AN INVESTIGATION ON 6^{TH} GRADE STUDENTS' MODELING PROCESS IN THE FERMI PROBLEMS CONSIDERING THE PROXIMITY OF THE PROBLEM CONTEXT TO STUDENTS' DAILY LIVES

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ABSTRACT

AN INVESTIGATION ON 6TH GRADE STUDENTS' MODELING PROCESS IN THE FERMI PROBLEMS CONSIDERING THE PROXIMITY OF THE PROBLEM CONTEXT TO STUDENTS' DAILY LIVES

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The purpose of this study is to investigate the modeling process of 6th grade students in the fermi problems, considering the proximity of the problem context to students' daily lives. The data were collected from 12 sixth students from public middle schools in different cities in Turkey. In this multiple-case study design, a selection test was applied to 20 students for selecting the case participants. Students' models and explanations were analyzed, and 12 students were selected and formed six groups based on their performances on the test as high achievers, medium achievers, and low achievers. This study involved two cases and cases were bounded by the order of the Fermi Problems asked. More specifically, Case 1 involved Fermi problems given in the order of close problem context first and remote problem contexts next, and Case 2 included Fermi problems given in the order of remote problem context first and closer problem contexts next. One group of sixth grade students from each performance category was placed in each of the two cases during the data collection. In other words, there were three groups at different achievement level in each case. Semi-structured clinical interviews with groups were carried out.

Students’ solution strategies were analyzed and compared with one another. Furthermore, researcher generated models were constructed based on students’ ways of solutions. Result of this study showed that the problems presented in a closer context to students' lives led to more detailed solutions because they could reconcile problems closer to their daily lives with the information they obtained from daily life. On the other hand, a noteworthy difference was not observed between Case 1 and Case 2. In other words, the order of the problems, whether it is from close problem context to remote problem context (Case 1) or from remote problem context to close problem context (Case 2) did not lead to an observable difference on sixth grade students’ way of solution. This indicated that teachers do not need to create a lesson plan by ordering the problems from close problem context to remote problem context in the studies they will do with Fermi problems.

Keywords: Fermi Problems, Mathematical Modeling, Students’ modeling performances, Context of the problem

ÖZ

6. SINIF ÖĞRENCİLERİNİN FERMİ PROBLEMLERİNDEKİ MODELLEME SÜREÇLERİNİN PROBLEM BAĞLAMININ GÜNLÜK HAYATLARINA YAKINLIĞINA GÖRE İNCELENMESİ

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Bu çalışmanın amacı, 6. sınıf öğrencilerinin Fermi problemlerine geliştirdikleri modelleri, problem bağlamının günlük yaşamlarına yakınlığı göz önünde bulundurarak incelemektir. Veriler, Türkiye'nin farklı illerindeki devlet ortaokullarında okuyan 12 altıncı sınıf öğrenciden toplanmıştır. Bu çoklu vaka çalışmasında, katılımcıları seçmek için 20 öğrenciye bir seçme testi uygulanmıştır. Öğrencilerin bu testte bulunan Fermi problemlerine geliştirdikleri modeller ve problem çözümüne dair açıklamaları incelenmiştir. 12 öğrenci seçilmiş ve testteki performanslarına göre yüksek başarılı, orta başarılı ve düşük başarılı olarak her seviyede iki grup olmak üzere toplam altı grup oluşturulmuştur. Bu çalışma iki vakayı içermektedir. Bu vakalar sorulan Fermi problemlerinin sırasına göre oluşturulmuştur. Vaka 1'de Fermi problemleri yakın problem bağlamından uzak problem bağlamına doğru sıralanmıştır. Vaka 2'de ise Fermi problemleri uzak problem bağlamından yakın problem bağlamına doğru sıralanmıştır. Farklı seviye gruplarındaki öğrenciler, veri toplama sırasında grup olarak iki vaka çalışmasından birine yerleştirmiştir. Diğer bir değişle, her vakada yüksek başarılı, orta başarılı ve düşük başarılı olmak üzere farklı başarı seviyesinde üç grup yer almıştır. Gruplarla yarı yapılandırılmış klinik mülakatlar yapılmıştır. Öğrencilerin çözüm stratejileri analiz edilmiş ve birbirleriyle karşılaştırılmıştır. Ayrıca, öğrencilerin çözüm yollarından yola çıkılarak araştırmacı tarafından genel matematiksel ifadeler içeren modeller oluşturulmuştur. Bu çalışmanın sonucundan, öğrencilerin yaşamlarına daha yakın bir bağlamda sorulan Fermi problemleri ile günlük yaşamdan edindikleri bilgiler arasında daha fazla bağlantı kurarak daha ayrıntılı çözümler geliştirdikleri görülmüştür. Öte yandan, vaka 1 ile vaka 2 arasında kayda değer bir fark gözlenmemiştir. Özetle, problemlerin sırası, ister yakın problem bağlamından uzak problem bağlamına (Vaka 1), ister uzak problem bağlamından yakın problem bağlamına (Vaka 2) doğru olsun altıncı sınıf öğrencilerinin çözüm yollarında gözlemlenebilir bir farklılığa yol açmamıştır. Bu durum öğretmenlerin Fermi problemleri ile yapacakları çalışmalarda problemleri yakın problem bağlamından uzak problem bağlamına doğru sıralayarak bir ders planı oluşturmalarına gerek olmadığını göstermiştir.

Anahtar Kelimeler: Fermi Problemleri, Matematiksel Modelleme, Öğrencilerin Modelleme Preformansı, Problem Bağlamı

To my mom

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ABBREVIATIONS

FP: Fermi Problem

MEAs: Model Eliciting Activities

CRC Group: Groups who solve the problems from the close problem context to the remote problem context.

RCC Group: Groups who solve the problems from the remote problem context to the close problem context.

NCTM: National Council of Teachers of Mathematics

CCSSM: Common Core State Standards for Mathematic

CHAPTER 1

INTRODUCTION

Since late 1960s, the study of mathematical modeling in mathematics education has been a constantly developing topic of research (Blum, 1995). This is because mathematical modeling has been proposed to improve students' ability to deal with real-world problems in the education (Lesh & Doerr, 2003).

In the last two decades, several studies have emphasized the challenges students have experienced while dealing with real word problems (Peter-Koop, 2005). The process of constructing a mathematical model to solve real-world problems is known as mathematical modeling and it is one way of utilizing mathematics to address real-world problems (Berry & Houston, 1995). A mathematical model is a set of meaningful constructions and representations that establish a link between mathematics and reality (Starfield et al., 1990). Mathematical modeling involved the process of solving real-world and non-routine problems.

Modeling problems are essential because they are open-ended instead of asking for a single valid response and solution and ready-made patterns (Kertil, 2008). The activities in the mathematical modeling process are constructed in the context of the students' interests in such a way that motivates them to examine and explain the real-world problem context (Doruk, 2011). Students construct models about the situation and analyze the application of their mathematical skills to examine problem situations that are connected to real life in the modeling process. Instead of a single correct response, these problems require several solutions. The growth of understanding, analyzing, and estimating variables is enhanced by this mathematical thinking process (Swan et al., 2007).

These open-ended problems, related to real world context, have multi-correct answers, and these types of problems are also known as non-routine problems. Non-routine problems are kinds of problems that allow students to utilize their creativity (Abay & Filiz, 2020). Fermi problems could be viewed as an example of non-routine problems. Before beginning to solve Fermi problems, students need to develop estimations about variables given in the problem (Abay & Filiz, 2020). Fermi questions use an educated estimation process that relies on making reasonable assumptions, rather than on searching for known data related to the question (Francisco & Anderson, 2010).

Fermi Problem computation could be used for a variety of reasons: (i) providing estimates for a study before it begins, (ii) allowing individuals to scope out the resources needed to complete it, (iii) estimating the feasibility of an opportunity, and (iv) determining whether an answer gained makes sense and providing the basis for a discussion (Abrams, 2011). The context of Fermi problems enables students in generating responses, deciding if a way of thinking is effective or not, and setting the conclusion into context familiar to them (English, 2015). Fermi problems based on mathematical modeling provide students a new perspective on mathematics, allowing them to see how the problem could be solved in different contexts (Albarracin & Gorgorió, 2012). Therefore, it is important to apply mathematical modeling activities based on Fermi problems.

Model eliciting activities (MEAs) are also found effective for improving essential and advanced thinking skills, and they provide a new and effective educational atmosphere for revealing deficiencies in existing theoretical knowledge and gaining new mathematical knowledge (Chamberlin, 2004). In terms of the benefits that MEAs bring to students and instructors, it is possible to argue that they are extremely essential. It has been observed that MEAs support the development of students as effective individuals (Doruk & Umay, 2011). The emphasis on MEA(s), which are implementations of real-life problem-solving examples by eliciting students' assessments of understanding, is a distinguishing feature of modeling (Lesh et. al., 2000). MEAs are effective for creating models suitable for problems. Besides, Fermi problems are also seen to be effective in creating models since they are open-ended and related to real life (Albarracín & Gorgorió 2014). Therefore, I, as the researcher of this study, have put forward a qualitative educational multiple-case study design to investigate the differences in 6^{th} grade students' models in the fermi problems considering the proximity of the problem context to their daily lives.

1.1 Purpose of the Study and Research Question

The main purpose of this study is to investigate the differences in 6^{th} grade students' models in the fermi problems considering the proximity of the problem context to their daily lives. Specifically, this research was conducted to understand the following research questions:

1. How do 6^{th} grade students' ways of solution (i.e. models) differ based on their initial performances in Fermi Problems?

1.1. What do high-achieving 6^{th} grade students develop as models in Fermi Problems?

1.2. What do mid-achieving 6^{th} grade students develop as models in Fermi Problems?

1.3. What do low-achieving 6^{th} grade students develop as models in Fermi Problems?

2. What is the role of the problem context in 6^{th} grade students' models in the fermi problems?

3. What is the role of the proximity of the problem context to students' daily lives in 6^{th} grade students' models in the fermi problems?

Based on the literature, a set of fermi problems was developed and implemented to accomplish this goal. Semi-structured clinical interview questions were used to investigate the differences in 6th grade students' models in the fermi problems considering the proximity of the problem context to their daily lives.

1.2 Significance of the Study

Despite the fact that mathematical modeling activities including non-routine problems have grown increasingly important, it has been noted that mathematics educators in schools do not employ them (Akgün, 2013). In order for mathematics educators to continue these studies, examples of these studies need to be included in the literature. Fermi problems are one of the types of problems that teachers can use to support the mathematical modeling process in the education (Filiz & Abay, 2020).

There have been many studies in the literature that aim to determine mathematical modeling process (e.g., Blum & Leiss 2007; Haines & Crouch 2010; Matsuzaki 2011, among others). However, I encountered a few studies dealing with Fermi problems that reveal mathematical modeling skills in the literature (e.g., Ärlebäck, 2009; Albarracín & Gorgorió, 2014; Efthimiou & Llewellyn, 2007; Peter-Koop, 2004, among others). Therefore, it is important to carry out this study in order to be a guide for teachers. Conducting this study will also encourage similar studies to be carried out. In this way, it can even be seen that there could make updates to the Turkish Education curriculum in the coming years. In addition to these, this study focuses on the role of the proximity of problem context and the sequence of the proximity of the problem context in Fermi problem-solving process. Some researchers focus on the context in their research based on Fermi problems (e.g., Albarracín & Gorgorió, 2014) and highlighted the role of the context on students' making sense of the problems and developing a model. Although students' way of solutions in the fermi problems considering the proximity of the problem context to their daily lives have been studied in some of these studies, there was no study that examined the fermi problems considering the sequence of the proximity of the problem context to their daily lives. Conducting this study allows teachers to explore the relationship between the models developed for Fermi problems and the closeness of their context to daily life. Teachers can plan lesson plans about Fermi problems by examining how important the sequence of the proximity of context is in their studies. In this way, they can determine the best way to develop the mathematical modeling process suitable for their students.

In the present study, fermi problems considering the proximity of the problem context to students' daily lives was studied. Therefore, this study aimed to contribute to the literature by presenting how students might find answers to Fermi problems in contexts at different proximities by using discussion, argumentation, and estimation in the real world. This study is a guide for teachers to include Fermi problems while preparing lesson plans that support the mathematical modeling process. Teachers can examine what gains students can achieve when they work with Fermi problems. They can examine the role of the proximity of context in models created for Fermi problems. They can examine the effect of students' levels on their work with Fermi problems, and they can form their study groups by taking this study into account. As a result of all this work, teachers can decide how to plan mathematical modeling processes using Fermi problems.

1.3 Motivation for the Study

This research can be seen as a good example of how teachers might contribute to the course process by including Fermi problems in the mathematical modeling process. The findings of this study can inform teachers whether it would be beneficial for their students to integrate Fermi problems into their lessons. They can plan their lessons in line with this study. One of the most distinguishing characteristics of Fermi problems is that they are based on real-life situations and do not include numbers (Peter-Koop, 2015). While working with these problems, students use the knowledge they have gained from their real-life experiences. Based on this information, they need to estimate realistic or approximate numerical values. In the teaching and learning of mathematics, Fermi Problems are related to estimation methods (Ärlebäck & Albarracín, 2019).

These estimation process needs to analyze their existing knowledge and interpret the relationship between students' information and the problem. Students develop their number sense during this estimation process (Ärlebäck & Albarracín, 2019). This process also contributes positively to the mathematical modeling process. If teachers do not use Fermi problems in their lessons, they miss a good opportunity for students to develop their estimation, argumentation, and problem-solving skills.

1.4 Definitions of Important Terms

Definitions of important terms for working are given in this section.

Model: Doerr and Lesh (2003) defined models as "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (p. 13). In this study, models indicated the structure of the students' ways of solutions, and the researcher generated expressions based on students' ways of thinking. Specifically, expressions generated in this research to explain the answers produced by students to Fermi problems led to generalizing them to the relevant conceptual systems.

Fermi Problems (FP): Ärlebäck (2009) defined non-standard problems as "an open problem requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations" (p.335). Three different Fermi problems were studied by students in this study.

Mathematical modeling: Mathematical modeling translates a real-world problem (Fermi Problems in the study) into a mathematical problem that can be solved by analyzing the mathematical assumptions (Czocher, 2017, p.129). In the study, students' mathematical modeling processes were investigated. Students' calculation and deduction processes are only part of what mathematical modeling involves. It is based on the observation of a pattern, the evaluation of conjectures, and the estimation of results (Schoenfeld, 1992).

The Proximity of the problem context: The proximity of the problem context creates a setting in which we may examine the evaluation of students' perceptions of the problem situation as their reasoning about important system components (Doerr & English, 2003, p.110). It is an evaluation of situations where students are less encountered to situations with more encountered in their real life. For example, the toilet paper roll question is related to the students’ everyday life experiences. But the question of number of shopping mall in Turkey is a situation that does not come up every day and that students may not even think about.

CHAPTER 2

THEOTETICAL FRAMEWORK

This chapter sets up the theoretical framework for the current study which investigates the differences in 6^{th} grade students' models in the fermi problems considering the proximity of the problem context to their daily lives. For this reason, the related studies on mathematical modeling in mathematics education, mathematical modeling approaches, Fermi problems, and model eliciting activities have been reviewed. The chapter also includes a part on Fermi problems that is based on model eliciting activities.

2.1 Mathematical Modeling in Mathematics Education

Problem-solving skills have an important role in helping students to cope with the difficulties they encounter. One of the goals of mathematics education is to develop these skills (Artun et. al., 2001). As a result, students need to learn mathematics in such a way that they can develop efficient solutions to real-world problems and utilize mathematics successfully in their daily lives (Gutiérrez & Gallegos, 2019). Contrary to this situation, students believe that the mathematics applied in real life problem-solving, and the mathematics studied in mathematics courses are significantly different. They believe that problem in mathematics lessons has only one correct answer and this correct answer can be reached with only one solution (Altun & Arslan, 2006). Students generally focus on the answer. They do not focus on the process of solution steps or different solutions to their questions. Students consider the purpose of math lessons as completing their workbooks, finding answers to a variety of problems, listening to the teacher's comments, and solving all questions as rapidly as possible. They do not see essential explanations for their solutions as part of the math process (Silver & Smith, 1996).

Educators consider the study of mathematical modeling as a teaching strategy to develop the students' idea of integrating mathematics with life and problem-solving skills in the real world (Gutiérrez & Gallegos, 2019). The effectiveness of mathematical modeling in developing real-world problem-solving skills and in establishing and retaining motivation to study mathematics are the two most fundamental reasons for using it as a teaching strategy (Czocher, 2016). To be more precise, mathematical modeling is utilizing mathematical methods to determine a real-life or real problem situation (Erbaş et al., 2014). One of the goals of using mathematics to solve real-world problems is to construct a mathematical model that properly represents the situation (Berry & Houston, 1995).

