ANALYSIS OF GEOMETRICAL OBJECTS TASKS IN MIDDLE SCHOOL MATHEMATICS TEXTBOOKS WITH PISA 2018 MATHEMATICS LITERACY FRAMEWORK

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

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The purpose of this study was to show to what extent middle school mathematics textbooks in Turkey include potential requirements of mathematics literacy for geometrical objects subject tasks in the PISA 2018 Mathematics Literacy Framework. Mathematics Literacy Framework includes four aspects: process, capabilities, contextualization, and proficiency levels. In total 220 tasks were coded. Regarding the process aspect, 88.1% of the tasks was about employing mathematical concepts, facts, and procedures, 7.8% was about formulating situations mathematically, 4.1% was about interpreting mathematical outcomes. Regarding the capability aspect, 44.4% of the tasks was about ‘‘Using Symbolic, Formal and Technical Language and Operations’’, 36% was about ‘‘Representation’’, 11% was about ‘‘Using Mathematical Tools’’ and 7% was about ‘‘Reasoning and Argument’’. In relation to contextualization, 85% of the tasks were decontextualized. In relation to six proficiency levels, 68% of the task’s proficiency levels was Level 1 and Level 2 implying to carry out routine procedures.

Keywords: Geometrical Objects, Textbook Analysis, Mathematics Literacy, Mathematical Processes, Context, Proficiency Levels, Geometrical Objects, Programme for International Student Assessment (PISA)

ÖZ

ORTAOKUL MATEMATİK DERS KİTAPLARINDA GEOMETRİK CİSİMLER GÖREVLERİNİN PISA 2018 MATEMATİK OKURYAZARLIĞI ÇERÇEVESİNDE ANALİZİ

Topuz, Zehra Betül
Yüksek Lisans, Matematik Eğitimi, Matematik ve Fen Bilimleri Eğitimi
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Bu çalışmanın amacı, ortaokul Türkiye'de kullanılan ortaokul Matematik ders kitaplarının, PISA 2018 Matematik Okuryazarlığı Çerçevesinde geometrik nesneler konusu görevleri için potansiyel matematik okuryazarlığı gereksinimlerine ne ölçüde yer verdiğini göstermektir. Matematik Okuryazarlığı Çerçevesi dört yönü içerir: süreç, yetenekler, bağlam sallaştırma ve yeterlilik seviyeleri. Toplamda 220 görev kodlanmıştır. Süreç boyutuyla ilgili olarak, görevlerin %88,1'i matematiksel kavramları, gerçekleri ve prosedürleri kullanmakla, %7,8'i durumları matematiksel olarak formüle etmekle, %4,1'i matematiksel sonuçları yorumlamakla ilgiliydi. Yetenek boyutu ile ilgili olarak, görevlerin %44,4'ü "Sembol, Biçimsel ve Teknik Dil ve İşlemleri Kullanma" ile ilgili, %36'sı "Temsil" ile, %11'i "Matematiksel Araçları Kullanma" ile ve %7'si ile ilgilidir. "Akıl yürütme ve Argüman" ile ilgiliydi. Bağlamsallaştırma ile ilgili olarak, görevlerin %85'i bağlamından arındırılmıştır. Altı yeterlilik düzeyiyle ilgili olarak, toplam görevlerin yeterlilik düzeylerinin %68'i, rutin prosedürlerin uygulandığı Düzey 1 ve Düzey 2'dir.

Anahtar Kelimeler: Ders Kitabı Analizi, Matematik Okuryazarlığı, Matematiksel Süreçler, Bağlam, Yeterlilik Düzeyleri, Geometrik Nesneler, Uluslararası Öğrenci Değerlendirme Programı (PISA)

To my beloved family…

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS

TIMMS: Trends in International Mathematics and Science Study

PIRLS: Progress in International Reading Literacy Study

PISA: Programme for International Student Assessment

TALIS: Teaching and Learning International Survey

PIAAC:Programme for the International Assessment of Adult Competencies

MoNE: Ministry of National Education

CR: Change and Relations

SS: Space and Shape

UD: Uncertainty and Data

Q: Quantity

F: Frequency

CHAPTER 1

INTRODUCTION

The Organization for Economic Co-operation and Development (OECD) makes policies to help governments with the aim of improving human life in various domains (Gurría, 2010). To achieve this, the OECD devises policy suggestions for social, economic, and political issues. For example, it prioritizes to achieve an upgraded education quality for each country because the level of human capital is accepted as a critical factor affecting economic development in the long term. Therefore, it has organized international exams to compare the cross-country educational outcomes. The Program for International Student Assessment (PISA), which is one of these exams, is a specific assessment and evaluation tool for different subjects and age groups (Engel, Rutkowski, & Thompson, 2019). Furthermore, while PISA examines 14 -15 years old children, OECD develops tests such as International Assessment of Adult Competencies (PIAAC) and Teaching and Learning International Survey (TALIS) for other age groups to expand its perspectives about education (Gurría, 2010). Some other international institutions also organize trends in International Mathematics and Science Study (TIMMS) and Progress in International Reading Literacy Study (PIRLS). These exams gain significance because they enlighten policymakers, educators, and academics when constructing educational program standards for students (Singer, Braun, & Chudowsky, 2018).

PISA assesses student achievement on different subjects and provides educational surveys in terms of several topics such as students' social status, socioeconomic condition, and daily lesson time. In addition, it gives information about other variables which may affect the education quality (Stacey & Turner, 2015a).

One of the features making PISA unique is literacy in different areas, which are mathematics, science, and financial literacy (OECD, 2019a).

Literacy is defined “capacity to apply knowledge and skills, and to analyze, reason and communicate effectively as they identify, interpret and solve problems in a variety of situations.” Specifically, "mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive. " (OECD, 2019a).

Mathematical literacy is becoming essential in changing world conditions. It helps one to comprehend the meaning of mathematics in everyday life and make logical decisions (Ojose, 2011). Therefore, mathematics literacy is one of the key concepts of PISA because mathematics learning goals overlap mathematics literacy. (Kamaliyah, Zulkardi, & Darmawijoyo, 2013). PISA indicators develop the mathematics literacy framework to examine students' level in various content and give more reliable results for countries to evaluate themselves (Stacey & Turner, 2015a). Given that education is a bridge to social and economic improvement, the countries pay attention to ranking exams conducted by international institutions to obtain various outcomes. These exams enable countries to compare and measure the education system's success and, if necessary, review the systems (Yalçın & Tavşancıl, 2014). In addition, the exams provide hints about the qualifications and potential of new generations who may change the country's destination (Hanushek & Woessmann,2011). Therefore, developed and developing countries give more importance to these exams to develop future strategies for education systems, affecting the level of human capital for any country (Hanushek & Woessmann, 2012).

As a developing country, in 2003, Turkey participated in PISA to measure the success of the education system and design a new set of policies (Yalçın & Tavşancıl, 2014). Turkey also considers these exam results to upgrade the quality of human capital and achieve significance economic developments (Worldbank, 2005). The results from PISA present some feedback on whether Turkey's educational system meets PISA standards or not. Since textbooks are essential concepts of academic procedures, their analysis with the PISA literacy framework gives us idea about these relationships (Gomez-Zwiep, 2017). This kind of an analysis lets educators and policymakers make better reforms than the ones that would be possible in the education system itself.

As mentioned previously, the PISA exam consists of reading literacy, science literacy, and math literacy. Math literacy, one of the most critical skills, measures students' ability to solve real-world problems using math knowledge. As gaining this skill is vital, most countries, including Turkey, care about the mathematics results to enhance their mathematics education. Unfortunately, the ratio of the students with a low proficiency level in mathematics is significantly higher than OECD, even though Turkey's 2018 PISA math score is higher than the ones in previous years (PISA 2018). In conclusion, since textbooks shape the quality of education and students' learning qualities (Murdaningsih & Murtiyasa, 2016), and mathematical abilities are also constructed with their content (Jäder, Lithner, & Sidenvall, 2019), textbook analysis of the international framework is essential to explore why students have a lower mathematics proficiency level.

PISA has four main content areas of mathematics: change and relationships (algebra), space and shape (geometry), quantity (number and operations), and uncertainty (data analysis) (OECD, 2019a). Because space and shape are related to spatial ability and spatial reasoning, they contribute to abstract thinking and making inferences from three and two-dimensional objects. (Novitasari, Nasrullah, Triutami, Apsari, & Silviana, 2021). Thus, the quality of the tasks related to this content area becomes significant in improving students' mathematics literacy, and it has an essential role in the mathematical thinking process (Jäder, Lithner, & Sidenvall, 2019).

1.1 Problem Statement and Rationale for the Study

An analysis of what extent Turkish middle school mathematics textbooks include the requirement of PISA's Mathematics Framework would be essential for teachers, textbook writers, and policymakers to reorganize the educational process for students mathematically. Therefore, the following research question guided this study.

Research Question: To what extent Turkish middle mathematics textbook include potential requirements of the mathematics literacy framework of PISA 2018 for geometrical objects subject?

1.2 Definition of Terms

Textbook: Textbooks are curriculum instrument give direction to the education not only students but also teachers. (H.Williams, 2014). In Turkey, school textbooks are prepared by Board of Education and approved by Turkish Ministry of National Education.

Worked-out example: Solutions and answers are presented with the questions (Delil, 2006). The tasks in the textbook under the heading of ‘example’ and ‘let’s do together’ were categorized as the worked-out examples.

Example: This kind of task gives students a model which is implemented in others (Gracin, 2018)

Let’s do together: This kind of task is presented with solution steps in textbooks

To-be-solved question: Students practices with this kind of questions and these questions do not present solutions (Delil, 2006). In this study, the tasks under the headings of ‘the evaluation of unit’, ‘exercise’, ‘it is your turn’, ‘activity’, ‘let’s think’, ‘leading question’, and ‘let’s apply’ were categorized the to-be-solved questions.

Leading Question: This kind of task enables student to think critically and reach solution through their reasoning (Wong, 2015)

The Evaluation of Unit: Task assessing all mathematical concepts in a unit.

Exercise: Task is used to reinforce mathematical learnings (Gracin, 2018)

Activity: Task encourage students in class exploration and application of the concept with hands (Bayazit, 2013).

Let’s Apply: Task is mostly used to reinforce mathematical formulas.

CHAPTER 2

LITERATURE REVIEW

2.1 PISA

PISA (Program for International Student Assessment) triennially measures 15-year-olds' ability to use their reading, mathematics and science knowledge and skills to meet real-life challenges since 2000, and it is organized by OECD (Schleicher, 2019). PISA includes different area surveys asking students socio- economic and socio-cultural levels, so it helps to understand how to distinguish educational outcomes for students from differing backgrounds (Schulz, 2005).

Since PISA is a large-scale examination for students, it gives an opportunity for educators and teachers to revise the educational system. For every three years after PISA scores are announced, they can evaluate effectiveness of their revisions. PISA has increasingly drawn attention of researchers because attendance of most countries provides an opportunity to compare different educational systems (Alacacı & Erbaş, 2010). Because of this, number of participant countries has increased each examination time, number of countries which attended PISA is given in Table 2.1.

Table 2.1. Number of countries attained PISA (Lockheed, 2015)

Examination date of PISA	Number of participants
2000/2001	43
2003	40
2006	57
2009/2010	65
2012	62
2015	72
2018	79

In 2018, 79 countries participated in PISA, and approximately 600.000 students completed the assessment 2018, representing about 32 million 15-year-olds in schools (OECD, 2019b). Thus, increasing the number of countries enables researchers to reach more generalizable results, critical for the educational system. In addition, PISA is a powerful assessment tool in a variety of ways given below:

> *Policy orientation*, which links data on student learning outcomes with data on students' backgrounds and attitudes towards learning, and with key factors that shape their learning, in and outside of school; by doing so, PISA can highlight differences in performance and identify the characteristics of students, schools and education systems that perform well,
>
> *Innovative concept of literacy*, which refers to students' capacity to apply their knowledge and skills in key areas, and to analyze, reason and communicate effectively as they identify, interpret, and solve problems in a variety of situations,
>
> *Relevance to lifelong learning*, as PISA asks students to report on their motivation to learn, their beliefs about themselves, and their learning strategies,
>
> *Regularity*, which enables countries to monitor their progress in meeting key learning objectives,
>
> *Breadth of coverage*, which, in PISA 2018, encompassed all 37 OECD countries and 42 partner countries and economies (OECD, 2019b, p.26).

As Stacey and Turner (2015b) mentioned, PISA results are widely used in different parts of countries to understand the direction of education.

2.2 The Concept of Math Literacy

Math literacy can be defined as mathematical proficiency for content and situations that can occur daily and in job-education lives (Özgen & Bindak, 2011). The people who are sufficiently math literate have high life quality in an increasingly growing technological society because all occupational areas relate to mathematical efficiency (Schoenfeld, 2002). As mathematical literacy is the ability to read and use the meaning of the mathematical concept, becoming significant for whole life, education policies are designed to achieve this goal (Marciniak, 2015). Also, mathematical literacy provides individuals with awareness of their role in changing the world, making judgments with reasoned arguments, and helping them make logical decisions. International exams give priority to these skills (Altun, Gümüş, Akkaya, Bozkurt, & Ülger, 2018). PISA definition has summarized the necessities of mathematical literacy as given below:

> Mathematical literacy is an individual's capacity to formulate, employ and interpret mathematics in various contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged, and reflective citizens (OECD, 2019a, p. 75).

2.3 Importance of Textbook

Textbooks are guidance instruments because they are essential for educational system in a variety of ways (Shield & Dole, 2013). Most countries, including Turkey, textbooks are used by teachers prepare for their lessons and lessons' activity and students benefits those books in individual studies in anywhere. (Massey & Riley, 2013). Thus, they give a direction for educational process not only for teachers but also learners. Rezat (2006) explained this condition and summarized the process with the model given below:

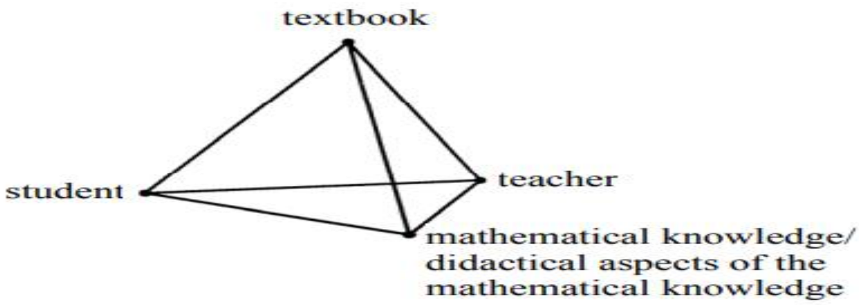


Figure 2.1.Fundamental model of didactical system relationships for student, teacher, and mathematics

> This model shows fundamental model of didactical system relationships for student, teacher, and mathematics. The mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons, by using the textbook in the lesson teachers

also mediate textbook use to students, and finally students learn from textbooks (Rezat, 2006, p.413).

Because of the given reasons textbooks have positive impact on effective learning (Wijaya, Heuvel-Panhuizen, & Doorman, 2015).Textbooks are linked to national curriculum and its effect on teaching. There are four levels developed by (A. Val verde, 2002) overlapping each other which is given:

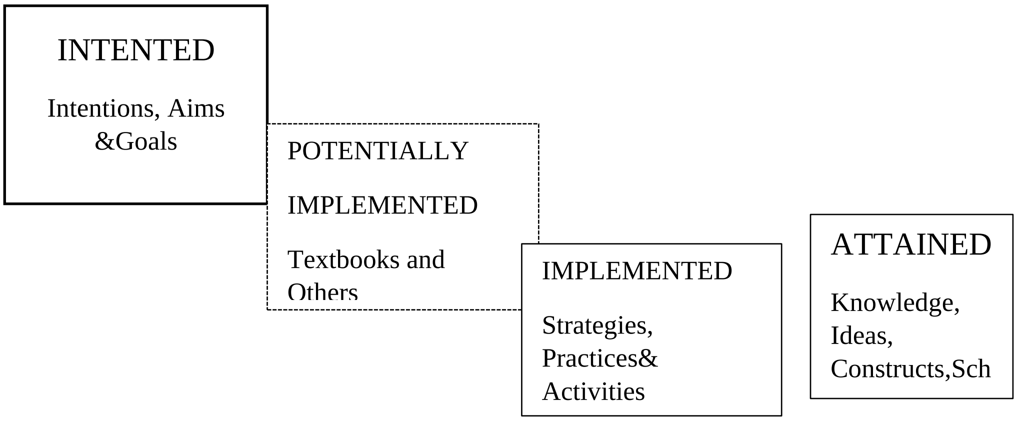


Figure 2.2. Textbooks and the tripartite model (A. Val verde, 2002)

The importance of textbooks is indicated in different directions in this modal. Also, analyzing the content of textbooks gives valuable information in exploring student achievement in mathematics (Törnroos, 2005).

Textbooks are also sources that provide benefits for students in terms of self-learning, so students are more successful in having higher grades with the additional textbooks (Piper, Zuilkowsk, Dubeck, Jepkemei, & King, 2018). Because of these reasons, the ministries of national education of countries enriched textbook content to raise more successful children for their future. Mathematic literacies are also affected positively by textbooks' content; if textbooks include a combination of real-life and mathematical content, the math literacy performance of students may increase (Draper, 2002).

2.4 PISA 2018 Math Literacy Framework

2.4.1 The Process Domain

Construction of knowledge is not gaining knowledge; it is a process of cognizing the subject (Steffe & Wiegel, 2009). There are elements of the mathematical cognitive process which are basic conceptual operations, action, and operation for transformation involved accommodation, and the role of social interaction in inducing these accommodations**.** According to Rico (2006), the mathematics learning model includes contextualized tasks, conceptual tools, and cognitive subjects**.** The PISA mathematics literacy framework contains these cognitive processes, but it may not fit the national curriculum. Although it is constructed by general framework of mathematical skills, it can be adapted and used by assessment of national lessons materials (Sáenz, 2009). This offers counties an opportunity to compare their developmental process in education.

Math literacy is a process of understanding and explaining mathematical concepts meaningfully and using them to make reasoning and interpretation in any part of a human's life (Altun, Gümüş, Akkaya, Bozkurt, & Ülger, 2018). Thus, Mathematics learning has a different process. According to the 2018 Math Literacy Framework, formulate, employ, and interpret are three elements of the process domain. These elements are defined below:

> The *formulating* process indicates how effectively students are able to recognize and identify opportunities to use mathematics in problem situations and then provide the necessary mathematical structure needed to formulate that contextualised problem into a mathematical form. The *employing* process indicates how well students are able to perform computations and manipulations and apply the concepts and facts that they know to arrive at a mathematical solution to a problem formulated mathematically. The *interpreting* process indicates how effectively students are able to reflect upon mathematical solutions or conclusions, interpret them in the context of a real-world problem, and determine whether the results or conclusions are reasonable (OECD, 2019a, p. 77).

2.4.2 The Context Domain

Context is essential for mathematics education as it motivates students and helps them with mathematical concepts by providing application and integration of the real world (Chapman, 2006). Furthermore, the mathematical process includes a description of context problem in mathematical terms, and their own mathematical activities help students reach a higher level of mathematics (Gravemeijer & Doorman, 1999).

According to the 2018 Math Literacy Framework, there are four elements of the context domain: Personel, Occupational, Societal and Scientific.
Definition of these four elements are given below:

> Problems classified in the personal context category focus on activities of one's self, one's family or one's peer group… Problems classified in the occupational context category are centred on the world of work… Problems classified in the societal context category focus on one's community (whether local, national or global)… Problems classified in the scientific category relate to the application of mathematics to the natural world and issues and topics related to science and technology (OECD, 2019a, p. 88).

Using these contexts categories in PISA assessment provides to us with an opportunity to determine if students have worldwide perspectives for mathematics or not since this cover every part of real life (OECD, 2019a). Each context category process offers a different view of mathematical knowledge for educators and policymakers.

