

VMD, chiral loops, σ -meson, and $\omega - \rho$ mixing in $\omega \rightarrow \pi^0\pi^0\gamma$ decay

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Abstract

In an attempt to explain the latest experimental result about the branching ratio of $\omega \rightarrow \pi^0\pi^0\gamma$ decay we reexamine the decay mechanism of this decay in a phenomenological framework in which the contributions of VMD, chiral loops, σ -meson intermediate state amplitudes and the effects of $\omega - \rho$ mixing are considered. We conclude that in order to obtain the experimental value of the branching ratio $B(\omega \rightarrow \pi^0\pi^0\gamma)$ σ -meson amplitude which makes a substantial contribution should be included into the reaction mechanism and the effects of $\omega - \rho$ mixing should be taken into account. We also estimate the coupling constant $g_{\omega\sigma\gamma}$ as $g_{\omega\sigma\gamma} = 0.11$ which is much smaller than the values suggested by light cone QCD sum rules calculations.

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The recent experimental study of $\rho \rightarrow \pi^0\pi^0\gamma$ and $\omega \rightarrow \pi^0\pi^0\gamma$ decays by SND Collaboration obtained the value $B(\omega \rightarrow \pi^0\pi^0\gamma) = (6.6_{-0.8}^{+1.4} \pm 0.6) \times 10^{-5}$ for the branching ratio of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay [1]. Their result is in good agreement with GAMS Collaboration measurement of $B(\omega \rightarrow \pi^0\pi^0\gamma) = (7.2 \pm 2.5) \times 10^{-5}$ [2], but it has a higher accuracy.

On the theoretical side, $\omega \rightarrow \pi^0\pi^0\gamma$ decay was first studied by Singer [3] who postulated that this transition proceeds through the $\omega \rightarrow (\rho)\pi^0 \rightarrow \pi^0\pi^0\gamma$ mechanism involving ρ -meson intermediate state. The contribution of intermediate vector mesons (VMD) to the vector meson decays into two pseudoscalars and a single photon $V \rightarrow PP'\gamma$ was also considered by Bramon et al. [4] using standard Lagrangians obeying the SU(3)-symmetry, and in particular for the branching ratio of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ they obtained the result $B(\omega \rightarrow \pi^0\pi^0\gamma) = 2.8 \times 10^{-5}$. The $V \rightarrow PP'\gamma$ decays have also been considered within the framework of chiral effective Lagrangians using chiral perturbation theory. Bramon et al. [5] studied various such decays using this approach and they noted that if chiral perturbation theory Lagrangians are used there is no tree-level contribution to the amplitudes for the decay processes $V \rightarrow PP'\gamma$, and moreover the one-loop contributions are finite and to this order no counterterms are required. They considered both $\pi\pi$ and $K\bar{K}$ intermediate loops. In the good isospin limit π -loop contributions to $\omega \rightarrow \pi^0\pi^0\gamma$ amplitude vanish, and the contribution of K-loops is two orders of magnitude smaller than the contribution of VMD amplitude. Therefore, the VMD amplitude essentially accounts for the decay rate of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay. Guetta and Singer [6] recently updated the theoretical value for the branching ratio $B(\omega \rightarrow \pi^0\pi^0\gamma)$ of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ as $B(\omega \rightarrow \pi^0\pi^0\gamma) = (4.1 \pm 1.1) \times 10^{-5}$. In their calculation they noted that when the Born amplitude for VMD mechanism is used the decay rate $\omega \rightarrow \pi^0\pi^0\gamma$ is proportional to the coupling constants $g_{\omega\rho\pi}^2$ and $g_{\rho\pi\gamma}^2$, and they assumed that the decay $\omega \rightarrow 3\pi$ proceeds with the same mechanism as $\omega \rightarrow \pi^0\pi^0\gamma$, that is as $\omega \rightarrow (\rho)\pi \rightarrow \pi\pi\pi$ [7]. They use the experimental inputs for the decay rates $\Gamma(\omega \rightarrow 3\pi)$, $\Gamma(\rho^0 \rightarrow \pi^0\gamma)$ and $\Gamma(\rho \rightarrow \pi\pi)$, and furthermore they employ a momentum dependent width for ρ -meson. If a constant ρ -meson width is used, then the value $B(\omega \rightarrow \pi^0\pi^0\gamma) = (3.6 \pm 0.9) \times 10^{-5}$ is obtained for the branching ratio of this decay. Therefore there appears to be a serious discrepancy between the theoretical result and the experimental value for the branching ratio of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay.

