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Monotonicity of Liquidity sensitive Option Prices with respect to Market Liquidity Parameters

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Abstract. Classical option pricing models assume that market prices of traded securities are given and independent of the actions of the traders. Recently, a number of option pricing models emerged where the impact of hedging transactions on prices of the traded securities is also taken into account in the pricing of the option. In this term project we study one of these models proposed by Gueant and Pu that is based on the optimal liquidation framework of Almgren and Chriss. The model includes the price impact of the hedging transactions and allows the option payoff to depend on the settlement of the option (physical or cash). We derive two results about this model in discrete time: the option price given by the model is monotone with respect to several parameters of appearing in the market impact function and with respect to the risk sensitivity parameter. Furthermore, when the increment of the underlying security has 0 expectation, the option price is always above the risk neutral price. We compute the liquidity based option price numerically for a call option with physical settlement and provide graphical examples to the derived results.

Keywords. Optimal Liquidation, Option pricing and hedging, market impact, transaction costs

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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Abstract

Classical option pricing models assume that market prices of traded securities are given and independent of the actions of the traders. Recently, a number of option pricing models emerged where the impact of hedging transactions on prices of the traded securities is also taken into account in the pricing of the option. In this term project we study one of these models proposed by Gueant and Pu that is based on the optimal liquidation framework of Almgren and Chriss. The model includes the price impact of the hedging transactions and allows the option payoff to depend on the settlement of the option (physical or cash). We derive two results about this model in discrete time: the option price given by the model is monotone with respect to several parameters of appearing in the market impact function and with respect to the risk sensitivity parameter. Furthermore, when the increment of the underlying security has 0 expectation, the option price is always above the risk neutral price. We compute the liquidity based option price numerically for a call option with physical settlement and provide graphical examples to the derived results.

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Chapter 1

Introduction

An option is a financial contract that promises a payoff over a time interval as a function of the price movements of another asset. The most basic and common examples are the European call/put options whose payoff is the positive/negative part of the difference between the price of the asset underlying the contract and the contract strike price at the maturity of the contract. Pricing and hedging options is one of the main areas of mathematical finance. The classical risk neutral option pricing approach (see e.g., [8, 9]) assumes market prices as given and independent of the actions of the traders, the price of the underlying security is independent of the hedging portfolio. The risk neutral pricing of options is very much based on this assumption. In real life, this independence may not be realistic; in the hedging of large option positions the transactions may be large enough to impact the price of the underlying security. We refer the reader to [6, subsection 9.1.2] for examples. Recently, a number of option pricing models emerged where the impact of hedging transactions on prices of the traded securities is also taken into account in the pricing of the option [4, 3, 5, 1, 10] and the book [6] and the references therein. The goal of this term project is to review one of these models proposed by Gueant and Pu [7]. This model is based on the Almgren-Chriss model of optimal liquidation, for this reason we will refer to it as the optimal-liquidation-pricing (OLP) model. The following aspects of the model are directly taken from the AC framework: the price and portfolio dynamics, the modification of the midprice as a function of the transaction, the formulation of the problem as utility maximization and optimal control. The only modification that the OLP model requires is the incorporation of a terminal transaction determined by the option contract.

An option is said to be cash settled if the terminal value of the option is paid in cash at the maturity of the contract. An option is said to be physically settled if the option holder and seller trade the underlying security at the strike price of the option at the maturity of the option. Because the OLP model includes transaction costs and market impact this terminal transaction leads to different prices for different type of settlements. We review the AC model in Chapter 2, the OLP model is reviewed in Chapter 3. The original OLP model is in continuous time, for a numerical computation a discrete version is useful, this is also reviewed in Chapter 3.

[6, Chapter 9] explores through several numerical examples how the hedging algorithm implied by OLP depends on model parameters and how it compares to the risk neutral hedging algorithm. In the same work there is also a one row table ([6, Table 9.2]) showing the dependence of the OLP price to one of the parameters (η, see (2.3)) appearing in the market impact function L. This table suggests that the price is increasing in η and it is always above the risk neutral price. In Chapter 4 we prove the following results 1) the monotonicity holds in general for a number of parameters of the model (Proposition 4.1) 2) the OLP price is always above the risk neutral price whenever the expected change in stock price at each step has 0 mean (Proposition 4.2. In Section 4.1, we give several numerical examples demonstrating the results above. For the numerical calculations we use the discrete dynamic programming equation (3.4). When the mean price change is nonzero, Proposition 4.1 may no longer hold, we provide a numerical example in Section 4.2. Chapter 5 (Conclusion) comments on future work.

