

# Liquidity and Asset Pricing under the Presence of Market Frictions; with an Analysis

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**Abstract.** This paper provides a literature review on the models explaining the relationship between liquidity and asset prices with an emphasis on the Vayanos and Wang's model (2012) which is one of the most significant contributions to this literature.

*Keywords.* market liquidity, market frictions, illiquidity measures, asset pricing

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TERM PROJECT

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Liquidity and Asset Pricing under the Presence of Market Frictions;  
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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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# 1 Introduction

The studies on the standard asset pricing models are rested on the friction-less assumption which refers to perfect liquid market in general. The concept of perfect liquid market means that there exists a trading ability of securities without any cost in all period-time as well as price-taker agents. On the other hand, the presence of friction-less market is impossible in the real world. There are many types of frictions in the market, which are asymmetric information, search frictions, inventory frictions some specific economic frictions, and the like. Before the Vayanos and Wang's (2012) studies, there exist few studies on the effects of a market friction on asset pricing in the literature. Asymmetric information is one of the market friction. Glosten and Milgrom [1] and Kyle [2] study the impact of asymmetric pricing, and the studies reveal the evidence on the presence of positive association of asymmetric information with market illiquidity through applying the bid-ask spread and by Kyle's lambda which is known as price impact. In addition to these studies, O'Hara [3] and Easley and O'Hara [4] analyse how asymmetric information creates an impact on expected returns of risky assets. This analysis is based on multi-asset extension of Grossman and Stiglitz (1980).

In the literature, market liquidity is seen as a complex concept. In general, the concept of market liquidity is defined as a market feature including the ability of agent to purchase or buy assets and securities at a determined current to get ready cash without impacting its market price /value. In brief, the market liquidity refers to the ease to trade a financial security [5]. And, the source of liquidity is discussed to a large extent in the literature. Demand pressure, inventory risk, exogenous transaction costs including brokerage fees, order-processing costs, or transaction taxes, and also location problems for the counter-party participating in trading a particular security are some of the reasons for the presence of market liquidity [5]. Additionally, Demsetz [6] discuss the determinants of liquidity, which are trading volume and the number of traders, firm size, price and the volatility. Moreover, the existence of a positive association of trading activity with liquidity is shown whereas the presence of an adverse association between trading activity and volatility is demonstrated in the studies of Tinic [7] and Benston and Hagerman [8].

The project paper is organized as follows. The section 2 covers the literature review in both theoretical and empirical studies on asset pricing models and market

liquidity measure methods. In the section 3, as an example model, the unified model of Vayanos and Wang (2012) is summarized in general. And also, finally, we present a summarizing part on the findings about the effects of market frictions on asset pricing and liquidity measures, using this unified model. The last section is the conclusion part.

## 2 Literature Review

### 2.1 Asset Pricing Models under The Market Frictions

There are quite a few theoretical studies on the liquidity-based asset pricing models under the existence of the market frictions.

Liu and Wang (2010) provide an analysis on the effect of asymmetric information, imperfect competition among market makers, and risk aversion on market illiquidity, trading volume, bid and ask prices, bid and ask spread, and welfare by introducing a tractable equilibrium model[9]. And this analysis shows remarkable findings. First finding is related to the decreasing effect of asymmetric information on the welfare loss because of the presence of decreasing impact of asymmetric information market power and market depth[9]. The second finding is that the market-making cost leads to a reduction in the equilibrium number of market makers whereas an increase in the equilibrium number of market makers occurs due to the existence of both trading volume and asymmetric information [9].

Capital gain taxation is another important issue in asset pricing and its effect on market liquidity. Sahm examines how the capital gain taxation, which is either accrual or realization based, affects asset prices and welfare within a simple general equilibrium model of an exchange economy [10]. The paper reveals the fact that much higher asset prices are obtained in the case of a realization tax, compared to the case of an accrual tax. However, the paper cannot reach a conclusive finding on the effect of a switch from an accrual to a realization system on welfare [10].