Guetta and Singer [6] noted that in the theoretical framework based on chiral perturbation theory and vector meson dominance one feature has been neglected. This is the possibility of $\omega - \rho$ mixing one consequence of which is the isospin violating $\omega \rightarrow \pi^+\pi^-$ decay with the branching ratio $B(\omega \rightarrow \pi^+\pi^-) = (2.21 \pm 0.30)\%$ [8]. The phenomenon of $\omega - \rho$ mixing has been observed in the electromagnetic form factor of the pion improving the standard VMD model result involving ρ -meson intermediate state. A recent review of $\omega - \rho$ mixing and vector meson dominance is given by O'Connell et al. [9]. Guetta and Singer [6] calculated the effect of $\omega - \rho$ mixing using the Born amplitude for VMD mechanism and showed that it increases the $\omega \rightarrow \pi^0\pi^0\gamma$ width only by 5%. They then combined all the improvements on the simple Born term of VMD mechanism, that is $\omega - \rho$ mixing, momentum dependence of ρ -meson width and the inclusion of the chiral loop amplitude as given by Bramon et al. [5], and using the resulting amplitude which includes all these effects they obtain the theoretical result $B(\omega \rightarrow \pi^0\pi^0\gamma) = (4.6 \pm 1.1) \times 10^{-5}$ for the branching ratio of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay. Palomar et al. [10] also analyzed the radiative $V \rightarrow PP'\gamma$ decays using vector meson dominance, chiral loops obtained using unitarized chiral perturbation

theory, and $\omega - \rho$ mixing. They obtained the result $B(\omega \rightarrow \pi^0\pi^0\gamma) = (4.7 \pm 0.9) \times 10^{-5}$ for this branching ratio. These theoretical results are still seriously less than the latest experimental result $B(\omega \rightarrow \pi^0\pi^0\gamma) = (6.6_{-0.8}^{+1.4} \pm 0.6) \times 10^{-5}$. Therefore, the possibility of additional contributions to the mechanism of $\omega \rightarrow \pi^0\pi^0\gamma$ decay should be investigated.

One such additional contribution to the $\omega \rightarrow \pi^0\pi^0\gamma$ decay may be provided by the amplitude involving scalar-isoscalar σ -meson as an intermediate state. The existence of the controversial σ -meson now seems to be established by the Fermilab (E791) Collaboration in their observation of the $D^+ \rightarrow \sigma\pi^+ \rightarrow \pi\pi\pi$ decay channel in which σ -meson is seen as a clear dominant peak with $M_\sigma = (438 \pm 31)$ MeV, and $\Gamma_\sigma = (338 \pm 48)$ MeV [11], where statistical and systematic errors are added in quadrature [12]. Two of present authors in a previous work [13], calculated the decay rate for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ by considering ρ -pole vector meson dominance amplitude as well as σ -pole amplitude in a phenomenological approach. By employing then the available experimental value for this branching ratio $B(\omega \rightarrow \pi^0\pi^0\gamma) = (7.2 \pm 2.5) \times 10^{-5}$ [2] which is somewhat less accurate than the present new value [1], they obtained for the coupling constant $g_{\omega\sigma\gamma}$ the values $g_{\omega\sigma\gamma} = 0.13$ and $g_{\omega\sigma\gamma} = -0.27$. They observed that σ -meson intermediate state amplitude makes an important contribution by itself and by its interference with the VMD amplitude. The same authors later estimated the coupling constant $g_{\omega\sigma\gamma}$ by studying $\omega\sigma\gamma$ -vertex in the light cone QCD sum rules method [14], and the value $|g_{\omega\sigma\gamma}| = (0.72 \pm 0.08)$ was deduced for this coupling constant. Aliev et al. [15] also used light cone QCD sum rules techniques to calculate the coupling constant $g_{\rho\sigma\gamma}$, and they obtained the value $g_{\rho\sigma\gamma} = (2.2 \pm 0.2)$ from which by using SU(3)-symmetry it follows that the coupling constant $g_{\omega\sigma\gamma}$ should have the value $g_{\omega\sigma\gamma} = 0.73$. Thus, there seems to be a serious discrepancy between the values obtained for the coupling constant $g_{\omega\sigma\gamma}$ using light cone QCD sum rules method and the phenomenological analysis of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay.

Therefore, in the present work, we reconsider the $\omega \rightarrow \pi^0\pi^0\gamma$ decay in a phenomenological framework in order to assess the role of σ -meson in the mechanism of $\omega \rightarrow \pi^0\pi^0\gamma$ decay and to recalculate the coupling constant $g_{\omega\sigma\gamma}$ utilizing the latest experimental value of the branching ratio $\omega \rightarrow \pi^0\pi^0\gamma$. For this purpose, we calculate the decay rate for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ by considering ρ -pole vector meson dominance amplitude, chiral loop amplitude, σ -pole amplitude and we also include the effects of $\omega - \rho$ mixing which was not taken into account in the previous analysis [13].

