversion coefficients of 2.25 and 3.

In case 1, the alternatives that change classes are the same in the comparisons. The highest change is observed when the loss aversion coefficient is taken as 2.25.

In case 2, no class change is observed in comparisons regarding the high difference of flow values of the reference profiles and the alternative since superiority relationships between the alternatives and the reference profiles do not change.

4.3.8 Mammographic

The mammographic data set from the UCI repository is about breast cancer screening by mammography. The data set includes 351 patients' attitudes to predict the severity (benign or malignant) of a mammographic mass lesion based on 4 criteria.

The classes are specified as below:

- Benign – Class-1
- Malignant – Class-2

Criterion values of each reference profile for Mammographic data set are shown in Appendix E.8. The criteria of mammographic data to evaluate the alternatives with their ranges and types are shown in Table 4.16.

Table 4.16: The criteria of mammographic data to evaluate the alternatives with their ranges and types

Criteria	**Range**	**Type**
Age	18-96	Higher the better
Shape	1-4	Lower the better
Margin	1-5	Lower the better
Density	1-4	Lower the better

The difference of total results between alternative classes comparing each run using mammographic data are shown in Table 4.17 for case 1 and case 2.

Table 4.17: The difference of total results between alternative classes comparing each run using Mammographic data

Compared Methods	**Number of Total Differences**	
	Case 1	**Case 2**
FlowSort-Proposed Method (λ=1.5)	6	0
FlowSort-Proposed Method (λ=2.25)	15	1
FlowSort-Proposed Method (λ=3)	9	1
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	9	1
Proposed Method(λ=1.5)-Proposed Method (λ=3)	5	1
Proposed Method(λ=2.25)-Proposed Method (λ=3)	6	0

In case 1, the highest difference is 15 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 2.25. The numbers of changes are 9 in the comparisons between FlowSort and the proposed method with loss aversion coefficient of 3, and between proposed method runs when the loss aversion coefficients are 1.5 and 2.25. Difference is 6 in the comparisons between FlowSort and proposed method with loss aversion coefficient of 1.5, and between proposed method runs when the loss aversion coefficients are 2.25 and 3. The difference is 5 between the proposed method results when the loss aversion coefficients are 1.5 and 3.

In case 1, the alternatives that change classes when the loss aversion coefficient is 1.5 change the classes when the loss aversion coefficient is 2.25 in the FlowSort and proposed method comparisons. The alternatives that have a class difference when the loss aversion coefficient is 3 change the classes when the loss aversion coefficient is 2.25 in the FlowSort and proposed method comparisons. 5 alternatives are the same in FlowSort comparison with the proposed method when the loss aversion coefficients are 1.5 and 3. The highest change is observed when the loss aversion coefficient is taken as 2.25.

In case 2, there is no difference in the comparisons between FlowSort and the proposed method with loss aversion coefficient of 1.5, and between the proposed method runs when the loss aversion coefficients are 2.25 and 3. In the remaining comparisons, the difference is equal to 1. The alternative that changes the class is the same in all comparisons.

4.3.9 General Remarks

In experiments maximum class change is obtained as 1 even if the number of classes are higher than 2. For the alternatives with class change in the proposed method, net flow value of alternative is close to the net flow values of one of the reference profiles. The class change is related to the superiority of the net flow value of the alternative and the reference profile, which has net flow value close to the alternative, over each other. The class of alternative can change if the superiority relationship between the net flows of alternative and the reference profile that has net flow value close to the alternative changes. The superiority relationships of alternative and other reference profiles do not change.

In case 2, the numbers of changes are observed as 1 or 0 in comparisons of all data sets except for CEV. In the 2-class structure, the change in the net flow values of the alternatives cannot reach the amount that will change the class by exceeding the net flow changes of the reference profiles. ERA, ESL and LEV data sets are re-examined with 4 classes for case 2 to analyze this attitude. Net flows of alternatives and reference profiles are closer to each other when 4 classes are determined comparing to the 2-class. According to the loss aversion coefficient value, the change in the net flow of the alternative can exceed the change in the net flow of reference profile and the class change of alternative is observed. The results of experiments with 4 classes are shown in Table 4.18.

Table 4.18: The total number of differences in case 2 analysis with 4 classes using ERA, ESL, LEV data sets

Compared Methods	Number of Total Differences		
	ERA	**ESL**	**LEV**
FlowSort-Proposed Method (λ=1.5)	0	4	31
FlowSort-Proposed Method (λ=2.25)	22	3	31
FlowSort-Proposed Method (λ=3)	47	1	31
Proposed Method(λ=1.5)-Proposed Method (λ=2.25)	22	1	0
Proposed Method(λ=1.5)-Proposed Method (λ=3)	47	3	0
Proposed Method(λ=2.25)-Proposed Method (λ=3)	25	2	0

Using ERA data set with 4 classes, the highest difference is observed as 47 in the comparison between FlowSort and the proposed method with the loss aversion coefficient of 3, and in the comparisons of proposed method when the loss aversion coefficient is 1.5 and 3. The number of differences is 25 in the comparisons of proposed method with loss aversion coefficient of 2.25 and 3. The number of changes is 22 in the comparisons between FlowSort and the proposed method with loss aversion coefficient of 2.25 and between proposed method runs when the loss aversion coefficients are 1.5 and 2.25. There is no class change comparison of FlowSort and the proposed method with loss aversion coefficient of 1.5.

The maximum change is 4, which is obtained by using ESL data set, in the comparison of FlowSort and the proposed method with loss aversion coefficient of 1.5. The number of differences is 3 in the comparisons of FlowSort with proposed method using loss aversion coefficient as 2.25 and in the comparison of proposed method with loss aversion coefficient of 1.5 and 3. The difference of proposed method comparison with loss aversion coefficient of 2.25 and 3 is 2. The changes are 1 in the comparisons between FlowSort and proposed method with loss aversion coefficient of 3, and between the proposed method comparisons with loss aversion coefficient of 1.5 and 2.25.

Using LEV data set, the number of differences is equal to 31 in FlowSort comparisons with proposed method when the loss aversion coefficients are 1.5, 2.25 and 3. There are no differences between proposed method comparisons. The alternatives that change the class with proposed method compared to FlowSort are the same when the loss aversion coefficients are 1.5, 2.25 and 3.

In the experiments, three different class attitudes of alternatives are observed. In the first attitude, the alternatives change their classes when 2.25 and 1.5 are used as the loss aversion coefficient values. They again change class when 3 and 2.25 are used as loss aversion coefficient values. In the second attitude, the alternatives stay in the same class whatever the loss aversion coefficient value is from 1.5 to 2.25. In the third attitude, the alternatives change their classes once. For the first, second and third attitudes, respectively the 8th alternative from CPU data, 17th alternative from ESL data and 193th alternative from CPU data are examined. The proposed method is applied with loss aversion coefficient values from 1.5 to 3 for 8th alternative of CPU data and 17th alternative of ESL and from 1 to 3 for 193th alternative of CPU by increasing 0.1 in each run.

Net flow and class values of 8th alternative from CPU data are given in Table 4.19 to illustrate the first attitude and the net flow diagram is shown in Figure 4.3.

Table 4.19: Net flow and class values of the 8th alternative from CPU data to illustrate the first attitude. (The first attitude: The alternatives change the class when 2.25 is used as the loss aversion coefficient value comparing when 1.5 is used and again changes the class when 3 loss aversion coefficient value is used comparing when 2.25 is used)

λ	**Net Flow Values**				**Class**
	Reference Profile-1 (x-1)	**Reference Profile-2 (x-2)**	**Reference Profile-3 (x-3)**	**Alternative (a)**	
1.50	0.65	-0.10	-0.76	-0.14	2
1.60	0.65	-0.13	-0.79	-0.15	2
1.70	0.65	-0.15	-0.82	-0.16	2
1.80	0.65	-0.17	-0.85	-0.18	2
1.90	0.65	-0.19	-0.88	-0.19	1
2.00	0.65	-0.21	-0.91	-0.20	1
2.10	0.65	-0.21	-0.91	-0.20	1
2.20	0.65	-0.21	-0.92	-0.21	1
2.30	0.65	-0.21	-0.92	-0.21	1
2.40	0.65	-0.21	-0.92	-0.22	2
2.50	0.65	-0.21	-0.93	-0.22	2
2.60	0.65	-0.21	-0.93	-0.23	2
2.70	0.65	-0.21	-0.94	-0.23	2
2.80	0.65	-0.21	-0.94	-0.23	2
2.90	0.65	-0.21	-0.95	-0.24	2
3.00	0.65	-0.21	-0.95	-0.24	2

