LONG-TERM PRODUCTION SCHEDULING OPTIMIZATION FOR A POLYMETALLIC OPEN PIT MINE

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ABSTRACT

LONG-TERM PRODUCTION SCHEDULING OPTIMIZATION FOR A POLYMETALLIC OPEN PIT MINE

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Open-pit production plannings are commonly performed as short-term and long-term production planning. Short-term production planning is generally conducted at the operational level on a weekly and daily basis, while long-term planning determines the production limits on an annual basis that can be divided into semiannual or quarter periods. Therefore, long-term plans set a boundary for short-term plans and require the inclusion of multiple parameters that should be evaluated attentively. The main intention of production planning in open pit mines is to determine ore and waste extraction sequences that satisfy geotechnical, processing, and other operational constraints so that the overall benefit of the block selections in different periods should be maximized. On this basis, production plans commonly aim to maximize the total amount of the final product or maximize net present value (NPV). Maximizing the final product is generally aimed in cases where market pressure or other commitments are available. The majority of long-term production plannings are carried out to maximize NPV.

Production in open-pit mining areas generally advances mainly in horizontal and vertical directions with the sequential excavation of multiple pit geometry called phases (pushbacks) from the inside out and is completed with reaching the ultimate pit geometry. Therefore, geotechnical, processing, and other operational constraints should be satisfied within the limits of each production phase and the ultimate pit. Decisions on the block and its excavation period can be more challenging if the blending of materials in multiple blocks is also required to keep the plant efficiency above target levels.

At this point, this study intends to present an integer programming model to optimize the long-term production scheduling of open-pit mines by maximizing the NPV value. The model was constructed in such a way that any constant pit slope value can be integrated into the model, and decisions on ore blocks are taken considering the periodic requirements of the plant and feeding material ranges that may entail blending of different blocks before the feeding stage. The model was computed in AMPL CPLEX Solver using NEOS Server for a real dataset acquired from an open-pit mine having Zn and Cu deposit, and that will produce ore for an eight-year mine life with three production phases. Besides, 30° overall pit slope was maintained in the model, and a Zn/Cu feeding requirement between one and six was considered. The integer model optimized the long-term production scheduling of the mine by maximizing the NPV to $11,489,800. Moreover, the sensitivity of the optimized production plans to the variations in operational cost, sales prices, and discount values was also evaluated. It was observed that the sales price is the most influential factor in NPV, while the changes in the discount rate and the operational cost values were detected to modify the block extraction sequence more when re-optimizing the production schedule.

Keywords: Optimization of Long-term Production Scheduling, Integer Programming, Blending, Multiple Phase Open-Pit Mine

ÖZ

BİR POLİMETAL AÇIK OCAK MADENİNİN UZUN VADELİ ÜRETİM PLANLAMA OPTİMİZASYONU

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Yüksek Lisans, Maden Mühendisliği
Tez Yöneticisi: Dr. Öğr. Üyesi Onur Gölbaşı

Şubat 2022, 88 sayfa

Açık ocak üretim planlaması, kısa dönem ve uzun dönem üretim planlaması olarak gerçekleştirilmektedir. Kısa dönem üretim planlamaları genellikle operasyonel seviyede haftalık ve günlük olarak yapılmakta, uzun dönem üretim planları ise üç aylık veya aylık dilimlere de ayrılacak şekilde genellikle yıllık düzeyde üretilecek alanların sınırlarını belirlemektedir. Böylelikle, uzun dönem planlar kısa dönem planları için bir sınır belirlemektedir ve dikkatlice değerlendirilmeleri gereken pek çok parametreyi bünyesinde barındırmaktadır. Açık ocaklardaki üretim planlamalarının esas amacı, jeoteknik, tesis ve diğer operasyonel kısıtları dikkate alarak ve farklı periyotlardaki üretim bloğu seçimlerinin nihai faydasını en üst düzeye çıkaracak şekilde, pasa ve cevher bloklarının üretim sırasını belirlemektedir. Bu bağlamda, üretim planları genellikle iki amacı hedeflemektedir: i) toplam nihai ürün miktarını en üst düzeye çıkarmak veya ii) net bugünkü değeri en üst düzeye çıkarmak. İlk belirtilen amaç genel itibariyle, piyasa baskısından veya şirketin diğer taahhütlerinden kaynaklı olarak gerçekleştirilebilir. Ancak, maden planlama çalışmalarının büyük çoğunluğunda net bugünkü kazancı azami seviyeye çıkarmayı amaçlanmaktadır.

Açık ocak maden sahalarında üretim ilerlemesi, çoğunlukla hem yatay hem de dikey düzlemde, ocak fazı adı verilen ve içten dışa doğru genişleyen ocak geometrilerinin sırasıyla üretilmesiyle gerçekleştirilmektedir. Bu nedenle, jeoteknik, tesis ve diğer operasyonel kısıtların hem her bir ocak fazı hem de nihai ocak geometrisi için sağlanıyor olması gerekmektedir. Üretim blokları üzerine alınan kararlar ve blokların üretim zamanları, tesisin hedeflenen verimi koruması için gerekli olabilecek harmanlama oranlarının dahil edildiği durumlarda daha da karmaşık hale gelebilmektedir.

Bu konuda, mevcut tez çalışması açık ocak madenlerin uzun dönem üretim planlamalarını, net bugünkü kazanç değerini azami seviyeye çekecek şekilde optimize edebilen bir tam sayı programlama modeli sunmaktadır. Bu model, farklı ocak şev açılarının modele entegre edilmesine ve tesise beslenilmeden önce farklı blokların farklı cevher tipleri için harmanlama gereksinimini dikkate alacak şekilde oluşturulmuştur. Sunulan model, NEOS sunucusu üzerinde AMPL CPLEX Solver ara yüzü kullanılarak, Çinko ve Bakır cevherine sahip, maden ömrü sekiz sene olan ve toplam üretimi üç fazda gerçekleştirmesi planlanan bir açık ocak madeninin gerçek verileri kullanılarak çözülmüştür. Aynı zamanda, yaklaşık 30 derecelik nihai şev açıları ve 1-6 arası sağlanması gereken Çinko/Bakır içerik oranı da modele sunulmuştur. Model, ilgili madenin uzun dönem üretim planlaması problemini, net bugünkü kazanç 11.489.900 $’a indirgenecek şekilde optimize etmiştir. Ayrıca, optimize edilen üretim planlarının, işletme maliyeti, nihai ürün satış fiyatları ve indirgenme (iskonto) oranındaki değişimlere hassasiyeti de irdelenmiştir. Analizlerden, ürün satış fiyatının, net bugünkü kazanç değerine en fazla etki ettiği tespit edilmiş, diğer bir yandan işletme maliyeti ve indirgenme oranındaki değişimlerin, blokların üretim sıralamalarında en önemli etkiye sahip olduğu belirlenmiştir.

Anahtar Kelimeler: Uzun Dönem Üretim Planlaması Optimizasyonu, Tam Sayı Programlama, Harmanlama, Çoklu Açık Ocak Üretim Fazı

To My Family

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

AMPL	A mathematical programming language
ALR	Augmented Lagrangian relaxation
FA	Firefly algorithm
GA	Genetic algorithm
G&A	General and Administrative
IP	Integer programming
LG	Lerch-Grossman
LOM	Life-of-mine
LP	Linear programming
LR	Lagrangian relaxation
LTPSP	Long-term production scheduling problem
LTPS	Long-term production scheduling
MILP	Mixed-integer linear programming
MIP	Mixed-integer programming
NPV	Net present value
NSR	Net smelter return
PSO	Particle swarm optimization
SIP	Stochastic integer programming

CHAPTER 1

INTRODUCTION

1.1 Background

The mining sector intends to recover ore deposits, which are not renewable resources, in the most economical way of extraction and processing. These resources can be extracted in two ways; underground mining and surface mining operations. Underground mining is generally implemented for deeper ore deposits. It includes more auxiliary operations than surface mining, like ventilation and ground support. However, both mining methods have basic activities of a cycle of drilling, blasting, loading, and dumping that require careful scheduling of block extraction since each extraction stage covers multiple operating cost measures that can lead to a remarkable change in the cost flow in case of non-optimum scheduling. On this basis, an optimal production-scheduling offers a process of extracting sequences of material generally by maximizing Net Present Value (NPV) or amount of final product for a given period.

Surface mining can be categorized into two main types; strip (open cast) mining and open pit mining (Figure 1.1). Strip mining is an operation where overburden material covering the economic minerals; the minerals which lies in flat or nearly-flat orientation are removed first using massive capacity earthmoving equipment. Simple side casting is the most common way of strip mining where the excavated overburden material is dumped to the areas where ore extraction is already completed. However, if the ore deposit locates in a deeper position with a steeper slope, this method becomes impractical. Open-pit mining is commonly used for these types of minerals, and overburden rehandling or filling the ore-recovered areas are not completed until mine closure and reclamation. Waste material is hauled and dumped to a separate area called dumpsite.

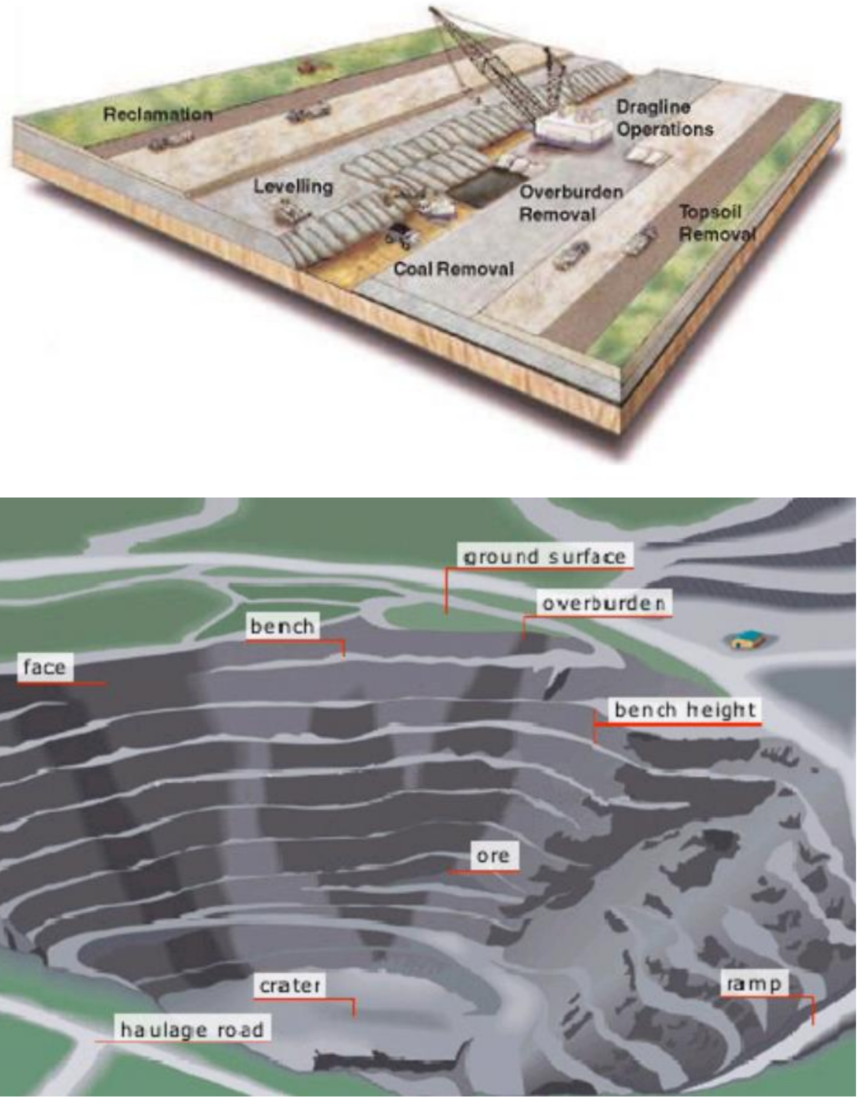


Figure 1.1 A representative illustration of strip coal mine above and open-pit mine below (National Research Council, 2007)

An ore deposit can be located beneath the surface in a changeable depth that can vary from ten to hundreds of meters. Here, production scheduling requires a preliminary determination of a pit optimization that points to ultimate pit limits and pushbacks. Ultimate pit limits set a constraint on which block should be extracted up to the end of mine life, but it does not concern about when and in which order the block should be produced. In addition, pushbacks specify multiple shells (also called pit limits or pit boundaries) when defining the advances in the pit. In brief, the last push-back pit shell is the ultimate pit. It divides the ultimate pit into more manageable units to schedule easily (Figure 1.2). At this point, production scheduling decides on which block should be mined when by maximizing the overall financial benefit from the extraction order or maximizing the amount of final product for a given period. The blocks covered within a pit shell can be either of a waste block, which does not include any valuable mineral or includes valuable mineral but under the economic threshold, or ore block, which includes valuable mineral whose unit financial return is higher than its unit total operating cost of mining, processing, and other related items. All these operations are expressed with a diagram in Figure 1.3.

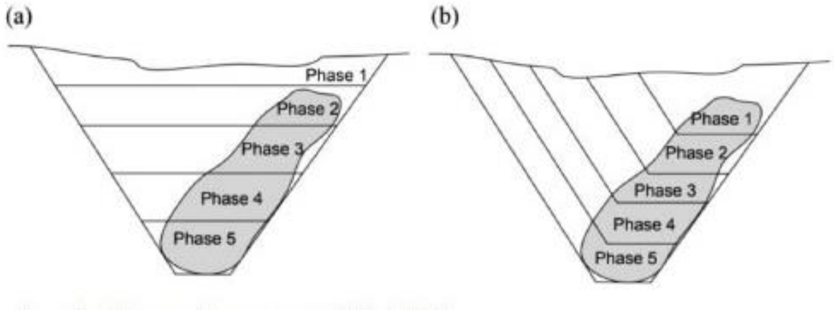


Figure 1.2 Different mining phases (Whittle, 2011)

An open-pit mine production scheduling problem can be expressed as a mathematical model of linear programming (LP), integer programming (IP), or mixed-integer programming (MIP) problem. To solve these problems, different approaches try to arrive at an exact optimal solution or nearly optimal solution. In some cases, having a large amount of data might cause reaching the exact optimal solution to be difficult due to the computational complexity.

The main problem in production scheduling is that any decision on the extraction of a block can be constrained by different factors such as geotechnical, financial, processing, or any other mining-related consideration. Besides, any favorable decision for mining a specific block in a definite period will affect the other decisions for the unmined blocks. These period-based decisions where the length of the period can be regarded in daily, monthly, quarterly, semi-annual, or annual should be given all at once in a mathematical model for hundreds of thousand blocks in a way to optimize the objective function constrained by multiple limits.

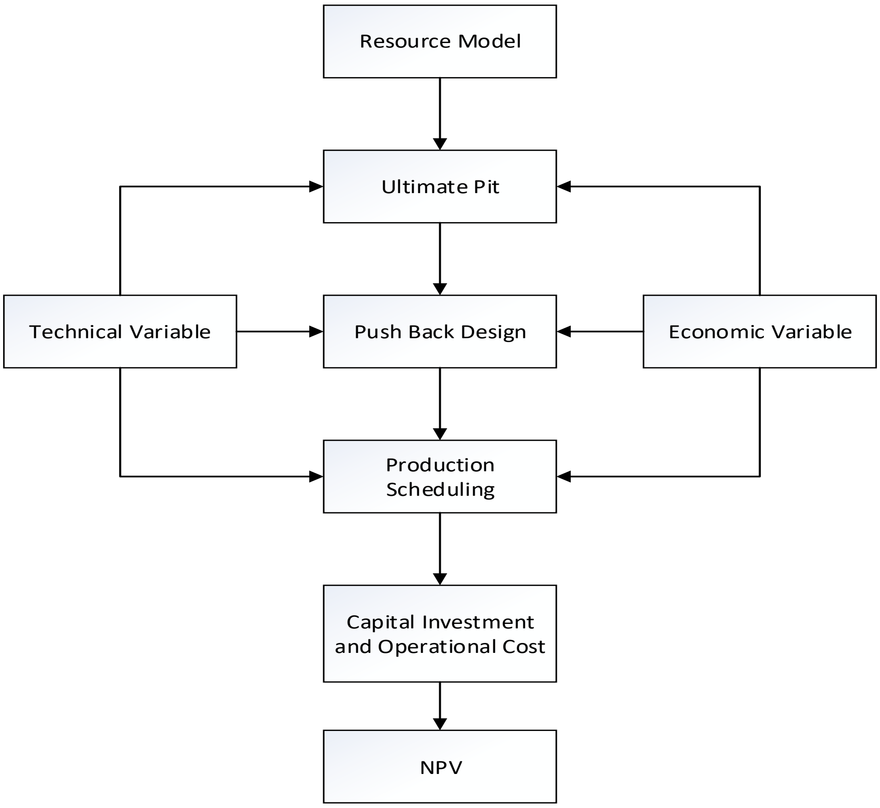


Figure 1.3 Long term mine planning process

This research study aims to develop a mathematical model to solve the open pit production-scheduling problem for polymetallic mines, where blending constraint is also included. The developed model will be applied for a poly-metallic base metal mine by using site-specific cost, ore grade, mining and processing factors, and geotechnical issues. In the literature, there is various research about scheduling problems. However, a blending constraint of two minerals that means the grade ratio of the minerals to each other has never been studied or at least never been reported in the literature.

1.2 Problem Statement

Production scheduling is limited by many factors stemming from geotechnical, financial, and processing considerations. Deciding on which ore block should be extracted when in an open-pit operation is critically important to achieve a high NPV amount for the profitability and sustainability of operations. Moreover, if ore blocks include more than one mineral and if the minerals in different blocks are blended in ore stockpiles and ROM pads with undesired amounts, this may cause some complications in the processing phase. Therefore, optimization of production scheduling needs to be performed carefully by including operational and geotechnical constraints in such a way that the extraction of specific blocks and their blending rates should not violate the processing and mining requirements. In addition, scheduling of the waste and ore block extraction should serve to optimize the problem by maximizing NPV or throughput amount. Otherwise, there will be observable drops in the processing recovery and financial return of the final product.

1.3 Objectives and Scopes of the Study

The main objective of this study is to develop a mathematical model to solve the open pit long-term production scheduling problem for poly-metal mines, where blending constraint is also included. Also, the developed model will be applied to an open-pit polymetallic mine that includes Cu-Zn sulfide minerals. Sub-objectives of the research can be listed as follows:

i. Determining problem limits: It is essential to define limits of the mathematical model that should be capable of solving production scheduling problems under given constraints in a reasonable computation time Otherwise, the problem may be insoluble or need extensive computational time. In a production scheduling problem, processing and mining constraints should be defined in detail to build up a proper boundary. Therefore, preliminary literature and site surveys are required to be completed.

ii. Developing a mathematical model of production scheduling: It should satisfy the constraints and maximize the NPV. Also, the problem should be solved in an acceptable computational time.

iii. Implementing the model to a mining company with real datasets: The model will be implemented for a polymetallic open-pit mine dataset.

Under the scope of the study, the implementation will be performed for a Cu-Zn open-pit mine and site-specific data of mine valid in 2021 will be used as input and in the block model validation phase. 1100 blocks and their specific mining and processing-related parameters will be considered in the application geotechnical, mine, and process plant capacity, blending, precedence block constraints are used as a basis for this model. Blending constraint refers to the Cu and Zn grade ratio limits when feeding into the processing plant that should not be violated to decrease the efficiency of the flotation plant, mineral processing facility in the mining area.

1.3.1 Research Methodology

The current study entails the completion of some major steps. These steps are summarized as follows:

i. First, the boundary of the production scheduling model will be determined to be able to decide on the parameters and the constraints that will be included in the optimization. In this respect, the ultimate pit limits and the concerned blocks will be discussed with the authorities of the mining company and an extensive literature review will be conducted to understand the major and minor aspects effective in both ultimate pit limit and production scheduling decisions. The technical and processing restrictions will also be evaluated and finalized after a holistic review and discussion phase.

ii. Second, in accordance with the study scope, the data acquisition process will be completed. At this point, all the required subjective (expert knowledge) and objective (technical and reached values) data will be acquired.

iii. Third, an integer mathematical model that is capable of optimizing the long-term production scheduling problem of a poly-metal mine with the blending constraint will be constructed in the AMPL optimizer.

iv. Fourth and the last, the developed model will be applied for the mine, where the data acquisition is performed, for the verification and validation of the model results. A detailed interpretation and discussion section will also be performed here to reveal the benefits and limitations of the model.

1.4 Expected Contributions of This Thesis

A number of studies about open pit production scheduling optimization have been carried out in the literature considering the factors of time, mineral type, blending, and stockpiling. The majority of the previous studies included one mineral when evaluating the economic excavability of blocks. In addition, the blending constraint of these types of problems is about mixing high-grade and low-grade ore to obtain the optimal feeding value for processing operations. However, there are many other mines that should evaluate more than one mineral when feeding into the processing mineral not to disturb the operational efficiency of processing plants and to sustain an optimized blending ratio of poly-minerals. Therefore, the ratio of two different metal content is included within the blending constraint of the current study. Developing a mathematical model that maximizes the NPV with this blending constraint and having an acceptable computational time are the aims of the study. In this way, both mining and processing considerations are included jointly. Future academic studies and the mining sector may utilize the discussions and model outcomes of the study when improving their long-term production scheduling models.