Process domain of three elements is related to context category and their requirement activities is given below before detailed framework:

Table 2.2. The Elements of the Mathematical Process and Activities related to Subsets (OECD,2019a)

The Elements of the Mathematical Process	Activities Related to Subsets
Formulating situations mathematically	• identifying the mathematical aspects of a problem situated in a real-world context and identifying the significant variables • simplifying a situation or problem • identifying constraints and assumptions behind any mathematical modeling • recognizing mathematical structure in problems or situations • representing a situation mathematically, using appropriate variables, symbols, diagrams, and standard models • representing a problem in a different way • understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language • understanding and explaining the relationships between the context-specific language of a problem and the symbolic and formal language • translating a problem into mathematical language or a representation • recognizing aspects of a problem that correspond with known problems or mathematical concepts, facts, or procedures • using technology (such as a spreadsheet or the list facility on a graphing calculator) to portray a mathematical relationship inherent in a contextualized problem.
Employing mathematical concepts, facts, procedures, and reasoning	• devising and implementing strategies for finding mathematical solutions • using mathematical tools1, including technology, to help find exact or approximate solutions • applying mathematical facts, rules, algorithms, and structures when finding solutions • manipulating numbers, graphical and statistical data, and information, algebraic expressions and equations, and geometric representations • making mathematical diagrams, graphs, and constructions, and extracting mathematical information from them • using and switching between different representations in the process of finding solutions • generalizing based on the results of applying mathematical procedures to find solutions • reflecting on mathematical arguments and explaining and justifying mathematical results.
Interpreting, applying, and evaluating mathematical outcomes	• interpreting a mathematical result back into the real-world context • evaluating the reasonableness of a mathematical solution in the context of a real-world problem • understanding how the real world impacts the outcomes and calculations of a mathematical procedure or model to make contextual judgements about how the results should be adjusted or applied explaining why a mathematical result or conclusion does, or does not, make sense given the context of a problem • understanding the extent and limits of mathematical concepts and mathematical solutions

2.4.3 The Capability Domain

Mathematical capability is related different student parameters for students such as self-efficacy, self-perception, and assessments applied to measure mathematical learnings (Pajares & Miller, 1997). Mathematical capability is shaped by knowledge, identity, and practice (Boaler, 2001).

PISA mathematics literacy framework combines fundamental capabilities in seven categories:

Table 2.3. Definition of Capabilities (OECD,2019a)

Capabilities:	Definition of Capabilities:
Communication	Mathematical literacy involves communication. The individual perceives the existence of some challenge and is stimulated to recognize and understand a problem situation. Reading, decoding, and interpreting statements, questions, tasks, or objects enables the individual to form a mental model of the situation, which is an important step in understanding, clarifying, and formulating a problem. During the solution process, intermediate results may need to be summarized and presented. Later, once a solution has been found, the problem solver may need to present the solution, and perhaps an explanation or justification, to others.
Mathematising	Mathematical literacy can involve transforming a problem defined in the real world to a strictly mathematical form (which can include structuring, conceptualizing, making assumptions, and/or formulating a model), or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem. The term mathematising is used to describe the fundamental mathematical activities involved.
Representation	Mathematical literacy frequently involves representations of mathematical objects and situations. This can entail selecting, interpreting, translating between, and using a variety of representations to capture a situation, interact with a problem, or to present one's work. The representations referred to include graphs, tables, diagrams, pictures, equations, formulae, and concrete materials.
Devising strategies for solving problems	Mathematical literacy requires using symbolic, formal, and technical language and operations. This involves understanding, interpreting, manipulating, and making use of symbolic expressions within a mathematical context (including arithmetic expressions and operations) governed by mathematical conventions and rules. It also involves understanding and utilizing formal constructs based on definitions, rules and formal systems and using algorithms with these entities. The symbols, rules, and systems used vary according to what particular mathematical content knowledge is needed for a specific task to formulate, solve or interpret the mathematics.

Table 2.3., Continued

Using symbolic, formal and technical language and operations:	Mathematical literacy requires using symbolic, formal, and technical language and operations. This involves understanding, interpreting, manipulating, and making use of symbolic expressions within a mathematical context (including arithmetic expressions and operations) governed by mathematical conventions and rules. It also involves understanding and utilizing formal constructs based on definitions, rules and formal systems and using algorithms with these entities. The symbols, rules, and systems used vary according to what mathematical content knowledge is needed for a specific task to formulate, solve, or interpret the mathematics.
Using mathematical tools	Mathematical tools include physical tools, such as measuring instruments, as well as calculators and computer-based tools that are becoming more widely available. In addition to knowing how to use these tools to assist them in completing mathematical tasks, students need to know about the limitations of such tools. Mathematical tools can also have an important role in communicating results

These capabilities are evident to varying degrees in each of the three mathematical processes. The ways in which these capabilities manifest themselves within the three processes are described in Table 2.4.

Table 2.4.Relationship between capabilities and process domain OECD (2019a)

	Formulating situations mathematically	**Employing mathematical concepts, facts, procedures and reasoning**	**Interpreting, applying and evaluating mathematical outcomes**
Communication	Read, decode, and make sense of statements, questions, tasks, objects or images, in order to form a mental model of the situation	Articulate a solution, show the work involved in reaching a solution and/or summarize and present intermediate mathematical results	Construct and communicate explanations and arguments in the context of the problem
Mathematising	Identify the underlying mathematical variables and structures in the real world problem, and make assumptions so that they can be used	Use an understanding of the context to guide or expedite the mathematical solving process, e.g., working to a context appropriate level of accuracy	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed

Table 2.4, Continued

Representation	Create a mathematical representation of real-world information	Make sense of, relate and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation or use; compare or evaluate two or more representations in relation to a situation
Reasoning and argument	Explain, defend, or provide a justification for the identified or devised representation of a real-world situation	Explain, defend, or provide a justification for the processes and procedures used to determine a mathematical result or solution Connect pieces of information to arrive at a mathematical solution, generalize or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution to a contextualized problem
Devising strategies for solving problems	Select or devise a plan or strategy to mathematically reframe contextualized problems	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion, or generalization	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a contextualized problem
Using symbolic, formal, and technical language and operations	Use appropriate variables, symbols, diagrams, and standard models to represent a real-world problem using symbolic/formal language	Understand and utilize formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution. Use this understanding to help interpret the solution in context and gauge the feasibility and possible limitations of the solution
Using mathematical tools	Use mathematical tools in order to recognize mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools that may assist in implementing processes and procedures for determining mathematical solutions	Use mathematical tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

2.4.4 The Proficiency Levels

Mathematic learning requires different cognitive steps, so assessments are organized to improve mathematical learning process by researchers, teachers (Schoenfeld, 2007). Considering these requirements has a significant effect on gaining mathematical thinking skills and this process can be named mathematical proficiency including different sitution for every student whose background is different than each other (Milgram, 2007). Assesing student mathematical proficiency levels provides an understanding of how students develop mathematical thinking skills and how teachers and researchers help them to improve themselves in this direction (Schoenfeld & Kilpatrick,2008).

The proficiency levels and their descriptions were explained in the Table 2.6. as follows:

Table 2.5. The proficiency levels and Their descriptions, OECD (2019a)

Levels	What a student can do at this level?
Level 6	At Level 6, students can conceptualize, generalize, and utilize information based on their investigations and modelling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.
Level 5	At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations, and insight pertaining to these situations. They begin to reflect on their work and can Formulate and communicate their interpretations and reasoning.
Level 4	At Level 4, students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic, linking them directly to aspects of real-world situations. Students at this level can utilize their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.
Level 3	At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions, and decimal numbers, and to work with proportional relationships. Their solutions Reflect that they have engaged in basic interpretation and reasoning.

Table 2.5., Continued

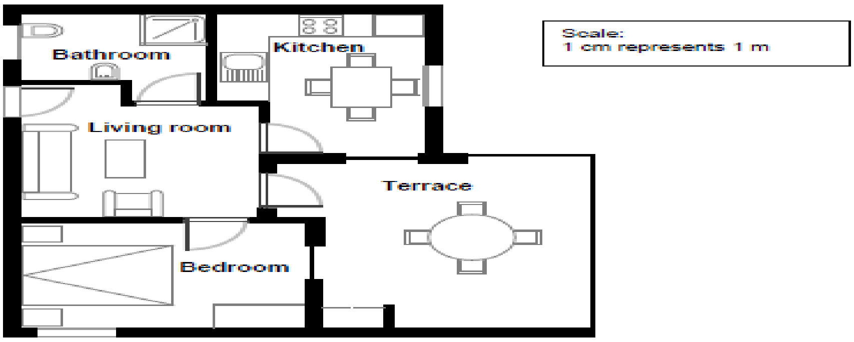
Level 2	At Level 2, students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers. They can make literal interpretations of the results.
Level 1	At Level 1, students can answer questions involving familiar contexts where all relevant information is present, and the questions are clearly defined. They can identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

2.5 Sample Questions Coded Concerning the PISA

PISA released some questions after each survey and some of questios are coded by trained item coders (OECD,2013). This provides hints about how to code these questions:

APARTMENT PURCHASE

This is the plan of the apartment that George's parents want to purchase from a real estate agency.



Translation Note: In this unit please retain metric units throughout.

Translation Note: Translate the term "real estate agency" into local terminology for businesses that sell houses.

Question 1: APARTMENT PURCHASE PM00FQ01 – 0 1 9

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together.

However, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the four lengths that are needed to estimate the total floor area of the apartment.

Figure 2.3. Apartment Purchase PISA question (OECD, 2013)

According to the PISA 2012 released items, this questions' from Figure 2.3 mathematical content area is ***Space and Shape*** and Context area is ***Personal*** which is related to people living area (OECD, 2013). This question process domain is ***Formulate***; "identifying of the mathematical aspects of a problem situated in a real-world context and representing a problem in a different way, including organizing it according to mathematical concepts and making appropriate assumptions" (OECD, 2019a, p. 78). This question proficiency level is ***Level 5***; "students can develop and work with models for complex situations, identifying constraints and specifying assumptions" (OECD, 2019a, p. 92).

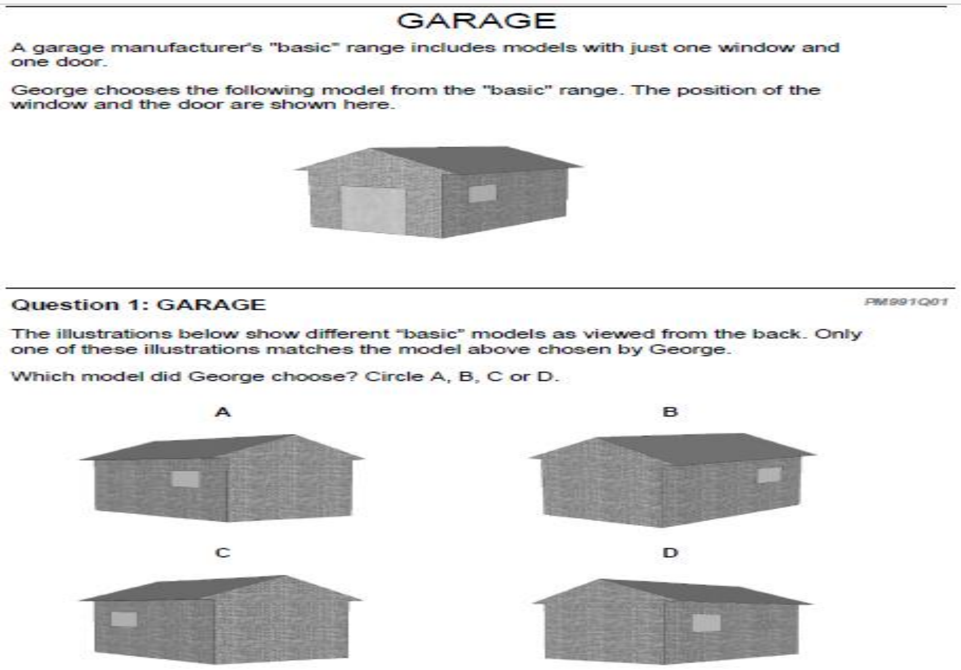


Figure 2.4. Garage question 1 from PISA 2012 (OECD, 2013)

According to the PISA 2012 released items, this questions' from Figure 2.4 mathematical content area is ***Space and Shape*** and Context area is ***Personal*** which is related to people living area (OECD, 2013). This questions process domain is ***Interpret; "***interpreting a mathematical result back into the real-world context and evaluating the reasonableness of a mathematical solution in the context of a real-world problem" (OECD, 2019a, p. 79). This question proficiency level is ***Level 5***; "students can develop and work with models for complex situations, identifying constraints and specifying assumptions" (OECD, 2019a, p. 92).

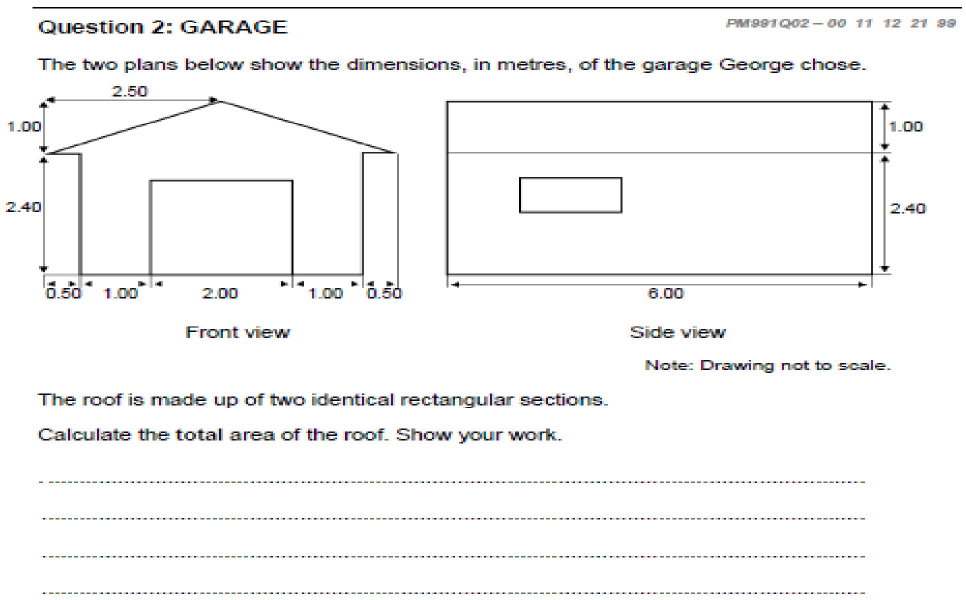


Figure 2.5. Garage question 2 from PISA 2012 (OECD, 2013)

According to the PISA 2012 released items, Garage question 1 from Figure 2.5 whose mathematical content area is ***Space and Shape*** and Context area is ***Occupational*** which is related to measuring (OECD, 2013). This questions process domain is ***Employ;*** "devising and implementing strategies for finding mathematical solutions" (OECD, 2019a, p. 79). This question proficiency level is ***Level 2***; "students can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers" (OECD, 2019a, p. 92).

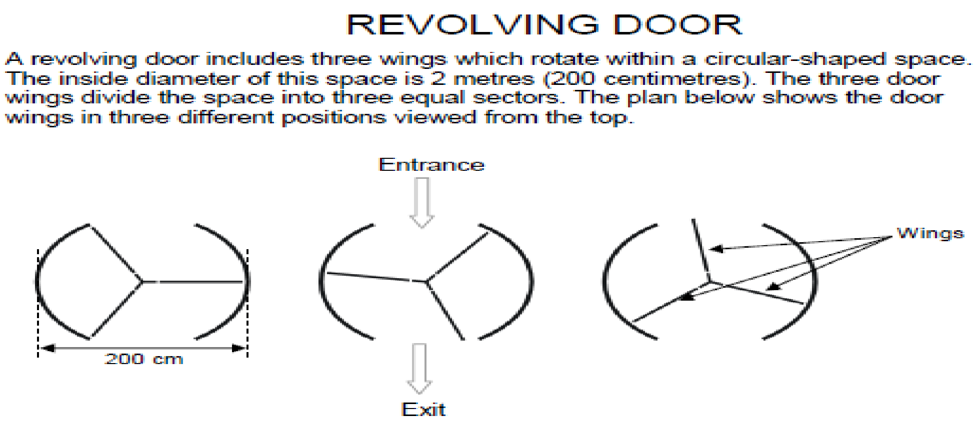


Figure 2.6. Revolving door question from PISA 2012 (OECD, 2013)

According to the PISA 2012 released items, Revolving door question from Figure 2.6 mathematical content area is ***Space and Shape*** and Context area is ***Scientific*** which is related to application of mathematics to the natural world. This questions process domain is ***Employ;*** **"**devising and implementing strategies for finding mathematical solutions" (OECD, 2019a, p. 79). This question proficiency level is ***Level 2***; "students can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers" (OECD, 2019a, p. 92).

2.6 Students Difficulties when learning Geometry

Geometry is one of the essential learning areas of mathematics and it includes spatial relationships and properties of shapes in K-12 educational process (MoNE, 2018c). Geometry has a significant role for all students in any age to improve their skill of reasoning in different area like science, technology, engineering, and mathematics (Chen, Li, & Zhang, 2021). Since spatial abilities and geometry subject have a strong relationship, mathematical literacy skill is also affected (Chen, Li, & Zhang, 2021).

According to Retnawati, Arlinwibowo, & Sulistyaningsih (2017), geometry learning needs a process for students, which includes understanding relationships of concepts and the communication of unknown amounts and values through signs, symbols, models, graphics, and mathematical terms. In addition to this, visualization skills, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical arguments, and proofing skills are supported during the geometry learning process because stimulating a wide range of skills mastery of geometry is becoming difficult for learners. Although complex skills are involved in this process, geometry can be misunderstood, so teachers should be aware of this, and their education is also regulated by considering this (Kuzniak & Rauscher, 2011).

In addition to these difficulties, especially geometry-related spatial abilities, three-dimensional thinking gains importance to become successful in geometry (Gal & Linchevski, 2010). As a result, students have higher scores in geometry. Still, they can face complexity in a three-dimensional object because the imagination of these objects is more complex compared to three-dimensional objects (Kusumah, Kustiawati, & Herman, 2020). Also, there is a lack of examination evaluating students' three-dimensional geometry concepts because these teaching processes and instruments in this content should be revised and examined to understand what causes the failure of the students (Hwang & Hu, 2013).

International examinations like PISA (Programme for International Student Assessment) and TIMSS (Trend in International Mathematics and Science Study) give place geometry subject because it is an integral part of mathematical ability (Retnawati, Arlinwibowo, & Sulistyaningsih,2017). PISA presents geometry questions in space and shape, which is one of four areas. TIMSS raises geometry questions, forming 30 percent of fourth grade in measurement and geometry area, which is one of the areas of three, and eighth grade in measurement and geometry area, which is one of the areas of four (OECD, 2019a; MoNE, 2020).

Definition of space and shape area in PISA is:

> Tasks related to space and shape entail understanding perspective, creating, and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives, and constructing representations of shapes. Space and shape are the "big idea" most closely related to geometry (OECD, 2016).

Definition of geometry area in TIMSS is:

> The geometric shapes and measures domain includes properties of geometrical figures such as lengths of sides, sizes of angles, areas, and volumes. Students should be able to identify and analyze the properties and characteristics of lines, angles, and a variety of geometric figures, including two- and three-dimensional shapes, and to provide explanations based on geometric relationships. This domain includes understanding informal coordinate systems and using spatial visualization skills to relate between two- and three-dimensional representations of the same shape (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009).

Although geometry is significant learning area in international examinations for the PISA mathematics space shape are mean score of Turkey very law compared to OECD average:

Table 2.6 PISA Scores of Turkeys (OECD 2016; OECD 2018)

Years	Shape and Shape Score	OECD Average
2012	447	481
2015	443	490
2018	439	495

2.7 Summary of Literature Review

International examination gains importance every passing day because their results are used to regulate educational process by researchers, educators, and policymakers. (Kamaliyah, Zulkardi, & Darmawijoyo, 2013). Because textbooks are the first source for national curriculum, analyzing them with international and national examination requirements and seeing how they present the subject may help to compare our country with other countries to improve our educational system. (H.Williams, 2014; Murdaningsih & Murtiyasa, 2016). Thus, reasons of students' failure in mathematics may be discovered and then educational system can be designed with considering these reasons (Törnroos, 2005). Although there are the other international assesment frameworks, PISA is more preferable (OECD, 2019a).

When reviewing the literature surrounding the topic of mathematics textbook analysis, there is a gap in the research focusing on one subject using the international framework in detail.

CHAPTER 3

METHODOLOGY

3.1 Research Design

The aim of this study was to show what extent Turkish Mathematics Textbooks includes requirements of mathematics literacy framework of PISA 2018 for geometrical objects subject. Content analysis was used as the method in this qualitative study. Content analysis can be defined as "a system program set for rigorous analysis, inspection and verification of the content of written data". (Fraenkel, Wallen, & Hyun, 2012)

To understand extension of Turkish Mathematics Textbooks requirements of mathematics literacy framework of PISA 2018 for geometrical objects subject, Turkish Mathematics Textbooks from fifth grade to eighth grade was analyzed with mathematics literacy framework of PISA 2018.