Chapter 2

Almgren and Chriss Model

In this Chapter we review the Almgren Chriss (AC) optimal liquidation model [2] as presented in [6, Chapter 3]. The option pricing model that we review in the next chapter is a simple modification of the AC model. The goal in optimal liquidation is the closing of a position q_0 on a financial asset; We consider the simplest formulation of this problem, which is called implementation shortfall (IS). Our presentation is based on [6, Chapter 3]. In IS the goal is to close the position q_0 before a future time T. $q_0 < 0$ is a short position and it is closed by buying the underlying asset while $q_0 > 0$ is a long position and is closed by selling. The position on the underlying asset at time t is denoted by q_t; q is assumed to be differentiable. We denote its derivative by q'. The midprice of the underlying asset is assumed to have the following dynamics:

$$S_t = S_0 + \sigma W_t + k(q_0 - q_t) \tag{2.1}$$

where W is a standard Brownian motion, σ is the price volatility and $k > 0$ is the permanent market impact factor. Let V_t denote the market volume process, i.e., we assume that in the time interval $[s, t]$ the total amount of transactions on the underlying security is assumed to be $\int_s^t V_u du$. The actual trade takes place at the following price:

$$S_t + \frac{q'_t}{V_t} L(q'_t/V_t); \tag{2.2}$$

the term $\frac{q'_t}{V_t} L(q'_t/V_t)$ represents the immediate/transitory impact of the transaction on the price, it can also be thought of as a transaction cost. The following assumptions are made on L:

1. $L(0) = 0$.
2. L is convex,
3. L is superlinear.

In the rest of this report we will assume L to be of the following form:

$$L(\rho) = \eta|\rho|^{1+\phi} + \psi|\rho|, \tag{2.3}$$

for some $\eta, \phi, \psi > 0$.

Given these assumptions, the money made from executing the trade corresponding to q is

$$X_t = X_0 - \int_0^t \left(q'_s S_s + V_s L(q'_s/V_s)\right) ds,$$

where X_0 denotes the initial cash position of the trader executing q.

The dynamics (2.1), (2.2) and integration by parts give

$$X_T = X_0 + q_0 S_0 - \frac{k}{2} q_0^2 + \sigma \int_0^T q_t dW_t - \int_0^T V_t L(q'_t/V_t) dt. \tag{2.4}$$

To formulate the liquidation problem as an optimal control problem we introduce the concave utility function $-e^{-\gamma x}$ where $\gamma > 0$ is the risk aversion parameter of the trader. The optimal liquidation problem is then the following stochastic optimal control problem:

$$\max_{q : q' \text{ exists}, q_T = 0} \mathbb{E}[-e^{-\gamma X_T}],$$

i.e., to choose the trading strategy q so that the position is closed at time T in such a way that the expected utility of terminal wealth is maximized. We refer the reader to [6, Chapter 3] for a solution to this problem. In the next chapter we will review how this problem can be easily modified to give a model of option pricing that takes into account the price impact of hedging operations.

Chapter 3

Optimal Liquidation based Option Pricing

In this Chapter we review the option pricing model of [7] as presented in [6, Chapter 9]. Trader with a long or short position in a European call or put option on a stock wants to hedge his position. For this, the trader could buy and sell stocks between time 0 and time T which is the expiration date of the option. Almgren-Chriss framework reviewed in the previous chapter can be used to model the trading involved in the hedging. This means that we assume (2.1) and (2.2) to model the price of the underlying security as well as the impact of the hedging trade on the price. In the option pricing context q_t denotes the risky component of the hedging portfolio; the interest rate is assumed to be 0. The cash position of the trader at time T, right before the settlement of the option is given by (2.4). Now we need to add to this expression terms representing the settlement of the option.

