FATIGUE LIFE ESTIMATION OF TUNED VIBRATION ABSORBERS WITH RESPONSE ESTIMATION METHOD

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FATIGUE LIFE ESTIMATION OF TUNED VIBRATION ABSORBERS WITH RESPONSE ESTIMATION METHOD

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ABSTRACT

FATIGUE LIFE ESTIMATION OF TUNED VIBRATION ABSORBERS WITH RESPONSE ESTIMATION METHOD

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Tuned Vibration Absorbers are designed and used in applications where any unwanted vibration is needed to be elimination from a vibrating system. In order to accomplish that goal TVAs are needed to be designed to absorb vibrational energy on its own at a specified frequency. Since the energy is absorbed from main system to the TVA, this energy will cause high vibration levels on the TVA. Due to the excessive vibration that occurs on the TVA, vibrational fatigue life is a concern for TVA design. Vibrational fatigue life tests or analyses can be carried out during design phase of the TVA. However, for the systems that are not easy to test or analyze because of its size or complexity, response estimation methods are required to ease calculation or testing of fatigue life of the TVA during design phase of it. Those response estimation methods are used to calculate response of the TVA as if it has attached to the main structure. Since the response of the TVA could be acquired from the estimation, fatigue life analysis or tests can be carried with the response that is estimated. Computational or testing time is saved with response estimation method. A previously developed response estimation method is evaluated and verified for fatigue life analysis and tests in this thesis. Two types of TVA have been designed for a basic structure as the first part of the thesis study. Those two types are cantilever beam type TVA and helical spring and mass type TVA. In order to design the TVAs modal analyses have been carried out for both main structure and TVAs. When design of the TVAs is finished, next step of the thesis study is to conduct fatigue life estimation analyses of whole structure that is main structure with TVA and only TVA with utilizing response estimation method. Effectiveness of response estimation method for estimating fatigue life of the TVA is compared by using analysis results. Furthermore, fatigue life experiments have been carried out for the beam type TVA design. Evaluation of the results that are acquired from analysis with utilization of response estimation method and whole structure are compared with the test results to compare analysis results with real life test. Hence, effectiveness of the response estimation method to estimate fatigue life of the TVAs is evaluated with analyses and tests.

Keywords: vibration control, tuned vibration absorber, identification, system modification, fatigue, fatigue life estimation, accelerated fatigue life test, response estimation

ÖZ

AYARLANMIŞ TİTREŞİM EMİCİLERİN OPERASYONEL CEVAPLARININ TAHMİNİ METODU İLE YORULMA ÖMRÜ TAHMİNİ

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Ayarlı Titreşim Soğurucular, titreşimli bir sistemden herhangi bir istenmeyen titreşimin giderilmesi gereken uygulamalarda tasarlanmış ve kullanılmaktadır. Bu amaca ulaşmak için TVA'ların belirli bir frekansta kendi başına titreşim enerjisini absorbe edecek şekilde tasarlanması gerekir. Enerji ana sistemden TVA'ya emildiği için bu enerji TVA'da yüksek titreşim seviyelerine neden olacaktır. TVA'da meydana gelen aşırı titreşim nedeniyle, titreşimsel yorulma ömrü TVA tasarımı için bir endişe kaynağıdır. TVA'nın tasarım aşamasında titreşimsel yorulma ömrü testleri veya analizleri yapılabilir. Ancak, boyutu veya karmaşıklığı nedeniyle test edilmesi veya analiz edilmesi kolay olmayan sistemler için, TVA'nın tasarım aşamasında yorulma ömrünün hesaplanmasını veya test edilmesini kolaylaştırmak için yanıt tahmin yöntemlerine ihtiyaç vardır. Bu tepki tahmin yöntemleri, TVA'nın tepkisini ana yapıya bağlıymış gibi hesaplamak için kullanılır. TVA'nın yanıtı tahminden elde edilebildiğinden, tahmin edilen yanıtla yorulma ömrü analizi veya testler yapılabilir. Yanıt tahmin yöntemi ile hesaplama veya test süresinden zaman kazanılır. Tez çalışmalarında yorulma ömrü analizi ve testleri için tepki tahmin yöntemi

değerlendirilmiştir. Tez çalışmasının ilk kısmı olarak temel bir yapı için iki tip TVA tasarlanmıştır. Bu iki tip konsol kiriş tipi TVA ve sarmal yay ve kütle tipi TVA'dır. TVA'ları tasarlamak için hem ana yapı hem de TVA'lar için modal analizler yapılmıştır. TVA'ların tasarımı tamamlandığında, tez çalışmasının bir sonraki adımı, ana yapı olan tüm yapının TVA ile ve sadece TVA'nın tepki tahmin yöntemi kullanılarak yorulma ömrü tahmin analizlerinin yapılmasıdır. TVA'nın yorulma ömrünü tahmin etmek için yanıt tahmin yönteminin etkinliği, analiz sonuçları kullanılarak karşılaştırılmıştır. Ayrıca, konsol kiriş tipi TVA için yorulma ömrü testleri yapılmıştır. Tepki tahmin yöntemi ve tüm yapı kullanılarak analizden elde edilen sonuçların değerlendirilmesi, analiz sonuçlarını gerçek hayat testi ile karşılaştırmak için test sonuçları ile karşılaştırılır. Bununla beraber, TVA'ların yorulma ömrünü tahmin etmede yanıt tahmin yönteminin etkinliği, analizler ve testler ile değerlendirilmiştir.

Anahtar Kelimeler: titreşim kontrolü, ayarlanmış titreşim sönümleyici, karakterizyon, sistem modifikasyon, yorulma, yorulma ömrü tahmini, hızlandırılmış yorulma ömrü testi, cevap tahmin etme

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For some of the experiments conducted in this thesis, some of the equipment (shaker and shaker control system) and test setups acquired through TÜBİTAK 1001 grant number 214M065 were used.

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ABBREVIATIONS	
3D	3 Dimensional
TVA	Tuned Vibration Absorber
SDOF	Single Degree of Freedom
MDOF	Multi Degree of Freedom
DOF	Degree of Freedom
FRF	Frequency Response Function
PSD	Power Spectral Density
FE	Finite Element
FEA	Finite Element Analysis

CHAPTER 1

INTRODUCTION

Tuned vibration absorbers (TVA)s are devices that are designed to absorb unwanted vibration occurred on a particular frequency or to reduce vibrating amplitude levels of a structure. TVAs consist of three main elements that are an inertial element, a stiffness element and an energy dissipation element. In order to tune the TVA at a particular frequency, it is required to adjust inertial, stiffness or damping element of the TVA. Since the TVAs are designed to absorb vibrational energy from the main structure, stiffness elements on the structure is expected to experience large dynamic deflections. Due to large deflections, stiffness elements on the TVA have high chance of failure due to fatigue loads. In order to eliminate chance of having fatigue failure on the TVA, during the design phase of the TVA fatigue life estimation analysis could be done. One of the acceleration methods is a response estimation method that is developed to estimate response of the TVA when it is attached on the main structure. Utilizing the response estimation method could accelerate design phase that is for to avoid fatigue failure on the TVA.

For the first part of the thesis study, two different types of TVA are designed to absorb vibration from a basic structure which is basically a beam. First of the two different type of TVA is cantilever beam type TVA. A cantilever beam is designed to work as a stiffness element in this TVA design. Additionally, masses are added to the TVA design. According to added mass and stiffness property from cantilever beam, TVA is tuned to the first natural frequency of the base structure. Second of the two different TVA is a helical spring type TVA. A set of helical springs are selected to be used in this type of TVA to be stiffness element. Additionally, a mass is also added to the TVA. Tuning frequency of the TVA is set by changing stiffness and mass elements of the TVA.

For the second part of the thesis study, a novel response estimation method is used to estimate response on TVA as if it is attached to the base structure. The response estimation method is using direct FRF of the point where TVA is attached, operational response of the base structure from where the TVA is attached and TVA's direct FRF from where it is attached to the base structure. By utilizing that three information about base structure and the TVA, response of the TVA at where it is attached to the base structure can be estimated. Furthermore, estimation of the response is used as base excitation to the TVA for fatigue life estimation analysis. Additionally, fatigue life estimation analysis of the whole structure which is base structure with TVA is also done to compare fatigue life estimation results with the results get from only TVA analysis by utilizing response estimation method.

For the third and last part of thesis study, series of fatigue life tests are run with both type of the TVA. Results of the tests are compared with the analysis results which are for whole structure which is base structure with TVA and for TVA only by utilizing response estimation method. Comparison of the analysis and test results are further discussed at conclusion section.

CHAPTER 2

LITERATURE SURVEY

Vibration can be defined as harmonic or random motion of particles of a material existing around equilibrium point of material due to elasticity of the material. Vibration is one of the root causes of most of engineering failures for machines, electronic components etc. Additionally, vibration can also affect performance of machines with unwanted oscillations. Due to this high potential of failure because of vibration, engineering world work on solving unwanted vibration and its avoidable results. Vibration can be eliminated from its source to eliminate unwanted effects on structure. However, if the source of the vibration cannot be eliminated, the next option would be to lowering the effects of vibration by reducing or controlling vibration amplitudes. Tuned vibration absorbers are one of the solutions for controlling and reducing vibration on structures. TVAs are mainly used to modify the structure in a way that, TVA would absorb most of the vibrational energy from the structure to itself. As a reason to that tuned vibration absorbers exposed to high level of vibration energy. This energy could lead to a phenomenon called fatigue failure on TVAs. In order to avoid fatigue failure on the TVAs, its design should be evaluated beforehand.

2.1 Tuned Vibration Absorber Theory

Mechanical systems could be subjected to unwanted vibrations due to its moving parts or they might be built on a vibrating base that could induce vibration on mechanical structure. In order to eliminate that unwanted vibrations one solution could be to modify the structure. However, using a TVA is usually preferable solution to reduce and control unwanted vibration on the system. Frahm has invented TVA in 1909

[2], after which TVAs are widely used to solve vibrational problems. A TVA can be considered as a single degree of freedom system that has a mass, a spring and a damping element. Equation of motion for this SDOF system can be written as;

$$m\ddot{y} + c\dot{y} + ky = f(t) \tag{2.1}$$

In this equation m refers for mass, k stands for stiffness and c stands for damping coefficient. Moreover, natural frequency and fraction of critical damping of the system can be defined as;

$$\omega_n^2 = \frac{k}{m} \tag{2.2}$$

$$\zeta = \frac{c}{2m\omega_n} \tag{2.3}$$

If the system is lightly damped $(\zeta << 1)$ equation of motion can be rewritten as;

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = \frac{f(t)}{m} \tag{2.4}$$

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = \frac{f(t)}{m} \tag{2.5}$$

By solving the differential equation transfer function written below can be get;

$$H(\omega) = \left(\frac{1}{m}\right)\left(\frac{1}{(\omega_n^2 - \omega^2) + 2i\zeta\omega\omega_n}\right) \tag{2.6}$$

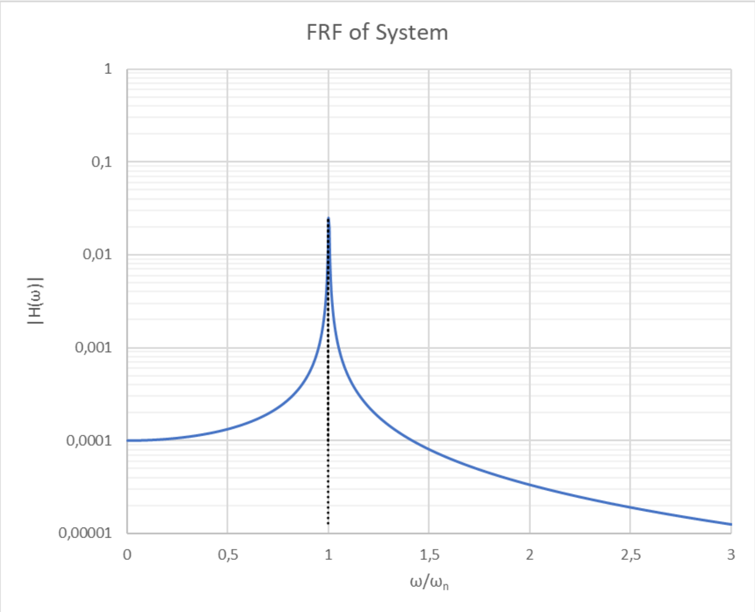

Figure 2.1: SDOF FRF Graph

FRF of the system is shown at 2.1. Resonance frequency can be observed from the FRF graph of the system. This system can be used as TVA for a host structure to reduce vibration around its natural frequency (ω_n). In order to visualize, consider a system with TVA as given at 2.2 with mass M, stiffness element K and forcing $P_0 sin\omega t$;

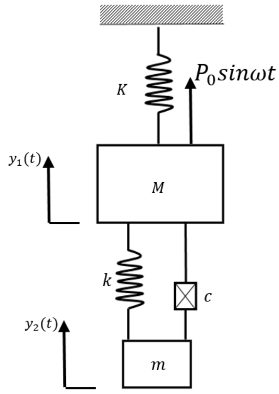

Figure 2.2: A Structure Modified with a TVA

For both of masses M and m force equilibrium are written as below;

$$m(\ddot{y_2}) + k(y_2 - y_1) + c(\dot{y_2} - \dot{y_1}) = 0 \tag{2.7}$$

$$M(\ddot{y_1}) + Ky_1 + k(y_1 - y_2) + c(\dot{y_1} - \dot{y_2}) = P_0 \sin \omega t \tag{2.8}$$

Steady state responses is considered for this case. As a result, equations can be modified as below;

$$-m\omega^2(Y_2) + k(Y_2 - Y_1) + i\omega c(Y_2 - Y_1) = 0 \tag{2.9}$$

$$-M\omega^2(Y_1) + KY_1 + k(Y_1 - Y_2) + i\omega c(Y_1 - Y_2) = P_0 \tag{2.10}$$

By using above equation, one can solve for y_1;

$$\frac{Y_1}{P_0} = H(\omega) \tag{2.11}$$

$$H(\omega) = \sqrt{\frac{(k - m\omega^2)^2 + \omega^2 c^2}{((-M\omega^2 + K)(-m\omega^2 + k) - m\omega^2 k)^2 + \omega^2 c^2 (-M\omega^2 + K - m\omega^2)^2}} \tag{2.12}$$

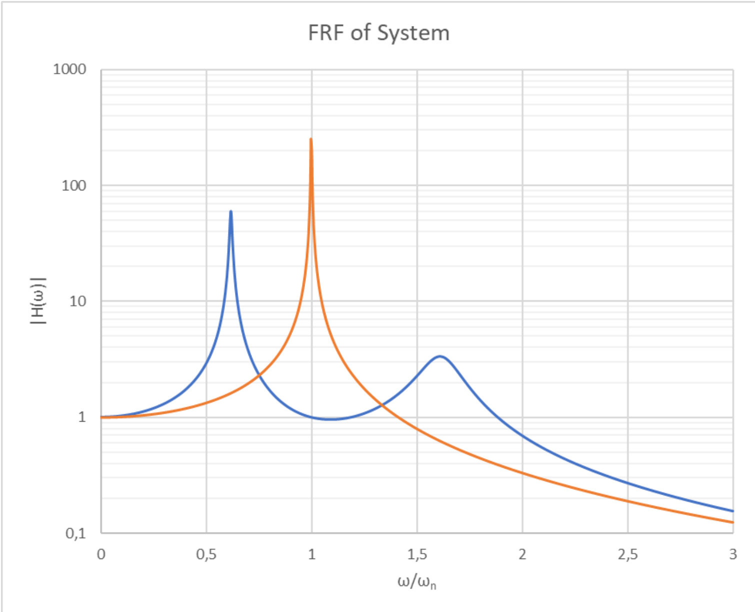

Figure 2.3: Modified system with TVA

From graph given at 2.3 it can be seen that response of the host structure reduced drastically at frequency where ($\omega = \omega_n$). Moreover, control over vibrational behavior of the system can be further altered via controlling damping (c) and mass ratio (m/M).

2.2 Response Estimation Methods

Performance of a TVA can be evaluated by using TVA response as well as host structure response data. For example, in order to evaluate fatigue life estimation of the TVA, one would need power spectrum responses for displacement, stresses and strains. In order to optimize design of the TVA for fatigue life, response of the TVA should be recalculated after every design step. FE modelling and analysis tools can be used for vibrational problems that dynamic properties of host structure and operational conditions are known. However, for the cases that host structures are complex which prolongs FE calculation times for each design steps, one need for a way to calculate response of the TVA faster. As a result, techniques that allows quick evaluation of response of the TVA are developed to allow use of system modification techniques that are based of frequency response functions. Wang and

Zhu studied on to find modified system in-situ frequency functions by utilizing a structural modification method with unknown excitation information during operation [3]. Assumption in their study is unchanged excitation source regardless of change on system. According to their study, they utilize operational response of the original system to estimate forcing by using inverse transfer functions. Furthermore, this data also used to calculate modified system response.

Another study for response estimation of vibrating bodies is completed by Gunnarsson [4]. In the study, a methodology of how to determine response a body that is excited by random sound pressure fields is investigated. A new tool is developed in this study by using MATLAB whose main purpose is to study structural response due to random sound pressure fields with different correlation methods.

Özden also come up with a method that enables to estimate response of a structure modification[1]. The method is utilizing operational response of base structure, direct FRFs of the base structure and TVA to estimate response of the system modification which is a TVA in that study.

2.3 Fatigue Life Estimation Methods

Cui examined different fatigue life estimation methods[5]. In the study, life estimation methods are studied in two groups. The first one is stated as cumulative fatigue damage theories. Stress-based, strain-based and energy -based approaches are in this group. Second one is stated as fatigue crack propagation. Under the second group, two different approaches are studied which are long crack growth and physically small crack growth.

Santecchia, Hamouda, Musharavati, Zalnezhad, Cabibbo, El Mehtedi and Spigarelli conduct a study to get a review on fatigue life prediction methods for metals [6]. According to the study, none of the proposed fatigue life prediction methods can be universally accepted. Each prediction method's effectiveness depends on the case itself. Features of an ideal fatigue life prediction model is given at Figure 2.4 according to their study[6].

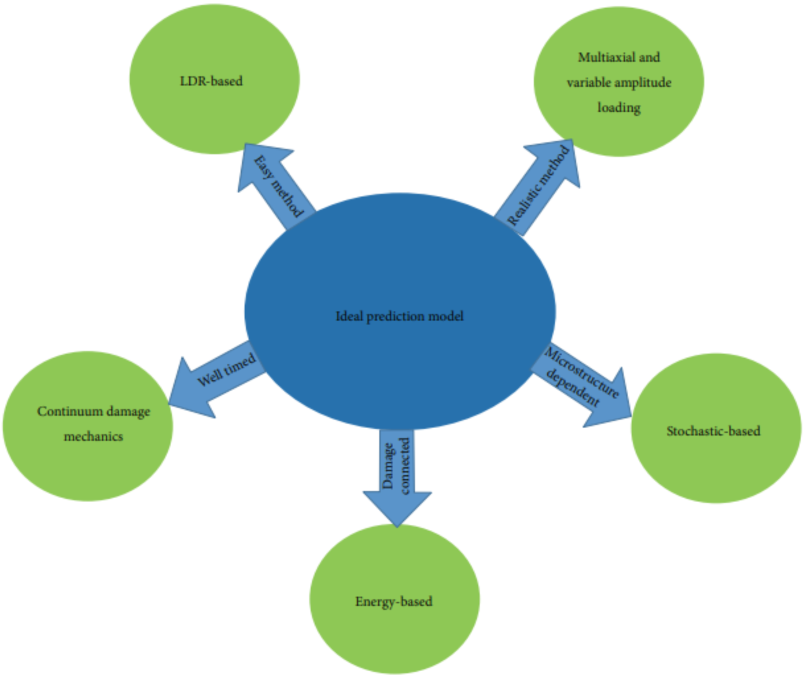

Figure 2.4: Ideal Fatigue Life Prediction Model Features

Eldoğan and Ciğeroğlu conducted a study by using different fatigue theories to estimate vibrational fatigue life of a cantilever beam[7]. Different fatigue life estimation methods such as Narrow-Band, Wirching, Tunna, Hancock, Kam and Dover, Steinberg, Dirlik methods are used to estimate vibrational fatigue life of the beam. Moreover, a series of fatigue life tests are conducted for notched beams to compare test results with different estimation methods results. According to results of this study made by Mr. Eldoğan and Mr. Ciğeroğlu, Dirlik method is estimating fatigue life of the subject closer to the test results among all methods.

Bishop and Woodward explain principal subjects that are used in fatigue life estimation in their article that is named as Fatigue Analysis of a Missile Shaker Table Bracket[8]. Fatigue life estimation can be done in time domain or frequency domain according to this article. Rain flow counting and Palmgren-Miner method is used in time domain solution. Moreover, narrowband solution and Dirlik method are given to use in frequency domain. Finally, a frequency based fatigue life estimation is calculated for a missile shaker table. Results of the fatigue life estimation are given in the article.

2.4 Fatigue Life Analysis Using FEA

Gümüş, Sen and Kalyoncu conducted a study about fatigue life analysis of a cantilever beam that is in transportation based on military standards with using FEA[9]. In order to calculate fatigue life of the beams they use Miner's Cumulative Damage technique. As a first step they calculate stresses on the beam due to random vibration using FEM. Moreover, that stress values are used to calculate fatigue life of the beam via Miner's Cumulative Damage technique. According to the evaluation, this technique is a time saving method to analyze fatigue life.

Another example of using FEA on the fatigue life evaluation is a study completed by Mulla[10]. In this article, Morrow's model and Smith-Watson Topper's model are used for fatigue life evaluation. Moreover, experimental results are also compared with FEA results for helical springs. ANSYS software is used for stress analysis during studies. Moreover, nCode software is used to calculate fatigue life of springs. According to the results of this study, Morrow's model gives more conservative results. On the other hand, Smith-Watson Topper's model results good fatigue life estimation.

A similar study made by Llano-Vizcaya, Rubio-Gonzalez, Mesmacque and Cervantes-Hernandez named as Multiaxial Fatigue and Failure Analysis of Helical Compression Springs[11]. Similarly, ANSYS is used for stress analysis and nCode is used for multiaxial fatigue analysis. However, different methods that are named as Fatemi-Socie, Wang-Brown and Coffin-Manson are used for evaluation. These methods are based on shear deformation. According to the results, Coffin-Mason model gives conservative results, Fatemi-Socie critical plane approach gives a good evaluation and Wang-Brown overestimate fatigue life of the spring.

2.5 Fatigue Life Tests

In the study named Random Vibration Test for Electric Assemblies Fatigue Life Estimation by Al-Yafawi, Patil, Yu, Park and Pitarresi make a comparison between simulation and experimental results of fatigue life estimation for electric assemblies[12]. In order to accomplish the comparison, a finite element model is developed via ANSYS. Additionally, this model is qualified to use in simulations by checking its mode shapes, natural frequencies and transmissibility functions with experimental results. Moreover, the model is used to obtain simulation results with sinusoidal and random vibration tests.

Another study made by Özsoy, Çelik and Kadıoğlu to come up with an approach for an analysis and testing program that includes accelerated life testing for an add-on unit of a helicopter[13]. First of all, loading data is acquired with operational flight tests. The loading data is used for simulation and testing. Fatigue life test are conducted with ALT profile that is synthesized from collected data in the laboratory. Strain gages are placed on critical points of add-on unit during the test. Fatigue life test is conducted until failure of add-on unit.

Jiang, Yun, Zhao and Tao conduct a study which is named as Experimental Design and Validation of an Accelerated Random Vibration Fatigue Testing Methodology to come up a novel test methodology for accelerated random vibration fatigue tests[14]. Several fatigue life tests are designed and conducted to show Gaussian and non-Gaussian random excitation on the vibration fatigue. As a first step, stress values at weak points on test specimens are measured with different loadings. Fatigue life is predicted with WAFO simulation technique with use of the stress values. Moreover, fatigue life tests are conducted to compare test results with simulation results. As conclusion WAFO for non-Gaussian vibration fatigue life prediction and accelerated fatigue life testing with non-Gaussian vibration excitation are verified experimentally.

CHAPTER 3

TVA DESIGN AND VALIDATION

In order to evaluate effectiveness of the response estimation method for fatigue life estimation analysis a case study with one base structure and two different Tuned Vibration Absorbers. Base structure selected for the thesis study is a simple aluminum beam with following dimensions.