The following order of this chapter is given below:

- Selection of the subject
- The matching process between the units of textbook and content areas of framework
- Selection of the textbooks
- Coding scheme used in this study
- Coding process and Some examples of coding from the study
- Reliability and validity issues of the research

3.2 Selection of The Subject:

PISA has four examinations of content areas which are change and relationships (CR), space and shape (SS), quantity (Q), Uncertainty, and data (UD). These content areas are not far from standard school curriculum because space and shape are connected to Geometry, quantity are connected to Number and Measurement, Uncertainty and Data' are connected to Probability and Statistics and Change and Relationships are connected to Algebra and Functions (OECD, 2019a). Space and shape (SS) content area were first chosen as a content area matching various objects given Table 3.1. The geometrical object is the selected subject for this research because this subject area, which is an essential part of PISA, is not easy to learn and become successful easily for students. (Yang, Tseng, & Wang, 2017). There have been rare studies on this subject (Yang, Tseng, & Wang, 2017). Matching Space and Shape Area with subjects in Turkish Curriculum. "Numbers and Operations", "Algebra'', "Geometry and Measurement", and "Data Analysis'', "Probability" are five main content areas of the Turkish curriculum. These three areas are placed from 5th grade to 8th grade with different subject and this construct (MoNE, 2018c).

3.3 Matching Space and Shape Area with subjects in Turkish Curriculum

Table 3.1. Matching Space and Shape Area with subjects in Turkish Curriculum (OECD, 2019a; MoNE, 2018c)

	5th grade	6th grade	7th grade	8th grade
Numbers and Algebra	Natural numbers and operations Fractions and operations Decimals Percentages	Natural numbers and operations Fractions and operations Decimals Percentages Groups Integers Ratio	Percentages Integers and operations	Multipliers and products Exponential numbers Square root expression
Algebra	-	Algebraic Expressions	Algebraic Expressions Equality and Equations	Linear equations Algebraic Expressions and Identities Inequalities
Geometry and Measurement	Basic geometrical concepts and drawings (SS) Triangles and Quadrilaterals (SS) Length and Time measurement Area measurement (SS) Geometrical shapes (SS)	Area measurement Geometrical shapes (SS) Angles (SS) Circles (SS) Liquid measurement (SS)	Lines and angles Circle (SS) Polygons (SS) Perspectives of objects with different sides	Triangles Geometrical shapes (SS) Transformation geometry (SS)
Data	Data collection and Analysis	Data collection and Analysis Data Analysis	Data Analysis	Data Analysis
Probability	-	-	-	Probability of Simple Events

Table 3.2.Necessities of ‘‘Space and Shape’’ area in PISA 2018 Framework_and Learning goals of Geometry and Measurement in Turkish Curriculum (OECD, 2019a; MoNE, 2018c)

Necessities of ‘‘Space and Shape’’ area in PISA 2018 Framework	**Learning goals of Geometry and Measurement in Turkish Curriculum:**
• Patterns, properties of objects, positions and orientations, representations of objects, decoding and encoding of visual information, navigation, and dynamic interaction with real shapes as well as with representations • Spatial visualization, measurement, and algebra • Understanding perspective creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspectives and constructing representations of shapes	• Explain quality of line segment, line and ray and draw them • Name and recognize polygons with quality of basic elements • Recognize length measurement units • Recognize geometrical objects and calculate their volume and surface area • Similarities and congruent for triangles • Pythagorean theorem

3.4 Selection of the Textbooks

PISA is a triennial exam for 15th age group, which is first high school class for Turkey (MoNE, 2019). In this study middle school textbooks are selected because mathematical skills are mostly shaped until the high school age. Textbooks are indispensable tool for mathematics teaching process (Shield & Dole, 2013). To comprise thought the mathematic success of students in PISA, middle school textbook suitability for PISA math literacy framework gains importance. Also, textbooks have a significant role in learning opportunities in many content areas in mathematics, procedural fluency, and conceptual understanding (N.Bieda, Ji, Drwencke, & Picard, 2013). In addition, these textbooks are mostly sources which is a reliable way of accessing information. Two or more textbooks are for each grade level every year in Turkey. All textbooks are inspected by an expert from the Ministry of National Education regarding content and suitability for learning outcomes etc. (MoNE, 2021). because of this, the content of every book published at every grade level is identical. I give examples of tasks in the different textbooks for all grades in Appendix A. One book each grade was chosen, and description of textbooks were given below:

- The fifth-grade textbook, *Imam-Hatip Middle School, and Middle School Textbook Mathematics 5* with 300 pages includes 6 chapters (Karakuyu, 2018).
- The sixth-grade textbook, *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 6* with 300 pages includes 6 chapters (Bektaş, Kahraman, & Temel, 2019).
- The seven-grade textbook *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 7* with 295 pages includes 6 chapters (Keskin-Oğan & Öztürk, 2019).
- The eight-grade textbook *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 8* with 328 pages includes 6 chapters (Serçifeli & Atmaz, 2018).

3.5 The Unit of Analysis

The fifth-grade textbook, *Imam-Hatip Middle School and* Middle *School Textbook Mathematics 5* with 300 pages, includes 6 chapters whose, one chapter is geometrical objects (between 272-286 pages). The tasks in this book are provided in the following parts: ‘‘Leading question’’, ‘‘Example’’ which are worked-out questions, ‘‘Activity’’, ‘‘Exercises’’ and ‘‘Unit Evaluation Questions’’ which are to be solved. The number of each task is given below: (Karakuyu, 2018).

Table 3.3. Distribution of the Tasks within Each Part of Fifth-Grade Textbook, (Karakuyu, 2018)

Leading Question	Example	Activity	Exercises	Unit Evaluation Questions	Total Number of Tasks
1	14	9	8	7	39

Sub-questions within a task were coded separately. These sub-questions were either different questions related to the text of the task or the questions that require the same thing based on different shapes.

For the ‘‘activity’’ (Karakuyu, 2018) provided in Figure 3.1, there are three questions, and these questions were coded separately as 1-a, 1-b, 1-c.

Materials: Medicine with rectangular or square base box, scissors, ruler.

1) Examine the medicine box and answer the following questions.

- How many faces does the box have?
- How many corners does the box have?
- Which geometric shape are the side faces of the box?

Figure 3.1. Activity Tasks coded separately from fifth-grade textbook (Karakuyu, 2018)

The sixth-grade textbook, *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 6* with 300 pages includes 6 chapters whose one chapter is geometrical objects (between 333-361 pages). The tasks in this book are provided in the following parts: ‘‘example’’ whose questions are worked-out questions , ‘‘Let’s Think’’, ‘‘It is Your Turn’’, ‘‘Exercises’’ and ‘‘Unit Evaluation Questions’’ whose questions are to be solved questions. Number of tasks in each part are given below: (Bektaş, Kahraman, & Temel, 2019)

Table 3.4. Distribution of the Tasks within Each Part of Sixth-Grade Textbook (Bektaş, Kahraman, & Temel, 2019)

Let's Think	Example	It is Your Turn	Exercises	Unit Evaluation Questions	Total Number of Tasks
1	6	8	22	5	42

For the ''It is Your Turn'' part (see Figure 3.2), there is one task that require to calculate volume of the three shapes (Bektaş, Kahraman, & Temel, 2019). Each volume calculation was coded separately as 1-a, 1-b, and 1-c as students deal with them separately. In addition,

For the ''It is Your Turn'' part, there are more than one shapes to draw different perspectives. These kinds of tasks are coded 1-a,1-b, 1-c because students draw views of shapes separately, effecting number of tasks.

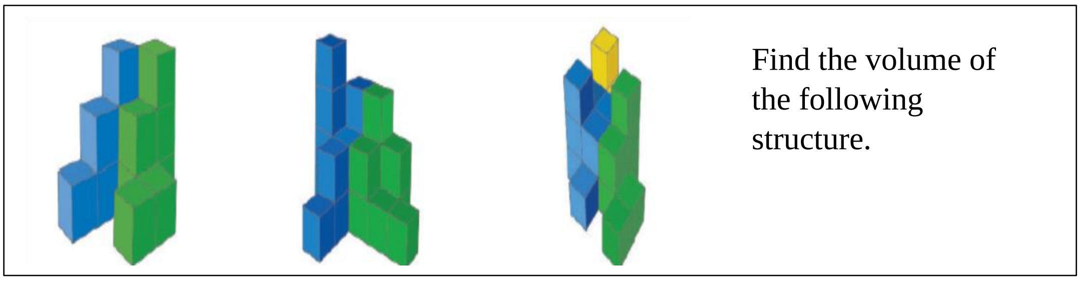


Figure 3.2 'It is Your Turn'' tasks coded separately from fifth-grade textbook (Bektaş, Kahraman, & Temel, 2019)

The seventh-grade textbook, *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 7* with 295 pages includes 6 chapters whose one chapter is geometrical objects (between 276-285 pages). The tasks in this book are provided in the following parts: 'Let's Do Together'' whose questions are worked-out questions, ''Leading Question'', ''It is Your Turn'', ''Activity'' and ''Unit Evaluation

Questions'' whose questions are to be solved questions. Number of each task are given below (Keskin-Oğan & Öztürk, 2019).

Table 3.5. Distribution of the Tasks within Each Part for Seventh-Grade Textbook (Keskin-Oğan & Öztürk, 2019)

Leading Question	Activity	Let's Do Together	It is Your Turn	Unit Evaluation Questions	Total Number of Tasks
1	6	5	9	3	24

The eight-grade textbook *Imam-Hatip Middle School, and* Middle *School Textbook Mathematics 8* with 328 pages includes 6 chapters (Serçifeli & Atmaz, 2018).). The tasks in this book are provided in the following parts: ''Example'' whose questions are worked-out questions, ''Let's Reminder'', ''It is Your Turn'', ''Let's Apply''; ''Problem'' ''Activity'', and ''Unit Evaluation Questions'' whose questions are to be solved questions. Number of each task are given below.

Table 3.6. Distribution of the Tasks within Each Part for Eight-Grade Textbook (Serçifeli & Atmaz, 2018)

Let's Reminder	Example	It's Your Turn	Let's Apply	Problem	Activity	Unit Evaluation Questions	Total Number of Tasks
3	23	20	22	6	22	19	115

3.6 Coding Scheme:

Table 3.7. The Domains of Coding Scheme(PISA 2018 Mathematics Liteacy Framework), (OECD, 2019a)

Domains of Mathematical Literacy	Subsets of The Domain	Key Words /Sentences for The Domains
Process	Formulate	• Recognize Mathematical Aspects and Structure • Simplify and Represent Situation or Problem • Translate Mathematical Language • Utilize Symbolic and Formal Language • Use Technology
	Employ	• Improve Strategies to Solve Mathematical Questions • Use Mathematical Tool • Utilize Mathematical Facts • Recognize Different Representations • Generalize Mathematical Procedures • Reflect Mathematical Arguments • Express Mathematical Results
	Interpret	• Interpret and Evaluate Logical Outcome of Mathematical Context in A Real Life • Connect How the Real World Impacts the Outcomes of Mathematical Procedure • Evaluate Limits of Mathematical Concepts and Solutions
Capability	Communication	• Recognize and understand a problem situation • Read, decode, and interpret statements • Present solution and explanation
	Mathematising	• Transform problem into real world • Evaluate mathematical outcomes and models • Include fundamental mathematical activities
	Representation	• Select/interpret and translate a situation • Involve graphs, diagrams, tables, pictures, formula equations, and concrete materials.
	Reasoning and Argument:	• Make deduction by exploring and connecting problem elements • Control a justification in the problem
	Devising Strategies for Solving Problems	• Formulate and solve problem critically • Select plan and strategy for problem • Any stage of problem-solving process

Table 3.7, Continued

	Using Mathematical Tools	• Include physical tools • Know how to use these tools
	Using symbolic, formal, and technical language and operations:	• Understand, interpret, manipulate, and make use of symbolic expressions within a mathematical context • understand and utilize formal constructs based on definitions, rules, and formal systems • use algorithms
Context	Personal	• Food preparation, games, shopping, personal transportation, personal health, sports, travel, personal scheduling, and personal finance
	Occupational	• Measuring, costing, and ordering materials for building, payroll/accounting, quality control, scheduling/inventory design/architecture and job-related decision making
	Societal	• Voting systems, public transport, government, public policies, demographics, advertising, national statistics, and economics. • Communities' issues
	Scientific	• Weather or climate, ecology, medicine, space science, genetics, measurement, and the world of mathematics
	Decontextualized	• Not include real-life context
Proficiency	LEVEL 6	• Conceptualize, generalize, and utilize information based on their investigations and modelling of complex problem situations • Use their knowledge non-standard contexts • Connect and translate different information and representations among them • Have advanced mathematical thinking and reasoning • Apply mastery of symbolic and formal mathematical operations and relationships • Develop new approaches and strategies • Reflect thinking and findings

Table 3.7, Continued

	LEVEL 5	• Develop and work with models for complex situations • Identify assumptions and constraints • Select, compare, and evaluate problem- solving strategies • Have well-developed thinking and reasoning skills • Link representations, symbolic and formal characterizations • Begin to reflect on their work • Formulate and communicate their interpretations and reasoning
	LEVEL 4	• Work models for complex situations • Select and integrate different representations • Link these situations to real world • Utilize limited range of skills • Construct and communicate explanations and arguments
	LEVEL 3	• Execute clearly described procedures • Interpret simple basic model • Apply simple problem-solving process • Interpret and use representations • Reflect basic interpretations and reasoning
	LEVEL 2	• Interpret and recognize situations in contexts that require no more than direct inference • Extract relevant information from one source • Use single representational mode • Employ basic algorithms, formula, procedures • Make literal interpretations of the results
	LEVEL 1	• Answer questions defined clearly including familiar context • Identify information and carry out routine procedures • Perform actions which are almost obvious • Follow actions from the given stimuli

Table 3.8. Relationship between mathematical processes (top row) and fundamental mathematical capabilities (left-most column) (OECD, 2019a)

	Formulating situations mathematically	**Employing mathematical concepts, facts, procedures and reasoning**	**Interpreting, applying, and evaluating mathematical outcomes**
Communicating	Read, decode, and make sense of statements, questions, tasks, objects, or images, to form a mental model of the situation	Articulate a solution, show the work involved in reaching a solution	Construct and communicate explanations and arguments in the context of the problem
Mathematising	Identify the underlying mathematical variables and structures in the real world problem	Use an understanding of the context to guide or expedite the mathematical solving process,	Understand the extent and limits of a mathematical solution that are a consequence of the mathematical model employed
Representation	Create a mathematical representation of real-world information	Make sense of, relate, and use a variety of representations when interacting with a problem	Interpret mathematical outcomes in a variety of formats in relation to a situation
Reasoning and argument	Explain, defend, or provide a justification for the identified or devised representation of a real-world situation	Connect pieces of information to arrive at a mathematical solution, generalize or create a multi-step argument	Reflect on mathematical solutions and create explanations and arguments that support, refute or qualify a mathematical solution
Devising strategies for solving problems	Select or devise a plan or strategy for problem with context	Activate effective and sustained control mechanisms across a multi-step procedure leading to a mathematical solution, conclusion or generalization	Devise and implement a strategy in order to interpret, evaluate and validate a mathematical solution to a problem with context

Table 3.8, Continued

Using symbolic, formal, and technical language and operations	Use appropriate variables, symbols, diagrams, and standard models	Understand and utilize formal constructs based on definitions, rules and formal systems as well as employing algorithms	Understand the relationship between the context of the problem and representation of the mathematical solution.
Using mathematical tools	Use mathematical tools to recognize mathematical structures or to portray mathematical relationships	Know about and be able to make appropriate use of various tools	Use tools to ascertain the reasonableness of a mathematical solution and any limits and constraints on that solution, given the context of the problem

3.7 Examples of coding tasks from the textbooks in data analysis process:

Table 3.7 given above was prepared by the 2018 PISA Mathematics Literacy Framework and for context domain one category was added named by ‘‘decontextualized’’ because most of tasks has no context (Bayraktar, 2019).Microsoft Excel Office Program was used data analysis process.

Firstly, I coded to context category if the tasks have a real-life situation; I coded a suitable context category from the coding scheme.

Secondly, I coded to process the domain of the tasks by considering what processes students face when dealing with these mathematical tasks. While tasks whose process domain is interpreted are obvious and rare in the middle school textbooks, examining tasks whose process domain is employ or formulate is more difficult. I used this to deal with this uncertainty: If the task requirement is based on using operations and applying formulas, I coded the process domain as employ. If the tasks’ have special requirements like formulation, explanations, or representations, I coded process domain as a formulate.

Leading question 1, page 272,5th grade textbook

All the faces of the skyscraper in the picture are covered with glass. How many square meters of glass in the construction of the building? How could the engineer have calculated that Please explain.

Figure 3.3. The Task Coded from the Page 272 of Karakuyu (2018) Fifth Grade Textbook

The context of the task provided in Figure 3.3 was coded as occupational because it is related to the engineering. Choosing its process domain was hard to decide as either ‘‘formulate’’ or ‘‘employ’’ because it leads both. One of the requirements of task is devising and implementing strategies to calculate total surface area of building which belongs to employ process, but also identifying the mathematical aspects of a problem situated in a real-world context which belongs to ‘‘formulate’’ is needed to achieve this task. This task was coded as ‘‘formulate’’ since measurement of building are unknown and logical estimation with explanation was needed.

After coding process domain, coding capability domain became easier with the help of Table 3.8. Relationship between mathematical processes and fundamental mathematical capabilities (OECD, 2019a).

Task from Figure 3.3 capability was *Reasoning and Argument* because its process domain was *formulate* and it ask students make explanations for answers. Only this capability requirement was these skills.

At the end I coded proficiency levels of the tasks with considering requirements of the proficiency levels and textbooks grade levels.

To code proficiency levels I consider following situation:

- If the tasks clearly described and its need is obvious to carry out routine procedures, coded as a Level 1.
- If the tasks' requirement is making *direct inference* and applying basic algorithms and formulas, coded as a Level 2.
- If the tasks' requirement is making inference and making basic interpretation and reasoning, coded as Level 3.
- If the tasks' requirement is making constructing and communicating explanations and arguments, coded as Level 4.
- If the tasks' requirement is reflecting on their work and constructing well developed-thinking skills, coded as Level 5.
- If the tasks' requirement is linking different information and reflecting and transforming their actions, coded as Level 6.

Proficiency level of the task given from Figure 3.4 was determined as Level 5 because it asked from the students construct and communicate explanations on covering building.

I give some examples tasks with different domains.

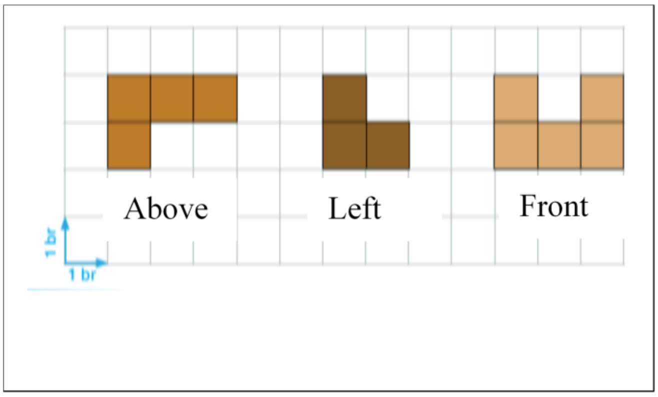
Exercises 1, page 282, 5th grade textbook

A square prism has a base area of 25 cm2 and a height of 9 cm. What is the lateral area of the prism?

Figure 3.4.The Task Coded from the Page 282 of Sixth Grade Textbook (Bektaş, Kahraman, & Temel, 2019)

Process of Task from the Figure 3.4 is *employ* because it needs applying mathematical fact to find lateral area. Context of Task is *decontextualization* because it has no real-life situation. Capability of Task is '*'Using symbolic, formal, and technical language and operations''* because it needs to utilize basic mathematical rules. Proficiency level of task is *Level 2* because when solving this student '' If the base is square with area 25 cm^2, one of edge is 5 cm and edges of prism is 5 cm ,5 cm and 9 cm, then find lateral area with these edges by using basic formulas'' .

Let's do together, page 279, 7th-grade textbook



From side, above, left, and front views are given. Let's construct the structure with identical cubes and find out how many pairs of cubes you need.