A model combines conceptual structures and their external forms used to analyze complex structures and mathematical systems in the mind (Lesh & Sriraman, 2005). In other words, the model includes both internal conceptual systems and the external representations systems used to expound complex systems (Lesh & Doerr, 2003). These conceptual systems could represent several tools like spoken or written language, graphs, concrete models, and metaphors (Lesh & Sriraman, 2005). On the other hand, Lehrer and Schauble (2003) describe the model as a connection between a system that we are not familiar and systems we know before.

The process through which there is an attempt to make the real-world systems meaningful with the help of these models is called mathematical modeling. Mathematical modeling means much more than calculation and deduction. It is based on making an observation of pattern, evaluation of conjectures and making an estimation related to results (Schoenfeld, 1992). Although this mathematical modeling process is still a controversial topic among the mathematical modeling community, the prevalent view presents it as a cyclic process (Cai et al., 2014). Many definitions have been made from the past to the present, describing this cyclic process structure of mathematical modeling (Blum & Leiß, 2007).

These cyclic processes of mathematical modeling were put forward by National Council of Teachers of Mathematics (NCTM, 1989). Five stages were determined to express a successful modeling. These stages emphasized the necessity to (*i*) construct a simplified and identified version of the real-world problem situation, *(ii)*construct a mathematical model, *(iii)* solve the model, *(iv)* evaluate model, and *(v)* confirm and use model (NCTM, 1989). There is model of the modeling process, which describes these five stages emphasized in mathematical modeling by NCTM as seen in Figure 2.1. If the model's conclusion does not "answer" the current problem, the problem solver moves back to stage (*ii)* and starts over. This modeling is commonly described as a cyclic process (Hodson, 1995).

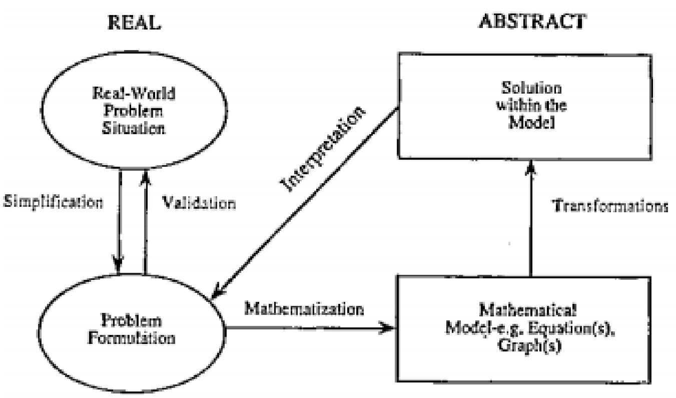

Figure 2.1 Mathematical modeling process (NCTM, 1989)

The mathematical modeling process starts with a problem encountered in a real-life problem, as seen in Figure 2.1. This problem situation could be broken down into simpler parts to make the problem more understandable. The problem formulation is modeled with appropriate mathematical expressions. This modeling process serves to understand the problem situation better and make it generalizable to other suitable problem situations. Whether it helps solve the problem is analyzed by interpreting the created model. The process and solution are validated if it is a model for solving a real-life problem. If this model does not serve to solve the problem, the whole process is renewed to create a suitable model.

Different studies have been done on this model of mathematical modeling. Blum and Leiss (2007) have reinterpreted the stages in the mathematical modeling process, as seen in Figure 2.2. In the first step of the mathematical modeling cycle, students need to understand the problem which occurs in the real world. Then, they construct the model. This model is called the situation model. In the next step, students simplify and structure the problem to create the real model. This model likely has internal and external components. The modeler illustrates the real mathematical model based on mathematical calculations. The results are then checked against the model to ensure that they are correct. Finally, researcher reveals or shares their model with others (Czocher, 2017).

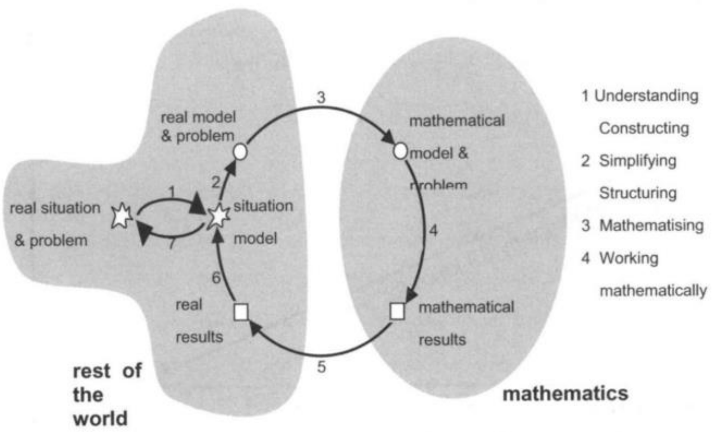

Figure 2.2 Blum and Leiss modeling cycle (Blum & Leiss, 2007, p. 131)

In addition to these, modeling is a K–12 mathematical practice requirement according to the Common Core State Standards for Mathematics (CCSSM, 2010). Mathematic modeling is also explained by CCSSM as “the process of choosing and applying proper mathematics and statistics to analyze empirical situations is the process” (CCSSI, 2010, p. 72). The modeling process of CCSSM includes six stages as seen in Figure 2.3.

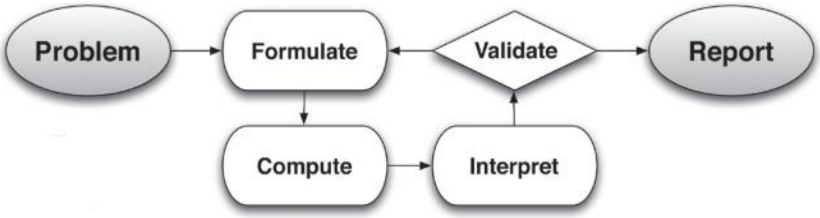

Figure 2.3 The mathematical modeling cycle of the Common Core State Standards for Mathematics (CCSSI, 2010, p. 72).

The first stage of the diagram involves identifying the variables and selecting them as a feature of the problem. In the second stage, a model is created. In the third stage, these models are analyzed, and necessary calculations are made. In the fourth stage, the result of the calculations is interpreted. In the fifth stage, a situation assessment is made with the results obtained and the models are updated if necessary. At the last stage of the modeling cycle, the whole process is reported. Modeling as a mathematical practice is promoted by understanding the elements that modeling problems contain. Therefore, examining these elements in detail is important to understand the mathematical modeling process. Anhalt and Cortez (2015) explained these elements in the model of mathematical modeling cycle in more detail as seen in Table 2.1

Table 2.1 Elements of f Mathematical Modeling and Description of the Work That They Entail (Oropesa & Cortez, 2015, p. 447)

Modeling Element	What It Entails
1. Analyze the situation or problem	•Identify a problem taken from an external context (often from an everyday life context) that must be solved or a situation that must be understood and explained. • Do background research if necessary. • Make sense of the situation or problem and understand the question.

Table 2.1 (continued)

2. Develop and formulate a model	•Determine all given information. • Determine what assumptions are necessary. • Translate the information given in the problem together with the assumptions into a mathematical problem that can be solved. • Use mathematics appropriate for the information given and assumed as well as the students' expertise
3. Compute a solution of the model	• Solve the mathematical problem stated in the model. • Analyze and perform operations in the model. • Check for correctness
4. Interpret the solution and draw conclusions	• Interpret the mathematical solution in terms of the original situation. • Draw conclusions that the solution implies about the original situation.
5. Validate conclusions	• Reflect on whether the mathematical answer makes sense in terms of the original situation (e.g., is the answer within a valid range of values?). • If the conclusions are satisfactory with regard to the accuracy needed, report the solution. If the conclusions are not satisfactory or need to be improved, go back to stage 2 ("Develop and formulate a model").

Table 2.1 (continued)

6. Develop and formulate a new or modified model	• Revise the assumptions made according to what was learned in the first solution and translate them into a new or modified mathematical problem that can be solved. • The type of mathematics in the current model may be different from the previous one. • Go through these stages again: Compute, Interpret, and Validate.
Report the solution	• Share your conclusions and the reasoning behind them.

Understanding the modeling elements that this table represents is an important part of understanding the modeling process. Analyzing the situation or the given problem is explained as the first step. At this stage, the problem situation should be clarified. After this stage, it is necessary to determine all the available information about the solution to the problem, decide which assumptions to make, and estimate them. In the third stage, it is expected that the model created in the previous stage will be implemented, and the problem would be solved. This solution is interpreted in the next step. This way, it is evaluated whether a solution suitable for the actual problem situation has been made. As a result of this evaluation, either the solution suitable for the model is verified, or a new model is created, and the process is repeated.

As a result, the decision making process that students go through while constructing a model is one of the features of mathematical modeling. Making assumptions is especially essential because it demands students to take proactive actions in the knowledge that their choices will aid in the development of an acceptable model. It is vital to distinguish between assumptions, which have a direct impact on the model, and procedural decisions. It is beneficial to engage the students in a debate about the influence of their assumptions as they are forming them (Anhalt & Cortez, 2015).

To sum up, the purpose of mathematical modeling is to predict, explain, describe, and understand different aspects of the real world. The goal of mathematical modeling activities is to produce tools that can be used to make decisions, rather than to solve the problem. (Lesh et al., 2000). Additionally, mathematical modeling is a problem situation with various possible solutions that expresses non-routine real-world situations, requiring students to mathematically interpret the situation and describe or formulate the process or method in order to aid individuals who will benefit from this situation (Lesh & Zawojewsky, 2007; Mousoulides, 2007).

2.2 Mathematical Modeling Approaches

Different approaches to modeling in mathematics education have been offered, each with its own theoretical perspective (Kaiser & Sriraman, 2006). To clarify the different perspectives on this issue, and the similarities and differences need to be elaborated (Erbaş et al., 2014). There are two main perspectives in mathematical modeling. The first method, which sees mathematics as a goal, and the second, which sees it as a tool, are two different approaches to mathematical modeling. (Erbaş, et al., 2014). Another classification of the modeling perspectives (Kaiser & Sriraman, 2006) shows six perspectives: realistic, contextual, educational, epistemological, socio-critical and cognitive as seen Table 2.2

Table 2.2 Classification of current perspectives on modelling (Kaiser, 2006, p. 302)

Name of the perspective	**Central aims**	**Relations to earlier perspectives**	**Background**
Realistic or applied modelling	Pragmatic-utilitarian goals, i.e.: solving real world problems, understanding of the real world	Pragmatic perspective of Pollak	Anglo-Saxon pragmatism and applied mathematics

Table 2.2 (continued)

Contextual modelling	Subject-related and psychological goals, i.e. solving word problems	Information processing approaches leading to systems approaches	American problem solving debate as well as everyday school practice and psychological lab experiments
Educational modelling; differentiated in a) **didactical** modelling and b) **conceptual** modelling	Pedagogical and subject-related goals: a) Structuring of learning processes and its promotion b) Concept introduction and development	Integrative perspectives (Blum, Niss) and further developments of the scientific-humanistic approach	Didactical theories and learning theories
Socio-critical modelling	Pedagogical goals such as critical understanding of the surrounding world	Emancipatory perspective	Socio-critical approaches in political sociology
Epistemological or theoretical modelling	Theory-oriented goals, i.e. promotion of theory development	Scientific-humanistic perspective of "early" Freudenthal	Roman epistemology
The following perspective can be described as a kind of meta-perspective:			
Cognitive modelling	Research aims: a) analysis of cognitive processes taking place during modelling processes and understanding of these cognitive processes		Cognitive psychology

Table 2.2 (continued)

	Psychological goals: b) promotion of mathematical thinking processes by using models as mental images or even physical pictures or by emphasizing modelling as mental process such as abstraction or generalization	

A realistic perspective aims to solve real life problems. It also focuses on understanding the process of mathematical modeling. On the other hand, contextual modeling, aims to work on a specific subject. Within the context of contextual modeling, modeling activities created a set of instructional principles. The activity needs to appear to be meaningful and allow for the construction of models (Zawojewski et al. 2003). Examining these activities could be important for understanding the mathematical modeling process. In the next section, model eliciting activities, which is one of these activities, will be examined. Another modeling perspective is educational modeling. Educational modeling is examined under two subheadings. These are didactical and conceptual modeling. Just like in contextual modeling, educational modeling also focuses on subject related issues.

Socio-critical modeling focuses more on the pedagogical-related subjects. Different from other perspectives, cognitive modeling perspective is examined under the meta-perspective. The table also includes details about the backgrounds of these perspectives.

In this study, students' models in Fermi problems are studied, taking into account the proximity of the problem context to their daily lives. The students solved the problem situations related to their daily lives. Therefore, it can be said that this study is carried out with a contextual modelling perspective. In line with the explanations as seen in the Table 2.2, the mathematical modeling perspective of this research can be clearly distinguished from other perspectives.

As mentioned earlier, modelling activities within the framework of contextual modelling is a good example for understanding the mathematical modeling process. Therefore, the next section will focus on these model eliciting activities to identify the mathematical modeling process.

2.3 Model Eliciting Activities

According to the modeling approach to problem solving, there is no unique powerful procedure between givens and aims, nor a set of techniques for overcoming any obstacles in these procedures between givens and aims to successfully solve a problem (Mousoulides et al., 2007). In addition, Polya (1991) stated that when a student solves a real-world problem using equations, he or she learns that mathematical concepts may be linked to realities, but that this relationship must be correctly worked out.

The many instruments being planned and built to facilitate students' and instructors' externalization of their thinking and knowledge of problem situations are referred to as Model Eliciting Activities (MEAs) by researchers (Lesh et al., 2003). MEAs, as well as its modifications, are a type of case-study that small groups commonly solve in one or two class periods. Because they entail mathematizing by quantifying, dimensioning, coordinating, classifying, algebraizing, and systematizing important things, connections, actions, patterns, and rules, MEAs can be intended to lead to major aspects of learning (Mousoulides et al., 2007). In other words, MEAs are a type of problem that small groups of three to five students try to solve across one or two class periods, simulating actual, real-world problems (Hamilton & Lesh, 2008). What the MEA is, how it is designed, and beyond the design principles, what are other important findings and premises about the MEA approach are explained in Table 2.3 in detail (Hamilton & Lesh, 2008).

Table 2.3 Characteristics of Model-Eliciting Activities (Hamilton & Lesh, 2008, p. 25)

What is an MEA?	An MEA is problem that simulates authentic, real-world situations that small teams of students work to solve over one or two class periods. The crucial problem-solving iteration of an MEA is to express, test and revise models that will solve the problem.
How is an MEA designed?	There are six design principles developed over several years of testing in mathematics and engineering courses.
Beyond the design principles, what are other important findings and premises about the MEA approach?	Elicitations of models and of systems thinking is emphasized in contrast to imparting ideas to be used in problem solving. Local concept development: A team's iterations through the "express-test-revise "cycle of model revision can yield new cognitive structures and understandings in the team members

Table 2.3 (continued)

	more effectively than single iteration application of textbook formula.
	The solution orientation of MEAs enables crucial development of complex Reasoning processes and suggests an alternative balance for how “product” and “process” are emphasized in the curriculum.

The aim of Model eliciting activities (MEA) is to encourage students to generate descriptions, explanations, and constructions that reveal how they interpret situations (Albarracín & Gorgorió, 2014). MEAs provide the critical development of complex problems in the context of their solution orientation.

Three categories of products are used in model eliciting activities: tools, constructs, and problems. Following examples could be given for tools and constructs. Models, descriptions and explanations, design and plans, and evaluation instruments are examples of tools that provide a functional role. Complex products, cases, and assessments are all examples of constructions (Mousoulides et al., 2007). MEAs are activities that encourage students to create models. They are established as open-ended problems that require students to construct models in order to solve complicated, real-life situations. Six key principles of MEAs are as seen in Table 2.4 They could be used to engage students in reasoning and thinking, as well as give researchers and teachers a better understanding of what students are thinking about.

Table 2.4 Six Principles to Write Effective Model-Eliciting Activities (Hamilton & Lesh, 2008, p. 25)

Principles for MEA Design	
Reality Principle (the "Personally Meaningful" Principle):	Could something like this happen in real life?
Model Construction:	Is it necessary to design, explain, estimate, or oversee a structural system?
Model Documentation:	Will students be required to express how they feel about the issue in their response? (aim, solution ways, givens, relations, objects)
Self-Evaluation:	Will pupils be able to judge when their replies are adequate on their own?
Model Generalization:	It is seldom useful to construct a conceptual tool (model) in "real life" contexts if the tool will only be utilized once. Model generalization principle stated that 'Is the model not just powerful but also shareable and reusable?'.
Simple Prototype:	Will the answer serve as models for understanding other conceptually related situations?