Let us consider what happens at the option expiration date T. Assume the trader who has sold a call option with strike K and nominal Q (i.e., the option concerns Q number of shares of the underlying security). In the case of a European call option we have the following possibilities: the stock price S_T is below the strike K or the stock price S_T is above the strike K. In the first case the option expires worthless. The terminal wealth of the hedging trader than is

$$X_T + q_T S_T - l(q_T)$$

where $l(q_T)$ is the risk-liquidity premium, which can be thought as the additional money the trader has to pay to liquidate the position q_T. We comment on how the function l is chosen below. For $S_T > K$ the terminal wealth of the hedging trader depends on the settlement type. There are two types of settlements : Physical Settlement and Cash Settlement.

Physical Settlement case: the counter part who has bought the call option pays QK for Q shares. At time T, if the trader does not have Q shares in his portfolio (i.e., if

$q_T < Q$) he must buy $Q - q_T$ shares. The final wealth in this case is

$$X_T + QK - (Q - q_T)S_T - l(Q - q_T).$$

Cash Settlement case: if the option is cash settled, the trader only pays the cash difference between the option price and the market price of the underlying, i.e., $Q(S_T - K)$. The final wealth in this case is:

$$X_T + q_T S_T - l(q_T) - Q(S_T - K).$$

A similar analysis can be carried out for the put option case. In general let us write the terminal wealth after settlement as

$$X_T + q_T S_T + \Pi(q_T, S_T)$$

where Π is a function which depends on the type of option, the type of position (long/short) and the type of settlement. We have two types of options (put/call) two types of settlements (physical/cash) and two types of positions (long/short). The Π function for each case is given in Table 3.1

The present liquidity based model has an additional assumption: it is assumed that the initial hedge q_0 is purchased at price S_0 from the counterparty. We refer the reader to [6, Chapter 9] for comments on how this type of transaction occurs in real life option contracts. A last important aspect of the option trade is the price P of the option, which the trader receives from the counter party; adding these to the terminal wealth, we get the following utility optimization problem:

$$\sup_{q \in A_{\rho max}} \mathbf{E}[-\exp(-\gamma\,(P - q_0 S_0 + X_T + q_T S_T + \Pi(q_T, S_T))]$$

If the trader doesn't do anything, the utility of the initial wealth X_0 will be

$$-e^{-\gamma X_0}.$$

The unknown price P is then determined by indifference pricing; that is, the optimal expected utility of the option trade must be at least equal to the utility of doing nothing, i.e., we would like

$$\sup_{q \in A_{\rho max}} \mathbf{E}[-\exp(-\gamma\,(P - q_0 S_0 + X_T + q_T S_T + \Pi(q_T, S_T))] = -e^{-\gamma X_0}.$$

By taking the log of both sides and dividing by γ, we get the following formula for the price P:

$$P = \inf_{q \in A_{\rho max}} \frac{1}{\gamma} \log \mathbf{E}[\exp\left(-\gamma(X_T - X_0 + q_T S_T - q_0 S_0 + \Pi(q_T, S_T))\right)]. \tag{3.1}$$

1. Call option:

   (a) Short position:

      i. Physical: $-Q(S_T - K)_+ - (l(Q - q_t)1_{S_T > K} + l(q_T)1_{S_T \leq K})$

      ii. Cash: $-Q(S_T - K)_+ - l(q_T)$

   (b) Long Position:

      i. Physical: $Q(S_T - K)_+ - (l(Q + q_t)1_{S_T > K} + l(q_T)1_{S_T \leq K})$

      ii. Cash: $Q(S_T - K)_+ - l(q_T)$

2. Put option:

   (a) Short position:

      i. Physical: $-Q(S_T - K)_- - (l(Q + q_t)1_{S_T > K} + l(q_T)1_{S_T \leq K})$

      ii. Cash: $-Q(S_T - K)_- - l(q_T)$

   (b) Long Position:

      i. Physical: $Q(S_T - K)_+ - (l(Q + q_t)1_{S_T > K} + l(q_T)1_{S_T \leq K})$

      ii. Cash: $Q(S_T - K)_- - l(q_T)$

Table 3.1: Terminal payoff of European options according to type, position and settlement

P is a function of the following parameters: q_0, S_0, X_0; one way to compute P is to derive the Hamilton Jacobi Belmann equation associated with the above control problem and solve it. This path is explored in [6, Chapter 9]. In this report we will follow another path which is also expounded in the same reference: writing a discrete version of the above problem and solve it numerically by dynamic programming. The next section presents the elements of this calculation.