Figure 3.5. The Task Coded from the Page 279 of Seventh Grade Textbook (Keskin-Oğan & Öztürk, 2019)

Process of Task from Figure 3.5 the is *interpret* because it needs explaining if the knowledge is sufficient to reach solution and use these views to make construction. Context of Task is *decontextualization* because it has no real-life situation. Capability of Task is '*'Representation''* because it needs to translate two-dimensional shapes to three-dimensional shape. Proficiency level of task is *Level 5* because when solving this, student is transforming appropriate representations to each other and linking information.

Problem, page 301, 8th grade textbook

A container with right circular cylindrical shape has a base radius of 10 cm and a height of 25 cm. 5/4 of it filled with salt water a remaining filled with vinegar to prepare pickle juice How much cm^3 of vinegar will be put in this container? (Let's take $\pi = 3$)

Figure 3.6.The Task Coded from the Page 301 of Seventh Grade Textbook (Serçifeli & Atmaz, 2018)

Process of Task from Figure 3.6 is *employ* because it needs to use the volume formula and exact volume proportion. The context of the Task from Figure 3.7 is *personal* because it is related to food preparation. The capability of Task from the is '*'Using symbolic, formal, and technical language and operation''* because it needs to use basic formulas to find the volume of the container and its proportion. The proficiency level of Task is *Level 3* because when solving this, the student is carrying out described procedures and working with proportional relationships.

Let's Think, Page 333,Sixth grade textbook	
	A toy company produces objects of three different colors and shapes by using unit cubes. A square prism-shaped box filled with them so that there is no space left. Is the space occupied by three bodies when they stand apart the same as when they are combined?

Figure 3.7. The Task Coded from the Page 282 of Sixth Grade Textbook (Bektaş, Kahraman, & Temel, 2019)

Process of Task from Figure 3.7 is *formulate* because it needs identifying the mathematical aspects of a problem situated in a real-world context. Context of Task from Figure 3.8 is *occupational* because it is related office work and its product. Capability of Task is '*'Reasoning and argument''* because it needs providing justification devised representation of objects for a real-world situation. The Proficiency level of task is *Level 3* because when solving this, the student is interpreting representations and making reasoning directly from the given shape.

3.8 Validity

Validity is defined by Fraenkel, Wallen, & Hyun, (2012) “appropriateness, correctness, meaningfulness, and usefulness of the inferences”. In other words, the consistency between the purposes of the study and the results drawn from the data is validity (Fraenkel, Wallen, & Hyun, 2012).

In this study, for “appropriateness” aspect , tasks of geometrical objects in middle school mathematics were determined, then the tasks were resolved and the skills to be possessed in solving the task were determined by researcher according to the requirements of PISA mathematics literacy framework prepared by OECD (2019).

After checking the relevance of the Turkish middle mathematics textbook, PISA 2018 Mathematics literacy framework developed by OECD (2019) is determined to appreciate then the goal of the research.

To check “meaningfulness” of the study, determining textbook requirements accordance with PISA mathematics literacy framework is essential to detect effect of textbook students’ performance. For instance, as Turkey has lower score than OECD average on every triennial examination. (OECD, 2019b : OECD, 2016), research is meaningful.

To check the “usefulness” of the research results are related to validity of the research. This kind of research which is examining Turkish textbooks requirements in terms of PISA framework give valuable information about current sitution of Turkish textbooks.

Due to this, people working in educational process like teachers, students, policymakers may benefit result of the research in a variety of ways. This is enough for usefulness of the study. Those mentioned above are valuable to carry out the research and conducting this research is valid.

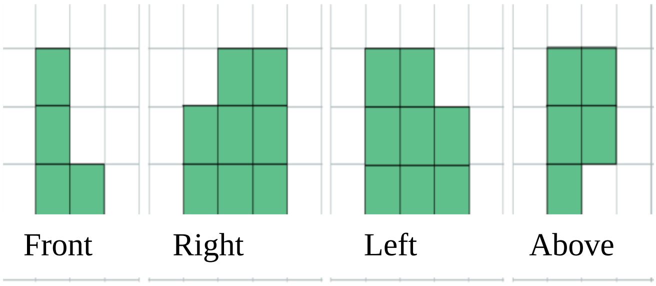
3.9 Reliability

Three criteria comprising the reliability of content analysis are: stability (the tendency for coders to consistently re-code the same data in the same way over a period), reproducibility (tendency for a group of coders to classify categories membership in the same way), and accuracy (extent to which the classification of text corresponds to a standard or norm statistically). The analysis of all the tasks in the selected textbooks was conducted using coding schema described Table 3.6. Each of the tasks was examined according to the coding schema and then coded into the corresponding category.

Firstly, the thesis supervisor trained the researcher who is middle school mathematics teacher about how to code the tasks using PISA 2018 Mathematics literacy framework. The supervisor first asked the researcher to code some tasks from the 5th grade textbook about geometric objects. Afterwards, they came together and discussed the coding of the researcher, and the process domain of the tasks was determined. Although tasks with interpret process were obvious, formulate and employ, process tasks are more complex. The thesis supervisor and researcher determined this criterion: If the requirement of tasks is employing an algorithm and using a formula coded as employ, the requirement of the tasks is a unique formulation or representation coded as a formulate. Secondly, determining proficiency levels is challenging because requirement of the levels is similar to each other. To overcome this, dominant requirement of the levels is detected, and grade levels of the tasks were considered while coding process. After this discussion, the supervisor asked to the researcher to code some other sample tasks from the 5th grade textbook and then they came together and went over the coding. In this training part, the criteria of the reliability were ensured by checking sample of the tasks with the thesis supervisor who is expert in task analysis and in geometry education. Then, the researcher and the supervisor reach consensus about coding of the tasks. After that the researcher coded rest of the tasks with the guidance of these discussions and consensus.

Upon completion of coding, to check the reliability again one independent coder who is a high school mathematics teachers and has written a master thesis on analysis of taks in 9th grade texbook with PISA 2012 Math Literacy Framework coded 7th grade tasks. 7th grade textbooks tasks were selected because there are tasks having different process, and proficiency levels. Researcher and the other coder had consensus on coding with process, context, and capability domain, but coding of two tasks proficiency levels were different than each other. For example, researcher coded tasks as Level 5, the other coder coded tasks as a Level 4. Example of the task coded differently by two coders given below.

Let’s do together, Page 279, seventh grade textbook



A front view left, and top views are given on the side. These views can be drawn so that the structure can be drawn. Is it enough? If that's enough, let's draw this structure.

Figure 3.8.Task whose proficiency level coded differently by coders (Keskin-Oğan & Öztürk, 2019)

After discussing students’ profiles in that grade, two coders accepted researcher codes because seventh-grade students need to reflect on their thinking and make logical reasoning.

CHAPTER 4

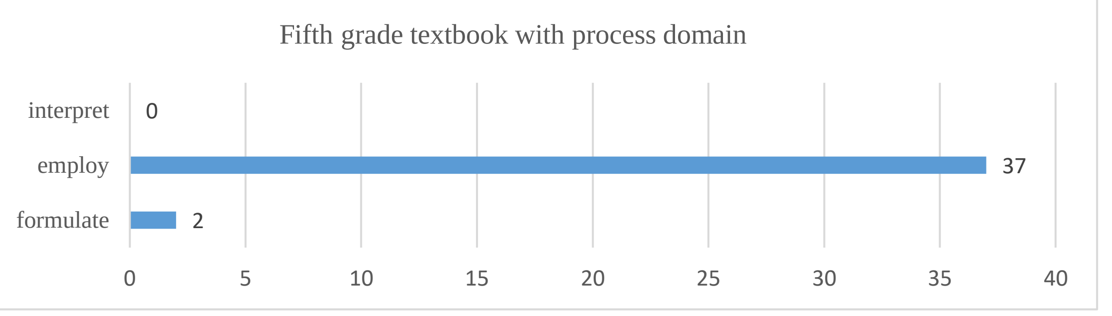
RESULTS

In this chapter, detailed results of the content analysis of the middle school mathematics textbooks written based on the fifth-grade textbook (Karakuyu, 2018), the sixth-grade textbook (Bektaş, Kahraman & Temel, 2019), the seven-grade textbook (Keskin-Oğan & Öztürk, 2019). The eighth-grade textbook (Serçifeli & Atmaz, 2018) are provided. These results answer the question to what extent geometrical object tasks in Turkish middle mathematics textbooks include potential requirements of the 2018 Pisa Mathematics Literacy Framework. I give results in the following order: Process, capabilities, contextualization, and levels of each grade individually. Tasks in mathematics textbooks are combined in two sections: questions and worked-out examples.

4.1 Overall Result of Process Domain of Middle Mathematics Textbook

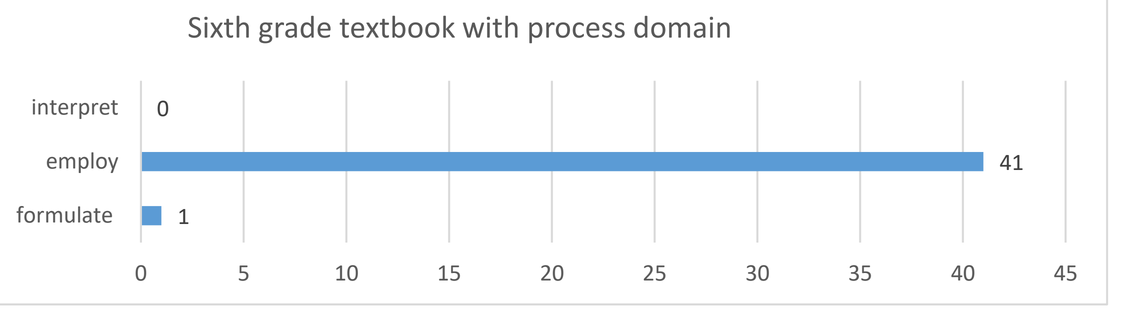
For the fifth-grade mathematics textbook, I coded 39 tasks with the process, capability, context, and proficiency levels. There are 25 tasks named as be-solved questions and 14 tasks named as worked-out examples. While 37 questions' process is *employ*, 2 tasks' process is formulate, there are no tasks whose process is *interpret.*

Table 4.1. Frequencies of Fifth Grade Textbooks' Tasks Regarding Process Categories of the Coding Scheme



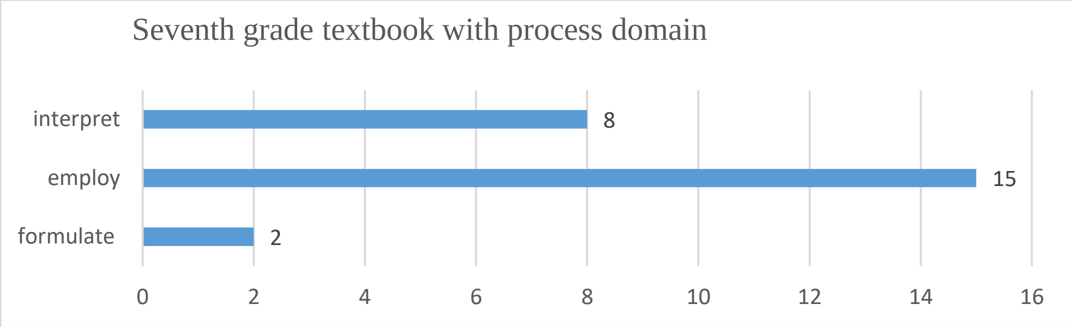
The sixth-grade textbook (Bektaş, Kahraman & Temel, 2019) has 36 to-be-solved questions and 6 worked-out examples, totaling 42 questions. While 41 tasks' process is *employ,* 1 tasks' process is *formulate*, there is no tasks whose process is *interpret*.

Table 4.2. Frequencies of Sixth Grade Textbooks' Tasks Regarding Process Categories of the Coding Scheme



The seventh-grade textbook (Keskin-Oğan & Öztürk, 2019) has 18 to-be-solved questions and 6 worked-out examples, totaling 24. While 15 tasks' process is employ, 8 tasks' process is *interpret*, and 2 tasks' process is *formulate.*

Table 4.3. Frequency of Seventh Grade Textbooks' Tasks Regarding Process Categories of the Coding Scheme



The eighth-grade textbook (Serçifeli & Atmaz, 2018) has 109 to-be-solved questions and 6 worked-out examples, totaling 115.While 101 tasks' process is *employ*, 1 tasks' process is *interpret*, and 13 tasks' process is *formulate.*

Table 4.4. Frequency of Eighth Grade Textbooks' Tasks with process domain

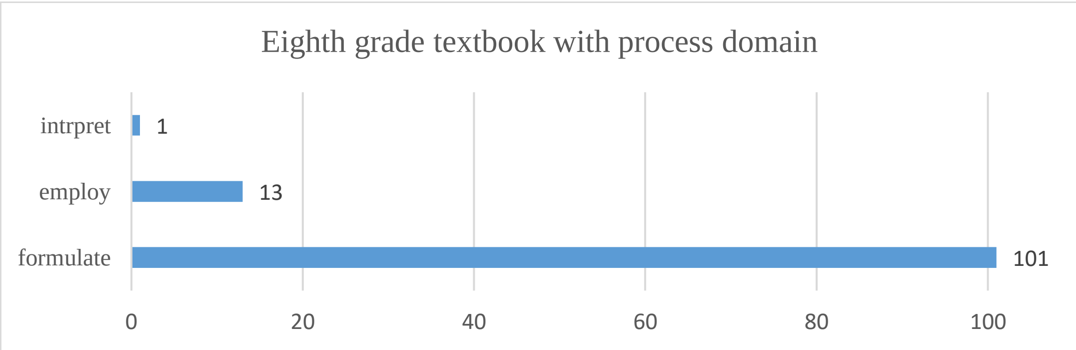
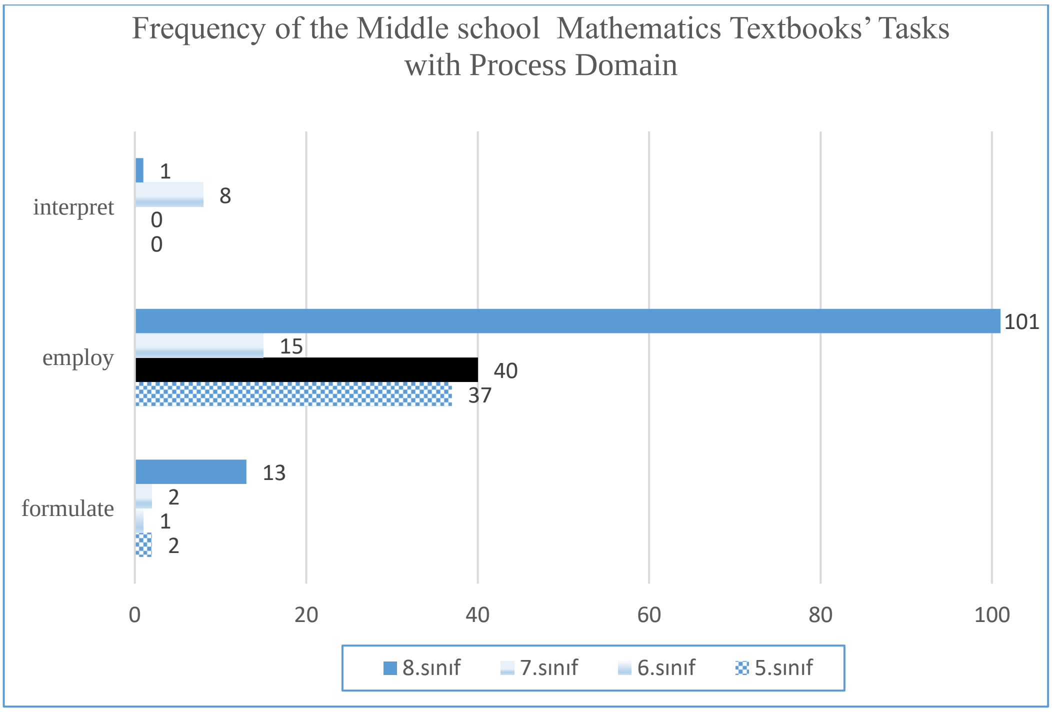


Figure 4.1. Frequency of the Middle school Mathematics Textbooks' Tasks with Process Domain

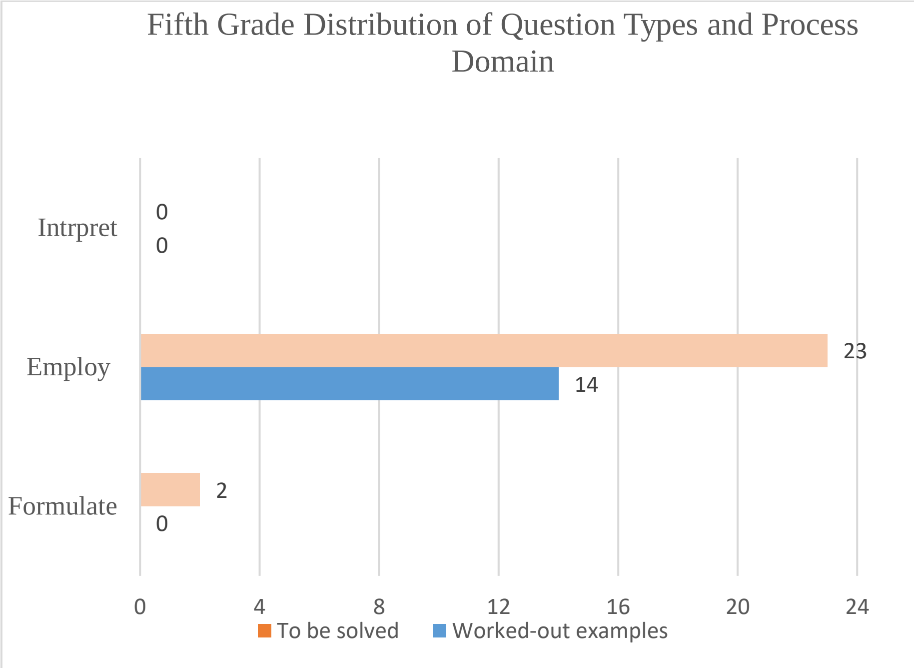


The total number of tasks with formulate process, occurring in all grade textbooks is higher than the number of tasks with interpret process. It is still little share in all tasks(18 tasks out of 220).

4.1.1 Results Regarding the Question Types and Process Domain

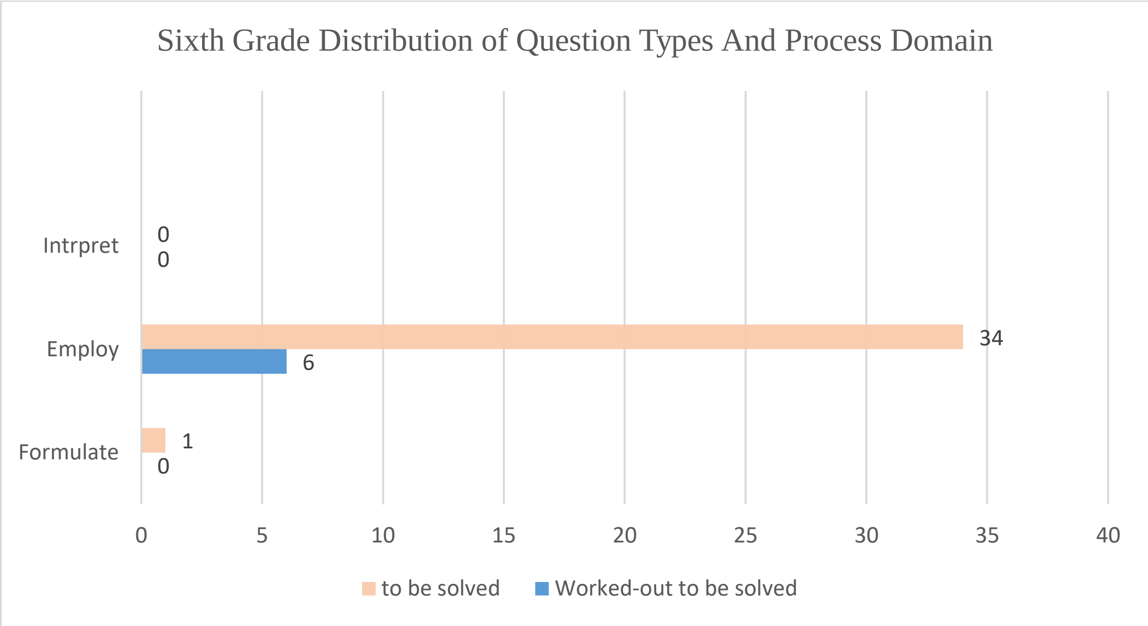
Task with different kind of process categories also provide information about educational process for students because different question types improve thinking skills (Smith & Holliday, 2010). The worked-out examples referred to the tasks with a solution. However, to-be-solved questions are the questions that have no solution and given for the practice of students (Delil, 2006).