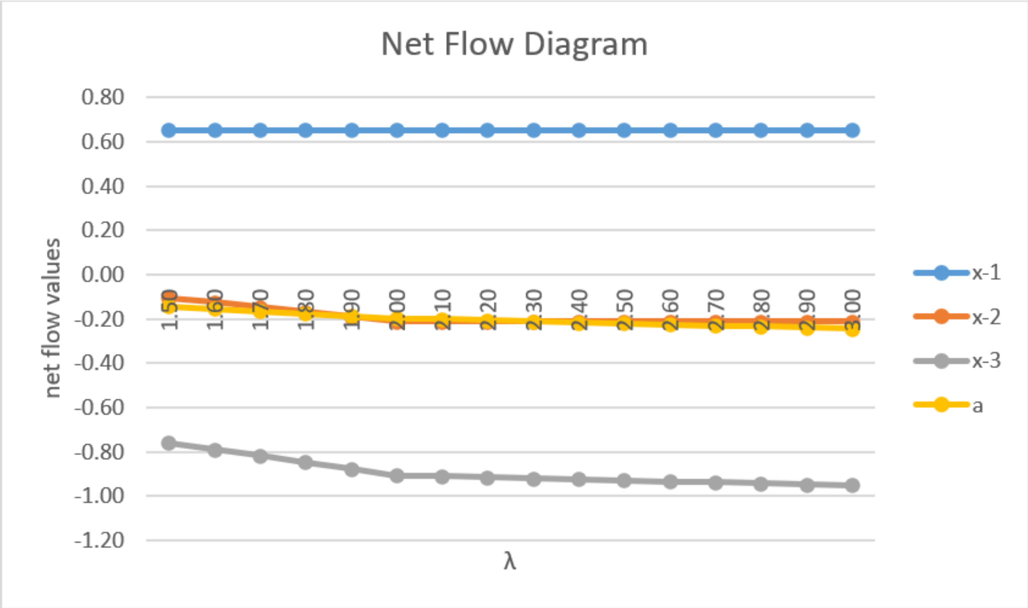


Figure 4.3: Net flow diagram of 8th alternative from CPU data

Net flow and class values of 17th alternative from ESL data are given in Table 4.20 to illustrate the second attitude and the net flow diagram is presented in Figure 4.4.

Table 4.20: Net flow and class values of the 17th alternative from ESL data to illustrate the second attitude. (The second attitude: The alternatives stay in the same class whatever the loss aversion coefficient value is from 1.5 to 2.25.)

λ	**Net Flow Values**				**Class**
	Reference Profile-1 (x-1)	**Reference Profile-2 (x-2)**	**Reference Profile-3 (x-3)**	**Alternative (a)**	
1.50	0.61	-0.11	-0.84	-0.06	1
1.60	0.61	-0.14	-0.86	-0.07	1
1.70	0.61	-0.16	-0.89	-0.08	1
1.80	0.61	-0.18	-0.92	-0.09	1
1.90	0.61	-0.20	-0.94	-0.10	1
2.00	0.61	-0.22	-0.97	-0.12	1
2.10	0.61	-0.23	-0.97	-0.13	1
2.20	0.61	-0.23	-0.98	-0.13	1
2.30	0.61	-0.23	-0.98	-0.14	1
2.40	0.61	-0.23	-0.98	-0.15	1
2.50	0.61	-0.24	-0.98	-0.16	1
2.60	0.61	-0.24	-0.99	-0.16	1
2.70	0.61	-0.24	-0.99	-0.17	1
2.80	0.61	-0.25	-0.99	-0.18	1
2.90	0.61	-0.25	-1.00	-0.19	1
3.00	0.61	-0.25	-1.00	-0.19	1

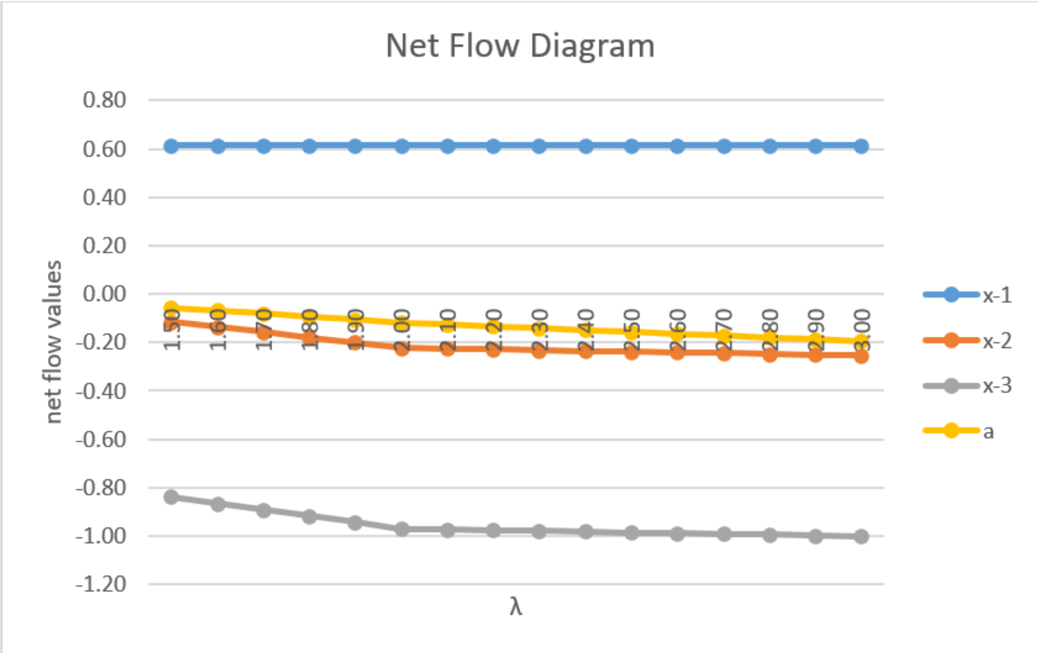


Figure 4.4: Net flow diagram of 17.th alternative from ESL data

Net flow and class values of the 193th alternative from CPU data are reported in Table 4.21 to illustrate the second attitude and the net flow diagram is shown in Figure 4.5.

Table 4.21: Net flow and class values of 93.th alternative from CPU data to illustrate the third attitude. (The third attitude: The alternatives change the class once.)

λ	Net Flow Values				Class
	Reference Profile-1 (x-1)	**Reference Profile-2 (x-2)**	**Reference Profile-3 (x-3)**	**Alternative (a)**	
1.00	0.643	-0.001	-0.623	-0.018	2
1.10	0.643	-0.023	-0.653	-0.041	2
1.20	0.643	-0.045	-0.683	-0.064	2
1.30	0.643	-0.066	-0.714	-0.083	2
1.40	0.643	-0.088	-0.744	-0.097	2
1.50	0.643	-0.109	-0.774	-0.110	2
1.60	0.643	-0.131	-0.804	-0.123	1
1.70	0.643	-0.152	-0.834	-0.136	1
1.80	0.643	-0.174	-0.864	-0.149	1
1.90	0.643	-0.196	-0.895	-0.161	1
2.00	0.643	-0.217	-0.925	-0.170	1
2.10	0.643	-0.218	-0.930	-0.174	1
2.20	0.643	-0.218	-0.935	-0.177	1
2.30	0.643	-0.218	-0.938	-0.181	1
2.40	0.643	-0.218	-0.940	-0.184	1
2.50	0.643	-0.218	-0.943	-0.188	1
2.60	0.643	-0.218	-0.946	-0.191	1
2.70	0.643	-0.218	-0.948	-0.195	1
2.80	0.643	-0.218	-0.951	-0.199	1
2.90	0.643	-0.218	-0.954	-0.203	1
3.00	0.643	-0.218	-0.956	-0.206	1