In order to calculate the effects of the $\omega - \rho$ mixing in the $\omega \rightarrow \pi^0\pi^0\gamma$ decay we need an amplitude characterizing the contribution of σ -meson to the $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay. In a previous work, two of the present authors [16] calculated the branching ratio $B(\rho^0 \rightarrow \pi^+\pi^-\gamma)$ in a phenomenological framework using pion bremsstrahlung amplitude and σ -meson pole amplitude, and they determined the coupling constant $g_{\rho\sigma\gamma}$ by using the experimental value of the branching ratio $B(\rho^0 \rightarrow \pi^+\pi^-\gamma)$. In a following work, these authors [17] calculated the branching ratio $B(\rho^0 \rightarrow \pi^0\pi^0\gamma)$ of the $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay using the value of the coupling constant $g_{\rho\sigma\gamma}$ they thus obtained again in a phenomenological approach in which the contribution of σ -meson, ω -meson intermediate states and of the pion-loops are considered. However, the branching ratio $B(\rho^0 \rightarrow \pi^0\pi^0\gamma)$ obtained this way was much larger than the experimental value. This unrealistic result was due to the constant $\rho^0 \rightarrow \sigma\gamma$ amplitude employed and consequently the large coupling constant $g_{\rho\sigma\gamma}$ that was deduced using the experimental branching ratio of the $\rho^0 \rightarrow \pi^+\pi^-\gamma$ decay. Therefore, it may be

concluded that it is not realistic to include σ -meson in the mechanism of radiative ρ^0 -meson decays as an intermediate pole state. On the other hand, the $V \rightarrow PP'\gamma$ decays were also studied by Marco et al. [18] in the framework of chiral perturbation theory. They used the techniques of chiral unitary theory [19], and by a unitary resummation of the pion loops through the Bethe-Salpeter equation they obtained the decay rates for various decays. They furthermore noted that their result for $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay could be interpreted as resulting from the $\rho^0 \rightarrow (\sigma)\gamma \rightarrow \pi^0\pi^0\gamma$ mechanism. Thus, it seems that a natural way to include the effects of σ -meson in the mechanism of radiative ρ^0 -meson decays is to assume that σ -meson couples to ρ^0 -meson through the pion-loop.

Our phenomenological approach is based on the Feynman diagrams shown in Fig. 1 for $\omega \rightarrow \pi^0\pi^0\gamma$ decay and in Fig. 2 for $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay. The direct terms shown in the diagrams in Fig. 1 b and in Fig. 2 b, c are required to establish the gauge invariance. The interaction term for two vector mesons and one pseudoscalar meson is given by the Wess-Zumino anomaly term of the chiral Lagrangian [20], we therefore describe the $\omega\rho\pi$ -vertex by the effective Lagrangian

$$\mathcal{L}_{\omega\rho\pi}^{eff} = g_{\omega\rho\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\vec{\rho}_\beta \cdot \vec{\pi} \quad , \quad (1)$$

which also conventionally defines the coupling constant $g_{\omega\rho\pi}$. This coupling constant was determined by Achasov et al. [21] through an experimental analysis as $g_{\omega\rho\pi} = (14.4 \pm 0.2) \text{ GeV}^{-1}$, who assumed that $\omega \rightarrow 3\pi$ decay proceeds with the intermediate $\rho\pi$ state as $\omega \rightarrow (\rho)\pi \rightarrow \pi\pi\pi$ and they used the experimental value of the $\omega \rightarrow 3\pi$ width to deduce the coupling constant $g_{\rho\omega\pi}$. Similarly we describe the $V\pi\gamma$ -vertices where $V = \rho, \omega$ with the effective Lagrangian

$$\mathcal{L}_{V\pi\gamma}^{eff} = g_{V\pi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu\partial_\alpha A_\beta\pi \quad . \quad (2)$$

We then use the experimental partial widths of the radiative $V \rightarrow \pi\gamma$ decays [8] to deduce the coupling constants $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$. This way for the coupling constants $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$ we obtain the values $g_{\omega\pi\gamma} = (0.706 \pm 0.021) \text{ GeV}^{-1}$ and $g_{\rho\pi\gamma} = (0.274 \pm 0.035) \text{ GeV}^{-1}$. The $\rho\pi\pi$ -vertex is described by the effective Lagrangian

$$\mathcal{L}_{\rho\pi\pi}^{eff} = g_{\rho\pi\pi}\vec{\rho}_\mu \cdot (\partial^\mu\vec{\pi} \times \vec{\pi}) \quad , \quad (3)$$

and the experimental decay width of the decay $\rho \rightarrow \pi\pi$ [8] then yields the value $g_{\rho\pi\pi} = (6.03 \pm 0.02)$ for the coupling constant $g_{\rho\pi\pi}$. We describe the $\sigma\pi\pi$ -vertex by the effective Lagrangian

$$\mathcal{L}_{\sigma\pi\pi}^{eff} = \frac{1}{2}g_{\sigma\pi\pi}M_\sigma\vec{\pi} \cdot \vec{\pi}\sigma \quad . \quad (4)$$

The experimental values for M_σ and Γ_σ $M_\sigma = (483 \pm 31) \text{ MeV}$ and $\Gamma_\sigma = (338 \pm 48) \text{ MeV}$, where statistical and systematic errors are added in quadrature [11,12] then results in the strong coupling constant $g_{\sigma\pi\pi} = (5.3 \pm 0.55)$. The effective Lagrangians $\mathcal{L}_{\sigma\pi\pi}^{eff}$ and $\mathcal{L}_{\rho\pi\pi}^{eff}$ are obtained from an extension of the σ model where the isovector ρ is included through a Yang-Mills local gauge theory based on isospin with the vector meson mass generated through the Higgs mechanism [22]. In order to describe the π^4 -vertex we again consider the