While designing the model eliciting activity, the selected context needs to be related to real life. The problem asked in the context of real-life needs to be analyzed, necessary estimations need to be made and a model should be created in this direction. Students need to express the solutions developed to the problem and their goals. They need to be able to make an individual evaluation of their modeling process. They need to discuss the validity of the model developed for solving the real-life problem for other similar situations and construct a general phrase according to the situation.

A local context that contextualizes the task is critical for MEAs. The context helps students generate answers, make decisions about whether a method of thinking is useful or poor, and put the outcome in perspective in a context that is familiar to them (English & Lesh, 2003). Model eliciting activities enable students to create a product for a specific context that can be used in different contexts and implemented with sequences which allow students develop their models.

In order to examine the mathematical modeling process, there are researchers who use Fermi problems as well as model eliciting activities. A wide review of Fermi problems made in this field can be found, for instance in Albarracin and Gorgorió (2014), and Arlacbeck (2009). Fermi problems were mentioned in the next section in detail.

2.4 Understanding Mathematical Modeling through Fermi Problems

Mathematical problems can be varied from problems with different ways of solving them to routine problems and with only one correct answer and open-ended, non-routine problems that every individual can interpret differently and whose outcomes vary from person to person. There is no single correct answer to open-ended problems and thus these are also called non-routine problems. These problems are important parts of the mathematics teaching program (Abay & Filiz, 2020). Based on this statement, problems should range from common mathematical problems to complicated problems with unknown solutions. The solutions of these complex problems are directly related to the mathematical thinking processes (Akay et al., 2006, p.129). Mathematical thinking process could be improved through mathematical modeling. Fermi problems could be employed to encourage and emphasize mathematical modeling processes (Peter-Koop, 2010)

Ärlebäck (2009) states Fermi problems as "Open, non-standard problems requiring the students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations" (p. 331). Palau et al. (2017) argue that this definition does not address Fermi problems completely. Fermi problems could be characterized as a form of activity that requires the simplification and mathematization of a reality requiring estimations (Palau et al., 2017). In other words, Fermi problems are the kind of problems that can be solved by dividing them into smaller parts and making the necessary estimations. This process is based on breaking problem into subproblems that can be solved separately utilizing their individual estimates (Albarracin & Gorgorio, 2014).

Ärlebäck (2011) states a number of reasons why working on Fermi problems could be a method to introduce modeling into the classroom.

(i) Students of all educational levels can access them.

(ii) To be realistic modeling, Fermi problems have a clear real-world relationship.

(iii) Problem solving needs the identification and categorization of important information and connections.

(iv) Students need to make reasonable estimates of relevant values because of the absence of numerical data.

(v) Fermi problems support working in a discussion environment.

These factors shows that Fermi problems provide an opportunity to examine students' modeling performance independent of existing modeling experience (Palau et al., 2017). Students are able to work on real-world context while they are solving Fermi Problems. The goal of presenting problems with real-life contexts is to move the mathematics classroom closer to reality and provide opportunity to practice multiple aspects of problem solving (Verschaffel et. al., 2002). Mathematical concept and creativity are needed to make the proper model for problems in real life context (Albarracín & Gorgorió, 2012). Students need to make estimations based on Fermi problems to construct models. Instead of searching for known facts relating to the issue, Fermi problems employ an informed estimating method that depends on establishing reasonable assumptions. Also, fermi problems enable students to work cooperatively with their friends (Zawojewski et al., 2003). Working in cooperative groups, students discover that the problem-solving knowledge and methods already exist inside the group (Clarke & McDonough, 1989). Working as groups is essential for the students to make discussions about estimations. The real-life context of the Fermi problem enables students to have more authenticity in their estimations. Additionally, fermi problems based on mathematical modeling provide students with a different view on mathematics, helping them to understand its utility in solving problems in their proximity (Albarracín & Gorgorió, 2012).

CHAPTER 3

METHODOLOGY

The purpose of this study is to investigate 6th grade students' models in the Fermi problems considering the proximity of the problem context to students' daily lives. In this chapter, the research design, the study participants, the data collection and analysis procedures, the role of the researcher, and the trustworthiness and credibility of the research are presented.

3.1 Research Design

This study utilized case-study research design that was utilized for understanding a phenomenon and explaining the case from different perspectives (Yin, 2009). The phenomenon under investigation were 6th grade students' models in the Fermi problems considering the proximity of the problem context to students' daily lives. According to Yin (2009), there are four types of case studies: a single-case holistic design, single-case embedded design, multiple-case holistic design, and multiple-case embedded design Studies are called single case or multiple cases with respect to the number of cases. If the research includes one case, it is called a single case design; if it contains more than one case, it is called the multiple-case design (Yin, 2009).

In the current study, there are two cases, and therefore, a multiple-case study design was used to examine how students interpreted and solved fermi problems in the order the questions were asked to them, with respect to their proximity of context. The cases of this study were determined by (i) the grade level of the students, (ii) their performances on the selection test of fermi problems, and (iii) the order of the fermi problems that they worked on during the main data collection process. The cases are as follows:

- Case 1: Three groups of two 6th grade students (six students in total) who worked on the Fermi problems given **in the order of close problem context first and remote problem context next**. Throughout the thesis, Case 1 was called "The Fermi Problems with the Sequence from Close Problem Context to Remote Problem Context" and referred as "Case 1: Fermi Problems in Close-to-Remote Context" in short.
- Case 2: Three groups of two 6th grade students (six students in total) who worked as three groups on the Fermi problems given **in the order of remote problem context first and close problem context next**. Throughout the thesis, Case 1 was called "The Fermi Problems with the Sequence from Remote Problem Context to Close Problem Context" and referred as "Case 2: Fermi Problems in Remote-to-Close Context" in short.

In each case, students are asked about three different fermi problems sets concerning the proximity of context.

In summary, the researcher conducted a multiple-case study design to examine how students interpreted and solved fermi problems in the order of which the questions were asked to them with respect to their proximity of context. All necessary permissions were obtained from the students, and their parents' consents were received in accordance with the ethics committee regulations (See Appendix A and Appendix B).

3.2 Participants

The present study was carried out with twelve 6th grade students (six girls and six boys), studying in public middle schools in different cities in Turkey. These twelve students worked in pairs, in six groups. Throughout the study, the students performed the tasks with their pairs. The study was carried out during a period when schools were online due to the covid 19 pandemic conditions and thus the study was carried out online.

A short meeting with colleagues (mathematics teachers in different cities in Turkey) was held to explain the purpose of the research. Upon their consent, the colleagues explained the details of the research to their students and asked for volunteers. Colleagues were asked to indicate to the volunteers that they should apply for the study in pairs. Teachers were the primary contact for initiating the communication with students. Twenty students volunteered as pairs. Only twelve students were selected. In other words, six groups were selected from among ten groups. The reason why twelve students were selected was that it is desired to continue the study in pairs with a total of six groups.

Three of these six groups are planned to work in Case 1 and the remaining three in Case 2**.** The participants of the study are from Kastamonu, Mersin, Batman, Muğla, Gaziantep, and Antalya. The reason why the students were chosen from these cities was that the colleagues in whom the study was explained were teaching in these cities. The students who were living in the same region were preferred to be in the same group. The reason was that they knew each other so that they could communicate better and make common interpretations based on their own lives and local situation in the city they lived.

Twelve students who participated in this study were selected among 20 students based on their performances to a test involving several Fermi Problems. The selection process of the students participating in the research is summarized as seen in the Figure 3.1.

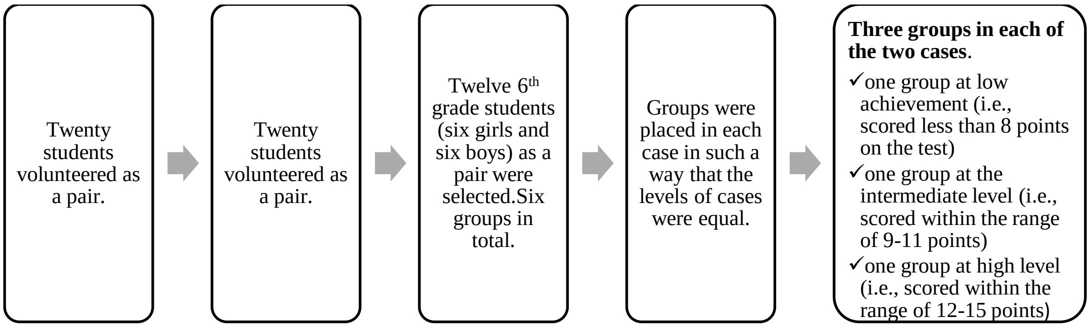

Figure 3.1 The selection process of the students as participants

The selection test was applied to select the participants among the student groups who volunteered for the research as seen in the Figure 3.1. Fermi problems asked in this selection test were similar to the fermi problems in the main study. In this selection assessment, students worked in pairs to solve Fermi problems in half an hour in an online video conference environment. Each pair tried to answer the question, "How much water is consumed in your home during a month?" given in Microsoft forms. The answers and the ideas of students produced as a group were evaluated by means of a rubric (see Appendix C for the rubric). Fermi problem and questions asked to the students during the selection test as seen in Figure 3.2.

- How much water can be consumed in your home during a month?
- The questions asked by the instructor to encourage the students to discuss for fermi problem are given below.
- Do you have enough information to solve the problem given above?
- No, the problem cannot be solved.
- Yes, the problem can be solved with appropriate estimates.
- Write down what information you might need to solve the problem.
- Make the most reasonable estimates with the information you need to solve the problem.
- Solve the problem by explaining where your guesses came from.

Figure 3.2 Fermi problems in the selection test

During the selection assessment, while the students were answering these questions on the Microsoft forms, the next question did not appear until the previous one was answered. To select the groups of students for each case of the main study, their performances were evaluated, and same average level of the students were grouped together. There were three groups in each of the two cases. Each case consists of one group demonstrated low achievement (i.e., scored less than 8 points on the test), one group at the intermediate level (i.e., scored within the range of 9-11 points), and one group at high level (i.e., scored within the range of 12-15 points). Hence, there were three groups at varying performances in Case 1: Fermi Problems in Close-to-Remote Context (CRC) and three groups in Case 2: Fermi Problems in Remote-to-Close Context (RCC). As the results were presented in the next chapter, the abbreviations of CRC-Group 1, CRC-Group 2, and CRC-Group 3 were used to identify the groups in the Case 1 and the abbreviations of RCC-Group 1, RCC-Group 2, and RCC-Group 3 were used to identify the groups in the Case 2. Information about the participants can be seen in Table 3.1

Table 3.1 Information about the participants

Case 1		**Case 2**	
CRC-Group 1	1 girl – 1 boy (12 years old.)	**RCC-Group 1**	1 girl – 1 boy (12 years old.)
CRC-Group 2	1 girl – 1 boy (12 years old.)	**RCC-Group 2**	1 girl – 1 boy (12 years old.)
CRC-Group 3	1 girl – 1 boy (12 years old.)	**RCC-Group 3**	1 girl – 1 boy (12 years old.)

In the main study, 2-4 hours was spent with each group while they were working on the Fermi Problems given in the particular order of the case that they were assigned. During this time, breaks were given at the middle of the period to ensure the well-being of the students.

3.3 Data Collection Procedures

This multiple-case study design involved collecting data through interviews with students, video recording of student ideas and problem-solving steps, which allowed to enhance in-depth understanding of qualitative data (Creswell, 2009).

In the initial stage of the research, to understand the methods the students use to improve a solution to the Fermi problems, students were questioned in a semi-structured clinical interview format. Students were given a flexible time frame to develop solutions to the Fermi problems. The students completed their work at different times because the discussions each group had on the problems varied. During the study, breaks were given in line with the student's request in order to ensure student well-being. The shortest period of students' work was two hours, while the longest lasted four hours. The students' discussion and problem-solving processes were audio and video recorded during the interview. The time range and date of the study with student groups are presented in Table 3.2

Table 3.2 Timeline of Working with Students on Fermi Problems

Date	**Time Interval**	**Content**	**Details of Content**
25.05.2021	2 hours	Implementation of the pilot study	Students answered the questions as a group on microsoft forms. Questions could be seen in Figure 3.1.

Table 3.2 (continued)

28.05.2021	2 hours	Implementation of main study	Students worked on Fermi problems with their groupmates. The researcher supported the studies of the students with semi-stucted clinical interview questions.
04.06.2021	2,5 hours		
09.06.2021	2 hours		
10.06.2021	4 hours (The work was completed in 3 parts with 2 break.)		
15.06.2021	3 hours (The work was completed in 2 parts with 1 break.)		Semi-stucted clinical interview questions., could be seen in Figure 3.2
25.06.2021	2 hours		

3.4 Clinical Interviews

The students in the groups were presented with three different fermi problems. They were informed about the expectations of the task. They were told that they would need to develop solutions to these problems through their discussions with their pairs. The researcher was present during the discussion process in order to understand the thinking process the students went through in solving the problems. In this way, the students were able to express their ideas, and the researcher reached their thinking steps. During this process, the researcher asked some clarifying and probing questions when needed. The semi-structured interview protocol prepared for this purpose was given in Figure 3.2. below. These interview questions were created with an appropriate structure with elements of mathematical modeling and a description of the work that they entail, as seen in Table 2.1 in Chapter 2 (Oropesa & Cortez, 2015).

- What do you understand from the problem?
- What information is given to you in the problem?
- Is the given information sufficient to solve the problem? Explain why?
- What information do you need to solve the problem?
- What can be the answers to this information you need?
- Explain where you got this information from?
- Solve the problem using this information?
- Was this information sufficient for you to solve the problem? Yes/ No
- Could there be other information/situations affecting the solution of the problem? Yes/ No

  If yes; What information/situations might affect the solution of the problem? How can it affect?
- Were you able to reach a definite conclusion of the problem? / Is it reachable?

  If your answer is yes,

  Why/how do you think you can get a definitive answer?

  If your answer is no,

  Why do you think you can't get a definitive answer?
- Can you reach the closest result to the result of the problem? Yes / No

  If yes, what should we pay attention to reach the closest result to the result of the problem?
- If you had solved the problem again,

  What would you pay attention to?

  What did you research?

Figure 3.3 Semi-structured Clinical Interview Questions

3.5 Data Analysis

Content analysis approach was used in the present study to derive codes and categories from the data (Elo & Kyngäs, 2008). Through content analysis, codes (i.e., students solution steps) and categories (i.e., solution approaches such as estimation vs algebraic) were drawn from students' written work and verbal explanations. Initially, the audio and video recordings were transcribed after each session with students. In carrying out the content analysis, the transcripts were read rigorously several times, different perspectives, such as the techniques students used to solve Fermi problems and the students' challenges were noted as codes and categories.

3.6 Role of the Researcher

In qualitative research, the researcher's ideas and perspectives can affect the research result (Johnson, 1997). The qualitative researchers should be aware of their biases and control them (Frankel, 2008) and be open and honest about her or his prior experiences as well as his or her connection with the volunteers (Creswell, 2009). Hence, qualitative researchers need to take some precautions for controlling the possible biases.

As a researcher of this study, I did not have any communication with the students before this study. The students were not my students, rather they were the students of my teacher colleagues living in a city other than the city I live in. Thanks to my colleagues, I reached the students who could voluntarily participate in this study. Before the study, a test was given to the students who volunteered. In this process, I met the student groups on the online platform and observed how they performed on the test. In the next stage, I continued working with the selected groups of students in an online environment. Online video and audio recordings were taken throughout the entire study process. I also told the students that the video and audio recordings would be kept strictly confidential and would not be shared with anybody and that the study would not utilize their real identities. As a result, recordings had as little influence on the students as possible, allowing them to behave naturally throughout the research.

In addition to these, I informed the students about the purpose of the study. The entire research process was video recorded and audiotaped. To continue the study away from my bias as a researcher, I didn't express my personal ideas about the solutions developed by the students and the calculations they made. To avoid altering the study's results, I did not offer any instruction when they tried to solve the problem or when they requested my confirmation. This description of my involvement as a researcher in this study is provided to assure the study's validity since researchers who are conscious of their responsibilities and keep a reflective research journal are more likely to be reliable.

3.7 Trustworthiness and Credibility

In qualitative studies, credibility, transferability, dependability, and confirmability are the four preoccupations emphasized to ensure validity and reliability (Lincoln & Guba, 1985).

To ensure the credibility and dependability, other researcher reviews and triangulation were utilized (Merriam, 1998). The researcher collaborated with a field specialist to analyze the findings in this study. They worked together on coding the transcripts of audio recordings, video recordings of clinical interviews, and verbal expressions. Triangulation refers to the collection of data using a variety of ways and the analysis of the same data by many academics and can be used establish conformability (Shenton, 2004). To arrive at more thorough and valid results, the researcher employed a variety of data sources, including video and audio recordings, interviews, and observations. The codes were triangulated by another researcher's codes, as previously mentioned. As a result, the researcher's involvement in this study was described in the section on the role of the researcher, and triangulation was used for ensuring the trustworthiness of the study.The ability of participants to ask questions of the researcher during data collection, the researcher's guidance of the practice, and the data collecting procedures to be stated as direct results were all used to assure the trustworthiness of this study.