Figure 4.2. Fifth Grade Textbook Distribution of Question Types and Process Domain



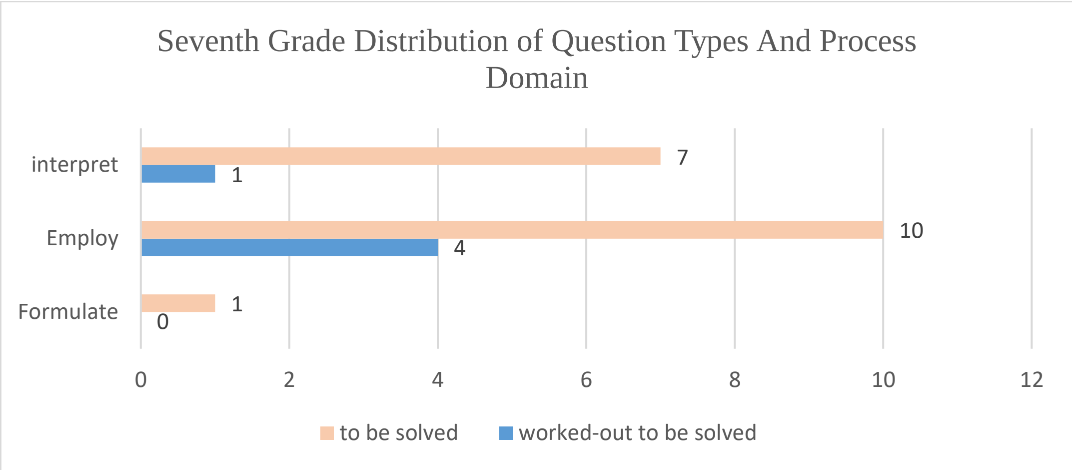
Fifth grade mathematics textbooks (Karakuyu, 2018) provided tasks under four different parts, which are ‘example’, ‘exercises’, ‘activity’, and ‘evaluation of the unit’. The task under the heading of ‘example' is categorized as worked-out examples where the task of ‘exercise’, ‘activity, and ‘evaluation of unit’ are categorized as to-be-solved questions. There are 39 tasks in textbook 14 of them are worked-out examples, 25 of them are to be solved questions. Whole worked out process of 14 tasks are *employ*, 2 tasks of to be solved questions’ process are formulate the others’ to be solved questions process are *employ*. There are no tasks whose process is interpret.

Table 4.5.Sixth Grade Textbook Distribution of Question Types and Process Domain



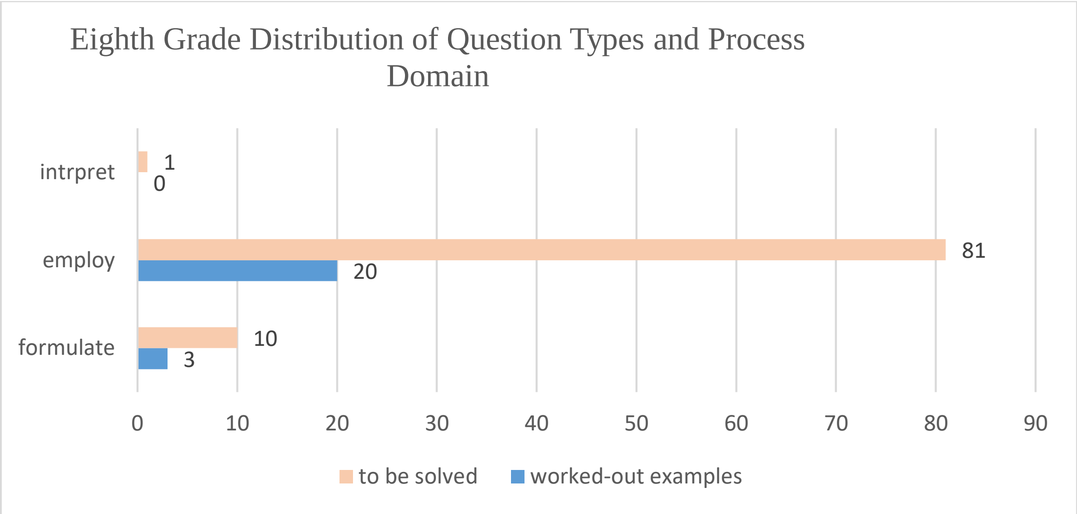
The sixth-grade textbook (Bektaş, Kahraman, & Temel, 2019) provided tasks under five parts which are ‘‘example’’, ‘‘Let’s Think’’, ‘‘It is Your Turn’’, ‘‘Exercises’’ and ‘‘Unit Evaluation Questions’’ whose tasks are to be solved questions. There are 42 tasks in textbook 6 of them is worked-out examples, 35 of them is to be solved questions. 34 to be solved questions process’ types is *employ*, 1 question of to be solved questions’ process is *formulate*, 6 worked out questions process are *employ*. . There are no tasks whose process is *interpret*.

Table 4.6. Seventh Grade Textbook Distribution of Question Types and Process Domain



The seven-grade textbook (Keskin-Oğan & Öztürk, 2019) provided tasks under five parts which are ‘‘Let’s Do Together’’ whose questions are worked-out questions , ‘‘Leading Question’’, ‘‘It is Your Turn’’, ‘‘Activity’’ and ‘‘Unit Evaluation Questions’’ whose questions are to be solved questions. There are 23 questions in textbook 5 of them are worked-out questions, 18 of them are to be solved questions.4 worked out examples process’ types are employ, 1 questions’ process is *interpret*, it is to first time for meeting this process domain until fifth grade. To be solved questions process is formulate the others’ process are *employ*. 7 to be solved questions’ process are interpret, 1 question’ process is *formulate*, process of the others is employ.

Table 4.7. Eight Grade Textbook Distribution of Question Types and Process Domain

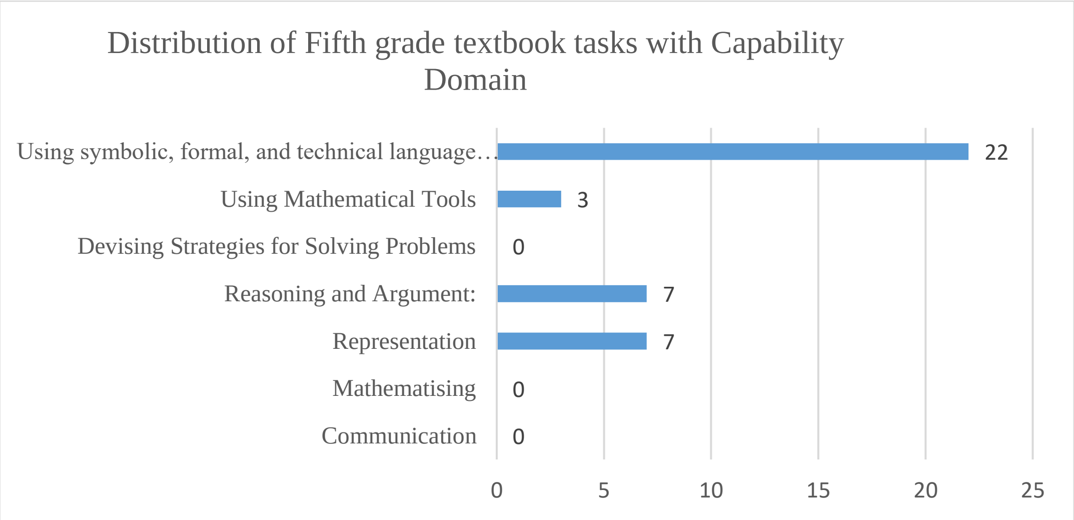


The eight-grade textbook are (Serçifeli & Atmaz, 2018) provided tasks under five parts which are ‘‘Example’’ whose questions are worked-out questions ‘‘Let's Reminder’’, ‘‘It is Your Turn’’, ‘‘Let's Apply ’’; ‘‘Problem’’ ‘‘Activity’’, and ‘‘Unit Evaluation Questions’’ whose questions are to be solved questions. . There are 115 tasks in textbook 23 of them are worked-out examples, 92 of them are to be solved questions. All worked-out examples process’ types is *employ*. While 1 to be solved questions’ process is *formulate* the others’ process are *employ*. 1 to be solved question process is interpret, 13 tasks’ process are *formulate*, the others’ (81) process are *employ*.

4.2 Overall Result of Capability Domain of Middle Mathematics Textbook

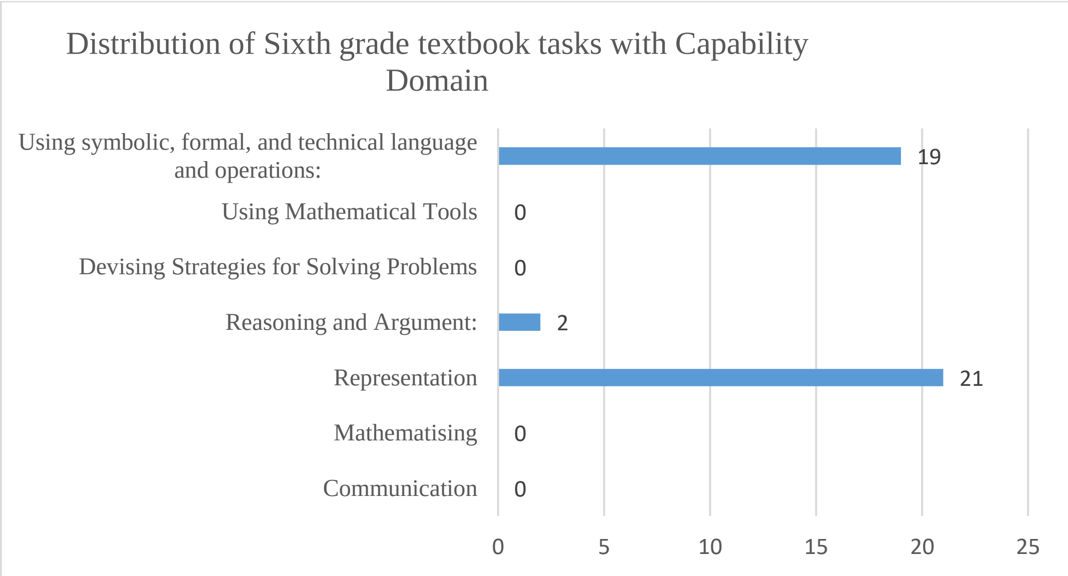
PISA has seven fundamental capacity which are ‘‘Communication, Mathematising, Representation, Reasoning and Argument, Mathematising, Devising strategies for solving problems, Using symbolic, formal and technical language and operations and Using mathematical tools ’’ (OECD, 2019a).

Table 4.8. Distribution of Fifth grade textbook tasks with Capability Domain



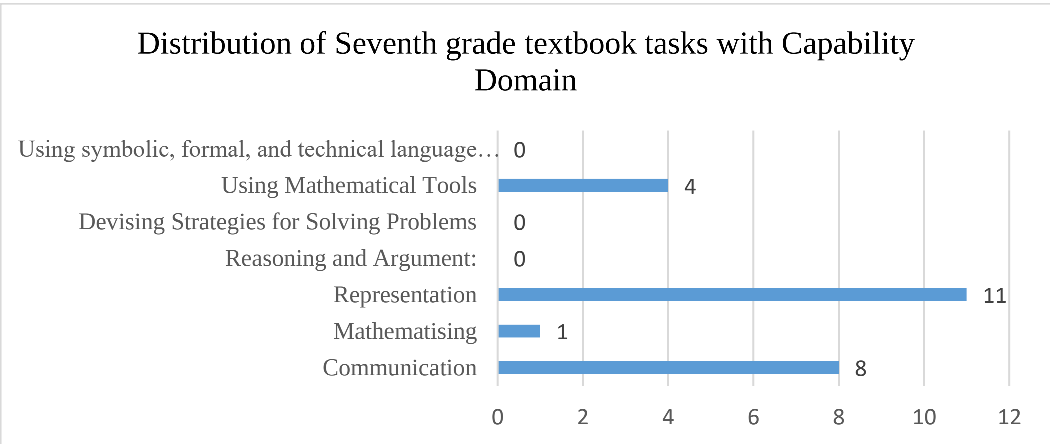
For fifth grade mathematics textbook there are 39 tasks coded. While 22 tasks’ capability is ‘‘Using symbolic, formal, and technical language and operations’’, 7 tasks’ capability is ‘‘Representation’’, 3 tasks’ capability is ‘‘Using mathematical tools’’ and 7 tasks’ capability ‘‘Reasoning and Argument’’. There are no tasks whose capability are ‘‘Devising Strategies for Solving Problems’’, ‘‘Mathematising’’, and ‘‘Communication’’

Table 4.9. Distribution of Sixth grade textbook tasks with Capability Domain



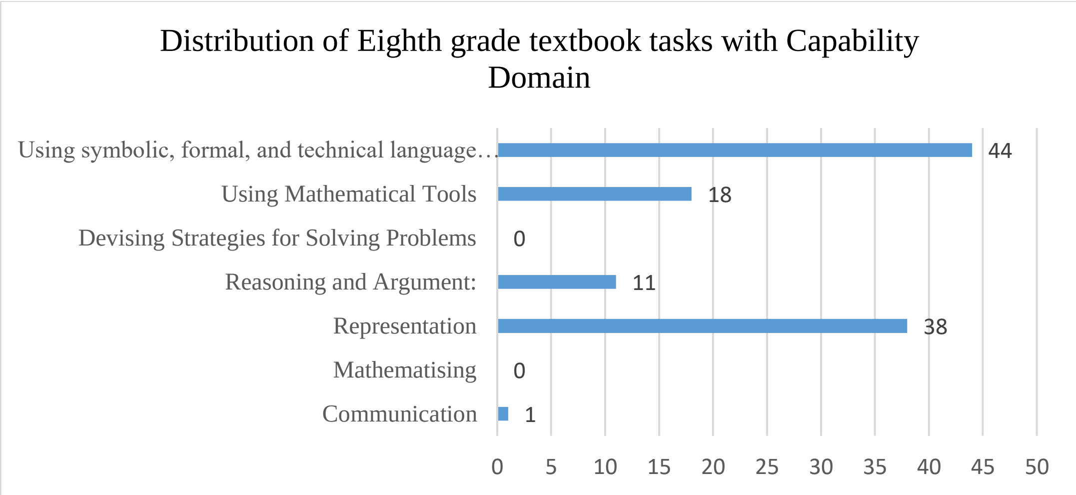
For the sixth-grade mathematics textbook, there are 42 tasks coded. There are 36 tasks to-be-solved questions and 6 tasks named by worked-out examples. While 19 tasks' capability is ''Using symbolic, formal, and technical language and operations'', 21 tasks' capability are ''Representation'', and 2 are ''Reasoning and Argument''. There are no tasks whose capabilities are ''Communication'', ''Mathematising'' and ''Devising strategies for solving problems.''

Table 4.10.Distribution of Seventh grade textbook tasks with Capability Domain



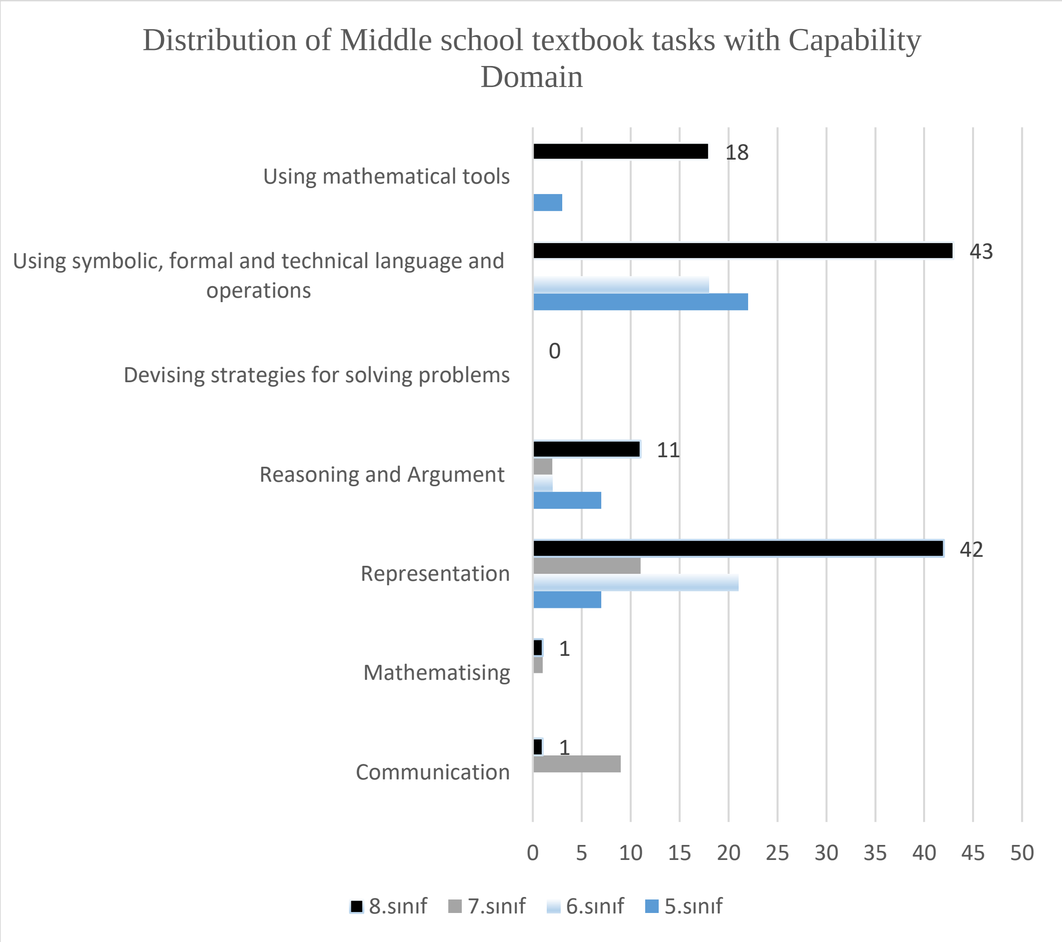
In the seventh-grade mathematics textbook, there are 24 tasks coded. While 1 task capability is ‘‘Mathematising’’, 11 tasks’ capability are ‘‘Representation, 8 tasks’ capability are ‘‘Communication’’, and 4 tasks’ capability are ‘‘Using mathematical tools’’. There are no tasks whose capabilities are ‘‘Reasoning and Argument’’, ‘‘Using symbolic, formal, and technical language and operations’’ and ‘‘Devising strategies for solving problems’’.

Table 4.11. Distribution of Eighth grade textbook tasks with Capability Domain



For the eighth-grade mathematics textbook, there are 115 tasks coded. While 44 tasks’ capability are ‘‘Using symbolic, formal, and technical language and operations’’ and 18 tasks’ capability are ‘‘Using mathematical tools’’, 11 tasks’ capability are ‘‘Reasoning and Argument’’, 42 tasks’ capability are ‘‘Representation’’, and 1 tasks capability is ‘‘Communication’’. There are no tasks whose capabilities are ‘‘Mathematising’’ and ‘‘Devising strategies for solving problems.’’.