For the liquidity premium l we will use the formulation in [6, Chapter 8] where l is computed from L and k by solving another optimal liquidation problem. The resulting function l is given as follows:

$$l(q) = \frac{k}{2}q^2 + \psi|q| + \frac{\eta^{\frac{1}{1+\phi}}}{\phi^{\frac{\phi}{1+\phi}}} \frac{(1+\phi)^2}{1+3\phi} \left(\frac{\gamma\sigma^2}{2V}\right)^{\frac{\phi}{1+\phi}} |q|^{\frac{1+3\phi}{1+\phi}}; \tag{3.2}$$

this formula is given in [6, section 8.3.3] and follows from [6, Corollary 8.1].

3.1 The discrete model

The discretization of the above model leads to the following formulation (as presented in [6, Section 9.3]):

$$S_{n+1} = S_n + \sigma\sqrt{\Delta t}\varepsilon_{n+1}, \tag{3.3}$$

where ε_n are iid with 0 mean and variance 1. The cash position of the trader evolves according to:

$$X_{n+1} = X_n - v_n S_n \Delta t - L(v_n/V_{n+1})V_{n+1}\Delta t.$$

A very important assumption here is the following: in the above display we assume $k = 0$. This assumption implies that the x dependence of the value function can be factored out. We come back to this point below.

As in the continuous case, the terminal wealth can be written as

$$X_N + q_N S_N + \Pi(q_N, S_N).$$

The change in the hedging position cannot be greater than the market volume available, i.e., $|q_{n+1} - q_n| \leq V_{n+1}\Delta t$. We slightly generalize this constraint to $|q_{n+1} - q_n| \leq CV_{n+1}\Delta t$ where $1 > C > 0$. With these modifications, the stochastic optimal control remains the same:

$$u^n(x, q_0, S) = \sup_{q\in\mathcal{A}} \mathbf{E}[-\exp(-\gamma\,(X_N + q_N S_N + \Pi(q_N, S_N)))\,|X_n = x, q_n = q_0, S_n = S],$$

where $\mathcal{A} = \{q : |q_{n+1} - q_n| \leq CV_{n+1}\}$. u^n satisfies the following dynamic programming equation (DPE):

$$u^n(x, q, S) = \sup_{|v|\leq CV_{n+1}} \mathbf{E}[u^{n+1}(x - vS\Delta t - L(v/V_{n+1})V_{n+1}\Delta t, q + v\Delta t, S + \sigma\sqrt{\Delta t}\epsilon_{n+1})]$$

and the terminal condition

$$u^N(x, q, S) = -\exp(-\gamma(x + qS + \pi(q, S))).$$

The assumption $k = 0$ implies that the x dependence of $u^n(x, q, S)$ can be factored out as follows:

$$u^n(x, q, S) = -\exp(-\gamma(x + qS - \theta^n(q, S))),$$

for some function θ. The function θ is then

$$\theta^n(q, S) = \frac{1}{\gamma}\log(-u^n) + (x + qS).$$

The DPE for u^n implies the following DPE for θ^n:

$$\begin{aligned}\theta^n(q,S) = \inf_{|v|\leq CV_{n+1}} \Bigg\{ & L(v/V_{n+1})V_{n+1}\Delta t \\ & + \frac{1}{\gamma}\log \mathbb{E}[\exp(-\gamma(q+v\Delta t)\sigma\sqrt{\Delta t}\epsilon_{n+1} \\ & + \gamma\theta^{n+1}(q+v\Delta t, S+\sigma\sqrt{\Delta t}\epsilon_{n+1}))]\Bigg\}.\end{aligned} \tag{3.4}$$

with the terminal condition $\theta^N(q,S) = -\pi(q,S)$. The optimizer in the last equation gives the hedging portfolio.

Chapter 4

Monotonicity of the OLP Price with respect to Liquidity Parameters

The OLP price has the following additional parameters: γ, η, ψ, ϕ, k and the volume process V_n. In the following proposition we indicate how the OLP price changes with γ, η, ψ and V_n.