CHAPTER 2

LITERATURE REVIEW

The study aims to construct a mathematical model on mine production scheduling with a blending constraint. Mining production is planned for varying periodic intervals called strategical, tactical, and operational planning. These types are also referred to as long-, mid-, and short-term planning, respectively. The period lengths may show a variation depending on the company policy. Extraction of mining blocks in long-term planning is performed generally on a yearly basis where short-term block extraction plans may provide details on a weekly or daily basis. Since the optimization of production scheduling for both short and long-term goals require solving an enormous number of equations, recent researches have generally relied on linear modeling (LP) or mixed-integer linear modeling (MILP). The majority of the models have intended to determine which mining blocks should be extracted and conveyed to a processing plant, stockpile, and dumpsite, in which time intervals. Besides, those models generally regarded a single commodity, i.e. economic mineral, by maximizing the net present values of mines. Blending constraint that considers mixing low- and high-grade ore for single or multiple commodities is not included in many models.

The literature section is structured to develop a background on the current study by discussing and evaluating the recent optimization studies in mining, production scheduling dynamics, and mathematical modeling principles. On this basis, theoretical knowledge, as well as particular applications, are presented in detail.

2.1 Mathematical Modeling of Engineering Problems

Engineering problems require the adaptation of multiple model types with varying complexity. Therefore, there is no unique holistic solution that can be used commonly for all problem types since each model covers different determinants, boundaries, and objective functions. Any real case can be realized in either simulations or mathematical models by imitating actual application' dynamics in a computational environment. Following a verification and validation step, these tools enable the iteration of various scenarios to reveal the set of all possibilities related to the case. Simulation models enable to analyze and monitor material-material or event flow interactions by revealing sequential dependencies and outcomes. On the other hand, mathematical models require multiple constraint functions and single or multiple objective functions intending to maximize or minimize a particular aspect. All the constraints need to be satisfied simultaneously where the parametric values of the optimized objective function should achieve the highest or the lowest value in the feasibility set depending on minimization or maximization requirement.

A properly-constructed mathematical model offers a detailed, systematic, and quantitative explanation of different real-world (physical, chemical, biological, and similar) systems. These models allow recognizing and evaluating the important qualitative characteristics of systems, arranging and processing data, and designing and improving complicated systems of engineering (Yatsenko and Hritonenko, 2005). A mathematical model needs a clear understanding of the process' complexity and familiarity with the available models and methods. A researcher must decide on the modeling method and the level of abstraction and aggregation required (Yatsenko and Hritonenko, 2005).

A basic mathematical model covers an objective function, a set of constraints, and a set of negativity or non-negativity constraints. Linear, integer, and mixed-integer linear are the most common types of mathematical modeling.

Models where the decision variables in the objective function are linear numbers, ie. fractional numbers, are linear. Linear programming is one of the most common ways of constructing models related to surface mining operations. The aim is to maximize or minimize a linear function which is called as the objective function. The results of the decision variables must satisfy all constraints. Also, these constraints have to be a linear equation or linear inequality. Moreover, there is a sign restriction for each variable. Variables cannot have a negative value.

Unlike the linear models, the models using integer decision variables in objective functions are called integer models. Binary models, where the decision variables take the value of 1 or 0, are also called integer models. This type of modeling offers an exact solution set but requires more computational time and power since integer numbers occupy more memory in computers than linear numbers. Some integer models, including stochasticity, have been performed in mining where at least one variable is unknown and offers a range of effective strategic planning to integrate different types of uncertainty (Birge and Louveaux, 2011). Ramazan and Dimitrakopoulos (2005) introduced its application in the mining context for the first time. The formulation aimed to improve the NPV of the project and reduce deviation from the ore, waste, and metal production, where binary extraction variables were the first stage variables. The second stage variables were used in each scenario to calculate deviations. Benndorf and Dimitrakopoulos (2013) advance the two-stage stochastic integer programming to optimize the multi-element mining with the constraint of complex blending criteria. When compared to the deterministic design, it is verified that the production schedule will provide high-quality material by reducing the grade variations of multiple elements in an iron ore deposit. Finally, annual production variations are minimized to a certain extent due to the increasing deviation penalty. Nevertheless, the production risk cannot be completely removed, and at some point, the marginal rise in penalty magnitude will no longer be beneficial.

Most of the graphical or network problems are non-deterministic polynominal problems and difficult to solve. In such a case, it is necessary to simplify the problem. Relaxation provides bounds on the optimal area. That is, it gives a result close to the optimal solution. One of the most common forms of relaxation is Lagrangian Relaxation. It is an approach to reduce the complexity of the problem by decomposing the constraints. The constraints are grouped by “the easy” and “the hard”. The hard ones are removed and transferred into the objective function. Then, easy constraints can be solved. As a result, a sub-optimal solution could be reached. This approach is used in IP. The last type of mathematical modeling is mixed-integer programming (MIP), where some decision variables are defined as linear, and the others are introduced as integer.

In production scheduling, optimization of large-scale mines especially with the constraints of blending and non-linear geo-metallurgical interactions in the processing of the refined bulk material can be comparatively tough in terms of modeling and computation. On this basis, Kawahata et al. (2016) proposed a multi-period MILP model to optimize large-scale open pits and underground mines with the process of blending in a stockpile, mining rates, and mill blending constraints. The model was applied for Newmont’s Twin Creeks mine (Figure 2.1). Although Newmont Company has two open-pit mines and one underground mine in the area, there are six different sources to feed the processing plant. Considering the high variability in the destination, production scheduling optimization was applied in the mine to sustain the amount of feed to the processing plant. In the basis, mining and processing capacities, processing blending, stockpile inventory balance at each time period, mining sequence, and some other mining constraints were included in the model. By applying this model for the operations in Twin Creeks of Newmont, the mine has increased the NPV by $120M and reduced the operating cost by $50M in three years (Kawahata et al. 2016).

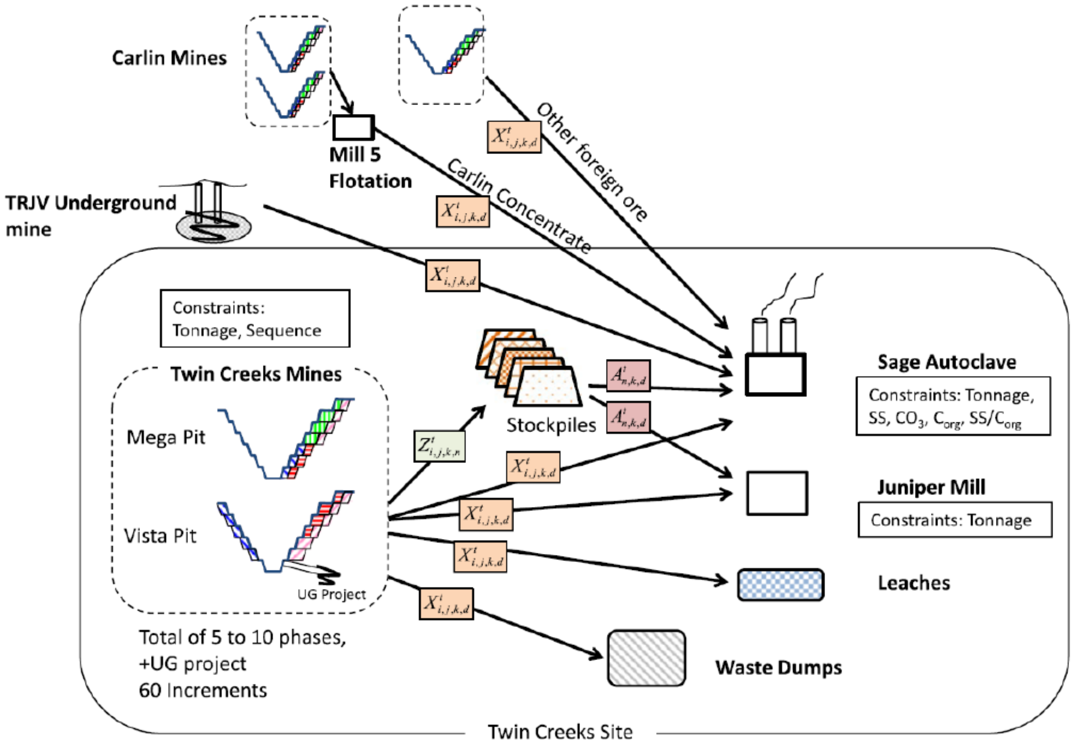


Figure 2.1 Twin Creeks optimization model (Kawahata et al., 2016)

In mathematical optimization, the heuristic approach is designed for solving problems more quickly than conventional methods. Also, it is hard to reach an optimal solution, an approximate result that is close to the exact solution can be found with the heuristic approach. The majority of metaheuristic algorithms are developed through studying natural events. Complex issues with considerably large dimensions may be solved optimally by metaheuristic algorithms. Where precise and analytical techniques may not be able to create improved answers within reasonable computing time, metaheuristic optimization algorithms have shown their efficacy in discovering near-optimal solutions (Glover & Kochenberger, 2006; Yang, 2010). Because the key characteristic of metaheuristics is that it is not issue-specific, any optimization problem may be handled with simplicity.

Tabu search (TS) is a type of a metaheuristic algorithm that checks its neighbors to find a better solution. It is based on the memory structure. Random initialization and solutions kick off the algorithm. A new solution is found in each iteration by making local movements over the current solution. The best of all (or a subset of) potential solutions in the neighborhood applying operator is the neighbor solution (Zhou et al., 2013). A genetic algorithm (GA) is a heuristic model and it mimics the process of natural selection. It generates solutions to optimization and search problems. Also, this algorithm has the ability to deal with complex problems and parallelism. GA works with the coding of the parameter set instead of working parameters themselves. Moreover, in this algorithm, probabilistic transition rules are used not deterministic ones (Yang, 2021).

2.2 Optimization of Ultimate Pit Limits

Ore deposits are classified as non-renewable resources located beneath the surface in varying depths. The amount and grade of deposits are crucial for operational feasibility and profitability. Technological growth in the mining industry and the increase in commodity demands and prices enable the economic extraction of low-grade deposits. Some mines may achieve a feasible production in very deep mines accordingly. Open pit mines, a particular type of surface mining, require an attentive extraction planning of ore and waste blocks so that the net present value (NPV) of the project is generally maximized considering net cash flows throughout the mine life. Optimization studies in open pits are classified into two main groups as ultimate pit and production scheduling optimization. This section concentrates on ultimate pit optimization and the related model dynamics.

When Hewlett (1961) formulated an LP problem to determine pit limits and ore reserve of an iron ore deposit, this study became the first LP model applied in mining. A combination of parametric programming, non-linear programming, and LP was also implemented in the study to determine the configuration of the economic limit for pit and mine planning.

The ultimate pit limits also called final or optimum pit limits, set the final pit geometry to determine which waste and ore blocks should be extracted in mining operations. In other words, an ultimate pit limit gives the mine size and shape at the end of its life (Caccetta and Giannini,1990). The ultimate pit limit primarily concerns about the pit slopes. Determination of an ultimate pit limit requires information mainly on block grade distribution, cost and revenue factors, geotechnical parameters, operational and processing restrictions, license area, and surface and underground water sources. Scheduling of extraction is not responded at the ultimate pit solutions. However, it can be concluded that the outcomes of an ultimate pit problem are used as an input to the production scheduling problem. One of the fundamental concerns when constructing an ultimate open-pit limit is that the mine design should be defined attentively to set proper sizing and planning of the mine-related facilities, such as processing plant and to constitute an effective basis for subsequent long-term planning (Johnson, 1969). Lerchs–Grossman (LG) Algorithm (Lerchs and Grossman, 1965) and the floating cone method (Laurich, 1990) are the common ways of optimizing pit design and determining the pit limits by yielding the highest NPV (Newman et al., 2010).

The Lerch-Grossman algorithm has been used for solving the ultimate pit problem. The method is guaranteed to produce an optimal solution. The method estimates the economic block value (EBV), cumulative block value (CBV), and temporary block values (TBV). The first step is dividing the 2-D section of orebody into blocks, B_{ij} [i=1, n;j=1, m]. Then, dummy blocks at the surface with zero values are provided. After that, EBV is estimated and cumulative column values for each block from top to bottom are computed. Cumulative column figure as the second value in the respective block is designated (Figure 2.2).

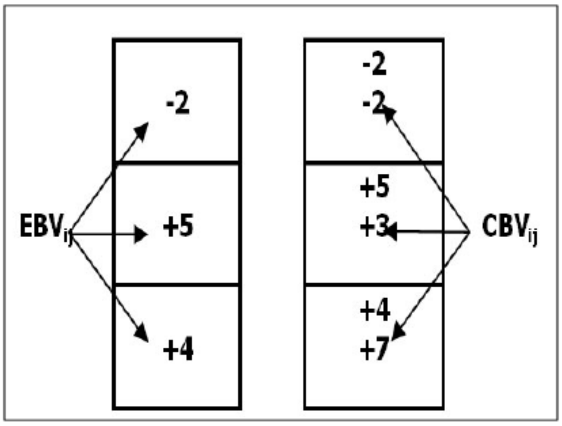


Figure 2.2 Illustration of calculating cumulative block value (CBV)

The equation of the CBV will be:

$$\mathrm{CBV_{ij}} = \sum_{t=0}^{i} \mathrm{EBV_{ij}}$$

The equation for the computing TBV_{ij} is as follows:

$$\mathrm{TBV_{ij}} = \mathrm{CBV_{ij}} + \max \begin{cases} \mathrm{TBV_{i-1,j-1}} \\ \mathrm{TBV_{i,j-1}} \\ \mathrm{TBV_{i+1,j-1}} \end{cases}$$

After designating TBV_{ij} for all the blocks column by column, the maximum value at the surface, P_{0j}, becomes the total value of the pit. The path followed in backtracking from this maximum P_{0j} is the optimal pit contour.

One of the most common intelligent algorithms is the moving (floating) cone algorithm. The cone is floated from left to right along the top row of blocks in the section. If there is a positive block, it is removed. After moving to the second row, it is started from the left and searched for the first positive block. If the sum of all blocks falling within the cone is positive, the blocks are mined. The floating cone process moving from left to right and top to bottom of the section is followed until no more blocks can be removed. Then the second iteration is processed. If during a given iteration no positive blocks can be mined, the profitability of the mined area can be found by adding the values of the blocks that are to be removed. Moreover, the overall stripping ratio can be determined by dividing the number of positive blocks by the total number of negative blocks. Basic description of the moving cone algorithm is shown in Figure 2.3.

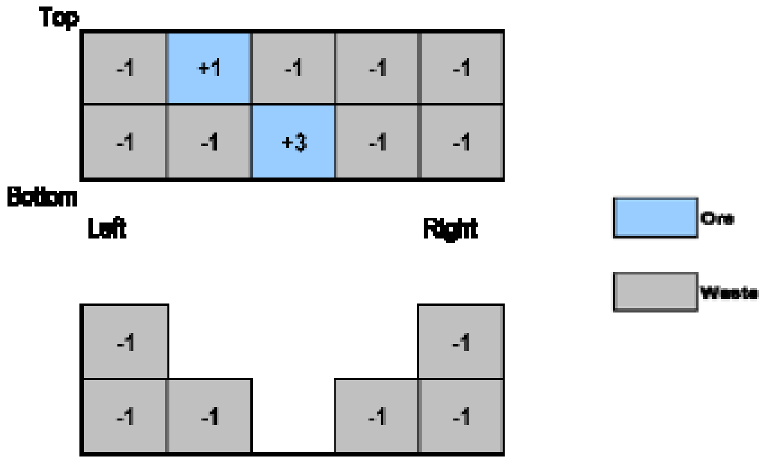


Figure 2.3 Moving cone basic procedure

The LG algorithm was modified by Zhao and Kim (1992) by expanding the model size so as to solve the problem for more mining blocks. The pit was represented by a directed graph where the arcs are generated between the positive (ore) and negative (waste) nodes (blocks) to show the sequencing relationships. The positive nodes were linked to negative nodes to achieve geotechnical stability. If an ore block cannot support the waste block, then an arc is guided as partial support from the waste block to the ore block. Yegulalp and Aries (1992) applied the excess-scaling algorithm of Ahuja and Orlin (1989) to solve the ultimate pit limit problem as a maximum flow problem. It was stated in the study that the LG algorithm in the Whittle software application has the ability to solve an ultimate pit limit problem in a shorter time, compared to the excess-scaling algorithm. In brief, most commercial and non-commercial solutions still use the LG algorithm when determining the ultimate limits of surface mines. The next stage of optimization is creating smaller pits within the ultimate pit limit that are called nested pit shells. These pit shells generate larger units known as phases or pushbacks (Figure 2.4) by grouping them (Whittle, 1988). It means that the latest pit shell in the order of occurrence points to the ultimate pit shell.

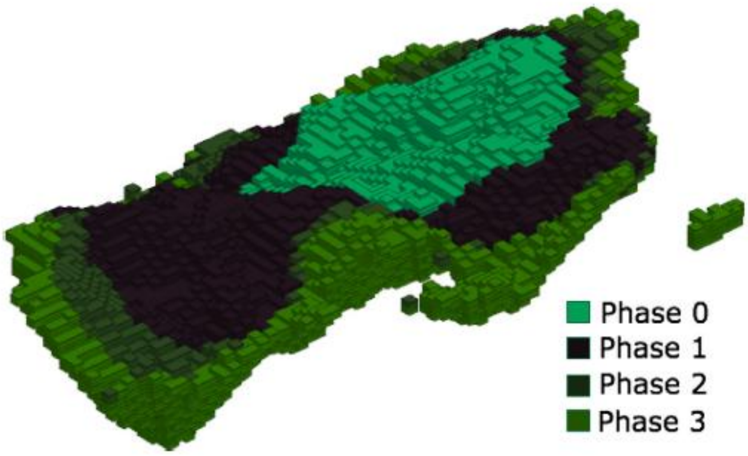


Figure 2.4 An example of a block model divided by pit shells (Rezakhah et. al.,2020)

Section 2.3 will detail how production scheduling within ultimate pit limits can be performed including long- and short-term scheduling factors.

2.3 Optimization of Open-Pit Mine Production Scheduling

In open pit mines, production scheduling is the sequencing and timing of waste and ore block extraction. This process directly affects the profitability and net cash flow of the operations throughout the mine life. Due to the large amount of data needed and the large number of constraints that have to be taken into account, developing a proper production schedule is generally challenging. Therefore, multiple software solutions specific to mining activities are available to offer fast and accurate production scheduling to increase the NPV of operations. These types of solutions embody various sub-modules, which intend to develop ultimate pit limits and mining pushbacks that are mentioned in Section 2.2, to construct an adequate and applicable basis for production scheduling particular to the mining area. The pushbacks develop boundaries for the scheduling operation.

In this sense, once the block grades and the design factors are available, then the ultimate pit geometry is determined and subdivided into multiple smaller shells called pushbacks. Pushbacks, also called phases, specify the extension shape and times of the pit sequentially up the ultimate pit geometry. The number of pushbacks is determined considering the ultimate pit size in lateral and axial dimensions and ore formation specifications. The first excavation area is called the initial pit and extended in three dimensions up to its limits, and then a new phase is started. Each sequential pushback generally covers the previously excavated area and the mining excavation that will be performed in the new phase boundary. The last pushback limits point to develop the ultimate pit geometry. As mentioned earlier, each subsequent pushback points to a boundary, i.e. three-dimensional shell, where the ore and waste blocks within this shell should be extracted in a pre-scheduled period before passing to the other pushback. Therefore, it can be referred that pushbacks determine the blocks to be evaluated under production scheduling so that the sequencing and timing of extracting these individual blocks can be achieved. At that point, when evaluating production scheduling in each pushback, various factors such as processing capacities, processing types, the convenience of block content for processing type, blending factors, mining and milling cut-off values, financial aspects, stockpile conditions, the operability of excavation, sequencing, and geotechnical parameters need to be appropriately considered to decide on timing and destination of each block so as to maximize the net present value (NPV) amount.

Johnson (1969) proposed a model to solve the mine production scheduling problem. The model decomposed the problem into a master problem and subproblems by using a technique called the Dantzig-Wolfe decomposition. The master problem covered linear programming of the blending problem such that total net income was maximized considering event occurrence possibilities for time t and satisfying constraint functions. The master problem solution ensures optimality between consecutive mine plans, while the subproblem uses the master problem's guide to create the mine plans for each cycle. It was observed in the study that available technology in those days created difficulties in the computational solution of the method offered. Chanda (2018) introduced a system LP model to improve production planning of underground and open pit mines by considering the associated metallurgical and processing facilities. The model aimed to minimize production and distribution costs for material flowing from the mines to the market. Although the method is efficient to model the material flow through the network, this technique uses the mining production results as an input parameter. It does not optimize the production sequencing and timing of mining blocks.