Table 4.12. Distribution of Middle School textbook tasks with Capability Domain

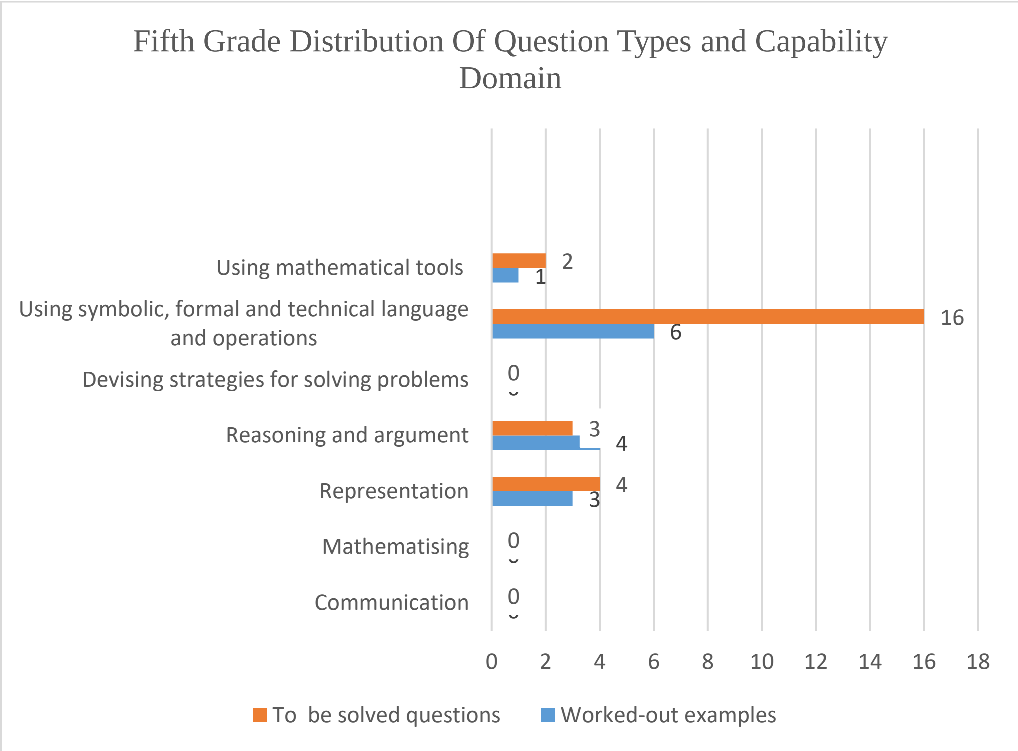


While ‘‘Devising strategies for solving problems’’ capability does not occur any tasks grade of middle school textbook, Number of tasks having capabilities of ‘‘Mathematising’’ and ‘‘Communication’’ capability is very low. Number of tasks having capabilities of ‘‘Using symbolic, formal, and technical language’’ and ‘‘Representation’’ is higher than the other because of nature of chosen subject.

4.2.1 Results Regarding the Question Types and Capability Domain

In the fifth-grade textbook, there are 25 tasks named by to be-solved questions and 14 tasks named by worked-out examples. Their capability domain is given below:

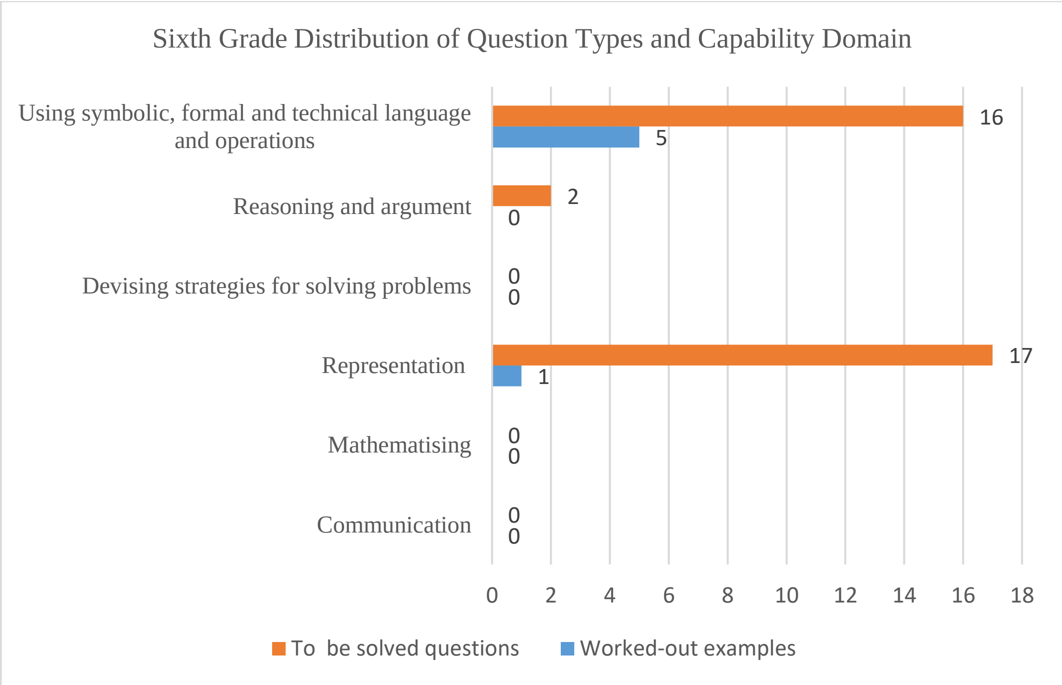
Table 4.16. Distribution of Question Types with Capability Domain for Fifth Grade Textbook



While the Capability of ''Using symbolic, formal, and technical language'' has the highest number for solved questions and worked-out examples, the Capability of ''Using mathematical tools'' has the lowest number for both question types. The number of the other capabilities has close to each other in both question types.

For sixth grade textbook, there are 14 worked out tasks, 25 to be solved questions. Their capability domain is given below:

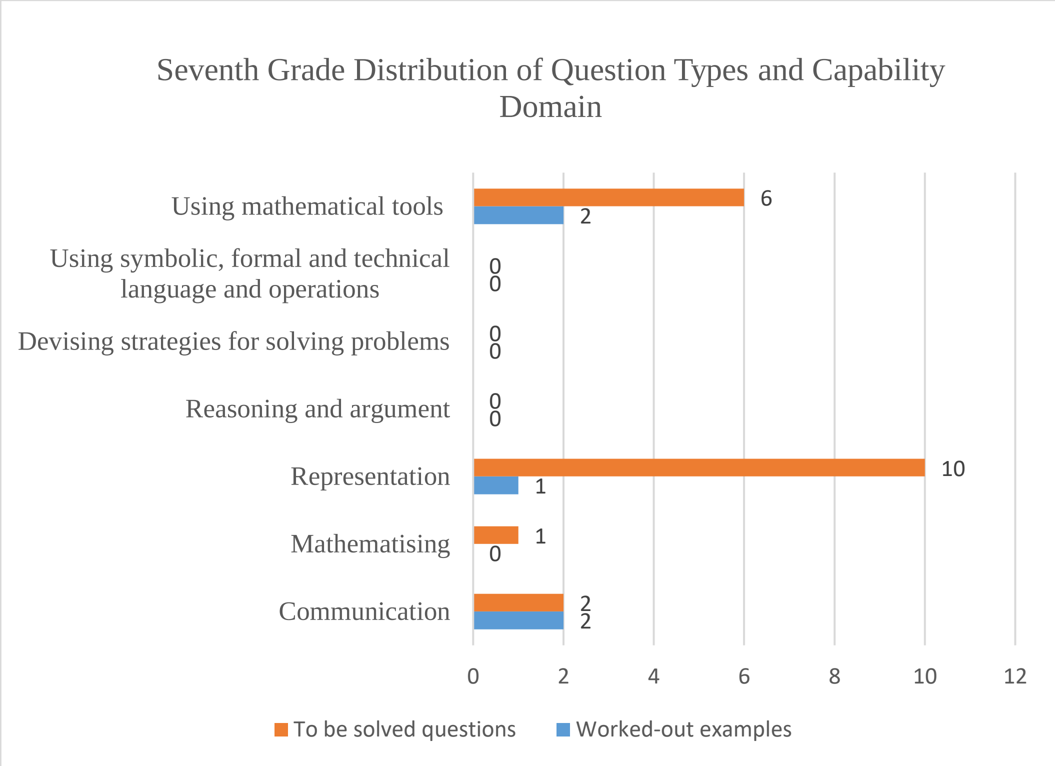
Table 4.13. Distribution of Question Types with Capability Domain for Sixth Grade Textbook



While capability of ‘‘Using symbolic, formal, and technical language’’ has the highest number for to be solved questions and worked-out examples, like 5th grade, ‘‘Reasoning and Argument’’ has lowest number for tasks, only occur in to be solved questions.

For seventh grade textbook, there are 5 worked out examples, 19 to be solved questions. Their capability domain is given below:

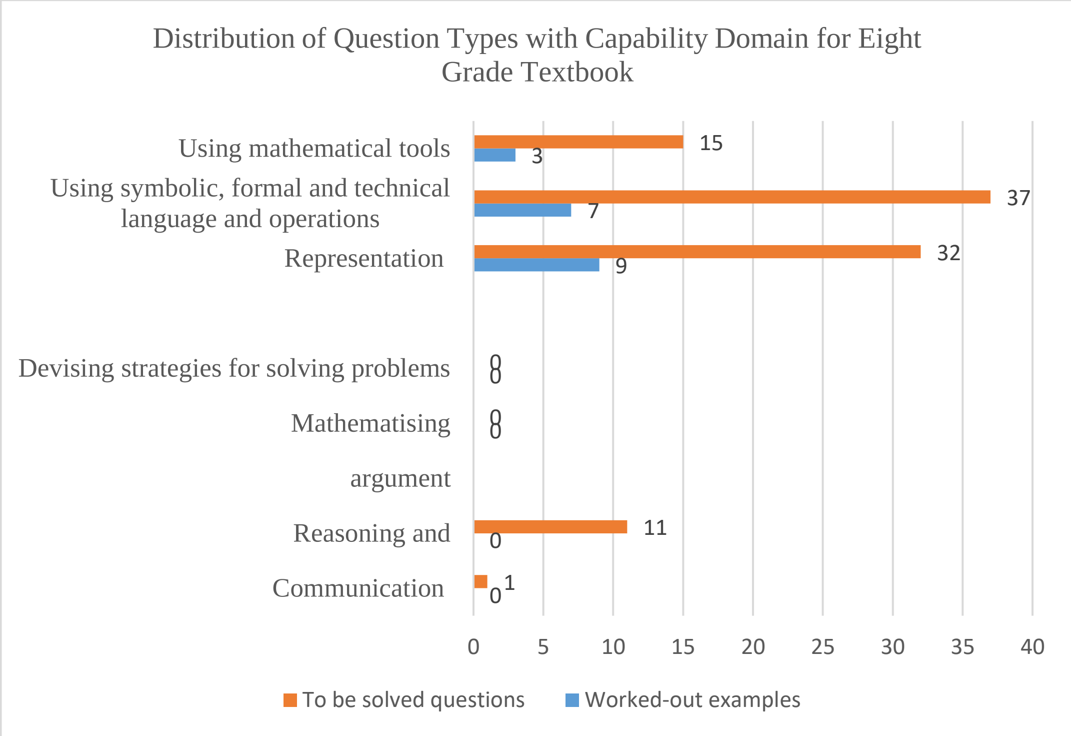
Table 4.20. Distribution of Question Types with Capability Domain for Seventh Grade Textbook



While capability of ‘‘Representation ’’ has the biggest number for to be solved questions, being nature of the objectives for seventh grade, ‘‘Mathematising’’ has lowest number for tasks, only occur in to be solved questions.

For eighth grade textbook, there are 24 worked out questions, 91 to be solved questions. Their capability domain is given below:

Table 4.14. Distribution of Question Types with Capability Domain for Eight Grade Textbook



While number of capabilities of ‘‘Using symbolic, formal, and technical language’’ has the biggest for to be solved questions and worked-out examples, like 5th and 7th grade, number of ‘‘Reasoning and Argument’’ capability has the lowest in the tasks.

4.3 Overall Result of Context Domain of Middle Mathematics Textbook

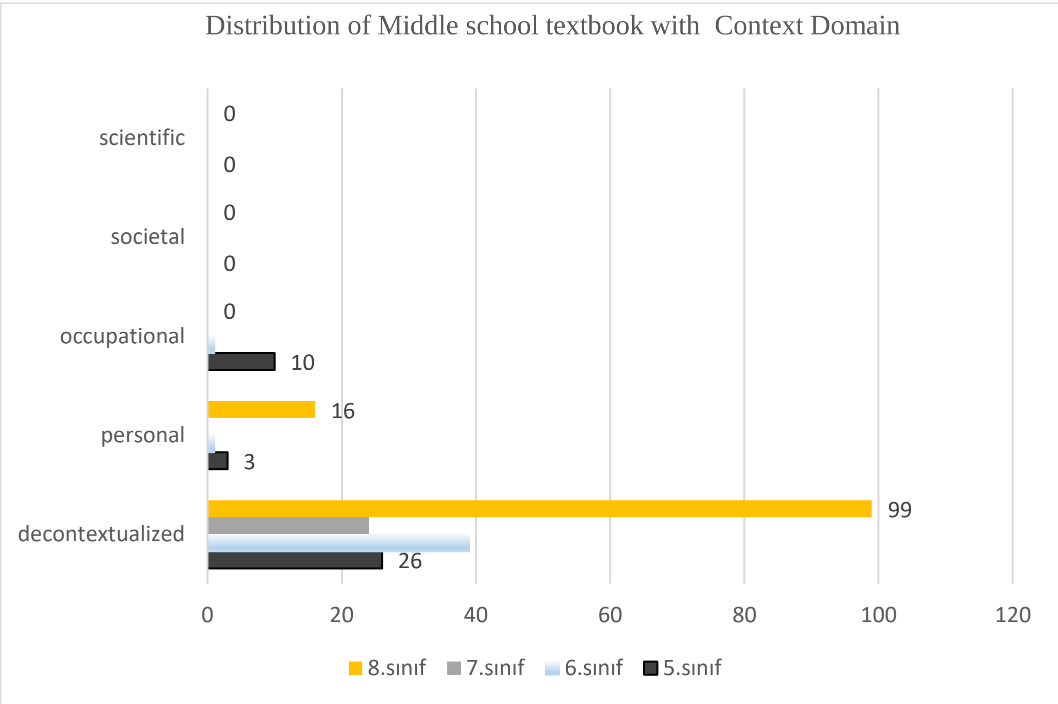
For purposes of the PISA 2018 mathematics framework, personal, occupational societal, and scientific categories are presented as context categories. (OECD, 2019a). In this study, some tasks are out of context named decontextualized. (Bayraktar, 2019)

For the fifth-grade mathematics textbook, there are 39 tasks coded. There are 26 tasks coded as decontextualized, 10 tasks coded occupational, and 3 tasks coded as personal. For the sixth-grade mathematics textbook, there are 42 tasks were coded.

There are 36 tasks named by to be-solved questions and 6 tasks named by worked-out examples.

While there are 39 tasks coded as decontextualized, 1 task coded occupational, and 1 task coded as personal. For seventh grade mathematics textbook there are 24 tasks were coded. There are 18 tasks named by to be-solved questions and 6 tasks named by worked-out examples. All 24 tasks coded as decontextualized, there is no tasks having context. For eighth grade mathematics textbook there are 115 tasks were coded. There are 91 tasks named by to be-solved questions and 24 tasks named by worked-out examples. There are 99 tasks coded as decontextualized, 16 tasks coded personal.

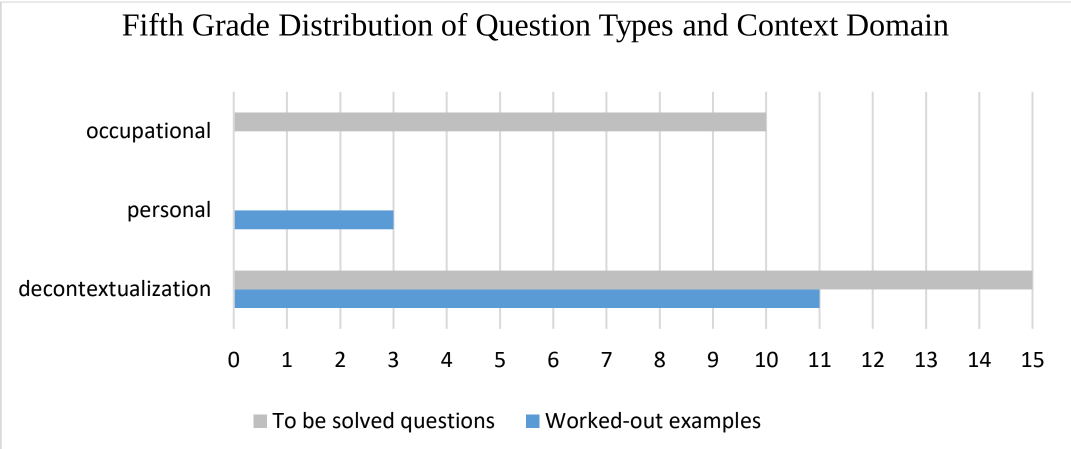
Table 4.15. Distribution of Middle school textbook with Context Domain



4.3.1 Results Regarding the Question Types and Context Domain

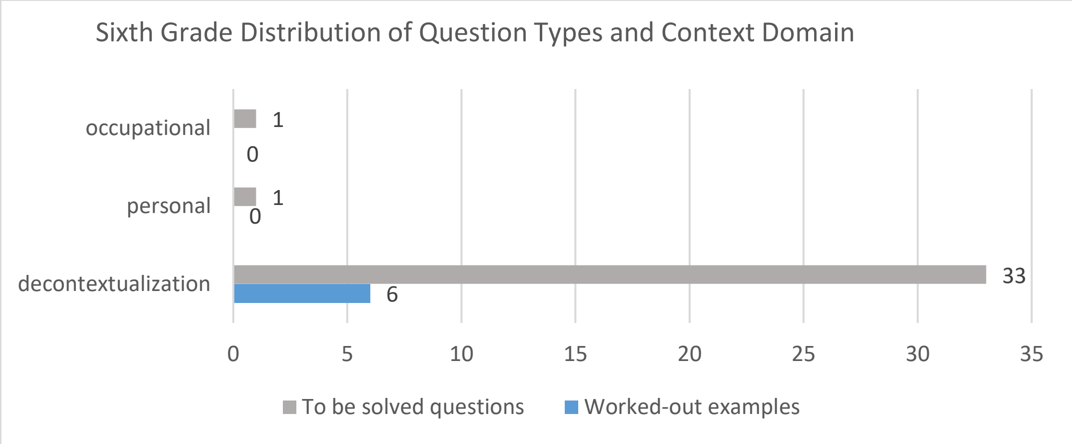
For fifth grade textbook, there are 14 worked out questions, 25 to be solved questions. Their context domain is given below:

Table 4.16. Distribution of Question Types with Context Domain for Fifth Grade Textbook



The number of tasks coded decontextualization is higher for two question types than for the others because the only context to be solved questions is occupational. The sixth-grade textbook has 6 worked-out questions and 36 to-be-solved questions. Their capability domain is given table.

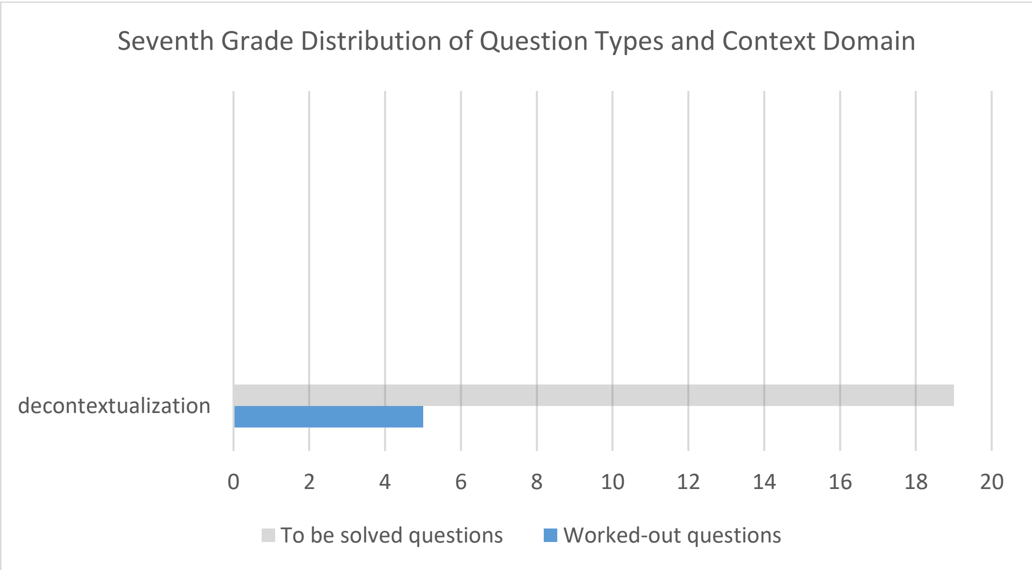
Table. 4.17. Distribution of Question Types with Context Domain for Sixth Grade Textbook



There are only two contexts occurred in only to be-solved questions which are occupational and personal.