Another study on risky asset pricing is Vayanos and Vila's [11]. They construct an OLG model with a risky asset, proportional trading costs and a liquid riskless asset in fixed supply. In this way, they endogenize the riskless interest rate. In the model, the risk-free asset is considered as another way for an investment in the case of the presence of higher trading cost for a risky asset[11]. And they conclude that this situation of increasing trading cost of the risky asset causes a rise in the equilibrium price of the risk-free asset[11]. The OLG model of Vayanos and Vila's [11]. is advanced by Heaton and Lucas [12]. They study an equilibrium model of incomplete risk-sharing. Heaton and Lucas reach two significant conclusion. First one is related to the presence of sizeable impacts in the cases of either larger trading costs or the small quantity of traded assets[12]. The second conclusive finding is that

an increase in the equity premium is due to trading costs whereas the decrease in the equity premium is due to riskless rate[12].

## 2.2 Market Illiquidity Measures

Market illiquidity is considered as a key concept in financial markets. And this results from a consequence of market frictions and accordingly, the illiquidity transitorily affects asset prices.

Based on the study of Backer (1996), Gabrielsen et. al. [13] summarizes three fundamental properties of a liquid market which are depth, breadth, and resiliency. Generally, these terms are explained as follows (Gabrielsen et. al., [13]): The depth of a market implies the ability of market so that it can absorb market orders with upper and lower of the trading asset prices. The breadth of a market refers to the volume degree of buying and selling orders. And the resiliency of a market means the adjustment of order flows in response to price changes.

The earliest models on different market liquidity measures are proposed by Glosten and Milgrom [1]. Their model is based on the bid-ask spread. In addition to this, the another market illiquidity measurement model which is introduced by Glosten and Milgrom [1] depends on the sensitivity of price to quantity. Easley and O'Hara [4] introduces a hybrid model. This model allows the private information of agents in stochastic time span. And there are some recent studies on the market liquidity measures based on the bid-ask spread. For instance, Foucault, Kadan, and Kandel [14], Goettler, Parlour, and Rajan [15] and Rosu [16] publish studies on the determination of the bid-ask spread which is arisen from the agents' preferences between market and limit orders, the expected time for limit orders to execute and the like.

In general, the measure approaches of market illiquidity are categorized into four fundamental groups including heuristic measures, price reversal measures, price impact measures and others. Basically, the heuristic measures are relied on such observed bond characteristics as age, maturity, issuance, turnover, number of traders and par value volume (Rayanakorn, [17]). The second type of illiquidity measure is the price impact ( $\lambda$ ). Vayanos and Wang state that the measure of price impact can seize both the permanent component and the transitory component of trade impact. The permanent component results from trade information while the transitory component results from risk aversion degree of liquidity suppliers. The return per volume



of Amihud [18] and the coefficient of return regression on signed volume of Vayanos and Wang are the examples of this type of liquidity measure. The third type of illiquidity measure is price reversal ( $\gamma$ ) which includes the bid-ask spread illiquidity measures introduced by Roll [19], and minus autocovariance introduced by Bao, Pan, and Wang [20]. The measure of price reversal consists of the transitory part of price changes. Furthermore, there exist other measure types of market illiquidity such as the volume-based liquidity measures, and the price-variability indices. Generally, the volume-based liquidity measures are based on the association of price with an asset quantity. The traded volume, conventional liquidity ratio, Martin's liquidity index [21], Hui and Heubel's liquidity index [22], the turnover ratio can be given as examples of volume-based liquidity-measures. The measure indices of price-variability, in general, depend on the changes in price behavior. The Marsh and Rock [23] liquidity ratio and the variance ratio, which is introduced by Lo and MacKinlay [24], are the most widely-used and known price-variability indices.

In the paper of Vayanos and Wang , the price reversal ( $\gamma$ ) calculated by minus autocovariance, and the price impact ( $\lambda$ ) calculated by the coefficient of return regression on signed volume are used. The findings related to market illiquidity, which is obtained from the application of Vayanos and Wang' model , are explained in the Section 3.

### 3 An Example Model

In this section, we will briefly explain the Vayanos and Wang's model as an example model [25].