Proposition 4.1. *The OLP price function θ^n is increasing in γ, η, ψ and decreasing in V_n for all European call and put options given in Table 3.1.*

Proof. All of the payoffs listed in Table 3.1 consist of two terms: the classical payoff of the option which is a function of Q, S_T and K and a liquidity premium term. Because l is symmetric around 0 and positive all of the payoffs are handled the same way; to ease notation we will only focus on a cash settled short position, for which the liquidity premium term is simply $l(q_T)$.

We begin by γ: we proceed by induction on n: for $N = n$ we have $\theta^N = -\Pi(q, S) = f(Q, S_T, K) + l(Q_T)$. The function l given in (3.2) is increasing in γ, which proves the claim for $n = N$. Now suppose that θ^{n+1} is increasing in γ; θ^n is computed from θ^{n+1} by the DPE (3.4). The only term that depends on γ on the right side of this expression is

$$\frac{1}{\gamma}\log\mathbb{E}[\exp(-\gamma(q+v\Delta t)\sigma\sqrt{\Delta t}\epsilon_{n+1}+\gamma\theta^{n+1}(q+v\Delta t, S+\sigma\sqrt{\Delta t}\epsilon_{n+1}))]$$

Let

$$Z = \exp(-(q+v\Delta t)\sigma\sqrt{\Delta t}\epsilon_{n+1}+\theta^{n+1}(q+v\Delta t, S+\sigma\sqrt{\Delta t}\epsilon_{n+1}))].$$

Then we can write the DPE (3.4)as

$$\theta^n(q, S) = \inf_{|v|\leq CV_{n+1}}\{L(v/V_{n+1})V_{n+1}\Delta t + \log(||Z||_\gamma)\} \tag{4.1}$$

where $||\cdot||_\gamma$ denotes the L_γ norm with respect to the distribution of ϵ_{n+1}. Z itself is increasing in γ by the induction hypothesis; furthermore for any random variable X,

$\gamma \mapsto ||X||_\gamma$ is increasing in γ. These imply that the right side of (3.4) is increasing in γ; taking the inf over v preserves this property. It follows that θ^n is increasing in γ.

The proof of monotonicity in η and ψ follow from a similar argument and the fact that L and l are both increasing in η and ψ. The monotonicity in V follows from again a similar induction argument and the following facts: 1) l is decreasing V and 2) by definition (see [6, page 43])

$$L(v/V_{n+1})V_{n+1}\Delta t = g(v/V_{n+1})v\Delta t$$

and g is monotone increasing in its argument. This implies that the right side of (4.1) is decreasing in V_{n+1} and once again the induction argument applies and allows one to transfer the decreasing in V_{n+1}, V_{n+2},...,V_N property of θ^{n+1} to θ^n. $\square$

The risk neutral price for the options in Table 3.1 is computed as follows. First, we assume that ϵ_n in (3.3) take only two values 1 and -1. This gives us a complete market. The symmetry of the increments and $r = 0$ imply that the risk neutral probabilities for this market is simply 1/2 for both increments. It follows that the risk neutral price at time n for any of the options is given by:

$$RP(s,n) = \mathbb{E}[f(s + \sigma\sqrt{\Delta t}(2B - (N-n)), K)].$$

where B is binomial with parameters 1/2 and $N - n$ and f is the classical payoff of the option. Conditioning on the first step of S_n gives

$$RP(s,n) = \mathbb{E}[RP(s + \sigma\sqrt{\Delta t}\epsilon_1, n+1)]. \tag{4.2}$$

The arguments in the previous proof allow us to compare the OLP price to the risk neutral price:

Proposition 4.2. *For all options listed in Table 3.1 we have*

$$\theta^n(s,q) \geq RP(s,n). \tag{4.3}$$

Proof. The proof once again proceeds by induction. For $n = N$, (4.3) follows from $l \geq 0$.