Another concern is transferability and could be established by providing a clear explanation and performing the study with enough data. The researcher took notes on all processes throughout the study. The methods used to choose participants, the data collection instruments, and the data analysis process were all outlined in the earlier sections of this study. As a result, providing a clear description of the study and conducting it with sufficient data could benefit other researchers in transferring the study's conclusions.

CHAPTER 4

FINDINGS

In this study, the analysis focused on how the proximity of the problem context to 6th grade students’ daily lives within the three different categories: low achievers (i.e., get less than 8 points on the participant selection test), intermediate (i.e., take the test in the range of 9-11 points), and high level (i.e., take the test in the range of 12-15 points) affect their models in the fermi problems.

4.1 CASE 1: The Sequence from Close Problem Context to Remote Problem Context

In this section, students studied in three groups to solve Case 1 Fermi problem from close problem context to remote problem context. Based on initial assessment, Group 1 consisted of low achievers. Intermediate level achievers made up Group 2. The final group, Group 3, consisted of high achievers. The problems were asked to these three groups from the close problem context to the remote problem context to students' daily lives. Groups which were worked the Sequence from Close Problem Context to Remote Problem Context are named as a CRC-Group #. Specifically, the groups were named as CRC-Group 1, CRC-Group 2, and CRC-Group 3 where CRC indicated the case of Close-to-Remote Context sequence of the problem. The purpose of asking the problems from the close to the remote context is to examine how the models developed for fermi problems differ among groups with regards to this problem sequence.

4.1.1 FP 1: Toilet Paper Roll

The first fermi problem is ‘How long can the toilet paper be used in a year in your home?' The problem does not specify how much toilet paper is used in a specific time interval. Therefore, students are expected to estimate the amount of toilet paper used in unit time intervals concerning their daily life observations. The three groups studied this problem in the order with from the close problem context to the remote problem context. CRC-Group 1 developed three different solutions. In the first solution, the students aimed to calculate the length of toilet paper used in a year from the length of toilet paper used in a month. In the second solution, the length of toilet paper used in a year is calculated by multiplying the length of toilet paper used in a week by the number of weeks in a year. In the last solution, they aimed to calculate the length of toilet paper used in a year from the length of toilet paper used in a day. They considered a shorter time (i.e. week, day) as the base unit in every solution they created. CRC-Group 2 developed two solutions. The first solution aimed to calculate the amount used in 12 months from the length of toilet paper used daily. While working on the first solution, they realized that they could reach the result by multiplying the amount of toilet paper used daily by the number of days in a year. Thus, they based their second solution on the first one and reached the result using this second solution. The third group, CRC-Group 3, developed a single solution. In this solution, they calculate how many pieces of toilet paper can be used in a year from the number of pieces of toilet paper used at each entrance to the toilet. In addition, they also took into account how much toilet paper can be used in a year for other needs.

The results showed that these three CRC-Groups, who wanted to calculate the length of toilet paper used in a year, focused on the shorter periods they could estimate rather than directly estimating a year in their solutions. Table 4.1 summarizes the solutions developed by these three groups to this problem.

Table 4.1 Solutions of CRC-Groups developed to FP 1: Toilet Paper Roll.

	Groups		
	CRC-Group1	**CRC-Group2**	**CRC-Group3**
Group's numerical solutions	**<u>1st solution</u>** 1 year = 12 months 36 m = An estimate of the length of the toilet paper roll for a month. **36 m × 12 = 432 m paper used in a year** **<u>2nd solution</u>** 1 year = 56 weeks 8 m = An estimate of the length of the toilet paper roll for a week **56 × 18 m = 972 meters paper used in a year** **<u>3rd solution</u>** 1 month = 30 days 18 m =An estimate of the length of the toilet paper roll for a day. **30 × 18 m = 540 m paper used in a month** **540 m × 12 = 6480 m paper used in a month**	**<u>1st solution</u>** 4 = number of people who use toilet. 4 = number of times for each person goes to the toilet in each day. **4 × 4 = 16 times entered to toilet** 16 = number of times for four people go to the toilet in each day. 5 = number of pieces of toilet paper used in each entered to toilet. **16 × 5 = 80 pieces used in 1 day** 80 = number of pieces of toilet paper used in one day. 10 cm = length of each piece of toilet paper. **80 × 10 cm = 800 cm toilet paper used in 1 day**	4 = number of people who use toilet. 2 = number of pieces of toilet paper used in each entered to toilet. **4 × 2 = 8 pieces of toilet paper in each entered to toilet for all 4 people** 4 = number of times for each person goes to the toilet in each day. **8 × 5 = 40 pieces of toilet paper used in 1 day** **40 × 7 =280 pieces used in a week.** **280 × 4= 1120 pieces used in a month** **1120 × 12 = 13 440 pieces used in a year to toilet needs**.

Table 4.1 (continued)

	800 cm × 30 = 24 000 cm toilet paper used in a month **<u>2nd solution</u>** 800 cm = length of toilet paper used in a day **800 cm · 365 = 292 000 cm toilet paper used in a year**	91= number of rolls of toilet paper for other needs in a year **91 rolls and 13 440 pieces of toilet paper are used in one year.** 180 = pieces of toilet paper in each roll. **180 × 91 = 16 380 pieces used in one year to other needs** **16 380 + 13 440 = 29 820 pieces of toilet paper are used in one year.**

As seen in Table 4.1, the first solution of the CRC-Group 1 included the estimation of the toilet paper used in a month. Their second and third solution included the estimation of the toilet paper used in a week and the toilet paper used in a day, respectively. They realized they could reach the result by determining the length of toilet paper used in shorter time intervals when they continued to solve their operations by discussing them in the group. In short, their solution included thinking about monthly, weekly, and daily usage of toilet paper, respectively.

When the solution of CRC- Group 1 examined in detailed, it was observed that started by making daily assumptions. Firstly, they thought that the length of toilet paper used in one day might be 20 centimeters. Then, they thought that the length of toilet paper used in one year could be 20 meters. Thereupon, the students checked the accuracy of these predictions on the Internet. As a result of their research, they learned that a roll of toilet paper is about 18 meters. Therefore, they decided to make their estimates based on the number of rolls used rather than the length of toilet paper used. Thus, they stated that the people living in the house and the guests could use two rolls of toilet paper in one day. They calculated the length of two toilet papers as 36 meters.

To sum up, students in CRC-Group 1 made their operations using the time intervals they determined, and the amount of toilet paper used in these time intervals. First solution included multiplication of the length of toilet paper used in a month by the number of months in a year. This was followed by the second solution which included multiplication of the amount used in a week by the number of weeks in a year. The third calculation involved multiplication of the length of toilet paper used in a day by the number of days in a year. The students reached a solution by focusing on a shorter period than the previous solution for each new solution they developed. Figure 4.1. below shows the researcher-generated mathematical expressions for this group's three solutions.

1st solution

$$\text{length of paper used per month} \times \text{number of months} = length\ of\ paper\ used\ in\ a\ year$$

2nd solution

$$\text{number of week} \times \text{length of paper used per week} = length\ of\ paper\ used\ in\ a\ year$$

3rd solution

$$\text{number of days of month} \times \text{length of paper used per day} \times \text{number of months in a year} = length\ of\ paper\ used\ in\ a\ year$$

Figure 4.1 Researcher-generated expressions of CRC-Group 1's models

Students in CRC-Group 2 searched the length of the toilet paper as 18,45 meters via internet. To get rid of the decimals, they converted the length from meters to centimeters. They applied the following conversion to the length measures: 18,45 meters = about 18 meters = 1800 cm. They assumed the length of each piece of toilet paper to be approximately 10 cm to facilitate their operation. The other information they needed to solve the problem was how many pieces of toilet paper a toilet paper roll contains. To answer this question, they divided 1800 cm, the length of a roll of toilet paper, by 10 cm, which they had searched as the length of a piece of toilet paper. As a result of this process, they determined that about 180 pieces of toilet paper came out of one toilet paper roll. They, then, assumed that five pieces of toilet paper could be used at each entrance to toilet. They made their calculations based on the length of this toilet paper they estimated. They calculated the number of toilet paper pieces used in a day by multiplying the number of toilet visits per day by the number of toilet paper pieces used each time they entered the toilet. They calculated the length of toilet paper used in a day based on this. They solved the problem by multiplying the length of toilet paper used in a day by the number of days in a year. As in CRC Group 1, the students reached the amount used in a year from the shortest time used toilet paper. Unlike CRC Group 1, they took the number of toilet paper pieces used to enter each toilet as the shortest unit of time. Similarly, CRC-Group 2 model is provided as a researcher-generated mathematical expression in Figure 4.2.

1st solution

number of people × number of entrances to toilet per person in a day ×
pieces of toilet paper which is used in each × number of days in a month ×
month of days in a year = $length\ of\ paper\ used\ in\ a\ year$

2nd solution

number of people × number of entrances to toilet per person in a day ×
pieces of toilet paper which is used in each × number of days in a year =
$length\ of\ paper\ used\ in\ a\ year$

Figure 4.2 Researcher-generated expressions of CRC-Group 2’s models

CRC-Group 3 students, on the other hand, calculated how many pieces of toilet paper were used when everyone entered the toilet at once. They multiplied the number of toilet paper pieces used in each entrance to the toilet with the number of people in the house. Based on this, they calculated the number of toilet paper pieces used per week, month, and year, respectively. Unlike CRC-Group 2, CRC-Group 3 calculated the smallest unit by multiplying the total number of people with the amount of toilet paper used in each toilet use. They calculated the unit of toilet paper used for all. Moreover, unlike CRC Group-1 and 2, CRC-Group 3 students also considered the amount of toilet paper that could be used other than toilet needs. Figure 4.3. presents the researcher-generated mathematical expressions for the CRC-Group 3.

(pieces of toilet paper used in each entered to toilet for all ×
number of toilets used in a day × number of days in a week
× number of months in a year) +
(amount of toilet paper roll used for per day × number of pieces in each paper roll)
= *pieces of toilet paper used in a year.*

Figure 4.3 Researcher-generated expressions of CRC-Group 3's models

To summarize, all three groups used unit rate to reach the amount of toilet paper that could be used in a year. Both CRC-Group 2 and CRC-Group 3 focused on the shorter time to calculate toilet paper used compared to CRC-Group 1. CRC-Group 2 and CRC-Group 3 tried to calculate the length of toilet paper used in a year by looking for an answer to how much toilet paper could be used at each toilet entrance.

Instead of telling the length of toilet paper used at each entrance to the toilet directly, they made observations from their own lives. CRC-Group 2 and CRC-Group 3 thought how many pieces of toilet paper would be used. The reason for this is that it's easier for them to estimate how many pieces of toilet paper they have used, rather than telling them how long they have used it each time. As a result of these calculation on toilet paper usage and the duration of using a certain amount of toilet paper, the students of all three groups reached these conclusions.

4.1.2 FP 2: Weight of the Students in the School

The three groups studied the second problem in the sequence of from close problem context to the remote problem context. The second fermi problem which was studied in this case was 'What would be the total weight, in kg, of all students studying at your middle school?' In this Fermi problem, the number of students in the school and the weights of the students was not specified. For this reason, students were expected to estimate the number of students population in the school and their weight.

CRC-Group 1 did not realize that to solve the problem, they must first estimate the appropriate values, and thus they could not solve the problem. For this reason, the researcher was able to examine the work of CRC-Group 2 and CRC-Group 3 in this problem. While CRC-Group 2 was solving the problem, they started by guessing the average weights of all students in the school. One student in the group said that the average weight of students in the school might be 40 kilograms, and another said 50 kilograms. The students could not explain why they estimated the average weight as 40 or 50. Since the two students in the group estimated different average weights, they decided to continue their studies with the average of these values (45 kg). After determining the average student weight in the school, they discussed what the total number of students in the school could be. They calculated what could be the total number of students in the school. They arrived at the total number of students by multiplying the approximate number of students in each class by the total number of classes. At the final stage of their processing, they multiplied the total number of students by the average weight of the students to reach the total weight of the students in the school.

Unlike CRC-Group 2, CRC-Group 3 calculated the average weights of the students for each grade level separately. Therefore, the CRC-Group 2 reached the result by multiplying the total number of students and the average of the weights of all students. On the other hand, CRC-Group 3 calculated the total weight of the students in the school by adding the approximately total weights of the students at each grade level. CRC-Group 3 calculated the total weights of students in each grade by multiplying the total number of students in each grade with the estimated average weights of the students in that grade. To do so, the students in the CRC-Group 3 estimated the weight of 5th grade student to be 40 kg, weights of 6th grade student to be 50 kg, weights of 7th grade student to be 55 kg, and the weight of 8th grade students to be60 kg on average. After determining the average weight in each grade, they discussed the total number of students in each grade. They stated that there were six classes at each grade level. There were approximately 30 students in each class. Therefore, they determined the number of students in each grade as 180. CRC-Group 3 determined the total weight of students at that grade level by multiplying the estimated weight by the number of students at that grade. After calculating the total weight for each level, they determined the total weight for the entire school. Table 4.2 presents the solution of CRC Groups.

Table 4.2 Solutions of CRC-Groups developed to FP 2: Weight of the Students in the School

	Groups		
	CRC-Group1	**CRC-Group2**	**CRC-Group3**
Group's numerical solutions	No solution	*40 kg and 50 kg are predictions of group members. They decided to make calculations with respect to their average for weight of students.* **Average of 40 kg and 50 kg is 45 kg** *18 = Number of classes* *25=Number of students in each class* **18 × 25 = 450 students in total.**	*30=Number of students in each class* *6 = Number of classes in each grade level.* **30 · 6 = 180 is the total number of students in each grade level** *40 = approximately weight of student in 5th grade.* **180 × 40 = 7200 kg**

Table 4.2 (continued)

	450 × 45 kg = **20 250 kg in total.**	**is a Weight of whole 5th grade students**
		50 = approximately weight of student in 6th grade. **180 × 50 = 9000 kg is a weight of whole 6th grade students**
		55 = approximately weight of student in 7th grade. **180 × 55 = 9900 kg is a weight of whole 7th grade students.**
		60 = approximately weight of student in 8th grade. **180 × 60 = 10800 kg is weight of whole 8th grade students.**
		7 200 kg + 9 000 kg + 9 990 kg + 10 800 kg = 36 900 kg is a total weight of students in school

As seen in Table 4.2, CRC-Group 1 could not develop an idea about the solution to the problem. These students did not find any estimations of the weights of the students in their school. It can be said that these students' ability to make inferences to reach a conclusion is at a lower level. The students in the second group, on the other hand, can make inferences even though they know that they cannot know the actual weight of each student. By estimating the number and average weights of all students, they tried to reach a value close to the total weight of the students. Figure 4.4. below shows the researcher-generated mathematical expressions for this group's three solutions.

$$\text{average weight of all students} \times \text{number of all class} \times \text{number of students in each class}$$
$$= \mathit{Total\ weight\ of\ the\ students\ in\ the\ school}$$

Figure 4.4 Researcher-generated expressions of CRC-Group 2’s models.

In addition to these, CRC-Group 3 used the average weight of students in each grade separately. These average weight values were determined by observation of students from themselves and their surroundings. Students are expected to find closer value to the actual values. Since the weights of students of the same age are relative to each other, calculating each grade level separately reduces the deviation from the mean value. Figure 4.5. below shows the researcher-generated mathematical expressions for this group’s solutions.

$$(\text{Number of students in each class} \times \text{number of classes in each grade level}$$
$$\times \text{average weight of each 5th grade students})$$
$$+(\text{Numberofstudentsineachclass} \times \text{numberofclassesineachgradelevel} \times \text{averageweightofeach6thgradestudents}) +$$
$$(\text{Number of students in each class} \times \text{number of classes in each grade level} \times$$
$$\text{average weight of each 7th grade students})$$
$$+ (\text{Number of students in each class} \times \text{number of classes in each grade level} \times$$
$$\text{average weight of each 8th grade students})$$
$$= \mathit{Total\ weight\ of\ the\ students\ in\ the\ school}$$

Figure 4.5 Researcher-generated expressions of CRC-Group 3’s models.

To summarize, the students in CRC-Groups 2 and 3 used the arithmetic mean to calculate the weights of students in their school. They supported their work with their estimations. Unlike CRC-Group 2, since CRC-Group 3 calculated the mean value for each grade level separately, it can be expected that the standard deviation will be smaller when calculating the total weights of the students.