The model consists of three periods,  $t = 0, 1, 2$ . Basically, the model is based on Grossman and Stiglitz's canonical framework. Vayanos and Wang replace the noise trades in Grossman and Stiglitz's model with rational hedgers. Financial market comprises of riskless and risky assets. In period 0, all agents are assumed to be identical, which means that there is no trade. And, the endowments of agents are the supply per capita of the riskless and risky asset in period 0. In Period 1, risk averse agents become heterogenous and start to trade riskless and risky assets that pay off in Period 2. It is assumed that heterogeneity exists through agents' endowment and information, but there is no preference heterogeneity due to the same utility function. In other words, in Period 1, agents are two types which are liquidity demanders and

liquidity suppliers. Liquidity demanders who will receive in Period 2 an endowment covarying with the risky asset's payoff whereas liquidity suppliers who will receive no endowment. Additionally, the covariance between the endowment and the risky asset's payoff is assumed to be private information only for liquidity demanders, and hence it is accepted as the source of trade.

### 3.1 Variables used in the model

The riskless asset is in supply of  $B$  shares and pay off one unit with certainty. And, the riskless asset is the numeraire. Moreover, the risky asset is in supply of  $\bar{\theta}$  shares and pay off  $D$  units, where  $D \sim N(\bar{D}, \sigma^2)$ .  $S_t$  represents price of risky asset at period  $t = 0, 1, 2$ . And the price of risky asset in Period 2 is assumed to equal to the pay off of risky asset, i.e.  $S_2 = D$ . As for the utility function of agents, it is defined based on consumption in Period 2 ( $C_2$ ) as follows:

$$U(C_2) = -exp(-\alpha C_2), \tag{1}$$

where  $\alpha > 0$  refers to the coefficient of absolute risk aversion.

The wealth of an agent in period  $t$  is represented by  $W_t$ . And, an agent's wealth in Period 2 is assumed to be equal to an agent's consumption in Period 2, i.e.  $W_2 = C_2$ .

In the model, there are key assumptions about receiving endowment. The portion  $1 - \pi$  of agents receives no endowment whereas the portion  $\pi$  of agents receives an endowment  $z(D - \bar{D})$  of consumption good, where  $z \sim N(0, \sigma_z^2)$  shows liquidity shock. Endowments are received in Period 2, but agents know whether they will receive it or not before trade in Period 1, which means that there is an interim Period  $t = 1/2$  when the agents are informed. Another important assumption is that only agents receiving the endowment can observe  $z$  in Period 1. It is important to note that under the normality condition of  $D$  and  $z$ , the endowment  $z(D - \bar{D})$  takes large negative value, which implies infinitely negative utility. Accordingly, in order to ensure finite utility, the variances of  $D$  and  $z$  satisfy the condition  $\alpha^2 \sigma^2 \sigma_z^2 < 1$ .

### 3.2 The computation of price function in the model

In order to compute the equilibrium, Vayanos and Wang (2012) use backward induction. At the first step, they we go backward from Period 2 to Period 1.

Initially, they start with considering the equilibrium for liquidity demanders. For liquidity demanders, the number of holding risky assets by liquidity demanders in Period 1 is represented by  $\theta_1^d$ . The consumption in period 2 is wealth of liquidity demanders in Period 2 which is equal to the sum of wealth in Period 1, capital gain from risky assets and the endowment. And,  $\theta_1^d(S_2 - S_1) = \theta_1^d(D - S_1)$  gives gain of risky assets for liquidity demanders. So, the budget constraint for liquidity demanders in Period 2 is given as:

$$C_2^d = W_1 + \theta_1^d(D - S_1) + z(D - \bar{D}) \quad (2)$$

In order to find the equilibrium for liquidity demanders, we need to maximize the utility function in the Equation (1) subject to the budget constraint in the Equation (2). And, through the utility maximization, the optimal liquidity demander's demand function for the risky asset in Period 1 is found as follows:

$$\theta_1^d = \frac{(\bar{D} - S_1)}{\alpha\sigma^2} - z. \quad (3)$$

And then, the authors continue with considering the equilibrium for liquidity suppliers. For liquidity demanders, the number of holding risky assets by liquidity suppliers in period 1 is represented by  $\theta_1^s$ . All calculations are the same, except for that the liquidity suppliers do not receive endowment, they cannot observe  $z$ ; thus,  $z = 0$ . Accordingly, the optimal liquidity supplier's demand function for the risky asset in period 1 is calculated as