To perform the induction hypothesis, we note:

$$\begin{aligned}\lim_{\gamma\to 0}\log(||Z||_\gamma) &= \mathbb{E}[\log(Z)]\\ &= \mathbb{E}[-(q+v\Delta t)\sigma\sqrt{\Delta t}\epsilon_{n+1} + \theta^{n+1}(q+v\Delta t, S + \sigma\sqrt{\Delta t}\epsilon_{n+1})]\\ &= \mathbb{E}[\theta^{n+1}(q+v\Delta t, S+\sigma\sqrt{\Delta t}\epsilon_{n+1})]\end{aligned}$$

where the limit follows from the Taylor expansion of the log function at 1 and in the last line we used $\mathbb{E}[\epsilon_{n+1}] = 0$. The induction hypothesis ($\theta^{n+1}(s,\cdot) \geq RP(n+1,s)$), the monotonocity of γ, (4.3) and the last limit imply

$$\log(||Z||_\gamma) \geq RP(n, s+1).$$

This and (4.1) imply $\theta^n(s,\cdot) \geq RP(n,s)$. $\square$

In the next section we give several numerical examples for the two results above.

4.1 Numerical Examples

The following parameter values are adapted from [6, Chapter 9]:

1. $Q = 2 \times 10^7$,
2. $\sigma = 0.6$.
3. Maturity $T = 63$ days,
4. The L function: $\phi = 0.75$.

As an example we consider a physically settled call option with strike $K = 45$. Figures 4.1, 4.2, 4.3 and 4.4 all provide examples of Propositions 4.1 and 4.2; in all of these figures the OLP price changes monotonically with the parameters γ, η, ψ and V and they are all above the risk neutral price.

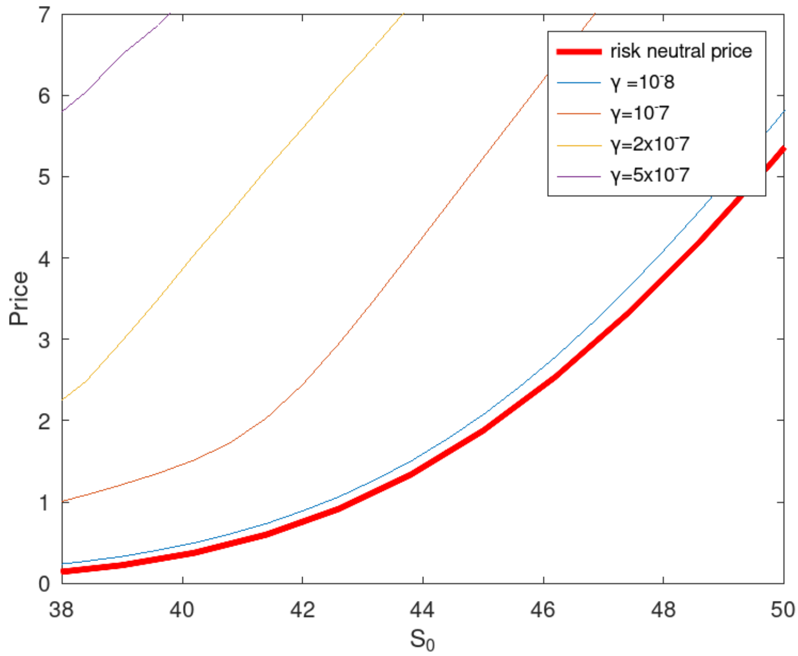

Figure 4.1: thin curves: $\theta^1(0.5Q, \cdot)$, as γ varies, thick curve: risk neutral price, $\eta = 0.2$, $\psi = 0.004$, $V = 4 \times 10^6$