σ -model with spontaneous symmetry breaking [23] and we describe the π^4 -vertex by the effective Lagrangian

$$\mathcal{L}^{eff} = \frac{\lambda}{4} (\vec{\pi} \cdot \vec{\pi})^2 , \quad (5)$$

where the coupling constant λ is given as $\lambda = -\frac{g_{\pi NN}^2}{2} \frac{M_\sigma^2 - M_\pi^2}{M_N^2}$ and we use $\frac{g_{\pi NN}^2}{4\pi} = 14$. We note that this effective interaction results in only isospin I=0 amplitudes. The small I=2 amplitudes were also neglected in previous calculations within the framework of chiral unitary theory [18]. The $\omega\sigma\gamma$ -vertex is described by the effective Lagrangian

$$\mathcal{L}_{\omega\sigma\gamma}^{eff} = \frac{e}{M_\omega} g_{\omega\sigma\gamma} \partial^\alpha \omega^\beta (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \sigma , \quad (6)$$

which also defines the coupling constant $g_{\omega\sigma\gamma}$.

In our calculation of the invariant amplitude, we make the replacement $q^2 - M^2 \rightarrow q^2 - M^2 + iM\Gamma$ in ρ -meson and σ -meson propagators. We use for σ -meson the momentum dependent width that follows from Eq. 4

$$\Gamma_\sigma(q^2) = \Gamma_\sigma \frac{M_\sigma^2}{q^2} \sqrt{\frac{q^2 - 4M_\pi^2}{M_\sigma^2 - 4M_\pi^2}} \theta(q^2 - 4M_\pi^2) , \quad (7)$$

and for ρ -meson we use the following momentum dependent width as conventionally adopted [9]

$$\Gamma_\rho(q^2) = \Gamma_\rho \frac{M_\rho}{\sqrt{q^2}} \left(\frac{q^2 - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2} \right)^{3/2} \theta(q^2 - 4M_\pi^2) . \quad (8)$$

Loop integrals similar to the ones appearing in Figs. 1 and 2 were evaluated by Lucio and Pestiau [24] using dimensional regularization. We use their results and, for example, we express the contribution of the pion-loop amplitude corresponding to $\rho^0 \rightarrow (\pi^+\pi^-)\gamma \rightarrow \pi^0\pi^0\gamma$ reaction in Fig. 2 b as

$$\mathcal{M}_\pi = -\frac{e g_{\rho\pi\pi} \lambda}{2\pi^2 M_\pi^2} I(a, b) [(p \cdot k)(\epsilon \cdot u) - (p \cdot \epsilon)(k \cdot u)] , \quad (9)$$

where $a = M_\rho^2/M_\pi^2$, $b = (p - k)^2/M_\pi^2$, $p(u)$ and $k(\epsilon)$ being the momentum (polarization vector) of ρ -meson and photon, respectively. A similar amplitude corresponding to $\rho^0 \rightarrow (\pi^+\pi^-)\gamma\sigma \rightarrow \gamma\pi^0\pi^0$ reaction can also be written. The function I(a,b) is given as

$$I(a, b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} \left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right) \right] + \frac{a}{(a-b)^2} \left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right) \right] \quad (10)$$

where

$$f(x) = \begin{cases} -\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2, & x > \frac{1}{4} \\ \frac{1}{4} \left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right]^2, & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}} \arcsin\left(\frac{1}{2\sqrt{x}}\right), & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}} \left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right], & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x} \left[1 \pm (1-4x)^{\frac{1}{2}} \right] . \quad (11)$$

Although the contribution of chiral kaon-loop diagram shown in Fig. 1 b to the decay rate of $\omega \rightarrow \pi^0\pi^0\gamma$ decay is small, we also include the corresponding amplitude of this diagram in our calculation for completeness. However, since we lack any experimental information to describe the ωK^+K^- -vertex and $K^+K^-\pi^0\pi^0$ amplitude, for the contribution of this diagram we use the amplitude given by Bramon et al. [5] derived using chiral perturbation theory. This may not be entirely consistent with the philosophy of our phenomenological approach, but since its contribution is shown to be small [5] we do not think that this way of including kaon-loop diagram into our calculation constitutes a serious inconsistency. Moreover, in Fig. 2 b in addition to pion-loop intermediate state there is also a contribution to $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay coming from $K\bar{K}$ intermediate state. However, as shown by Bramon et al. [5], these kaon-loop intermediate states give a contribution to $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay which is 10^3 times smaller than the contribution coming from the charged-pion loops. Therefore, in our calculation we do not take the kaon-loop amplitude in $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay into account.