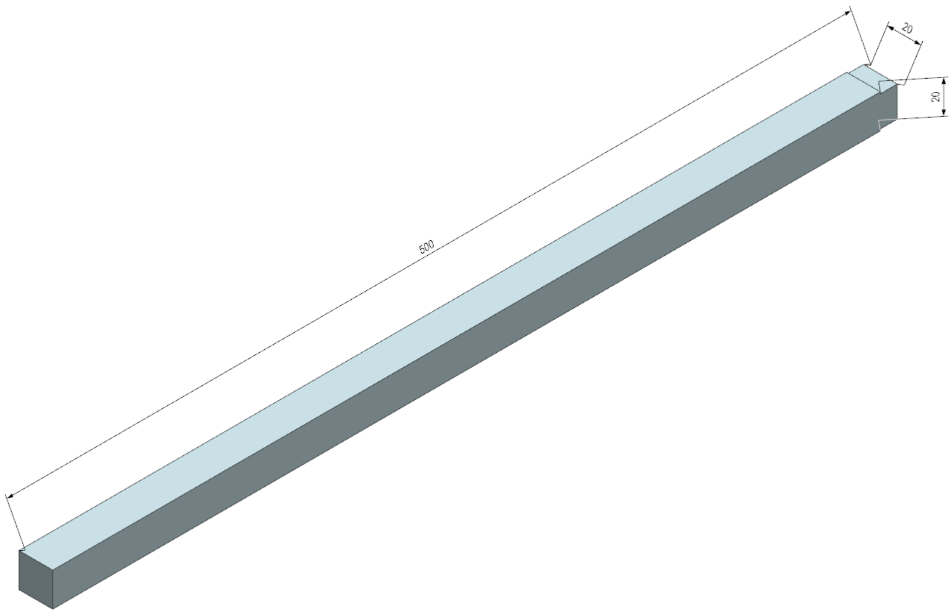

Figure 3.1: Base Structure with Dimensions

Modal analysis of the base structure is completed in order to find tuning frequency of the base structure. Second mode of the base structure is selected to be the tuning frequency. First six modes of the base structure can be seen at Table 3.1. Total deformation for the second mode of the base structure can be seen at Figure 3.2.

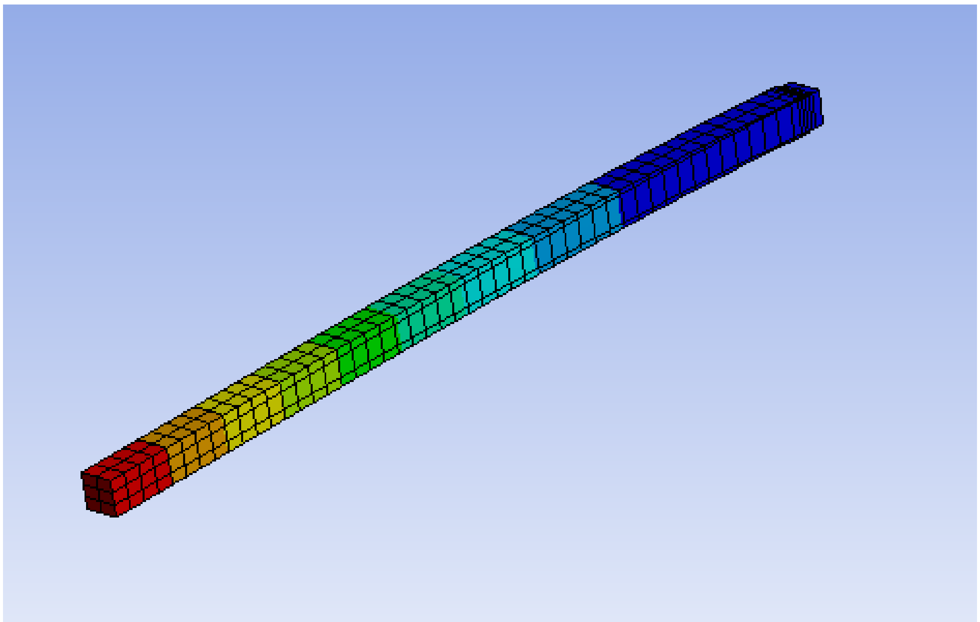

Figure 3.2: Second Mode Total Deformation of Base Structure

Since required modal frequency information about the base structure is acquired, next step of the study is TVA design.

Table 3.1: Base Structure Modal Frequency Table

Mode	Frequency [Hz]
1	66.882
2	68.438
3	416.07
4	425.64
5	1151.6
6	1177.6

3.1 Cantilever Beam Type TVA Design

Cantilever beam type TVAs are consists of cantilever beams as stiffness elements, a mass element and attachment structure of the TVA. In order to tune cantilever beam type of TVA, cantilever beam's dimensions and mass can be altered in design phase.

Additionally, notches are designed on the cantilever beams to decrease fatigue life of the TVA to shorten fatigue life tests that took place in following sections. Design of the cantilever beam type TVA can be seen at Figure 3.3.

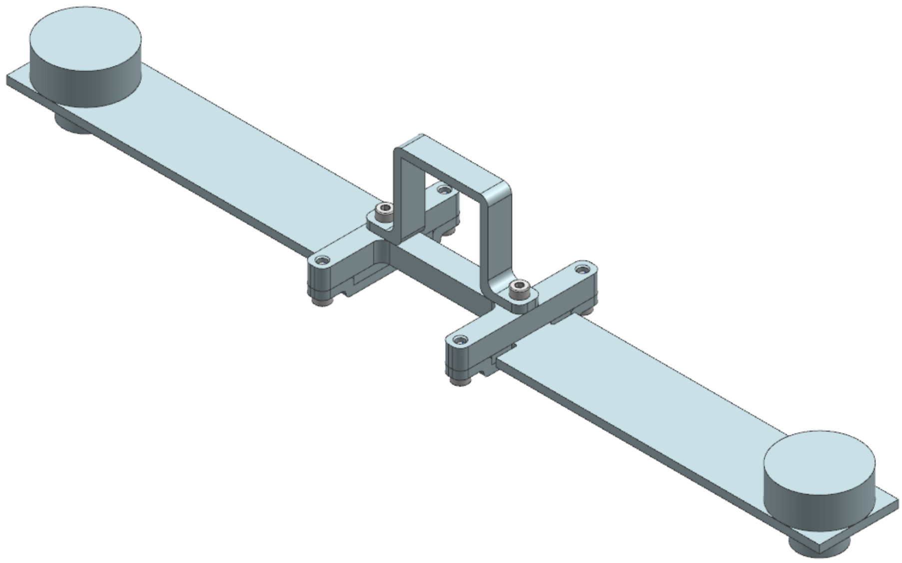

Figure 3.3: Cantilever Beam Type TVA

Material of the parts are selected as aluminum except masses added on tips of the beams which are copper. Additionally, geometry of the part where the TVA is attached to the base structure designed with as light as possible to avoid ineffective mass that is added to the base structure. Dimensions of the cantilever beam type TVA are decided according to modal frequency analysis. Dimensions are changed after every modal frequency analysis to optimize dimensions to get desired tuned frequency. Modal frequency analysis results are given in Table 3.2. Furthermore, effectiveness of the TVA is checked with harmonic analysis if tuning of the TVA is correctly done. Frequency response functions of base structure is compared with base structure with the TVA. Comparison graph is given at Figure 3.4

Table 3.2: Cantilever Beam Type TVA Modal Frequency Table

Mode	Frequency [Hz]
1	66.792
2	67.506
3	236.88
4	257.87
5	617.42
6	620.53

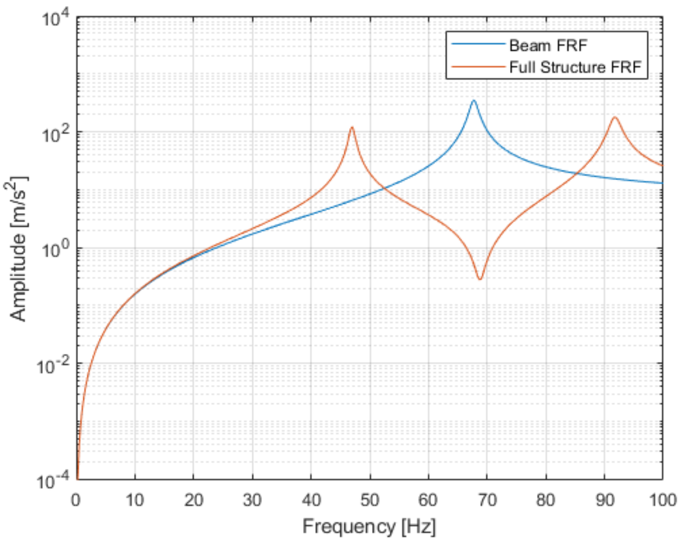

Figure 3.4: Base Structure vs. Full Structure (Base+TVA) FRF Comparison

3.2 Helical Spring Type TVA Design

Helical spring type TVAs are basically spring mass systems that are designed to absorb vibration at tuned frequencies. In order to tune TVA to a particular frequency, its natural frequency is needed to be set at tuned frequency by changing stiffness and mass properties of the TVA. Design of the helical spring type TVA basically consists of two spring, one copper mass and two linear bearings to make mass element slide on two aluminum cylinders. Rest of the design are for to attach the TVA to the base structure. Design of the helical spring type TVA can be seen at Figure 3.5.

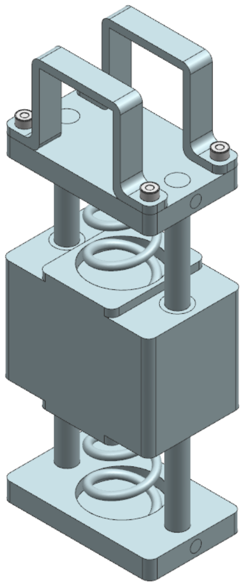

Figure 3.5: Helical Spring Type TVA

Material of the parts that are designed in helical spring type TVA are mainly selected as aluminum except spring, mass and linear bearing elements. Copper and steel are selected to be materials for those parts. Dimensions of the helical spring type of TVA is changed to arrange tuning frequency of the TVA. Additionally, mass of the attachment parts of the TVA are design as light as possible to eliminate ineffective mass effects on the base structure as in cantilever type of the TVA. Modal frequency analysis results are given in Table 3.3. Finally, tuning of the helical spring

Table 3.3: Helical Spring Type TVA Modal Frequency Table

Mode	Frequency [Hz]
1	67.751
2	1249.3
3	1251.4
4	1478.2
5	1479.1
6	1513.9

type of TVA is checked with harmonic analysis if the tuning is correctly done or not. Frequency response functions of the base structure is compared with frequency response function of base structure with helical spring type of TVA. Comparison graph is given at Figure 3.6.

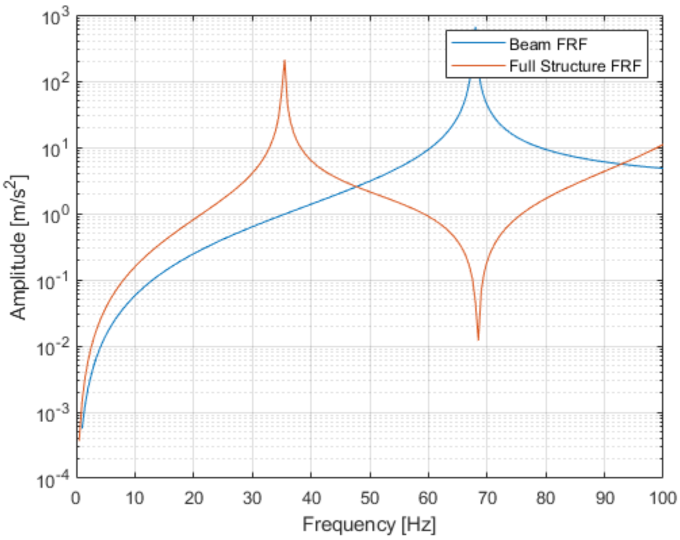

Figure 3.6: Base Structure vs. Full Structure (Base+TVA) FRF Comparison

According to the results of comparison made between frequency response functions of base structure with and without TVAs designs of the TVAs are correctly done. Furthermore, analysis to estimate response and fatigue life of the TVA can be run with designed models of the cantilever beam type and helical spring type of TVAs.

CHAPTER 4

FATIGUE LIFE ESTIMATION OF SELECTED TVAS USING RESPONSE ESTIMATION METHOD

In this chapter response estimation method developed by Özden[1] is implemented to estimate the fatigue life of two TVAs designed for the reference base structure. Results are presented in this chapter are all FE simulations. A random forcing is defined to represent an actual forcing acting on the reference base structure and the response estimation method by Özden[1] is utilized to estimate the response of the TVA designs. In order to execute response estimation method for the TVAs, a list of things is required for base structure and TVAs. These are operational response of the base structure caused by the actual force input in operational conditions when the TVA is not present, direct FRF of the base structure at the connection DOF of the TVA and direct FRF of the TVA at the connection DOF. Response of the TVA's attachment point is calculated with using the gathered information and the response estimation method by Özden [1]. Furthermore, response of the TVA is used in a fatigue life analysis to estimate fatigue life of the TVA. In order to verify the fatigue life estimate obtained using estimated TVA response, full body (base structure + TVA) fatigue life analysis of the TVA is calculated with the same given input that is used in analysis which is made to find operational response of the base structure. Comparison of the fatigue life estimates gives information for how good response estimation method to use in fatigue life analysis. Summary of what is completed in analysis are given at Figure 4.1.

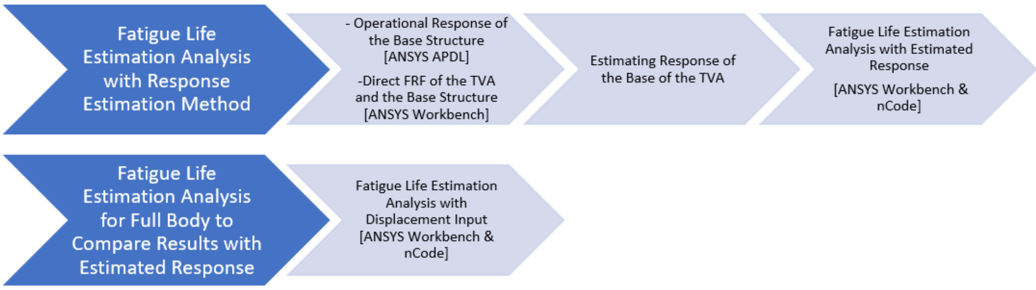

Figure 4.1: Summary of Analysis Process

4.1 Response Estimation Method Theory (Özden[1])

According to response estimation method developed by Özden[1], there are two main assumptions; system is linear and regardless of the modification, excitation source remain same after the TVA attachment. Consider 2DOF linear system given at Figure 4.2 below. Random excitation is applied at first degree of freedom.

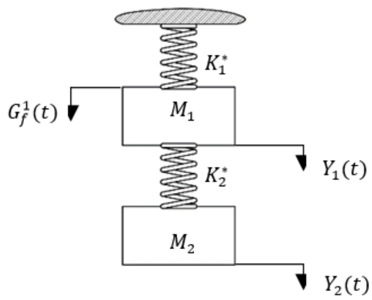

Figure 4.2: 2 DOF Structure.

For vibrational suppression at a certain frequency band, TVA is going to be attached to structure via 2nd DOF, consisting of 1DOF system with mass M_t and K_t (Figure 4.3).

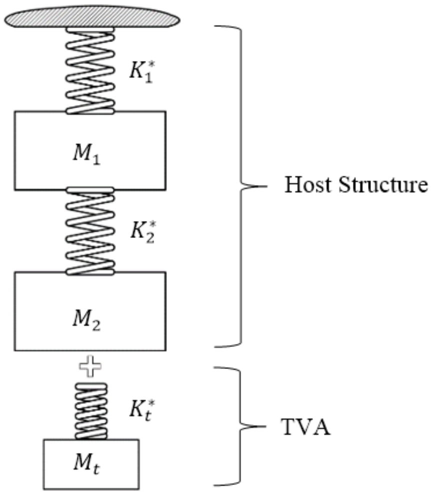

Figure 4.3: 3 DOF Structure.

System matrixes of TVA alone are as follows:

$$[M] = \begin{bmatrix} M_t & 0 \\ 0 & 0 \end{bmatrix} [K_t] = \begin{bmatrix} K_t^* & -K_t^* \\ -K_t^* & K_t^* \end{bmatrix} \tag{4.1}$$

The system matrix of TVA is defined as follows:

$$[H^t] = [-\omega^2[M] + [K_t]]^{-1} \tag{4.2}$$

$$[H^t] = \begin{bmatrix} H_{1,1}^t & H_{1,2}^t \\ H_{2,1}^t & H_{2,2}^t \end{bmatrix} \tag{4.3}$$

$$[H^t] = \begin{bmatrix} \frac{1}{(-\omega^2 M_t)} & \frac{1}{(-\omega^2 M_t)} \\ \frac{1}{(-\omega^2 M_t)} & \frac{-\omega^2 M_t + K_t)}{(-\omega^2 M_t)} \end{bmatrix} \tag{4.4}$$

Where H_{22}^o (2^{nd} DOF direct FRF of host structure) is a known property as result of measurement output of host structure. Now modified structure's direct FRF at connecting DOF can be calculated by impedance coupling method using H_{22}^o and H_{22}^t. FRF of modified 2nd DOF can be calculated as follows:

$$[\widetilde{H_{22}^m}] = \frac{H_{22}^o(\omega) H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)} \tag{4.5}$$

Where $[\widetilde{H}_{22}^m]$ stands for modified structure 2nd DOF direct FRF.

Generalized hypothetical forcing is given as below[1];

$$G_f^h(\omega) = \frac{G_{nn}^Y(\omega)}{|H_{nn}^o(\omega))|^2} \tag{4.6}$$

Point response can be calculated using FRF and forcing function acting that point.

$$\widetilde{G_{22}^Y}^m(\omega) = |\widetilde{H}_{22}^m(\omega)|^2 G_f^H(\omega) \tag{4.7}$$

Where $\widetilde{G_{22}^Y}^m(\omega)$ stands for nodified point (2nd DOF) response estimation.

Open form of the above equation is given at below;

$$\widetilde{G_{22}^Y}^m(\omega) = \left|\frac{H_{22}^o(\omega)H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)}\right|^2 \frac{G_{22}^Y(\omega)}{|H_{22}^o(\omega)|^2} \tag{4.8}$$

$$\widetilde{G_{22}^Y}^m(\omega) = \left|\frac{H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)}\right|^2 G_{22}^Y(\omega) \tag{4.9}$$

Full solution of the original and modified structure has been given. Second DOF response in PSD form is given at below;

$$G_{22}^Y = |H_{21}^o|^2 G_f^1 \tag{4.10}$$

Inserting the eqn.4.10 into eqn.4.9, estimation as a function of input forcing as given below,

$$\widetilde{G_{22}^Y}^m = |H_{21}^o|^2 G_f^1 |\frac{H_{22}^t}{H_{22}^o + H_{22}^t}|^2 \tag{4.11}$$

Modified structure (3 DOF) system matrixes are defined. Second DOF exact reaction is;

$$G_{22}^{Y\,m} = |H_{21}^m|^2 G_f^1 \tag{4.12}$$

In order to check analytical relationship between exact results of the second DOF reaction with the estimation, following steps are followed;

$$G_{22}^{Y\,m} = \widetilde{G_{22}^{Y\,m}} \tag{4.13}$$

$$|H_{21}^m|^2 G_f^1 = |H_{21}^o|^2 G_f^1 \left|\frac{(H_{22}^t)}{(H_{22}^o + H_{22}^t)}\right|^2 \tag{4.14}$$

With omitting input forcing from both sides, above equation reduced to;

$$|H_{21}^m| = \left|\frac{H_{21}^o H_{22}^t}{H_{22}^o + H_{22}^t}\right| \tag{4.15}$$

In order to check above identity one should calculate modified structure cross point FRF(H_{21}^m) in terms of original structure FRFs (H_{21}^o, H_{22}^o) and TVA FRF (H_{22}^t).

4.2 Response Estimation Method Applied to the Base Structure with Cantilever Beam Type TVA

Harmonic and random vibration analysis are needed to be carried out to estimate response of the cantilever beam type TVA attached to the reference base structure according to response estimation method. Harmonic analysis are required to calculate direct FRF of TVA at point where it is connected to the base structure and to calculate direct FRF of base structure where the TVA will be connected. By utilizing these two FRF with eqn. 4.15, a coupled FRF is calculated. Moreover, random vibration analysis with given forcing input is run to calculate operational response of the base structure. According to response estimation method by Özden [1], in order to find base excitation of the TVA, coupled FRF and operational response of the base structure are used in eqn. 4.8. Analysis which are run on base structure and the TVA are constructed in a way that ineffective mass on the TVA is added to the base structure. Forcing input that is decided to be used in the analysis is given at Figure 4.4 and Table 4.1. Location of the point where the PSD forcing input will be applied to the structure is given at Figure 4.5.

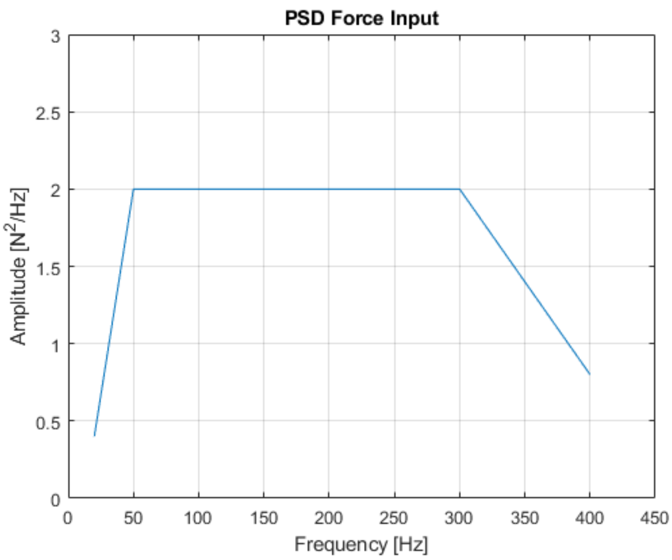

Figure 4.4: PSD Force Input for Structure

Table 4.1: PSD Force Input Values for Structure

Frequency $[Hz]$	Amplitude $[N^2/Hz]$
20	0.4
50	2
300	2
400	0.8

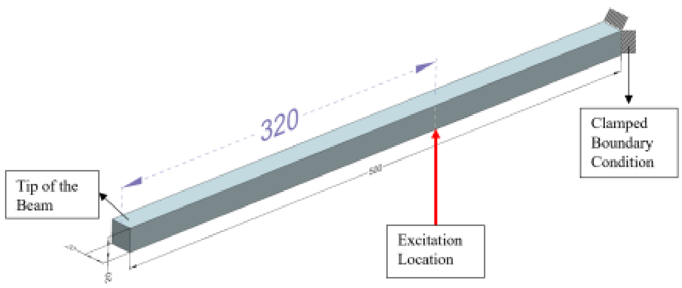

Figure 4.5: Application Point for PSD Forcing on the Structure

ANSYS Mechanical APDL analysis program is used to calculate operational response of the structure with given input. Meshing and material assignments to the parts are completed in ANSYS Workbench. Input file is created from ANSYS Workbench with model, material and meshing information. Forcing PSD is defined and applied to the given point in the ANSYS Mechanical APDL analysis program at Figure 4.6.

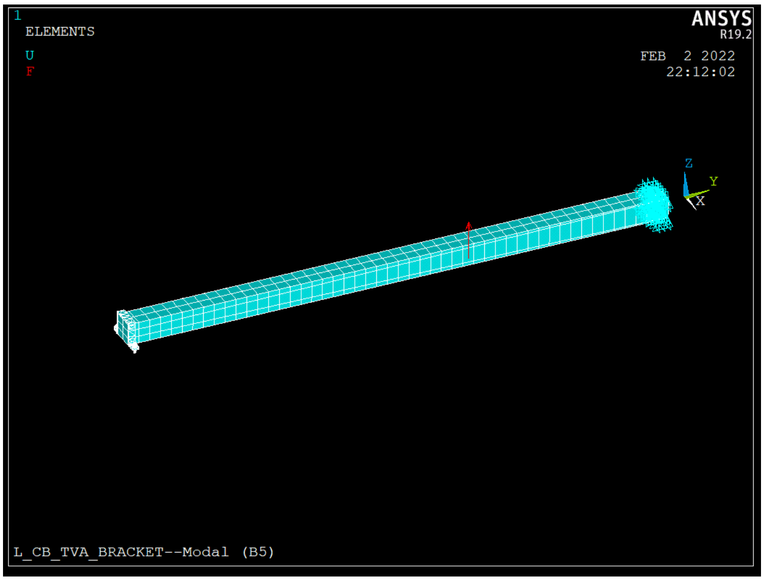

Figure 4.6: Force Application on the Base Structure for Random Vibration Analysis

Spectrum analysis is completed with defined inputs. Operational response of the tip of base structure is extracted from solution. Operational response is generated as acceleration PSD. Operational response data is used as $G_{22}^{Y}(\omega)$ in equation 4.8.

