THE STRUCTURE AND UNITY OF ATOMIC FREGEAN THOUGHTS: AN EXPLICATION AND EMENDATION

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ABSTRACT

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This thesis is about the structure and unity of atomic Fregean Thoughts as abstract structured entities which are the meanings and the primary truth-bearers of sentences as well as the objects of attitudes. We explicate Frege's theory of *Thoughts* to argue that it is the most adequate semantic theory for explaining the meaning and truth of natural language expressions. However, there are certain problems concerning the structure and unity of Fregean Thoughts. Our initial conjecture is that Frege's theory is incomplete in the sense of being vulnerable to certain puzzles and paradoxes, hence the problems concerning the structure and unity of atomic Thoughts demonstrate the apparent need of certain interpretations and supplementations to Frege's semantic framework. For the problems concerning the structure, we focus on the apparent tension between the Context Principle and the Compositionality Principles, and Frege's conflicting theses regarding the structural analysis and decomposition of Thoughts. For the problems concerning the unity, we deal with a unique problem, namely the concept *horse* paradox. In this thesis, we present our

emendatory framework to interpret the sense and denotation functions as lambda-abstracts and provide our solution to the paradox. We claim that a grammatical subject, i.e., the concept *horse*, can denote a concept occurring in a context in which it is mentioned. Hence, the occurrence of the concept *horse* denotes an unsaturated concept. To conclude, we provide a satisfactory semantic theory of Thoughts and preserve the nature of Fregean intuitions about the philosophy of language.

Keywords: Fregean Thoughts, structured propositions, unity of propositions, the concept *horse* paradox, lambda-abstraction

ÖZ

FREGECİ ATOMSAL DÜŞÜNCELERİN YAPISI VE BİRLİĞİ:
BİR AÇIMLAMA VE GÜÇLENDİRME

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Bu tez, soyut yapısal varlıklar olarak cümlelerin anlamları, birincil doğruluk taşıyıcıları ve aynı zamanda önermesel tutumların nesneleri olan Fregeci atomsal Düşüncelerin yapısı ve birliği hakkındadır. Frege'nin Düşünceler teorisini, onun doğal dil ifadelerinin anlamını ve doğruluğunu açıklamak için en uygun anlambilimsel kuram olduğunu iddia ederek için açımlıyoruz. Ne var ki, Fregeci Düşüncelerin yapısı ve birliği ile ilgili bazı sorunlar vardır. İlk kestirimimiz, Frege'nin kuramının belirli problemlere ve paradokslara karşı savunmasız olma anlamında bazı eksikleri olduğudur, bu nedenle atomsal Düşüncelerin yapısı ve birliği ile ilgili problemler, Frege'nin anlambilim kuramı çerçevesinde belirli yorumlamaların ve düzeltici eklemelerin ihtiyacını zorunlu kılmaktadır. Yapıyla ilgili problemler için, Bağlam İlkesi ile Bileşim İlkeleri arasındaki soruna ve de Frege'nin Düşüncelerin yapısal çözümlemesi ve ayrıştırılmasına ilişkin çelişkili tezlerine odaklanıyoruz. Düşüncelerin birliğiyle ilgili problemler için *at* kavramı paradoksuna odaklanıyoruz. Tezimizde, duyum ve gönderge fonksiyonlarını lamda-

soyutlaması olarak yorumlayarak paradoksa bir çözüm sağlamak için güçlendirici çerçevemizi sunuyoruz. Dilbilgisel bir öznenin, yani *at* kavramının, söz edildiği bir bağlamda bir kavramı gösterebileceğini iddia ediyoruz. Dolayısıyla bu bağlamlarda *at* kavramı doygun olmayan bir kavrama gönderimde bulunmaktadır. Sonuç olarak, yeterli ve uygun bir anlambilimsel *Düşünceler* kuramı ortaya koyarak Frege'nin dil felsefesi üzerine olan sezgilerinin doğasını koruyoruz.

Anahtar Kelimeler: Fregeci Düşünceler, yapısal önermeler, önermelerin birliği, *at* kavramı paradoksu, lamda-soyutlaması.

to the memory of my mother, Nefise Akçelik

to my father, İbrahim Ethem Akçelik

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CHAPTER 1

INTRODUCTION

The subject matter of this thesis is Gottlob Frege's theory of *Thoughts*. For Frege, *Thoughts* are abstract structured entities which are sharply distinguished from mental acts of thinking. He conceived *Thoughts* as the meanings of sentences which denote the truth-values. In this respect, Frege's notion of *Thought* has an indispensable role in his philosophy of language. However, certain problems emerge in his semantic theory concerning the structure and unity of thoughts. In this thesis we aim to provide an explication and emendation of the structure and unity of atomic Fregean *Thoughts*.