4.1.3 FP 3: The Number of Malls in Turkey

The third fermi problem was 'How many shopping malls can there be in Turkey?'. The problem did not make a specific definition of what shopping malls were meant. Also, the problem did not specify the number of shopping malls in a particular region for reference. Therefore, students were expected to estimate the number of the shopping mall. The three groups studied this problem in the sequence of close-to-remote problem contexts. CRC-Group 1 made direct predictions without establishing any relationship. They did not make any explanations as to the number they stated as their solution.

Contrary to CRC-Group 1, CRC-Group 2 and 3 found a solution to the problem by making an explanation. CRC-Group 2 students stated that there could not be an equal amount of malls in all cities in Turkey, so they wanted to group provinces as small and big. They divided provinces into two groups in this way thinking that there would be fewer malls in small cities compared to big cities. They assumed that there are 30 big and 51 small provinces in Turkey. They determined the number of provinces in this way since they thought that more than half of the 81 provinces in Turkey were small cities. Based on this inference, CRC-Group 2 students calculated the estimated number of shopping centers in small cities by multiplying the estimated number of small cities in Turkey with the average number of malls. They applied the same procedures to calculate the number of shopping malls in big cities. In the last step of their solution, they aimed to reach the total number of shopping malls in Turkey by adding the number of shopping malls in small and big cities. On the other hand, CRC-Group 3 developed a solution to the problem based on an advertisement they observed. Based on the advert "1000 units of 101 (name of supermarkets in Turkey), all over Turkey"; they stated that there could be 1000 malls in each province. Since there are 81 provinces in Turkey, the calculations of the total number of malls were done by multiplying these two values. Unlike the students in the other groups, CRC-Group 3 students considered supermarkets as shopping centers.

The results showed that these three CRC-Groups used different methods to find the number of shopping malls in Turkey. Table 4.3 summarizes the solutions developed by these three groups to this problem.

Table 4.3 Solutions of CRC-Groups developed to FP 3: The Number of Malls in Turkey

	Groups		
	CRC-Group1	**CRC-Group2**	**CRC-Group3**
Group's numerical solutions	'There should be 550 shopping malls in Turkey.'	In big cities, there may be 20 shopping malls as an average. Small cities can have 5 shopping malls as an average. There are 30 big and 51 small cities. **20 × 30 = 600** malls in big cities **51 × 5 = 255** malls in small cities There are 855 in total.'	**81 × 1000 = 81 000** shopping malls in Turkey. 81 = number of provinces in Turkey 1000 = number of markets in each province

As seen in Table 4.3, to find the number of shopping malls in Turkey, CRC-Group 1 utilized direct estimation. They indicated that the number of shopping malls in Turkey was 550 without any explanations.

Although CRC-Group 1 students directly predicted the result, CRC-Group 2 and 3 established a relationship between quantities they identified to their solutions. CRC-Group 2 estimated the number of malls in smaller units, small and big provinces, to develop a solution to the fermi problem. They tried to reach the result by establishing a ratio between the approximate number of malls in one small province to the number of malls in all small provinces. Also, they tried to establish a ratio between the approximate number of malls in one big province to the number of malls in all big provinces, one big province to all big provinces. It can be said that they reached the number of shopping malls in Turkey by estimating the number of shopping malls in a small or large city. Therefore, this group determined the unit rate and reached the whole. Figure 4.6. below shows the researcher-generated mathematical expressions for this group's solutions

$$(\text{Number of small cities in Turkey} \times \text{average number of mall in small cities}) + (\text{Number of big cities in Turkey} \times \text{average number of mall in big cities}) = number\ of\ malls\ in\ Turkey$$

Figure 4.6 Researcher-generated expressions of CRC-Group 2's models

CRC-Group 3 also established a ratio while developing solutions to their problems. They determined the number of shopping centers in the province by evoking an advertisement. They have reached the number of shopping malls in all provinces in Turkey from the number of shopping malls in one province with respect to a unit rate. They thought there were 1000 shopping malls in each province and stated that there are 81000 shopping malls in total. In doing so, they used unit rate. Figure 4.7. below shows the researcher-generated mathematical expressions for this group's solutions

$$\text{number of markets in each province} \times \text{number of province in Turkey} = number\ of\ malls\ in\ Turkey$$

Figure 4.7 Researcher-generated expressions of CRC-Group 3's models

To summarize, CRC-Group 1 made a direct estimation to find the number of shopping malls. Contrary to CRC-Group 1, students in CRC-Groups 2 and 3 used a unit rate to calculate the number of shopping malls in Turkey. CRC-Group 2 estimated the number of shopping malls for small and big provinces. From here, they calculated how many shopping malls could be located in Turkey. Unlike CRC-Group 2, CRC-Group 3 calculated the number of shopping malls by using the slogan on advertising and they estimated that there 1000 shopping malls in every province as a unit rate.

4.1.4 Students' Overall Performance for Case 1

Those students who scored 8 points in the selection test were categorized as low achievers and thus worked in CRC-Group 1. CRC-Group 1 developed three different solutions for the first problem of the sequence from close problem context to remote problem context. The first problem was "How long can the toilet paper be used in a year in your home?" In the first solution, CRC-Group 1 estimated how much toilet paper could be used in a month. Then, they calculated the length of toilet paper used in a year. In the second solution, they calculated the length of toilet paper that could be used in a week. From there, they determined the length of toilet paper that could be used in a year. In the third solution, they calculated the length of toilet paper that could be used in a day. Then, they determined the length of toilet paper that could be used in a year. The students produced three different solutions to this question, which involved a situation they encountered every day. They aimed to find closer value to the actual result from the amount of toilet paper used in a shorter time in each solution. They did not develop a solution for the second question, which is 'What would be the total weight, in kg, of all students studying at your middle school? ' The third problem of case 1 was 'How many shopping malls can there be in Turkey?'. CRC-Group 1 directly predicted the result of the third problem without making any estimation for the solution and establishing relationship. CRC-Group 1 had more difficulty interpreting problems from the close problem context to the remote problem context. They showed the best performance in the context of the closest problem.

CRC-Group 2, which received 10 points on the participant selection test, worked as an intermediate group in case 1.CRC-Group 2 developed two different solutions for the first problem of case 1. In the first solution, they estimated how much toilet paper could be used at each entrance to the toilet. They found the amount of toilet paper used in a day, a month, and a year. In the second solution, CRC-Group 2 solved the problem by directly multiplying the length of toilet paper used daily by the number of days in a year. The students produced two related solutions to this question, which was about a situation they encountered every day. For the second problem, CRC-Group 2 estimated the average weights of middle school students. Then, they multiplied average weight by the estimated total number of students. Finding the average weights of the students in different grade levels took them away from the actual value, according to the solutions of the students who found the average weight at each grade level separately. As for the third problem of case 1, CRC-Group 2 aimed to reach the total number of shopping malls by determining how many shopping malls there are in big and small cities. Categorizing all cities in Turkey as small and large led them to find a more distant result than groups that take large, medium, and small cities into account. The CRC-Group 2 group found more general results from the close problem context to the remote problem context. It is expected that the closest value to the actual result could be found in the closest context because the students have set the smaller unit rate. Thus, they reached the solution from the value of the smaller units.

CRC-Group 3 worked as a high-level group in case 1 as they scored 13 points in the participant selection test. CRC-Group 3 used only one solution in the first question of case 1. By estimating how much toilet paper each person living in the house uses in a day, they calculated how many pieces of toilet paper were used in a week, month, and year, respectively. They also considered that toilet paper would also be used for other needs. Instead of calculating the length of the total toilet paper used, they have reached the solution of how many toilet paper pieces are used. The reason for this is that they want to reach a value closer to the actual result by using their experiences in their daily lives. They stated that they could interpret how much toilet paper they used each day more easily than what length they use it. In the second problem, they computed the average weight for each grade level independently before calculating the total weight of students. They arrived at the total weights of the students in the entire school by adding the approximate weights determined for each grade level. Determining the average weights of students at each grade level separately helped to find a value closer to the actual weights of students in the same age group. For the third problem, they found the total number of shopping malls by using an advertisement slogan they encountered in their daily lives. CRC-Group 3 calculated the quantities in smaller units for the three problems and used their daily life experience to solve the problems professionally. Although they worked successfully in all three problems, it can be said that estimations and calculations are expected to be closer to the actual value in the closest problem context to daily life. Thus, they established a more relevant relationship between quantities they identified with their daily lives in estimated values to solve the problem.

4.2 CASE 2: The Sequence from Remote Problem Context to Close Problem Context

In this section, students studied in three groups to solve Case 2 Fermi problems from remote problem context to close problem context.' The groups working in case 1 and case 2 were three different groups from each other. Based on the initial assessment, Group 1 consisted of low achievers. Intermediate level achievers made up Group 2. The final group, Group 3, consisted of high achievers. The problems were asked to these three groups from the remote problem context to the close problem context to students' daily lives. Groups which were worked the Sequence from Remote Problem Context to Close Problem Context are named as an RCC-Group # (i.e., RCC-Group 1, RCC-Group 2, and RCC-Group 3.) The aim of asking the problems from the remote to the close context is to examine how the models developed for fermi problems differ among groups regarding this problem sequence.

4.2.1 FP 1: The number of malls in Turkey

The first fermi problem is 'How many shopping malls can there be in Turkey?'. As in case 1, the problem does not comprehensively describe the meaning of shopping malls. Moreover, the problem does not reference the number of shopping malls in a specific location. Therefore, the researcher expected students to estimate the number of shopping malls. The three groups investigated this problem in the order of the remote problem context to the close problem context.

RCC-Group 1 students argued that instead of finding the number of shopping malls in Turkey directly, a more specific region should be discussed. They indicated that they could estimate the number of shopping malls in the district where they live. They guessed that there were 100 shopping malls in that district. RCC-Group 1 estimated that there are 92 districts in Turkey. RCC-Group 1 has assumed that there could be approximately 100 shopping malls in each district. Therefore, they calculated the total number of shopping malls in Turkey as $92 \times 100 = 92\ 000$ shopping malls. They said that they could make this calculation for 81 provinces, but they did not make any adjustments or additions to their solutions.

RCC-Group 2 group stated that the ratio of the number of shopping malls in each province or district to whole provinces or districts should be known. They stated that knowing the average number of shopping malls in each district or province would help them solve the problem. Unlike RCC Group 1, RCC-Group 2 students did not estimate numerical values, rather they continued their calculations with algebraic expressions. They accepted the number of shopping malls in Muğla as 'x' and the number of shopping malls in İstanbul as 'y'. The range between these values was accepted as z. They claimed that the mean value could be calculated with the below-mentioned algebraic expression.

$\frac{x+z}{2}$ or $\frac{x-z}{2}$

RCC-Group 2 stated that they had reached the algebraic expression for the average number of shopping malls in this calculation. They stated that this is a very general expression for the number of shopping malls in Turkey.

RCC-Group 3 group stated that they needed to know the number of shopping malls in a province to solve the problem. Students also stated that there could not be an equal number of shopping malls in the east and west of Turkey. They emphasized that there are fewer shopping centers in the east of Turkey than in the west. Moreover, RCC-Group 3 indicated that the number of shopping malls in the provinces could differ depending on the size of the population. Therefore, they preferred to group the provinces in Turkey as big, medium, and small cities according to the size of their populations in their calculations. In this way, they stated that they could find a value closer to the actual result. While RCC-Group 3 estimated the number of shopping malls in cities, they took the number of shopping malls in their city as a reference. They estimated the approximate number of shopping malls in big, medium, and small cities as fifteen, six, and one, respectively. In addition, they stated that there could be three large, twenty-three medium, and fifty-five small provinces in Turkey. Therefore, they calculated that there are fifty-five shopping malls in small-sized cities, a hundred thirty-eight in medium-sized cities, and forty-five in large cities.

The results showed that these three RCC-Groups used different methods to find the number of shopping malls in Turkey. Table 4.4 summarizes the solutions developed by these three groups to this problem.

Table 4.4 Solutions of RCC-Groups developed to FP 1: The Number of Malls in Turkey

	Groups		
	RCC-Group 1	**RCC-Group 2**	**RCC-Group 3**
Group's numerical solutions	**100 × 92 = 9200** 100 = number of shopping malls 92 = number of districts	x for an amount of shopping mall in Muğla. y for an amount of shopping mall in İstanbul. If z in the range, average is: $\frac{x+z}{2}$ or $\frac{x-z}{2}$	55 × 1 = 55 shopping malls in small-sized cities 3 × 15 =45 shopping malls in large-sized cities 23 × 6 =138 shopping malls in medium-sized cities 55 + 45 + 138 = 238 shopping mall in Turkey

As seen in Table 4.4, to find the number of shopping malls in Turkey, unlike RCC-Group 2, RCC-Group 1 and 3 tried to solve the fermi problem from a unit to a whole. Therefore, it can be said that they are trying to establish a part-whole relationship. In other words, they tried to reach the number of shopping malls in Turkey by basing their calculations to the number of shopping malls in a district or province. Therefore, it can be said that these groups use unit rate in their calculations.

RCC-Group 1 students stated that there could be 100 shopping centers in their province. The students in this group live in a rural area of a province that can be considered medium-sized in Turkey. This may be the reason why they chose to multiply the number of shopping malls they estimated for the province by the number of districts in Turkey. At the last stage of solving the problem, they said these procedures could be implemented in 81 provinces. However, in their calculation, they never mentioned the difference between using the number of provinces or the number of districts in Turkey. Figure 4.8. below shows the researcher-generated mathematical expressions for this group's solution.

approximately number of shopping center in a province × number of districts or province = *the number of shopping mall in Turkey*

Figure 4.8 Researcher-generated expressions of RCC-Group 1's models

RCC-Group 2 students preferred to use algebraic expressions because the exact values were unknown. The mean value was calculated with the variables expressing the number of shopping malls in a small city and a large city in Turkey. The range between the number of shopping malls in these cities could not be used efficiently when calculating the mean value. In addition, the average value they determined could not be generalized to find the number of shopping malls in Turkey. In other words, after calculating the average number of shopping malls, they did not calculate how many shopping malls there could be in Turkey. Unlike other RCC-Groups, RCC-Group 2 used the algebraic expression. However, they could not use this expression efficiently in the solution stages of the problem; therefore, researcher-generated mathematical expression could not be written for this group.

RCC-Group 3 estimated the number of shopping malls in smaller units, small, medium, and big provinces, to develop a solution to the fermi problem. They tried to reach the result by establishing a ratio between the approximate number of malls in one small province to the number of malls in all small provinces. Also, they tried to establish a ratio between the number of malls in a big province to the number of malls in all big provinces. It can be said that they reached the number of shopping malls in Turkey by estimating the number of shopping malls in small, medium, and large provinces. Therefore, similar to RCC-Group 2' s solution found in case 1, RCC-Group 3 determined the unit rate and reached the whole. Figure 4.9. below shows the researcher-generated mathematical expressions for RCC-Group 3’s solution.

(number of small sized provinces × average number of shopping center in small sized province + (Number of medium sized provinces) × (average number of shopping center in medium sized province) + (Number of big sized provinces × average number of shopping center in big sized province) = *the number of shopping malls in Turkey*

Figure 4.9 Researcher-generated expressions of RCC-Group 3’s models

To summarize, RCC-Group 2 tried to find algebraic expression of number of shopping malls in Turkey directly. Contrary to RCC-Group 2, students in RCC-Groups 1 and 3 estimated unit values. RCC-Groups 1 and 3 used a unit rate to calculate the number of shopping malls in Turkey. RCC-Groups 1 estimated that there could be 100 shopping malls in a district and calculated the number of shopping malls in all districts in Turkey. Unlike RCC-Group 1, RCC-Group 3 estimated the number of shopping malls for small, medium, and big provinces, and then, they calculated how many shopping malls could be in Turkey.

4.2.2 FP 2: Weight of the students in the school

The second fermi problem of the second case is 'What would be the total weight, in kg, of all students studying at your middle school?' In this fermi problem, like in case 1, the number of students in the school and the weights of the students are not specified. For this reason, RCC-Groups are expected to estimate the number of students enrolled in the school and their weight.

RCC-Group 1 and RCC-Group 3 calculated the total weight of the students in the school by adding the approximately total weights of the students at each grade level. Unlike these two groups, RCC-Group 2 first calculated the average weight of the students in the whole school and then multiplied this value by the total number of students.

RCC-Group 1 started to solve the problem by estimating the weights of the students in the school based on their grade level. RCC-Group 1 students determined the weights of fifth, sixth, seventh, and eighth-grade students as 48 kg, 58 kg, 68 kg and 58 kg, respectively. They did not explain why they determined these weights as these many kilograms. RCC-Group 1 stated that there were 12 students in the 5th grade, 10 students in the 6th grade, 16 students in the 7th grade, and 18 students in the 8th grade. Based on these values, they calculated the total weight of the students in the school.