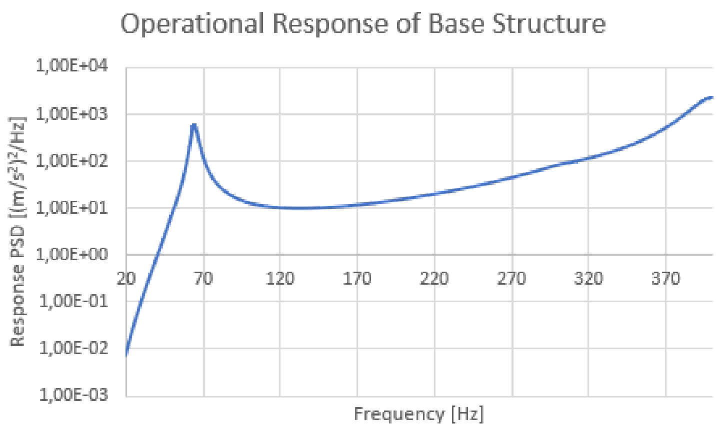

Figure 4.7: Operational Response of the Base Structure at the Tip (TVA Connection DOF)

Second step for calculating of response estimation is to get direct FRF of the TVA from where it is connected to the base structure. ANSYS Workbench Harmonic Response analysis is used to calculate direct FRF of the TVA.

Unit force is applied to the surface where the TVA is attached to the base structure. Frequency response function is calculated from same surface to calculate direct FRF of the TVA. Direct FRF of the TVA is calculated at Figure 4.9. Direct FRF of the TVA data is used as $H^t_{22}(\omega)$ in equation 4.8.

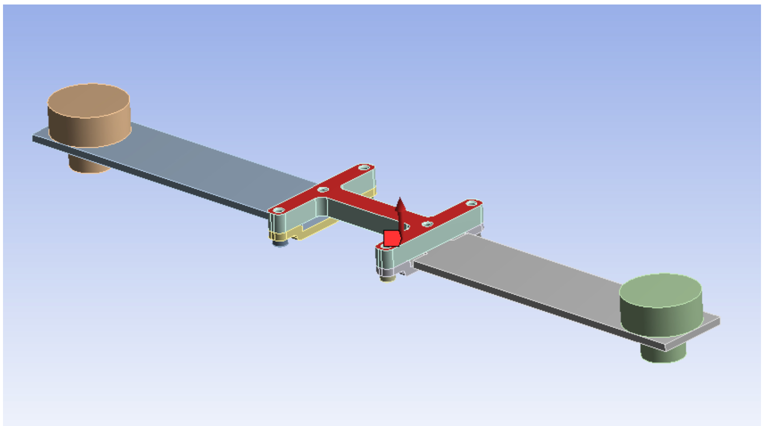

Figure 4.8: Applied Unit Force on the TVA

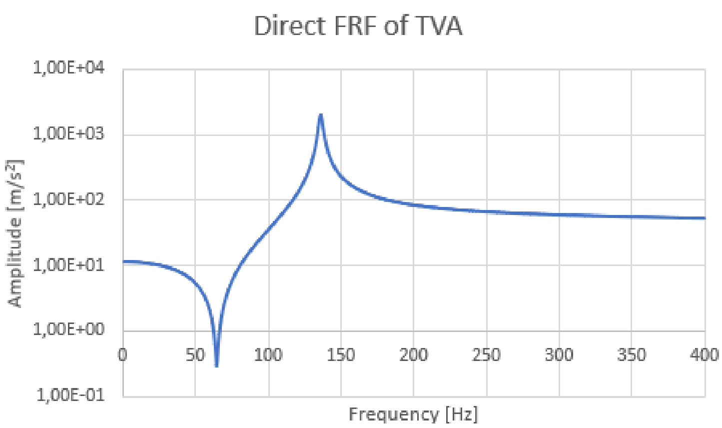

Figure 4.9: Direct FRF of the TVA at the Connection DOF (Tip of the Base Structure)

Third step for calculating response estimation method is to calculate direct FRF of the base structure from where the TVA is connected. Similar procedure of calculating direct FRF of the TVA with ANSYS Workbench is carried. Direct FRF of the base structure is calculated at Figure 4.10. Direct FRF of the base structure data is used as

$H^{o}_{22}(\omega)$ in equation 4.8.

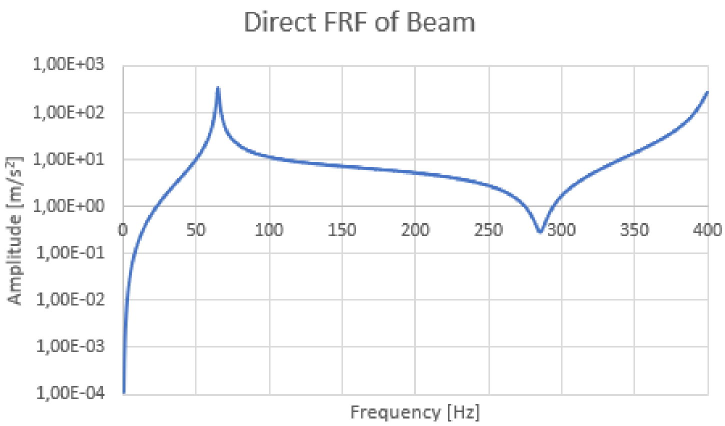

Figure 4.10: Direct FRF of the Base Structure

Since all the information that is required to calculate estimated response at the base of the TVA is calculated, calculation procedure of the method is carried out. Results is compared with full body analysis which are the analysis that run on base structure with the TVA. Comparison of FRFs from estimation and full body analysis is given at Figure 4.11. Furthermore, estimated response of the point where TVA is connected to base structure is compared to the full body random vibration analysis at Figure 4.12.

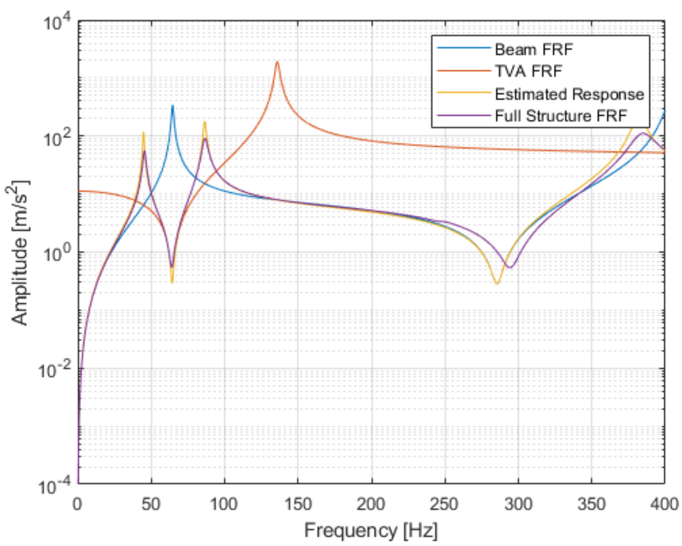

Figure 4.11: Comparison of FRFs for TVA, Base Structure, Estimated Response and Full Body

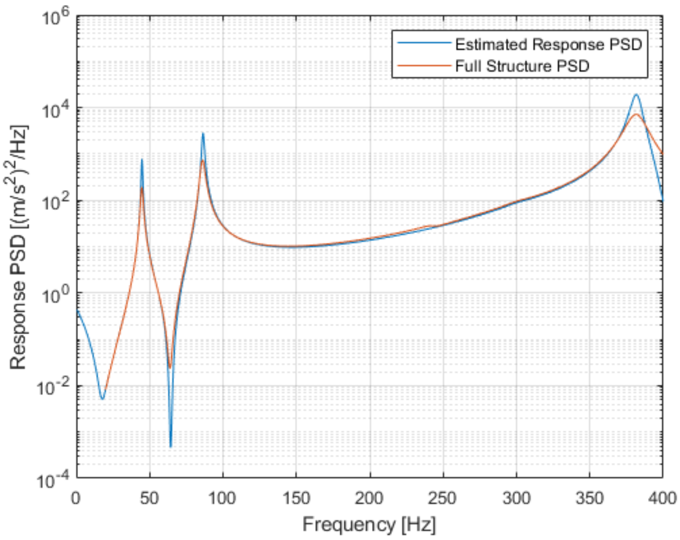

Figure 4.12: Comparison of Estimated Response and Full Body Analysis

4.3 Fatigue Life Estimation of Cantilever Beam Type TVA

Fatigue life analysis with estimated response of the TVA is the next step of the procedure. Analysis program which is named as nCode is used to estimate fatigue life of the TVA. Firstly, full body fatigue life analysis is carried out. Finite element model with fixed support and nodal forcing application point given at Figure 4.13 for full body analysis. Moreover, support and base excitation of only TVA analysis of FE model is given at Figure 4.14. Secondly, fatigue life estimation of the TVA with the base excitation that is calculated with response estimation method is carried out. FE model, FRF of the structure with unit input and estimated base acceleration excitation is entered to the analysis. Moreover, as PSD cycle methods, four options are available in nCode. Those are Narrow Band, Lallane, Dirlik and Steinberg. Fatigue life analysis are run for four different PSD cycle counting methods. Firstly, Dirlik PSD cycle count method is used. Result of the analysis are given at Figure 4.16 and Figure 4.17. Secondly, Lalanne PSD cycle counting method is used. Results of this analysis are given at Figure 4.18 and Figure 4.19. Thirdly, Narrow Band PSD cycle counting method is used. Analysis results are given at Figure 4.20 and Figure 4.21. Finally, Steinberg method is used. Result for this method are given at Figure 4.22 and Figure 4.23. Summary of the four different analysis with four different PSD cycle count method are given at Table 4.2.

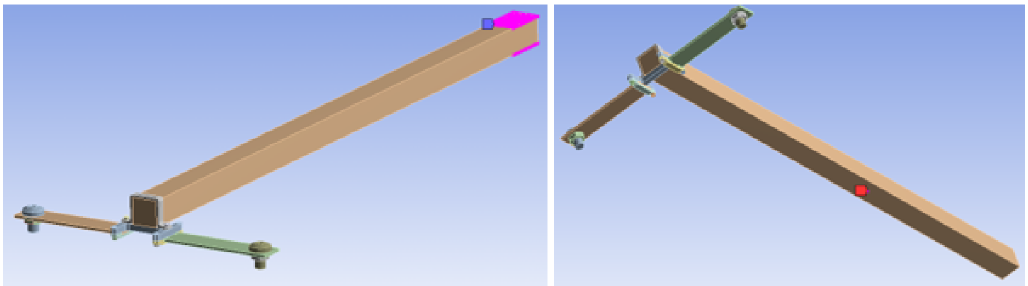

Figure 4.13: Fixed Support and Forcing Application Node of the FE Model for Full Body Analysis

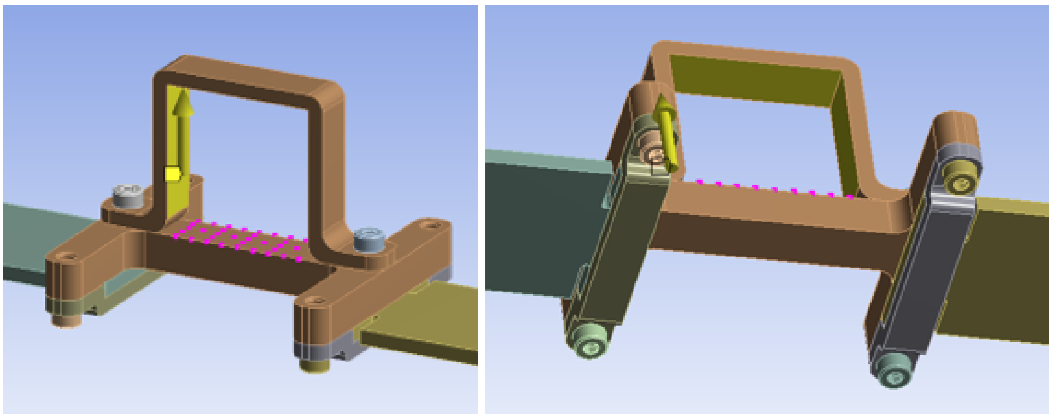

Figure 4.14: Fixed Support and Base Excitation Application Nodes of the FE Model for Only TVA Analysis

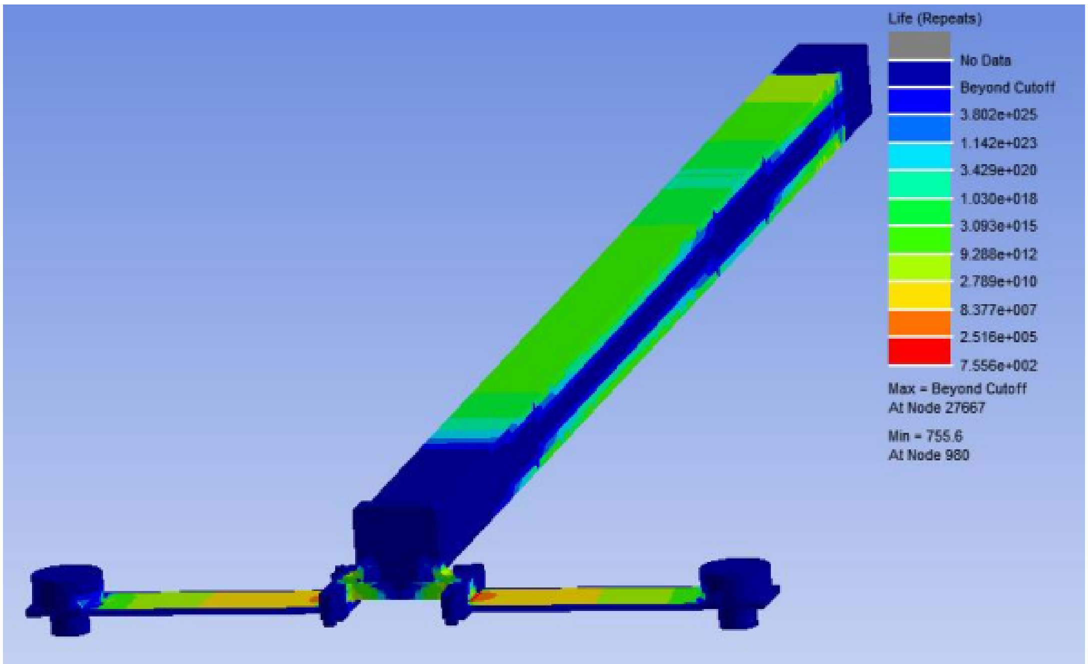

Figure 4.15: Fatigue Life Estimation Analysis Result for Full Body with Dirlik PSD Cycle Counting Method

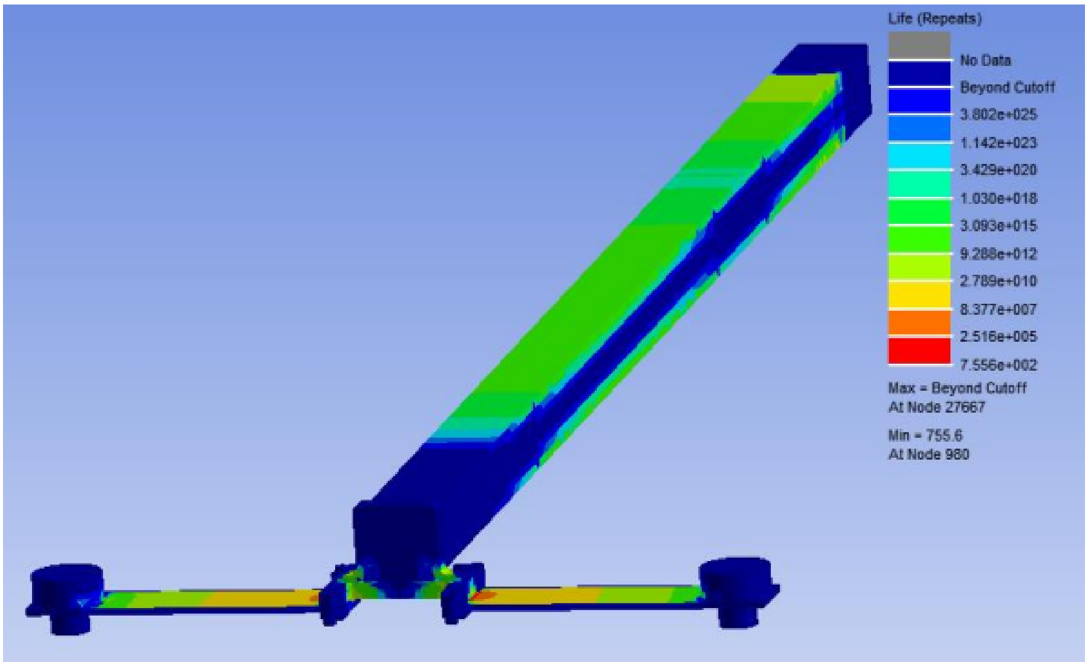

Figure 4.16: Fatigue Life Estimation Analysis Result for Full Body with Dirlik PSD Cycle Counting Method

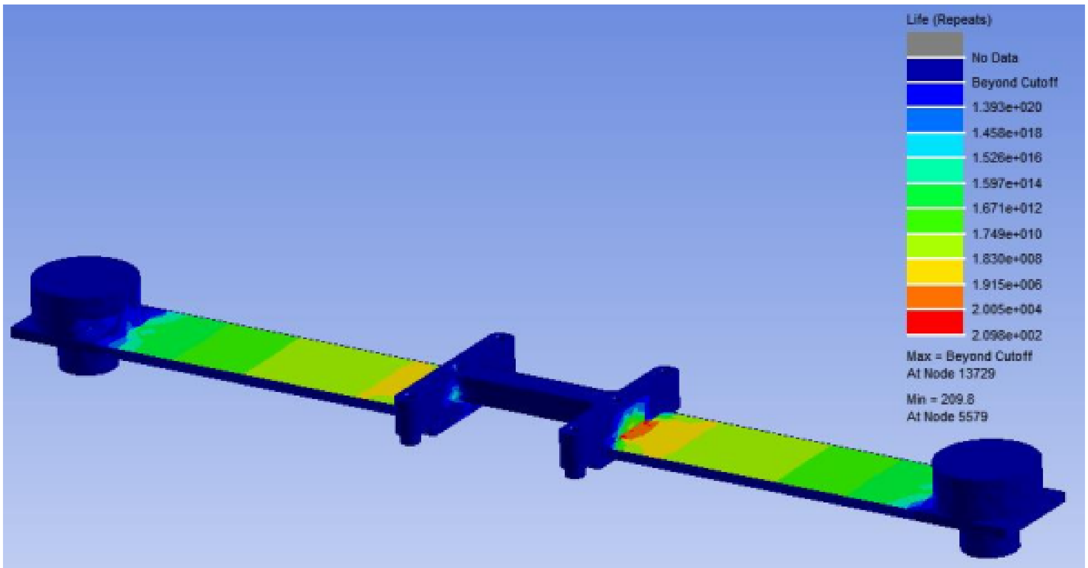

Figure 4.17: Fatigue Life Estimation Analysis Result for the TVA with Dirlik PSD Cycle Counting Method

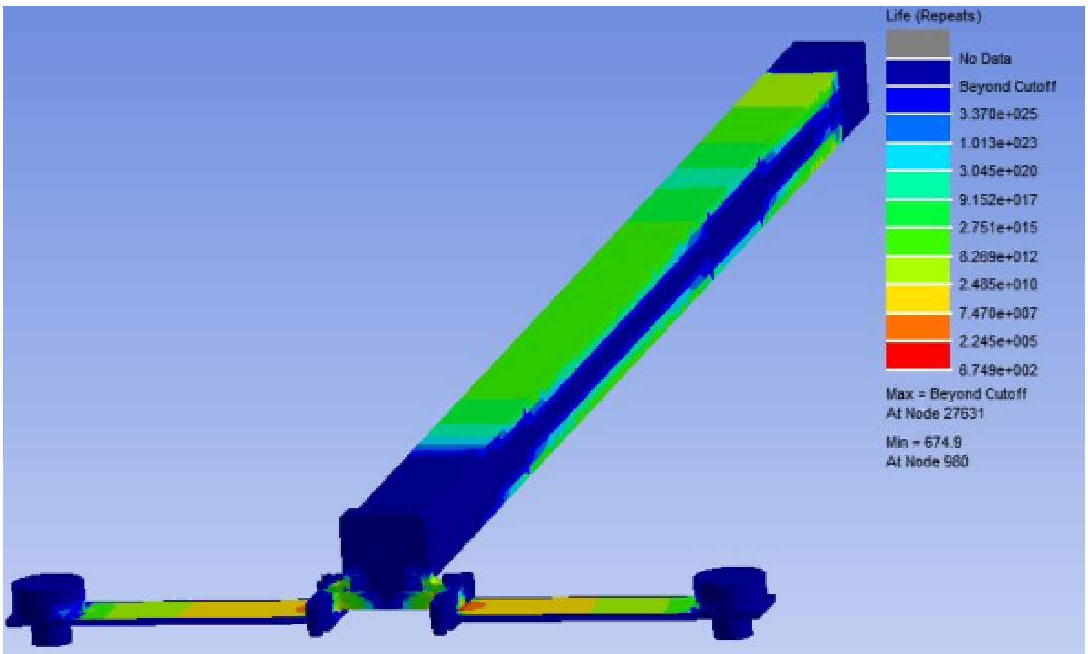

Figure 4.18: Fatigue Life Estimation Analysis Result for Full Body with Lalanne PSD Cycle Counting Method

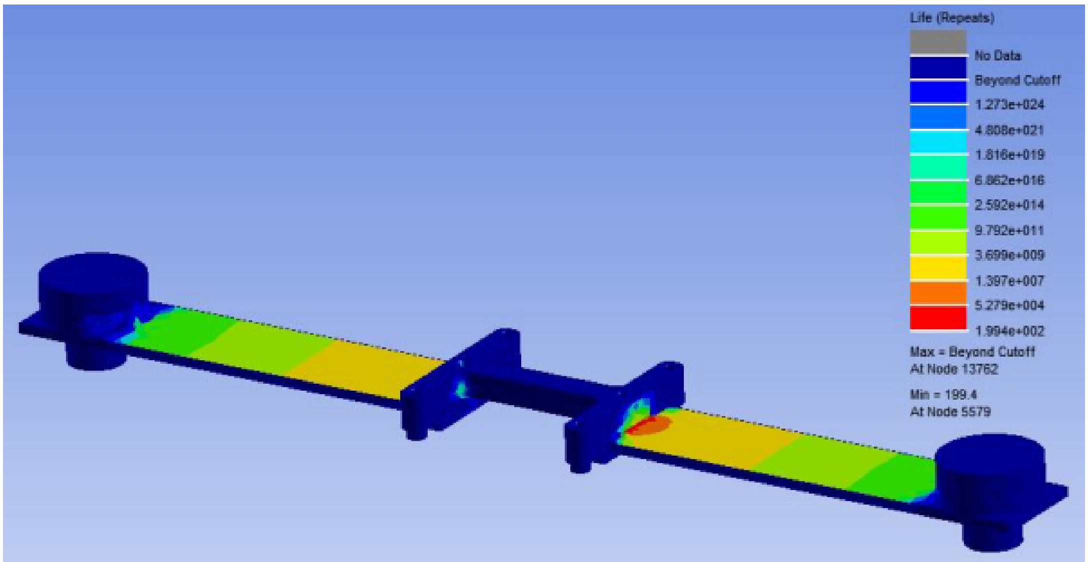

Figure 4.19: Fatigue Life Estimation Analysis Result for the TVA with Lalanne PSD Cycle Counting Method