We describe the $\omega - \rho$ mixing by an effective Lagrangian of the form

$$\mathcal{L}_{\rho-\omega}^{eff} = \Pi_{\rho\omega}\omega_\mu\rho^\mu \quad , \quad (12)$$

where ω_μ and ρ_μ denote pure isospin field combinations. The corresponding physical states can therefore be written as [9]

$$\begin{aligned} |\rho\rangle &= |\rho, I=1\rangle + \epsilon |\omega, I=0\rangle \\ |\omega\rangle &= |\omega, I=0\rangle - \epsilon |\rho, I=1\rangle \quad , \end{aligned} \quad (13)$$

where

$$\epsilon = \frac{\Pi_{\rho\omega}}{M_\omega^2 - M_\rho^2 + iM_\rho\Gamma_\rho - iM_\omega\Gamma_\omega} \quad . \quad (14)$$

O'Connell et al. [9] determined $\Pi_{\rho\omega}$ from fits to $e^+e^- \rightarrow \pi^+\pi^-$ data as $\Pi_{\rho\omega} = (-3800 \pm 370) \text{ MeV}^2$. Then, using the experimental values for M_V and Γ_V , the mixing parameter ϵ is obtained as $\epsilon = (-0.006 + i0.036)$. Another effect of $\omega - \rho$ mixing besides the mixing of the states is that it modifies the ρ -propagator in diagrams in Fig. 1 a as

$$\frac{1}{D_\rho(s)} \rightarrow \frac{1}{D_\rho(s)} \left(1 + \frac{g_{\omega\pi\gamma}}{g_{\rho\pi\gamma}} \frac{\Pi_{\rho\omega}}{D_\rho(s)} \right) \quad (15)$$

where $D_\rho(s) = s - M_\rho^2 + iM_\rho\Gamma_\rho(s)$. The amplitude of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ can then be written as $\mathcal{M} = \mathcal{M}_0 + \epsilon\mathcal{M}'$ where \mathcal{M}_0 includes the contributions coming from the diagrams shown in Fig. 1 for $\omega \rightarrow \pi^0\pi^0\gamma$ and \mathcal{M}' represents the contributions of the diagrams in Fig. 2 for $\rho^0 \rightarrow \pi^0\pi^0\gamma$.

We calculate the invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ this way for the decay $\omega \rightarrow \pi^0\pi^0\gamma$ from the corresponding Feynman diagrams shown in Fig. 1 and 2 for the decays $\omega \rightarrow \pi^0\pi^0\gamma$ and $\rho^0 \rightarrow \pi^0\pi^0\gamma$, respectively. The differential decay probability for an unpolarized ω -meson at rest is then given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M_\omega} |\mathcal{M}|^2, \quad (16)$$

where E_γ and E_1 are the photon and pion energies respectively. We perform an average over the spin states of the vector meson and a sum over the polarization states of the photon. The decay width is then obtained by integration

$$\Gamma = \left(\frac{1}{2}\right) \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad (17)$$

where the factor $(\frac{1}{2})$ is included because of the $\pi^0\pi^0$ in the final state. The minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M_\omega^2 - 4M_\pi^2)/2M_\omega = 341$ MeV. The maximum and minimum values for pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_\omega - M_\omega^2)} [-2E_\gamma^2 M_\omega + 3E_\gamma M_\omega^2 - M_\omega^3 \pm E_\gamma \sqrt{(-2E_\gamma M_\omega + M_\omega^2)(-2E_\gamma M_\omega + M_\omega^2 - 4M_\pi^2)}] .$$

The theoretical decay rate for $\omega \rightarrow \pi^0\pi^0\gamma$ decay that we calculate results in a quadric equation for the coupling constant $g_{\omega\sigma\gamma}$, and using the experimental value for this decay rate [1], we obtain the values $g_{\omega\sigma\gamma} = (0.11 \pm 0.01)$ and $g_{\omega\sigma\gamma} = (-0.21 \pm 0.02)$ for the coupling constant $g_{\omega\sigma\gamma}$. These values are smaller than the values $g_{\omega\sigma\gamma} = 0.13$ and $g_{\omega\sigma\gamma} = -0.27$ that were obtained in the previous phenomenological analysis which did not include the effects of $\omega - \rho$ mixing [12]. We may therefore conclude that $\omega - \rho$ mixing does make a reasonably substantial contribution to the $\omega \rightarrow \pi^0\pi^0\gamma$ decay amplitude when σ -meson intermediate state is taken into account which then results in a reduced value for the coupling constant $g_{\omega\sigma\gamma}$.

The resulting $\pi^0\pi^0$ -invariant mass distribution for the $\omega \rightarrow \pi^0\pi^0\gamma$ decay if we use the coupling constant $g_{\omega\sigma\gamma} = 0.11$ is plotted in Fig. 3 where we also indicate the contributions coming from the different amplitudes. The interference term between the different amplitudes is positive over the whole region. The contribution of the VMD amplitude calculated from the diagrams in Fig. 1 a does not change appreciably if we take into account the effect of $\omega - \rho$ mixing by including the contribution coming from the diagrams in Fig. 2 a. The situation changes somewhat if we consider VMD and chiral loop amplitudes and $\omega - \rho$ mixing as well. However, the significant modification is obtained when we include VMD, chiral loop, and σ -meson intermediate state amplitudes with $\omega - \rho$ mixing.

