A Low-complexity Policy for Outage Probability Minimization with an Energy Harvesting Transmitter

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***Abstract*—Outage probability in an energy harvesting (EH) block-fading communication system is studied in the finite-horizon online setting. First, the offline version of the problem is considered, and formulated as a mixed integer linear program (MILP). Then, the infinite-horizon online problem (IIL) is considered relaxing the battery constraints. Solutions of these two problems provide lower bounds on the finite-horizon online problem, for which we provide a low-complexity heuristic scheme, called the fixed threshold transmission (FTT) scheme. Numerical results show that the FTT scheme achieves an outage performance close to the MILP lower bound for a wide range of operation regimes, and close to IIL when the EH rate is low. It is also observed that the power allocated by the FTT scheme resembles the optimal offline solution with high probability, despite the lack of information about future channel states and energy arrivals.**

***Index Terms*—energy harvesting, outage probability, power allocation, fading channels.**

I. Introduction

In this paper we study a point-to-point block-fading channel under strict delay constraints with an EH transmitter. A message of rate R is generated at each channel block, and due to strict delay constraints it has to be delivered by the end of that block; otherwise, the message becomes stale. Minimum required power to deliver a message depends on the state of the channel, which is constant within a block, and independent and identically distributed (i.i.d.) across blocks. When the actual transmit power is below the minimum required power for reliable transmission, an *outage* is declared.

We assume perfect causal channel state information at the transmitter (CSIT) and receiver. Rate adaptation is not possible, but the transmitter can adjust its power to minimize the *probability of outage*. In this model, transmitting at a power below the minimum required value is useless. Under a long-term average power constraint, a threshold-type power allocation policy is known to minimize the outage probability [1]. However, with an EH transmitter, the battery state becomes important for the optimal strategy. Substantial gains can be achieved by adapting the rate and power according to the channel and energy states [2]. In general, the performance and solution techniques depend on the causal or noncausal availability of CSIT and the energy state information (ESI) at the transmitter (ESIT). If the application is not delay-limited, under noncausal CSIT and ESIT, directional waterfilling optimizes the throughput [3], [4]. For causal CSIT and ESIT, the problem is modeled as a Markov decision process (MDP), and stochastic optimization techniques should be used [2].

In an EH system with delay constraints, energy unavailability becomes another reason for outages (in addition to deep channel fading). This is exacerbated with limited battery capacity. In [5], *average* outage probability is studied with an infinite battery when there is only channel distribution information (CDI) at the transmitter. Similarly, with only CDI at the transmitter, [6] extends [5] to a system with EH transmitter and receiver. In [7], an infinite battery is assumed and a general framework is set for utility maximization in EH communication systems. In [8], outage probability is studied in a delay-limited EH system with finite and discrete energy, channel and battery states. The problem is modeled as an infinite-horizon finite-state MDP, and the optimal strategy is characterized in the high signal-to-noise ratio regime.

In this paper, we study the finite-horizon outage probability, or equivalently the throughput, with causal CSIT and ESIT, a finite-capacity battery, and continuous energy, channel and battery states. We propose a low-complexity online scheme to minimize the outage probability. We also solve the offline and the infinite-horizon online problems to establish lower bounds on the performance. Numerical results show that the proposed scheme achieves an outage probability very close to the lower bound. We also investigate the effects of the battery size and number of blocks on the outage probability.

II. System Model

We consider delay-limited communication over a block fading channel. The channel gain in the nth block, $h_n, n = 1, \ldots, N$, is i.i.d. complex Gaussian. Complex additive noise is also Gaussian, with zero mean and unit variance. A message at rate R is generated at each channel block, and an *outage* is declared if it cannot be delivered within that block. Each block is large enough to invoke Shannon theoretic arguments.

We denote the size of the random energy packet harvested at the beginning of the nth block by $Q_n, n = 1, \ldots, N$, which are i.i.d. with mean Q_{av}. Harvested energy is first stored in a rechargeable battery of capacity E_{max}, and become available in the current block. We assume $Q_n \leq E_{max}$ without loss of generality. The state of the battery right after the arrival of the energy packet Q_n is denoted by E_n, where $Q_1 = E_1$ is the initial energy in the battery. The state of the system at block n is denoted by $S_n \triangleq (h_n, E_n)$.