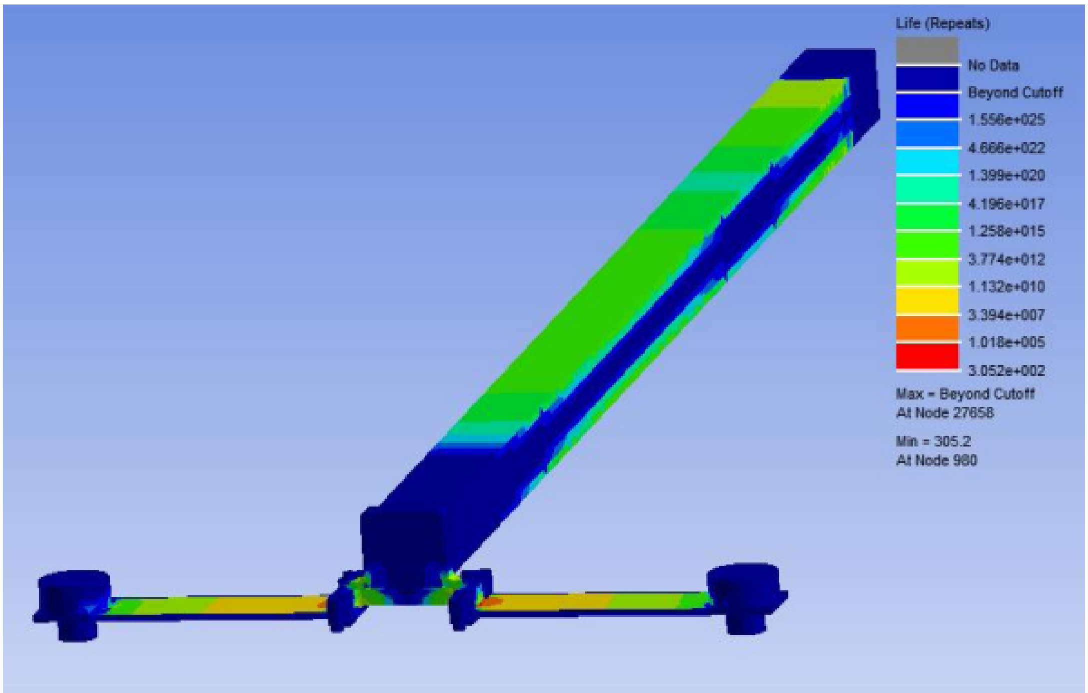

Figure 4.20: Fatigue Life Estimation Analysis Result for Full Body with Narrow Band PSD Cycle Counting Method

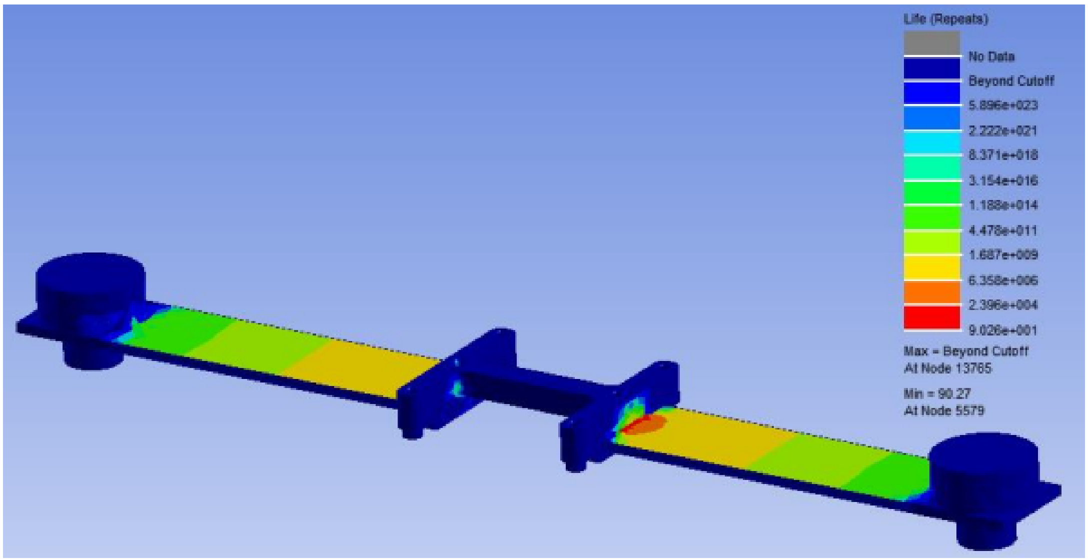

Figure 4.21: Fatigue Life Estimation Analysis Result for the TVA with Narrow Band PSD Cycle Counting Method

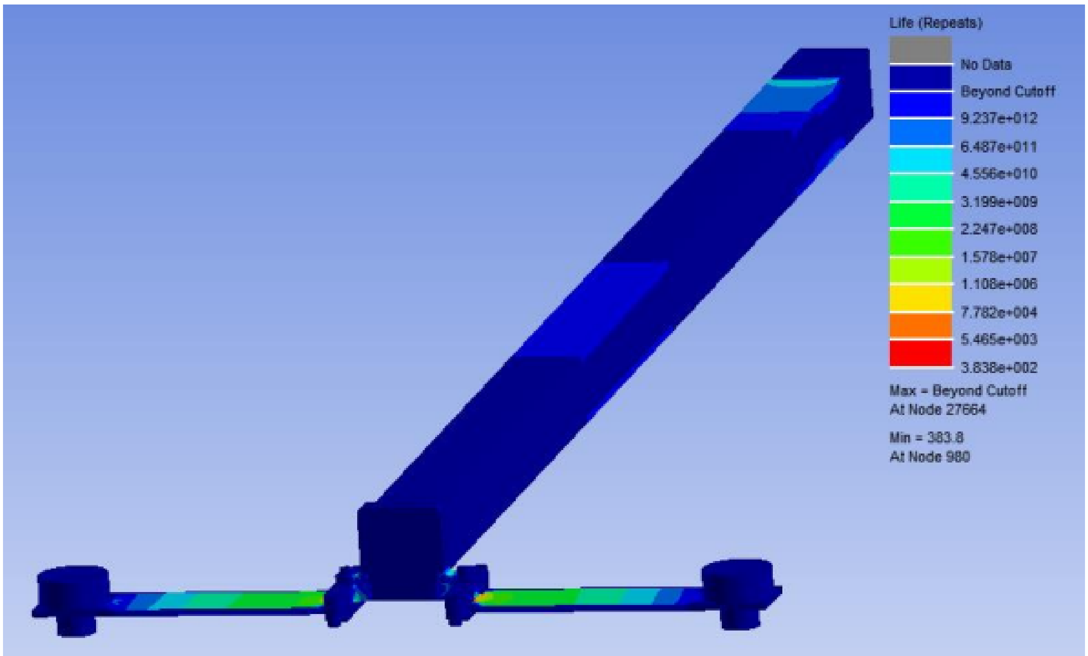

Figure 4.22: Fatigue Life Estimation Analysis Result for Full Body with Steinberg PSD Cycle Counting Method

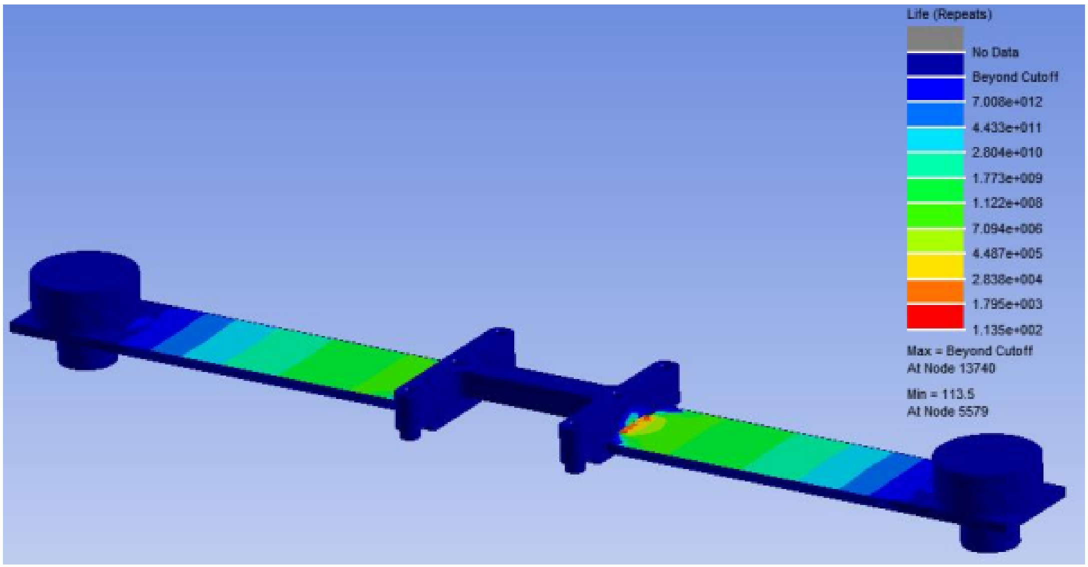

Figure 4.23: Fatigue Life Estimation Analysis Result for the TVA with Steinberg PSD Cycle Counting Method

Table 4.2: Summary Table for PSD Cycle Counting Methods for Cantilever Beam Type TVA

	PSD Cycle Counting Method			
	Dirlik	Lalanne	Narrow Band	Steinberg
Full Body Analysis Results (seconds)	755.6	674.9	305.2	383.8
Only TVA Analysis Results (seconds)	209.8	199.4	90.27	113.5
Ratio Between Analysis Results	3.6015	3.3847	3.381	3.3815

According to study completed above Narrow Band method is selected PSD cycle counting method with selected input. In order to decrease ratio between analysis results for only TVA and full body analysis, resolution of harmonic response and random vibration analysis increased greatly. Result of the fatigue life estimation analysis with increased resolution is given at Figure 4.24.

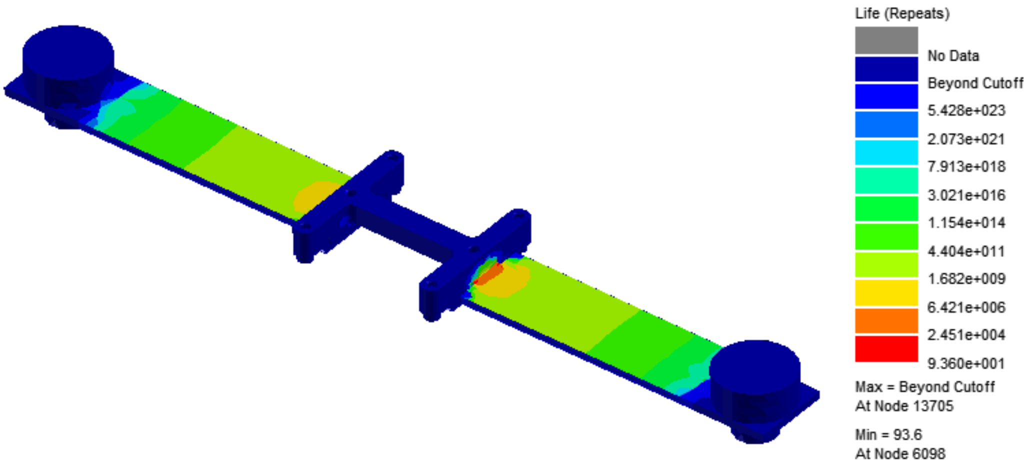

Figure 4.24: Fatigue Life Estimation Analysis Result for the TVA

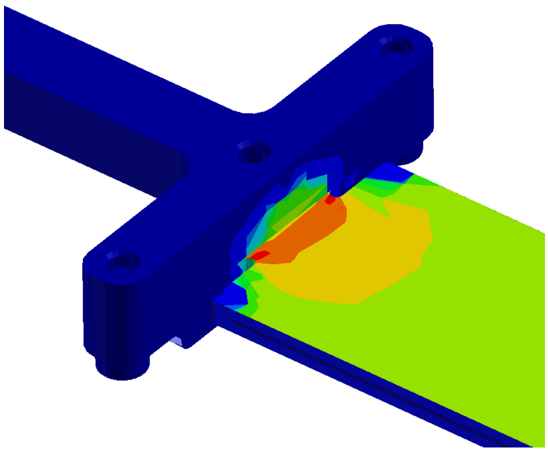

Figure 4.25: Detailed View of Fatigue Life Estimation Result of Notched Area

Estimated life of the TVA is 93,6 repeats. Since, exposure duration is selected as 1 second fatigue life estimation of the TVA with estimated response is 93,6 seconds. As a final step of the analysis for cantilever beam type TVA, fatigue life analysis for the full body which is base structure with the TVA is carried out. Similar to the analysis of the TVA; FE model, FRF of the structure with unit input and given input is entered into the analysis. Narrow Band method is also selected for this analysis. Result of the fatigue life analysis of the full body is given at Figure 4.26.

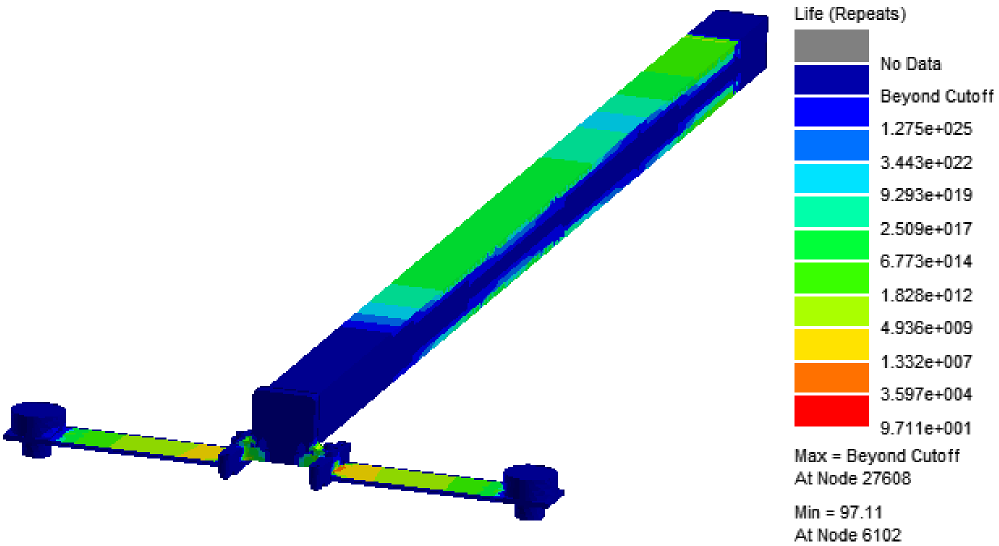

Figure 4.26: Fatigue Life Estimation Analysis Result for Full Body

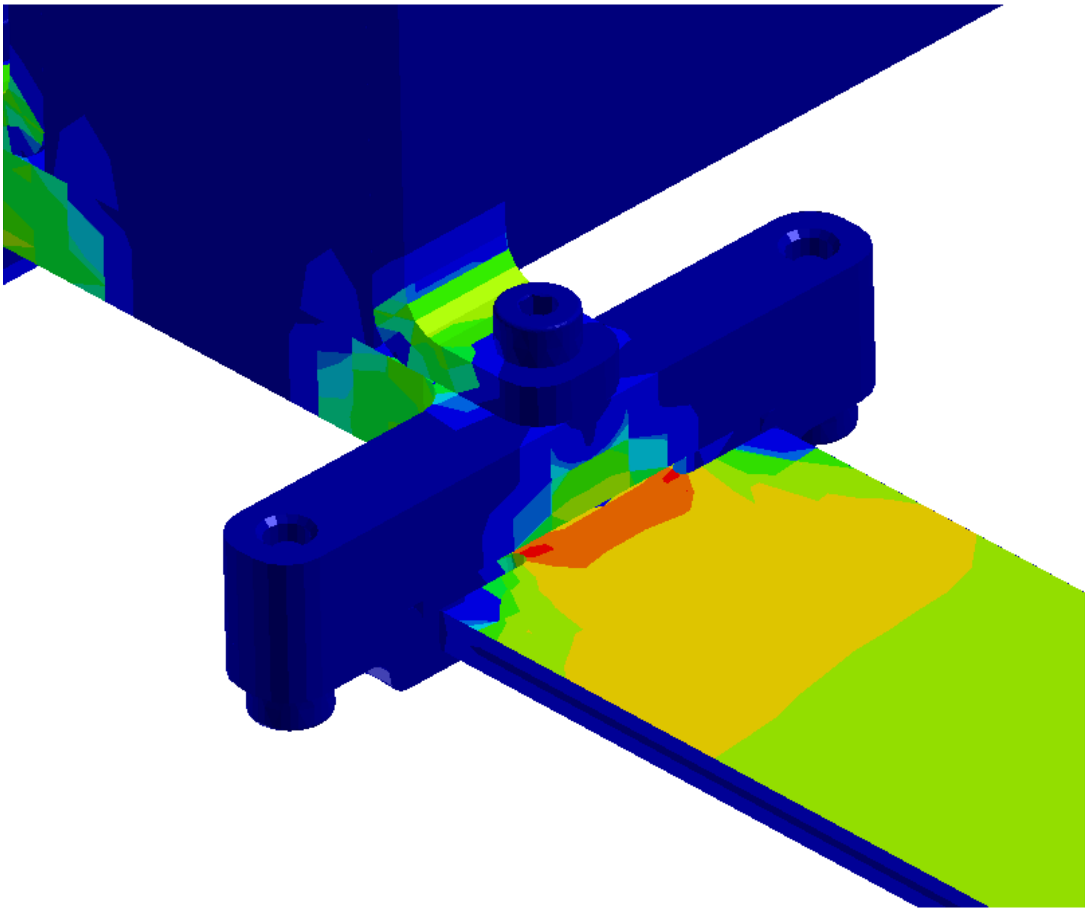

Figure 4.27: Detailed View of Fatigue Life Estimation Result of Notched Area

Fatigue life estimation for the full body is 97,11 repeats. Exposure duration is also selected as 1 second for this analysis. Fatigue life estimation for full body is estimated as 97,11 seconds.

4.4 Response Estimation Method Applied to the Structure with Mass and Spring Type TVA

Similar procedure with cantilever type TVA is applied to the case with base structure with a mass and spring type TVA. Firstly, operational response of the base structure is gathered. Furthermore, direct FRFs of the base structure and the TVA are gathered. Moreover, response estimation method is executed with gathered data from analysis. Finally, fatigue life analysis for full structure and the TVA is carried.

PSD force input for the mass and spring type TVA is given at Figure 4.28 and Table 4.3.

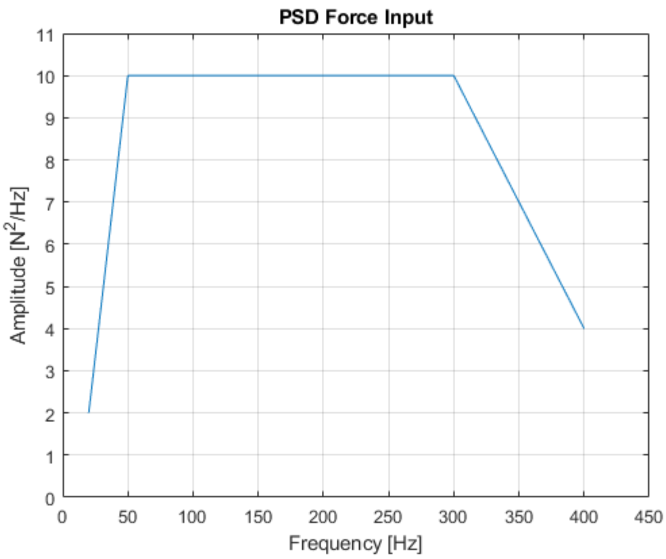

Figure 4.28: PSD Force Input for Structure

Location of PSD input is same as it is given in cantilever beam type TVA. Operational response of the base structure is calculated with an analysis that is run using ANSYS Mechanical APDL. Result of the analysis is given at Figure 4.29. Direct FRF of the TVA is gathered from harmonic response analysis of the mass and spring type TVA.

Table 4.3: PSD Force Input Values for Structure

Frequency $[Hz]$	Amplitude $[N^2/Hz]$
20	2
50	10
300	10
400	4

Direct FRF is given at Figure 4.31. Moreover, direct FRF of the base structure is gathered as given at Figure 4.32 from harmonic response analysis that is run with base structure model.

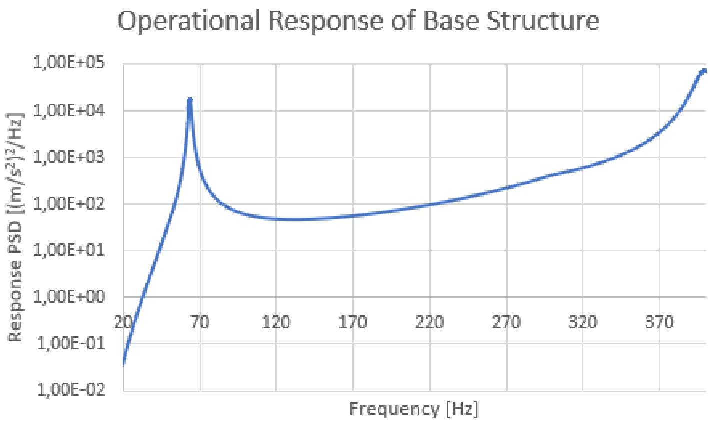

Figure 4.29: Operational Response of the Base Structure

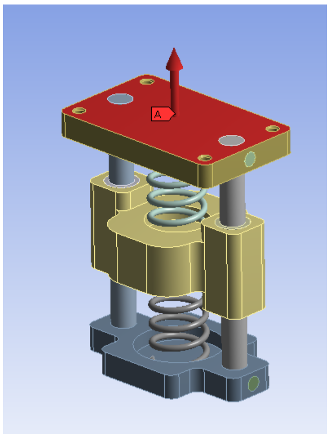

Figure 4.30: Applied Unit Force on the TVA

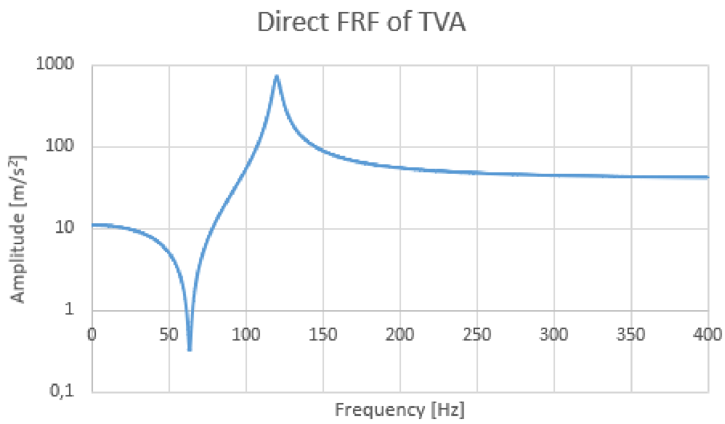

Figure 4.31: Direct FRF of the TVA

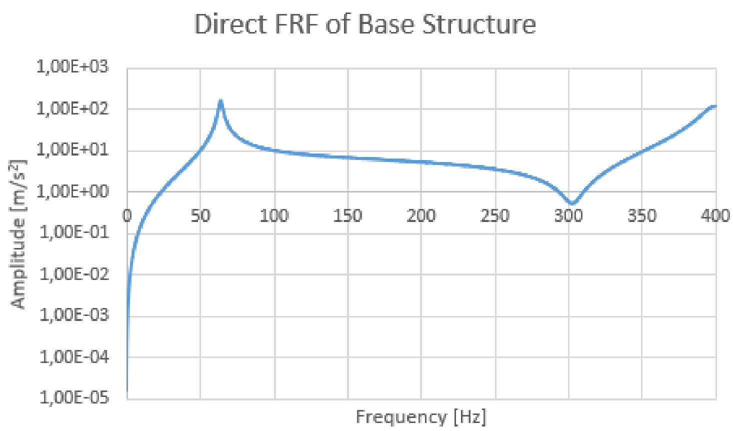

Figure 4.32: Direct FRF of the Base Structure

According to the response estimation method, all necessary information is gathered to execute the method at this point. Furthermore, response estimation method is executed and result of the estimation is compared full structure analysis results at Figure 4.33.

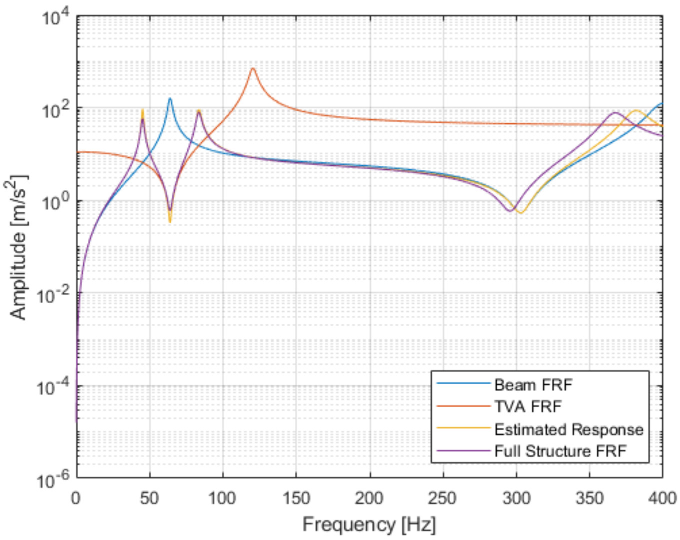

Figure 4.33: Comparison of FRFs for TVA, Base Structure, Estimated Response and Full Body

Moreover, base excitation that will be at the point where the TVA is attached to base structure is compared for response estimation method and full structure analysis results. Comparison is given at Figure 4.34.