Although the shapes of the various curves are quite similar, as it can also be seen the corresponding branching ratios are considerably different. If we use only the VMD amplitudes we obtain for the branching ratio $B(\omega \rightarrow \pi^0\pi^0\gamma)$ the values $B(\omega \rightarrow \pi^0\pi^0\gamma) = 3.96 \times 10^{-5}$ and $B(\omega \rightarrow \pi^0\pi^0\gamma) = 4.22 \times 10^{-5}$ without and with the effect of the $\omega - \rho$ mixing, respectively. These results are in agreement with the calculation of Guetta and Singer [6]. When we consider VMD amplitudes and chiral loop amplitudes, the resulting values for the branching ratio are $B(\omega \rightarrow \pi^0\pi^0\gamma) = 3.98 \times 10^{-5}$ and $B(\omega \rightarrow \pi^0\pi^0\gamma) = 4.67 \times 10^{-5}$ without and with the effect of the $\omega - \rho$ mixing included, respectively. Again these calculated values are consistent with the previous results, in particular, with the results of Bramon et al. [5] and Palomar et al. [10]. However, the main difference with previous results arises when we consider the contribution of σ -meson intermediate state. Indeed, the branching ratio $B(\omega \rightarrow \pi^0\pi^0\gamma)$ that we calculate using the full amplitude including the contributions of VMD, chiral loop, and σ -meson intermediate state diagrams which is $B(\omega \rightarrow \pi^0\pi^0\gamma) = 7.29 \times 10^{-5}$ is reduced to

the value $B(\omega \rightarrow \pi^0\pi^0\gamma) = 6.6 \times 10^{-5}$ when the effect of the $\omega - \rho$ mixing is included. We thus observe that in this case the effect of $\omega - \rho$ mixing on the amplitude of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay is reasonably pronounced and moreover the contribution of the σ -meson intermediate state is quite substantial. We may therefore conclude that σ -meson intermediate state and $\omega - \rho$ mixing should be included in the analysis of $\omega \rightarrow \pi^0\pi^0\gamma$ decay in order to explain the latest experimental result [1].

In Fig. 4, we plot the $\pi^0\pi^0$ -invariant mass distribution we obtain if we use the coupling constant $g_{\omega\sigma\gamma} = -0.21$. In this case also the σ -meson intermediate state amplitude makes a very significant contribution, but the interference term resulting from the σ -meson and VMD and chiral loop amplitudes is negative over some regions of the spectrum. Moreover, the overall shape is quite different from the previous case.

In our work, we consider a new approach to the mechanism of $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay. We assume that the σ -meson couples to ρ^0 -meson through the pion-loop and thus we neglect a direct $\rho\sigma\gamma$ -vertex. The assumed mechanism of $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay is shown by the Feynman diagrams in Fig. 2. We can therefore calculate the branching ratio of $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay based on these Feynman diagrams. This way we obtain for the branching ratio of $\rho^0 \rightarrow \pi^0\pi^0\gamma$ decay the value $B(\rho^0 \rightarrow \pi^0\pi^0\gamma) = (3.81 \pm 0.63) \times 10^{-5}$ which is in a good agreement the latest experimental result $B(\rho^0 \rightarrow \pi^0\pi^0\gamma) = (4.1_{-0.9}^{+1.0} \pm 0.3) \times 10^{-5}$ [1]. Scalar σ -meson effects in radiative ρ^0 -meson decays are studied in a recent work by two of the present authors in detail [25]. In that work, the standard $\pi^+\pi^- \rightarrow \pi^0\pi^0$ amplitude of chiral perturbation theory is used in the loop diagrams, and the value $B(\rho^0 \rightarrow \pi^0\pi^0\gamma) = (4.95 \pm 0.82) \times 10^{-5}$ is obtained for the branching ratio. If we use the same amplitude in the present work we then obtain the value $B(\omega \rightarrow \pi^0\pi^0\gamma) = 6.12 \times 10^{-5}$ using the full amplitude including the contributions of VMD, chiral loop, and σ -meson intermediate state diagram as well as the effects of $\omega - \rho$ mixing, which is not very different from $B(\omega \rightarrow \pi^0\pi^0\gamma) = 6.6 \times 10^{-5}$ that is obtained employing the effective Lagrangian given in Eq. 5 to describe the π^4 -vertex.

An essential assumption of our work, therefore, is that there is no SU(3) vector meson-sigma-gamma vertex. Thus, the $\omega\sigma\gamma$ -vertex cannot be related to the $\rho\sigma\gamma$ -vertex. The $\omega\sigma\gamma$ -vertex that we use may be considered as representing the effective final state interactions in the $\pi\pi$ -channel. The small value of the coupling constant $g_{\omega\sigma\gamma}$ that we obtain leads to a change in the Born amplitude of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay which is of the same of the magnitude, as it is typical of final state interactions. As a matter of fact, Levy and Singer [26] in a study of the $\omega \rightarrow \pi^0\pi^0\gamma$ decay using dispersion-theoretical approach shown that final state interactions resulting in a decay rate of the same order of magnitude as the one calculated from the Born term can be parameterized with the effective-pole approximation.