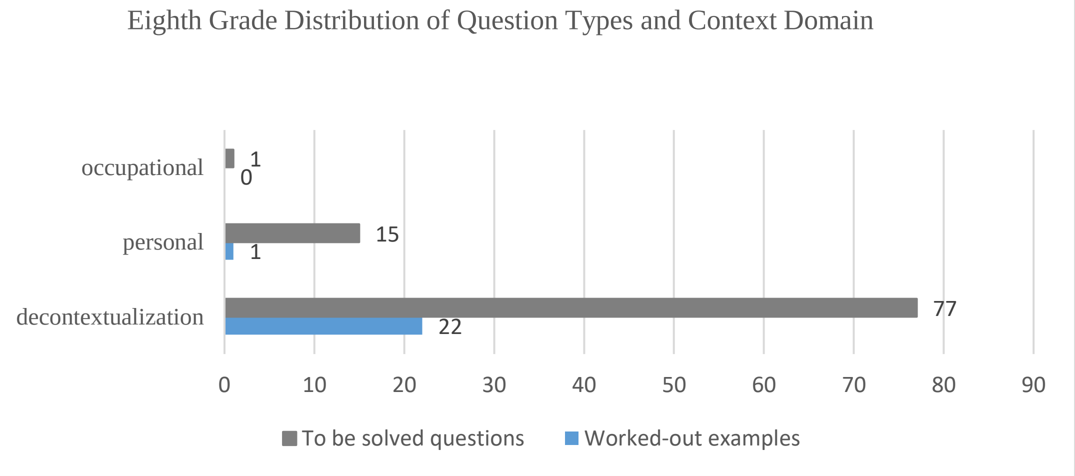
For seventh grade textbook, there are 5 worked out questions, 19 to be solved questions. Their context domain is given table:

Table 4.18. Distribution of Question Types with Context Domain for Seventh Grade Textbook



For seventh-grade textbooks, there are no tasks having context. The eighth-grade textbook has 24 worked-out questions and 91 to-be-solved questions. Their context domain is given table:

Table.4.19. Distribution of Question Types with Context Domain for Eight Grade Textbook



Most of the tasks have no contexts coded as decontextualization for both question types and there are few tasks coded only occupational and personal.

4.4 Result of Proficiency Level Domain of Middle School Mathematics Textbook

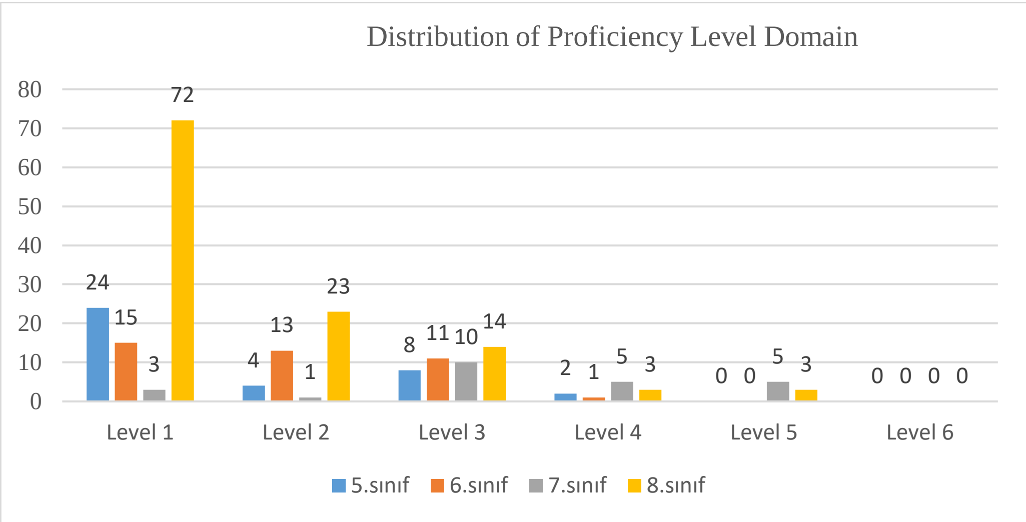
PISA has six proficiency level (OECD, 2019a). For fifth grade mathematics textbook there are 39 tasks were coded. There are 25 tasks named by to be-solved questions and 14 tasks named by worked-out examples. While 18 tasks' proficiency level is Level 1, 4 tasks' proficiency level is Level 2, 8 questions' proficiency level is Level 3 and 2 tasks' proficiency level is Level 4. Also, there is no tasks whose proficiency level is 5 or 6.

For sixth grade mathematics textbook there are 42 tasks were coded. There are 35 tasks named by to be-solved questions and 6 tasks named by worked-out examples. While 15 tasks' proficiency level is Level 1, 13 tasks' proficiency level is Level 2, 12 tasks' proficiency level is Level 3 and 1 tasks proficiency level is Level 4.Also, there is no questions whose proficiency level is 5 or 6.

For the seventh-grade mathematics textbook, there are 24 tasks coded. There are 18 tasks named by to-be-solved questions and 6 tasks named by worked-out examples. While 2 tasks proficiency level is Level 1, 1 tasks proficiency level is Level 2, 10 tasks' the proficiency level is Level 3, 5 tasks' the proficiency level is Level 4, and the 5 tasks' proficiency level is Level 5. Also, there are no tasks whose proficiency level is 6.

For the eighth-grade mathematics textbook, there are 115 tasks coded. There are 91 tasks named by to-be-solved questions and 24 tasks named by worked-out examples. While proficiency level of 72 tasks' is Level 1, proficiency level of 23 tasks is Level 2, proficiency level of 14 tasks is Level 3, proficiency level of 3 tasks is Level 5. Also, there are no questions whose proficiency level is 6.

Table 4.20. Distribution of Proficiency Level Turkish Middle Mathematics Textbook

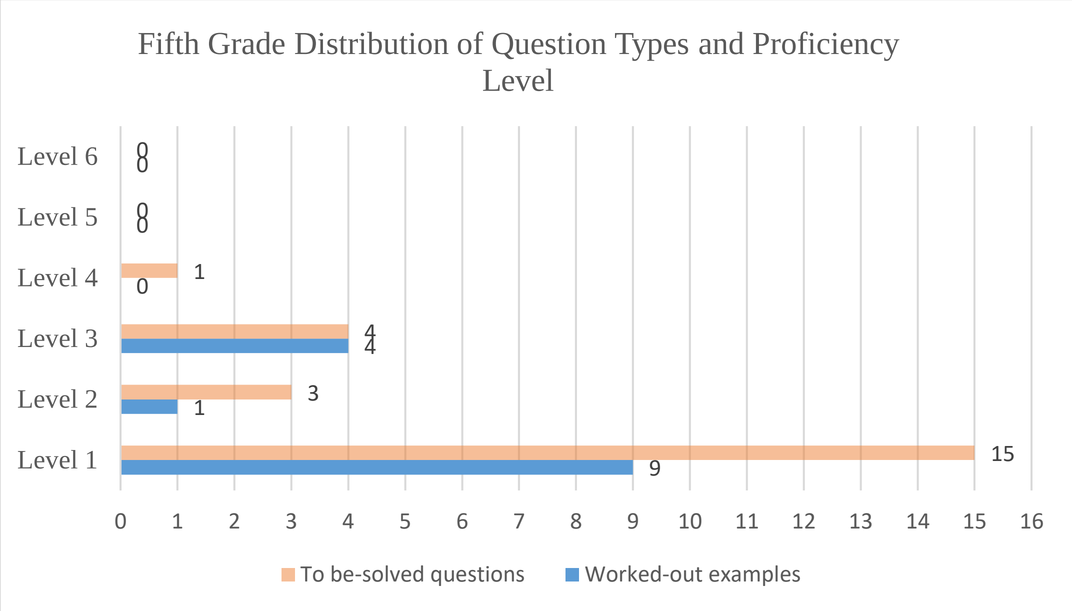


While Proficiency level of most of the tasks are accumulated in Level 1 and Level 2, especially in fifth and sixth grade , there are few tasks in 7th and 8th grade having higher proficiency level.

4.4.1 Results Regarding the Question Types and Proficiency Level

For fifth grade textbook, there are 14 worked out questions, 25 to be solved questions. Their proficiency level domain is given below.

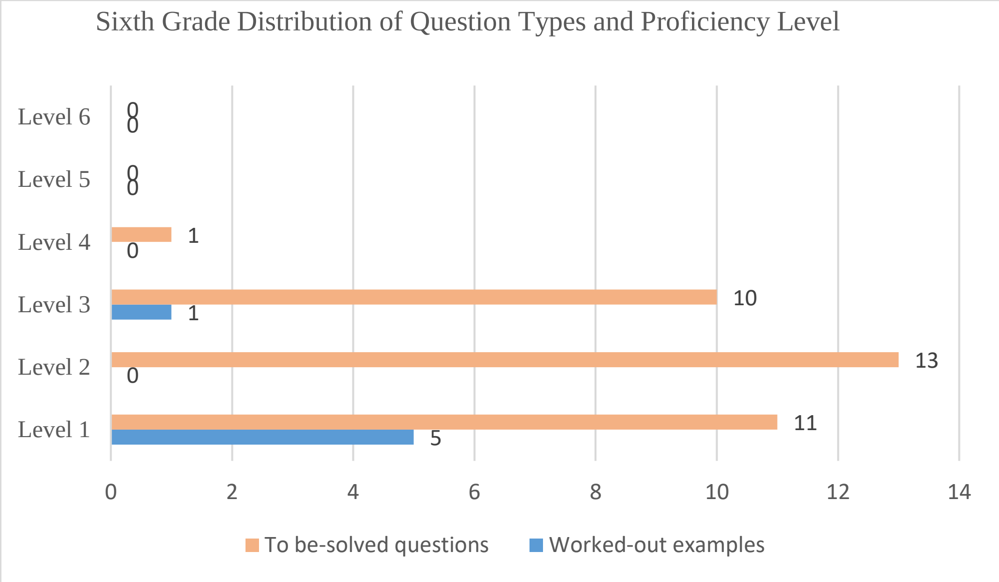
Table 4.21.Distribution of Question Types with Proficiency Level for Fifth Grade Textbook



For fifth grade textbook, lower proficiency level is mostly occurred for both question types. Every proficiency level except Level 4 occurred in this grade level is occurred both question types.

For sixth grade textbook, there are 6 worked out questions, 36 to be solved questions. Their capability domain is given table:

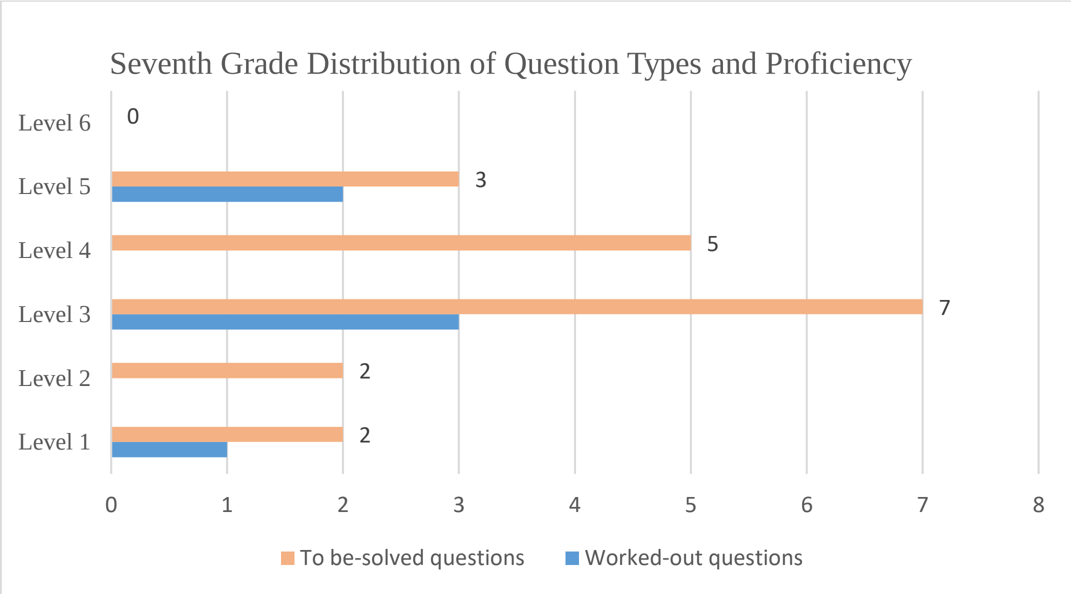
Table 4.22.Distribution of Question Types with Proficiency Level for Sixth Grade Textbook



While number of tasks having proficiency level of Level 1, Level 2, and Level 3 is approximately to each other in to be solved question type, worked-out examples have only two proficiency level which are Level 1 and Level 3.

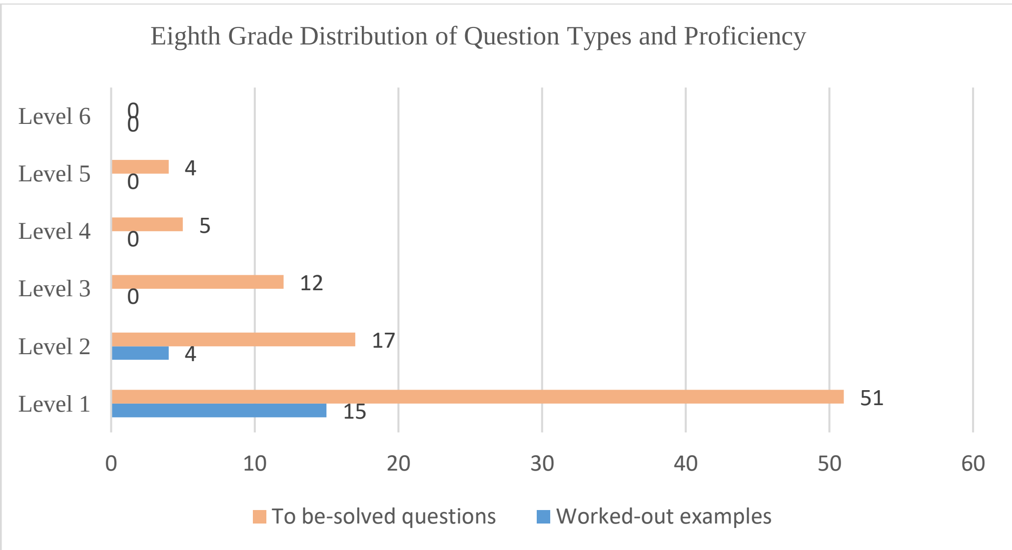
For seventh grade textbook, there are 5 worked out questions, 19 to be solved questions. Their context domain is given table:

Table 4.23. Distribution of Question Types with Proficiency Level for Seventh Grade Textbook



Task with Level 5 is firstly occurred both question types in this grade level, the proficiency level of the tasks are normally distributed compared to the other grade levels. For eighth grade textbook, there are 24 worked out examples, 91 to be solved questions. Their context domain is given table:

Table 4.24. Distribution of Question Types with Proficiency Level for Eighth Grade Textbook



While worked out examples have Level 1 and Level 2, to be solved questions have different proficiency level starting from Level 1 to Level 5.

CHAPTER 5

DISCUSSION

This research aims to examine the requirements of geometry problems in Turkish middle school mathematics textbooks using the PISA 2018 Mathematics Literacy Framework. The results were presented in the following parts: process domain, context domain, capability, and proficiency level of the tasks. The findings of the research demonstrated not only characteristics of the textbook in selected subject but also the relationship between textbooks and national curriculum targets. Therefore, the results of this study enlighten not only educators and curriculum developers but also policy makers.

First of all, in terms of the mathematical process category involving employ, formulate, and interpret, 194 of 220 tasks (88.1%) enable employing mathematical concepts, facts, and procedures, 17 tasks (7.8%) enable formulating situations mathematically, and 9 tasks (4.1%) enable interpreting, applying, and evaluating mathematical outcomes. This means that most of the tasks imply computations and application of rules. According to the PISA 2018, formulating process is a defined link between the real world and mathematical structure, representations, and specificity (OECD, 2019a).Therefore, 189 of 220 tasks were decontextualized. Stacey & Turner (2015b) underlined that student who want to be mathematically literate should construct relations between real life and the representation of the mathematical structure. These findings are compatible with the literature (e.g., İncikabı & Tjoe,2013; Kar, Güler, ¸Sen, & Özdemir, 2018; Delil, 2006; Yalçın & Tavşancıl, 2014; Altun, Gümüş, Akkaya, Bozkurt, & Ülger, 2018).

On the other hand, tasks whose process is interpreting mathematical outcomes are few (only 4.1% of all tasks). As mentioned by PISA 2018 Framework, interpreting mathematical outcomes is defined as translating mathematical solutions or making logical reasonings about the solution (i.e., determining the meaningfulness of the solution and the contexts of the problem) (OECD, 2019a). This process domain is first seen in the seventh-grade textbooks of all the textbooks which are analyzed in this study. Therefore, it can be related to the objectives of seventh grade in the Turkish Mathematics Curriculum. (MoNE, 2018c). These objectives are given below:

- Draw two-dimensional views of three-dimensional objects from different perspectives.
- Construct the structures whose drawings are given regarding their views from different perspectives.

To achieve these objectives, tasks with interpret process domain can be beneficial because while drawing, students interpret and make reasoning about its correctness. However, the objectives of the mathematics curriculum for geometrical objects of fifth and sixth grade are more suitable for the other process domain. Which are given below:

- Recognize rectangular prism and determine its basic elements (5^{th} grade).
- Draw the surface expansions of the rectangular prism and decide whether the given different expansions belong to the rectangular prism (5^{th} grade).
- Solve problems that require calculating the surface area of a rectangular prism. (6^{th} grade)
- Comprehend that volume is the number of unit cubes placed in the rectangular prism in such a way that there is no space, calculate the volume of the given object by counting the unit cubes (6^{th} grade).
- Construct different rectangular prisms with a given volume measure their prisms in unit cubes, explain that volume is the product of the area and the height with justification (6^{th} grade).

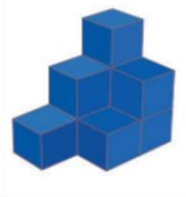
Given those objectives of the fifth and sixth grades above, they are based on knowing features of the shapes and its elements and calculation of the volume and surface area. Therefore, these behaviors are more related to *formulate* and *employ* process domain. In addition, *interpreting mathematical structure* is one of the essential goals of the Ministry of National Education. To achieve this, they have reformed the high school examination called LGS 2018 (MoNE, 2018c). LGS, which is centrally applied, is an assessment and evaluation system. Its purpose is to prepare students for international examination standards and increase qualifications of education from middle schools to high schools. (Obay, Demir, & Pesen, 2021). However, although the examination system changed, any reforms have not occurred to textbooks and other materials, so the intended success for this purpose has not been achieved yet (Gür, Öztürk, Özer, & Suna, 2021). This important goal is not only for Turkey but also for other countries with similar conditions (Hong & Choi, 2014). Interpreting the mathematical structure process is one of the ways to achieve this goal. However, unfortunately, Turkish middle school mathematics textbooks have only 9 tasks whose process is interpret among 220 tasks. This situation contradicts the targets of Ministry of National Education.