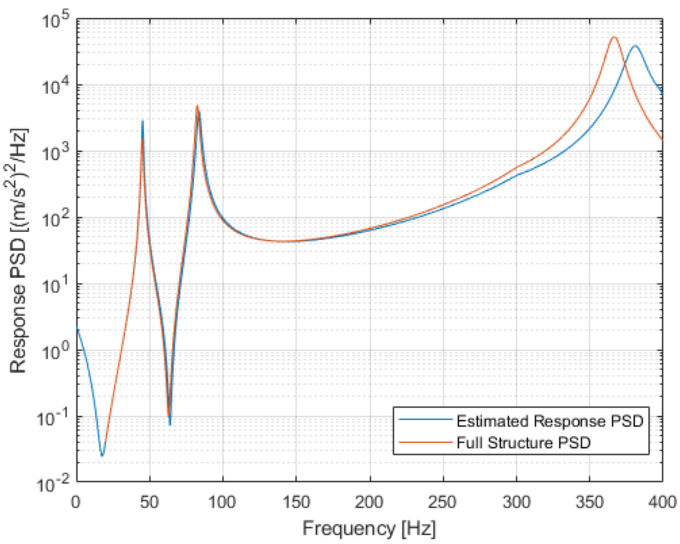

Figure 4.34: Comparison of Estimated Response and Full Body Analysis

4.5 Fatigue Life Estimation of Mass and Spring Type TVA

Similar to the study completed for cantilever beam type TVA, PSD cycle count method analysis are completed for mass and spring type TVA with four different methods. Firstly, Dirlik PSD cycle count method is used. Result of the analysis are given at Figure 4.35 and Figure 4.36. Secondly, Lalanne PSD cycle counting method is used. Results of this analysis are given at Figure 4.37 and Figure 4.38. Thirdly, Narrow Band PSD cycle counting method is used. Analysis results are given at Figure 4.39 and Figure 4.40. Finally, Steinberg method is used. Result for this method are given at Figure 4.41 and Figure 4.42. Summary of the four different analysis with four different PSD cycle count method are given at Table 4.4.

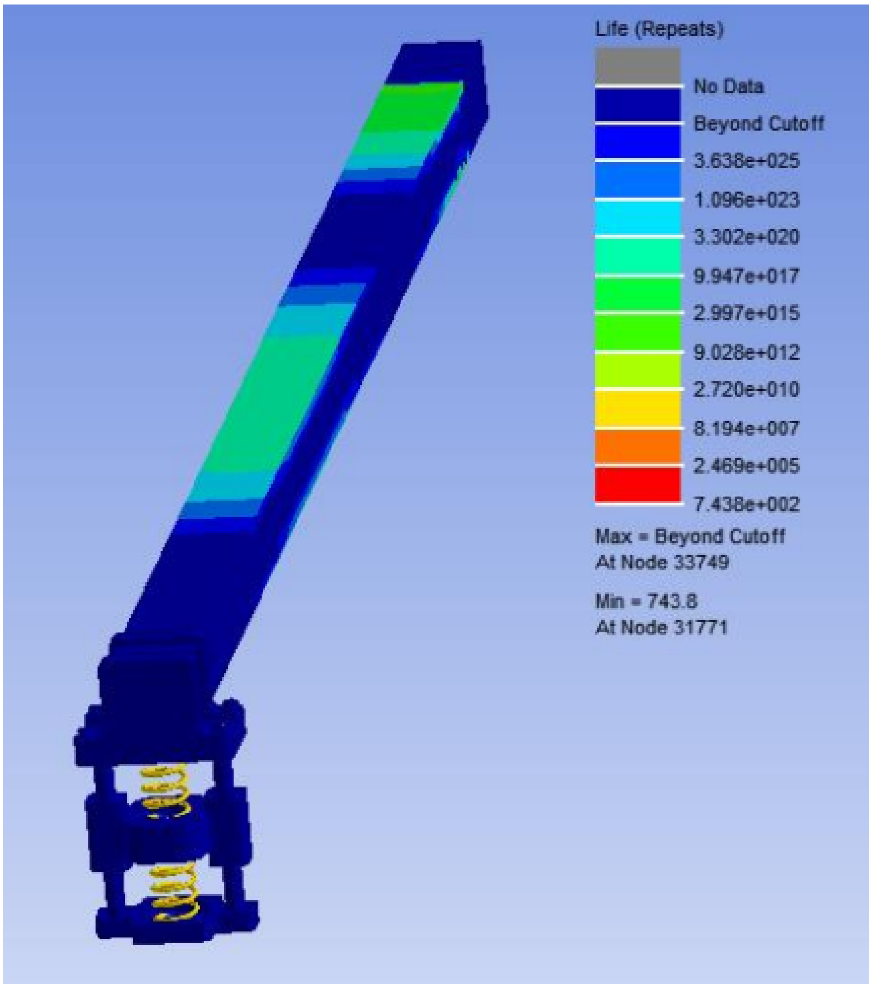

Figure 4.35: Fatigue Life Estimation Analysis Result for Full Body with Dirlik PSD Cycle Counting Method

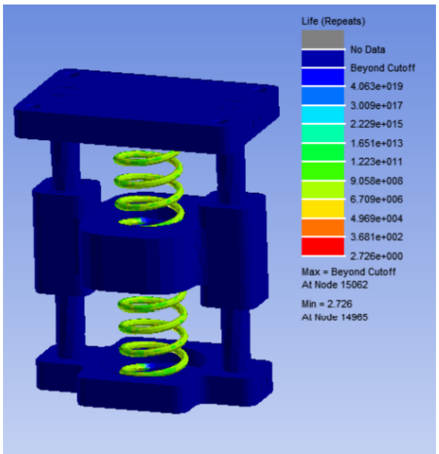

Figure 4.36: Fatigue Life Estimation Analysis Result for the TVA with Dirlik PSD Cycle Counting Method

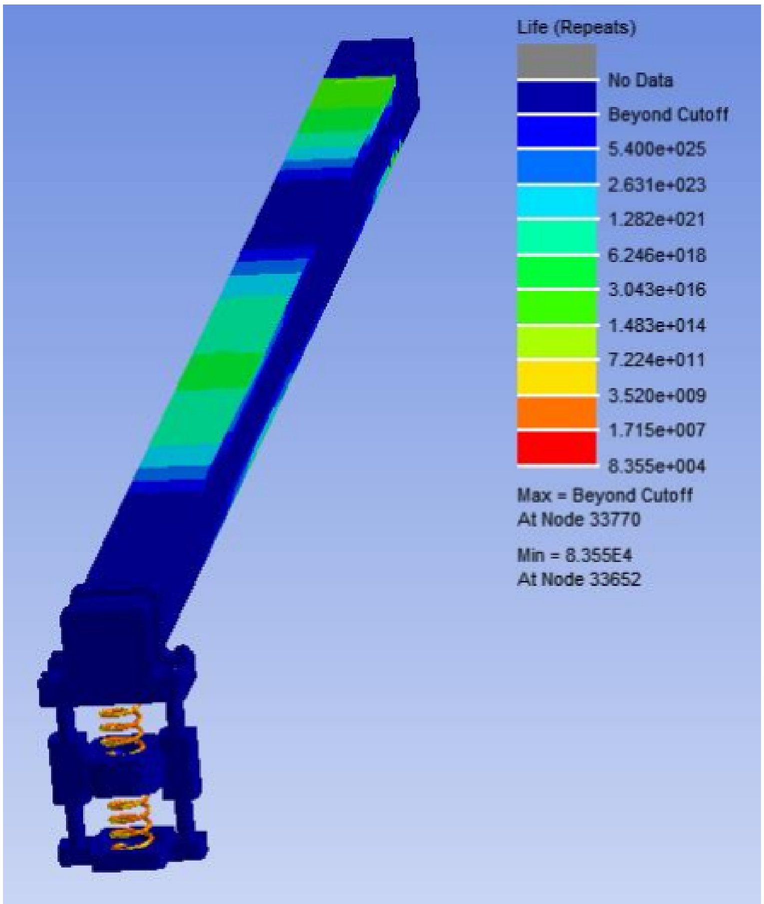

Figure 4.37: Fatigue Life Estimation Analysis Result for Full Body with Lalanne PSD Cycle Counting Method

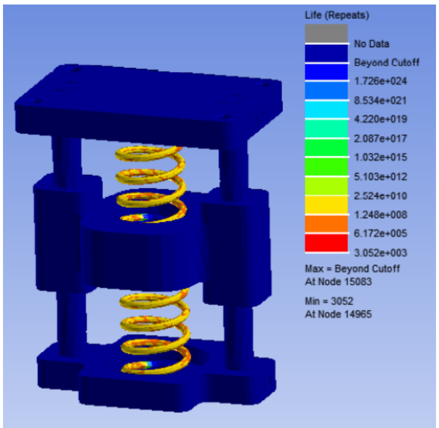

Figure 4.38: Fatigue Life Estimation Analysis Result for the TVA with Lalanne PSD Cycle Counting Method

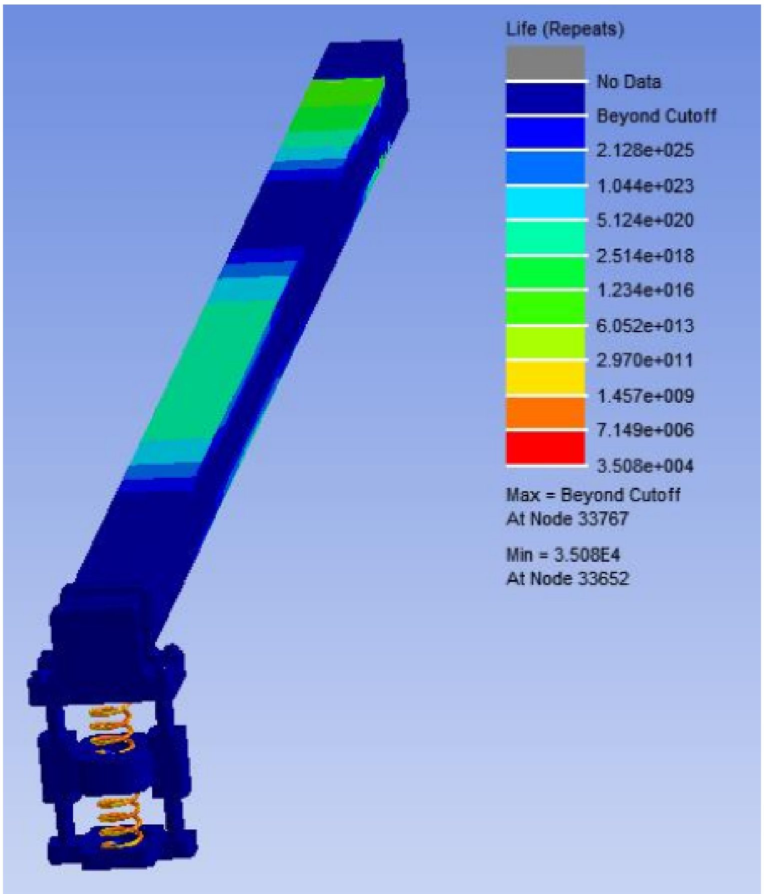

Figure 4.39: Fatigue Life Estimation Analysis Result for Full Body with Narrow Band PSD Cycle Counting Method

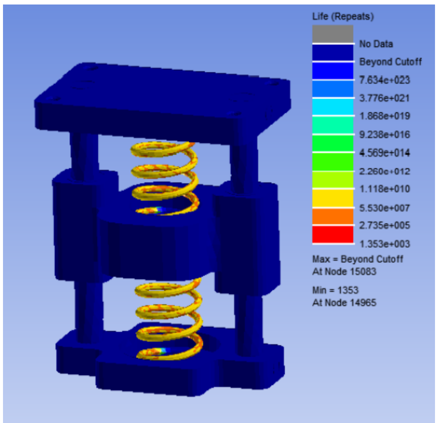

Figure 4.40: Fatigue Life Estimation Analysis Result for the TVA with Narrow Band PSD Cycle Counting Method

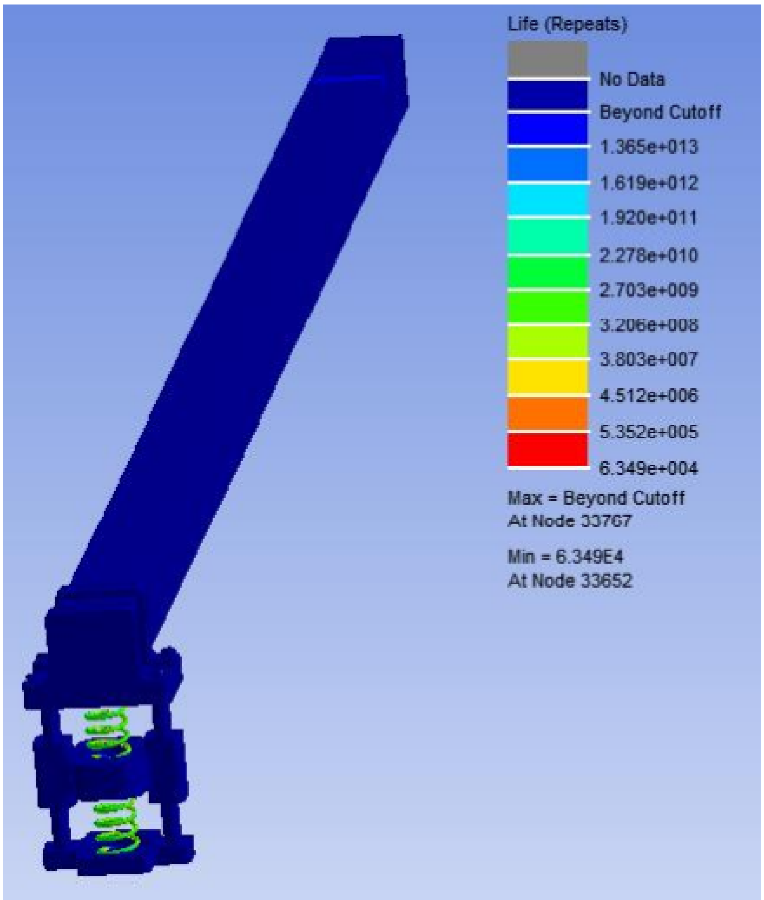

Figure 4.41: Fatigue Life Estimation Analysis Result for Full Body with Steinberg PSD Cycle Counting Method

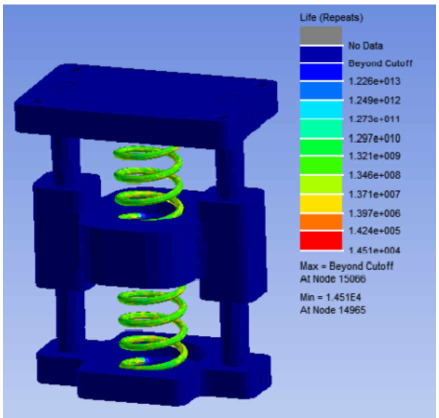

Figure 4.42: Fatigue Life Estimation Analysis Result for the TVA with Steinberg PSD Cycle Counting Method

Table 4.4: Summary Table for PSD Cycle Counting Methods for Mass and Spring Type TVA

	PSD Cycle Counting Method			
	Dirlik	Lalanne	Narrow Band	Steinberg
Full Body Analysis Results (seconds)	743.8	8.355E4	3.508E4	6.349E4
Only TVA Analysis Results (seconds)	2.726	3052	1353	1.451E4
Ratio Between Analysis Results	272.854	27.375	25.928	4.376

According to study completed above Steinberg method is selected PSD cycle counting method with selected input. In order to decrease ratio between analysis results for only TVA and full body analysis, resolution of harmonic response and random vibration analysis increased. Result of the fatigue life estimation analysis with increased resolution is given at Figure 4.43.

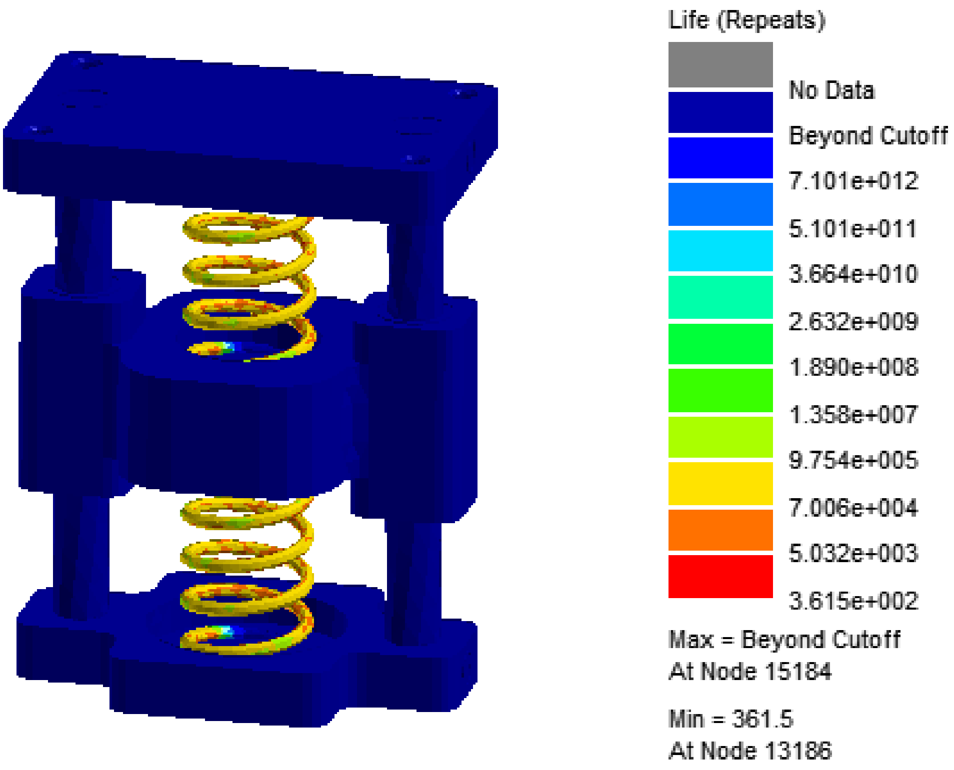

Figure 4.43: Fatigue Life Estimation Analysis Result for the TVA

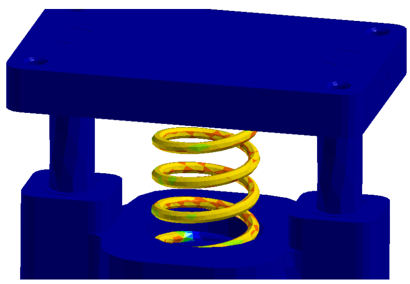

Figure 4.44: Detailed View of Fatigue Life Estimation Result of Most Critical Area

Estimated life of the TVA is 361,5 repeats. Since, exposure duration is selected as 1 second as in cantilever beam type TVA fatigue life estimation of the TVA with estimated response is 361,5 seconds.

Final step for the mass and spring type TVA analysis is to carry out fatigue life analysis for full structure to compare results with the fatigue life estimation with response estimation method. PSD cycle count method for full structure fatigue life estimation analysis is Steinberg. Result of the fatigue life estimation analysis for full structure is given at Figure 4.45.

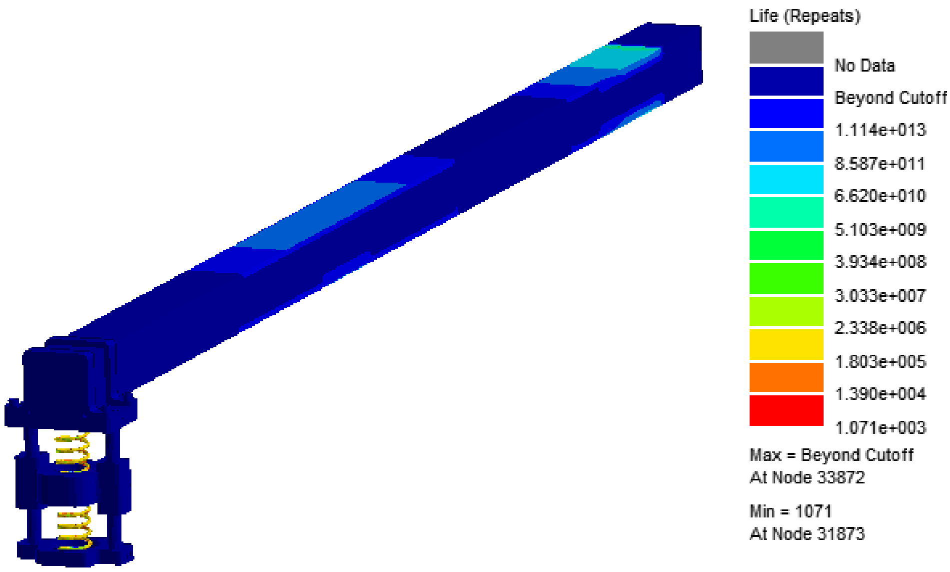

Figure 4.45: Fatigue Life Estimation Analysis Result for Full Body

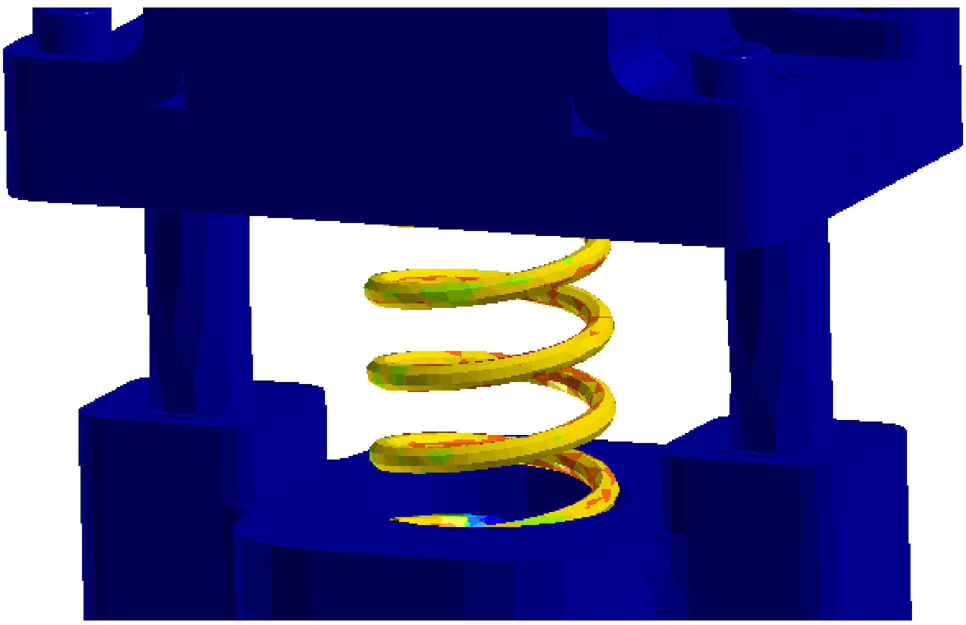

Figure 4.46: Detailed View of Fatigue Life Estimation Result of Most Critical Area

Fatigue life estimation for the full body is 1071 repeats. Exposure duration is also selected as 1 second for this analysis. Fatigue life estimation for full body is estimated as 1071 seconds. Since the difference between full body analysis and only TVA analysis is too wide, fatigue life tests for mass and spring type TVA is decided to not carried out.

4.6 Response Estimation of Cantilever Beam Type TVA with Two Inputs

Response estimation method theory is executed with only single input in the study completed by Özden [1]. In this chapter, in continuation to his work, response estimation method for the case where two PSD force inputs executed on the base structure is carried. Executing response estimation method theory with two inputs as follow. TVA is going to be attached to structure via 3rd DOF, consisting of 1DOF system with mass M_t and K_t (Figure 4.47).

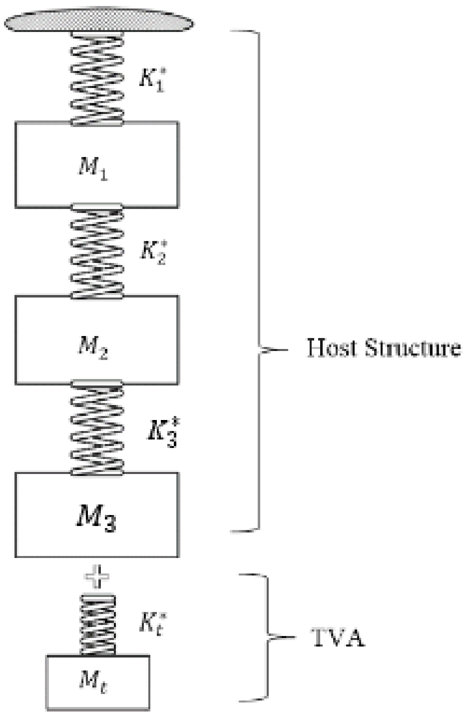

Figure 4.47: 4 DOF Structure.