As a final comment, we like to mention that our analysis suggests that the coupling constant $g_{\omega\sigma\gamma}$ has actually a much smaller value than obtained by light cone QCD sum rules calculations.

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FIGURES

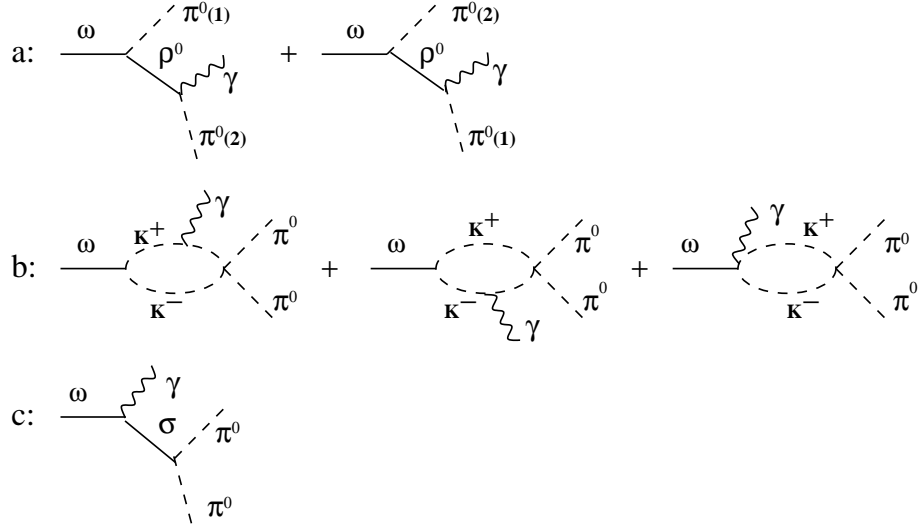


FIG. 1. Feynman diagrams for the decay $\omega \rightarrow \pi^0 \pi^0 \gamma$.

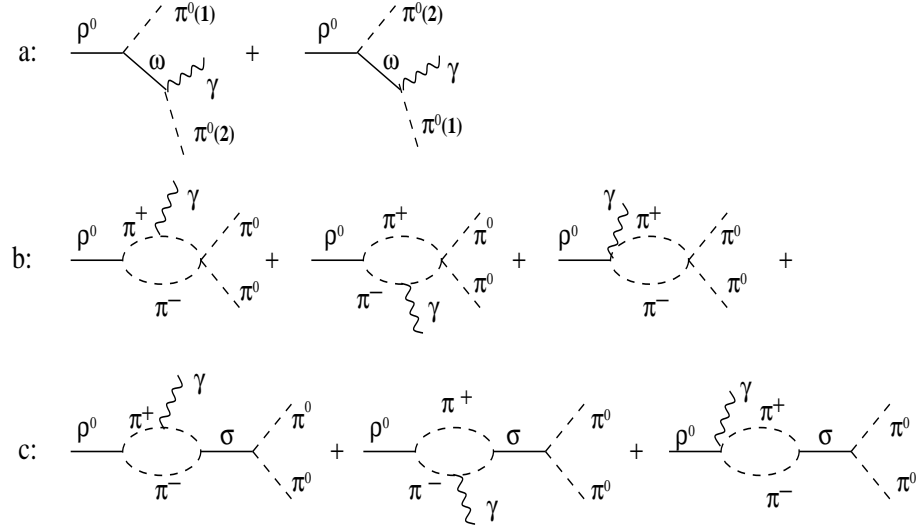


FIG. 2. Feynman diagrams for the decay $\rho^0 \rightarrow \pi^0 \pi^0 \gamma$.