We consider *online optimization* of the outage probability over N channel blocks, i.e., CSI (h_n) and ESI (E_n) are causally available at the transmitter and receiver at the beginning of each block. The transmitter adapts its transmission power, P_n, based on the current and past CSIs, $h_1^n \triangleq (h_1, \ldots, h_n)$, and ESIs, $E_1^n \triangleq (E_1, \ldots, E_n)$. We have $0 \leq P_n \leq E_n$. The outage probability can be expressed as

$$\mathbb{P}_{out} = \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}(\log(1 + |h_n|^2 P_n) < R), \tag{1}$$

where $\mathbb{1}(x) = 1$ if x holds, and 0 otherwise. We normalize the channel block duration such that consumed energy over one block is equal to the transmit power. Accordingly, the battery state is updated as follows

$$E_{n+1} = \min\{E_n + Q_{n+1} - P_n, E_{max}\}. \tag{2}$$

This is a finite-horizon continuous-state MDP, where the goal is to find the optimal action $\pi_n^*(S_n)$, $\forall n = 1, \ldots, N$, which maps each state S_n to a transmission power P_n. Before presenting a low-complexity solution for this MDP, we present two lower bounds on the performance.

III. Lower Bounds

A. *Offline Optimization*

In the offline version of the problem all the channel gains and energy arrivals are assumed to be known in advance. For each realization, i.e., for N channel blocks, the offline problem is stated as follows:

$$\min_{P_1,\ldots,P_N} \quad \mathbb{P}_{out} \tag{3}$$

$$\text{s.t.:} \quad \sum_{n=1}^{k} P_n \leq \sum_{n=1}^{k} Q_n \quad k = 1, \ldots, N, \tag{4}$$

$$\sum_{n=1}^{k} Q_n - \sum_{n=1}^{k-1} P_n \leq E_{max} \quad k = 1,\ldots, N, \tag{5}$$

$$0 \leq P_n \quad k = 1,\ldots, N. \tag{6}$$

We first highlight that the above offline formulation is significantly different from the offline throughput optimization studied in [9], [10], and other follow-up papers. The objective in (3) is not an increasing concave function of the transmission power P_n. In contrast, no gains can be obtained by increasing P_n beyond $P_{req,n}(h_n)$, where

$$P_{req,n}(h_n) \triangleq \frac{2^R - 1}{|h_n|^2} \tag{7}$$

is the minimum transmission power that guarantees successful decoding in block n. Therefore, as opposed to [9], [10], objective function cannot be always improved upon by increasing P_n to prevent overflows, and energy may be wasted inevitably; however, the problem formulation in (3)-(6) is still without loss of optimality.

We can rewrite the problem in (3) - (6) by replacing the indicator function in the objective with a simple sum by introducing N new binary variables $a_1^N \triangleq (a_1, \ldots, a_N)$, and $N-1$ energy state variables E_2^N to track the available energy for transmission. Thanks to this transformation, the equivalent optimization problem can be presented as a mixed integer linear program (MILP):

$$\min_{a_1^N, E_2^N} \quad \frac{1}{N}\sum_{n=1}^{N} a_n \tag{8}$$

$$\text{s.t} \quad E_n - E_{n-1} \leq Q_n - (1-a_{n-1})P_{req,n-1}, \tag{9}$$

$$(1-a_n)P_{req,n} \leq E_n, \quad n = 2, \ldots, N, \tag{10}$$

$$E_n \leq E_{max} \quad n = 1, \ldots, N, \tag{11}$$

$$a_n \in \{0, 1\}, \quad n = 1, \ldots, N. \tag{12}$$

Note that $Q_1 = E_1$ by definition, and hence, $N-1$ energy state variables, E_2^N, are sufficient. Constraints in (12) impose that the optimal power allocation assigns only two power levels to each block, either 0 or $P_{req,n}$. Due to the binary constraints, the optimization problem is a MILP, which is known to be NP-hard. It can be solved using the branch-and-bound algorithm. In order to obtain a lower bound on the online problem, we solve the offline problem for each realization of the energy and channel processes, and take their average.

Algorithm 1: Fixed Threshold Transmission Scheme

input : $P_{th}, E_{max}, \{Q_i\}_{i=1}^n, \{h_i\}_{i=1}^n$
output: $\{E_i\}_{i=1}^n, \{P_i\}_{i=1}^n$

$E_1 = Q_1$
for $n := 1$ **to** $N-1$ **do**
  $P_{req,n} = (2^R - 1)/|h_n|^2$
  if $P_{req,n} < \min\{P_{th}, E_n\}$ **then**
    $P_n = P_{req,n}$
  end
  else
    $P_n = 0$
  end
  $E_{n+1} = \min\{E_n + Q_{n+1} - P_n, E_{max}\}$
end
$P_N = E_N$