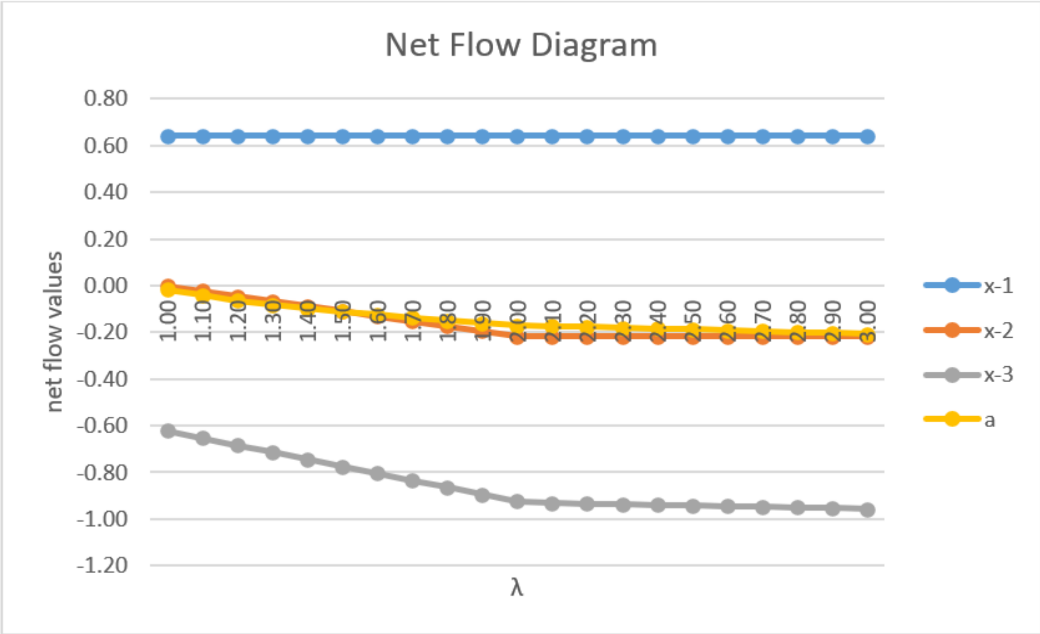


Figure 4.5: Net flow diagram of 193.th alternative from CPU data

The reason for all three attitudes is that the loss aversion coefficient value affects the negative flow values, hence the net flow values of alternatives and reference profiles. Net flow values decrease since negative flow values increase, and positive flow values stay the same when the loss aversion coefficient value increases. However, the decrease in net flow values continues until the difference between the alternative and reference profiles exceed the preference threshold values. Sometimes alternatives reach that point earlier or sometimes reference profiles do. This relation affects the class change attitude of alternatives.

Considering all results of experiments, when the alternatives have the class changes with comparison of the FlowSort and the proposed method results, the alternatives can be assigned to the better or worse classes in the proposed method assignments. Negative flows, hence net flows of both alternatives and reference profiles change. If the decrease in net flow value of reference profile is higher than the decrease in net flow value of alternative, the alternative can be assigned to a better class in proposed method. If the decrease in net flow value of alternative is higher than the decrease in net flow value of reference profile, the alternative can be assigned to a worse class in proposed method.

4.4 ANALYSIS OF WEIGHTS

Computational results of all data sets are calculated using equal weights. To analyze the class changes of alternatives using different weight values, CEV data set with 6 criteria and 1728 alternatives from the UCI Machine Learning Repository is used. In each experiment, the class of alternatives are determined using FlowSort and the proposed method, and the differences are examined. The loss aversion coefficient is used as 2.25 as suggested by Kahneman and Tversky (1976), in all experiments. Weight values are determined in the following ways in experiments. Firstly, one criterion weight value is taken as 0.5, others are taken as 0.1 to observe the results when one of the criteria weights is dominant. Secondly, two criteria weight values are taken as 0.3, others are taken as 0.1 to analyze the results when two of the criteria weights are dominant. The weight values of experiments are shown in Table 4.22. The results of weight analysis are reported in Table 4.23.

Table 4.22: Weight values of experiments

Experiments	**Weight Values**					
	Price	**Maintenance cost**	**Number of doors**	**Number of person that can be carried**	**Luggage boot capacity**	**Safety**
Experiment 1	0.5	0.1	0.1	0.1	0.1	0.1
Experiment 2	0.1	0.5	0.1	0.1	0.1	0.1
Experiment 3	0.1	0.1	0.5	0.1	0.1	0.1
Experiment 4	0.1	0.1	0.1	0.5	0.1	0.1
Experiment 5	0.1	0.1	0.1	0.1	0.5	0.1
Experiment 6	0.1	0.1	0.1	0.1	0.1	0.5
Experiment 7	0.3	0.3	0.1	0.1	0.1	0.1
Experiment 8	0.3	0.1	0.3	0.1	0.1	0.1
Experiment 9	0.3	0.1	0.1	0.3	0.1	0.1
Experiment 10	0.3	0.1	0.1	0.1	0.3	0.1
Experiment 11	0.3	0.1	0.1	0.1	0.1	0.3
Experiment 12	0.1	0.3	0.3	0.1	0.1	0.1
Experiment 13	0.1	0.3	0.1	0.3	0.1	0.1
Experiment 14	0.1	0.3	0.1	0.1	0.3	0.1
Experiment 15	0.1	0.3	0.1	0.1	0.1	0.3
Experiment 16	0.1	0.1	0.3	0.3	0.1	0.1
Experiment 17	0.1	0.1	0.3	0.1	0.3	0.1
Experiment 18	0.1	0.1	0.3	0.1	0.1	0.3
Experiment 19	0.1	0.1	0.1	0.3	0.3	0.1
Experiment 20	0.1	0.1	0.1	0.3	0.1	0.3
Experiment 21	0.1	0.1	0.1	0.1	0.3	0.3

Table 4.23: The results of weighting analysis experiments

Experiments	**Total Difference**
Experiment 1	248
Experiment 2	248
Experiment 3	248
Experiment 4	283
Experiment 5	283
Experiment 6	283
Experiment 7	322
Experiment 8	322
Experiment 9	301
Experiment 10	301
Experiment 11	301
Experiment 12	322
Experiment 13	301
Experiment 14	301
Experiment 15	301
Experiment 16	301
Experiment 17	301
Experiment 18	301
Experiment 19	313
Experiment 20	313
Experiment 21	313

The differences are higher when two criteria weight values are used as 0.3 and other weights are 0.1 with comparison to one criterion weight value is used as 0.5 and other weights are 0.1. The highest changes are observed as 322 when the weight value of any two of the criteria price, maintenance cost, and number of doors are equal to 0.3 and other weight values are 0.1. The next higher-class differences are equal to 313 when the weight value of any two of the criteria number of person that can be carried, luggage boot capacity, and safety are equal to 0.3 and other weight values are 0.1. Total differences are observed as 301 when the weight value of one of the criteria price, maintenance cost or number of doors and the weight value of one of the criteria number of person that can be carried, luggage boot capacity or safety are equal to 0.3. When the weight values of criteria number of person that can be carried, luggage boot capacity or safety are equal to 0.5 and other criteria's weight values are 0.1, total class change is equal to 283. Total difference is observed as 248 when the weight values of criteria price, maintenance cost or number of doors are equal to 0.5 and others are 0.1.

The reason for the class change difference between FlowSort and the proposed method in experiments using the different weight values is the distribution of the alternative values. Criteria price, maintenance cost and number of doors have values between 1 and 4 whereas criteria number of person that can be carried, luggage boot capacity and safety have values between 1 and 3 in CEV data set. The alternatives that are placed in different classes by comparing two methods are more when two criteria weight values are 0.3 and others 0.1 than one criteria value is 0.5 and others 0.1. The differences that occur due to the change in weight values are all related to the data distribution.

CHAPTER 5

CONCLUSION AND FUTURE WORK

In this study, the integration of the prospect theory into the ranking and sorting methods based on dominance relations has been studied by using the well-known multi criteria outranking method PROMETHEE and the sorting method FlowSort. The prospect theory asserts that the losses have a higher impact than gains for the same amount. In PROMETHEE/FlowSort methods which ranks/sorts the alternatives based on the flow values that are calculated by using pairwise comparisons by evaluating the criterion value with the same importance in order to determine whether the criterion value of the alternative is better or worse. From the perspective of the loss effect, the impact of having a worse criterion value is increased in PROMETHEE and FlowSort as it is in the prospect theory by using a steeper slope in preference functions for the negative flow calculation. The loss aversion coefficient of the prospect theory is used in order to obtain the preference functions with a steeper slope.

The proposed method for ranking/sorting is a generalization of PROMETHEE/FlowSort. PROMETHEE/FlowSort is a special case of the proposed methodology since the proposed method and PROMETHEE/FlowSort give the same results when the loss aversion coefficient value is equal to 1. Therefore, the proposed method can be used where PROMETHEE/FlowSort is applicable. Additionally, the proposed method is appropriate when the choice behavior of the decision maker cannot be modelled using the MAUT due to the reason that the DM gives more importance to losses than gains. The justification of PROMETHEE/FlowSort is valid for the proposed method.

The proposed method for ranking is compared with PROEMETHEE, PT-PROMETHEE and the weight sum method using THE World University Ranking

2019 and 2020 data sets. In PT-PROMETHEE calculations, three different reference alternatives are used. The weighted sum method is considered in the comparisons as THE uses the weighted sum method for university ranking. The results from each method are compared in pairs. The maximum order change of alternative, the average change and standard deviation of the changes are analyzed.