$$\theta_1^s = \frac{(\bar{D} - S_1)}{\alpha\sigma^2}. \quad (4)$$

After obtaining the optimal demand functions the risky asset in period 1 for both types of agents, the market clearing condition is considered. The market clearing condition implies that

$$\pi\theta_1^d + (1 - \pi)\theta_1^s = \bar{\theta}. \quad (5)$$

When we integrate the optimal demand functions in the Equations (3) and (4) into the Equation (5), the price of the risky asset in Period 1 is obtained as

$$S_1 = \bar{D} - \alpha\sigma^2(\pi z + \bar{\theta}). \quad (6)$$

The price of the risky asset in Period 1 depends on the liquidity shock  $z$ . That's, the price of the risky asset in Period 1 decreases when the liquidity shock  $z$  occurs.

As the second step, the authors go to the Period 0. The model assumes that all agents are identical in Period 0, and so, both types of agents have the same number of holding risky assets in Period 0 ( $\theta_0$ ). The wealth of agents in period 1, which is equal to the sum of wealth in period 0 and total capital gain from risky asset ( $\theta_0(S_1 - S_0)$ ), is expressed as

$$W_1 = W_0 + \theta_0(S_1 - S_0). \quad (7)$$

The expected utility functions for liquidity suppliers and demanders are respectively obtained as

$$U^s = -exp\left\{-\alpha(W_0 + \theta_0(\bar{D} - S_0) - \theta_0\alpha\sigma^2(\pi z + \bar{\theta}) + \frac{1}{2}(\pi z + \bar{\theta})^2(\alpha\sigma^2))\right\} \quad (8)$$

$$U^d = -exp\left\{-\alpha(W_0 + \theta_0(\bar{D} - S_0) - \theta_0\alpha\sigma^2(\pi z + \bar{\theta}) + \frac{1}{2}(\pi z + \bar{\theta})^2(\alpha\sigma^2) - z\alpha\sigma^2(\pi z + \bar{\theta}))\right\} \quad (9)$$

An agent's expected utility in period 0 is calculated as

$$U = (1 - \pi)U^s - \pi U^d \quad (10)$$

After inserting the Equations (8) and (9) into the Equation (10), the re-written utility function  $U$  is maximized with respect to  $\theta_0$ , and the price of the risky asset in Period 0 is found as follows:

$$S_0 = \bar{D} - \alpha\sigma^2\bar{\theta} - \frac{\pi M}{1 - \pi + \pi M}\Delta_1\bar{\theta}, \quad (11)$$

where

$$M = \exp\left(\frac{1}{2}\alpha\Delta_2\bar{\theta}^2\right) \sqrt{\frac{1 + \Delta_0\pi^2}{1 + \Delta_0(1 - \pi)^2 - \alpha^2\sigma^2\sigma_z^2}}$$

$$\Delta_0 = \alpha^2\sigma^2\sigma_z^2 \text{ and } \Delta_1 = \frac{\alpha\sigma^2\pi\Delta_0}{1 + \Delta_0(1 - \pi)^2} - \Delta_0 \text{ and } \Delta_2 = \frac{\alpha\sigma^2\Delta_0}{1 + \Delta_0(1 - \pi)^2 - \alpha^2\sigma^2\sigma_z^2}$$

It is important to note that  $\alpha\sigma^2\bar{\theta}$  gives the risk discount which is independent of  $\sigma_z^2$ , and  $\frac{\pi M}{1 - \pi + \pi M}\Delta_1\bar{\theta}$  shows illiquidity discount. The illiquidity discount consists of two parts which are the risk-neutral probability of being a liquidity demander ( $\frac{\pi M}{1 - \pi + \pi M}$ ) and the discount that an agent would require conditional on being a demander ( $\Delta_1\bar{\theta}$ ).