System matrixes of TVA alone:

$$[M] = \begin{bmatrix} M_t & 0 \\ 0 & 0 \end{bmatrix} [K_t] = \begin{bmatrix} K_t^* & -K_t^* \\ -K_t^* & K_t^* \end{bmatrix} \tag{4.16}$$

For system matrix of TVA

$$[H^t] = [-\omega^2[M] + [K_t]]^{-1} \tag{4.17}$$

$$[H^t] = \begin{bmatrix} H_{1,1}^t & H_{1,2}^t \\ H_{2,1}^t & H_{2,2}^t \end{bmatrix} \tag{4.18}$$

$$[H^t] = \begin{bmatrix} \frac{1}{(-\omega^2 M_t)} & \frac{1}{(-\omega^2 M_t)} \\ \frac{1}{(-\omega^2 M_t)} & \frac{-\omega^2 M_t + K_t)}{(-\omega^2 M_t)} \end{bmatrix} \tag{4.19}$$

One can calculate the FRF of modified 2nd DOF as follows:

$$[\widetilde{H_{22}^m}] = \frac{H_{22}^o(\omega) H_{22}^t(\omega)}{H_{22}^o(\omega) + H_{22}^t(\omega)} \tag{4.20}$$

Where $[\widetilde{H}_{22}^m]$ stands for modified structure 2nd DOF direct FRF.

Forcing on the other DOFs will be simulated as if there is only one source of forcing that is acting on the attachment point, in this case that's 3^{rd} DOF.

$$[G] = [H^o][G_f][H^o]^H \tag{4.21}$$

Where $[G]$,$[H^o]$ and $[G_f]$ are refer to system response, system matrix and forcing matrix respectively. Forcing is assumed to be acting only second DOF therefore forcing matrix is represented by eqn. 4.22

$$[G_f] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_f^h \end{bmatrix} \tag{4.22}$$

$$\begin{bmatrix} G_{11}^Y & G_{12}^Y & G_{13}^Y \\ G_{21}^Y & G_{22}^Y & G_{23}^Y \\ G_{31}^Y & G_{32}^Y & G_{33}^Y \end{bmatrix} = \begin{bmatrix} H_{11}^o & H_{12}^o & H_{13}^o \\ H_{21}^o & H_{22}^o & H_{23}^o \\ H_{31}^o & H_{32}^o & H_{33}^o \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_f^h \end{bmatrix} \begin{bmatrix} H_{11}^o & H_{12}^o & H_{13}^o \\ H_{21}^o & H_{22}^o & H_{23}^o \\ H_{31}^o & H_{32}^o & H_{33}^o \end{bmatrix}^H \tag{4.23}$$

It is observed that hypothetical forcing is only function of direct FRF of interested DOF and operational response of same DOF as well. Therefore higher degrees of freedom linear system as given in the eqn. 4.24.

$$G_f^h(\omega) = \frac{G_{nn}^Y(\omega)}{|H_{nn}^o(\omega))|^2} \tag{4.24}$$

For this case this equation will become;

$$G_f^h(\omega) = \frac{G_{33}^Y(\omega)}{|H_{33}^o(\omega))|^2} \tag{4.25}$$

Therefore, point response can be calculated using FRF and forcing function acting that point.

$$\widetilde{G_{33}^Y}^m(\omega) = |\widetilde{H}_{33}^m(\omega)|^2 G_f^H(\omega) \tag{4.26}$$

Where $\widetilde{G_{33}^Y}^m(\omega)$ stands for nodified point (3rd DOF) response estimation.

Open form of the above equation is given at below;

$$\widetilde{G_{33}^Y}^m(\omega) = \left| \frac{H_{22}^t(\omega)}{H_{33}^o(\omega) + H_{22}^t(\omega)} \right|^2 G_{33}^Y(\omega) \tag{4.27}$$

Full solution of the original and modified structure has been made and given in the following steps. 3 DOF original structure mass and stiffness matrixes are formed:

$$[M_s] = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} [K_s] = \begin{bmatrix} K_1^* + K_2^* & -K_2^* & 0 \\ -K_2^* & K_2^* + K_3^* & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix} \tag{4.28}$$

$$H_s(\omega) = [-\omega^2[M_s] + [K_s^*]]^{-1} \tag{4.29}$$

Where H_s denotes ("ω" omitted from both sides for the sake of simplicity);

$$[H_s] = \begin{bmatrix} H_{11}^o & H_{12}^o & H_{13}^o \\ H_{21}^o & H_{22}^o & H_{23}^o \\ H_{31}^o & H_{32}^o & H_{33}^o \end{bmatrix} \tag{4.30}$$

*Where superscript 'o' means original system. Since PSD function of forcing input is random stationary process, response of the 2 DOF system under random excitation can be found as;

$$\begin{bmatrix} G^Y_{11} & G^Y_{12} & G^Y_{13} \\ G^Y_{21} & G^Y_{22} & G^Y_{23} \\ G^Y_{31} & G^Y_{32} & G^Y_{33} \end{bmatrix} = \begin{bmatrix} H^o_{11} & H^o_{12} & H^o_{13} \\ H^o_{21} & H^o_{22} & H^o_{23} \\ H^o_{31} & H^o_{32} & H^o_{33} \end{bmatrix} \begin{bmatrix} G^1_f & 0 & 0 \\ 0 & G^2_f & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H^o_{11} & H^o_{12} & H^o_{13} \\ H^o_{21} & H^o_{22} & H^o_{23} \\ H^o_{31} & H^o_{32} & H^o_{33} \end{bmatrix}^H \tag{4.31}$$

Where G^1_f, G^Y_{11}, G^Y_{22}, G^Y_{12} and G^Y_{21} refer to PSD of input force, PSD response of DOF 1, PSD response of DOF 2, coherence of responses at DOF 1 and DOF 2. Hence third DOF response in PSD form is given at below;

$$G^Y_{33} = |H^o_{31}|^2 G^1_f + |H^o_{32}|^2 G^2_f \tag{4.32}$$

Inserting the eqn.4.32 into eqn.4.27, estimation as a function of input forcing as given below,

$$\widetilde{G^Y_{33}}^m = |H^o_{31}|^2 G^1_f + |H^o_{32}|^2 G^2_f \frac{H^t_{22}}{H^o_{33} + H^t_{22}}|^2 \tag{4.33}$$

Modified structure (3DOF) system matrixes are defined at below,

$$[M^m_s] = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \\ 0 & 0 & 0 & M_t \end{bmatrix} \tag{4.34}$$

$$[K^m_s] = \begin{bmatrix} K^*_1 + K^*_2 & -K^*_2 & 0 & 0 \\ -K^*_2 & K^*_2 + K_3 & -K_3 & 0 \\ 0 & -K^*_3 & K^*_3 + K_t & -K_t \\ 0 & 0 & -K_t & K_t \end{bmatrix} \tag{4.35}$$

$$H^m_s(\omega) = [-\omega^2 [M^m_s] + [K^{*m}_s]]^{-1} \tag{4.36}$$

$$[H^m_s] = \begin{bmatrix} H^m_{11} & H^m_{12} & H^m_{13} & H^m_{14} \\ H^m_{21} & H^m_{22} & H^m_{23} & H^m_{24} \\ H^m_{31} & H^m_{32} & H^m_{33} & H^m_{34} \\ H^m_{41} & H^m_{42} & H^m_{43} & H^m_{44} \end{bmatrix} \tag{4.37}$$

Where $[H^m_s]$ and $[G^{Ym}]$ refer to modified system FRF matrix and modified system response matrix.

$$[G^{Y^m}] = \begin{bmatrix} H_{11}^m & H_{12}^m & H_{13}^m & H_{14}^m \\ H_{21}^m & H_{22}^m & H_{23}^m & H_{24}^m \\ H_{31}^m & H_{32}^m & H_{33}^m & H_{34}^m \\ H_{41}^m & H_{42}^m & H_{43}^m & H_{44}^m \end{bmatrix} \begin{bmatrix} G_f^1 & 0 & 0 & 0 \\ 0 & G_f^1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_{11}^m & H_{12}^m & H_{13}^m & H_{14}^m \\ H_{21}^m & H_{22}^m & H_{23}^m & H_{24}^m \\ H_{31}^m & H_{32}^m & H_{33}^m & H_{34}^m \\ H_{41}^m & H_{42}^m & H_{43}^m & H_{44}^m \end{bmatrix}^H \tag{4.38}$$

Third DOF exact reaction is denoted in the above matrix as;

$$G_{33}^{Y\,m} = |H_{31}^m|^2 G_f^1 + |H_{32}^m|^2 G_f^2 \tag{4.39}$$

Below equations4.40and 4.41 is expected to be hold, thus, they are inserted in to 4.39 check if response estimation method holds;

$$|H_{31}^m| = \left| \frac{H_{31}^o H_{22}^t}{H_{33}^o + H_{22}^t} \right| \tag{4.40}$$

$$|H_{32}^m| = \left| \frac{H_{32}^o H_{22}^t}{H_{33}^o + H_{22}^t} \right| \tag{4.41}$$

Since after insertion equation 4.42 holds, it can be said that response estimation method is also valid for two input case.

$$G_{33}^{Y\,m} = \widetilde{G_{33}^{Y\,m}} \tag{4.42}$$

Moreover, analysis of an example with two input has been carried followingly. Application location of the force inputs are given at Figure 4.48. PSD force inputs for the analysis are given at Figure 4.49, Figure 4.50, Table 4.5 and Table 4.6.

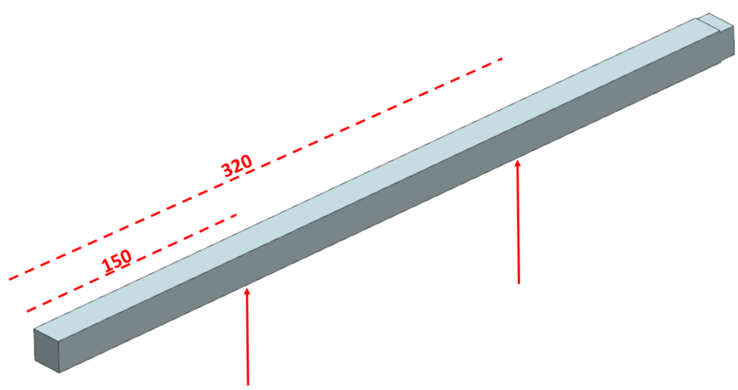

Figure 4.48: Application Point for PSD Forcings on the Structure

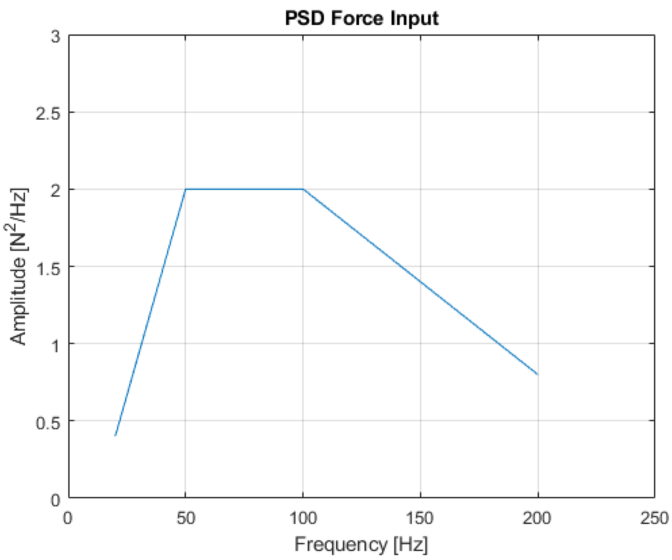

Figure 4.49: First PSD Forcing Applied at 150 mm Location

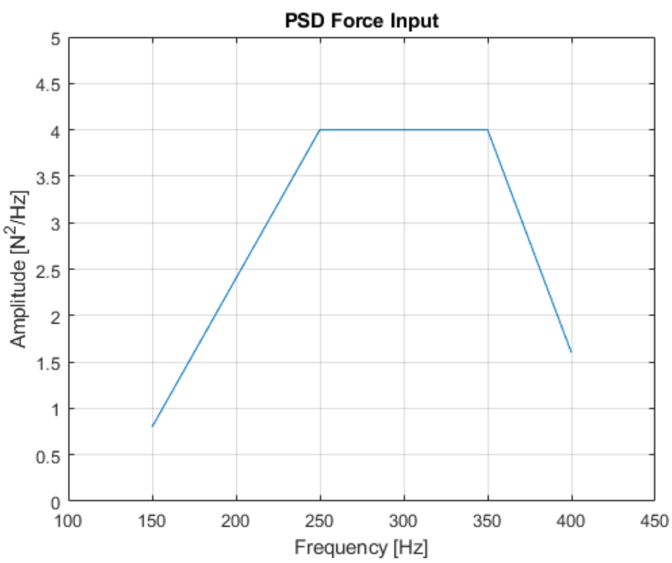

Figure 4.50: Second PSD Forcing Applied at 320 mm Location

Operational response of the base structure with two forcing input is gathered from analysis. Result of the analysis is given at Figure 4.51.

Table 4.5: First PSD Force Input Values for Structure

Frequency $[Hz]$	Amplitude $[N^2/Hz]$
20	0.4
50	2
100	2
200	0.8

Table 4.6: Second PSD Force Input Values for Structure

Frequency $[Hz]$	Amplitude $[N^2/Hz]$
150	0.8
250	4
350	4
400	1.6

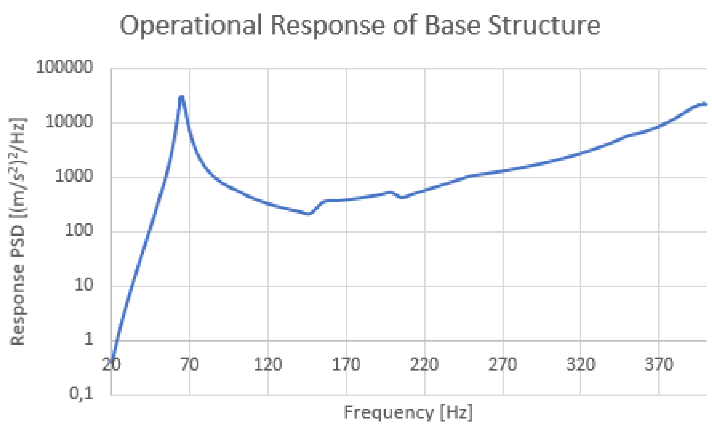

Figure 4.51: Operational Response of the Base Structure

Response estimation method is carried out with gathered FRF data and recently gathered operational response data. Result of the response estimation method is given at Figure 4.52. According to the results given at Figure 4.52, it can be seen that response estimation method which is generated by Özden [1] gives fairly accurate results with two input case.

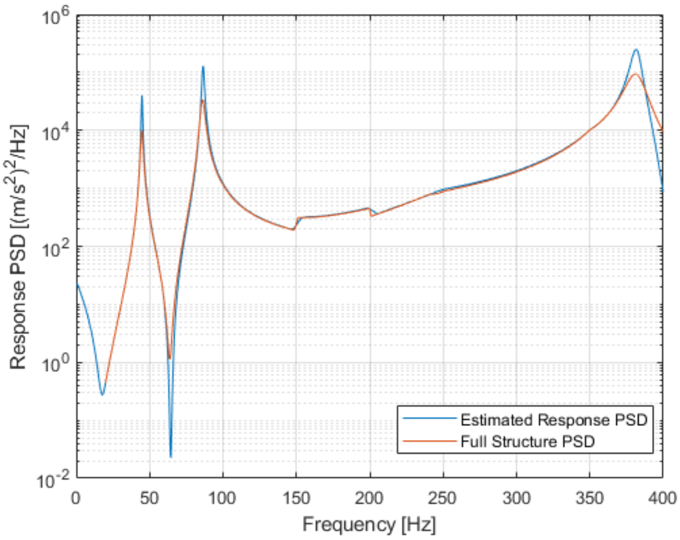

Figure 4.52: Comparison of Estimated Response and Full Body Analysis

CHAPTER 5

FATIGUE LIFE TESTS OF TUNED VIBRATION ABSORBERS

Fatigue life tests of the designed TVA is conducted to compare test results with the analysis results that is run utilizing response estimation method. Scenario of the tests are decided to have PSD forcing input on the base structure. Fatigue life tests is conducted for full structure which is base structure with the TVA and for only TVA. Test that are run with full structure is to get a comparison between only TVA fatigue life test result. Total fatigue life of the TVAs is measured with number of specimen to come up with an average fatigue life for the TVAs. Comparison of the test results and fatigue life estimation analysis with response estimation method is done in this chapter.

5.1 Cantilever Beam Type TVA Tests

Prior to the fatigue life test of the TVA, tests to come up with characterization and checking vibrational properties of the TVA and base structure are carried out. In order to gather those data on the TVA and base structure three different measurements are taken from test setup. Those measurements are operational response of the base structure with given input, direct FRF of the base structure and transmissibility of the TVA. Test step for these measurements consists with components which are given at Table 5.1;

Table 5.1: Test Setup Components

1)	Aluminum Beam (20mmx20mmx500mm)
2)	Designed TVA
3)	Accelerometers (3)
4)	Force transducer (1)
5)	Stringer (1)
6)	Dynamic Shaker and Control Unit
7)	Data Acquisition Device (DEWESOFT)
8)	Laptop
9)	Polyamide Tip Impact Hammer

Test setups with given components are presented in Figure 5.1, 5.2 and 5.2.

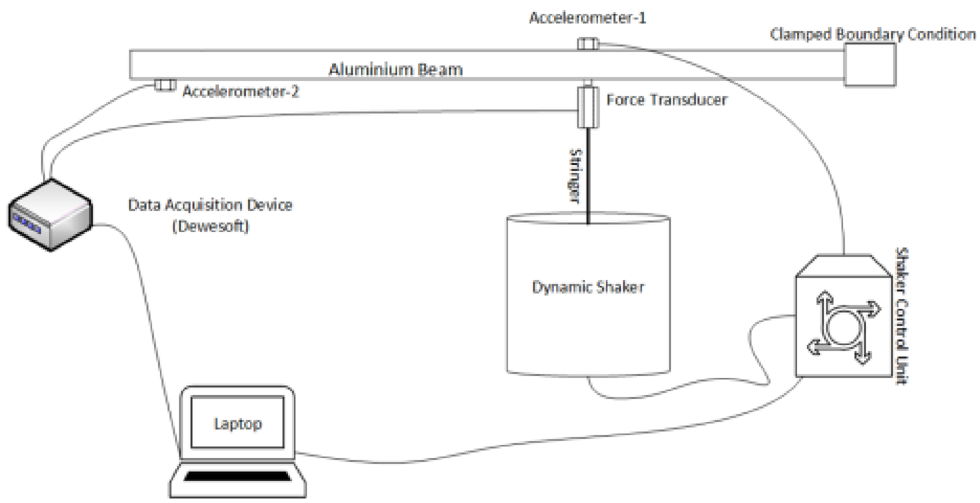

Figure 5.1: Test Setup for Operational Response Measurement of the Base Structure

First experimental setup is presented in Figure 5.1. In this configuration operational response of the base structure from tip of the structure is measured with accelerometer.

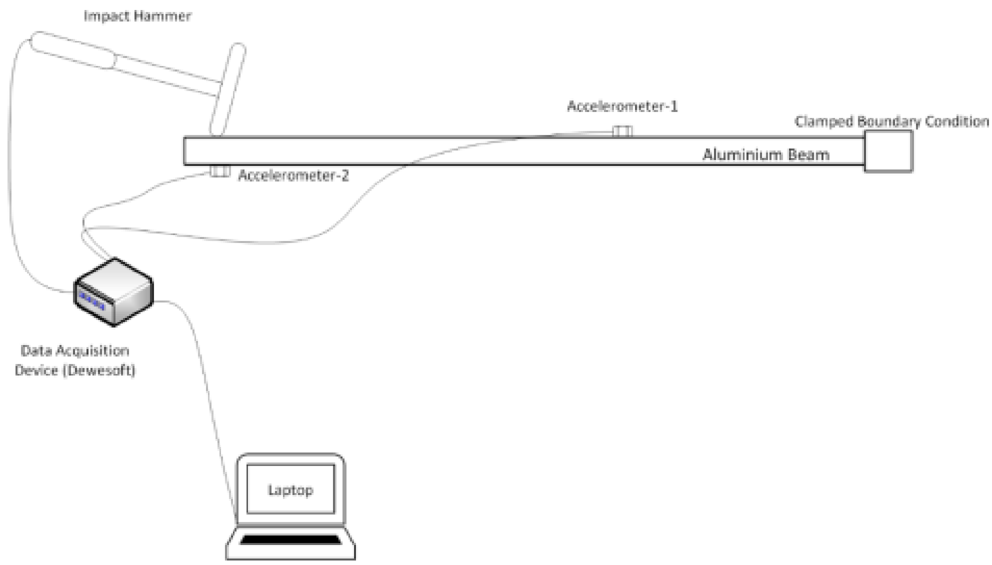

Figure 5.2: Test Setup for Tip Point Direct FRF Measurement of the Base Structure

Second experimental setup is given at Figure 5.2. Direct FRF of the tip point of the base structure is measured with accelerometer and polyamide tip impact hammer.

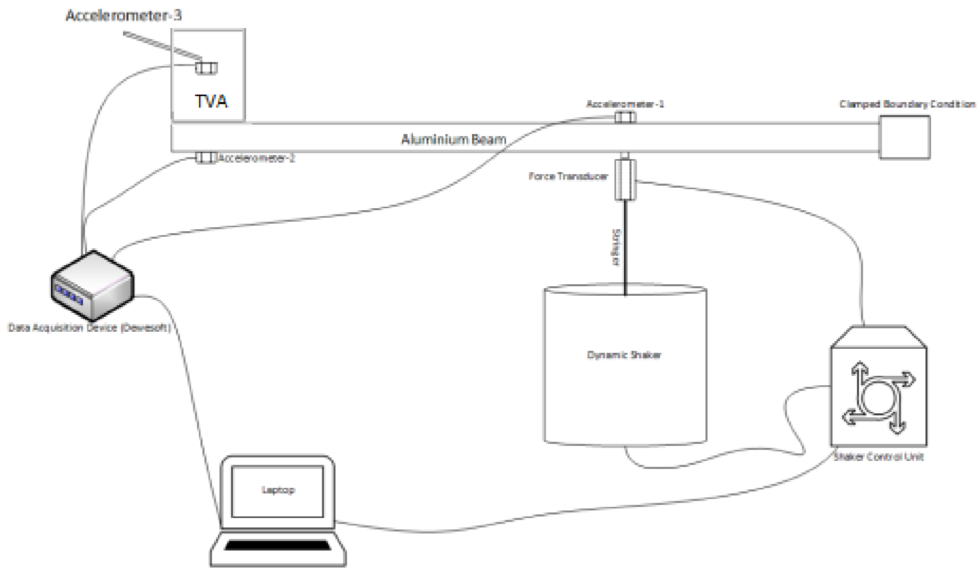

Figure 5.3: Test Setup for Tip Point FRF and Transmissibility Measurement of the TVA

Third test setup is given at Figure 5.3. Tip point FRF and Transmissibility of the TVA is measured by using accelerometers on tip of the base structure and the TVA.

Image of the data acquisition device is given at Figure 5.4. Moreover, impact hammer and vibration control device given at Figure 5.4.

Figure 5.4: Data Acquisition Device

Figure 5.5: Impact Hammer and Vibration Control Device

5.1.1 Operational Response Measurement from Tip of the Base Structure

Random vibration analysis of the base structure was carried at previous sections with given PSD force input at Figure 4.4. In order to verify analysis results, following test setup given at Figure 5.6 is used to get operational response of the base structure. Operational response of the base structure is gathered by using accelerometer at the tip point with data acquisition device. Gathered data given at Figure 5.7. Comparison between analysis result and test data is given at Figure 5.8.