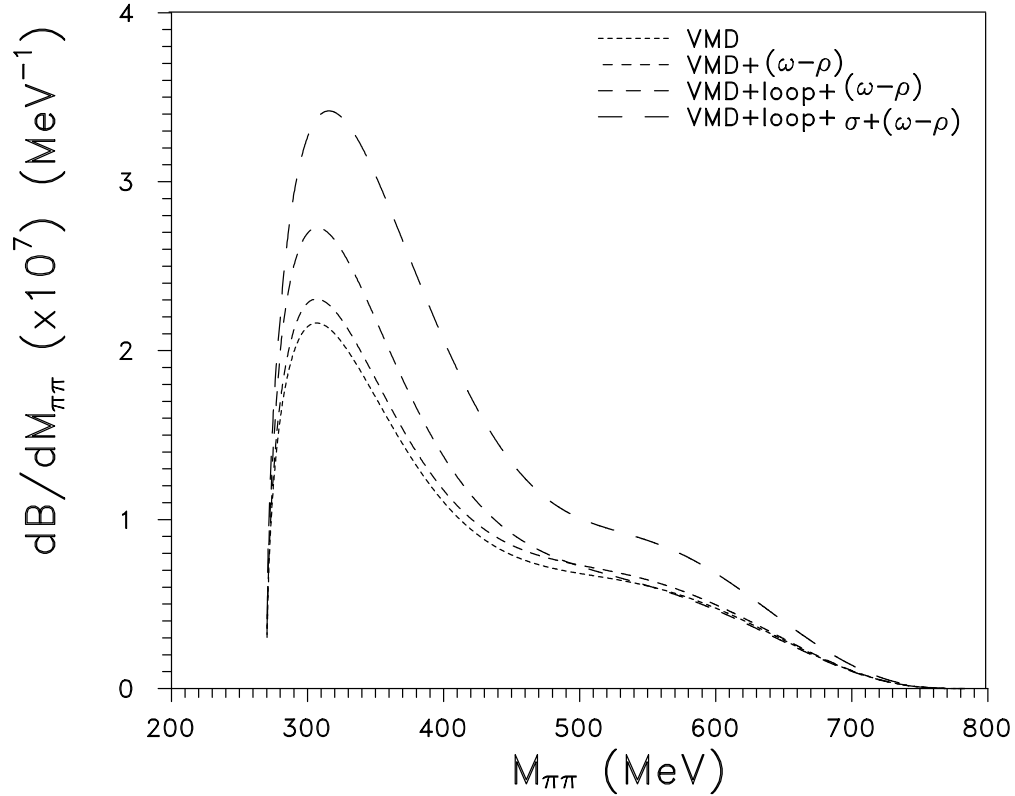


FIG. 3. The $\pi^0\pi^0$ invariant mass spectrum of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} = 0.11$. The separate contributions resulting from the amplitudes of VMD; VMD and $\omega - \rho$ mixing; VMD, chiral loop, $\omega - \rho$ mixing; VMD, chiral loop, σ -meson intermediate state, $\omega - \rho$ mixing are shown.

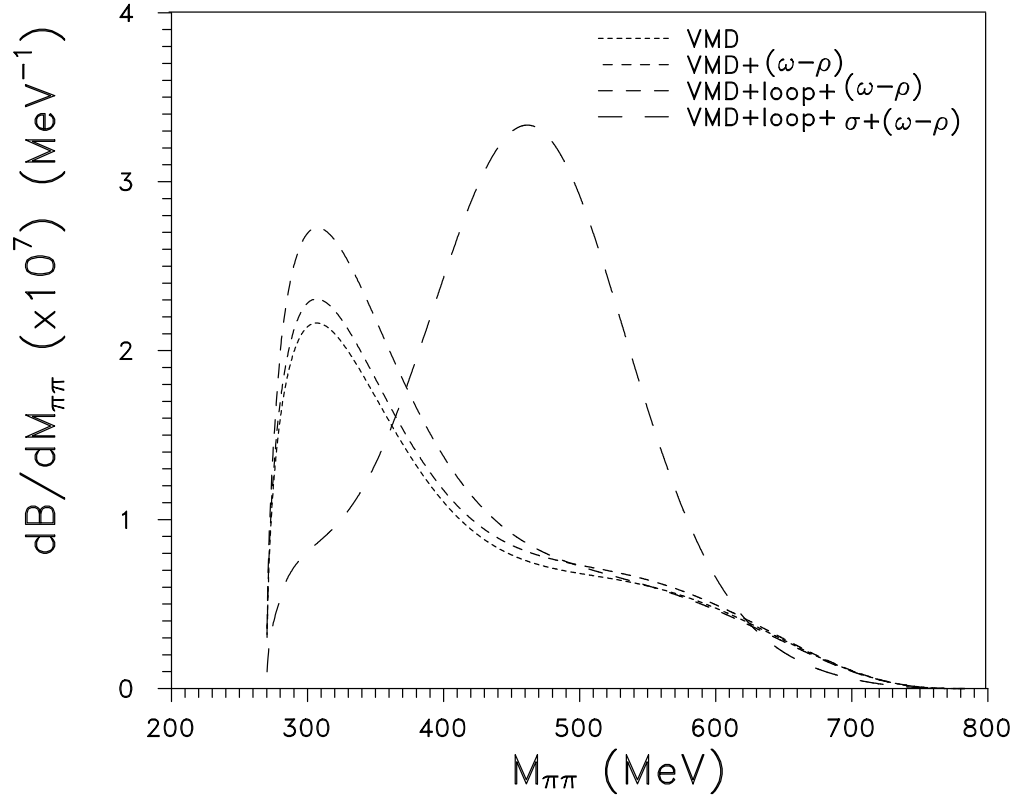


FIG. 4. The $\pi^0\pi^0$ invariant mass spectrum of the decay $\omega \rightarrow \pi^0\pi^0\gamma$ for $g_{\omega\sigma\gamma} = -0.21$. The separate contributions resulting from the amplitudes of VMD; VMD and $\omega - \rho$ mixing; VMD, chiral loop, $\omega - \rho$ mixing; VMD, chiral loop, σ -meson intermediate state, $\omega - \rho$ mixing are shown.