The results of THE World University Ranking 2019 and 2020 data sets are similar. The highest differences are obtained from comparisons of the weighted sum method. The alternatives are ranked based on the scores obtained from criteria values of alternatives and the weight values of criteria in the weighted sum method. Pairwise comparisons are not used in the weighted sum method. Since the calculation technique is different than the other methodologies. the highest differences are obtained from the weighted sum comparisons.

Significant differences are obtained from the comparisons in which the proposed method is applied. The proposed method ranks the alternatives from a different perspective compared to the existing methods in the literature. Since the proposed method is a generalization of PROMETHEE and gives the same result as PROMETHEE when the loss aversion coefficient is 1, the difference between the proposed method and PROMETHEE shows that the ranking changes of the alternatives when the impact of losses is increased. The increase in the difference when the impact of losses is increased shows how much the superiority of the alternatives to each other can change according to the value that the decision-maker determines between loss and gain.

The proposed method for sorting is compared with FlowSort using the data sets that have different number of alternatives, criteria, and classes from the literature that are used for sorting techniques. The results obtained from the proposed method with different loss aversion coefficients are compared with the FlowSort method and among themselves. In these comparisons, the class changes of alternatives are analyzed. For the analysis of weights, the proposed method and FlowSort are compared with various weight value combinations using the CEV data set.

Even if the number of classes is more than two in the comparisons, the alternatives can leap to one better or worse class. The maximum level of class change for an alternative is 1. The net flow values of the alternatives are close to only one reference profile. A class change occurs if the superiority relation between the alternative and the closest reference profile to the alternative changes with a decrease in net flows in the proposed method. Since the net flow value of the alternative is distant from other reference profiles, there is no change in the superiority relationship with other profiles.

Classes of alternatives obtained by the proposed method may be better or worse than the classes obtained by FlowSort. If the decrease in the net flow of the alternative is more than that of the reference profile it is close to, the alternative can be placed in the worse class; and on the contrary if it is less, in the better class.

When the number of classes is low, the alternatives can be placed in the same classes by the proposed method and FlowSort where the number of class changes can be zero even if the calculation method is different. To analyze this attitude, ERA, ESL, and LEV data sets are extended up to four classes, and the proposed and FlowSort are compared. The results show that the number of alternatives that change class increases. When there are two classes, the distance between the reference profiles increases. Relatively, the superiority relationship between the alternatives and reference profiles does not change even if the net flows decrease in the proposed method since the distance between the reference profiles are high. The basis of all the results obtained in the proposed method for sorting is the change in the superiority relationship of the alternative with the reference profiles.

The reason for the class change differences between the proposed method and FlowSort in experiments with different combinations of weight values is the distribution of alternative values. In cases where the weight of two criteria on the data is 0.3 and others are 0.1, the class changes of alternatives are more than the case where the weight of one criterion is 0.5 and the others are 0.1. Differences with the changing weight values are all about data distribution.

The weight and the loss aversion coefficient values are the essential point of the proposed methodology. In this study, the same weight values with THE are used for the ranking problem and equal weights are used for the sorting problems in computational experiments. Weight values can be determined using the mixture experiment design as a future work. To satisfy the preferences of the DM, a methodology such as AHP or ANP can be used to determine the weight values. Additionally, to analyze similarities between the proposed method and the other methodologies in literature, weight values that produce similar results with other ranking methodologies such as the weighted sum method can be determined.

The loss aversion coefficient value of the prospect theory is suggested as 2.25 by Kahneman and Tversky (1976). In this study, the loss aversion coefficient values are used as 2.25 for the ranking part and as 1.5, 2.25 and 3 for the sorting part. 1.5 and 3 are used since they provide lower and upper bounds on 2.25. As a future work, a method that determines the loss aversion coefficient values according to the problem types and DM's preferences can be developed. Hu et al. (2014) defined multiple reference intervals by extending the reference points of the prospect theory to provide solutions that satisfy the DM's preferences. Using Hu et al. (2014)'s perspective, a method can be developed to provide appropriate weight and loss aversion coefficient intervals according to the problem types or DM's preferences.

In this study, linear preference function with indifference area is used in all experiments. The effect of using different preference functions can also be studied as a future work.

REFERENCES

Abdullah, L., Chan, W., Afshari, A. (2018). Application of PROMETHEE method for green supplier selection: a comparative result based on preference functions. Journal of Industrial Engineering International (2019) 15:271–285

Alali, F., Tolga, A. C. (2019). Portfolio allocation with the TODIM method. Expert Systems With Applications 124 (2019), 341-348.

Almeida-Dias, J., Figueira, J.R., Roy, B.: Electre Tri-C: a multiple criteria sorting method based on characteristic reference actions. Eur. J. Oper. Res. 204(3), 565–580 (2010).

Almeida-Dias, J., Figueira, J.R., Roy, B.: A multiple criteria sorting method where each category is characterized by several reference actions: the electre Tri-nC method. Eur. J. Oper. Res. 217(3), 567–579 (2012).

Araújo, R. M. (2015). Multicriteria Classification with TODIM-FSE. Procedia Computer Science, 55, 559–565.

Araz, C., & Ozkarahan, I. (2007). Supplier evaluation and management system for strategic sourcing based on a new multicriteria sorting procedure. International Journal of Production Economics, 106 (2), 585–606.

Autran Monteiro Gomes, L. F., and González, X. I. (2012). Behavioral multi-criteria decision analysis: further elaborations on the todim method. Foundations of Computing and Decision Sciences, 37(1), 3–8.

Aytaç Adalı, E. (2016). Personnel Selection In Health Sector With EVAMIX and TODIM Methods. Alphanumeric Journal, 0-0.

Bai, C., and Sarkis, J. (2017). Improving green flexibility through advanced manufacturing technology investment: Modeling the decision process. International Journal of Production Economics, 188, 86–104.

Baillon, A., Bleichrodt, H., and Spinu, V. (2019). Searching for the Reference Point. Management Science.

Barberis, N., Huang, M., and Santos, T. (2001). Prospect Theory and Asset Prices. The Quarterly Journal of Economics, 116(1), 1–53.

Barberis NC (2013) Thirty years of prospect theory in economics: A review and assessment. J. Econom. Perspect. 27(1):173–196.

Behzadian, M., Kazemzadeh, R. B., Albadvi, A., and Aghdasi, M. (2010). PROMETHEE: A comprehensive literature review on methodologies and applications. European Journal of Operational Research, 200(1), 198–215. doi:10.1016/j.ejor.2009.01.021.

Boujelben, M. A. (2016). A unicriterion analysis based on the PROMETHEE principles for multicriteria ordered clustering. Omega, S0305048316305126–. doi:10.1016/j.omega.2016.08.007

Bozkurt, A. (2007). Multi-Criteria Decision Making With Interdependent Criteria using Prospect Theory, Master thesis, Middle East Technical University, Ankara, Turkey [online] http://etd.lib.metu.edu.tr/upload/12608408/index.pdf

Brans, J.P., Mareschal, B. (1992). Promethee-V – MCDM problems with segmentation constraints. INFOR 30(2), 85–96.

Brans, J.P., Mareschal, B., 1994a. The PROMETHEE GAIA decision support system formulticriteria investigations. Investigation Operative 4 (2), 107–117.

Brans, J.P., Mareschal, B., 1994b. PROMCALC and GAIA decision support system formulticriteria decision aid. Decision Support Systems 12, 297–310.

Brans, J.P., Mareschal, B. (1995). The PROMETHEE VI procedure. How to differentiate hard from soft multicriteria problems. J. Decis. Syst. 4, 213–223.

Brans JP, Mareschal B, Vincke PH (1984) PROMETHEE: a new family of outranking methods in multicriteria analysis. In: Brans JP (ed) OR'84.

Brans, J.P., Vincke, Ph., and Mareschal, B. (1986). How to select and how to rank projects: The PROMETHEE method. European Journal of Operational Research 24 (1986) 228-238.

C. Macharis, J.P. Brans and B. Mareschal Journal of Decision Systems, 7 (1998), pp. 283-307.

Campos, A. C. S. M., Mareschal, B., & de Almeida, A. T. (2015). Fuzzy-FlowSort: An integration of the FlowSort method and Fuzzy Set Theory for decision making on the basis of inaccurate quantitative data. Information Sciences, 293, 115–124.

Cavalcante, C.A.V., De Almeida, A.T., 2007. A multi-criteria decision-aiding modelusing PROMETHEE III for preventive maintenance planning under uncertainconditions. Journal of Quality in Maintenance Engineering 13 (4), 385–397

CHEN, H.-C., & HU, Y.-C. (2011). SINGLE-LAYER PERCEPTRON WITH NON-ADDITIVE PREFERENCE INDICES AND ITS APPLICATION TO BANKRUPTCY PREDICTION. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 19(05), 843–861. doi:10.1142/s021848851100726x

Chen, R.-H., Lin, Y., and Tseng, M.-L. (2015). Multicriteria analysis of sustainable development indicators in the construction minerals industry in China. Resources Policy, 46, 123–133.