Ramazan and Dimitrakopoulos (2004) stated that MIP is insufficient in generating applicable optimal solutions for mining schedules and dealing with the in-situ variable orebodies. This study applied two models that optimize the production schedule for multi-element deposits in open-pit mines. The first application of traditional MIP verified its weakness, while an alternative MIP formulation was then applied to the same deposit. The results showed that this alternative formulation overcomes the disadvantage of MIP and generate more applicable and practical mining schedules. Boland et al. (2009) aimed to solve the open pit mining production scheduling problem by formulating a MIP model. It was mentioned in the study that block numbers in large mines should be decreased by increasing block size since a large number of blocks and precedence constraints linking them may cause a computational issue. At this point, small mining units enable selective mining but are difficult to be solved. An iterative disaggregation method that refines aggregates was proposed in the study. The refined aggregates method is defended to solve a very large problem in a reasonable time. A life-of-mine optimizer called Bodor, which is particular to deposit information, was developed for BHP's Boddington Bauxite mine in Western Australia (Zuckerberg et al., 2011). The study used MIP formulation to minimize net present costs, achieve mixing goals, comply with a complex collection of environmental and operational constraints. This is achieved by optimizing the extraction sequence of the bauxitepod, the size and usage of the fleet, and the facilities of the crusher and conveyor. Chicoisne et al. (2012) presented a new decomposition method to solve the LP relaxation of the C-PIT model (Johnson, 1969) with a single capacity constraint per period of time. This algorithm is based on manipulating the problem structure of the priority constrained knapsack and runs in O(mn log n) in which n is the number of blocks and m is a function of the relationships of precedence in the mine.

Production scheduling is performed in strategical, tactical, and operational levels, which points to production planning in short-, medium- and long-term horizons. At this point, there are various methods and mathematical approaches that are capable of solving the mine production scheduling problems within certain limits and optimality gaps. It is observed that one of the most challenging parts of the models is capacity constraint since an enormous number of constraints should be solved simultaneously to optimize the extraction schedules such that this number increases remarkably with the increase in the period numbers. Integer models intending to find out exact solutions and mixed-integer models capable of solving models with near-optimal approaches are frequently used in the literature to solve the problems. Heuristic techniques are used more commonly than the exact solutions to overcome the models' capacity constraints (Chicoisne et al., 2012).

For production scheduling optimization in mining, some basic algorithms are used. One of these algorithms is network-flow-based. It represents the problem as a graphical or network (Figure 2.5). This algorithm aims to create a flow with numerical values on each edge that follow capacity limits and that have incoming and outgoing flow equal at all vertices for certain chosen nodes. Network flow problems can be discussed as the maximum flow problem and the minimum cost circulation problem (Goldberg et al., 1989).

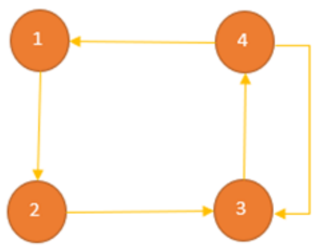


Figure 2.5 Example of a network

An other algorithm is branch-and-bound. It is formed for discrete and combinational optimization problems. In this algorithm, branches of the tree represent subsets of the solution. The tree is created with all possible solutions. When reaching the possible solution of a branch, the branch is checked upper and lower estimated bounds on the optimal solution. If there is no solution better than found, it means the final optimal solution. This method also solves the problem by LP relaxation of IP. If it is assumed that all decision variables are integer in the optimal solution to the LP relaxation, then the solution is the optimal solution for IP (Morrison et al., 2016). Figure 2.6 shows the algorithm tree.

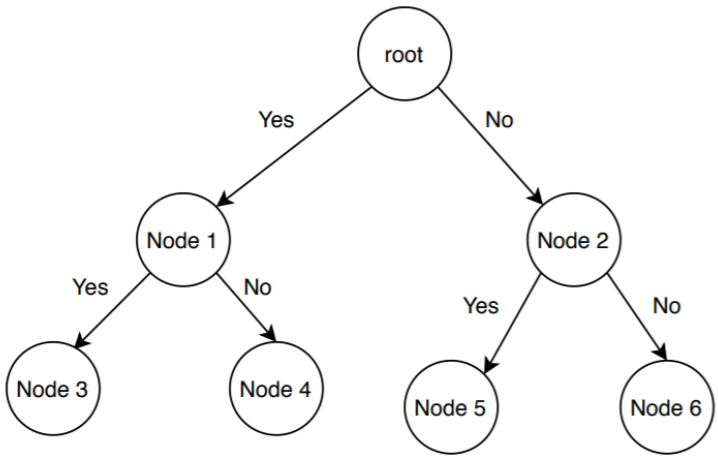


Figure 2.6 Representation of branch-and-bound algorithm tree (Datta, 2020)

Particle swarm optimization (PSO) performs iterations on the candidate solutions to optimize the problem. It is a computational method. It solves a problem by generating a population of possible solutions, which are referred to as particles, and moving them around in the search space using a simple mathematical formula based on their position and velocity. The movement of each particle is controlled by its local best-known position, but it is also directed toward the best-known positions in the search space, which are updated when better places are discovered by other particles (Yang, 2021). In Figure 2.7 flowchart of the algorithm is shown.

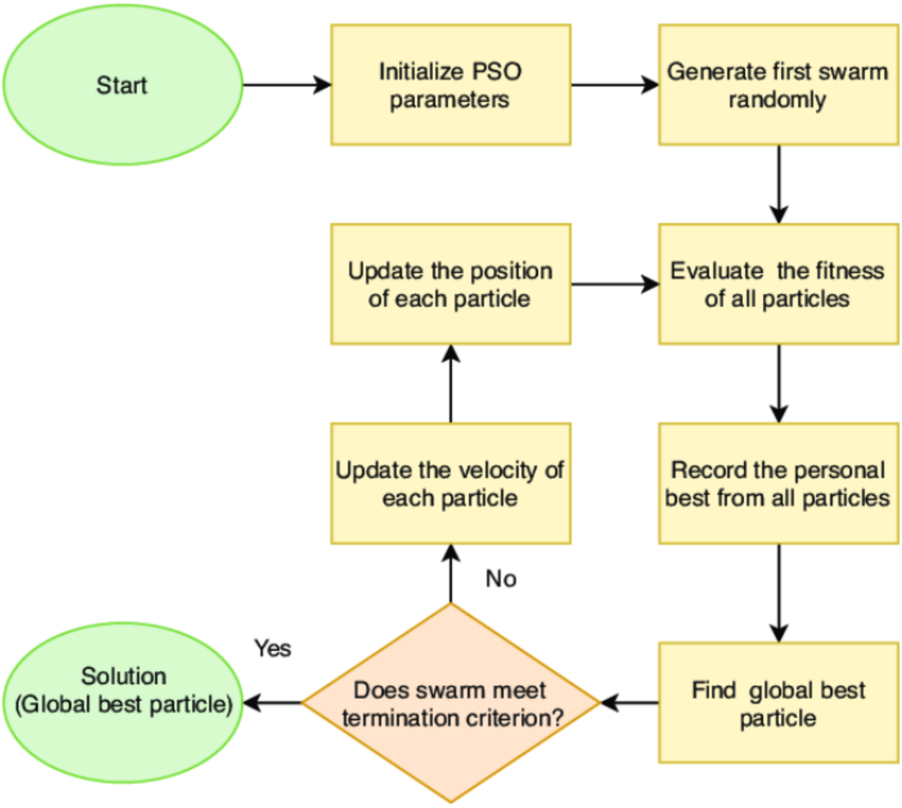


Figure 2.7 Flowchart of PSO algorithm (Masaracchia et al., 2019)

Sections 2.3.1 and 2.3.2 introduce the details of long-term and short-term production scheduling models and the related studies conducted in the literature.

2.3.1 Long-Term (Strategical) Production Scheduling

Long-term production planning is a crucial step in mining plans that should be performed attentively to achieve an increased NPV by regarding the net cash flows throughout the mine life. Long-term planning is also called life-of-mine (LOM) planning or strategical planning and determines the limit of mining advances. This type of scheduling is carried out before the start-up of the mining operations and during the ongoing operations, mainly for budgeting purposes by revealing expected revenue and operating cost flows in the following years. The time interval value of long-term scheduling and its determinants are affected by technological constraints, economic constraints, equipment selection, and scheduling, ore-waste balance, risk appetite and production policy of the corporation, and precedence jobs. The annual mining sequence and which blocks should be extracted in the strategical planning step under the geological and resource constraints (L'Heureux et al., 2013).

Periodic intervals in long-term plans are generally considered on an annual basis. Therefore, the sequence of extracting ore and waste materials is determined with yearly increments. Cash flow estimation details, equipment requirements, equipment procurement schedules, staff requirements over time, and environmental impact assessments allow the construction of long-term plans more realistically (Johnson, 1968). A large-scale optimization problem involving large sets of data, multiple constraints, and uncertainty in the inputs that need to be solved in a reasonable period, is a long-term production scheduling issue (LTPSP). Although there is a great effort to solve the LTPSP, it has not yet been well resolved (Tolouei et al., 2020). Some recent studies on the long-term scheduling of open-pit mines are discussed in this section.

Caccetta and Hill (2003) discussed the production scheduling techniques and their limitations and offered a new branch and cut algorithm. This algorithm was developed as a MILP model and was implemented in the C++ environment. The objective function of the model was set to be maximizing NPV. Mill throughput, the volume of the material extracted per period, blending constraints, stockpile-related limitations, and logistic limitations are considered as the model constraints. The model results were compared with the results in MineMac software for the same inputs. It was defended in the study that the model increased the NPV value by 13.1%, compared to the software results.

Ramazan and Dimitrakopoulos (2008) proposed a two-stage stochastic integer programming (SIP) approach to handle the long-term mine production scheduling problem with the constraint of uncertain geology. The annual tonnage and grade variations of multiple scenarios are considered in the recourse decisions. A solution that optimizes expected NPV is determined while decreasing scenario-dependent production deviations, resulting in a more profitable and risk-resistant schedule. This SIP model is stated to be used with commercial optimization software, which is helpful because it shows how close the solution is to the theoretical optimum. Moreover, Ramazan and Dimitrakopoulos (2013) used their two-stage SIP in a gold deposit and the result shows a 10% increase in the projected NPV compared to the deterministic design.

Askari-Nasab et al. (2011) aimed to develop, implement, and verify deterministic MILP formulations for long-term and large-scale open pit production scheduling problems. In the study, the problem is tried to be solved in a deterministic structure. The model intends to maximize the NPV while satisfying the grade blending, mining and milling capacities, and extraction sequence. In this respect, four different MILP models were proposed in the study where two of them were stated to be the modified models, and the other two were new MILP models. The developed models were solved in TOMLAB/CPLEX. The models were compared in terms of NPV, practical mining production constraints (overall pit slopes, mining extraction sequence, capacities, blending constraints, and minimum mining width), size of mathematical programming formulations, the number of required integer variables, and the computational time. The research findings were also highlighted with a case in an iron ore mine. The formulation of Model 01 is similar to the model developed by Ramazan and Dimitrakopoulos (2004) except for the varying slope angles. In this model, only a single stage period was considered for processing an ore block. For Model 02, the formulation is based on the model by Caccetta and Hill (2003) where some other decision variables dealing with extraction and processing at the block level and controlling the precedence of extraction are included additionally. The study of Boland et al. (2009) is considered in formulating Model 03. Finally, Model 04 is developed by integration of Model 02 and Model 03, which means that extraction sequence, mining, and processing are controlled at the mining cut level. The results showed that Model 02 could not be used practically in long-term scheduling since it causes an out-of-memory problem just after the computation. In addition, it was seen that Model 02 and Model 03 could be used in short-term scheduling efficiently. On the other hand, Model 04 was defended to ensure all the constraints mentioned above while maximizing NPV in a very effective manner.

Vallejo and Dimitrakopoulos (2018) studied an application of long-term stochastic planning for strategic risk management at the Quebec, Canada for KéMag iron ore deposit. The study first quantified both the volumetric and multi-element grade uncertainty of the deposit by producing a range of orebody scenarios equally possible. In the case study, a pattern-based wavelet simulation algorithm set the limits of the lithologies (volumetric uncertainty) describing the iron ore deposit. Using the direct block minimum/maximum autocorrelation factors, the related grade properties, namely, head iron, Davis Tube weight recovery, Davis Tube concentrate iron, and silica content (multi-element grade uncertainty) were jointly simulated. Davis tube is an instrument for separating small samples of strongly magnetic ores into strongly magnetic and weakly magnetic fractions. In order to control and reduce the risk related to the geological volatility of the deposit while meeting production targets, the developed stochastic optimization model was used by building up a mining sequence of extraction that maximizes the net present value. The case study results are observed to measure the available risks related to the overall output of iron, and the estimated discounted annual cash flows.

Senécal and Dimitrakopoulos (2019) introduced a new two-stage mathematical model on mine production scheduling with several processing types regarding the uncertainty of mineral availability. The destination of each block is introduced as a model variable. Besides, this method seeks to change the mining block extraction cycles, destinations, and periods. The model objective function maximizes the NPV where some penalties are applied for the deviations from output goals. A parallel multi-neighborhood Tabu search metaheuristic was used for optimizing the objective function. It is implemented using memory structures defining the visited solutions or user-provided sets of rules. Near-optimal solutions were achieved in the study, and it was observed that parallelism could simultaneously increase the project value. The model was applied for a gold deposit and solved in a reasonable time. Besides, the application results' risk profiles were revealed to evaluate the method's ability to meet the production targets.

The research by Saliba and Dimitrakopoulos (2019) represents an application of a stochastic optimization application for mining, destination, and processing decisions of a multi-pit and multi-processor gold mining complex. Also, the simultaneous stochastic optimization model developed by Goodfellow and Dimitrakopoulos (2016) is adapted to this gold mining complex. The model objective function was set to maximize the sales value of the commodity while minimizing the deviations from the production target. The constraints such as capacity, reserve, slope, destination policy, and processing streamflow are included in to the model. Processing material is conveyed from two separate pits. High-grade oxide ore is extracted from Pit B while sulfide ore is extracted from Pit A. The sulfide ore requires a preliminary pressure oxidation process. Then, both the treated sulfide material and the oxide material are fed to the autoclave, which is a horizontal cylindrical pressure vessel. Blending amounts of the material affect the performance of autoclaves, directly. Therefore, some geochemical constraints were also introduced to the model. The flow of the materials between the destinations can be seen in Figure 2.8. The model is stated to improve the sales rate by optimizing the cut-off grade and satisfying the processing requirement. It is also mentioned in the study that the model has the capability to generate effective long-term scheduling with less complexity.

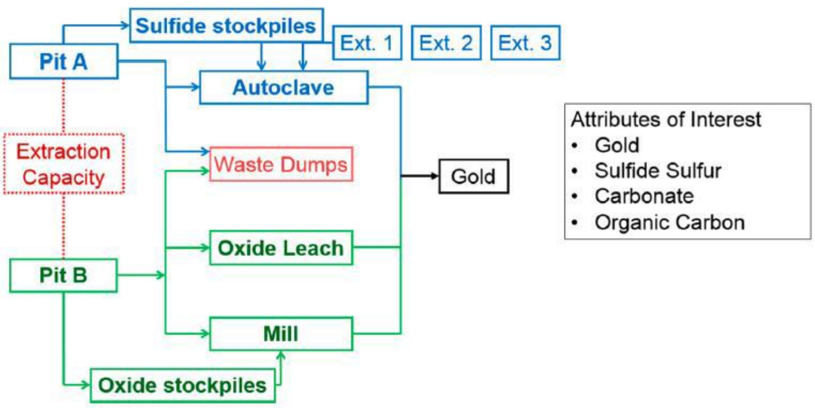


Figure 2.8 Flow of the ore material in the mine (Saliba & Dimitrakopoulos, 2019)

In the research by Tolouei et al. (2020a), some hybrid models considering grade uncertainty were offered by integrating the Lagrangian relaxation (LR) approach with meta-heuristic techniques, bat algorithm, and particle swarm optimization to solve LTPSP. It is concluded in the study that the LR-Bat algorithm can be an effective alternative to optimize long-term production plants so that NPV and average ore grade in the application could be estimated for the computational time over a 12-year production cycle. A near-optimal solution with a rational time was achieved in the study. The flowchart of the proposed approach is shown in Figure 2.9.

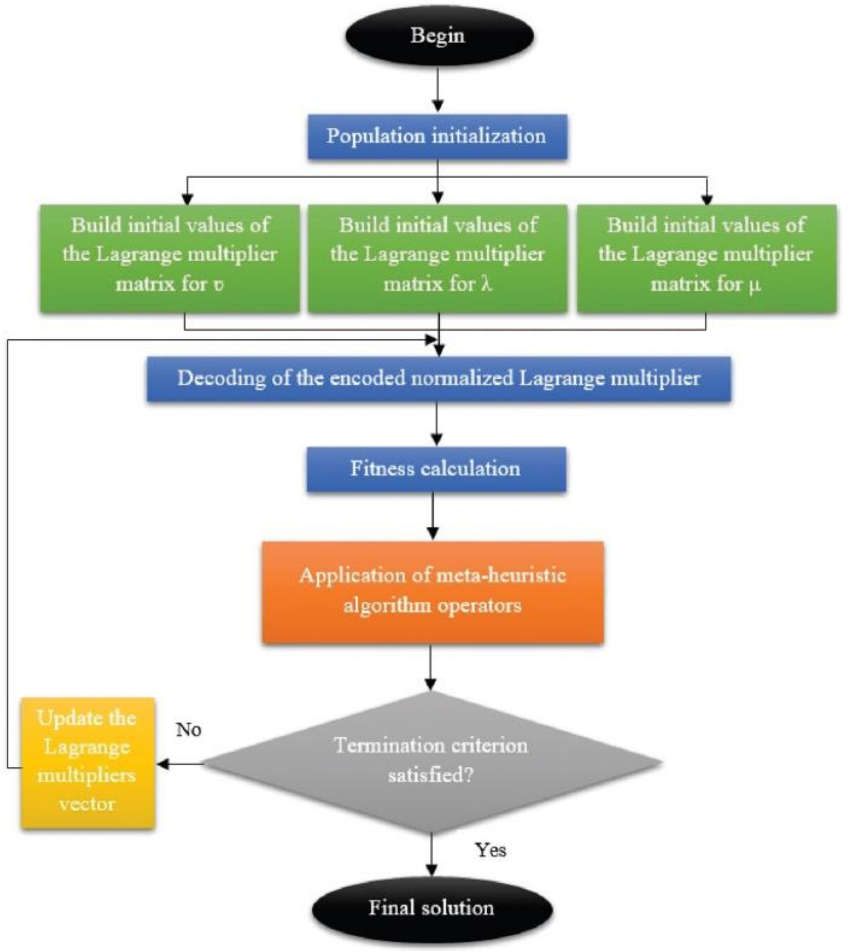


Figure 2.9 Flowchart of the proposed approach (Tolouei et al., 2020a)

Tolouei et al. (2020b) introduced hybrid models with metaheuristic methods, firefly algorithm (FA), and bat algorithm to elucidate the LTPS problem concerning grade uncertainty by using LR and augmented Lagrangian relaxation (ALR). The model's objective function was specified to maximize profitability. The model constraints are grade blending ratio, reserve amount, processing capacity, mining capacity, and pit slope. The results showed that the ALR-FA model offers some advantages over the traditional models with a near-optimal solution in a reasonable solution time.

Lotfian et al. (2021) built up a new method based on block clustering. Since there are thousands or millions of blocks in the real block models, there may be some computational restrictions in solving the models. Therefore, this study tries to reduce the model size by clustering the blocks. Blocks are first grouped into clusters called mother clusters with the clustering principles in mathematical models and solved using GA. In the second part, the mother clusters are converted to mining clusters considering the practicality problems. The implementation results on a real deposit showed that the number of binary variables could be reduced from 476,100 to 5,652 by block clustering. A computational time reduction from 372 hours to 23.4 seconds was achieved compared to the original block model.

Gilani et al. (2020) studied a stochastic particle swarm-based model with geological uncertainty. In this respect, as inputs, a set of equiprobable orebody scenarios and two new block models called risk block model and EType were derived. EType block model is created by averaging the metal grade of orebody simulations. A stochastic integer programming (SIP) model was then developed to incorporate the geological uncertainty. Finally, a PSO-based algorithm was developed to solve the SIP model. Figure 2.10 shows the framework to solve long-term production planning using the PSO algorithm. According to the population topology and how to use the risk block model, four different strategies were developed. The topology of the population determines the subset of particles. The proposed model was implemented at the Sungun copper mine in Iran. The application created a unique plan, including geological uncertainties by maximining NPV and minimizing deviations from production targets.