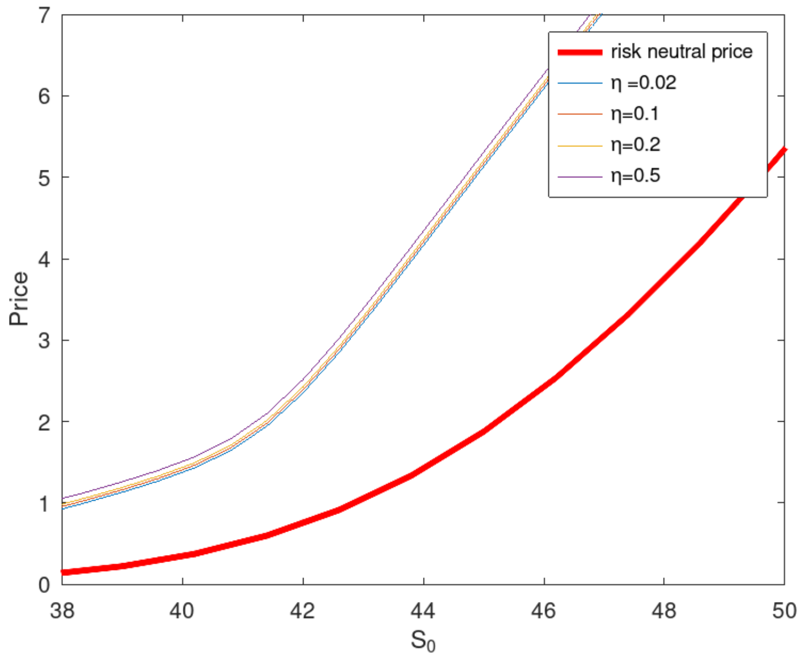

Figure 4.2: thin curves: $\theta^1(0.5Q, \cdot)$, as η varies, thick curve: risk neutral price, $\gamma = 10^{-7}$, $\psi = 0.004$, $V = 4 \times 10^6$

4.2 Impact of the actual distribution of S on the OLP price

If $\mathbb{E}[\epsilon_n] \neq 0$, Proposition 4.2 may no longer hold, that is θ^n can be both below or above the risk neutral price as the moneyness of the option changes. We provide an example in Figure 4.5; Several comments on Figure 4.5:

1. Even for small deviations from $\mathbb{E}[\epsilon_n] = 0$, the OLP price can be below the risk neutral price.

2. Intuitively, $\mathbb{E}[\epsilon_n] > 0$ appears to be additional risk for the seller of the call option, as the short call position can be seen as a bearish bet. However, as can be seen in Figure 4.5 it in fact reduces the price of the option for out of the money calls.

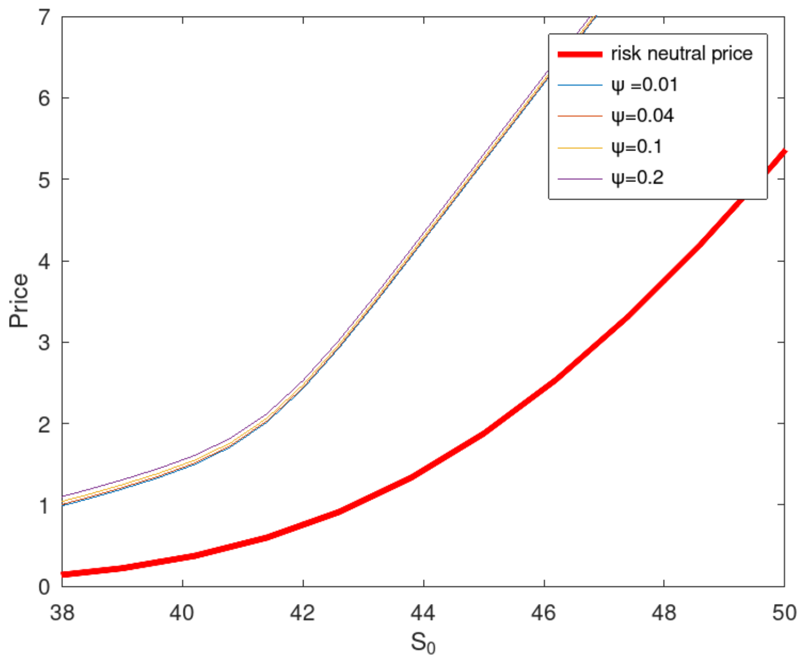

Figure 4.3: thin curves: $\theta^1(0.5Q, \cdot)$, as ψ varies, thick curve: risk neutral price, $\gamma = 10^{-7}$, $\eta = 0.2$, $V = 4 \times 10^6$