Clemen, R. T. and Reilly, T. (2013). Making Hard Decisions with Decision Tools. 3rd Edition, Chapter 15.

Collier, Z. A., & Lambert, J. H. (2018). Time Management of Infrastructure Recovery Schedules by Anticipation and Valuation of Disruptions. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 4(2), 04018012. doi:10.1061/ajrua6.0000961.

Cunbin, L., Jiahang, Y., and Xia, F. (2016). Linguistic assessment information risky multi-criteria decision-making about wind power investment. Journal of Intelligent & Fuzzy Systems, 30(5), 3017–3023.

De Lima Silva, Silva, Silva, Ferreira & de Almeida-Filho (2018) proposed a sorting method based on PROMETHEE in order to assign sovereign credit securities into three different risk categories.

De Smet, Y. (2013). P2CLUST: An extension of PROMETHEE II for multicriteria ordered clustering. 2013 IEEE International Conference on Industrial Engineering and Engineering Management. doi:10.1109/ieem.2013.6962531.

Dong, Y., Luo, N., and Liang, H. (2015). Consensus building in multiperson decision making with heterogeneous preference representation structures: A perspective based on prospect theory. Applied Soft Computing, 35, 898–910.

Durbach I. (2009). On the estimation of a satisficing model of choice using stochastic multicriteria acceptability analysis. Omega, 37(3), 497-509.

Doumpos, M., Zopounidis, C. (2004). Developing sorting models using preference disaggregation analysis: An experimental investigation. European Journal of Operational Research, 154 (3), pp. 585–598.

Doumpos, M., Zopounidis, C. (2004). A multicriteria classification approach based on pairwise comparisons. European Journal of Operational Research 158, 378–389.

Eisenführ F., Weber M. and Langer T. (2010). Rational Decision Making, Springer Verlag, Berlin, 2010.

Fan, Z.-P., Zhang, X., Chen, F.-D., and Liu, Y. (2013). Multiple attribute decision making considering aspiration-levels: A method based on prospect theory. Computers & Industrial Engineering, 65(2), 341–350.

Fan, Z.-P., Zhang, X., Zhao, Y.-R., and Chen, F.-D. (2013). Multiple Attribute Decision Making with Multiple Formats of Attribute Aspirations: A Method Based on Prospect Theory. International Journal of Information Technology and Decision Making, 12(04), 711–727.

Ferreira de Lima Silva, D., & Teixeira de Almeida Filho, A. (2020). Sorting with TOPSIS through Boundary and Characteristic Profiles. Computers & Industrial Engineering, 106328. doi:10.1016/j.cie.2020.106328.

Figueira J, De Smet Y, Brans J (2004) MCDA methods for sorting and clustering problems: Promethee TRI and Promethee CLUSTER, VUB. Tech Rep.

Figueira, J., de Smet, Y., Brans, J.P., 2004. MCDA methods for sorting and clusteringproblems: Promethee TRI and Promethee CLUSTER, Université Libre deBruxelles. Service deMathématiques de la Gestion, Working Paper 2004/02.http://www.ulb.ac.be/polytech/smg/indexpublications.htm

Gao, J., and Liu, H. (2015). Interval-valued intuitionistic fuzzy stochastic multi-criteria decision-making method based on Prospect theory. Kybernetes, 44(1), 25–42.

Gomes L.F.A.M. and Lima M.M.P.P. (1992). TODIM: Basic and Application to Multicriteria Ranking of Projects with Environmental Impacts, Foundations of Computing and Decision Sciences, 16, 1992a, 113-127.

Gomes L.F.A.M. and Lima M.M.P.P. (1992). From Modelling Individual Preferences to Multicriteria Ranking of Discrete Alternatives: A Look at Prospect Theory and the Additive Difference Model, Foundations of Computing and Decision Sciences, 17 (3), 1992b, 171–184.

Gomes, L. F. A. M., Araujo, R. M., Machado, M. A. S. (2014). Retail Location Analysis with TODIM-FSE. Recent Advances in Theory, Methods, and Practice of Operations Research, pp. 311-318, UANL

Gomes, L. F. A. M., Machado, M. A. S., da Costa, F. F., and Rangel, L. A. D. (2013). Criteria Interactions in Multiple Criteria Decision Aiding: A Choquet Formulation for the TODIM Method. Procedia Computer Science, 17, 324–331.

Gomes, L. F. A. M., Machado, M. A. S., and Rangel, L. A. D. (2013). Behavioral multi-criteria decision analysis: the TODIM method with criteria interactions. Annals of Operations Research, 211(1), 531–548.

Gomes, L. F. A. M., Rangel, L. A. D., and Maranhão, F. J. C. (2009). Multicriteria analysis of natural gas destination in Brazil: An application of the TODIM method. Mathematical and Computer Modelling, 50(1-2), 92–100.

Han, J., Wang, Q. L., Ye, S. X., Chai, J., Lai, K. K. (2016). Selecting Medical Service Scheme: a Novel Method based on Prospect Theory. Filomat 30:15(2016),4049–4058.

Hoyer, W. D., Herrmann, A., and Huber, F. (2002). When buyers also sell: The implications of pricing policies for customer satisfaction. Psychology and Marketing, 19(4), 329–355.

Hu, J., and Yang, L. (2011). Dynamic stochastic multi-criteria decision making method based on cumulative prospect theory and set pair analysis. Systems Engineering Procedia, 1, 432–439.

Hu, J., Chen, P., Chen, X. (2014). Intuitionistic random multi-criteria decision-making approach based on prospect theory with multiple reference intervals. Scientia Iranica E (2014) 21(6), 2347-2359.

HU, Y.-C. (2013). A NOVEL FLOW-BASED METHOD USING GREY RELATIONAL ANALYSIS FOR PATTERN CLASSIFICATION. International Journal of Information Technology & Decision Making, 12(01), 75–93. doi:10.1142/s021962201350004

Hu, Y.-C. (2016). Single Criterion Net Flow Using Concordance and Discordance Relations. Cybernetics and Systems, 47(3), 220–235. doi:10.1080/01969722.2016.1158552.

Jati, H., Dominic, D. D. (2017). A New Approach of Indonesian University Webometrics Ranking Using Entropy and PROMETHEE II. Procedia Computer Science, 124(), 444–451. doi:10.1016/j.procs.2017.12.176

Janssen, P., & Nemery, P. (2013). An extension of the FlowSort sorting method to deal with imprecision. 4OR, 11 (2), 171–193.

Ji, P., Zhang, H., and Wang, J. (2016). A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. Neural Computing and Applications, 29(1), 221–234.

Jiang, Y., Liang, X., and Liang, H. (2016). An I-TODIM method for multi-attribute decision making with interval numbers. Soft Computing, 21(18), 5489–5506.

Kadziński, M., & Słowiński, R. (2012). DIS-CARD: a new method of multiple criteria sorting to classes with desired cardinality. Journal of Global Optimization, 56(3), 1143–1166. doi:10.1007/s10898-012-9945-9

Kadzinski, M., Ciomek, K. (2016). Integrated framework for preference modeling and robustness analysis for outranking-based multiple criteria sorting with ELECTRE and PROMETHEE. Information Sciences 352-353 (2016) 167-187.

Kahneman, D., and Tversky, A. (1976). Prospect theory: an analysis of decisions under risk. Econometrica, 47(2), 263–292.

Kahneman, D., and Tversky, A. (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. Journal of Risk and Uncertainty, 5, 297-323.

Karasakal, E., Karasakal, O., Bozkurt, A. (2019). AN APPROACH FOR Extending PROMETHEE to Reflect Choice Behaviour of the Decision Maker. Journal of Industrial Engineering 30(2), 123-140, 2019.

Keeney, R. L., H. Raiffa (1976). Decisions with Multiple Objectives. John Wiley and Sons, New York.

Korhonen, P., Wallenius, J., and Zionts, S. (1984). Solving the discrete multiple criteria decision problem using convex cones. Management Science, 30, 1336-1345.

Korhonen, P., Moskowitz, H., and Wallenius, J. (1990). Choice Behavior in Interactive Multiple-Criteria Decision Making. Annals of Operations Research 23(1990)161-179.

Krohling, R. A., and de Souza, T. T. M. (2012). Combining prospect theory and fuzzy numbers to multi-criteria decision making. Expert Systems with Applications, 39(13), 11487–11493.