### 3.3 Illiquidity Measures

#### 3.3.1 Price Impact ( $\lambda$ )

Vayanos and Wang (2012) define the value of lambda as the regression coefficient of the price change between Periods 0 and 1 on liquidity demanders' signed volume in Period 1. And accordingly, the value of lambda characterizes the price impact of liquidity demanders' trades.

Firstly, the authors consider the perfect-market benchmark. And the lambda is defined from the two aspects including liquidity demanders and liquidity suppliers. From the aspect of liquidity demanders, the regression equation is expressed as follows:

$$(S_1 - S_0) = \lambda * \pi(\theta_1^d - \bar{\theta}) + e, \quad (12)$$

where  $\pi(\theta_1^d - \bar{\theta})$  shows the signed volume of liquidity demanders in Period 1,  $(S_1 - S_0)$  shows the asset's return between Periods 0 and 1, and  $\lambda$  represents the coefficient of a regression of the asset's return between Periods 0 and 1 on the signed volume of liquidity demanders in Period 1. Considering the rules of a linear regression,  $\lambda$  is calculated by the following formula:

$$\lambda = \frac{\text{cov}(S_1 - S_0, \pi(\theta_1^d - \bar{\theta}))}{\text{var}(\pi(\theta_1^d - \bar{\theta}))}. \quad (13)$$

In the model, the asset price in Period 1 and the number of holding risky assets by liquidity demanders are found in the Equations (6) and (3). Based on these equations, it can be obtained that

$$S_1 - S_0 = \bar{D} - \alpha\sigma^2(\pi z + \bar{\theta}) - S_0$$

$$\pi(\theta_1^d - \bar{\theta}) = -(1 - \pi)\pi z$$

Accordingly, it can be computed that

$$cov(S_1 - S_0, \pi(\theta_1^d - \bar{\theta})) = \alpha\sigma^2\pi^2(1 - \pi)\sigma_z^2$$

$$var(\pi(\theta_1^d - \bar{\theta})) = \pi^2(1 - \pi)^2\sigma_z^2$$

So,

$$\lambda = \frac{\alpha\sigma^2}{1 - \pi} \tag{14}$$

In the case of larger  $\alpha$  value, the asset is implied to be riskier ( $\sigma^2$  is larger), or agents are more risk averse which indicates higher price impact ( $\lambda$ ).

As for the aspect of liquidity suppliers, the regression equation is expressed as follows:

$$(S_1 - S_0) = \lambda * [-(1 - \pi)(\theta_1^s - \bar{\theta})] + e. \tag{15}$$

And the rest of the calculations are the same. And the price impact for liquidity suppliers is obtained to be the same as that of liquidity demanders.

### 3.3.2 Price Reversal ( $\gamma$ )

Price reversal is calculated by minus of the autocovariance of price changes. While calculating price reversal, Vayanos and Wang (2012) consider the perfect-market benchmark initially. Considering the price functions of risky asset in Period 1 and 2, the returns in Period 1 and 2 are calculated as follows:

$$S_2 - S_1 = D - \bar{D} + \alpha\sigma^2(\pi z + \bar{\theta}) - S_0$$

$$S_1 - S_0 = \bar{D} - \alpha\sigma^2(\pi z + \bar{\theta}) - S_0$$

Accordingly, the price reversal is found as follows:

$$\gamma = -cov(S_2 - S_1, S_1 - S_0) = \alpha^2\sigma^4\pi^2\sigma_z^2 \quad (16)$$

So, the cases which price reversal  $\gamma$  is higher are listed as follows:

- Agents are more risk averse ( $\alpha$  high),
- the asset is riskier ( $\alpha^2$  high),
- there are more liquidity demanders ( $\pi$  high),
- liquidity shocks are larger ( $\sigma_z^2$  large).

So far, these sections summarize the findings on the model of Vayanos and Wang (2012) [25].