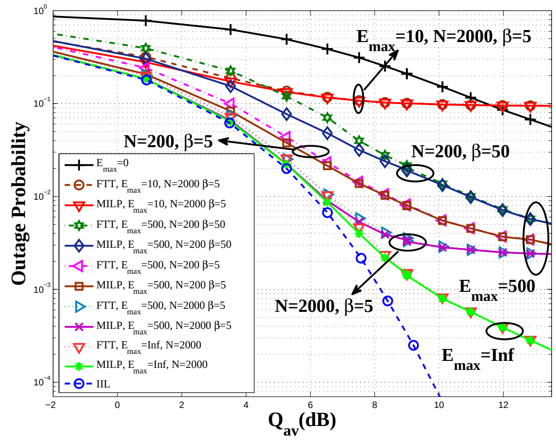

Fig. 1. Outage probability vs. average harvested energy for the IIL and MILP bounds and the FTT scheme under different battery constraints and block lengths.

B. *Infinite-Horizon Infinite-Battery Lower Bound (IIL)*

For the offline problem a trivial lower bound on the outage probability is obtained when all the harvested energy is assumed to be available initially, and the battery constraint is ignored. This is equivalent to solving (3) only under the total energy constraint, i.e., the constraint in (5) for $k = N$. It is not difficult to see that the optimal solution for this offline problem is threshold type; that is, the transmitter orders the channels such that $P_{req,o(1)} \leq P_{req,o(2)} \leq \cdots \leq P_{req,o(N)}$. Power is allocated to the K best channels that satisfy $\sum_{n=1}^{K} P_{req,o(n)} \leq \sum_{n=1}^{N} Q_n \leq \sum_{n=1}^{K+1} P_{req,o(n)}$.

More interestingly, this threshold-type policy achieves the optimal online solution with an infinite battery, when N, the number of channel blocks goes to infinity, thanks to the law of large numbers. Hence, we name this as the *infinite-horizon infinite-battery lower bound* (IIL). If the required transmission power for the nth block, $P_{req,n}$ is below a threshold value, P_{th}, then we set $P_n = P_{req,n}$. Otherwise, the transmitter remains silent, and $P_n = 0$. Since $|h_n|^2$ is a one-to-one function of $P_{req,n}$, we can equivalently consider a threshold value on the channel gain, denoted by ν_{th}. We have $\nu_{th} = (2^R - 1)/P_{th}$. For given Q_{av} ν_{th} is calculated by solving $Q_{av} = \int_{\nu_{th}}^{\infty} f(h) P_{req}(h) dh$, where we dropped the subscript n, as fading is i.i.d., and $f(h)$ is the probability density function of the channel gain h. Thus, for each Q_{av} there is a unique P_{th}, which determines the minimum outage probability [1].

IV. Proposed Heuristic Transmission Scheme

Common approach to solve the MDP characterized in Section II would be to discretize the state space and apply dynamic programming (DP) [3]-[8]. However, an accurate solution requires fine discretization of both the CSI and ESI, which, together with increasing N, explodes the complexity of the DP. Instead, we propose a low-complexity solution, called the *fixed threshold transmission* (FTT) scheme, and show that its performance is very close to the optimal. The motivation behind the FTT scheme is the following: We expect that the optimal online policy obtained through DP will be of threshold-type; that is, for each channel block, the optimal policy will transmit a message if the required energy is below a certain threshold, and drop it otherwise. However, the optimal policy may have a different optimal threshold value for each channel block and battery state. Considering the computational complexity of characterizing all these threshold values, we use a single threshold value, which is the optimal one when both N and E_{max} are large. We expect this scheme to perform well for large N values, and when the battery capacity is relatively large compared to the average EH rate. The FTT scheme mimics the optimal solution of IIL discussed in Subsection III-B. It assumes the same threshold value, P_{th}, to determine whether transmission will take place or not. If $P_{req,n} < \min\{P_{th}, E_n\}$ then the transmission takes place, and $P_n = P_{req,n}$. If $P_{req,n} > P_{th}$,no transmission is allowed, and $P_n = 0$. That is, if the channel gain in block n is of poor quality, and saving energy for future-channel blocks is a wiser choice. Similarly, if $P_{req,n} > E_n$, no power is allocated for transmission, i.e., $P_n = 0$. In both cases, the residual energy is saved, and the battery state is updated according to (2). The algorithm of FTT is given in Algorithm 1.

The FTT scheme is based on forward search, and its complexity is $\mathcal{O}(N)$. Note that FTT is an *online policy*, and only requires causal CSIT and ESIT.