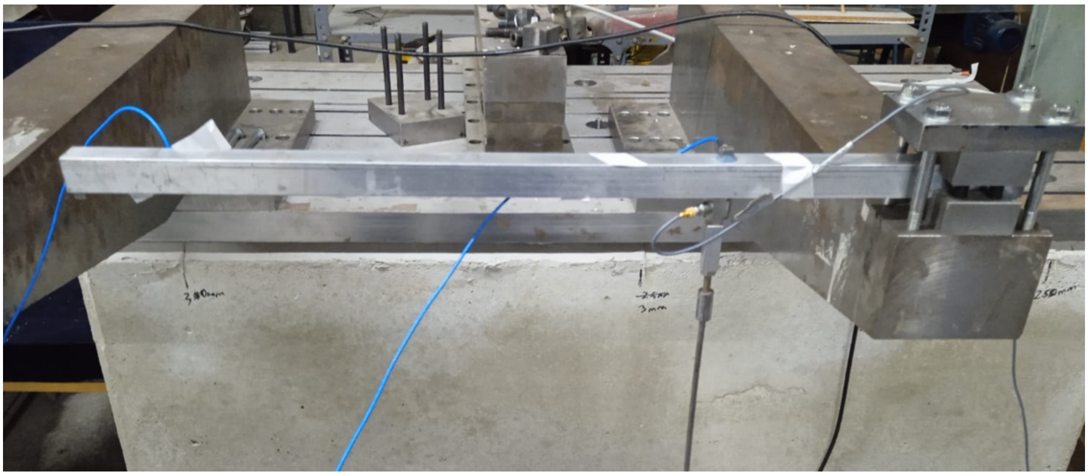

Figure 5.6: Test Setup for Operational Response Measurement of the Base Structure

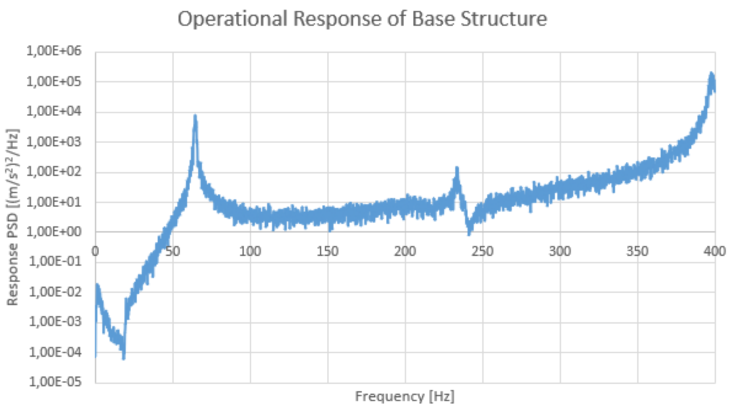

Figure 5.7: Operational Response at Tip Point of the Base Structure

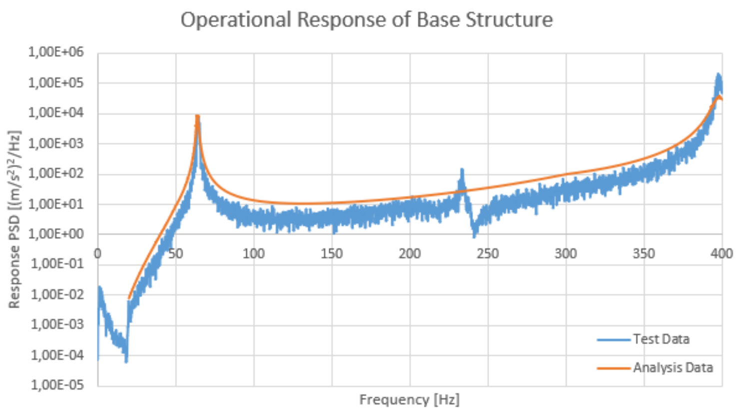

Figure 5.8: Comparison of Operational Response at Tip Point of the Base Structure with Analysis and Test Data

5.1.2 Direct FRF Measurement from Tip of the Base Structure

Similar to the operational response measurements of the base structure, direct FRF at tip tip of the base structure is also measured to verify analysis results which are given at the previous section. Test setup for the direct FRF measurement includes impact hammer and accelerometers. Direct FRF at the tip of the base structure data that is gathered from the test is given at Figure 5.9. Comparison between analysis and test data also given for direct FRF at Figure 5.10.

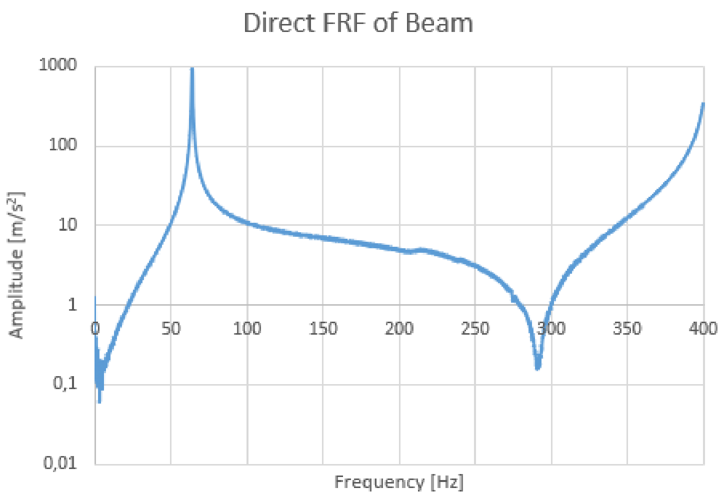

Figure 5.9: Direct FRF at Tip Point of the Base Structure

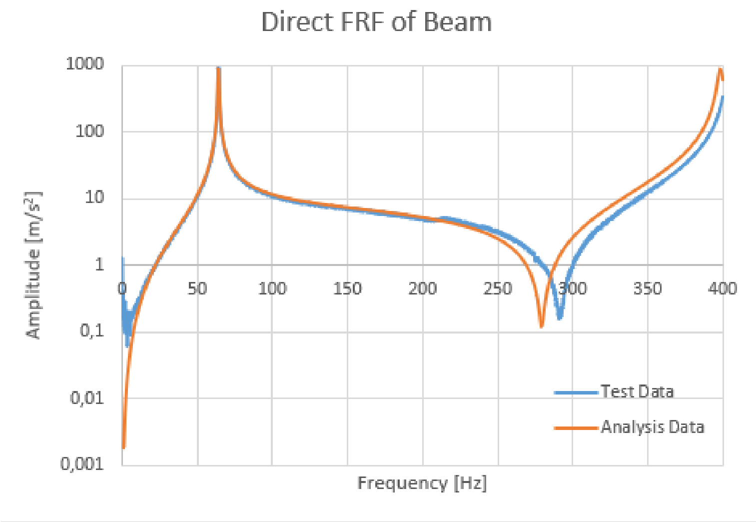

Figure 5.10: Comparison of Direct FRF at Tip Point of the Base Structure with Analysis and Test Data

5.1.3 Tip Point FRF and Transmissibility Measurement of the TVA

Cantilever beam type TVA design completed at previous section to reduce vibrating level of the base structure. However, as in the base structure, cantilever beam type TVA is also needed to be verified to use in fatigue life tests. In order to verify the TVA design, tip point FRF of the full structure and resonance frequency of the TVA should be measured. This measurement is done with two accelerometers attached on base of the TVA and mass of the TVA. Ratio between two accelerometer data is giving transmissibility function of the TVA. Test setup for the transmissibility measurements are given at Figure 5.11 and 5.12. Measurement result of the tip point FRF of full structure is given at Figure 5.13. Comparison between base excitation that is calculated with response estimation method test data is given at Figure 5.14. Measurement result for the TVA transmissibility is given at Figure 5.15.

Figure 5.11: Test Setup for TVA Transmissibility Measurement

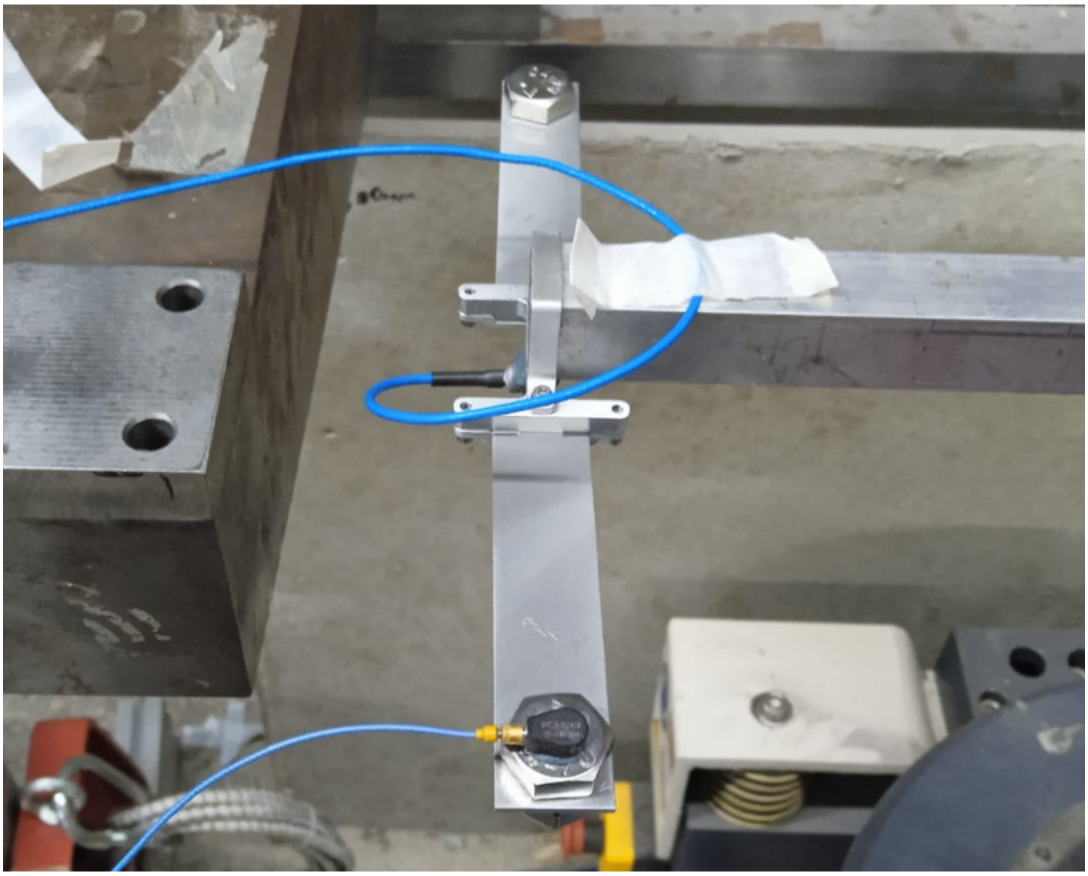

Figure 5.12: Detail View of the TVA at Transmissibility Measurement

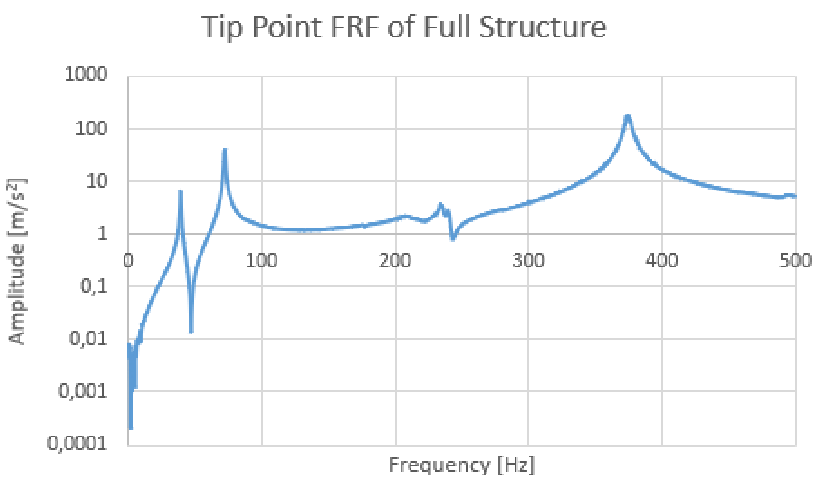

Figure 5.13: Tip Point FRF Measurement Result of the Full Structure

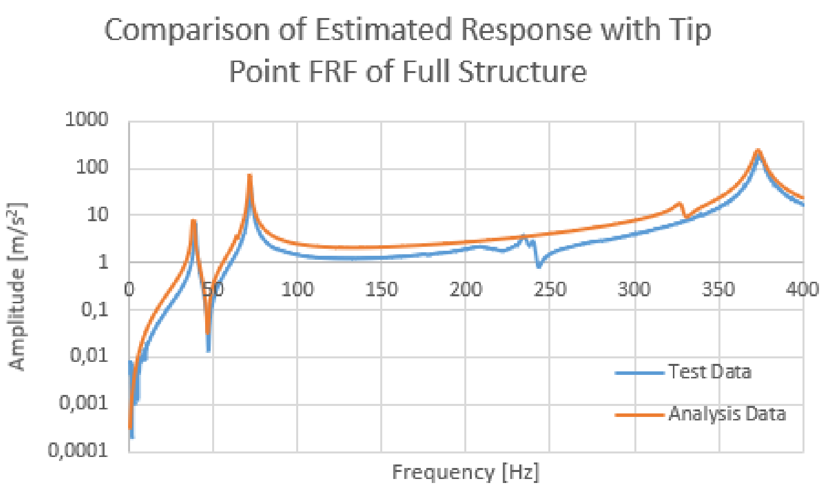

Figure 5.14: Comparison of Tip Point FRF Measurement Result for Analysis and Test of the Full Structure

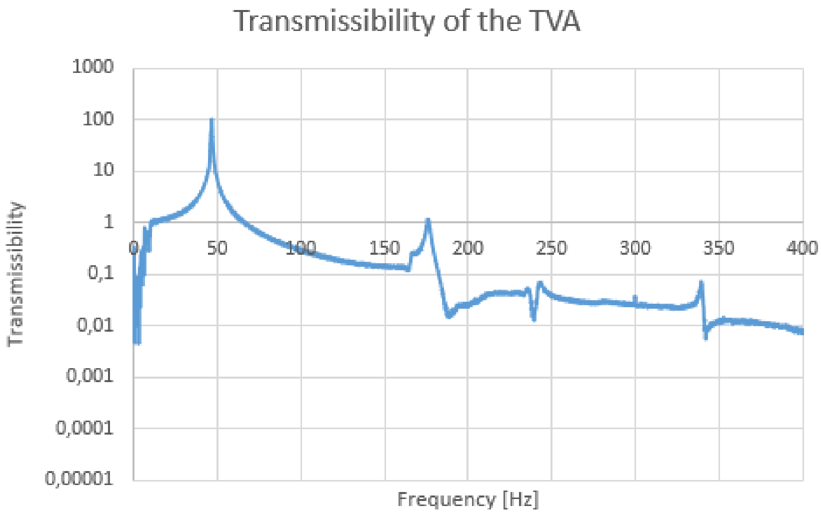

Figure 5.15: Transmissibility Measurement of the TVA

According to the test results, first natural frequency of the cantilever beam type TVA is 46.5 Hz. This is probably due to effects of connection brackets of the TVA. There are three brackets on the TVA design that enable assembly of TVA itself and assembly to the base structure. Due to production imperfections, those brackets could not properly hold the parts in their places. Thus, connections are not stiff as in the analysis. In order to be able to make a comparison between analysis results and test results for fatigue life estimations, analysis of cantilever beam type TVA should be revised at this point. This revision is accomplished with changing elastic modulus of the brackets to the level that, first natural frequency of the cantilever beam type TVA would be close to 46.5 Hz.

Response estimation method and fatigue life estimation analysis rerun with revised TVA model. Result of the fatigue life estimation analysis is given at Figure 5.16.

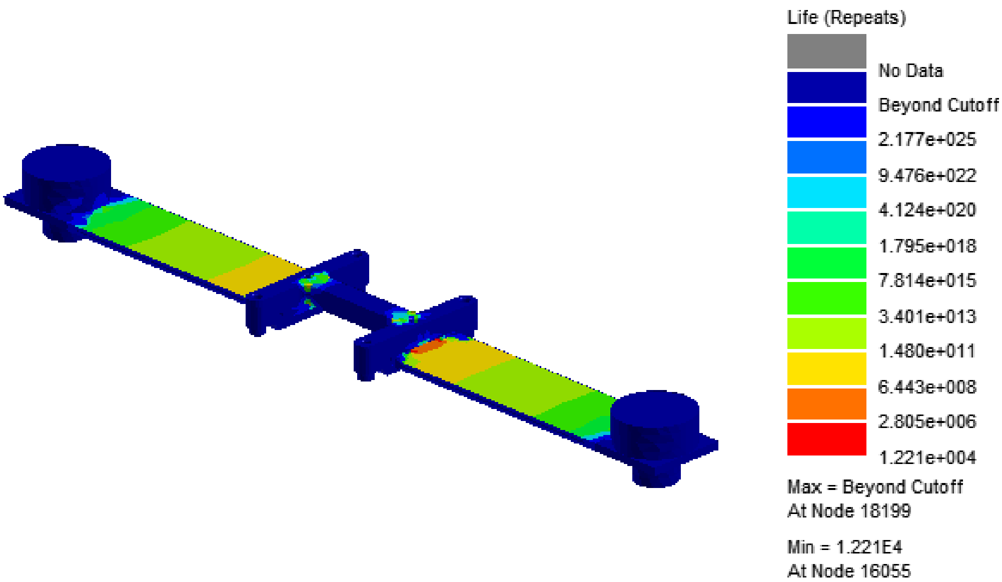

Figure 5.16: Fatigue Life Estimation Analysis Result for the TVA

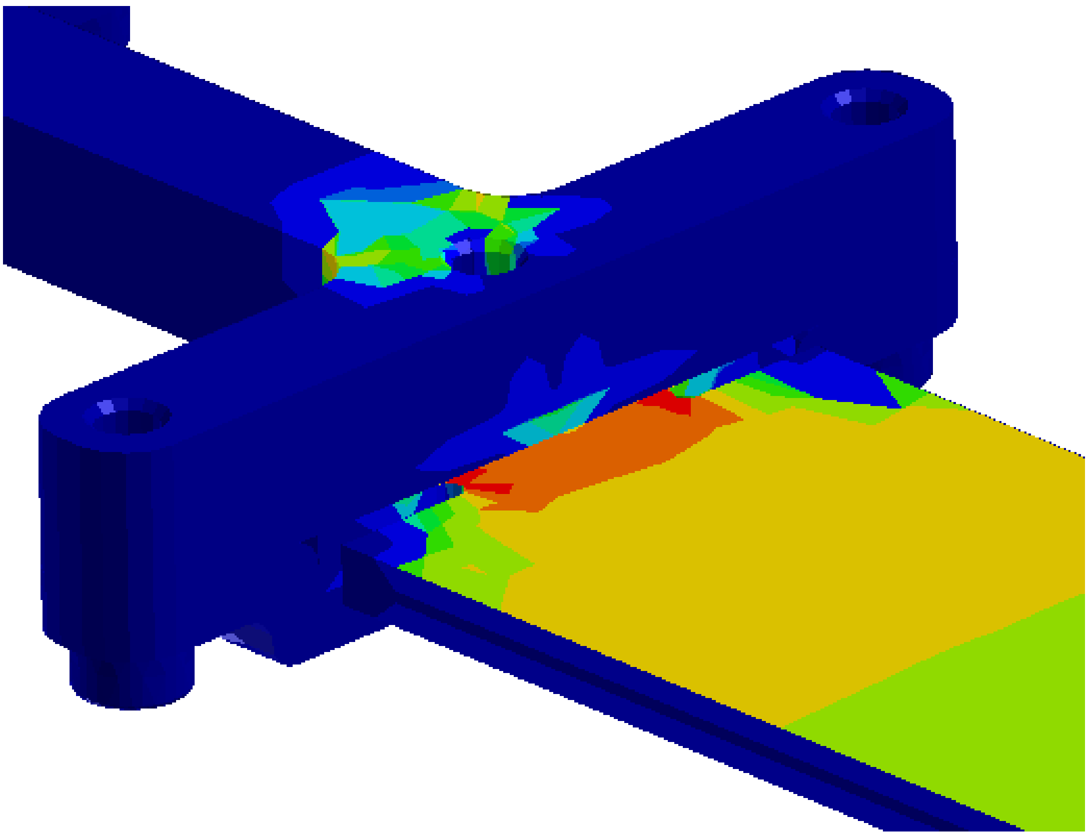

Figure 5.17: Detailed View of Fatigue Life Estimation Result of Notched Area

Estimated life of the TVA is 12210 repeats with revised model. Since, exposure duration is selected as 1 second fatigue life estimation of the TVA with estimated response is 12210 seconds.

Result of the fatigue life analysis of the full body is given at Figure 5.18.

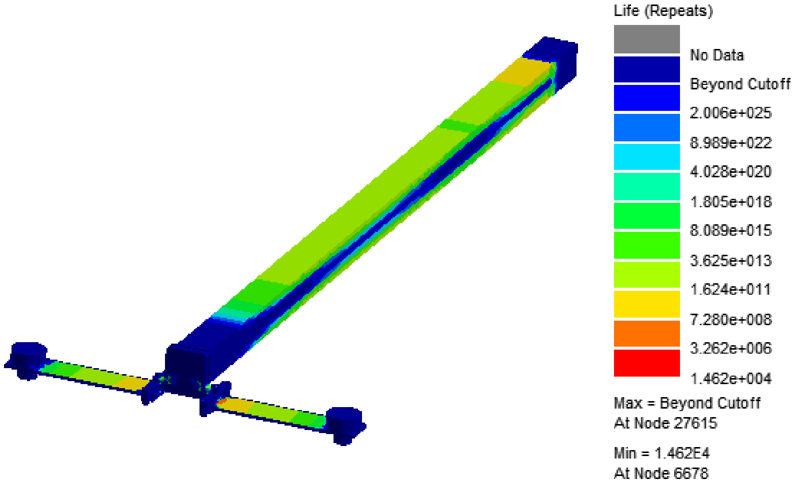

Figure 5.18: Fatigue Life Estimation Analysis Result for Full Body

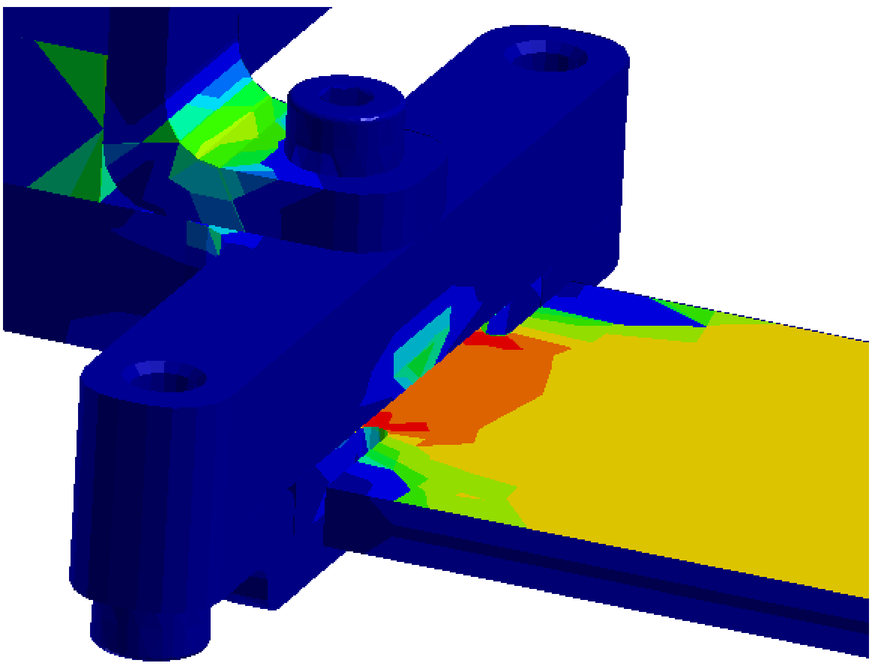

Figure 5.19: Detailed View of Fatigue Life Estimation Result of Notched Area

Fatigue life estimation for the full body is 14620 repeats with revised model. Exposure duration is also selected as 1 second for this analysis. Fatigue life estimation for full body is estimated as 14620 seconds.

5.1.4 Redesign of the Cantilever Beam Type TVA

According to the results of fatigue life analysis with rerun model, it is not practical to run the tests since required time is too long to execute the tests. Main purpose on the design of the TVA is to reduce expected fatigue life to shorten test duration. However, at the current design, expected fatigue life is too long to conduct tests due to effects of connection brackets of the TVA. In order to shorten the expected fatigue life of the TVA with current design, masses that are placed on the tip of the cantilever beam is changed with lighter masses. As a result, with lighter masses, tuned frequency of the TVA rearranged to the first resonance frequency of the base structure. Mass on the tip of the cantilever beam is changed from 30 gr to 14,5 gr for each. New selected masses are given at Figures 5.20 and 5.21.

Figure 5.20: Measurement of Single Selected Mass

Figure 5.21: Measurement of Two Selected Masses

Response estimation method and fatigue life estimation process with changed mass executed again to get fatigue life estimation for revised TVA. Measurements taken from the base structure is used without change since they are only related with the base structure. However, direct FRF of the TVA is needed to be recalculated to be used in response estimation method. In order to calculate direct FRF of the TVA effective mass of the TVA is gathered from modal analysis of the TVA. Effective mass of the TVA is given at Figure 5.22.