The analysis of capabilities of tasks in the middle school mathematics textbook indicated that students face tasks whose capability requirements are different from each other. 2018 Mathematics literacy of PISA has seven fundamental mathematical capabilities which are *‘‘Communication, Mathematising, Representation, Reasoning and Argument, Devising Strategies for Solving Problems, Using Mathematical Tools and Using symbolic, formal, and technical language and operations’’*. There are 220 tasks in middle school mathematics textbooks, and approximately 44% of the tasks’ capability is ‘‘Using symbolic, formal, and technical language and operations’’, approximately 36% of the tasks’ capability is *‘‘Representation’’*, approximately 11% of the tasks’ capability is *‘‘Using Mathematical Tools’’*, approximately 7% of the tasks’ capability is *‘‘Reasoning and Argument’’*, approximately 0.04% of the tasks’ capability is communication. According to the results obtained from the analysis, tasks based on ‘‘Using symbolic, formal, and technical language and operations’’ and ‘‘Representation’’. ‘‘Using symbolic, formal, and technical language and operations’’ capability is mostly based on computation and application of the rules. When students know the rules and procedures, they can easily carry out solution process. Also, as worked-out examples given with their solutions' capability is ‘‘Using symbolic, formal, and technical language and operations'', and this may lead the students to imitate solution process without considering mathematical structures (Tall, 2008). Besides these risks, examples should take a part in a textbook because they provide students opportunities to improve computational skills and relationships between rules and procedures. If the worked-out examples' and to be solved questions' capabilities have homogeneity, students' learning process on their own are affected positively. Another high ratio of the capability in middle mathematics textbooks was ‘‘Representation'' whose requirement is creating mathematical representation of real-world information, connecting, and comparing different representations (OECD, 2019a). As a nature of selected subject in this study ‘‘Representation'' and ‘‘Using symbolic, formal, and technical language and operations'' capability occur more than others because geometrical object subject is based on representations relations and application of some rules like finding volume and surface area of the object. Representation capability is critical for gaining 21st-century skills because it enables students to connect with mathematical structure and enhance problem-solving skills, activating meta-cognition skills (Nizaruddin, Muhtarom, & Murtianto, 2017). Due to the high number of tasks whose capability is ‘‘Representation'', transition of the subject becomes easier for not only students but also teachers. Another capability is ‘‘Using Mathematical Tools'' with the ratio of 11% whose requirements are using mathematical tools such as a ruler, protractor, or dynamic software, GeoGebra, Sketch-up and so on. This capability is essential for the students, especially in terms of geometry subject, to activate different parts of their brain for effective learning since geometry subject is based on drawings, representations, and animations (Kuzniak, Nechache, & Drouhard, 2016). Increasing usage of physical manipulatives and technological tools in geometry education also enables students to have a more meaningful and permanent learning. (Reimer & Moyer, 2005). The other capabilities were found to have a lower ratio or not existing in Turkish middle school textbooks, but these capabilities are as much crucial as placed capabilities. These capabilities with tasks enrich the mathematics classroom, so textbook developers and educators should understand their significance. The analysis of capabilities of tasks in the middle school mathematics textbook indicated that students face tasks whose capability requirements are different from each other.

According to the findings of this study, tasks of middle school mathematics textbooks in the selected subject are mostly based on Level 1 and Level 2 with the ratio of 68% (147 tasks in overall 220 tasks) whose requirements are basic skills in mathematics such as applying rules, following procedures, or making direct inference from the tasks. It is known that specific objectives are considered while preparing the textbooks, and the educators who work as book writers also carry out their studies in line with these objectives of the curriculum (MoNE, 2021). It can be thought that the grade levels of the tasks and the objectives may be directly proportional to each other; in other words, if the grade level increases, the level of the tasks may also increase. Also, the scope of objectives is expanded with an increment in grade level. (MoNE, 2018c). This is consistent with the findings of this study because the proficiency level of the tasks in the fifth and sixth-grade textbook is lower than other levels, as they are mostly Level 1 and Level 2.Although the proficiency level of these grades is mostly lower than others, examples contradict this. For example, for the sixth grade, tasks for objectives of ‘‘Comprehend that volume is the number of unit cubes placed in the rectangular prism in such a way that there is no space, calculate the volume of the given object by counting the unit cubes.’’ are generally Level 1 and Level 2, but there is a task having Level 3.

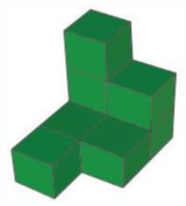
Samples of tasks are given below:

Example, page 336,6th



Let's find the volume of the construction formed by unit cubes on the side.

Exercises, page 339,6th



Find out at least how many cubes should be added to construction formed by unit cubes to form a square prism?

Figure 5.1. Samples of tasks for the same objectives with different proficiency level (Bektaş, Kahraman, & Temel, 2019)

While the textbook task from Figure 5.1 on the left needs only counting unit cubes to find volume, carrying out the explicit process, the other task in the textbook on the right does not focus only on calculating the volume of construction, but also on how many cubes are needed to complete the structure of square prism. For this task, the students should know the qualities of square prisms and be aware that a cube is also a square prism. Thus, the students should relate representations based on information from different sources to each other and make reasoning directly to reach a solution. To improve students' mathematical thinking skills, these kinds of tasks should be placed in a lower grade.

Besides lower proficiency level placed primarily on the fifth and sixth-grade textbook, there are 56 tasks with a ratio of 25% whose proficiency levels were Level 3 and Level 4, whose requirements was basic reasoning and argument skills (OECD, 2019a). The number of tasks with Level 5 and Level 6 proficiency levels requiring advanced mathematical thinking skills was 16, with a 7% ratio. This condition is disadvantageous for textbook users because focusing only on basic mathematical skills does not help improve the students' meta-cognition skills. (Milgram, 2007). Also, it is not consistent with the objective of the national curriculum; the national curriculum aims for students to enhance their mathematical communication, reasoning, and problem-solving skills, which is implying high proficiency levels (MoNE, 2018c). Although further levels, which are Level 5 and 6, require advanced skills such as reasoning argument and proof, there are few questions whose potential skills are Level 5 and Level 6. Having no questions with high proficiency levels may lower scores in international and national examinations. In 2018, the high school examination system was changed from TEOG, whose questions' requirement were knowledge and basic calculations, to LGS, whose questions' requirement are application and analysis (Biber & Tuna, 2017; Ekinci & Bal, 2019). In addition to this, LGS questions' requirements are consistent with PISA Mathematics Literacy Framework, but mathematics textbook questions are not (Öztürk, 2020). Therefore, there are fewer tasks in eight-grade textbooks whose need is similar to national and international examinations. The examples of tasks in eighth-grade textbook and high school examination (LGS) with the same objectives are given below:

- Recognize the right pyramid, determine its basic element, construct and draw its nets.

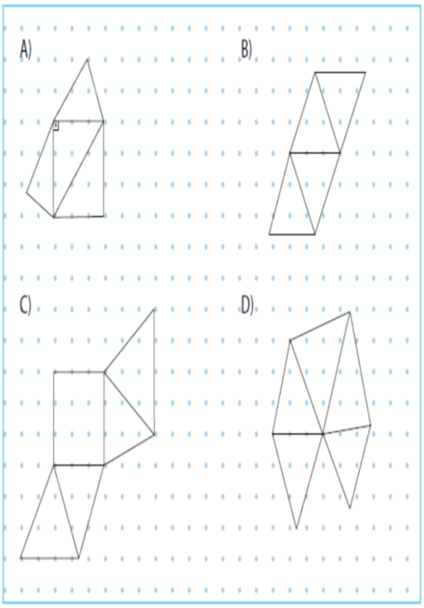
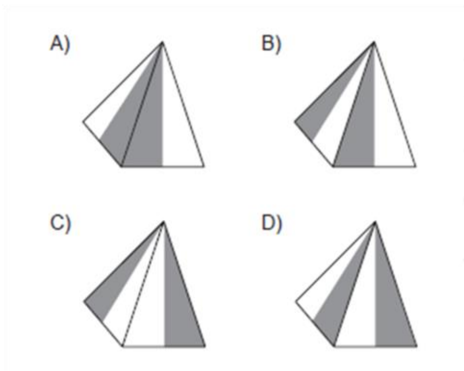
Let's Apply, page 313,8th grade, (Serçifeli & Atmaz, 2018)	**LGS 2018- 8th grade, (MoNE,2018)**
Which of the following nets could belong to an equilateral triangular pyramid? 	 A part of the outer surface of a square vertical pyramid made of white cardboard is painted gray. Which of the following is not a view of this pyramid? 

Figure 5.2. Samples of tasks for same objectives with different proficiency level for eighth grade

While the task given on the left in the eighth-grade textbook only assesses if students know nets of the equilateral triangular pyramid or not, the task given on the right in LGS assesses students' reasoning and well-developed thinking skills. However, fewer questions in the textbooks focus on these skills, as seen in the example. This condition also creates a contradiction between textbooks' content and examination. Students ' performance may increase if mathematics textbooks have high proficiency level questions. (Hong & Choi, 2014; Öztürk, 2020).

Up to this point, the mentioned 220 questions within the 2018 Mathematics literacy framework domains,which are process, capability, context, and proficiency levels, were criticized. Turkish middle mathematics textbooks for geometrical objects subjects do not have high-level thinking questions, and they are based on basic computational and application skills. Ministry of National Education changes high school entrance examinations without considering current conditions such as students' backgrounds, teacher training, and textbook contents. Also, This may cause a decrease in students' performance in the national examination. The average number of correct answers in mathematics test was 8.42 in the 2017 high school examination called TEOG(42.05 points calculated out of 100 scores, correct answers were transformed into several correct answers). In 2018, the number of correct answers in Mathematics decreased to 6.99(MoNE, 2017; MoNE, 2018b). This study's findings also support these results. Therefore, for international and national success, Turkey should revise educational instruments starting form to textbooks.

5.1. Implications of the study

The striking finding obtained from the textbook research is that the textbooks do not contain all the six-proficiency levels and contexts determined by PISA. There were mostly Level 1 and Level 2 tasks, which are defined as low level. To overcome that deficiency, the proficiency levels should be increased by preparing the tasks in the textbooks following the daily life and multiple entry points problems involving context, which require the students to work more with concrete materials and encourage group work.

The research findings revealed that the distribution of domain of the tasks within the framework domain in the eighth-grade textbooks is much more balanced than the other grade levels, which is implying this distribution becomes unorganized in the lower grade levels. Although an increasing number of tasks and expansion of objectives may cause this, the other grades' textbooks must examine the subject in more detail and include tasks with different domain requirements in PISA, just like eighth-grade textbooks. Also, the lower grade textbooks should be prepared considering this. Therefore, improving mathematical thinking skills in lower grades is more significant (Sak & Maker, 2010).

This research is limited to the geometrical objects subject and one textbook in each grade. For more contributions to literature, content analysis of textbooks used in this study may be applied to subjects like numbers and operations, measurement, and data analysis. Also, the PISA 2021 Mathematics Literacy Framework or different international frameworks like the TIMMS Mathematics framework may enlighten the condition of textbooks with international standards.

As a result, I have demonstrated to what extent middle school textbook includes PISA 2018 Mathematics Literacy Framework requirements in geometrical objects tasks. Students' difficulty with solving geometrical objects tasks may come from the limited learning and practice environment facilities, and textbooks have a critical impact on them. Therefore, this research significantly contributes to the literature on these subjects.

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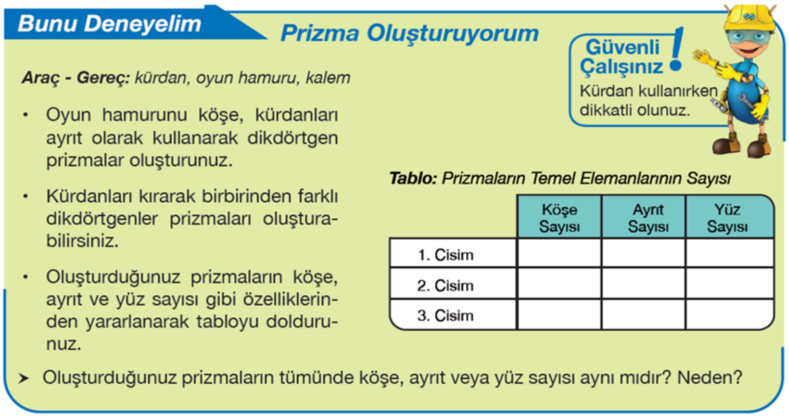
APPENDIX

Examples of The Other Middle School Textbook



Bunu Deneyelim — *Prizma Oluşturuyorum*

Araç - Gereç: *kürdan, oyun hamuru, kalem*

- Oyun hamurunu köşe, kürdanları ayrıt olarak kullanarak dikdörtgen prizmalar oluşturunuz.
- Kürdanları kırarak birbirinden farklı dikdörtgenler prizmaları oluşturabilirsiniz.
- Oluşturduğunuz prizmaların köşe, ayrıt ve yüz sayısı gibi özelliklerinden yararlanarak tabloyu doldurunuz.

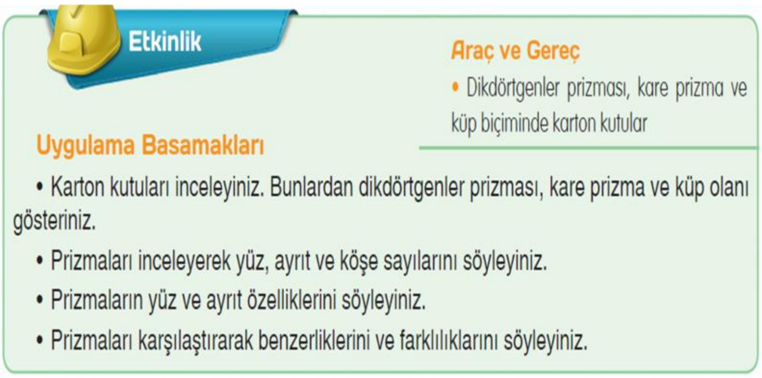
Güvenli Çalışınız! Kürdan kullanırken dikkatli olunuz.

Tablo: *Prizmaların Temel Elemanlarının Sayısı*

	Köşe Sayısı	Ayrıt Sayısı	Yüz Sayısı
1. Cisim			
2. Cisim			
3. Cisim			

➤ Oluşturduğunuz prizmaların tümünde köşe, ayrıt veya yüz sayısı aynı mıdır? Neden?

(Cırıtcı, et al, 2018)



Etkinlik

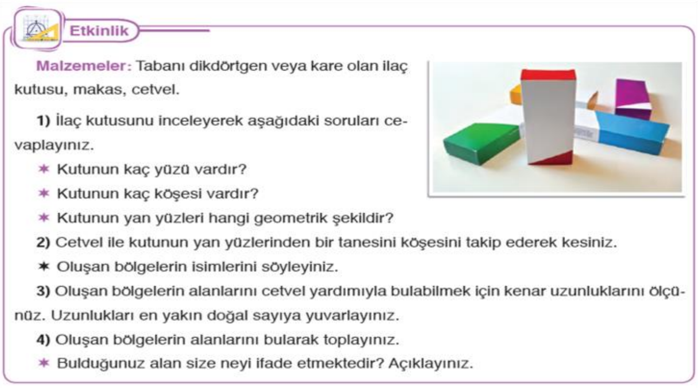
Araç ve Gereç

- Dikdörtgenler prizması, kare prizma ve küp biçiminde karton kutular

Uygulama Basamakları

- Karton kutuları inceleyiniz. Bunlardan dikdörtgenler prizması, kare prizma ve küp olanı gösteriniz.
- Prizmaları inceleyerek yüz, ayrıt ve köşe sayılarını söyleyiniz.
- Prizmaların yüz ve ayrıt özelliklerini söyleyiniz.
- Prizmaları karşılaştırarak benzerliklerini ve farklılıklarını söyleyiniz.

(Erenkuş & Eren-Savaşkan,,2019)



Etkinlik

Malzemeler: Tabanı dikdörtgen veya kare olan ilaç kutusu, makas, cetvel.

1) İlaç kutusunu inceleyerek aşağıdaki soruları cevaplayınız.

- ✶ Kutunun kaç yüzü vardır?
- ✶ Kutunun kaç köşesi vardır?
- ✶ Kutunun yan yüzleri hangi geometrik şekildir?

2) Cetvel ile kutunun yan yüzlerinden bir tanesini köşesini takip ederek kesiniz.

- ✶ Oluşan bölgelerin isimlerini söyleyiniz.

3) Oluşan bölgelerin alanlarını cetvel yardımıyla bulabilmek için kenar uzunluklarını ölçünüz. Uzunlukları en yakın doğal sayıya yuvarlayınız.

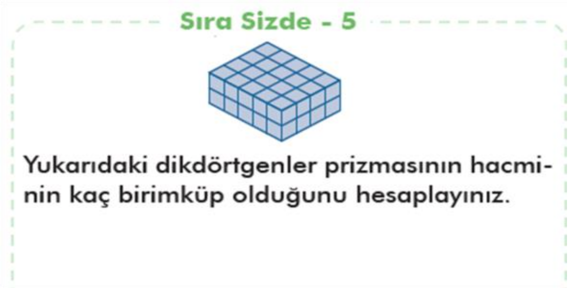
4) Oluşan bölgelerin alanlarını bularak toplayınız.

- ✶ Bulduğunuz alan size neyi ifade etmektedir? Açıklayınız.

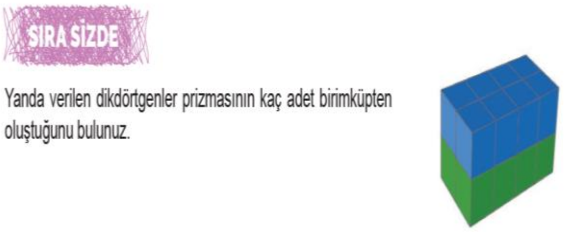
The Selected Textbook

(Karakuyu, 2018)

Figure A.1. Examples of Fifth grade



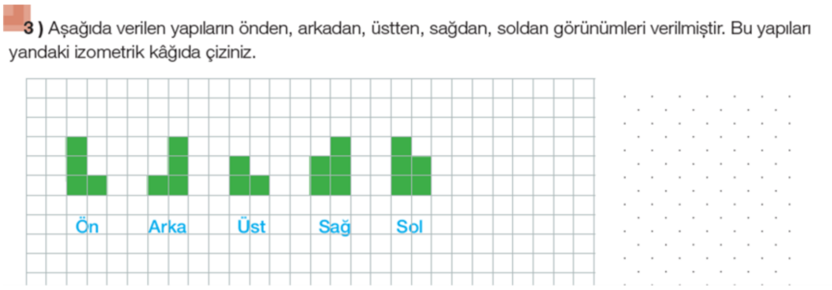
(Çağlayan, Dağıstan, & Korkmaz, 2018



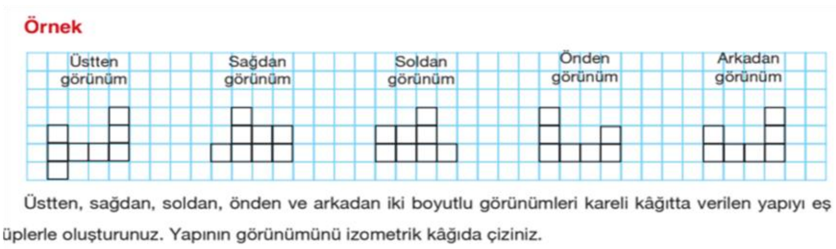
The Selected Textbook

(Bektaş, Kahraman, & Temel, 2019)

Figure A.2. Examples of Sixth grade



(Akbulut, 2018)



The selected textbook (Keskin-Oğan & Öztürk, 2019)

Figure A.3. Examples of Seventh grade

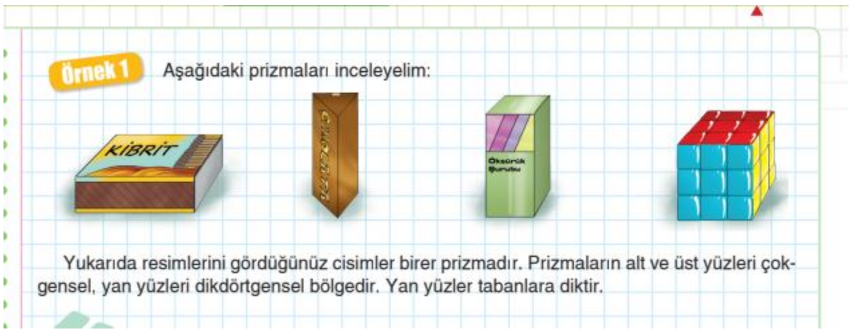
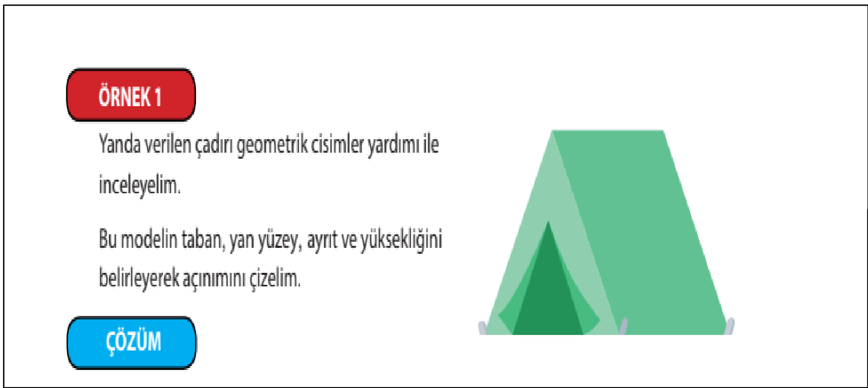
	(Erenkuş & Savaşkan, Middle and Imam-Hatip Middle School Mathematics Textbook 5 , 2018)
	(Çetin, Aksakal, Ertürk, Şay, & Tığlı, 2019)
	The selected textbook (Serçifeli & Atmaz, 2018)

Figure A.4. Examples of Eighth grade