While RCC-Group 3 was solving the problem, they started by estimating the total number of students in the school. They stated that each grade level has approximately six classes. They argued that there were $6 \times 30 = 180$ students in the school, as they thought there were about 30 students in each class. Students made inferences from their own weights in order to determine the weights of other students in the school. They thought that the average weight of students in 6th grade could be 45 kilograms. One of their friends in the 5th grade stated that her/his weight was 38 kg. Also, RCC-Group 3 stated that the weight of 7th and 8th grade students would be more than the weight of 5th and 6th grade students. Therefore, they accepted the approximate weights of 7th and 8th grade students as 50 kg and 55 kg, respectively. RCC-Group 3 has prepared a table that indicates the number of students by grade level in the school and the average weight of each student. (See Table 4.5)

Table 4.5 Average student weight and number of students by grade level determined by RCC-Group 3

Grade	Average Weight	Number of Students
5^{th} grade	38 kg	180
6^{th} grade	45 kg	180
7^{th} grade	50 kg	180
8^{th} grade	55 kg	180

After RCC-Group 3 found the total weight of students at each grade level, they added them to calculate the total weight of students in the school. Contrary to RCC Group 1 and 3, RCC-Group 2 calculated the total weights of students by multiplying the total number of students with the estimated average weights of the students in the school.

Based on students' own observations, RCC- Group 2 stated that there could be 32 students in each class at the school they also stated that there were eight classrooms each from the 5^{th} and 6^{th} grades and nine classrooms each from the 7^{th} and 8^{th} grades in the school. In addition, RCC-Group 2 stated that the average weight of students belonging to the fifth, sixth, seventh, and eighth-grade levels could be 40 kg, 45 kg, 55 kg, and 60 kg, respectively. RCC-Group 2 calculated the average weights of students in the whole school by averaging these values for each grade level. By multiplying the average weight of all students in the school with the total number of students in the school, they calculated the total weight of the students in the school. The results showed that RCC-Group 1 and 3 used similar methods to the total weight of the students in the school. RCC-Group 2 used a separate way from others. Table 4.6 summarizes the solutions developed by these three groups to this problem.

Table 4.6 Solutions of RCC-Groups developed to FP 2: Weight of the students in the school

	Groups		
	RCC-Group 1	RCC-Group 2	RCC-Group 3
Group's numerical solutions	**$12 \times 48 = 576$** **$10 \times 58 = 580$** **$16 \times 68 = 1088$** **$18 \times 58 = 1044$** Total weight for students in each grade level. 576 + 580 + 1088 + 1044 = 3288 kg kg total weight	**264 + 264 + 297 + 297 = 1122** total number of students. **40+45+55+60 =200** total weight according to closest weight for each student in each grade level. **$200 \div 4 = 50$ kg** average weight for all grades **$11122 \times 50 = 56100$** total weight	**$180 \times 38 = 6840$ kg** **$180 \times 45 = 8100$ kg** **$180 \times 50 = 9000$ kg** **$180 \times 55 = 9900$ kg** Total weight for students in each grade level. **6840 + 8100 + 9 000 + 9900 = 33 840** kg total weight

As seen in Table 4.6, RCC-Group 1 and 2 used similar methods to find the total weight of the students in the school. RCC-Group 1 directly estimated both the number of students in the school and the weights of the students without any explanation. At each grade level, they increased the weights of the students by 10. They thought that 78 kg would be too much at the eighth-grade level. Therefore, instead of leaving the weight of the 8^{th} grade students the same as the 7^{th} grade students, they lowered it even more and accepted it as the same as the 6^{th} grade students. It can be seen that these students' ability to make inferences to conclude is at a surface level. In this process, students thought additively. Considering calculating the average weight at each grade level indicates that they realize they need to use the arithmetic average. Figure 4.10. below shows the researcher-generated mathematical expressions for this group's solution.

(number of students in 5^{th} grade × estimated weight of student who study in 5^{th} grade) + (number of students in 6^{th} grade × estimated weight of student who study in 6^{th} grade) + (number of students in 7^{th} grade × estimated weight of student who study in 7^{th} grade) + (number of students in 8^{th} grade × estimated weight of student who study in 8^{th} grade) = *Total weight of the students in the school*

Figure 4.10 Researcher-generated expressions of RCC-Group 1's models

RCC-Group 2 made estimations about the average weight of students in the school and the total number of them. The total weight of the students in the school was reached by multiplying the average weight of the students in the whole school by the number of students in the school. Like RCC-Group 1, RCC- Group 2 calculated the average weight at each grade level indicating that they realize they needed to use the arithmetic average. Like RCC-Group 1, RCC Group 2 also used the arithmetic mean. Unlike RCC-Group 1, the group continued the operations not separately for each grade level, but by taking into account the arithmetic average of the students in all grade levels. Figure 4.11 below shows the researcher-generated mathematical expressions for this group's solution.

Total number of students × [(total weight according to closest weight for each student in each grade level) ÷ number of grade levels] = *Total weight of the students in the school*

Figure 4.11 Researcher-generated expressions of RCC-Group 2's models

In addition to these, RCC-Group 3 used the average weight of students in each grade separately. These average weight values were determined based on their and their friends' weights. RCC-Group 3 was expected to find a closer total weight to the actual total weight. Since the weights of students of the same age are relative to each other, they made their calculations based on actual weight values. They also used the arithmetic mean while they were calculating. Calculating each grade level separately reduces the deviation from the mean value. Figure 4.12. below shows the researcher-generated mathematical expressions for this group's solution.

[(number of students in 5^{th} grade) × (estimated weight of student who study in 5^{th} grade)] + [(number of students in 6^{th} grade) × (estimated weight of student who study in 6^{th} grade)] + [(number of students in 7^{th} grade) × (estimated weight of student who study in 7^{th} grade)] + [(number of students in 8^{th} grade) × (estimated weight of student who study in 8^{th} grade)] = *Total weight of the students in the school*

Figure 4.12 Researcher-generated expressions of RCC-Group 3's models

To summarize, all RCC-Groups used the arithmetic mean to calculate the weights of students in their school. They made their calculations with their estimated values. Unlike RCC-Group 2, RCC-Group 1 and 3 calculated the mean value for each grade level separately.

4.2.3 FP 3: Toilet Paper Roll

The third fermi problem is 'How long can the toilet paper be used in a year in your home?' This problem also does not specify how much toilet paper is used in a specific time interval. Therefore, students are expected to estimate the amount of toilet paper used in unit time intervals concerning their daily life observations. The three groups studied this problem as the third Fermi Problem in Case 2.

RCC-Group 1 aimed to calculate the length of toilet paper used in a year from the number of boxes containing toilet paper used in a month. The students estimated how many toilet paper rolls were used in a month by multiplying the number of toilet paper rolls boxes bought in a month by the number of toilet paper rolls in each box, inferring from the shopping done at home. They calculated the number of toilet paper rolls used in a year by multiplying the number of toilet paper rolls used in a month by the number of months in a year. Since the fermi problem asked the length of toilet paper used in one year, they determined that they needed to know how many meters of toilet paper was in a toilet paper roll. Therefore, they directly assumed that there were three meters of toilet paper in a toilet paper roll. They calculated the length of toilet paper used in a year by multiplying the number of toilet paper rolls used in a year by the length of toilet paper in a toilet paper roll.

Unlike RCC-Group 1, RCC-Group 2 and RCC-Group 3 aimed to calculate the amount used in a year from the length of toilet paper used daily. RCC-Group 2 realized that they could reach the result by multiplying the amount of toilet paper used daily by the number of days in a year. While calculating the length of toilet paper used in a day, attention was paid to the number of people living in the house using the toilet in a day, how long toilet paper was used at each entrance to the toilet, and how long toilet paper was used for other needs other than the toilet in a day.

Unlike RCC-Group 2, RCC-Group 3 has examined toilet paper in three groups the length of toilet paper used in the toilet, non-toilet cleaning needs, and other needs. In addition, instead of directly multiplying the length of toilet paper used in a day by the number of days in a year, they first found the length of toilet paper used in a month and then calculated the length of toilet paper used in a year. The results showed that these three RCC-Groups, who wanted to calculate the length of toilet paper used in a year, focused on the shorter periods they could estimate rather than directly estimating a year in their solutions. Table 4.7 summarizes the solutions developed by these three groups to this problem.

Table 4.7 Solutions of RCC-Groups developed to FP 3: Toilet Paper Roll

	Groups		
	RCC-Group1	**RCC-Group2**	**RCC-Group3**
Group's numerical solutions	**2 × 20 = 40** 2=number of boxes used in a month 20=number of toilet paper roll in each box **12 ×40 = 480**	**4 × 4 =16** 4 = number of people 4 = number of entrances to toilet for each person 16 = entrance to toilet in a day **100 cm × 16 =1600**	**3×12 cm = 36 cm** 3 = number pf toilet paper pieces used in each entrance to toilet 12 cm = length of each toilet paper piece

Table 4.7 (continued)

	12= number of month in a year 40= number of toilet paper roll used in a month 480= number of toilet paper roll used in a year **480 × 3 m =1440 m'** 3m = length of a toilet paper roll 1440 m= length of toilet paper roll used in a year	100 cm = length of toilet paper used in each entrance to toilet 1600 cm = length of toilet paper used in a day **50 cm × 4 = 200** 50 cm = length of toilet paper used in other needs for each person 4 = number of people 200 = length of toilet paper used in other needs in a day **200 cm + 1600 cm = 1800 cm = 18 m** 18 m = length of toilet paper used in a day **18× 365 = 6 570 m** 365 = number of days in a year 6570 m = length of toilet paper used in a year	36 cm = length of toilet paper used in each entrance toilet **4 × 5 = 20** 4= number of entrance to toilet for each person in a day 5= number of people 20 = number of entrance to toilet in each day **20 × 36 = 720 cm** 720= length of toilet paper used in a day for toilet needs **5 × 12 = 60 cm** ɔer of pieces of toilet sed in other needs 12 cm = length of each toilet paper piece 60 cm= length of toilet paper used in a day for other needs **10 × 5 = 50 cm** 50 cm=length of toilet paper used in a day for other cleaning needs **720 + 60 + 50 = 830 cm** **830 × 30 = 24 900 cm**

Table 4.7 (continued)

	length of toilet paper used in a month **24 900 × 12 = 298 800 cm** length of toilet paper used in a year

As seen in table, RCC-Group 1 estimated that if two boxes of twenty toilet paper each are used in a month, 40 toilet rolls per month will be used. They stated that if 40 toilet papers are used in a month and since a year has twelve-month, 12 × 40 = 480 toilet papers will be used in a year. They stated that there could be 3 meters of toilet paper in each toilet paper roll. Thus, they calculated that 480 toilet rolls could contain a total of 1440 meters of toilet paper. Figure 4.12. below shows the researcher-generated mathematical expressions for this group's solution.

number of boxes which used in a month × number of toilet paper roll in each boxes × number of month in a year × length of toilet paper in a roll = $length\ of\ paper\ used\ in\ a\ year$

Figure 4.13 Researcher-generated expressions of RCC-Group 1's models

Unlike RCC-Group 1, RCC-Group 2 aimed to calculate the amount used in 365 days from the length of toilet paper used daily. They stated that a person could use the toilet four times a day. Since both students, who continued their group work, lived at home with a total of four people, they focused on the length of toilet paper used by four people. Therefore, they said that if each person living in the house used the toilet on average four times a day, the toilet could be used 4 × 4 = 16 times in a day. Also, they assumed that if a person used 100 cm of toilet paper to each entrance to the toilet, then 1600 cm toilet paper were used in a day. They also estimated that each person could use 50 cm toilet paper for other needs in a day. Therefore, they found the total length of toilet paper used in a day by adding these values. Then RCC-Group 2 solved the problem by multiplying the length of toilet paper used in a day by the number of days in a year. Like RCC-Group 1, RCC-Group 2 reached the amount used in a year from the shortest time used toilet paper. Figure 4.14 below shows the researcher-generated mathematical expressions for this group's solution.

(number of people × length of toilet paper which a person used in a day) + (number of people × length of toilet paper which a person used in a day for other need × number of days in a year) = $length\ of\ paper\ used\ in\ a\ year$

Figure 4.14 Researcher-generated expressions of RCC-Group 2's models

Unlike RCC Group-1 and 2, RCC-Group 3 students calculated the length of toilet paper used in the toilet in a day from pieces of toilet paper used at each toilet entrance. They multiplied the number of toilet paper pieces used in each entrance with the length of toilet paper. They investigated each piece of toilet paper as 12 cm. Also, they admitted that one person used three pieces of toilet paper at each entrance to the toilet. Therefore, they stated that they thought they could calculate how many pieces of toilet paper could be used in one day. If everyone uses the toilet four times a day, a family with five people uses 20 × 36 = 720 cm toilet paper. They also calculated the amount of toilet paper used for other needs of all individuals in a day. They calculated the length of toilet paper used in a month and a year, respectively, from the total length of toilet paper spent in a day. Figure 4.15 below shows the researcher-generated mathematical expressions for this group's solution.

[(pieces of toilet paper which is used in each entrance × length of each pieces of toilet paper × number of times to used toilet in each day for a person × number of people) + (pieces of toilet paper used in the kitchen in a day for each person × number of people) + (length of paper used in other needs for a person in a day × number of people)] × number of day in a month × number of a month in a year= $length\ of\ paper\ used\ in\ a\ year$

Figure 4.15 Researcher-generated expressions of RCC-Group 3's models

To summarize, all three groups used unit rate to reach the amount of toilet paper that could be used in a year. Both RCC-Group 2 and RCC-Group 3 focused on the shorter time to calculate toilet paper used than RCC-Group 1. RCC-Group 2 and RCC-Group 3 tried to calculate the length of toilet paper used in a year by looking for an answer to how much toilet paper could be used at each toilet entrance. RCC-Group 2 and RCC-Group 3 thought how many pieces of toilet paper would be used for other needs. This is because toilet paper could be used outside of the toilet. The students of all three groups reached these conclusions through calculations of toilet paper usage and its duration.

4.2.4 Students' Overall Performance for Case 2

RCC-Group 1 worked as low achievers in case 1, receiving 6 points (out of 15 points) from participant selection test. RCC-Group 1 developed a solution for the first problem of the sequence from remote problem context to close problem context. In order to reach the solution in the first problem, they multiplied the number of districts (estimated number) in Turkey by the number of shopping malls in each district (estimated number). However, since they did not establish a relationship in the estimated values, this situation made them question the closeness of the result to the real solution. In the second problem, the approximate weights of the students belonging to each grade level were estimated. Then, they calculated the total weight of whole students. Since the weights of the students at each grade level were calculated separately, the solution was closer to the actual result. On the other hand, the inconsistency of the average estimated weights of students in different grades makes the closeness of the result to the actual result questionable. Because 7^{th} grade students were determined to be heavier than students at all grade levels. At the same time, despite the age difference between the 6^{th} and 8^{th} grades, it is noteworthy that the weights were determined the same. In the third problem, it was estimated that two boxes of toilet paper rolls were used in a month. Each boxes included 20 toilet paper rolls. After that, RCC-Group 1 found the amount of toilet paper used in a year. Then, RCC-Group 1 calculated the length of these toilet paper rolls used in one year. Based on the number of toilet paper rolls used per month, they focused on a larger unit than groups calculated how much toilet paper was used at each toilet entry. The RCC- Group 1 had difficulties interpreting problems though the close problem context to the remote problem context. They showed the best performance in the context of the closest problem.

RCC-Group 2 worked as intermediate group in case 2. They scored 9 from the participant selection test. Since the values are not known in the first question, they wanted to make an algebraic calculation. Finding the general expression with algebraic expressions was a good point of view. However, the error in calculation of mean value prevented them from making a logical generalization. In the second problem, they found the average weight of all students in the school. They aimed to reach the total weight by multiplying this value with the total number of students. Estimating the mean weights of different age groups may have led to a value far from the real solution compared to the groups that calculate the average weight separately at each grade level. In the third question, they calculated how much toilet paper they would use in total by using the length to be used in a day. They also took into account the length of toilet paper to be used for other needs. At this point, the fact that a relationship between quantities they identified was not established during the estimation of the length of the toilet paper used at the entrance to each toilet makes the closeness of the result to the real solution questionable. RCC-Group 2 had difficulty interpreting problems in the context of the remote problem relative to the close problem context. They showed the best performance in the context of the closest problem.