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	63.3560	0.15784E-01	0.89847E-01	0.492720	0.807252E-02	0.195345	0.130771
2	63.6288	0.15716E-01	0.18235	1.000000	0.332513E-01	0.999988	0.538655
3	84.5552	0.11827E-01	-0.44805E-03	0.002457	0.200749E-06	0.999993	0.325204E-05
4	84.7157	0.11804E-01	-0.53730E-03	0.002947	0.288690E-06	1.00000	0.467664E-05
5	615.900	0.16236E-02	-0.14218E-04	0.000078	0.202159E-09	1.00000	0.327488E-08
6	618.248	0.16175E-02	0.13642E-03	0.000748	0.186111E-07	1.00000	0.301491E-06
sum					0.413244E-01		0.669434

Figure 5.22: Effective Mass of the TVA

Moreover, effective stiffness of the TVA is calculated with utilizing Equation 5.1.

$$\omega_n = \sqrt{\left(\frac{k}{m}\right)} \tag{5.1}$$

Effective stiffness of the TVA is calculated as $k = 5315 N/m$ with given effective mass and resonance frequency.

TVA system matrices can be written as below:

$$[M] = \begin{bmatrix} M_t & 0 \\ 0 & 0 \end{bmatrix} [K_t] = \begin{bmatrix} K_t^* & -K_t^* \\ -K_t^* & K_t^* \end{bmatrix} \tag{5.2}$$

For system matrix of TVA

$$[H^t] = [-\omega^2[M] + [K_t]]^{-1} \tag{5.3}$$

$$[H^t] = \begin{bmatrix} H^t_{1,1} & H^t_{1,2} \\ H^t_{2,1} & H^t_{2,2} \end{bmatrix} \tag{5.4}$$

$$[H^t] = \begin{bmatrix} \frac{1}{(-\omega^2 M_t)} & \frac{1}{(-\omega^2 M_t)} \\ \frac{1}{(-\omega^2 M_t)} & \frac{-\omega^2 M_t + K_t)}{(-\omega^2 M_t)} \end{bmatrix} \tag{5.5}$$

Since direct FRF of the TVA is the $H^t_{2,2} = \frac{-\omega^2 M_t + K_t)}{(-\omega^2 M_t)}$, it is calculated as given at Figure 5.23.

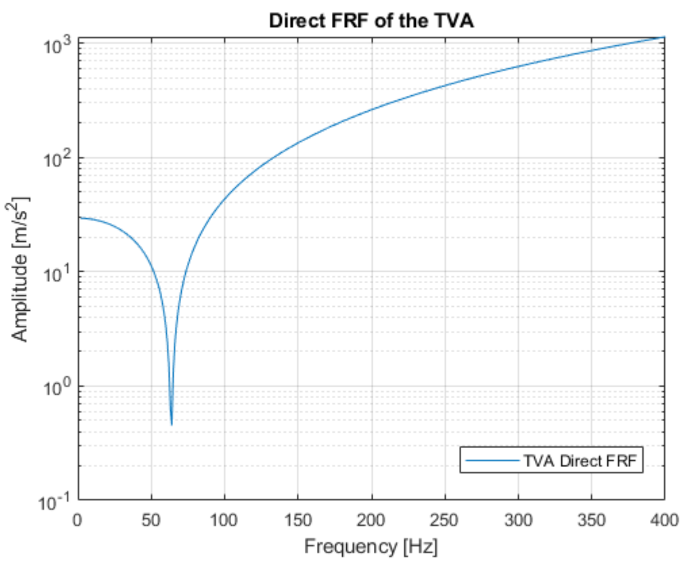

Figure 5.23: Direct FRF of the TVA

Moreover, PSD force input also changed since with the old loading fatigue life estimation analysis result is lower than one seconds. New loading for the case is given at Figure 5.24 and Table 5.2.

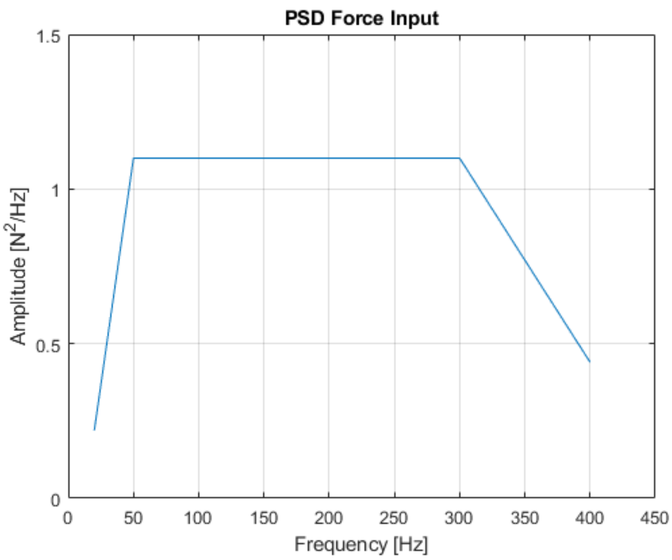

Figure 5.24: Changed PSD Force Input

Table 5.2: Changed PSD Force Input Values for Structure

Frequency $[Hz]$	Amplitude $[N^2/Hz]$
20	0.22
50	1.1
300	1.1
400	0.44

With calculation of the response estimation method, fatigue life analysis is completed. Results of the fatigue life estimation analysis is given at Figure 5.25 and 5.26.

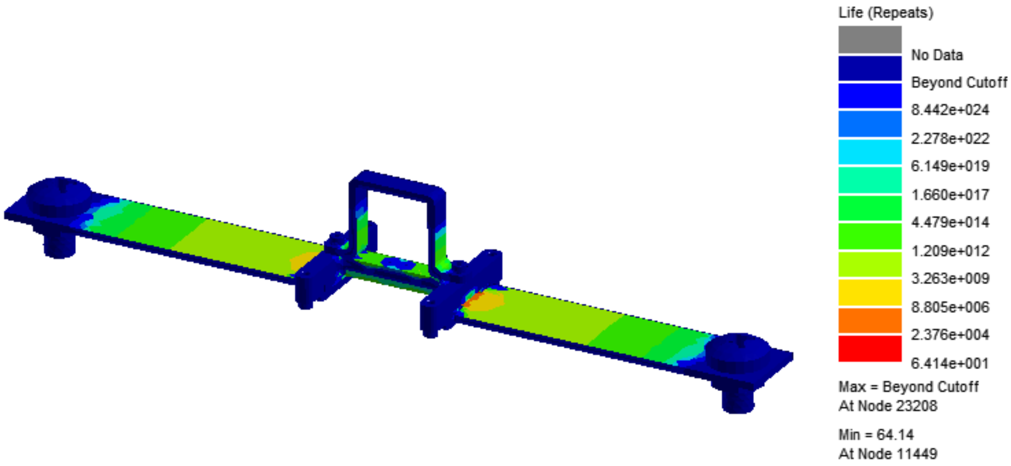

Figure 5.25: Fatigue Life Estimation Analysis Result for the TVA

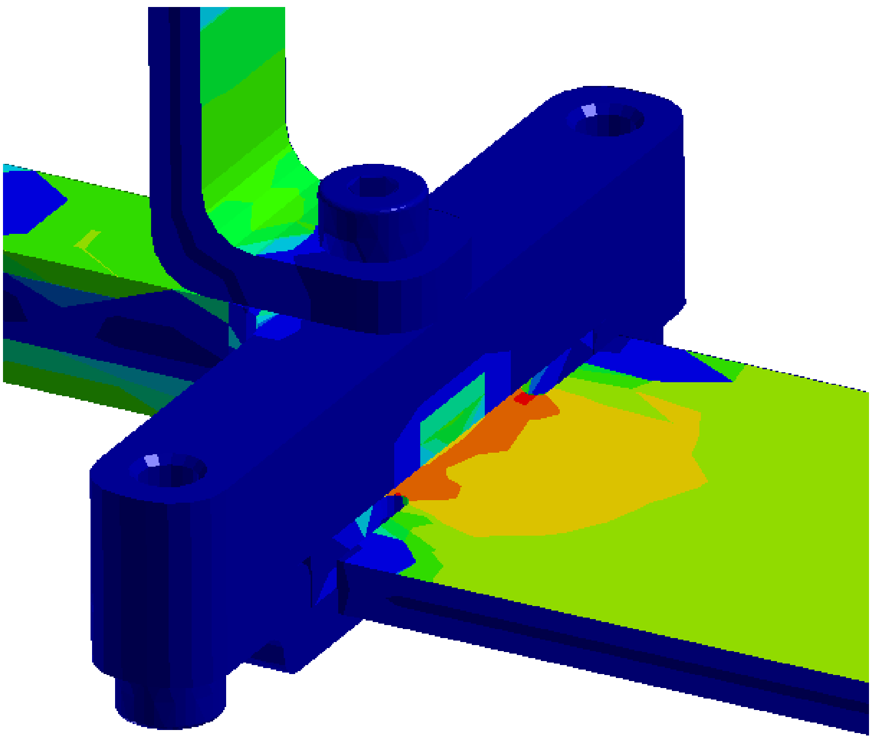

Figure 5.26: Detailed View of Fatigue Life Estimation Result of Notched Area

Estimated life of the TVA is 64.14 repeats with the TVA with changed mass. Since, exposure duration is selected as 1 second fatigue life estimation of the TVA with estimated response is 64.14 seconds.

Moreover, full body fatigue life estimation analysis is also carried out. Results of the fatigue life estimation analysis for full body is given at Figure 5.27 and 5.28.

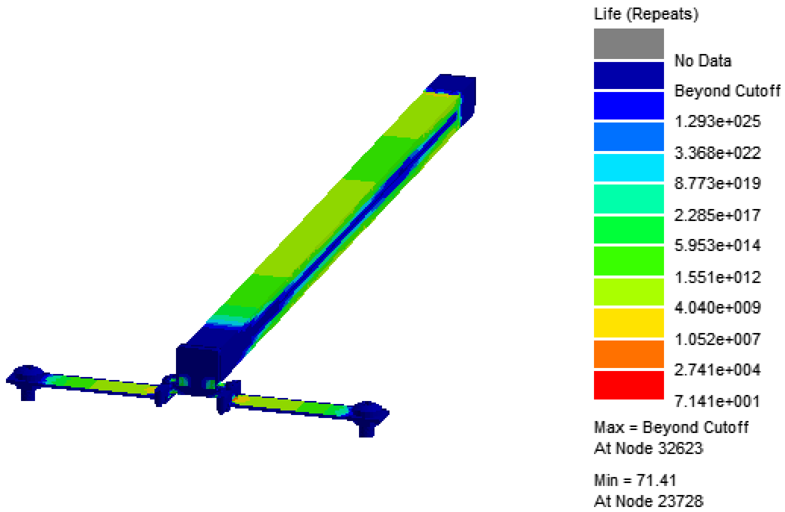

Figure 5.27: Fatigue Life Estimation Analysis Result for Full Body

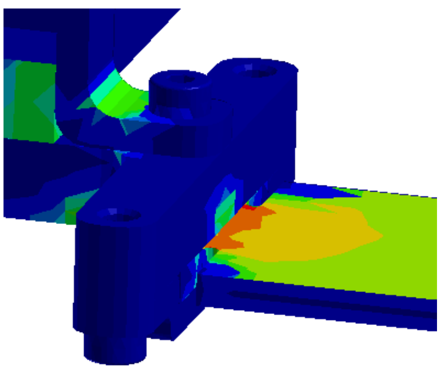

Figure 5.28: Detailed View of Fatigue Life Estimation Result of Notched Area

Fatigue life estimation for the full body is 71.41 repeats with changed mass. Exposure duration is also selected as 1 second for this analysis. Fatigue life estimation for full body is estimated as 71.41 seconds.

5.1.5 Fatigue Life Tests of the Cantilever Beam Type TVA

After completion of fatigue life analysis of the newly designed TVA, operational response at the tip point of the base structure with new given PSD force input stated at Table 5.2 is completed. Operational response measurement is required to recalculate base excitation of the TVA to where it is attached to the base structure. Measured operational response of the base structure is given at Figure 5.29.

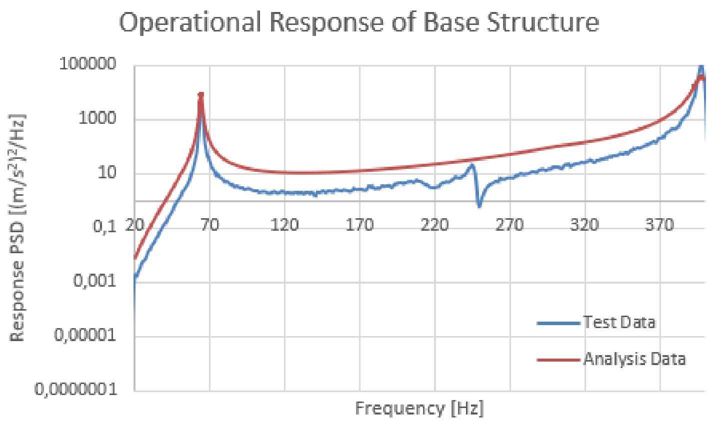

Figure 5.29: Operational Response with New PSD Force Input

Transmissibility measurement is repeated with redesigned TVA with new masses. It is checked if the redesigned TVA is tuned to the target frequency with mass changes. Transmissibility measurement setup is given at Figure 5.30 and Figure 5.31. Result of the transmissibility measurement is given at Figure 5.32.

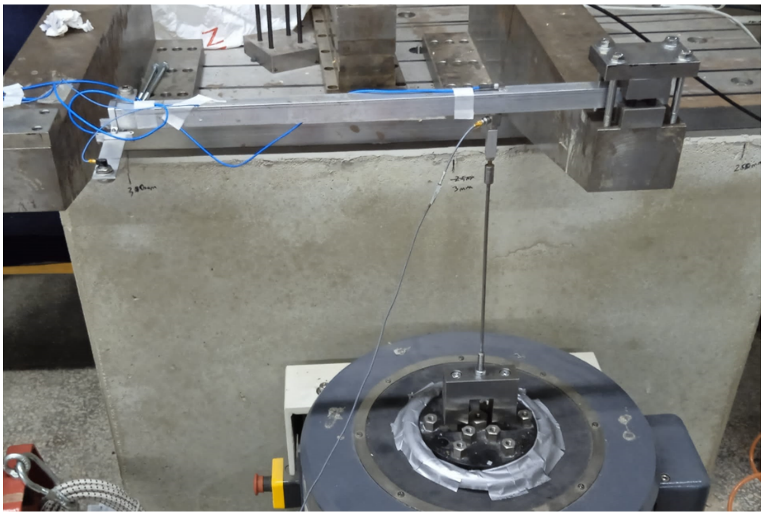

Figure 5.30: Transmissibility Measurement Setup with Redesigned TVA

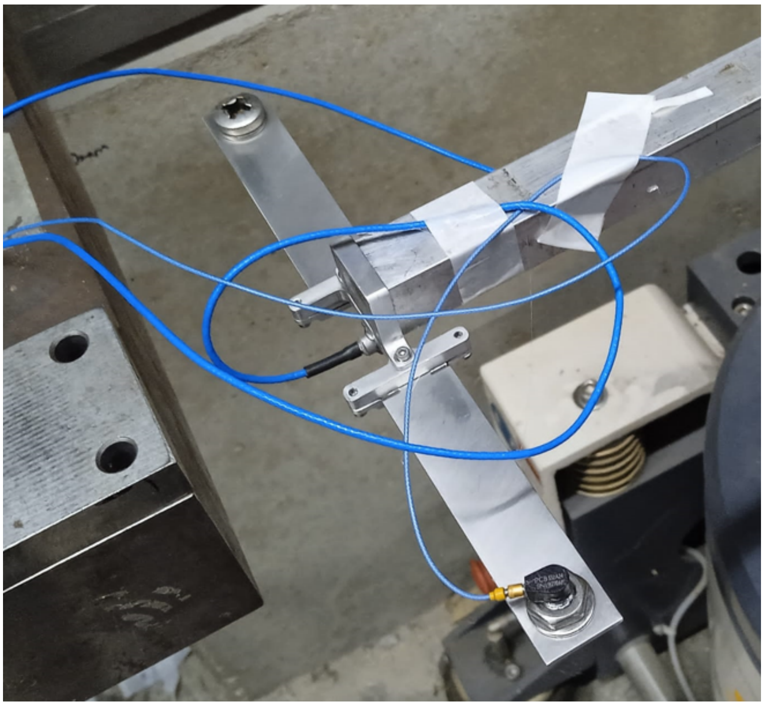

Figure 5.31: Detail View of Redesigned TVA During Transmissibility Measurement

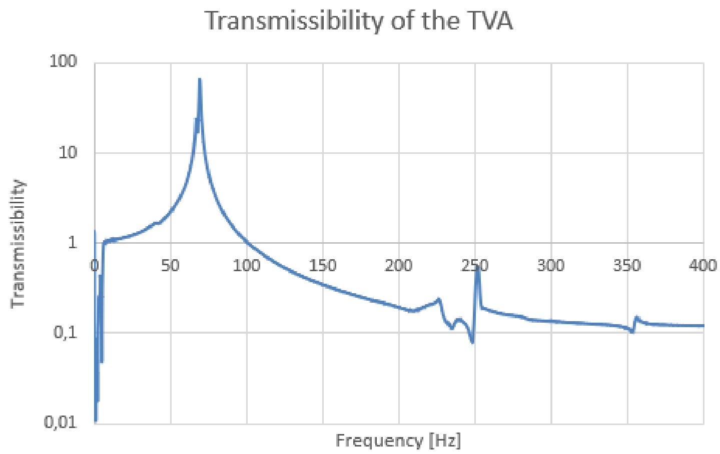

Figure 5.32: Transmissibility Measurement of the Redesigned TVA

According to the transmissibility measurement of the TVA, tuned frequency of the TVA is 69.824 Hz with changed masses. Although, the aim of re-tuning of the TVA is to tune it to 63.6288 Hz, due to in efficient design of the TVA in means of the connection parts, tuned frequency can not be accomplished. However, with the redesigned TVA, tuned frequency is much closer than previous case. Since, much closer tuned frequency with the TVA is accomplished, fatigue life tests with full body and only TVA setups are executed. Full body fatigue life tests are executed with setups shown at Figure 5.30 and Figure 5.31. PSD force input that is used for full body fatigue life test is given at Table 5.2. Fatigue life test are conducted for three hours without a failure with given input and setup. Since, no failure is observed on the TVA, it is decided to carry on with only TVA fatigue life test. In order to start only TVA fatigue life test, response estimation method which is developed by Özden [1] is executed with remeasured operational response with new PSD force input and mistuned redesigned TVA (the one which is tuned to 69.824 Hz). Result of the response estimation method with comparison to PSD acceleration data which is measured from at the point where the TVA is connected to the base structure during full body test is given at Figure 5.33.

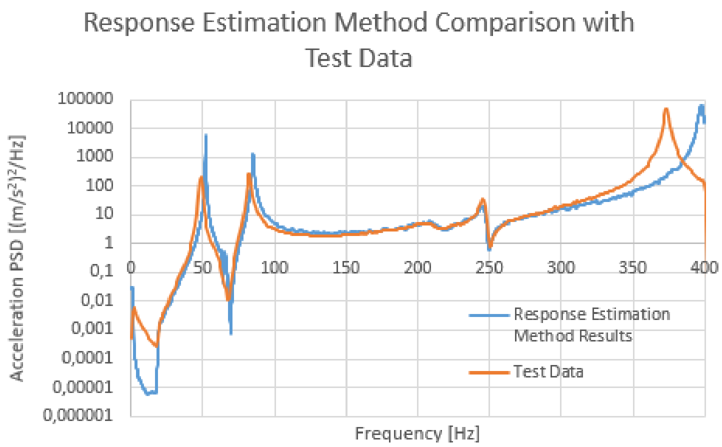

Figure 5.33: Comparison of Response Estimation Method Results with Test Data

Fatigue life test of the TVA with the setup which is shown at Figure 5.34 and with the response estimation method result which is shown at Figure 5.33 is started. However, after three hours passed with given input that is acquired with response estimation method, no failure is observed on the TVA. At this point, tests are stopped with only TVA and full body setups. Fatigue life tests will be continued as a future work.

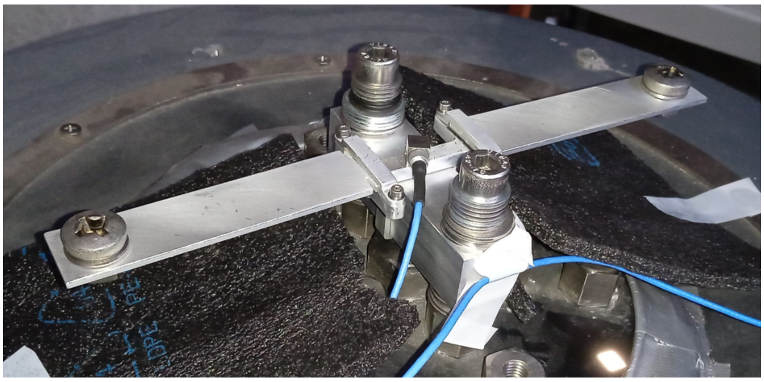

Figure 5.34: Only TVA Fatigue Life Test Setup

CHAPTER 6

CONCLUSIONS

In this chapter whole evaluation for the effectiveness of the response estimation method on fatigue life estimation analysis is evaluated with the given analysis and test results. Response estimation method is developed by Özden [1] during his Master's Thesis study. Stress PSDs for a given case are compared as a final step in his study. In this thesis study, the aim is to continue his work as using response estimation method to execute fatigue life analysis on a TVA to see its effectiveness on fatigue life estimation. According to analysis results, for the cantilever beam type TVA difference between the results of the fatigue life analyses for full body and TVA only structures are very low. However, this is heavily dependant on the resolution of the Harmonic Response and Fatigue Life Estimation analysis. For the lower resolution levels difference between fatigue life estimation for full body and the TVA can be higher. Furthermore, tests of the cantilever beam type TVA is carried out. During the tests of the cantilever beam type TVA, first natural frequency of the TVA is observed different from what it is acquired during analysis. This is due to poor connections of the designed TVA. Moreover, material properties of the connecting elements are changed in the analysis. With altered parts analysis are repeated. According to analysis results, fatigue life is expected to be very long with revised connection parts. Since, expected fatigue life is too long with current design at this stage, it is decided to redesign the cantilever beam type TVA by changing masses that are present at the tips of cantilever beams. By using different types of bolt and nuts as masses on the TVA, it is again tuned to the target frequency which is natural frequency of the base structure. Fatigue life test are started again with redesigned TVA to see effectiveness of the response estimation method. Since, design of the TVA is changed, PSD force input for the test is also decided to reduced to get meaningful results from fatigue life analysis with redesigned TVA. Operational response measurement and transmissibility measurement are completed with redesigned TVA. According to the transmissibility measurement, it is observed that TVA is again mistuned. However, this time tuned frequency is closer to the target frequency according to the previous case. So that, it is decided to continue with fatigue life test with slightly mistuned TVA. Firstly, fatigue life test is started with full body setup. According to the analysis result and literature survey, it was expected to have fatigue failure on the TVA around half an hour. However, after two hours of fatigue life test for full body setup, no failure was observed on the TVA. At this point, it is decided to stop with full body analysis and continue with only TVA fatigue life test. Operational response with new PSD force input and TVA FRF which is generated from transmissibility measurement of the TVA is used to calculate estimated response as acceleration at the connection of the TVA to the base structure. Comparison of the estimated response with measured acceleration data from full body test is completed and good estimation results are observed. Furthermore, estimated response is used as a base excitation for a test setup where the TVA is directly attached to the shaker. However, same situation with the full body fatigue life test is observed also for only TVA fatigue life test. Since, no damage is observed on the TVA during only TVA fatigue life test, it is decided to stop after two hours. Continuation of the tests are considered as a future work for to get a comparison between analysis and test results.

Fatigue life estimation analysis with response estimation method has been also completed for mass and spring type TVA. Results of the analysis that is run for mass and spring type TVA shows different behaviours than cantilever beam type TVA. Although, response estimation method shows promising results for cantilever beam type TVA, for mass and spring type TVA results of full body and only TVA fatigue life estimation analysis are very different. Since the difference between full body analysis and only TVA analysis is very wide for mass and spring type TVA, fatigue life tests for the mass and spring type TVA is decided to not carried out. Further investigating on the reasons why response estimation method gives inaccurate results for mass and spring type TVA can be a future study topic for this study.

Thirdly a case study with two PSD force input is also executed. Response estimation method originally was corrected with a single input by Özden[1] during his Master's

Thesis study. In order to observe its effectiveness on estimation response of the TVA, a case study is conducted with same base structure and the cantilever beam type TVA. On the same base structure two different PSD force input is executed and random vibration analysis is executed. Operational response at the tip point of the base structure which is gathered from random vibration analysis is used with FRF of base structure and TVA to get a response estimation of a point on the TVA where it is attached to the base structure. From analytic equations and analysis example of two force PSD input, it can be concluded that response estimation method's effectiveness is also valid for multiple input cases. According to the analytical equations it can also be seen that, increased number of input will also be valid to use response estimation method.

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