Krohling, R. A., Pacheco, A. G. C., and Siviero, A. L. T. (2013). IF-TODIM: An intuitionistic fuzzy TODIM to multi-criteria decision making. Knowledge-Based Systems, 53, 142–146.

Król, A., Księżak, J., Kubińska, E., and Rozakis, S. (2018). Evaluation of Sustainability of Maize Cultivation in Poland. A Prospect Theory—PROMETHEE Approach. Sustainability, 10(11), 4263.

Lahdelma R., and Salminen P. (2001). SMAA-2: Stochastic multicriteria acceptability analysis for group decision making. Operation Research, 49(3), 444-54.

Lahdelma, R., Salminen, P., and Kuula, M. (2003). Testing the efficiency of two pairwise comparison methods in discrete multiple criteria problems. European Journal of Operational Research, 145(3), 496–508.

Lahdelma, R., and Salminen, P. (2009). Prospect theory and stochastic multicriteria acceptability analysis (SMAA). Omega, 37(5), 961–971.

Lee, Y.S., and Shih, H.S. (2015). A Study of Behavioral Considerations based on Prospect Theory for Group Decision Making. International Journal of Information and Management Sciences, 26 (2015), 103-122.

Lee, Y.-S., and Shih, H.-S. (2015). Incremental analysis for generalized TODIM. Central European Journal of Operations Research, 24(4), 901–922.

Lerche, N., and Geldermann, J. (2015). Integration of prospect theory into PROMETHEE - a case study concerning sustainable bioenergy concepts. International Journal of Multicriteria Decision Making, 5(4), 309.

Li, C.-B., Gao, P.-F., Qi, Z.-Q., and Feng, X. (2016). A risky multi-criteria decision-making approach under language environment. Intelligent Decision Technologies, 10(1), 69–79.

Li, P., Chen, X., Qu, X., and Xu, Q. (2018). The Evaluation of Mineral Resources Development Efficiency Based on Hesitant Fuzzy Linguistic Approach and Modified TODIM. Mathematical Problems in Engineering, 2018, 1–9.

Li, Y.-L., Ying, C.-S., Chin, K.-S., Yang, H.-T., and Xu, J. (2018). Third-party reverse logistics provider selection approach based on hybrid-information MCDM and cumulative prospect theory. Journal of Cleaner Production, 195, 573–584.

Li, X., and Chen, X. (2014). Extension of the TOPSIS method based on prospect theory and trapezoidal intuitionistic fuzzy numbers for group decision making. Journal of Systems Science and Systems Engineering, 23(2), 231–247.

Li, X., Wang, F., and Chen, X. (2014). Trapezoidal Intuitionistic Fuzzy Multiattribute Decision Making Method Based on Cumulative Prospect Theory and Dempster-Shafer Theory. Journal of Applied Mathematics, 2014, 1–8.

Liao, H., Wu, D., Huang, Y., Ren, P., Xu, Z., and Verma, M. (2018). Green Logistic Provider Selection with a Hesitant Fuzzy Linguistic Thermodynamic Method Integrating Cumulative Prospect Theory and PROMETHEE. Sustainability, 10(4), 1291.

Liu, J., Liu, P., Liu, S.-F., Zhou, X.-Z., and Zhang, T. (2014). A study of decision process in MCDM problems with large number of criteria. International Transactions in Operational Research, 22(2), 237–264.

Liu, J., Zhao, H.-K., Li, Z.-B., and Liu, S.-F. (2017). Decision process in MCDM with large number of criteria and heterogeneous risk preferences. Operations Research Perspectives, 4, 106–112.

Liu, H., Song, Y., and Yang, G. (2018). Cross-efficiency evaluation in data envelopment analysis based on prospect theory. European Journal of Operational Research.

Llamazares, B. (2018). An analysis of the generalized TODIM method. European Journal of Operational Research, 269 (2018), 1041–1049.

Lolli, F., Ishizaka, A., Gamberini, R., Rimini, B., & Messori, M. (2015). FlowSort-GDSS – A novel group multi-criteria decision support system for sorting problems with application to FMEA. Expert Systems with Applications, 42(17-18), 6342–6349. doi:10.1016/j.eswa.2015.04.028.

Lourenzutti, R., and Krohling, R. A. (2013). A study of TODIM in a intuitionistic fuzzy and random environment. Expert Systems with Applications, 40(16), 6459–6468.

Lourenzutti, R., and Krohling, R. A. (2015). TODIM Based Method to Process Heterogeneous Information. Procedia Computer Science, 55, 318–327.

M. A. Boujelben, Y. De Smet, A multicriteria ordered clustering algorithm to determine precise or disjunctive partitions, International Journal of Multicriteria Decision Making 6 (2016) 157–187.

Macharis, C., Brans, J.P., Mareschal, B., 1998. The GDSS PROMETHEE procedure – aPROMETHEE–GAIA based procedure for group decision support. Journal of Decision Systems 7, 283–307.

Markowitz HM (1952) The utility of wealth. J. Political Econom. 60(2): 151–158.

Nemery, P., & Lamboray, C. (2008). Flowsort: A flow-based sorting method with lim- iting or central profiles. TOP, 16 (1), 90–113.

Nemery, P., Campos, A. C. S. M., Mareschal, B., & de Almeida, A. T. (2015). Addendum on: "Fuzzy FlowSort: An integration of the FlowSort method and Fuzzy Set Theory for decision making on the basis of inaccurate quantitative data." Information Sciences, 315, 54–55. doi:10.1016/j.ins.2015.04.021

Özerol, G., and Karasakal, E. (2007). A Parallel between Regret Theory and Outranking Methods for Multicriteria Decision Making Under Imprecise Information. Theory and Decision, 65(1), 45–70.

Passos, A.C., Gomes, L.F.A. (2014). TODIM-FSE: A MULTICRITERIA CLASSIFICATION METHOD BASED ON PROSPECT THEORY. Multiple Criteria Decision Making, Vol=9, 123-139.

Pelissari, R., Oliveira, M.C., Amor, S. B., Abackerli, A., J. (2019). A new FlowSort-based method to deal with information imperfections in sorting decision-making problems. European Journal of Operational Research

Peng, J.-J., Wang, J.-Q., and Wu, X.-H. (2016). Novel Multi-criteria Decision-making Approaches Based on Hesitant Fuzzy Sets and Prospect Theory. International Journal of Information Technology & Decision Making, 15(03), 621–643.

Ph. Nemery, On the use of multicriteria ranking methods in sorting problems, PhD Thesis, Université Libre de Bruxelles, Brussels, Belgium, 2008, pp. 203–209.

Qin, Q., Liang, F., Li, L., Chen, Y.-W., and Yu, G.-F. (2017). A TODIM-based multi-criteria group decision making with triangular intuitionistic fuzzy numbers. Applied Soft Computing, 55, 93–107.

Qin, Q., Liang, F., Li, L., and Wei, Y.-M. (2017). Selection of energy performance contracting business models: A behavioral decision-making approach. Renewable and Sustainable Energy Reviews, 72, 422–433.

Ravaja, N., Korhonen, P., Köksalan, M., Lipsanen, J., Salminen, M., Somervuori, O., and Wallenius, J. (2016). Emotional–motivational responses predicting choices: The role of asymmetrical frontal cortical activity. Journal of Economic Psychology, 52, 56–70.

Ren, P., Xu, Z., and Gou, X. (2016). Pythagorean fuzzy TODIM approach to multi-criteria decision making. Applied Soft Computing, 42, 246–259.

Ren, P., Xu, Z., and Hao, Z. (2017). Hesitant Fuzzy Thermodynamic Method for Emergency Decision Making Based on Prospect Theory. IEEE Transactions on Cybernetics, 47(9), 2531–2543.

Roy, B. (1996). Multicriteria Methodology for Decision Aiding. Kluwer Academic Publishers, Dordrecht.

Roy B, Bouyssou D (1993) Aide multicritère à la décision: méthodes et cas. Economica, Paris

Roy, B., Figueira, J. R., and Almeida-Dias, J. (2014). Discriminating thresholds as a tool to cope with imperfect knowledge in multiple criteria decision aiding: Theoretical results and practical issues. Omega, 43, 9–20.

Royne, M. B., Martinez, J., Oakley, J., and Fox, A. K. (2012). The Effectiveness of Benefit Type and Price Endings in Green Advertising. Journal of Advertising, 41(4), 85–102.

Saaty, T.L. (1990). How to make a decision: The analytic hierarchy process. European Journal of Operational Research, 48, 9-26.

Saaty, T. L. (1999). Fundamentals of the analytic network process. The International Symposium on the Analytic Hierarchy Process, Kobe, Japan.