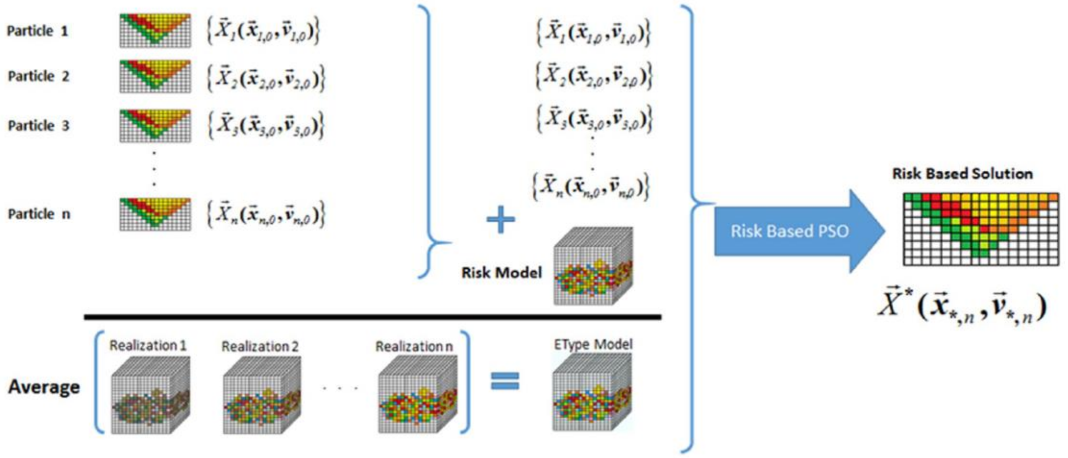


Figure 2.10 The framework of solving LTPP using an uncertainty-based PSO algorithm (Gilani et al., 2020)

2.3.2 Short-Term (Operational) Production Scheduling

Detailed design of a mine is one of the challenges encountered in the operation areas and short-term production planning is applied in micro-scale time intervals, compared to the long-term production planning, to obtain block extraction decisions with “when and which block” inquiries. The detailed design deals with i) working equipment efficiently, ii) ensuring the production advance, iii) satisfying the bench slope angles, iv) providing haul road access to working benches, and finally, v) maintaining the ore blend desirable from the plant. The production limit is previously determined by long-term production scheduling. In brief, short-term scheduling makes long-term plans operationally applicable. Therefore, the main purpose of short-term scheduling is the planning of waste and ore extraction and grade control (Smith, 1998).

Short-term scheduling intends to develop daily, weekly, or monthly production plans and generally retrieve input data from the long-term plans. Shift-to-shift or day-to-day decisions on which block patterns to be blasted and excavated can be generated according to the geological sequence and the weekly or daily production requirements (Blom et al., 2018). Short-term planning is performed at an operational level and divides orebody into small blocks with the estimated grades. Some particular restrictions such as precedence relationships between the blocks for geotechnical concerns, precedence relationships between the operational activities of drilling, blasting, transport, processing capacities, and equipment availability and locations are included in short-term production scheduling (L'Heureux et al., 2013). Therefore, data variability and requirement are higher than the long-term planning, and it may need some constraints like grade control values that are not included in long-term models. Various research studies have been carried out to improve short-term production scheduling decisions.

Kumral and Dowd (2002) proposed multi-objective simulated annealing (MSSO) metaheuristic to optimize the short-term mine production scheduling problem of an iron deposit. Using a Lagrangian structured model and enhanced using MSSO, a sub-optimal solution is produced by MSSO. The objective of the problem is set to satisfy the total ore tonnage mined and reduce grade variations for the elements. The perturbation acceptance criterion in this MSSO technique is not just a binary choice since the proposed solution may enhance one objective while deteriorating another. The research stated to over 250 solutions in a short time interval and allow manual selection of the best five scenarios. The following are mentioned to be the limitations of the model: i) subjectivity in the decisions, ii) failure to obtain a single optimal production schedule and iii) unavailability of some short-term production constraints such as equipment access and relocation.

Vargas et al. (2008) aimed to produce a production scheduling tool that can generate multiple decision scenarios in a reasonable time. The model concentrated on the blending constraints related to the ore's geo-metallurgical characteristics and stock dynamics, geotechnical constraints, and plant and mine capacities. The developed tool was applied to a real mine of BHP Billiton Spence in Chile. The results were integrated with the probability maps of individual block's extraction duration, graphical analysis of the output reliability, selectivity and functionality of mine design, and data clustering size.

Eivazy and Askari-Nasab (2012) developed and tested a MILP model to optimize short-term open pit mine production scheduling. The objection function of the model is minimizing the overall cost of mining, processing, haulage, rehandling, and rehabilitation costs. The branch-and-cut method is used in the algorithm, and it is solved by using TOMLAB/CPLEX optimizer. The model is constrained by various factors such as buffer and blending stockpiles, mining in a horizontal direction, and geotechnical parameters. In addition, a hypothetical dataset and a real dataset from an iron mine in Iran are used to validate the capability of the model. In this application, different destinations that connect the pit, process plants, stockpiles, and waste dumps are considered (Figure 2.11). Also, there are three different directional mining scenarios where the model is implemented i) without imposing any horizontal direction mining, ii) with imposing northwest to southeast-northwest horizontal extraction, and iii) with imposing southeast to northwest horizontal extraction. The optimal solution in the study is determined by comparing all the achieved scenarios.

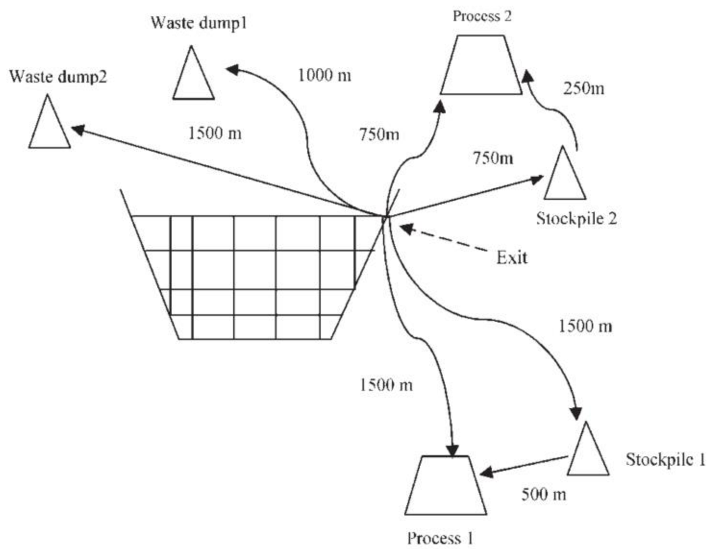


Figure 2.11 Ore and waste block paths in the algorithm (Eivazy & Askari-Nasab, 2012)

Mousavi et al. (2016) studied a comprehensive mathematical model for a short-term block sequencing problem in an open-pit mine. The purpose of the model is to achieve the optimum extraction sequence of real size blocks with the constraints of precedence relationship, capacity, grade, processing, and stockpile in a short time interval. The method offers a hybrid computation of branch-and-bound and simulated annealing algorithms when solving the problem.

Blom et al. (2017) proposed an algorithm to generate multiple short-term production schedules of an open-pit mine with multiple objectives. It was mentioned in the study that the traditional short-term planning forces a planner to run heuristic scenarios several times to construct a single schedule where loading/excavating equipment needs to be used in proper compliance with the grade and tonnage requirements of production. Therefore, multiple iterations are required to be performed to realize the final scenario. This situation necessitates the professional intuition and experience of a planner for assessing the consistency and mineability of the production schedules. Generating multiple schedules to detect extractable ore and waste materials in particular sequences are compared manually by short-term planners. The study algorithm was applied to a real mine with an annual mining rate of up to one million ton. It is observed in the results that short-term schedules could be achieved in minutes but need some more time to correlate the parameters for achieving more applicable solutions.

Both and Dimitrakopoulos (2020) aimed to develop a simultaneous optimization model of short-term planning and fleet management. The metaheuristic method was used in the model where the objective function was built to maximize production and profitability. It was then applied to a real mine in a case study. The extracted ore was transformed into multiple process facilities and stockpiles. Uncertainty arising from various aspects was integrated into the mathematical model. When the study results were compared to the conventional method neglecting fleet management, the costs due to shovel movement were detected to be reduced by 56%. Besides, the production loss of shovel relocation was reduced by 54%, whereas a cost reduction of 3.1% was achieved in truck operations.

To sum up, short-term planning is concerned with satisfying processing, mining, and operational limits while maximizing the profit or produced metal amount. Also, it must match with the long-term production schedule. Therefore, Nelis and Morales (2020) proposed an optimization model that is based on representative selective mining units to solve the operational and scheduling problems at the same time. The mining cut form and the production schedule are defined in the model. The model was tested by a real case study. The study concluded that generating mining cuts and extraction sequences satisfy the mining, processing, and operational limits. Mining cuts are mentioned to be useful for short-term planning by setting boundaries.

2.4 Mathematical Modeling of Other Mining Related Problems

Mining problems are not only about ultimate pit or scheduling but also about fuel consumption, dispatching, or maintenance optimization. Yan et al. (2008) studied an integrated model that combines ready mixed concrete which is primarily material for infrastructure works production scheduling and truck dispatching in the same framework. The formulation of the model is a mixed-integer network flow problem. It is an NP-hard problem. In addition, the model is implemented in a real mine that is located in Taiwan. Firstly, the model is run CPLEX MIP, embedded in the simplex method with branch-and-bound. Since achieving the solution takes time, the model is reduced. After that, near-optimal solutions are found with CPLEX within the acceptable time. Figure 2.12 shows the solution flowchart. The solution model can be suitable for large-scale problems.

Mining projects, particularly surface mines, are capital-intensive endeavors with significant running expenses. Haulage and materials handling account for around 50% of operational expenses in open-pit mines, and even 60% in big open-pit mines. Hauling has the greatest operational cost of any material handling operation in open-pit mines. As a result, optimizing operational mining planning and fleet management has a substantial influence on operational efficiency (Afrapoli and Askari-Nasab, 2017).

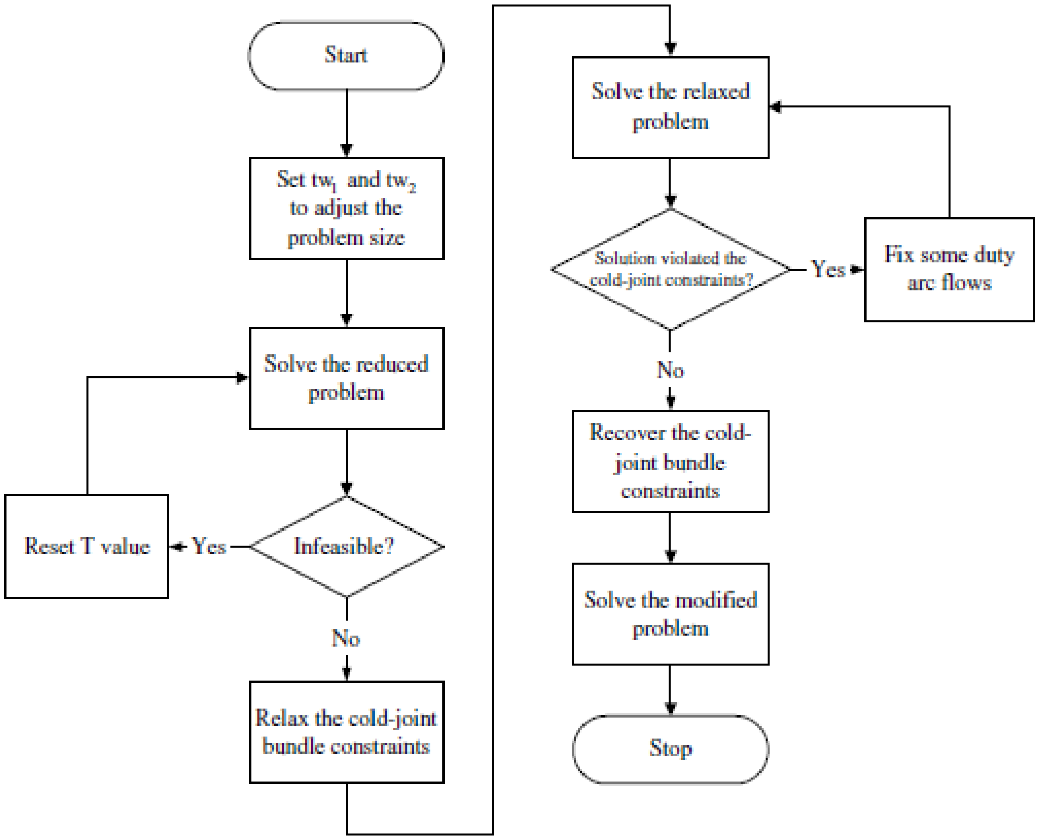


Figure 2.12 Flowchart of the solution for a mixed-integer network flow problem (Afrapoli and Askari-Nasab, 2017)

Gurgur et al. (2011) developed a LP model for truck allocation that can be used in connection with the existing MIP model for short and long-term mine planning. The model is evaluated and run simultaneously and interactively with a MIP model to solve the truck allocation problem. The suggested truck allocation technique addresses the inadequacies of previous models by accounting for economic characteristics, multiple time periods, and the unpredictability of load and trip times and ore grades. Also, the model is implemented to the McLaughlin gold mine.

2.5 Mathematical Modelling in AMPL Optimizer

AMPL (A Mathematical Programming Language) is an algebraic modeling language developed by Robert Fourer, David Gay, and Brian Kernighan at Bell Laboratories to define and solve high-complexity problems in large-scale mathematical computing (i.e., large-scale optimization and scheduling-type problems). Open source and commercial software solvers, including CBC, CPLEX, FortMP, Gurobi, MINOS, IPOPT, SNOPT, KNITRO, and LGO, are enabled by AMPL. Mathematical models can be transferred from files to solvers. According to NEOS Server, the most commonly used mathematical modeling language is AMPL (Figure 2.13).

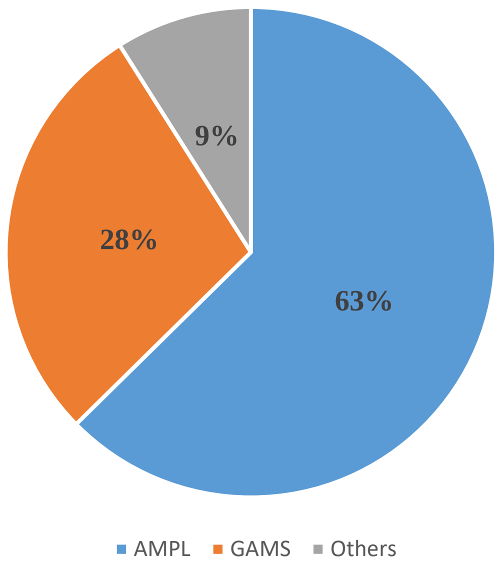


Figure 2.13 Mathematical modeling language distribution (NEOS Statistics)

There are some basic files that are .mod, .dat, and .run in AMPL. The .mod file includes variables, objective function, constraints, sets and parameters. The .dat file defines the data for the model while the .run file defines the variable configurations and uses calculating the objective function. Set is a term in AMPL that is used to define collections of items. These components, which might be numerical or non-numerical, are commonly employed as indexes. Parameters are often crucial in a mathematical model. The majority of model analyses are centered on parameters. Parameters are stated as param in AMPL. It is used to keep track of constant values. Variables are the main elements that must be solved in a mathematical model. The keyword var represents variables in the AMPL.

A considerable effort to formulate the model and generate the necessary computational data structures must be performed before optimizing any routine. AMPL is a modern language designed to facilitate this process with a user-friendly interface having a comprehensive debugging tool. The symbolic algebraic notations used by many modelers to describe mathematical programs are closely similar to AMPL. However, the generality of syntaxes and the variety of its indexing operations can be achieved practically in the AMPL environment. A translator is embedded to take a linear AMPL model and associated data as input and produces output acceptable for regular linear optimizers of programming (Fourer et al., 1990). One benefit of the algebraic approach used in AMPL is its familiarity. The researchers involved in large-scale programming may construct their model with well-known syntaxes, which can be converted easily from the manual formulation of a model (Fourer et al., 1990).

In brief, the purpose of AMPL is to make model development by utilizing an algebraic language that allows for the creation of understandable, efficient, and simply debugged and updated models. AMPL converts the model into a format that the optimization software can understand, as well as providing display capabilities for presenting solution outcomes. An optimization algorithm is chosen that is appropriate for the problem at hand (Smith, 1998).

Moreover, there are some studies that used AMPL solver. Ore quality generally varies by depth, seam, and distance along the strike. Phosphorus recovery is very sensitive to these changes. Therefore, to minimize that effect, feeding ore to the plant has to be arranged. It can happen by blending mine production material and stockpiled material. The model not only optimizes the short-term production scheduling but also includes the stockpile. However, inventory control becomes a non-linear factor in the equation. A partial linear solution is modeled and solved using AMPL/Cplex, and solution methodologies for similar large-scale problems are presented (Smith, 1999).

Moreno et al. (2017) built up a scheduling model to NPV by including spatial precedence constraints and resource capacities. The stockpile was also considered in the model. They proposed linear-integer and non-linear-integer models to solve the scheduling problem in open-pit mines by comparing the solution quality and tractability. The effect of stockpile parameters into the models was evaluated by both including and excluding stockpiling. The models were coded in AMPL (2014) and solved by using CPLEX (2009). The numerical analysis showed that the proposed models are computationally solvable.

2.6 Summary

It is observed that optimization has great importance in the mining industry. Therefore, researchers focus on mathematical modeling to optimize the problems. Ultimate pit optimization and scheduling optimization are the main objects. The ultimate pit is the base of the scheduling. Also, pit parameters are evaluated at that stage. It provides a limit of the working area. Scheduling optimization can be for long-term or short-term production. Long-term production scheduling describes yearly block extraction by looking from a broad perspective. Detailed studies are in the short-term production scheduling. It may be daily, weekly, or monthly.

CHAPTER 3

DEVELOPMENT OF THE INTEGER PROGRAMMING MODEL

3.1 Introduction

The mining industry is continuously exposed to financial risks due to the uncertainties in reserve amount and content, market price, and the values of required capital and operating cost flows. Therefore, optimization of mining-related activities is vital for efficient and sustainable production. In open-pit mining, mathematical optimization works can be divided into two main parts: ultimate pit and production scheduling optimization. This study concentrates on production scheduling where predetermined pushback (production phase) and ultimate pit design limits will be used as input. The long-term mine production scheduling aims to determine the production block extraction sequencing in a way to maximize NPV or the amount of final product throughout the mine life. In this section, a mathematical optimization model of the open-pit mine production scheduling of multiple pushbacks with the material blending constraint is presented together with a comprehensive discussion on the objective function and other operational and mine design constraints in addition to the blending constraint.

3.2 Model Problem Statement

Mine production scheduling basically decides which ore/waste block should be extracted in which time interval in compliance with the objective function under some particular constraints. Mineral processing recoveries of products and/or by-products in the ore blocks and sales prices of those products in the market can differ considerably and are pretty critical in the production scheduling decisions. At this point, where the sales prices are affected by the commodity's supply and demand rates that can change domestically or globally, mineral processing recovery is generally concerned with the blending aspects of the feeding material. The blending concept is generally related to the chemical composition of the feeding material that needs to be ensured within a pre-specified range to keep the working efficiency of the plant as high as possible. In the availability of multi-products to be recovered, these products may require a specific mixture ratio before feeding the blended material into the plant. If more than one metal is recovered in the same production and beneficiation cycle, these mine types are called poly-metal mines. In that case, ore block production rate and sequence should be maintained attentively due to the requirement of mixing metals in a particular ratio and the limitations in plant capacities. Therefore, the current model considers the blending constraint, and the mine block production sequence problem is solved using integer programming (IP) by the CPLEX solver embedded in the AMPL interface.

3.2.1 The Mathematical Model Definition and Formulation

A typical block model including three-dimensional ore and waste blocks with dimensions varying in x, y, and z directions is shown in Figure 3.1. Block dimensions are determined mainly considering ore mineral composition, achievable production height of excavators, and smallest mining unit (SMU).

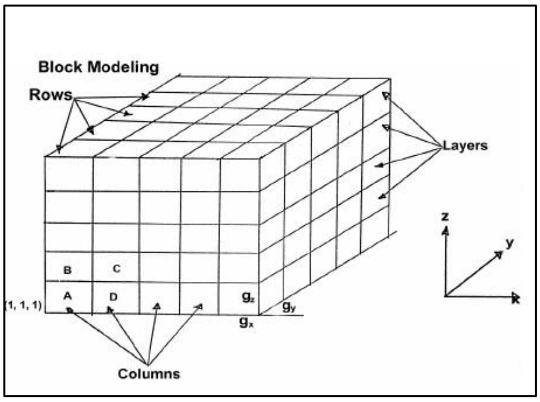


Figure 3.1 Block model identification

Determining waste or ore production blocks for a given period should generally satisfy either of the following objective functions: i) maximizing net present value (NPV) of the investment, or ii) maximizing the amount of final product. Maximizing NPV requires attentive decisions on block selection so that its financial return at present should yield the highest value. In such cases, the model can decide to produce waste blocks for a long time without any positive cash flow if this production decision will bring more benefit in the future regarding all the decisions throughout the mine life. If the amount of final product is required to be maximized, the block selection decisions may slightly differ from the NPV selection criteria. Here, a company may desire to produce the final product as much as possible for a specific period without considering its long-term adverse effects. It is generally observed for the changing conditions where market demands and expectations should be ensured. This production strategy prioritizes the revenue flow timetable more than net cash flow, taking the risk of increasing unit operating costs. In general, long-term production scheduling with NPV maximization is more frequently observed in the strategical plans of mining companies. The current study will consider NPV maximization in the objective function. The decision model will include the blocks in two-dimension for the computational restrictions. The indices to be used in the construction of the model are given in Table 3.1.