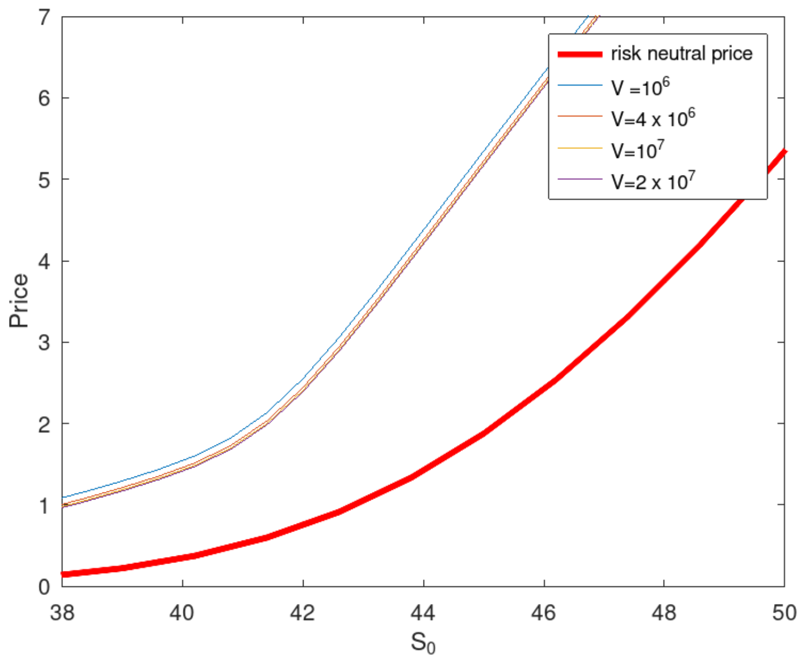

Figure 4.4: thin curves: $\theta^1(0.5Q, \cdot)$, as V varies, thick curve: risk neutral price, $\gamma = 10^{-7}$, $\eta = 0.2$, $\psi = 0.04$

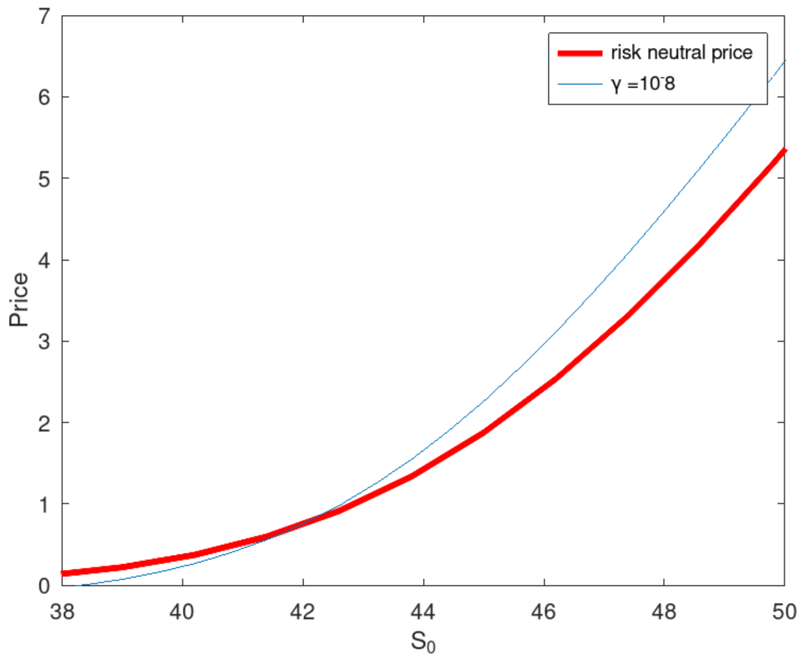

Figure 4.5: thin curve: $\theta^1(0.5Q, \cdot)$, thick curve: risk neutral price, $\gamma = 10^{-7}$, $\eta = 0.2$, $\psi = 0.04$, $V = 4 \times 10^6$, $\mathbb{E}[\epsilon_n] = 0.04$

Chapter 5

Conclusion

In this project we reviewed an option pricing model (OLP) proposed by Gueant and Pu that is based on the Almgren Chriss liquidation framework that explicitly takes into account the impact that the hedging operations may have on the price of the underlying security. In Chapter 4 we proved that 1) the option price in this model is monotone increasing in the parameters η, ψ (parameters determining the transaction costs/temporary price impact) and γ (risk averseness) and decreasing in V (market volume) and 2) when the price increment of the underlying has 0 mean, the OLP price is always above the risk neutral price. In the same Chapter we provided several numerical examples of these results. Future work can try to derive similar results for the parameters k and ϕ. Another interesting direction for future work is an analysis of the actual performance of the hedging algorithms implied by OLP.

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