### 3.4 Frictions in The Model

Vayanos and Wang also publish two studies to analyze six different types of market frictions [26] and [27]. In this part, we mention about these findings on the effects of market frictions by using their model which is explained in the previous part. In period 0, perfect market assumptions hold whereas market frictions are related to trade in period 1. Firstly, the study of Vayanos and Wang (2010) take asymmetric information into consideration. The case of asymmetric information is modeled through a private signal  $s$  about the risky asset payoff  $D$  that Liquidity demanders can observe in Period 1. And the signal is defined as  $s = D + \epsilon$ , where  $\epsilon \sim N(0, \sigma_{\epsilon^2})$  is independent of  $(D, z)$ . It is assumed that only the agents receiving the endowment can observe this private signal whereas liquidity suppliers are all uninformed. And, the price functions and the measures of market illiquidity including price reversal and price impact are calculated. The expression of the price impact under asymmetric information include learning effect, which implies that the liquidity suppliers seek for learning signals from the price. Secondly, Vayanos and Wang (2012) consider the case of imperfect competition in their study. The imperfect competition is modeled through assuming collusion of liquidity demanders and assuming that they

Type of Imperfections	Impact of Imperfection		
	Lambda	Price Reversal	Expected Return
Asymmetric information	+	+/-	+
Non-competitive behavior	0/+	+/-	+/-
Participation costs	+	+	+
Transaction costs	+	+	+
Leverage constraints	+	+	+
Search frictions	+/-	+/-	+/-

Figure 1: The Efficient Frontiers [26]

exert market power in period 1, which means that liquidity demanders can collude and behave as a single monopolist in Period 1 whereas liquidity suppliers exhibit monopolistic behaviors. The price impact and price reversal under the case of imperfect competition are found to be the same as the lambda value under asymmetric information case.

In addition to this paper published in 2010, Vayanos and Wang extend this analysis about the effect of market friction on asset pricing and market illiquidity measures by considering such following market imperfections as participation costs, transaction costs, leverage constraints, non-competitive behavior and search frictions. The effects of each imperfection on market illiquidity measured by both lambda and price reversal, and expected returns are summarized in the Figure 1.

Participation cost means that when agents decide to enter a trade in the interim period once they are informed about the status of receiving an endowment, they encounter with a cost in Period 1. That's to say, participation is assumed to be an ex-ante decision. An increase in participation cost leads to an increase in lambda, price reversal and expected return. Furthermore, transaction costs are also taken into consideration, which is introduced into the model in period 1. Vayanos and Wang consider the costs as a separate market imperfection, and the model assumes both proportional and fixed costs. Proportional transaction costs account for proportionality to transaction size whereas fixed costs ignore the dependency of transaction size. The transaction cost is assumed to subject to the change in the price in Period



1. And also, transaction costs result in a bid-ask spread in Period 1, which means that when an agent buys one share, s/he pays an effective bid price that is equal to the price of risky asset plus the transaction cost, or when an agent sells one share, s/he receives an effective bid price that is equal to the price of risky asset minus the transaction cost. Transaction costs increase  $\lambda$ , price reversal and expected return. The reason why an increase in transaction costs causes an increase in price impact is due to the dissuasive feature of transaction costs from trading for liquidity suppliers. In addition to transaction cost, leverage constraints is accounted for. And leverage constraints briefly refer to putting limit to agents' borrowing ability. And the limitation of agents' leverage is considered as a function of capital. For that, the model assumes that agents can cover losses on levered positions in full. Leverage constraints also increase  $\lambda$ , price reversal and expected return. Moreover, search frictions are explained as the case that agents must search for counter-parties in period 1. More explicitly, it means that there is a bilateral bargaining between a liquidity demander and a supplier over the terms of trade including the number of shares traded and the share price. In the case that liquidity demanders have the most of the bargaining power in their bilateral meetings with suppliers, they can reach the negotiation on the better price compared to the case of centralized market; therefore,  $\lambda$  and price reversal drop due to search frictions. In addition, search frictions have a decreasing effect on expected return of the risky assets.

## 4 Conclusion

The markets with no friction and perfect liquidity assumptions is not valid for real world. All types of frictions play an important role in in all markets because it reduce liquidity. The theory of asset pricing models under market imperfections is growing. Vayanos and Wang contribute to this literature to a large extent by studying many different concepts on this issue. The most significant study is the paper of Vayanos and Wang (2012). In this study, we summarize their all significant studies and conclusive findings.

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