RCC-Group 3 worked as high-level group in case 2, receiving a score of 14 from the participant selection test. In the solution of the first problem of case 2, the total number of shopping malls in Turkey was calculated according to the number of small, medium and large cities. Since it was studied in smaller units, the expectation was to find a result closer to the real value. At this point, the number of shopping malls determined according to the size of the cities was important in terms of finding a value close to the actual result. In the solution of the second problem, after determining the average weight of the students at each grade level, the total weight of all students was calculated. They used their daily life experiences while estimating the weights of the students. Therefore, it could be thought that the problem was close to the actual result. In the last problem of case 2, they calculated the total length of toilet paper used in a year from the number of toilet paper pieces used at each toilet entrance. Here, they tried to reach the result by calculating the observations in daily life and the actual length of a piece of toilet paper from real data. In addition, the length of toilet paper used for cleaning and other needs was calculated separately. Since they solved problem by dividing it into many parts, they calculated smaller units. It can be said that they could find a result close to the real result. RCC-Group 3 estimated the data by establishing relationship between quantities they identified in all three problem cases. They have worked to reach a close result by combining experiences and real values in daily life. Although they solved problems by successfully discussing all problems, it could be said that they set clear values with more confidence while making calculations and estimations in the closer context to their daily lives. Thus, it can be said that they were able to express their knowledge in daily life with specific values in the close context problem to their daily lives.

4.2.5 Comparison of Students' Performance in Case 1 and Case 2

In this study, the student groups at three different levels in case 1 and case 2 were examined in detail. In this part of the study, the solutions developed by the student groups at the same level to the problems were compared.

The low achiever groups in case 1 and case 2 are named as CRC-Group 1 and RCC-Group 1, respectively. CRC-Group 1 encountered the toilet paper roll problem as the first problem. CRC-Group 1 developed three different solutions to the problem. They determined the method that could find a value closer to the actual result in each next solution. They tried to solve the problem by estimating the length of toilet paper used in a shorter time. RCC-Group 1 stated how many boxes of toilet paper were used in a month. They calculated the number of toilet paper rolls used in one year from the number of toilet paper rolls used in a month. The students stated that they observed this inference during shopping for their homes. They calculated the length of toilet paper used in a year based on this. CRC-Group 1 tried to solve the question by focusing on smaller and smaller units, while RCC-Group1 solved the problem using their daily experience directly.

Both CRC-Group 1 and RCC-Group 1 encountered the weight of the students in the school problem as the second problem. CRC-Group 1 was unable to develop a solution. RCC-Group 1 estimated a student's weight at each grade level and calculated the total weight of the students. However, it was noteworthy that the estimated weights were lower in older age groups (for 8th grades).

While CRC-Group 1 could not develop a solution to the shopping mall problem, RCC-Group 1 stated that they could find a result close to the actual value with approximate values. CRC-Group 1 encountered the number of shopping malls in Turkey as the third problem. RCC-Group 1 encountered that problem as first the first problem. CRC-Group 1 directly predicted the result of the problem. On the other hand, RCC-Group 1 estimated how many districts and shopping malls will be in each district in Turkey. From here, they calculated the total number of shopping malls. When the solutions of these three groups were examined, it was not found that asking the problem from a close problem context to a remote problem context in daily life had an apparent effect on the students' solution development.

The intermediate level achievers in case 1 and case 2 were named as CRC-Group 2 and RCC-Group 2, respectively. The toilet paper roll problem was encountered by CRC-Group 2 as the first problem and RCC-Group 2 as the third problem. CRC-Group 2 developed two different solutions to this problem. These two solutions were based on how many pieces of toilet paper they use in a day. Similarly, RCC-Group 2 solved the problem based on how long toilet paper is used daily. Unlike CRC-Group 2, it also considered the amount of toilet paper that could be used for other needs.

Both groups encountered the weight of the students in the school problem in the second place. CRC-Group 2 and RCC-Group 2 have made very similar solutions. Both estimated the average weight of all students in the school, multiplied by the number of students in the school. Then, CRC-Group 2 and RCC-Group reached the solution. RRC-Group 2 considered the weight values they encountered in daily life while estimating the weights. On the other hand, CRC-Group 2 calculated the average weight of all students by calculating the average of an approximate range of values.

The problem related to the number of malls in Turkey was presented to CRC-Group 2 as the third problem, and to RCC-Group 2 as the first problem. CRC-Group 2 solved the problem while calculating the number of shopping malls in Turkey by focusing on the number of shopping malls in small and big cities. They concluded by breaking down the problem into smaller subunits. RCC-Group 2 stated that the problem could not be solved with unknown data, and they wanted to express the mean value with algebraic expressions. However, the methods of calculating the arithmetic mean contained errors. When these three groups' responses were analyzed, it was seen that moving sequencing the problem from a close problem context to a remote problem context had no apparent role on the students' solution development.

The high achiever groups in case 1 and case 2 are named as CRC-Group 3 and RCC-Group 3, respectively. The toilet paper roll problem was encountered by CRC-Group 3 as the first problem and RCC-Group 3 as the third problem. CRC-Group 3 developed two different solutions to this problem. CRC-Group 3 considered how many times a day the toilet would be used and how many pieces of toilet paper would be used for each use. CRC-Group 3 first focused on the number of toilet paper pieces used and then calculated the length of these pieces. In addition, the CRC-Group 3 calculated the amount of toilet paper that could be used for other needs in the solution steps of the problem. They directly stated how many toilets rolls they would use for different needs in a year with a more general expression. A direct value has been estimated without focusing on the shorter period for the amount of toilet paper to be used for other needs. On the other hand, RCC-Group 3’s response was based on the length of toilet paper to be used at each toilet entrance. RCC-Group 3 also calculated the amount of toilet paper used for other cleaning needs and other needs other than cleaning. Unlike CRC-Group 3, RCC-Group 3 calculated the toilet paper used for different needs, from the length used in a day to the length used in a year. In other words, RCC-Group 3 estimated the shorter period and reached the whole. In this solution, it could be said that RCC-Group 3 could find a result closer to the actual result than CRC-Group 3.

Both groups encountered the weight of the students in the school problem in the second place. Both groups found very similar solutions to each other. First, both determined the number of students at each grade level and the average weight. Then, both groups calculated the weights of the students at that grade level. By adding these values, they calculated the total weight of the students in the school. The weights estimated by the students were also very close to each other. Therefore, it could be said that based on the weights of the students in their own schools, these students found values close to the actual value.

The number of malls in Turkey problem was presented to CRC-Group 3 as the third problem, and RCC-Group 3 as the first problem. CRC-Group 3 calculated the total number of shopping malls from the number of shopping malls in each province by using real-life experiences on the slogan in an advertisement. On the other hand, RCC-Group 3 examined the provinces in Turkey under the subtitle of the small, medium, and large cities. They calculated the number of shopping malls in provinces of this size by interpreting the number of shopping malls in their own living areas. They calculated the total number of shopping malls as a result of these transactions. Both groups developed a solution to the problem using their daily observations. When these three groups' responses were analyzed, it was discovered that moving sequencing the problem from a close problem context to a remote problem context in daily life had no apparent effect on the students' solution development.

The levels of CRC-Groups directly influenced the solutions they developed to problems. Compared to the low-level group, a high-level group combined the problem with their daily life experiences and solved the problem by establishing a relationship. Understanding the problem and developing solutions in student groups showed parallelism with the students' levels. The order of the problems did not appear to have a considerable effect. In conclusion, the order of the questions doesn't matter. Regardless of the order of the questions, the closer the context of the problem to students' daily lives, the more engaged the students became, the closer estimations they could make, and the better solutions they could produce.

CHAPTER 5

CONCLUSION AND DISCUSSION

The purpose of present study is to investigate 6th grade students' models in the fermi problems considering the proximity of the problem context to their daily lives. The study's conclusions are summarized and discussed in this chapter. Furthermore, the study's limitations, recommendations for further research, and implications for educational process are discussed.

5.1 The differences between students' models considering the sequence of the Fermi problems

The study's conclusions were drawn regarding the similarities and differences between students' models in the fermi problems, considering the proximity of the problem context to their daily lives. A detailed examination was made of students' comprehension of Fermi problems, development of solution methods to these problems, and estimation of the appropriate numerical values for solution.
The selection test determined the case the student groups would study. The groups were formed so that each case was at the same average level. The fact that students at the same level make similar inferences and establish a relationship between an estimated value and the quantity at the same rate shows that the levels are closely distributed.

The study results showed that the sequence of studied the Fermi Problems does not matter. The results also indicated that the fact that when the context of problems is close to the daily life of the students, it has a positive influence on the students' model regardless of the sequence of the problems, considering the proximity of the problem context to their daily lives.

During the study, the student groups in cases 1 and 2 established more meaningful relationships while solving Fermi problems in a closer context to their daily lives. For instance, the RCC-Group 3 found a solution to the problem related to the student weight in the school by inferring from weights of their friends. They stated that one of their friends’ weights in the 5th grade was 38 kg. They made inferences by using their daily life experiences to estimate the values they needed to solve the problem. The students' developing a solution in the group discussion environment and determination of the sub-parts of the problem positively affected the solution phase of the problem. Thus, the Fermi problems, which were solved based on estimations and did not have a single correct answer, pushed the students in this study to question, evaluate, and be creative, which was consistent with the findings of other studies (e.g., Abay & Filiz, 2020). According to Abay and Filiz (2020), include Fermi problems in the curriculum would allow students to develop creativity and critical thinking skills.

In the study, groups at all levels could make inferences regardless of the order in which the problem was asked for the toilet paper roll problem. The CRC (i.e., Case 1: The Sequence of Close-to-Remote Problem Contexts) and RCC (i.e., Case 2: The Sequence of Remote-to-Close Problem Contexts) groups have made estimations by considering the problem for smaller periods. In other words, they solved the problem by dividing it into smaller parts. By combining the results, they obtained from these small parts, they calculated how long toilet paper they could use in a year. The inferences of CRC and RCC groups at different levels on this problem made a difference in the intensity of the connections established between daily life skills and the problem's solution.

Students in the higher-level groups used more information from their daily lives to solve the problem. It was noteworthy that as the levels of the groups' increased, they tried to reach the general results by getting smaller parts in the solution of the problem. For this reason, the models developed by groups at different levels for

Fermi problems can be examined on a level-specific basis. To provide further insight, groups at different levels will be discussed below by focusing on a few examples specific to the level.

5.1.1 High Achievers' Models in Case 1 & Case 2

High achiever groups' work was more extensive than that of other groups. The students had detailed discussions on Fermi problems throughout the group work. They shared the information they gained from their daily life experiences with each other in this discussion environment. This sharing process led them to combine their knowledge and make the most reasonable estimations for Fermi problems. The students divided the problem into parts by discussing it. They included all the critical issues for the problem's solution by working on these parts. As an example, in Case 1 the CRC-Group 3 divided the toilet paper roll problem into two parts to calculate the length of toilet paper used in one year. These were the "length of toilet paper used in the toilet" and "the length of toilet paper used for non-toilet needs."

Similarly, in Case 2 RCC-Group3 examined the problem under three parts. *The length of the toilet paper used in the toilet, the length of the toilet paper used for other cleaning needs,* and 'the length of the toilet paper used for needs other than cleaning' were three parts of the problem. The groups aimed to get closer to the actual result by breaking down the Fermi problems into smaller parts. They discussed each part in detail and made the appropriate estimations. In addition, while calculating the amount of toilet paper used for the toilet, the groups calculated the toilet paper used for the smallest unit of time (each toilet entry). Starting from this, they calculated the amount of toilet paper used in the toilet in a year. Also, the solution to the number of the shopping mall in Turkey problem can be given as an example of dividing the problem into smaller parts for the solution. For instance, the number of shopping malls in these provinces was calculated by RCC-Group 3 in Case 2 based on the size of the provinces in Turkey. The pupils divided the problem into smaller sections. They arrived at the numbers by considering the size of the provinces in where they live and the number of shopping malls in that province.

As can be understood from these examples, the students preferred to work by breaking the problem into smaller parts instead of directly solving it. In this way, they made the problem interpretable. Thanks to their comments about the problem, they could estimate the numerical values that they could use to solve the problem. Working by dividing the problem into smaller parts ensured that the numerical values they calculated were close to the actual values. Similarities and differences in High Achievers' models can be seen in Table 5.1.

Table 5.1 Similarity and differences of High Achievers’ Models in Case 1 & Case 2

	CRC-Group 3	**RCC-Group 3**
Time Periods	*Each entrance to toilet *They referenced the toilet paper used in the shortest time frame.*	*Each entrance to toilet *They referenced the toilet paper used in the shortest time frame*
Uses of toilet paper	*1120 × 12 = 13 440 pieces used in a year to toilet needs *180 × 91 = 16 380 pieces used in one year to other needs *It was accepted that it was used toilet needs and other needs (other cleaning needs)*	*1600 cm = length of toilet paper used in a day for toilet needs. *200 = length of toilet paper used in other needs in a day *It was accepted that it was used toilet needs and other needs.*
Method for determining the amount of toilet paper used	*1120 × 12 = 13 440 pieces used in a year to toilet needs.	*18× 365 = 6 570 m paper used in a year *They calculated the toilet paper used in meters.*

Table 5.1 (continued)

	*91 rolls and 13 440 pieces of toilet paper are used in one year. *16 380 + 13 440 = 29 820 pieces of toilet paper are used in one year. *They used both the number of pieces and the number of toilet rolls. They expressed the result of the process as the number of pieces of toilet paper used.*	

* These sentences were taken directly from the students' studies and put as evidence for the situation.

As an example, the toilet paper problem was examined in the table above. CRC and RCC-Group 3 have estimated the amount of toilet paper used by focusing on a smaller time frame for problem-solving. Both groups considered how much toilet paper they could use each time they entered the toilet. In addition, both took into account that toilet paper can be consumed both in the toilet and in other areas of use. In addition, CRC-Group 3 expressed the amount of toilet paper used in terms of the number of pieces, while RCC-Group 3 expressed directly in meters.

5.1.2 Medium Achievers’ Models in Case 1 & Case 2

Medium achievers’ models were given less detail than models of high achievers. This might be because they had not reconciled their daily life experiences to solving Fermi problems as much as high achievers. For example, CRC-Group 2 has accepted that toilet paper is used only in the toilet. They did not consider other uses. RCC-Group

2 has also considered how much toilet paper will be used for different needs. Both groups found the length of toilet paper used in the toilet. Firstly, they calculated how many times the toilet could be used in a day. Then, they estimated the amount of toilet paper used each time a toilet was used.

Similar to high achiever groups, medium achiever groups' working in case 1 (CRC-Group 2) estimated how many pieces of toilet paper would be used in each toilet visit with respect to their daily life experiences. They found the length of each piece of toilet paper by researching. In this way, they combined real-life information with the calculations they made from their research. It is expected that this would lead them to find a closer result to the actual result. Medium achievers working in case 2 (RCC-Group 2) directly estimated how long of a toilet paper could be used each time a toilet was used. The estimated length of 1 meter indicates that the students did not fully associate the length of toilet paper with their real-life knowledge.

When the studies of medium achiever groups were examined, they solved the problem by associating them with daily life less than high achiever groups. They examined the problem in smaller parts, as in the high achiever groups. However, medium achiever groups remained more superficial while examining Fermi problems. They were limited in relating the problem they were working on to the real world. They elaborated on the problem less in the discussion process. To give an example of this situation, the solution to the toilet paper problem can be examined as seen in Table 5.2.

Table 5.2 Similarity and differences of Medium Achievers' Models in Case 1 & Case 2

	CRC-Group 2	**RCC-Group 2**
Time Periods	*Each entrances to toilet *They referenced the toilet paper used in the shortest time frame.*	*Each entrances to toilet *They referenced the toilet paper used in the shortest time frame*
Uses of toilet paper	*4 = number of people who use toilet. *4 = number of times for each person goes to the toilet in each day. *It was accepted that it was used only for toilet needs.*	*1600 cm = length of toilet paper used in a day for toilet needs. *200 = length of toilet paper used in other needs in a day *It was accepted that it was used toilet needs and other needs.*
Method for determining the amount of toilet paper used	*16 × 5 = 80 pieces used in a day *80 × 10 cm = 800 cm toilet paper used in a day *They first calculated the number of pieces of toilet paper used, and then calculated the length in cm.*	*18× 365 = 6 570 m paper used in a year *They calculated the toilet paper used in meters.*

* These sentences were taken directly from the students' studies and put as evidence for the situation.

To solve the problem, CRC-Group 2 and RCC-Group 2 first estimated the amount of toilet paper used at each toilet entrance. This is the similarity between these two groups. When focusing on the usage area of toilet paper, differences can be observed in the studies of these two groups. CRC-Group 2 has considered the use of toilet paper only in the toilet.

Although RCC-Group 2 stated that toilet paper could be used for different purposes, they did not specify as many uses as RCC-Group 3. In addition, it can be examined by which unit of measurement these two groups express the amount of toilet paper used. CRC-Group 2 first calculated the number of pieces of toilet paper used and then expressed this length in cm. RCC-Group 2 was calculated in meters. The fact that CRC-Group 2 determines the amount of toilet paper used according to the number of pieces shows that it uses its observations in daily life more in solving the problem. Calculating the number of toilet paper pieces used based on the information they have gained from their own life experience will give a closer result than estimating the length directly.