Safari, H., Sadat Fagheyi, M., Sadat Ahangari, S., Reza Fathi, M. (2012). Applying PROMETHEE Method based on Entropy Weight for Supplier Selection. Business Management and Strategy, 3(1), –. doi:10.5296/bms.v3i1.1656

Salminen, P., and Wallenius, J. (1993). Testing Prospect Theory in a Deterministic Multiple Criteria Decision-Making Environment. Decision Sciences, 24(2), 279–294.

Salminen, P. (1994). Solving the discrete multiple criteria problem using linear prospect theory. European Journal of Operational Research, 72(1), 146–154.

Sang, X., and Liu, X. (2016). An interval type-2 fuzzy sets-based TODIM method and its application to green supplier selection. Journal of the Operational Research Society, 67(5), 722–734.

Sarrazin, R., & De Smet, Y. (2016). Solving a Multicriteria Road Design Problem: A Practical Example. Multiple Criteria Decision Making, 113–136. doi:10.1007/978-3-319-39292-9_7.

Sarrazin, R., De Smet, Y., & Rosenfeld, J. (2018). An extension of PROMETHEE to interval clustering. Omega, 80, 12–21. doi:10.1016/j.omega.2017.09.001.

Sen, D. K., Datta, S., and Mahapatra, S. S. (2016). Application of TODIM (Tomada de Decisión Inerativa Multicritero) for industrial robot selection. Benchmarking: An International Journal, 23(7), 1818–1833.

Sepulveda, J., Gonzalez, J., Alfaro, M., Camargo, M. (2010). A Metrics-based Diagnosis Tool for Enhancing Innovation Capabilities in SMEs. International Journal of Computers, Communications & Control, ISSN 1841-9836, E-ISSN 1841-9844, 5(5), 919-928.

Sepúlveda, J. M., & Derpich, I. S. (2014). Automated Reasoning for Supplier Performance Appraisal in Supply Chains. Procedia Computer Science, 31, 966–975. doi:10.1016/j.procs.2014.05.349

Sepúlveda, J. M., Derpich, I. S. (2015). Multicriteria Supplier Classification for DSS: Comparative Analysis of Two Methods. International Journal of Computers Communications & Control, 10(2) 238-247. doi: 10.15837/ijccc.2015.2.1755

Shyur, H., Yin, L., Shih, H., and Cheng, C. (2015). A Multiple Criteria Decision Making Method Based on Relative Value Distances. Foundations of Computing and Decision Sciences, 40(4), 299–315.

Soni, N., Christian, R. A., Jariwala, N. (2016). Pollution Potential Ranking of Industries Using Classical TODIM Method. Journal of Environmental Protection, 2016, 7, 1645-1656.

Sun, R., Hu, J., and Chen, X. (2017). Novel single-valued neutrosophic decision-making approaches based on prospect theory and their applications in physician selection. Soft Computing.

Tan, C., Ip, W. H., and Chen, X. (2014). Stochastic multiple criteria decision making with aspiration level based on prospect stochastic dominance. Knowledge-Based Systems, 70, 231–241.

Tan, C., Jiang, Z.-Z., and Chen, X. (2015). An extended TODIM method for hesitant fuzzy interactive multicriteria decision making based on generalized Choquet integral. Journal of Intelligent & Fuzzy Systems, 29(1), 293–305.

Thillaigovindan, N., Shanthi, S. A., and Naidu, J. V. (2016). New Method for Solving a General Multiple Criteria Decision-Making Problem Under Risk in Fuzzy Environment. International Journal of Information Technology & Decision Making, 15(05), 1157–1179.

Tseng, M.-L., Tan, K.-H., Lim, M., Lin, R.-J., and Geng, Y. (2013). Benchmarking eco-efficiency in green supply chain practices in uncertainty. Production Planning & Control, 25(13-14), 1079–1090.

Tversky, A. (1969). Intransitivity of preferences. Psychological Review, 76, 31-48.

Wang, J., Cao, Y., and Zhang, H. (2017). Multi-Criteria Decision-Making Method Based on Distance Measure and Choquet Integral for Linguistic Z-Numbers. Cognitive Computation, 9(6), 827–842.

Wang, J., and Sun, T. (2008). Fuzzy Multiple Criteria Decision Making Method Based on Prospect Theory. 2008 International Conference on Information Management, Innovation Management and Industrial Engineering.

Wang, J., Wang, J.-Q., Zhang, H.-Y., and Chen, X.-H. (2017). Distance-Based Multi-Criteria Group Decision-Making Approaches with Multi-Hesitant Fuzzy Linguistic Information. International Journal of Information Technology & Decision Making, 16(04), 1069–1099.

Wang, J.Q. and Gong, L. (2009). Interval probability stochastic multi-criteria decision-making approach based on set pair analysis. Control and Decision. 24(12) (2009) 1877-1880 (In Chinese).

Wang, R., and Li, Y. (2018). A Novel Approach for Green Supplier Selection under a q-Rung Orthopair Fuzzy Environment. Symmetry, 10(12), 687.

Wang, W., Liu, X., Qin, Y., and Fu, Y. (2018). A risk evaluation and prioritization method for FMEA with prospect theory and Choquet integral. Safety Science, 110, 152–163.

Wang, Y., Wang, J.-Q., and Wang, T.-L. (2018). Fuzzy stochastic multi-criteria decision-making methods with interval neutrosophic probability based on regret theory. Journal of Intelligent & Fuzzy Systems, 1–14.

Wei, C., Ren, Z., and Rodríguez, R. M. (2015). A Hesitant Fuzzy Linguistic TODIM Method Based on a Score Function. International Journal of Computational Intelligence Systems, 8(4), 701–712.

Wei, L., Hou J., Qin, T., Yuan, Z., Yan, Y. (2016). Evaluation of Grid Energy Storage System Based on AHP-PROMETHEE-GAIA. Proceedings of the 35th Chinese Control Conference July 27-29, 2016, Chengdu, China.

Wei, L., Yuan, Z., Yan, Y., Hou, J., Qin T. (2016). Evaluation of Energy Saving and Emission Reduction Effect in Thermal Power Plants Based on Entropy Weight and PROMETHEE Method. 2016 Chinese Control and Decision Conference (CCDC).

Wu, J.-Z., and Tiao, P.-J. (2018). A validation scheme for intelligent and effective multiple criteria decision-making. Applied Soft Computing, 68, 866–872.

Wu, Y., Xu, C., and Zhang, T. (2018). Evaluation of renewable power sources using a fuzzy MCDM based on cumulative prospect theory: A case in China. Energy, 147, 1227–1239.

Y. De Smet, P. Nemery, R. Selvaraj, An exact algorithm for the multicriteria ordered clustering problem, Omega 40 (2012) 861–869.

Yan, S., and Liu, S. (2016). Multi-stage group risk decision making with grey numbers based on grey target and prospect theory. Grey Systems: Theory and Application, 6(1), 64–79.

Yu, S., Wang, J., and Wang, J. (2016). An extended TODIM approach with intuitionistic linguistic numbers. International Transactions in Operational Research, 25(3), 781–805.

Yu, W. (1992) ELECTRE TRI: Aspects méthodologiques et manuel d'utilisation. Document du Lamsade, Université Paris-Dauphine, 74.

Yua, H., Liub, P., Jinb, F. (2014). Research on the stochastic hybrid multi-attribute decision making method based on prospect theory. Scientia Iranica E (2014) 21(3), 1105-1119.

Zhang, W., Du, J., and Tian, X. (2018). Finding a promising venture capital project with TODIM under probabilistic hesitant fuzzy circumstance. Technological and Economic Development of Economy, 24(5), 2026-2044.

Zhang, W., Ju, Y., and Gomes, L. F. A. M. (2017). The SMAA-TODIM approach: Modeling of preferences and a robustness analysis framework. Computers & Industrial Engineering, 114, 130–141.

Zhang, W., Ju, Y., Liu, X., and Giannakis, M. (2017). A mathematical programming-based method for heterogeneous multicriteria group decision analysis with aspirations and incomplete preference information. Computers & Industrial Engineering, 113, 541–557.

Zhao, H., Guo, S., and Zhao, H. (2018). Comprehensive assessment for battery energy storage systems based on fuzzy-MCDM considering risk preferences. Energy.

Zhou, H., Wang, J., Zhang, H. (2017). Grey Stochastic Multi-criteria Decision-making Approach Based on Prospect Theory and Distance Measures. The Journal of Grey System Volume 29 No.1, 2017.