Table 3.1 The model indices

Indices	**Definition**
t	Period Index (integer, $t = 1..T$)
i	Level Index (integer, $i = 1..I$)
j	Column Index (integer, $j = 1..J$)
p	Phase Index (integer, $p = 1..P$)
m	Element types (integer, $m = 0..M$)

The considerations included in the model when deciding on the extraction of block $B(i, j)$ are given as follows:

i. Each block is labeled according to its location in level and column and its expected extraction phase and referred $B_{i,j}^{p}$. Level index, i, is associated with the depth of the block. It is valued one at the surface and incremented by 1 up to I, which refers to the maximum depth of the ultimate pit. Column index, j, refers to the order of a block in horizontal. It is labeled as 1 for the surface block at a particular corner of the ultimate pit and incremented by 1 up to J, the last surface block at the opposite corner.

ii. The decision variable, $X_{i,j}^{t}$, determines whether or not $B_{i,j}^{p}$ should be mined at time t where $t = \{1..T\}$. Time index is the extraction period and can be taken as a year, month, week, day, hour, or different scale. Long-term production scheduling is generally considered monthly or yearly, while a weekly or daily period is generally included for short-term production scheduling. It should be remembered that equipment information such as the configuration of haul and excavating equipment, their availabilities and cycle times, and length and condition of haul roads should be considered with up-to-date data in short-term planning. On the other hand, long-term production scheduling generally does not include detailed on-site information.

$$X_{i,j}^{t} = \begin{cases} 1, & \text{if the block is mined in period t} \\ 0, & \text{otherwise} \end{cases}$$

iii. One of the most fundamental constraints in block selection is that each block can be selected only once throughout mine life. Therefore, once a waste or ore block is decided to be extracted, then all the decisions for other blocks should be given accordingly.

iv. The other model constraint is the precedence constraint that refers to which blocks should be excavated previously to enable extraction of a specific block. This constraint relates mainly to the geotechnical limitation in pit slope that should be sustained for mine safety. It determines the blocks $B_{i,j}^{p}$ as a set of blocks required to be mined at a period between $[0..t]$ if $X_{i,j}^{t} = 1$.

Basically, the blocks at the above level, $X^{tt}_{(i-1),jj}$ for tt in $[0..t]$ and jj in $[(j-n)..(j+n)]$, needs to be satisfied for $X^{t}_{i,j} = 1$. Here, the parameter n is the number of the above blocks at only one-side slope requiring extraction for geotechnical concerns. Figure 3.2 illustrates a block production sequence regarding the precedence conditions where the bold line points to the ultimate pit limits.

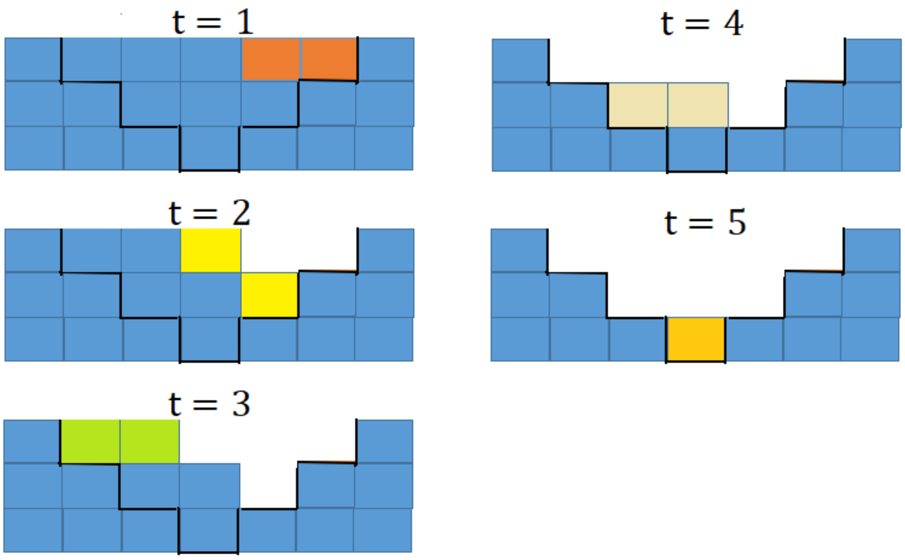


Figure 3.2 Block production precedence conditions

v. Mining production generally does not advance identically same with the horizontal and vertical extensions in the ultimate pit. There can be multiple production phases designed throughout the life of a mine where the last phase determines the ultimate pit limits. A phase design directly affects the direction of ore and waste production sequencing since an extraction hierarchy among a particular number of blocks $\left(B^{p}_{i,j} \text{ within Phase}\{P\}\right)$ for the allowable excavation start $\left(t^{p}_{min}\right)$and ending $\left(t^{p}_{max}\right)$periods of the phase are constructed. The periods for different phases can be overlapped. It means that when producing material within the limits of one phase, an extraction in another phase can be performed simultaneously.

vi. Minimum and maximum-achievable mining or processing capacity values should be considered in the model. Minimum mining capacity is the periodic production target of the company, while the maximum-achievable mining capacity is the maximum earthmoving capacity of the equipment fleet employed in the operation area for the corresponding period. Production targets, production geometry, and selectivity are the key parameters when determining the excavation equipment configuration. Mine design geometry, excavators available in the market with good maintenance and spare parts support, strategical production targets of the company, and the other geological and operational factors affect each other when determining the achievable and available mining production rates. Among the parameters, selectivity may play a critical role in deciding excavator capacity since dilution can be avoided by choosing small capacity excavators, especially for disseminated ore deposits. In these circumstances, the number of excavators can be increased to sustain production. Long-term production scheduling is only interested in minimum and maximum achievable production rates, whereas short-term production planning deals with up-to-date performances of individual excavating equipment. Therefore, the earthmoving equipment fleet's minimum and maximum-achievable production rates for given period intervals are regarded in long-term production decisions.

vii. On the other hand, processing plant capacity generally sets a limitation for mine production since technological and feeding capacity constraints of the plant equipment allow treating an allowable amount of material even though the mining rate, in some periods, achieves a higher capacity. Therefore, ore mining capacity should be slightly higher than the minimum and maximum processing capacities since the plant operation should not be halted due to insufficient feeding material. Minimum processing capacity here points to the target amount of processed material for a given period. On the other hand, maximum processing capacity is the maximum achievable capacity of the processing plant considering their technological, maintenance, and other plant-related limitations.

viii. Mining and processing capacity constraints are generally explained in terms of tonnes. Block weights are determined by block dimensions and ore density values obtained from laboratory results. It is conventional to take representative density values in the long-term planning stage.

ix. For $B_{i,j}^{p}$ including multiple valuable minerals, if extraction of $B_{i,j}^{p}$ is decided for a specific period $\left(X_{i,j}^{t} = 1\right)$, then this block may require an additional evaluation before feeding it to the plant if a particular range of blending should be satisfied by mixing with other ore blocks. If the block fits the plant's feeding range requirements, the block can be fed directly without blending. Otherwise, unmined ore blocks should be evaluated for their convenience of blending with an already excavated block in the given period. This situation enforces the model to apply excavation limitation for the blocks not eligible for blending in the active period (t) even though their economic block values are high.

x. The blending process is generally performed in ore stockpiles close to the primary crusher, the plant-feeding point. Ore stockpiles are used to store the excavated material if the instant capacity of the plant requires no further feeding and/or if the plant requires blending previously. Depending on the composition of ore blocks and the technical specifications of processing plants, a feeding material with compositions in a particular value range can be necessitated for plant efficiency. Therefore, ore stockpiles offer a buffer zone between the plant's mining area. Hence, ore stockpile capacity can also be included as another constraint.

xi. Processing recovery is another parameter that should be evaluated in the model since the processing recoveries of multi-minerals in the same plant can be different. In addition, if there are varying compositions for the same commodity types and/or if there is more than one processing plant, different processing recoveries can be achieved again. For instance, a valuable mineral may be located in oxide and sulfide deposits. Then, they can require different processing techniques such as heap leach and pressure oxidation methods, respectively. Therefore, introducing the recoveries specific to the mineral type itself will help estimate NPV more accurately since final product amounts and unit operating costs will differentiate. Therefore, including processing type and recovery into the model will affect the decisions on block production sequence.

3.2.2 Mathematical Model

This section will discuss the mathematical model regarding the objective function, model constraints, parameters, and variables. Accordingly, the following notations and definitions will be considered for the formulation of the long-term production scheduling problem.

***Indices and Sets*:**

$t \in T$: Time period, $T: 1 \ldots \mathrm{NoPeriod}$, $T \subseteq \mathbb{Z}^+$

$i \in I$: Block Level Index, $I: 1 \ldots \mathrm{MaxRow}$, $I \subseteq \mathbb{Z}^+$

$j \in J$: Block Column Index, $J: 1 \ldots \mathrm{MaxColumn}$, $J \subseteq \mathbb{Z}^+$

$m \in M$: Material Label, $M: 0 \ldots \mathrm{MaterialNo}$, $M \subseteq \mathbb{N}$

$p \in P$: Phase ID, $P: 1 \ldots \mathrm{PhaseNo}$, $P \subseteq \mathbb{Z}^+$

$g \in G$: Blending Groups, $G: 1 \ldots \mathrm{GroupNo}$, $G \subseteq \mathbb{Z}^+$

Blocks^p: Set of all (i, j) value of $B_{i,j}^p$ excavated in phase p

Parameters:

Bmax^{g}	Maximum blending ratio for blending group g
Bmin^{g}	Minimum blending ratio for blending group g
$\mathrm{BV}_{i,j}$	Economic Value of Block (i,j) at $t = 0$
d^{t}	Discount rate for period t
$g_{i,j,m\neq 0}$	Grade Value of Block (i,j) for $m = 1 \dots M$
$g_{i,j,m=0}$	Ore/Waste Identifier of Block (i,j) in $[0,1]$ where $0\!:\mathrm{Waste}$ and $1\!:\mathrm{Ore}$
iMax^{p}	Arrived level i of pit limits after the end of phase p
jMax^{p}	Maximum arrived j value of pit limits after the end of phase p
jMin^{p}	Minimum arrived j value of pit limits after the end of phase p
MCmax^{t}	Maximum Mining Capacity at period t
MCmin^{t}	Minimum Mining Capacity at period t
n	Number of blocks that should be excavated at any level $(i-1)$ for only at one side of $B^{p}_{i,j}$, i.e. $\frac{\text{number of the blocks}-1}{2}$
NB^{p}	Total number of blocks included in phase p
PCmax^{t}	Maximum Processing Capacity at period t
PCmin^{t}	Minimum Processing Capacity at period t

$tMax^{p}$	The latest period to end up the production of phase p
$tMin^{p}$	The earliest period to start the production of phase p
$W_{i,j}$	Weight of Block (i, j)

***Decision Variables*:**

$$X_{i,j}^{t}: \begin{cases} 1, \text{if block } (i,j) \quad \text{at period t is extracted} \\ 0, \text{if block } (i,j) \text{ at period t is not extracted} \end{cases}$$

***Objective Function*:**

$$\text{Maximize NPV} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} BV_{i,j} * X_{i,j}^{t} * (1+d)^{-t} \tag{3.1}$$

***Constraints*:**

C1: Unique Block Extraction throughout the Production Period

$$\sum_{t=1}^{T} X_{i,j}^{t} = 1 \tag{3.2}$$

$$\text{for } \forall i = \{1 \dots \mathbb{Z}^{+}\} \text{ and } \forall j = \{1 \dots \mathbb{Z}^{+}\}$$

C2: Slope Stability and Sequencing According to Phase Designs

$$\sum_{j-n}^{j+n}\sum_{t=1}^{T} X_{(i-1),j}^{t} \geq (2n+1)X_{i,j}^{t} \tag{3.3}$$

$$\text{for } \forall p = \{1 \dots P\}, \forall i = \{2 \dots iMax^{p}\},$$

$$\forall j = \{(jMin^{p}+n) \dots (jMax^{p}-n)\}, \forall t = \{tMin^{p} \dots tMax^{p}\}$$

C3: Phase Production Blocks

$$\sum_{i}\sum_{j}\sum_{t} X_{i,j}^{t} \geq NB^{p} \tag{3.4}$$

$$\text{for } \forall p = \{1 \dots P\}, \forall (i,j) \epsilon Blocks^{p}, \text{and } \forall p \ = \{tMin^{p} \dots tMax^{p}\}$$

C4: Ore Blending Limitations

$$Bmin^{g} \leq \sum_{i=1}^{I}\sum_{j=1}^{J} X_{i,j}^{t}\, W_{i,j} g_{i,j}^{m'} \Bigg/ \sum_{i=1}^{I}\sum_{j=1}^{J} X_{i,j}^{t}\, W_{i,j} g_{i,j}^{m''} \leq Bmax^{g} \tag{3.5}$$

$$\text{for } \forall t = \{1 \dots T\}$$

C5: Mining Capacity

$$MCmin^{t} \leq \sum_{i}^{I}\sum_{j}^{J} X_{i,j}^{t} W_{i,j} \leq MCmax^{t} \tag{3.6}$$

$$\text{for } \forall t = \{1 \dots T\}$$

C6: Processing Capacity

$$PCmin^{t} \leq \sum_{i}^{I}\sum_{j}^{J} X_{i,j}^{t} W_{i,j} g_{i,j}^{(m=0)} \leq PCmax^{t} \tag{3.7}$$

$$\text{for } \forall t = \{1 \dots T\}$$

C7: Decision Variable

$$X_{i,j}^{t} \ \in [0,1] \tag{3.8}$$

A sample illustration of the phase designs is shown in Figure 3.3. The objective function (Equation 3.1) aims to maximize the NPV of the block extraction decisions. $BV_{i,j}$ refers to the economic value of block (i, j) at period $t = 0$ and is calculated considering the operational expenses and revenue of the block. The operational cost of an ore block excavation covers ore mining cost, which includes the expenses of blasting, loading, and hauling of unit material, processing cost, and general and administrative (G&A) cost. On the other hand, the operational cost of a waste block excavation covers only waste mining costs. On the other hand, revenue is a function of the grade of valuable mineral(s) within the ore block (i, j), the market price of the final product(s), processing and mining recoveries, and dilution. In brief, the economic value of any block (i, j) is evaluated its own positive and negative cash flows at period $t = 0$ and inputted into the mathematical model. Depending on the excavation year of the block, which will be decided by the model, the related $BV_{I,j}$ value will be discounted with a rate of $(1 + d)^{-t}$. Therefore, the model is enforced to mine the blocks having higher block values with priority as well as satisfying the limitations highlighted by the constraints.

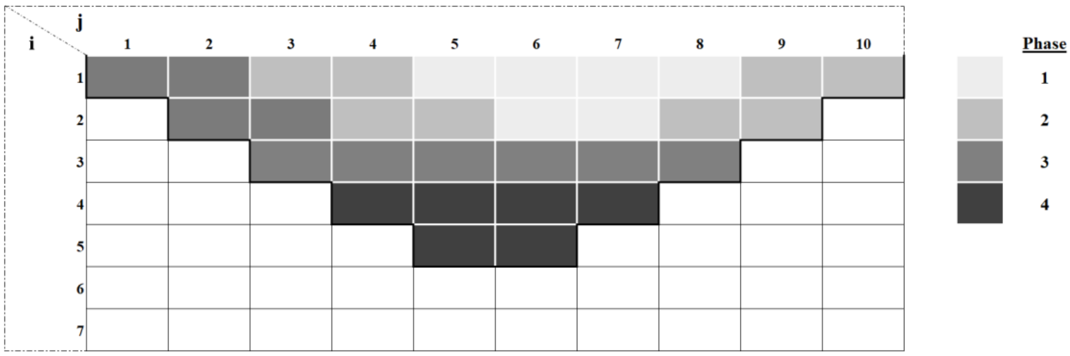


Figure 3.3 A Sample 2D Ultimate Pit with Multiple Production Phases

Constraint (3.2) ensures that each block can be extracted one time at most considering all time periods $t = \{1 \dots T\}$. Therefore, cumulative of $X_{i,j}^{t}$ for a specific block (i, j) should be less than or equal to 1. Constraint (3.3) satisfies pit slope considerations for each $X_{i,j}^{t} = 1$ decision by evaluating the extraction of the upper-level blocks $\big((i-1), j\big)$. The parameter n is the number of blocks in horizontal at only one slope at $(i-1)$ level and provides flexibility when inputting geotechnical factors into the model. For instance, if the extraction of block $(i = 3, j = 5)$ is evaluated at period t, and if the block $(i = 2, j = 4)$ and block $(i = 2, j = 6)$ should be extracted previously or at the same period $(t' = 0 \ldots t)$, then the parameter n will take the value of 1. In that case, $(2n + 1 = 3)$ blocks should be excavated at the top of the block to be mined. Thus, it offers a multiple overall slope angles for different cases. It means that it is usable for different slope angles with different n values Constraint (3.4) will ensure that the stability constraint is satisfied within the related phase limits by enforcing the model to excavate the phase blocks in pre-determined phase production periods. Constraint (3.5) decides whether the ore blocks mined in the same period satisfy the plant-feeding requirements. Here, minimum and maximum blending values will keep the blended material of multiple blocks within the allowable limits. Constraint (3.6) will satisfy that ore and waste material mining rate is between the minimum required and maximum allowable values. As discussed previously, the minimum value points to the periodic production target, while the maximum value refers to the mechanically-achievable production capacity of the earthmoving system in a given period. On the other hand, Constraint (3.7) considers the ore block production since minimum target processing rate and maximum achievable processing limits are evaluated jointly using this equation. Constraint (3.8) represents $X_{i,j}^{t}$ is a binary variable $[0,1]$.

CHAPTER 4

IMPLEMENTATION OF THE MODEL FOR AN OPEN-PIT CU-ZN MINE IN TURKEY

4.1 Introduction

This section will implement the mathematical model for a dataset acquired from an open pit mine project in Turkey. Accordingly, Chapter 4.2 will explain the related aspects of the study area in detail. The input dataset will also be introduced in this chapter. Chapter 4.3 will show and discuss the implementation results.

4.2 Study Area and Data Acquisition

The developed model will be applied and tested for a poly-metal mine to be operated with an open pit production method located in the northeastern part of Turkey. The mineralization of the production area contains copper (Cu) and zinc (Zn) minerals together. Besides, a certain amount of silver is occasionally located in the production blocks. This study will optimize the production scheduling model for a representative 2D ultimate pit design covering three production phases in varying production and processing requirements. Before implementing the model, a detailed explanation of the study is presented.

The project license area is close to the border between the Eastern Anatolia and the Black Sea Regions, approximately 13km away from Ispir District of Erzurum. The main skarn deposit is about 2.5km away from the highway. The distance between the license area and Trabzon Port is about 141km. The project will be performed at an altitude changing 2,000m to 2,400m. The location map of the project area is shown in Figure 4.1.

Figure 4.1 Location map of the project area in Turkey

The mining activities will be started with i) removing the topsoil, ii) arranging the storage areas for topsoil, and iii) opening access roads. Some part of the waste rock material will be used to construct the tailing storage facility. Some part of the waste rock will be used as filling material of the open pit mine (by casting method). The ore production will be started approximately two years after initiating waste extraction, and the waste and ore production will be continued simultaneously until the end of mine. The ore material will be processed at a flotation plant within the license area. The mine life is around eight years. The project layout is shown in Figure 4.2. Process plant area is placed at the highest location in the mine site with an elevation of 2,400m. Access to the process facility is provided by two roads which are ore transportation, and plant access roads. The ore transportation road is used to supply the ore to the plant. On the other hand, plant access road aims to transport personnel, equipment, and concentrate transportation. Thus, these two roads can decrease traffic congestions and prevent accidents. The waste dump is located on the northern side of the open pit mine. Also, the waste dump will support the tailing storage facility construction with a body height of 185m, and this facility will recycle the water to the process plant.