V. Numerical Results

For numerical simulations we assume that the energy arrivals, Q_n, are i.i.d. gamma random variables with a

TABLE I
PERCENTAGE OF DISSIMILARLY ALLOCATED BLOCKS BETWEEN MILP AND FTT SCHEMES ($\beta = 5$)

$Q_{av}[dB]$	0	2	4	6	8
$E_{max} = 10$	2.65	2.11	1.53	0.87	0.63
$E_{max} = 500$	2.25	1.81	1.23	0.67	0.53
$E_{max} = \inf$	1.90	1.41	1.08	0.55	0.41

mean value of Q_{av} and a variance of βQ_{av}. The channel gain follows a Rayleigh distribution. In Fig. 1 the outage performances of the FTT scheme, offline MILP solution and the IIL are compared under different N, β and E_{max} values. Fig. 1 shows that the FTT scheme performs very close to the offline lower bound for all number of channel blocks, N, and battery capacity, E_{max}, when $\beta = 5$. A large β value indicates a fast changing EH process, and being able to store and adaptively reallocate energy packets among blocks becomes more important. As the FTT scheme uses a constant threshold for all β values, the gap between MILP and FTT is larger for larger β and moderate Q_{av} values. A typical value for β is reported to be less than 1 in [11]. Moreover, for high Q_{av}, battery capacity dominates the outage performance, and FTT again attains the MILP solution for all β values. The performance of FTT approaches that of IIL performance for low to moderate Q_{av} values as well. The figure also displays $E_{max} = 0$ curve for comparison. This curve assumes that arriving energy is immediately used in each block, i.e., $P_n = Q_n$, for all n.

Fig. 1 also shows that for finite battery capacity, both the proposed scheme and the offline solution exhibit an outage floor as Q_{av} increases. As the energy delivered is first stored in the battery, in this regime, most of the energy is wasted, and the outage probability is primarily determined by the battery capacity. If the system is modified such that the arriving energy packets can be used immediately for transmission and the residual energy is stored in the battery; then, as $Q_{av} \to \infty$ the outage performance of the FTT scheme would converge to the $E_{max} = 0$ curve in Fig. 1.

In Fig. 1, two curves for different number of blocks when $E_{max} = 500$ are also shown. For low to moderate Q_{av} values, outage events due to poor channel conditions are dominant. When battery constraint permits, saved energy is adequate to attain an outage performance close to the offline lower bound. In other words, the FTT scheme mimics the optimal threshold-type power control IIL implements, and achieves a very similar performance. When energy is available in advance, channel can be inverted for all channel gains larger than ν_{th}. However, when energy is delivered in packets, for some blocks, sufficient energy is not available, and the performance of an average energy-constrained system cannot be attained with an EH transmitter for finite N values. Outage events due to instantaneous energy insufficiency are likely to occur at the beginning until sufficient energy is accumulated in the battery. For shorter N, this *transient* period is more dominant, and the outage probability is larger for medium Q_{av} values. As Q_{av} goes to infinity, the outage probability eventually reaches its floor determined by E_{max}. For larger N, outage performance of both the MILP and FTT schemes improve. In other words, the number of blocks sets the speed of convergence to the outage floor. Note that, the probability of outage due to instantaneous energy insufficiency goes to zero when N goes to infinity, and the outage performance of the FTT scheme converges to IIL performance under the infinite battery capacity assumption.

Finally, the MILP and FTT schemes can designate different blocks for outage. Table I shows the percentage of such blocks, which is at most 3%. As the battery size and Q_{av} increase, this difference becomes negligible.

VI. CONCLUSION

We have studied the outage probability in a delay limited EH system, taking the finite battery constraint and the number of channel blocks into account. Due to the computational complexity of classical DP solution, a low-complexity *online* transmission scheme, called FTT, is proposed, and compared both with the offline solution, and the infinite-horizon system with an infinite capacity battery (IIL). A mixed integer linear programming (MILP) solution is presented for the offline optimization problem. The low-complexity online FTT scheme mimics the optimal threshold-type power control of IIL: If the channel quality is above a certain threshold and enough energy is available, the channel is inverted, and communication takes place; otherwise, an outage is declared, and no energy is wasted. The FTT scheme is shown to perform very close to the offline lower bound for practically relevant EH rates. For low to moderate EH rates, it also approaches the IIL performance. Our results also reveal that when the EH rate is high, limited battery capacity results in an outage floor, and the outage probability is larger for smaller block lengths.

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