Zopounidis, C., Doumpos, M. (2002). Multicriteria classification and sorting methods: A literature review. European Journal of Operational Research, 138(2), pp. 229–246

APPENDICES

A. The Weight and the Threshold Values of each Criterion for Case 1 and 2

Table A.1: The Weight and the Threshold Values of each Criterion for Case 1

Criteria	**Min/Max**	**Parameters**			**Weight**
		q	**p**	**λ**	
Teaching	Max	2	14.89	2.25	30%
Research	Max	2	17.70	2.25	30%
Citations	Max	2	28.54	2.25	30%
Industry Income	Max	2	16.39	2.25	2.50%
International Outlook	Max	2	23.35	2.25	7.50%

Table A.2: The Weight and the Threshold Values of each Criterion for Case 2

Criteria	**Min/Max**	**Parameters**			**Weight**
		q	**p**	**λ**	
Teaching	Max	5	29.78	2.25	30%
Research	Max	5	35.40	2.25	30%
Citations	Max	5	57.07	2.25	30%
Industry Income	Max	5	32.78	2.25	2.50%
International Outlook	Max	5	46.70	2.25	7.50%

B. Criterion Values of each Reference Alternative for each Data Set

Table B.1: Criterion Values of each Reference Alternative for THE World University Ranking 2019 Data Set

Reference Alternatives	Criteria				
	Teaching	**Research**	**Citations**	**Industry Income**	**International Outlook**
I	28.67	24.27	48.56	46.45	47.27
II	43.56	41.97	77.09	62.84	70.62
III	13.78	6.57	20.02	30.06	23.92

Table B.2: Criterion Values of each Reference Alternative for THE World University Ranking 2020 Data Set

Reference Alternatives	Criteria				
	Teaching	**Research**	**Citations**	**Industry Income**	**International Outlook**
I	28.23	23.98	48.11	46.48	47.11
II	42.37	41.51	75.84	62.74	70.39
III	14.08	6.45	20.39	30.21	23.83

C. Criterion Values of each Reference Alternative for each Data Set

Table C.1: The number of alternatives that are placed in the same rank in the comparisons using THE World University Ranking 2019 data sets

Compared Methodologies	2019 Case I	2019 Case II
Weighted Sum & PROMETHEE	11	26
Weighted Sum & PROPOSED	11	18
Weighted Sum & PT-PROMETHEE (ref=ave.)	12	22
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	15	22
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	11	31
PROMETHEE & PROPOSED	115	67
PROMETHEE & PT-PROMETHEE (ref=ave.)	394	325
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	134	93
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	1114	1101
PROPOSED & PT-PROMETHEE (ref=ave.)	99	65
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	116	69
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	111	71
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	902	880
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	477	410
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	217	186

Table C.2: The number of alternatives that are placed in the same rank in the comparisons using THE World University Ranking 2020 data sets

Compared Methodologies	2020 Case I	2020 Case II
Weighted Sum & PROMETHEE	11	25
Weighted Sum & PROPOSED	11	17
Weighted Sum & PT-PROMETHEE (ref=ave.)	11	19
Weighted Sum & PT-PROMETHEE (ref=ave.+st.dev.)	11	16
Weighted Sum & PT-PROMETHEE (ref=ave.-st.dev.)	9	24
PROMETHEE & PROPOSED	102	100
PROMETHEE & PT-PROMETHEE (ref=ave.)	407	358
PROMETHEE & PT-PROMETHEE (ref=ave.+st.dev.)	155	97
PROMETHEE & PT-PROMETHEE (ref=ave.-st.dev.)	1240	1234
PROPOSED & PT-PROMETHEE (ref=ave.)	87	86
PROPOSED & PT-PROMETHEE (ref=ave.+st.dev.)	92	83
PROPOSED & PT-PROMETHEE (ref=ave.-st.dev.)	97	100
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.+st.dev.)	1023	989
PT-PROMETHEE (ref=ave.) & PT-PROMETHEE (ref=ave.-st.dev.)	488	419
PT-PROMETHEE (ref=ave.+st.dev) & PT-PROMETHEE (ref=ave.-st.dev.)	238	176

D. Threshold Values of each Criterion for each Data Set

Table D.1: Indifference Threshold (q) Values of each Criterion for each Data Set for both Cases

Data Set	c-1	c-2	c-3	c-4	c-5	c-6	c-7
CPU	148.3	3193.6	6393.6	25.6	5.2	17.6	-
Auto MPG	0.5	879	18.4	352.7	87	1.2	0.2
ESL	0.9	0.9	0.6	0.6	-	-	-
ERA	1.4	1.4	1.3	1.4	-	-	-
LEV	0.4	0.4	0.4	0.4	-	-	-
CEV	0.3	0.3	0.3	0.2	0.2	0.2	-
Breast Cancer	0.2	1	0.6	0.1	0.2	0.1	-
Mammographic	6.9	0.3	0.4	0.3	-	-	-

Table D.2: Preference Threshold (p) Values of each Criterion for each Data Set for Case I

Data Set	c-1	c-2	c-3	c-4	c-5	c-6	c-7
CPU	1483	31936	63936	256	52	176	-
Auto MPG	5	8790	184	3527	870	12	2
ESL	9	9	6	6	-	-	-
ERA	14	14	13	14	-	-	-
LEV	4	4	4	4	-	-	-
CEV	3	3	3	2	2	2	-
Breast Cancer	2	10	6	1	2	1	-
Mammographic	69	3	4	3	-	-	-

Table D.3: Preference Threshold (p) Values of each Criterion for each Data Set for Case II

Data Set	**c-1**	**c-2**	**c-3**	**c-4**	**c-5**	**c-6**	**c-7**
CPU	1483*λ	31936*λ	63936*λ	256*λ	52*λ	176*λ	-
Auto MPG	5*λ	8790*λ	184*λ	3527*λ	870*λ	12*λ	2*λ
ESL	9*λ	9*λ	6*λ	6*λ	-	-	-
ERA	14*λ	14*λ	13*λ	14*λ	-	-	-
LEV	4*λ	4*λ	4*λ	4*λ	-	-	-
CEV	3*λ	3*λ	3*λ	2*λ	2*λ	2*λ	-
Breast Cancer	2*λ	10*λ	6*λ	λ	2*λ	λ	-
Mammographic	69*λ	3*λ	4*λ	3*λ	-	-	-

E. Criterion Values of each Reference Profile for each Data Set

Table E.1: Criterion Values of each Reference Profile for CPU Data Set

Reference Profiles	MYCT	MMIN	MMAX	CACH	CHMIN	CHMAX
r-1	17	32000	64000	256	52	176
r-2	758.5	16032	32032	128	26	88
r-3	1500	64	64	0	0	0

Table E.2: Criterion Values of each Reference Profile for Auto MPG Data Set

Reference Profiles	Cylinders	Displacement	Horse power	Weight	Acceleration	Model year	Origin
r-1	3	1010	46	1613	80	82	3
r-2	5.5	5405	138	3376.5	515	76	2
r-3	8	9800	230	5140	950	70	1

Table E.3: Criterion Values of each Reference Profile for ESL Data Set

Reference Profiles	Criterion-1	Criterion-2	Criterion-3	Criterion-4
r-1	9	9	8	8
r-2	6.75	6.75	6.5	6.5
r-3	4.5	4.5	5	5
r-4	2.25	2.25	3.5	3.5
r-5	0	0	2	2

Table E.4: Criterion Values of each Reference Profile for ERA Data Set

Reference Profiles	Criterion-1	Criterion-2	Criterion-3	Criterion-4
r1	14	14	13	14
r2	7	7	6.5	7
r3	0	0	0	0

Table E.5: Criterion Values of each Reference Profile for LEV Data Set

Reference Profiles	Criterion-1	Criterion-2	Criterion-3	Criterion-4
r-1	4	4	4	4
r-2	2	2	2	2
r-3	0	0	0	0

Table E.6:Criterion Values of each Reference Profile for CEV Data Set

Reference Profiles	Price	Maintenance cost	Number of doors	Number of person that can be carried	Luggage boot capacity	Safety
r1	4	4	4	3	3	3
r2	3.25	3.25	3.25	2.5	2.5	2.5
r3	2.5	2.5	2.5	2	2	2
r4	1.75	1.75	1.75	1.5	1.5	1.5
r5	1	1	1	1	1	1

Table E.7: Criterion Values of each Reference Profile for Breast Cancer Data Set

Reference Profiles	Menopause	Tumor Size	Inv-nodes	Node-caps	Deg-malig	Irradiant
r-1	3	11	7	2	3	2
r-2	2	6	4	1.5	2	1.5
r-3	1	1	1	1	1	1

Table E.8: Criterion Values of each Reference Profile for Mammographic Data Set

Reference Profiles	Age	Shape	Margin	Density
r-1	88	4	5	4
r-2	53.5	2.5	3	2.5
r-3	19	1	1	1