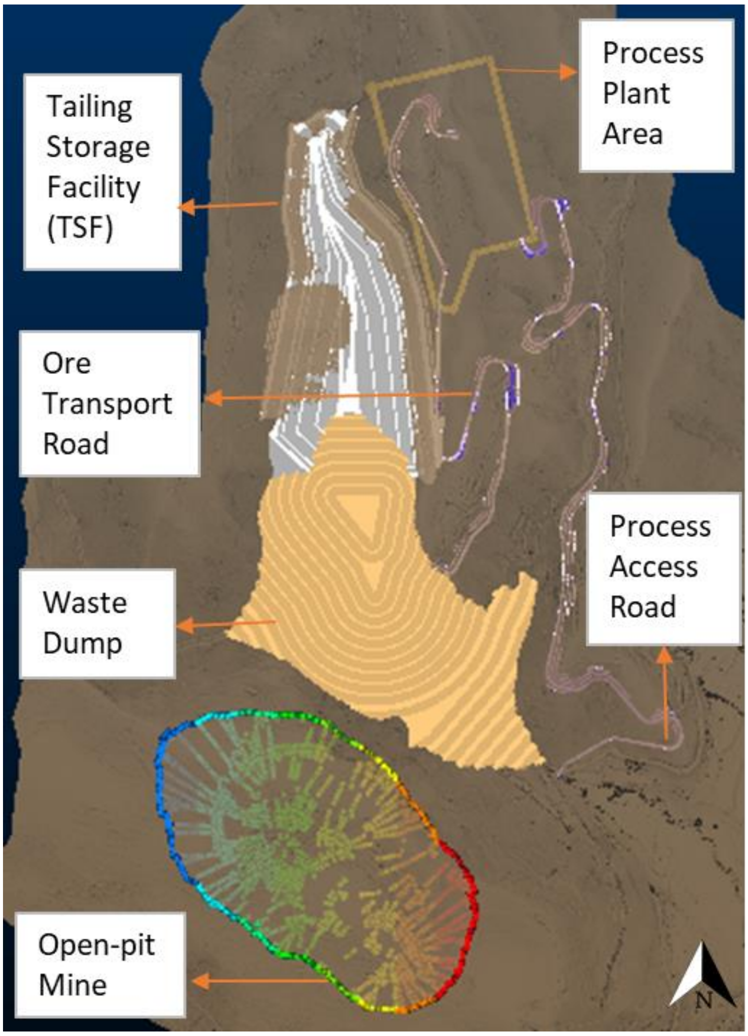


Figure 4.2 Mine site layout

4.2.1 Block Model Information

The block model was created using the following block size (length x width x height): 3x3x2.5 m. Mining will be carried out with CAT 349 or 374, and Hitachi excavator with capacities of 50 tonnes and 70 tonnes, and hauling operation will be performed by 31 to 41 tonnes MAN 4 axle highway trucks. The smaller block models separate out better the internal waste and reduce mining losses and mine dilution at the ore-waste contact, which in real mine operation can be separated. As the geotechnical report is based on a safety berm every 10m face height, it was decided to use the 3x3x2.5 m block model. On the other hand, 10x10x10 m block dimensions is employed in this study with re-block operation from 3x3x2.5 m block dimensions due to reduce the computation time, and data preparation process.

4.2.2 Pit Design Criteria

According to the JORC Resource Report (2019), the resource estimation is approximately 5.6 Mt. The rock types for the ore are classified as oxide, transitional, and Cu-rich fresh rock and Zn-rich fresh rock. The waste materials in the area are classified into two types, rock and till. Till is a weak formation type and occupies a large portion of the license area. Therefore, rock types effectively determine the geotechnical parameters in the pit design stage. Accordingly, various geotechnical tests and analyses were conducted to find out safe pit slope values. Table 4.1 summarizes the resource and the slope design parameters, and Figure 4.3 shows the top view of the ultimate pit design along the Section A-A'.

Table 4.1 Resource estimation of the deposit (CSA Global, 2019)

Resource Class	Tonnage (Mt)	Cu (%)	Cu (t)	Zn (%)	Zn (t)	Ag (g/t)	Ag (Moz)
Measured	1.34	1.68	22,466	2.48	33,098	37.66	1.62
Indicated	3.09	0.83	25,636	4.55	140,765	23.19	2.31
Inferred	1.13	1.13	12,828	3.73	42,195	32.67	1.19
Total	5.56	1.1	60,930	3.89	216,058	28.59	5.11

Sector	Face Angle (°)	Berm Width (m)	Overall Slope Angle (°)
West	65	6	29.9
South	65	6	33.5
East	65	8	29.9
North	65	6	30.6

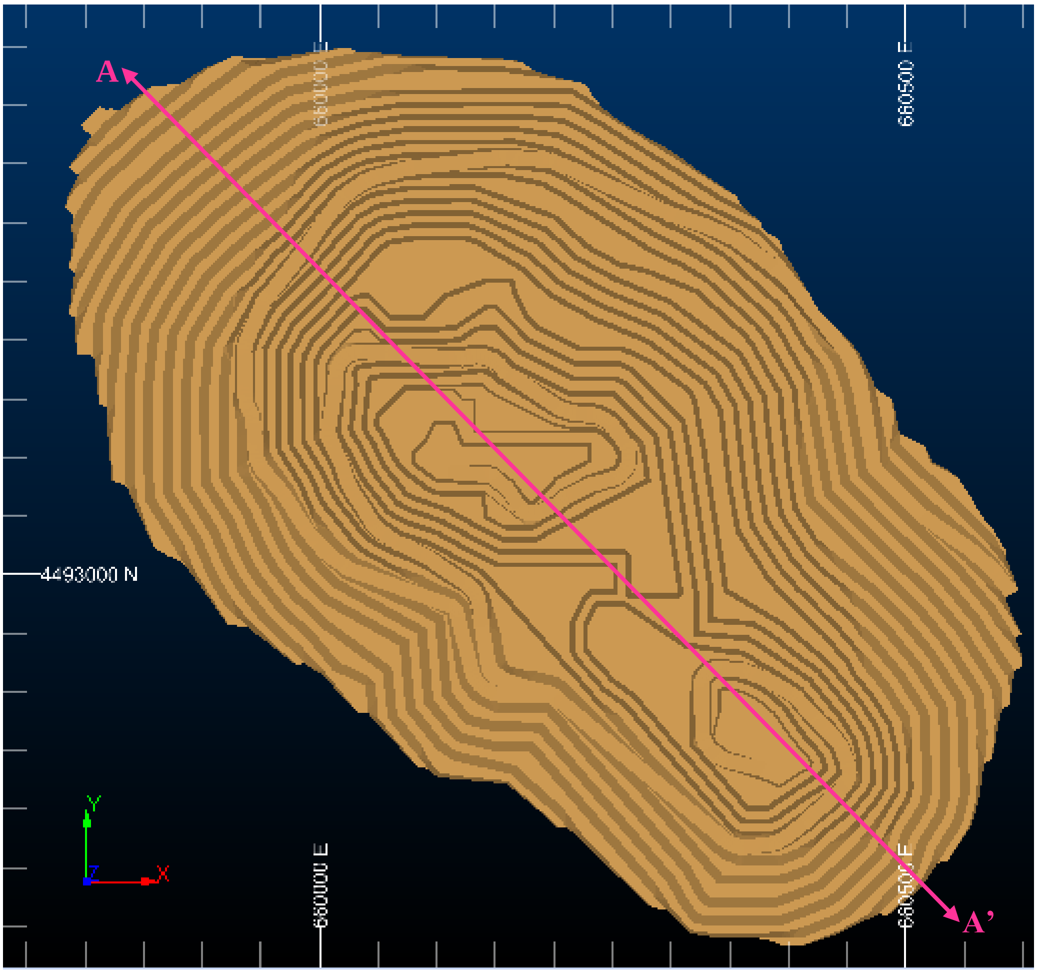


Figure 4.3 Top view of the designed pit

4.2.3 Mineralization in the Area

The project is located in the eastern Pontides porphyry belt, where mineralization is associated with Palaeocene or Eocene porphyry intrusions. The project includes a traditional porphyry-skarn mineralizing system. Within porphyry intrusions, copper-molybdenum mineralization occurs, and replacive skarn mineralization occurs in the adjacent host rocks, primarily in calcareous units in the Palaeozoic basement.

The skarn deposit replaces the Palaeozoic basement with sulfide-rich mineralization and gangue, including high-temperature prograde silicate minerals and substantial lower-temperature retrograde overprint. Mineralization is strata-bound and dominated by marble and impure calcareous units in the basement layers, whereas endo-skarn-type mineralization replaces clastic and intrusive rocks. Because the deposit is located in a shallow-dipping basement stratigraphic region, the strata-bound sulfide-replacement bodies are shallow-dipping to the southwest. The deposit is hidden by Quaternary moraine deposits and may only be seen from the northeast up-dip edge.

Marble and calcareous clastic precursor skarn replacement are significantly more widespread than high-grade sulfide mineralization. Most sulfide mineralization is thought to have occurred during early retrograde skarn alteration.

4.2.4 Ore Processing Information

The concentrator will receive ore from the open pit mine via trucks and will process 850 kt per annum of the skarn, polymetallic ore (~0,91% Cu and ~3.62% Zn) through a sequential Cu/Zn flotation circuit to attain separate Cu and Zn concentrates of saleable grade. There are four different types of ore; oxide, transition, Cu-rich fresh, and Zn-rich fresh. Only oxidized ore will be stockpiled. Since the greater part of ore (transitional ore and fresh ore) contains primary sulfide minerals, this type of ore will not be kept in the ore stockpile for more than a week. Otherwise, its processing specifications may be altered due to the start of oxidization if the stockpiling of the sulfide material takes more than one week. Besides, the recovery of the oxidized material is very low. The plant's final product will be 25% copper concentrate with the bonus grade of silver and 52% of zinc concentrate. Table 4.2 summarizes the process recovery values according to the ore zone types.

Table 4.2 Process recovery values of different zones

Zone	Cu Process Recovery (%)	Zn Process Recovery (%)
Oxide Zone	45	60
Transition Zone	70	60
Fresh Cu-Rich Zone	91	83
Fresh Zn-Rich Zone	68	92

4.2.5 Model Input Dataset

The data file used in the model requires a pre-formatting of the raw data acquired from the mining area, according to the mathematical model discussed in Section 3.2.2. A representative illustration of the 2D ultimate pit and phase design limits valid in the mining area is shown in Figure 4.4. Small blocks were reunited to develop 10mx10m blocks for faster computation. The production will start from the south pit in Phase-1 and expends mainly in the lateral direction toward the north pit in Phase-2. The ultimate pit limits are achieved in Phase-3 by producing blocks both in horizontal and vertical directions.

Taking multiple slices from the actual pit showed that the phase design limits and the location of the zones illustrated in Figure 4.4 are 2D representative of the actual pit condition. It illustrates the section A-A' of the ultimate pit in Figure 4.4. After deciding on the phase and block info, a representative grade distribution was achieved by evaluating the available 3D block model in terms of the mineralization zones, grade distribution behaviors per zone, percentile weights of the ore block for both Zn and Cu, and percentile weight of waste material per zone. Percentile weights of Zn and Cu-included blocks in 3D block modeling are stated in Table 4.3. This information will determine how many blocks in the 2D model should cover Cu-only, Zn-only, and Cu and Zn together. The block sets ($Blocks^p$) are given in Appendix A.

Table 4.3 Zonal Block Statistics in 3D Block Model

Zone	**Total Block (#)**	**Zonal Block (%)**	**Cu Weight (%)**	**Zn Weight (%)**
Waste	984,405	93.88	-	-
Oxide	3,352	0.32	81.53	52.36
Transition	5,762	0.55	52.36	26.94
Cu-rich sulfide	25,592	2.44	69.70	66.06
Zn-rich sulfide	29,640	2.81	28.43	13.13

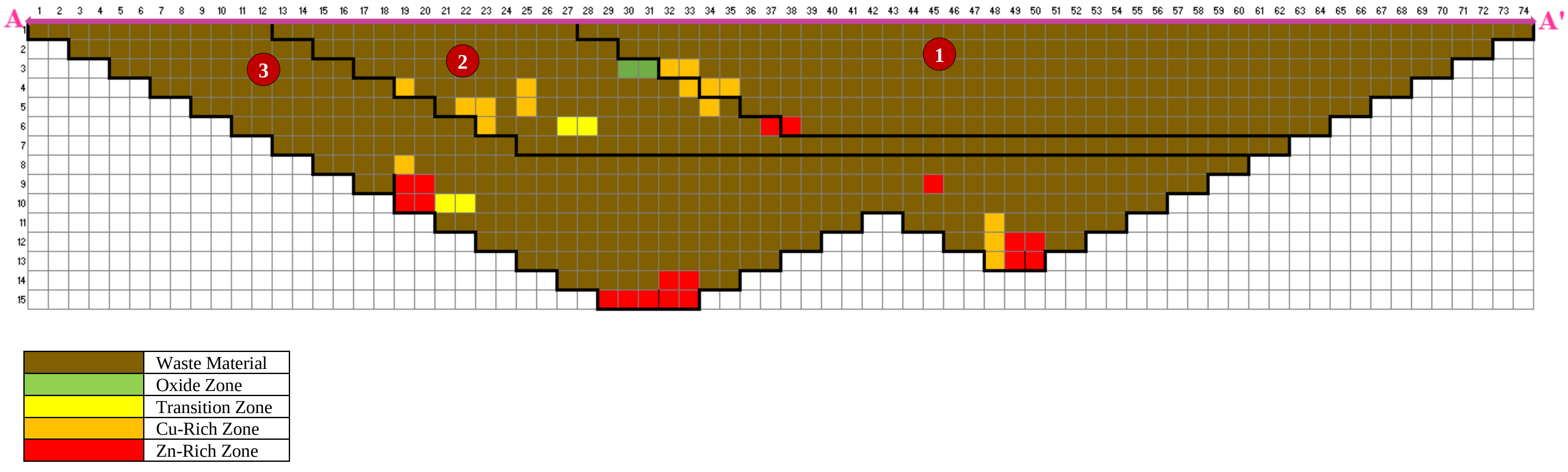


Figure 4.4 Phase Designs Available for the Mining Area Section A-A’

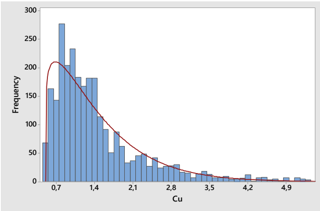
After deciding Cu- and Zn-included blocks in the model, allocation of grades is achieved from the best-fit distributions of Cu and Zn mineral separately for each zonal area. Accordingly, parametric values of eight different 3P Weibull, which were detected as best-fit distribution, were determined as given in Figure 4.5. 3P Weibull distribution has three descriptive parameters called scale parameter (characteristic life), shape parameter (slope), and location parameter (lower bound). The shape parameter determines the distribution behavior where the distribution reduces to 2P Exponential distribution if this parameter is equal to 1.0 and reduces to normal distribution if it takes the value of 3.0. On the other hand, the scale parameter determines the range of distribution, and the location parameter points to the minimum allowed value of 0.5 and 1.0 as Cu and Zn cut-offs in the distributions.

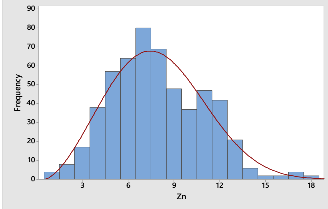
The economic block values of blocks are determined by the related revenue and operating cost items given in Table 4.4. All these values have been the same throughout the life of mine. OPEX of a unit weight of the waste block is calculated using waste mining cost, increased when moving deeper in lower levels. On the other hand, ore block OPEX is estimated by ore mining, processing, and general and administrative (G&A) costs of unit ore production. Equations 4.1 and 4.2 are utilized in revenue calculations of ore and waste materials, respectively.

$$\mathrm{BV_{i,j}}^{\mathrm{ore}} = \left(\sum_{\mathrm{m}=1}^{2} \mathrm{W_{i,j}}^{\mathrm{ore}} \times \mathrm{g_{i,j,m}} \times \mathrm{Rec_m} \times \mathrm{Price_m}\right) - \left(\mathrm{W_{i,j}}^{\mathrm{ore}} \times \mathrm{OreMC} + (\mathrm{i}-1) \times 0.05\right) \quad (4.1)$$

$$\mathrm{BV_{i,j}}^{\mathrm{waste}} = -\left(\mathrm{W_{i,j}}^{\mathrm{waste}} \times \mathrm{WasteMC} + (\mathrm{i}-1) \times 0.05\right) \quad (4.2)$$

In the equations, $\mathrm{BV_{i,j}}^{\mathrm{ore/waste}}$ is the economic value of ore or waste block (i,j), $\mathrm{W_{i,j}}^{\mathrm{ore/waste}}$ is the unit weight of the related block, $\mathrm{g_{i,j,m}}$ is the grade of material m available in the block, $\mathrm{Rec_m}$ is the overall recovery of material m, $\mathrm{Price_m}$ is the market price of processed material m, OreMC and WasteMC are the ore and waste mining costs of a unit tonne of material, and i is the block's excavation level (row).

<table>
<tr><th></th><th>Copper Grade</th><th>Zinc Grade</th></tr>
<tr><td>Oxide</td><td>

<table><tr><th></th><th>Shape</th><th>Scale</th><th>Threshold</th></tr><tr><td>3PWeibull</td><td>1.16</td><td>1.0</td><td>0.5</td></tr></table></td><td>

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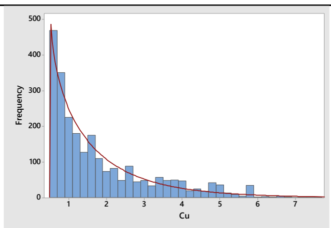
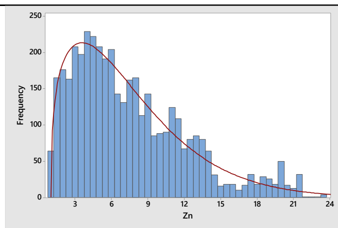
<tr><td>Transition</td><td>

<table><tr><th></th><th>Shape</th><th>Scale</th><th>Threshold</th></tr><tr><td>3PWeibull</td><td>0.90</td><td>1.27</td><td>0.50</td></tr></table></td><td>

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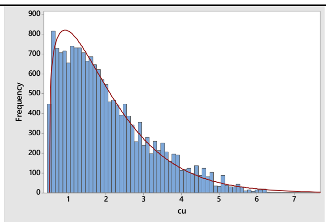
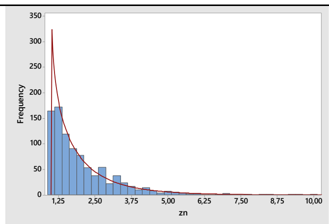
<tr><td>Cu-rich sulfide</td><td>

<table><tr><th></th><th>Shape</th><th>Scale</th><th>Threshold</th></tr><tr><td>3PWeibull</td><td>1.23</td><td>1.62</td><td>0.50</td></tr></table></td><td>

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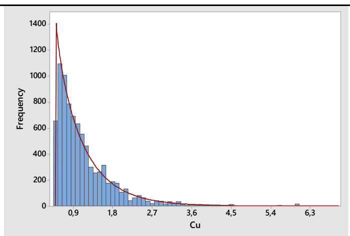
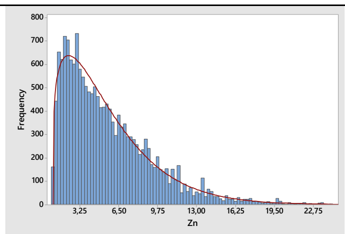
<tr><td>Zn-rich sulfide</td><td>

<table><tr><th></th><th>Shape</th><th>Scale</th><th>Threshold</th></tr><tr><td>3PWeibull</td><td>0.96</td><td>0.64</td><td>0.50</td></tr></table></td><td>

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</table>

Figure 4.5 Copper and Zinc distributions for different zones in the orebody

Table 4.4 Operating Cost and Revenue Parameters

Parameters	Value	Unit
Ore mining cost	1.5	\$/tonne
Stripping cost	0.9	\$/tonne
Processing cost + G&A	21.24	\$/tonne
Cu MP	8,000	\$/tonne
Zn MP	3,000	\$/tonne
Process Recovery – Cu (oxide)	45	%
Process Recovery – Cu (transition)	70	%
Process Recovery – Cu (sulfide Cu-rich)	91	%
Process Recovery – Cu (sulfide Zn-rich)	68	%
Process Recovery – Zn (oxide)	60	%
Process Recovery – Zn (transition)	60	%
Process Recovery – Zn (sulfide Cu-rich)	83	%
Process Recovery – Zn (sulfide Zn-rich)	92	%
Mining recovery	95	%
Dilution	10	%
Block dimensions	10x10x10	m
Overall slope angle	30	°
Ore density (in-situ)	3.2	tonne/m^3
Waste density (in-situ)	2.6	tonne/m^3

In addition to $BV_{i,j}{}^{ore/waste}$, the model requires introducing multiple data about capacity constraints, phase production information, blending ranges, and other data (Table 4.5). Capacity constraints are minimum and maximum capacities of mining production and ore processing rates. Here, actual production targets using the 3D model was correlated for the 2D model comparing the block numbers when determining the $MCmin^t$ values, while there is no set $MCmax^t$ for this application because of the decision of authorized people from the company. The processing capacities were also correlated with the actual target and limitations. On the other hand, pit geometry values and target phase production periods were also determined for each phase design. Accordingly, Phase-1, Phase-2, and Phase-3 are expected to be completed in three years, two years, and three years, respectively. That means the equipment fleet is assumed to be the same until the end of the mine life. Blending constraint is between six ($Bmax^g$) and one ($Bmin^g$). Besides, each ore and waste blocks are 3,210 and 2,600 tonnes, respectively. Last, the discount rate and the slope parameter were taken 10% in terms of USD, and $n = 2$ (for 30° slope).