5.1.3 Low Achievers' Models in Case 1 & Case 2

Models of low achievers were given less detail than models of medium achievers. Both CRC-Group 1 and RCC-Group 1 groups considered toilet paper to be used only in the toilet. While calculating the length of toilet paper used, RCC-Group 1 used the data obtained from their daily life experiences. They remembered the number of toilet paper boxes they bought at home during their monthly shopping with their family. They accepted the number of toilet paper in these boxes as the amount of toilet paper used in a month. They solved the problem by calculating the length of these toilet papers. CRC-Group 1 wanted to calculate the length of toilet paper used in a year by estimating the lengths of toilet paper used in different periods (annual-weekly-monthly). However, the fact that they claimed that they would use shorter toilet paper for long periods led the solution of the problem to be remote from the actual result. Low achiever groups did not specify for what purpose the toilet paper used in their study was used. Based on this, it could be said that they associated the problem with real-life experiences less and examined the problem more superficially. In addition to these, CRC-Group 1 expressed the length of toilet paper used in meters, while RCC-Group 1 determined the number of pieces of toilet paper used first and then expressed this length in cm. All these examples could be seen in

Table 5.3 Similarity and differences of Low Achievers' Models in Case 1 & Case 2

	CRC-Group 1	RCC-Group 1
Time Periods	* Monthly Weekly Daily *they took the amount of toilet paper used in less and less time as a reference.*	* Monthly *They referenced the toilet paper boxes used in the monthly*
Uses of toilet paper	*No specific purpose was specified.*	*No specific purpose was specified.*
Method for determining the amount of toilet paper used	*36 m × 12 = 432 m paper used in a year *They calculated the toilet paper used in meters.*	*3= number of toilet paper pieces used in each entrance to toilet *12 cm = length of each toilet paper piece *36 cm = length of toilet paper used in each entrance toilet *They first calculated the number of pieces of toilet paper used, and then calculated the length in cm.*

* These sentences were taken directly from the students' studies and put as evidence for the situation.

The low achiever group was less productive in the discussion process compared to the other groups. Although they tried to use the information they gained from their daily life experiences, they could not carry out a detailed study as other groups. They did not sufficiently reconcile the information they obtained from daily life with the solution of the problem. It can also be explained by the number of shopping mall problem that low achievers have more general solutions compared to other groups. For instance, CRC-Group 1 (low achievers in case 1) directly predicted the result without establishing a relationship between quantities in this problem. RCC-Group 1 (low achievers in case 2) tried to reach the total number of shopping malls from each district's number of shopping malls. However, they directly estimated the numerical values without establishing a relationship. Compared to the other groups, the low achiever group was the group that established the least meaningful relationship in problem solving.

The solutions the groups developed to problems were more similar when the problems were asked in contexts more remote from their daily lives. For instance, most of the students made inferences about the solution to the student weight in the school problem. CRC-Group 1 stated that the problem could not be solved without real data. When the groups' solutions that continue to work were examined, it could be said that they tried similar solution methods. That is, the CRC and RCC groups solved this problem by inferring their own weights and their friends' weights. They discussed whether the weights of the groups that were not their age were less or more than their own weight. The students focused on each grade in order to calculate the weight of the students in the school. This shows that the students tried to solve the problem in smaller parts. Also, in these small parts, data could be estimated more easily.

The number of malls in Turkey was a problem asked in the most remote context of students' daily lives. Groups developed different solutions to this problem, which were asked in the more remote context of their daily lives. No difference was observed regarding the questions in case 1 and case 2 being asked in a different sequence. Here, CRC-Group 1 directly predicted the result without establishing a relationship in this problem. RCC-Group 1 tried to reach the total number of shopping malls from each district's number of shopping malls. However, they directly estimated the numerical values without establishing a relationship. CRC-Group 2 and RCC-Group 3 estimated the number of shopping malls in these provinces according to the size of the provinces in Turkey. The students solved the problem by dividing smaller parts in this solution method. They calculated the numerical values by considering the size of the provinces they live in and the number of shopping malls in this province.

On the other hand, CRC-Group 3 estimated the number of shopping malls in each province with the data obtained from their daily life experiences. They reached the result by adding the number of shopping malls in all provinces. Unlike all groups, RCC-Group 2 stated that the problem could not be solved without numerical data.

5.2 The differences between students' models considering the proximity of the Fermi problems' context to students' daily lives

Considering the proximity of the problem context to their daily lives, when the students' models in Fermi Problems were examined, the sequence in which the problems were asked did not have a noticeable role. It was observed that groups were more creative when working with close context to their daily life. The most difficult part in this process was establishing an acceptable model to solve Fermi problem because it necessitates a deep understanding of the context, as well as a high degree of creativity the related mathematical concepts (Albarracín & Gorgorió, 2015). Students' motivation was enhanced while creating a mathematical model to solve a real-world mathematics problem (English et al., 2003). They could discuss the problem productively, which was closer to their daily life. The closer the problem situation is to their real-life experience, the more information they can find to discuss the situation. Students were motivated by studying mathematical problems in their daily lives, debating different solutions, and engaging in diverse educational activities (Tzanakis & Arcavi, 2000). Discussing the problem enabled them to notice the smaller parts that needed to be considered in solving the problem. In this way, they were able to examine the factors affecting the problem's solution in detail.

In problem situations close to their daily lives, they could make more consistent estimations by explaining the reason. This is because students could look at the problem context from different perspectives by working in small groups and using different methods (Albarracin & Gorgorio, 2014). Each student shared their problem-solving skills and knowledge that could be used in solving the problem with their group friends. Therefore, these types of problems encouraged students to work in groups and use various methods in different ways to solve the problem context (Albarracin & Gorgorio, 2014). Therefore, students were able to develop better solutions for problem situations that were close to their real life, compared to problem situations that are far from their real lives. The sequences proximity of the problem context in cases did not make a noticeable contribution to the solutions students develop to Fermi problems. For this reason, it could be inferred that teachers do not need to create a lesson plan by ordering the problems from close problem context to remote problem context in the studies they would do with Fermi problems.

While studying the Fermi Problem, the CRC and RCC groups utilized mathematical abilities, logic, critical thinking, life experiences, and the ability to divide complex situations into easy, solvable parts (Abrams, 2011). In this sense, the modeling problems encouraged students in improving their mathematical thinking by allowing them to debate solutions to problems. Additionally, other researchers have confirmed similar findings (English et al., 2003). Also, according to Oropesa and Cortez (2015), analyzing the problem, which is one of the first elements of mathematical modeling (can be seen in Table 2.1.), develops this mathematical thinking skill, because students identify Fermi problems and search for necessary information by making sense. In addition to improve mathematical thinking skills, studying with Fermi problems also contributes to the development of different thinking skills of students. The reason of this, students need to consider many variables when they solve Fermi problems. Therefore, including Fermi problems in lesson plans has an important place in the development of students' mathematical modeling, thinking and problem-solving skills.

This research indicated that Fermi Problems could be employed to encourage and emphasize the mathematical modeling process (Peter-Koop, 2010). In addition, thanks to this study, how the level groups affect working with Fermi problems has been clearly examined. Differences and similarities were revealed in the interpretation of Fermi problems according to the level groups of the students. As the level groups of the students increased, the solutions they developed to the Fermi problems and the mathematical models were getting improved. Thanks to this study, teachers or researchers who will carry out similar studies can create study groups in accordance with their goals.

The activities in the mathematical modeling process are constructed in the context of the students' interests in such a way that motivates them to examine and explain the real-world problem context (Doruk, 2011). Therefore, Fermi problems in the context need to be constructed that allow students to analyze, detail the problem by breaking it into smaller parts, and estimate the required numerical values. In this way, students could solve Fermi Problems more efficiently by discussing them.

Contexts are better to be chosen that support students to use their experiences in their daily lives in their work with Fermi problems. If a problem context close to students' daily lives is chosen, students can make more inferences and conduct more discussions. This supports the mathematical modeling process. This study determined that presenting the Fermi Problems contexts from close problem context to remote problem context or from remote problem to close problem context has no noteworty role in students’ models.

5.3 Limitations, Recommendations and Implications

This study aimed to identify how students' models in fermi problems are similar and different depending on problem's proximity to their daily lives. The results of the present study were presented in detail in the previous chapter. However, there are several limitations to this qualitative study. The study's limitations, recommendations for further research, and implications for educational practices are discussed in this section.

First, three fermi problems have been used in this study. The implementation of the fermi problems was planned based on a literature study. There was a limited time to study with students. If more time was invested and more exercises were conducted, richer data regarding the students' Fermi Problems model could be presented. Therefore, this study suggests studies that are designed to include longer time period for students' getting familiar to Fermi Problems' structure because they were quite different than other mathematics problems in that they did not include any numerical values.

Second, the groups studied in the research consisted of students living in different regions of Turkey. Although the selection test determined the levels of the student groups, the students might have solved the problems from different perspectives because they came from different backgrounds (Ferrando & Albarracín, 2019). Third, the study with the groups of students were conducted through the online session due to the covid 19 pandemic conditions. This process has challenged the students. In the online sessions, the students drew the drawings related to the solutions on the computer screen and this process may have put some strain on them. Regarding this limitation, I suggest studies using fermi problems with students but in a face-to-face learning environment, which may increase students' interaction with one another and ease their mathematical model development process.

With reference to educational implications, considering that students' benefits in developing mathematical insights through Fermi Problems, I recommend textbook writers including these problems to assist students in making links between their academic and their daily lives. Furthermore, in light of this study, teachers can plan their lectures by arranging activities since addressing Fermi problems by creating mathematical models increased students' comprehension of mathematics. Fermi problems are problems that do not contain numbers. Solving these problems in the discussion environment can make a different contribution to students' perception of mathematics. Due to the nature of the Fermi problems (not containing numbers), these problems require estimation-based solutions. Thus, these problems may increase students’ estimation skills and may help them develop an insight on how to use estimations when building mathematically sound solution to a problem.

According to the results of this study, problems presented in a closer context to students' lives led to more detailed solution because they can reconcile problems that are closer to their daily lives with the information they obtain from daily life. Therefore, the findings of this study suggest teachers select problem contexts to be closer to the students’ lives. Not only students can benefit from the Fermi Problems, but also pre-service teachers do. In other words, Fermi problems may also be effective instructional tools in teacher education courses where prospective teachers may understand the value and contribution of those problems to students’ estimation skills and mathematical modeling process.

In addition, students are involved in a process where they actively use their thinking skills while working with Fermi problems. Actively solving problems by inferring from their existing knowledge allows students to be involved in the process voluntarily. Therefore, they are more motivated to solve the problem. In addition, working with their friends, bringing different ideas together and creating a common solution positively affects students' perceptions of problem solving. Therefore, incorporating Fermi problems into mathematical modeling processes has an important role in both improving students' mathematical thinking skills and incorporating them into the learning environment in a motivated way.

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APPENDICES

A. PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

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24 Mayıs 2021

Konu : Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi : İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Dr. Öğr. Üyesi Şerife SEVİNÇ

Danışmanlığını yürüttüğünüz Burcu Arıcan'ın "Günlük hayatta edinilen bilgilerinin fermi problemlerine çözüm geliştirilmesindeki etkisi" başlıklı araştırmanız İnsan Araştırmaları Etik Kurulu tarafından uygun görülmüş ve **236-ODTU-2021** protokol numarası ile onaylanmıştır.

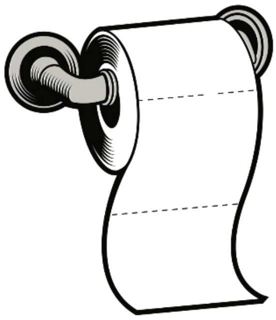
Saygılarımızla bilgilerinize sunarız.

Dr.Öğretim Üyesi Ali Emre TURGUT
İAEK Başkan Vekili

B. INTERVIEW QUESTIONS WITH FERMI PROBLEMS

Aşağıda verilen problemlere grup arkadaşlarınız ile tartışarak çözüm arayınız.

1) Evinizde bir yılda ne kadar uzunlukta tuvalet kağıdı kullanılıyor olabilir?



- Size verilen problemden ne anlıyorsunuz?
- Problemde size verilen bilgiler nelerdir?
- Bu bilgiler problemi çözmeniz için yeterli mi? Nedenini açıklayınız?

• Problemi çözmek için ihtiyaç duyduğunuz bilgiler nelerdir?	• İhtiyaç duyduğunuz bu bilgilerin cevapları neler olabilir?

- Bu bilgilere nereden ulaştığınızı açıklayınız?
- Bu bilgileri kullanarak problemi çözünüz?
- Bu bilgiler problemi çözebilmeniz için yeterli oldu mu? Evet / Hayır
- Problemin çözümünü etkileyen başka bilgiler/durumlar olabilir mi? Evet / Hayır
  Evet ise; hangi bilgiler/durumlar problemin çözümünü etkileyebilir? Nasıl etkileyebilir?

- Problemin kesin bir sonucuna ulaşabildiniz mi? / ulaşılabilir mi?

  Cevabınız evet ise;

  - Kesin cevaba neden / nasıl ulaşabileceğinizi düşünüyorsunuz?

  Cevabınız hayır ise;

  - Neden kesin cevaba ulaşamayacağınızı düşünüyorsunuz?

  - Problemin sonucuna en yakın sonuca ulaşabilir miyiz? Evet / Hayır

    Evet ise, problemin sonucuna en yakın sonuca ulaşabilmek için nelere dikkat etmeliyiz?

  - Problemi tekrardan çözseydin;

    Neye dikkat ederdin?

    Neyi araştırmak isterdin?

2) Ortaokulunuzda okuyan tüm öğrencilerin kg cinsinden toplam ağırlığı ne olabilir?

- Size verilen problemden ne anlıyorsunuz?

- Problemde size verilen bilgiler nelerdir?

- Bu bilgiler problemi çözmeniz için yeterli mi? Nedenini açıklayınız?

• Problemi çözmek için ihtiyaç duyduğunuz bilgiler nelerdir?	• İhtiyaç duyduğunuz bu bilgilerin cevapları neler olabilir?

- Bu bilgilere nereden ulaştığınızı açıklayınız?

- Bu bilgileri kullanarak problemi çözünüz?

- Bu bilgiler problemi çözebilmeniz için yeterli oldu mu?   Evet / Hayır
- Problemin çözümünü etkileyen başka bilgiler/durumlar olabilir mi? Evet / Hayır
- Evet ise; hangi bilgiler/durumlar problemin çözümünü etkileyebilir? Nasıl etkileyebilir?
- Problemin kesin bir sonucuna ulaşabildiniz mi? / ulaşılabilir mi?

  Cevabınız evet ise;
  - Kesin cevaba neden / nasıl ulaşabileceğinizi düşünüyorsunuz?

  Cevabınız hayır ise;
  - Neden kesin cevaba ulaşamayacağınızı düşünüyorsunuz?
- Problemin sonucuna en yakın sonuca ulaşabilir miyiz?   Evet / Hayır

  Evet ise, problemin sonucuna en yakın sonuca ulaşabilmek için nelere dikkat etmeliyiz?

- Problemi tekrardan çözseydin;
  Neye dikkat ederdin?
  Neyi araştırmak isterdin?

3) Türkiye'de kaç tane alışveriş merkezi olabilir?

- Size verilen problemden ne anlıyorsunuz?

- Problemde size verilen bilgiler nelerdir?

- Bu bilgiler problemi çözmeniz için yeterli mi? Nedenini açıklayınız?

• Problemi çözmek için ihtiyaç duyduğunuz bilgiler nelerdir?	• İhtiyaç duyduğunuz bu bilgilerin cevapları neler olabilir?

- Bu bilgilere nereden ulaştığınızı açıklayınız?

- Bu bilgileri kullanarak problemi çözünüz?

- Bu bilgiler problemi çözebilmeniz için yeterli oldu mu?   Evet / Hayır

- Problemin çözümünü etkileyen başka bilgiler/durumlar olabilir mi? Evet / Hayır
  Evet ise; hangi bilgiler/durumlar problemin çözümünü etkileyebilir? Nasıl etkileyebilir?

- Problemin kesin bir sonucuna ulaşabildiniz mi? / ulaşılabilir mi?
  Cevabınız evet ise;
    - Kesin cevaba neden / nasıl ulaşabileceğinizi düşünüyorsunuz?

  Cevabınız hayır ise;
    - Neden kesin cevaba ulaşamayacağınızı düşünüyorsunuz?

- Problemin sonucuna en yakın sonuca ulaşabilir miyiz?   Evet / Hayır

  Evet ise, problemin sonucuna en yakın sonuca ulaşabilmek için nelere dikkat etmeliyiz?

- Problemi tekrardan çözseydin;
  Neye dikkat ederdin?