Table 4.5 Capacity, Phase, and Blending Parameters

						Annual Capacity Data
		Year	$MCmin^t$ (t)	$MCmax^t$ (t)	$PCmin^t$ (t)	$PCmax^t$ (t)
		1	228,350	M*	-	-
		2	177,410	M	3,210	40,000
		3	158,440	M	12,840	40,000
		4	148,200	M	19,260	40,000
		5	150,800	M	22,470	40,000
		6	239,200	M	25,680	40,000
		7	236,600	M	22,470	40,000
		8	236,600	M	22,470	40,000
						Phase Production Data
Phase	$iMax^p$ (level)	$jMin^p$ (column)	$jMax^p$ (column)	$tMin^p$ (year)	$tMax^p$ (year)	NB^p (block)
1	6	28	74	1	3	222
2	7	13	74	4	5	128
3	15	1	74	6	8	296
						Other Data
	$Bmax^g$	$Bmin^g$	$W_{i,j}^{ore}$ (t/block)	$W_{i,j}^{waste}$ (t/block)	d^t (%)	n (block)
	6.0	1.0	3,210	2,600	10	2

*M: A relatively very high value

4.3 Implementation Results

This section discusses the sequencing of production blocks for the available dataset given in Section 4.2 and performs sensitivity analysis for the decisions considering the variations in critical parameters such as discount rate, commodity prices, and operational cost (mining, processing, and G&A cost). In this way, it aims to highlight the optimal production scheduling that maximizes NPV for the current and potential future conditions. The model was solved in the AMPL CPLEX module of the NEOS Server, and the results, including sensitivity analysis, are presented in Figures 4.6 to 4.11. In these figures, the blocks with X represent the ore blocks. Also, the result of the yearly production amount is explained in Table 4.6.

Table 4.6 Yearly Production Amount

Period	Waste Amount (t)	Waste Volume (m^3)	Ore Amount (t)	Cu Grade (%)	Zn Grade (%)	Stripping Ratio (m^3/t)
1	228,800	88,000	-	-	-	-
2	174,200	67,000	3,210	0.56	1.36	20.87
3	161,200	62,000	16,050	1.57	2.33	3.86
4	156,000	60,000	19,260	1.60	4.45	3.12
5	145,600	56,000	22,470	0.89	2.73	2.49
6	215,800	83,000	25,680	1.53	4.23	3.23
7	228,800	88,000	22,470	1.23	5.73	3.92
8	267,800	103,000	22,470	1.38	7.41	4.58
Total	1,578,200	607,000	131,610	1.33	4.50	4.61

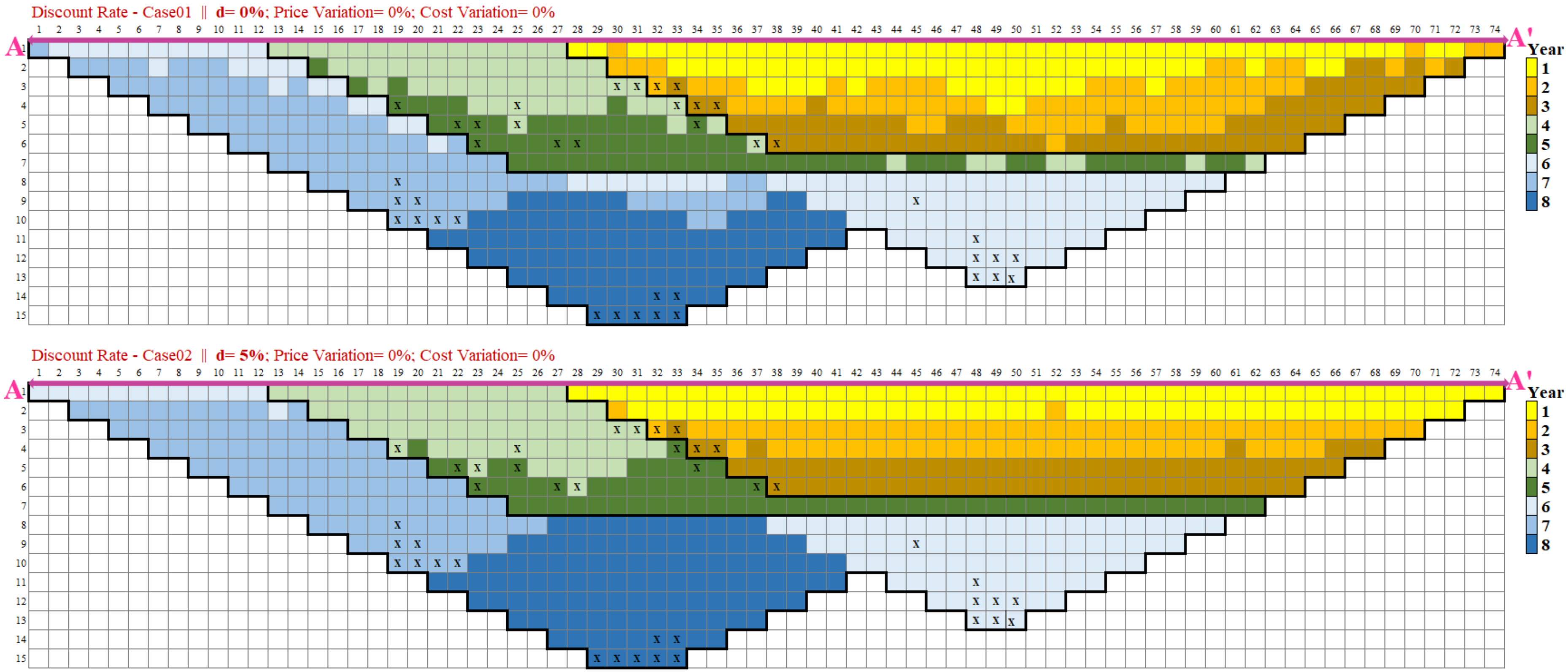


Figure 4.6 Changes in Production Scheduling due to the Variation in Discount Rate (Cases for d:0% and d:5%)

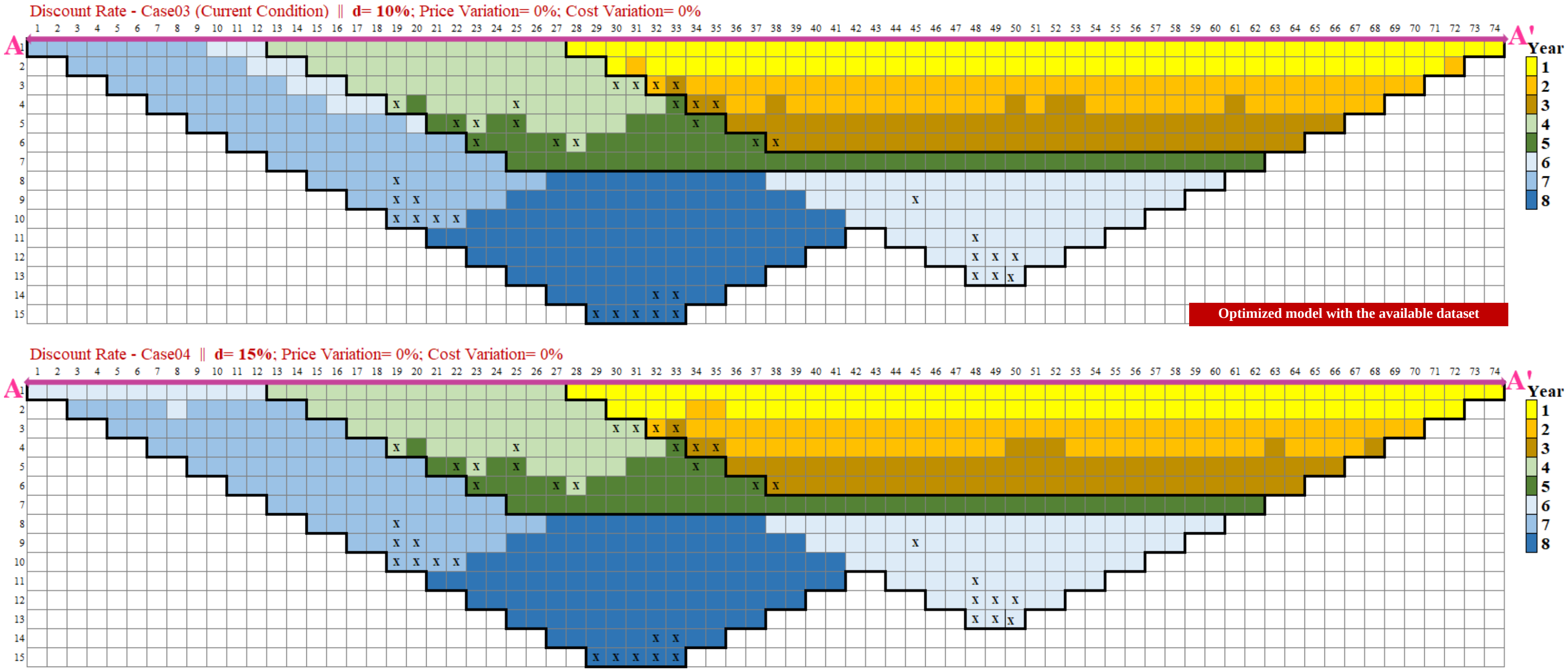
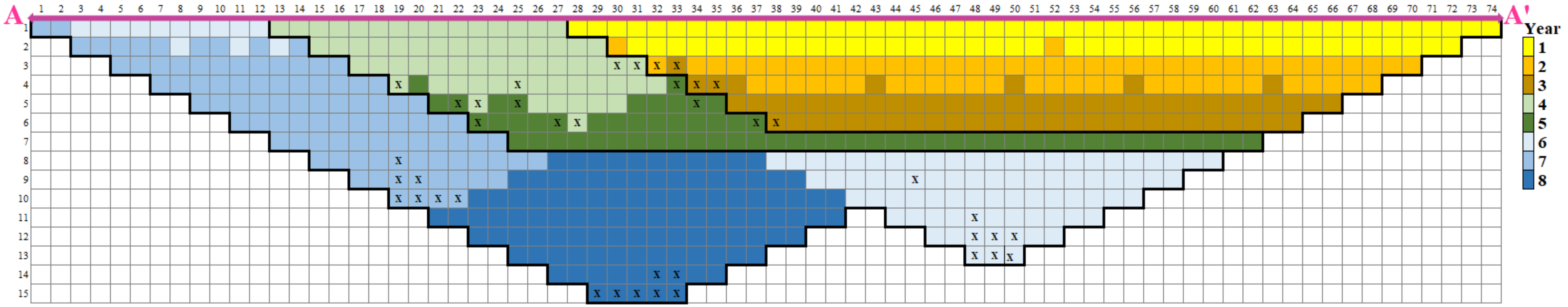


Figure 4.7 Changes in Production Scheduling due to the Variation in Discount Rate (Cases for d:10% and d:15%)

Sales Prices - Case01 || d= 10%; **Price Variation= -20%**; Cost Variation= 0%



Sales Prices - Case02 || d= 10%; **Price Variation= -10%**; Cost Variation= 0%

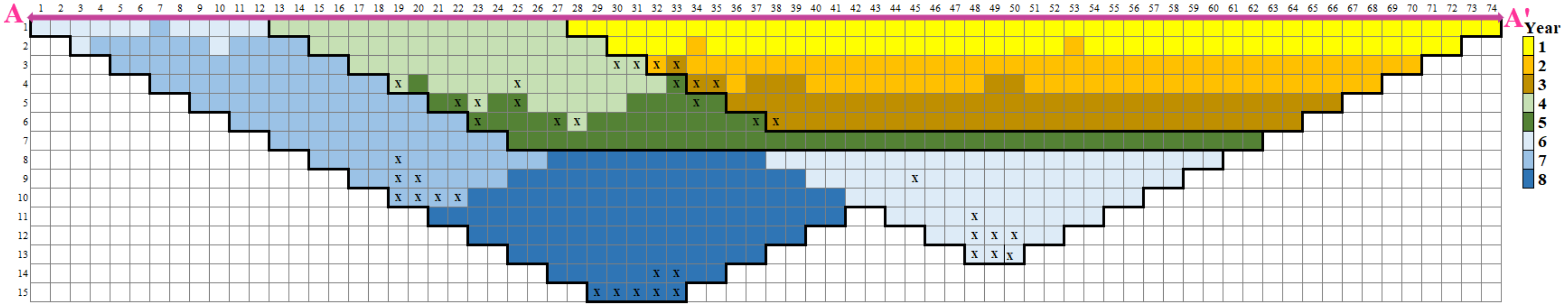
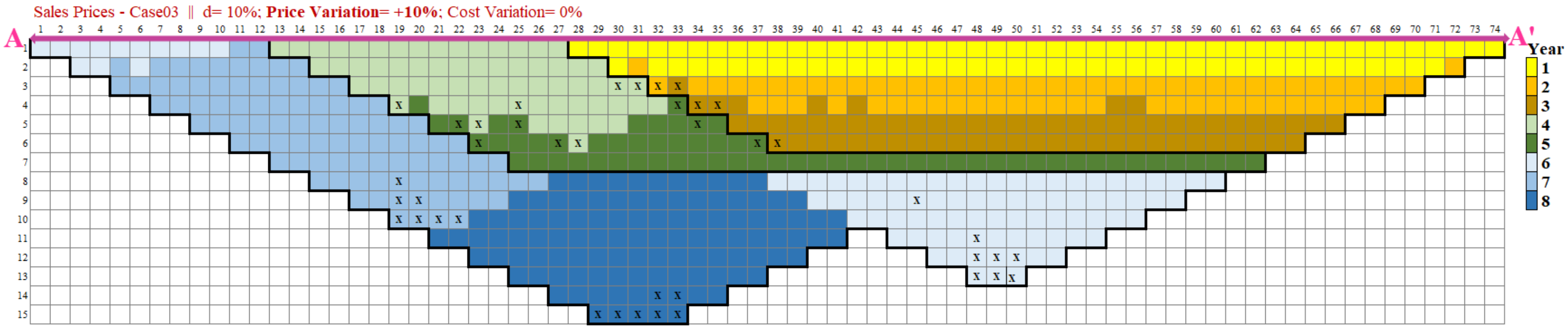


Figure 4.8 Changes in Production Scheduling due to the Variation in Metal Prices (Cases for Price Variations: -20% and -10%)



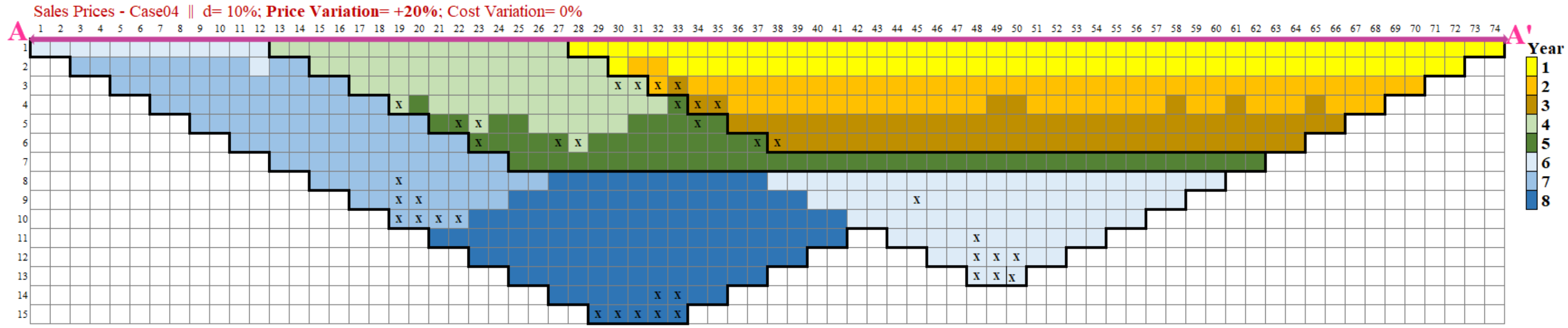


Figure 4.9 Changes in Production Scheduling due to the Variation in Metal Prices (Cases for Price Variations: +10% and +20%)

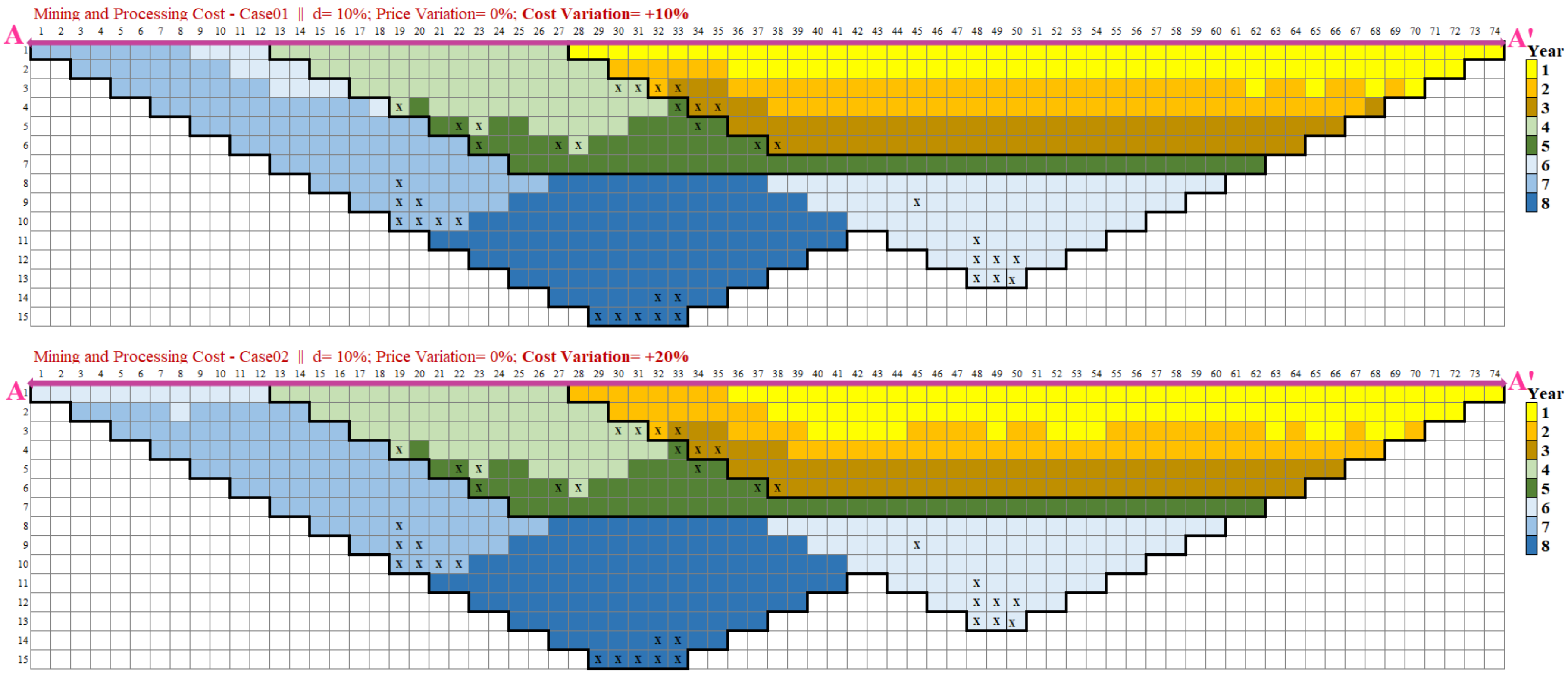
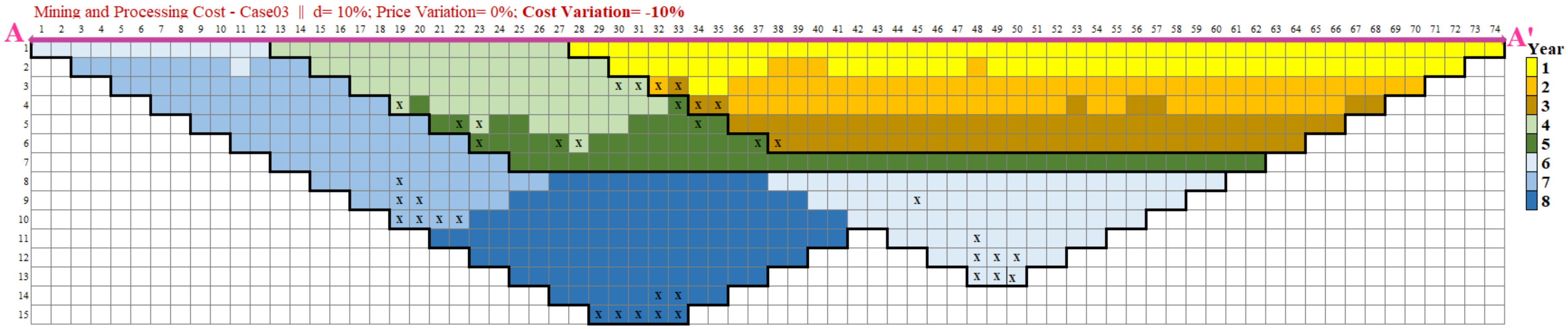


Figure 4.10 Changes in Production Scheduling due to the Variation in Mining and Processing Cost Items (Cases for Cost Variations: +10% and +20%)



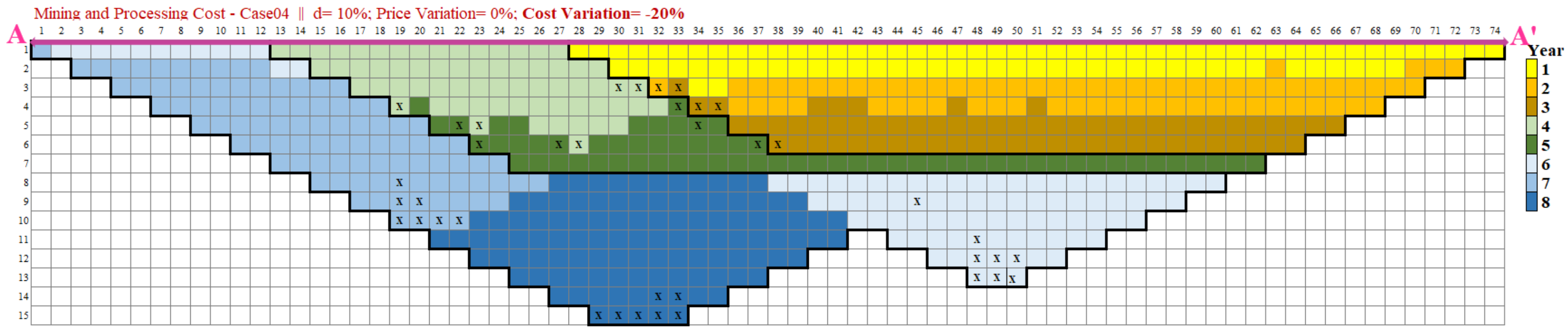


Figure 4.11 Changes in Production Scheduling due to the Variation in Mining and Processing Cost Items (Cases for Cost Variations: -10% and -20%)

The model is solved with 8,822 variables and 5,410 constraints in each run. Since the model is solved in NEOS Server, there is no information about the exact computing time and capacity required for the model. However, the model was observed to be solved in less than 60 seconds in the server. First, the long-term production optimization was achieved for the available capacity, phase, and blending parameters in Table 4.6. Since the discount rate for the mine is 10%, the optimized block production for three-phase design limits for the LOM of eight years can be viewed from the case of d:10% in Figure 4.7. It is observed from Figures 4.6 and 4.7 where the discount rate was changed 0 to 15% by 5% increments; the increasing discount rate leads to a pressure on the model to extract the positive block in earlier years to improve the NPV. On the other hand, the variation of metal prices ±20% with 10% increments caused comparatively less pressure on the model in changing the sequence of block production (Figures 4.8 and 4.9). At this point, under the available 10% discount rate for all cases, the increase in Zn and Cu metal prices allows producing some of the ore blocks in earlier stages. Since the mathematical model considers the overall financial benefit of the decisions throughout the total period, it was observed that an increase in metal prices does not block production order remarkably compared to the optimized current production case. Last, it is seen from Figures 4.10 and 4.11 that mining, processing, and G&A cost variations have an observable effect on block production sequencing. Here, Phase-1 is exposed to more sensitive selection on the blocks in each period when the cost items of both waste and ore blocks are increased by 20%. If the cost values are decreased, then the model applies the decisions relying more on the revenue parameters.

Moreover, it can be seen that the current application does not have any positive stress for completing the extraction of the same-level block before passing to the sequential level for the given period. Therefore, the model may leave some blocks of a specific level to be extracted in the following periods. This condition can be overcome practically by extracting all same-level blocks in the same period by dividing phase limits into multiple production limits. However, there is not any such limitation for the current application area.

Computational results of the cases are summarized in Table 4.7 and shown in Figure 4.12. It is seen that the pit production scheduling is optimized by minimizing the NPV to $11,489,800 for an eight-year production with an available discount rate of 10 percent. The sensitivity analysis showed that even though the cost variation causes some observable changes in the periodic block selections, it least affects the resultant NPV value. On the other hand, discount rate and price variation are expected to influence the NPV most considering the total mine life. In the figure, the discount rate line was extended between -100% and 50% since the rate itself is a percentile value, and these values were converted to the variation percentages.

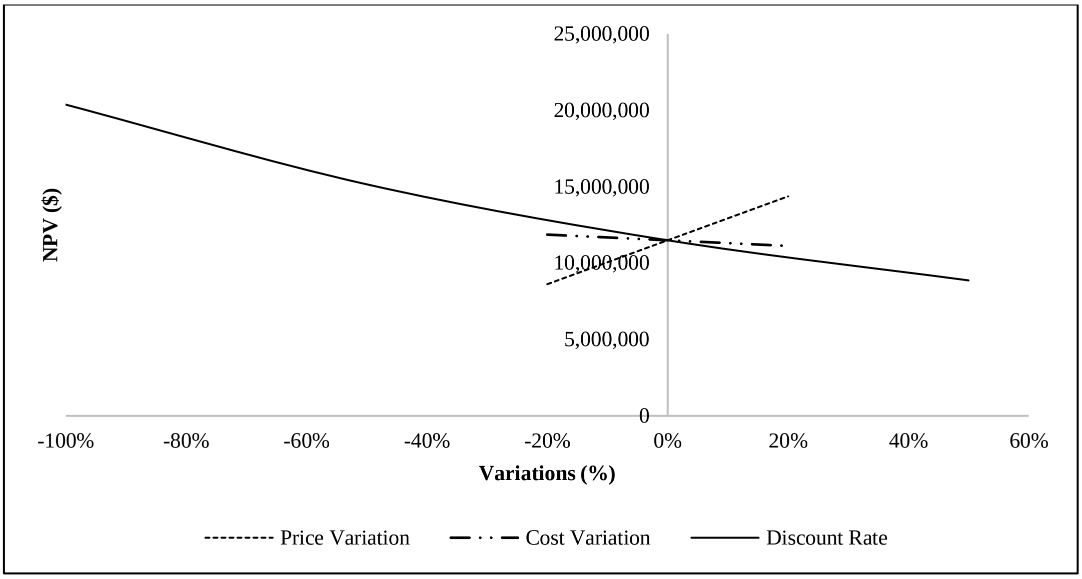


Figure 4.12 Sensitivity Analysis for the Optimized Cases with Varying Parameters

Table 4.7 The NPV Values of the Optimized Production Scheduling for the Sensitivity Parameters

Sensitivity Parameter	**Value/Change (%)**	**Optimality Gap Percentage**	**NPV Value ($) (Obj Function)**
Discount Rate	0%	1.82907e-16	20,367,100
	5%	9.4568e-05	15,152,000
	10%	***9.40812e-05***	***11,489,800***
	15%	9.57988e-05	8,864,006
Price Variation	(-) 20%	9.50472e-05	8,616,840
	(-) 10%	8.38455e-05	10,053,400
	0%	***9.40812e-05***	***11,489,800***
	(+) 10%	7.07566e-0	12,926,400
	(+) 20%	6.6498e-05	14,363,000
Cost Variation	(-) 20%	8.18349e-05	11,860,900
	(-) 10%	8.19781e-05	11,675,400
	0%	***9.40812e-05***	***11,489,800***
	(+) 10%	9.2883e-05	11,304,400
	(+) 20%	7.39806e-05	11,119,300

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Mine production models generally intend to maximize the amount of final product periodically or maximize the net present production value. These two objective functions can show variation since the first model does not consider the increase in operating cost. Therefore, maximizing the final product cannot always generate the highest financial value. In some circumstances where a market pressure or commitment is available, the companies may choose this objective function since other options may lead to other external side effects. Otherwise, maximizing NPV (net present value) is the most common objective in mine production planning. Regardless of the objective function, multiple limitations (constraints) should be included when deciding about the production blocks. Moreover, the limitations should be regarded in each phase by evaluating inter-phase interactions jointly.

This study presents an integer programming model to optimize long-term production planning of open-pit mines with multiple phases with varying production periods and capacity requirements. In addition, geotechnical, financial, processing, and mining-related data were introduced into the model. Besides, the model also considers the blending constraint of blocks covering multiple ingredients that should be kept in a pre-determined range for plant efficiency. The developed integer programming model was applied for an open-pit poly-metal mine, including Zn and Cu deposits. The mine is expected to produce ore in three phases with changeable production periods for the formation divided into three: oxide, transition, Zn-rich sulfide, and Cu-rich sulfide. Each zone has different grade distribution values and

Cu and Zn-included blocks. In addition, a certain range of Zn/Cu content should be maintained by blending materials not to drop the plant efficiency. Accordingly, a representative 2D pit and phase designs of the mine were determined, and the corresponding grades were assigned to the blocks in different zones using their own zonal distribution functions. After computing the model for the currently valid data, additional comparative results were derived by a sensitivity analysis. At this point, potential variations in the discount rate, cost values, and commodity prices and their effects on the production scheduling decisions were revealed and discussed.

The model was optimized by maximizing the NPV of eight-year production to 11,489,800 with a negligible optimality gap. The variations in the model parameters showed that the cost has the least effect on the NPV even though it may lead to observable changes in periodic block selections. Metal prices are seen to be the most influential parameter on the resultant NPV value even in the optimized production.

5.2 Recommendations

An integer programming model was developed to optimize an open-pit mine long-term production scheduling under geotechnical, blending, and other operational constraints. The model was solved in the AMPL environment using a CPLEX solver. Following aspects can be considered in future studies for further improvement of the model:

i. The two-dimensional model can be extended to a three-dimensional model that can be more representative of the pit geometry and deposit varying in different lengths in different orientations

ii. Multiple slopes requiring different numbers of block excavation in upper levels can be introduced into the model.

iii. Variations in cost and revenue parameters can be evaluated in the model by sensitivity analysis. On this basis, the model can be constructed more dynamically to forecast uncertainties in financial values and evaluate the effect of uncertainties on production planning decisions.

iv. Multi-stockpile option can be included into the model. Individual stockpile(s) of each pit and joint stockpiles can be considered for mines where multi-pit production is available or joint stockpile(s) is used by multiple different operation.

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APPENDICES

A. Block and Grade Information of the Phase Designs

Block (i,j) Sets for the Phase Designs 1-2-3:

$Blocks^1$: **{** (1,28) ; (1,29) ; (1,30); (1,31); (1,32); (1,33); (1,34); (1,35); (1,36); (1,37); (1,38); (1,39); (1,40); (1,41); (1,42); (1,43); (1,44); (1,45); (1,46); (1,47); (1,48); (1,49); (1,50); (1,51); (1,52); (1,53); (1,54); (1,55); (1,56); (1,57); (1,58); (1,59); (1,60); (1,61); (1,62); (1,63); (1,64); (1,65); (1,66); (1,67); (1,68); (1,69); (1,70); (1,71); (1,72); (1,73); (1,74); (2,30); (2,31); (2,32); (2,33); (2,34); (2,35); (2,36); (2,37); (2,38); (2,39); (2,40); (2,41); (2,42); (2,43); (2,44); (2,45); (2,46); (2,47); (2,48); (2,49); (2,50); (2,51); (2,52); (2,53); (2,54); (2,55); (2,56); (2,57); (2,58); (2,59); (2,60); (2,61); (2,62); (2,63); (2,64); (2,65); (2,66); (2,67); (2,68); (2,69); (2,70); (2,71); (2,72); (3,32); (3,33); (3,34); (3,35); (3,36); (3,37); (3,38); (3,39); (3,40); (3,41); (3,42); (3,43); (3,44); (3,45); (3,46); (3,47); (3,48); (3,49); (3,50); (3,51); (3,52); (3,53); (3,54); (3,55); (3,56); (3,57); (3,58); (3,59); (3,60); (3,61); (3,62); (3,63); (3,64); (3,65); (3,66); (3,67); (3,68); (3,69); (3,70)(4,34); (4,35); (4,36); (4,37); (4,38); (4,39); (4,40); (4,41); (4,42); (4,43); (4,44); (4,45); (4,46); (4,47); (4,48); (4,49); (4,50); (4,51); (4,52); (4,53); (4,54); (4,55); (4,56); (4,57); (4,58); (4,59); (4,60); (4,61); (4,62); (4,63); (4,64); (4,65); (4,66); (4,67); (4,68) (5,36); (5,37); (5,38); (5,39); (5,40); (5,41); (5,42); (5,43); (5,44); (5,45); (5,46); (5,47); (5,48); (5,49); (5,50); (5,51); (5,52); (5,53); (5,54); (5,55); (5,56); (5,57); (5,58); (5,59); (5,60); (5,61); (5,62); (5,63); (5,64); (5,65); (5,66)(6,38); (6,39); (6,40); (6,41); (6,42); (6,43); (6,44); (6,45); (6,46); (6,47); (6,48); (6,49); (6,50); (6,51); (6,52); (6,53); (6,54); (6,55); (6,56); (6,57); (6,58); (6,59); (6,60); (6,61); (6,62); (6,63); (6,64) **}**

$Blocks^2$: **{** (1,13); (1,14); (1,15); (1,16); (1,17); (1,18); (1,19); (1,20); (1,21); (1,22); (1,23); (1,24); (1,25); (1,26); (1,27); (2,15); (2,16); (2,17); (2,18); (2,19); (2,20); (2,21); (2,22); (2,23); (2,24); (2,25); (2,26); (2,27); (2,28); (2,29); (3,17); (3,18); (3,19); (3,20); (3,21); (3,22); (3,23); (3,24); (3,25); (3,26); (3,27); (3,28); (3,29); (3,30); (3,31); (4,19); (4,20); (4,21); (4,22); (4,23); (4,24); (4,25); (4,26); (4,27); (4,28); (4,29); (4,30); (4,31); (4,32); (4,33); (5,21); (5,22); (5,23); (5,24); (5,25); (5,26); (5,27); (5,28); (5,29); (5,30); (5,31); (5,32); (5,33); (5,34); (5,35); (6,23); (6,24); (6,25); (6,26); (6,27); (6,28); (6,29); (6,30); (6,31); (6,32); (6,33); (6,34); (6,35); (6,36); (6,37); (7,25); (7,26); (7,27); (7,28); (7,29); (7,30); (7,31); (7,32); (7,33); (7,34); (7,35); (7,36); (7,37); (7,38); (7,39); (7,40); (7,41); (7,42); (7,43); (7,44); (7,45); (7,46); (7,47); (7,48); (7,49); (7,50); (7,51); (7,52); (7,53); (7,54); (7,55); (7,56); (7,57); (7,58); (7,59); (7,60); (7,61); (7,62) **}**

$Blocks^3$: **{** (1,1); (1,2); (1,3); (1,4); (1,5); (1,6); (1,7); (1,8); (1,9); (1,10); (1,11); (1,12); (2,3); (2,4); (2,5); (2,6); (2,7); (2,8); (2,9); (2,10); (2,11); (2,12); (2,13); (2,14); (3,5); (3,6); (3,7); (3,8); (3,9); (3,10); (3,11); (3,12); (3,13); (3,14); (3,15); (3,16); (4,7); (4,8); (4,9); (4,10); (4,11); (4,12); (4,13); (4,14); (4,15); (4,16); (4,17); (4,18); (5,9); (5,10); (5,11); (5,12); (5,13); (5,14); (5,15); (5,16); (5,17); (5,18); (5,19); (5,20); (6,11); (6,12); (6,13); (6,14); (6,15); (6,16); (6,17); (6,18); (6,19); (6,20); (6,21); (6,22); (7,13); (7,14); (7,15); (7,16); (7,17); (7,18); (7,19); (7,20); (7,21); (7,22); (7,23); (7,24); (8,15); (8,16); (8,17); (8,18); (8,19); (8,20); (8,21); (8,22); (8,23); (8,24); (8,25); (8,26); (8,27); (8,28); (8,29); (8,30); (8,31); (8,32); (8,33); (8,34); (8,35); (8,36); (8,37); (8,38); (8,39); (8,40); (8,41); (8,42); (8,43); (8,44); (8,45); (8,46); (8,47); (8,48); (8,49); (8,50); (8,51); (8,52); (8,53); (8,54); (8,55); (8,56); (8,57); (8,58); (8,59); (8,60); (9,17); (9,18); (9,19); (9,20); (9,21); (9,22); (9,23); (9,24); (9,25); (9,26); (9,27); (9,28); (9,29); (9,30); (9,31); (9,32); (9,33); (9,34); (9,35); (9,36); (9,37); (9,38); (9,39); (9,40); (9,41); (9,42); (9,43); (9,44); (9,45); (9,46); (9,47); (9,48); (9,49); (9,50); (9,51); (9,52); (9,53); (9,54); (9,55); (9,56); (9,57); (9,58); (10,19); (10,20); (10,21); (10,22); (10,23); (10,24); (10,25); (10,26); (10,27); (10,28); (10,29); (10,30); (10,31); (10,32); (10,33); (10,34); (10,35); (10,36); (10,37); (10,38); (10,39); (10,40); (10,41); (10,42); (10,43); (10,44); (10,45); (10,46); (10,47); (10,48); (10,49); (10,50); (10,51); (10,52); (10,53);

(10,54); (10,55); (10,56); (11,21); (11,22); (11,23); (11,24); (11,25); (11,26); (11,27); (11,28); (11,29); (11,30); (11,31); (11,32); (11,33); (11,34); (11,35); (11,36); (11,37); (11,38); (11,39); (11,40); (11,41); (12,23); (12,24); (12,25); (12,26); (12,27); (12,28); (12,29); (12,30); (12,31); (12,32); (12,33); (12,34); (12,35); (12,36); (12,37); (12,38); (12,39); (13,25); (13,26); (13,27); (13,28); (13,29); (13,30); (13,31); (13,32); (13,33); (13,34); (13,35); (13,36); (13,37); (14,27); (14,28); (14,29); (14,30); (14,31); (14,32); (14,33); (14,34); (14,35); (15,29); (15,30); (15,31); (15,32); (15,33); (11,44); (11,45); (11,46); (11,47); (11,48); (11,49); (11,50); (11,51); (11,52); (11,53); (11,54); (12,46); (12,47); (12,48); (12,49); (12,50); (12,51); (12,52); (13,48); (13,49); (13,50) }

Ore Grade ($g_{I,j,m}$) Sets for Cu, Zn and Ore/Waste Binary Information:

CU Grade Information in % (Block (I, j): $g_{i,j,(m=1)}$): { (8,19): 2.09; (9,19): 1.49; (10,19): 0.72; (4,19): 2.6; (9,20): 1.06; (10,20): 0.51; (10,21): 2.12; (10,22): 0.6; (5,22): 1.11; (5,23): 1.56; (6,23): 1.4; (4,25): 0.72; (5,25): 0.54; (6,27): 0.51; (6,28): 2.16; (11,48): 0.94; (12,48): 5.3; (13,48): 1.36; (12,49): 1.29; (13,49): 0.93; (12,50): 1.2; (13,50): 0.59; (3,30): 1.54; (3,31): 1.03; (3,32): 0.56; (3,33): 3.69; (15,29): 0.66; (15,30): 1.71; (15,31): 1.08; (15,32): 2.2; (15,33): 1.44; (14,32): 0.76; (14,33): 1.82; (4,33): 1.06; (4,34): 0.73; (4,35): 0.74; (5,34): 1.02; (6,37): 0.57; (6,38): 1.11; (9,45): 0.59 }

ZN Grade Information in % (Block (i, j): $g_{i,j,(m=2)}$): { (3,30): 1.54; (3,31): 1.03; (3,32): 0.56; (3,33): 3.69; (4,19): 2.6; (4,25): 0.72; (4,33): 1.06; (4,34): 0.73; 4,35): 0.74; (5,22): 1.11; (5,23): 1.56; (5,25): 0.54; (6,37): 0.57; (5,34): 1.02; (6,23): 1.4; (6,27): 0.51; (6,28): 2.16; (6,38): 1.11; (8,19): 2.09; (9,19): 1.49; (9,20): 1.06; (11,48): 0.94; (9,45): 0.59; (10,19): 0.72; (10,20): 0.51; (10,21): 2.12; (10,22): 0.6; (14,32): 0.76; (14,33): 1.82; (12,48): 5.3; (12,49): 1.29; (12,50): 1.2; (15,29): 0.66; (15,30): 1.71; (15,31): 1.08; (15,32): 2.2; (15,33): 1.44; (13,48): 1.36; (13,49): 0.93; (13,50): 0.59 }

Binary Ore Information Block (i, j): $g_{i,j,(m=0)} = 1$): { (9,45); (3,30); (3,31); (3,32); (3,33); (4,19); (4,25); (4,33); (4,34); (4,35); (5,22); (5,23); (5,25); (5,34); (6,23); (6,27); (6,28); (11,48); (12,48); (12,49); (12,50); (6,37); (6,38); (8,19); (14,32); (14,33); (13,48); (13,49); (13,50); (9,19); (9,20); (15,29); (15,30); (15,31); (15,32); (15,33); (10,19); (10,20); (10,21); (10,22) }