Frege's theory of *Thoughts* is constructed on the following key notions: the function-argument analysis, the object-concept dichotomy, and the sense-denotation distinction. Frege has revolutionized logic and provided an important tool for the analysis of natural language expressions. Frege's notion of function-argument analysis reveals the logico-semantic composition of the *Thoughts*. Frege argues that all terms and well-formed formulas are denoting expressions and he distinguished simple expressions from complex expressions. For Frege, sentences as complex expressions are formed with the unsaturated, i.e., functional or predicative, expressions which are completed with the corresponding saturated expressions, i.e., singular terms. Frege has systematically introduced the ontological categorization of entities into objects and concepts, which constitutes an exhaustive division. Furthermore, all saturated entities are objects, and all unsaturated entities are concepts. Finally, and most importantly, Frege has introduced the distinction between sense and denotation of expressions to account for the cognitive difference between identity statements. The senses of singular terms are the modes of

presentations of their objects, and their denotations are the objects themselves. Frege has applied this distinction to sentences. Frege conceived *Thoughts* as the senses of sentences and their denotations are truth values. Accordingly, Fregean *Thoughts* are the meanings or semantic values and the primary bearers of truth values of sentences, as well as the objects of attitudes, such as knowledge, belief, desire etc. Frege presents the structure of Thoughts by composition of their singular terms and functional or predicative expressions. In this respect, the saturated senses of singular terms complete the unsaturated senses of functional expressions, as a result it is the composition of saturated and unsaturated parts that provides the unity of *Thoughts*, i.e., holding the constituents together.

In this thesis, we explicate Frege's Theory of *Thoughts* to achieve two primary goals. First, we argue that Frege's theory of *Thoughts* is the most adequate semantic theory for explaining the meaning and truth of natural language expressions.

However, there are certain problems concerning the structure and unity of Fregean *Thoughts* which brings us to the second primary goal of this thesis. We investigate the set of problems of particular importance for the structure and unity of *Thoughts* among other problems of Frege's theory of logic and ontology. For the problems concerning the structure of atomic Fregean *Thoughts,* we will focus on two important problems. The first problem is the apparent tension between the Context Principle and the Compositionality Principles. The second problem concerning the structure of atomic *Thoughts* stems from Frege's seemingly conflicting theses regarding the analysis and decomposition of *Thoughts*. Frege has been criticized for holding two initially incompatible theses. The first thesis states that a *Thought* is isomorphic with the sentence which expresses it. On the other hand, the second thesis states that two structurally different (i.e., non-isomorphic) sentences can express the same atomic *Thought*. The problem poses a serious threat to compositional structure of atomic *Thoughts* because it implies an inconsistency between unique analysis of *Thoughts* on the one hand and multiple decomposition of *Thoughts* on the other.

For the problems concerning the unity of atomic Fregean *Thoughts,* there is a unique and the most important problem namely the concept *horse* paradox. This paradox arises when we consider the atomic *Thought* expressed by the following sentence

(H) The concept *horse* is not a concept.

Frege treats the phrase 'the concept *horse*' as a singular term, thus it denotes an object. However, according to Frege, concept-words cannot denote an object, hence the negation of (H) must be true:

(¬H) The concept *horse* is a concept.

Thus, the paradox demonstrates that we are left with the conclusion that "the concept *horse*" is not a concept but an object. As we have argued above, Frege has theorized the unity of *Thoughts* on the functional composition of saturated and unsaturated expressions. Nevertheless, the paradoxical sentence shows that "the concept *horse*" can be both an unsaturated and a saturated expression. In this respect, the paradox compromises the ontological distinction between objects and concepts which are supposed to be mutually exclusive for Frege. In other words, an expression cannot be both a singular term, i.e., has a saturated sense and denotes an object; and a predicative or functional expression, i.e., has an unsaturated sense and denotes a concept. As a result, the paradox inevitably juxtaposes the unity of *Thoughts*. Therefore, we consider the concept *horse* paradox as the most important problem for the unity of atomic *Thoughts*.

Our conjecture is that Frege's semantic theory of *Thoughts* is incomplete in the sense of being vulnerable to certain puzzles and paradoxes. We see that the problems concerning the structure and unity of atomic *Thoughts* demonstrate the apparent need of certain elaborations and supplementations to Frege's semantic framework. In this thesis, we shall present our emendations to Frege's semantic theory to successfully solve these problems. Nevertheless, we aim to provide minimal revisions to Frege's theory of *Thoughts* in order to preserve the nature of Frege's intuitions for philosophy of language. To conclude, we aim to achieve a

satisfactory theory of *Thoughts* in Fregean spirit by our modifications in an interconnected framework of semantics and metaphysics of natural language.

We shall mention some preliminary remarks on our methodology. Our first remark is that we include only atomic sentences within our framework in this thesis. In other words, we exclude the sentences of the molecular and quantificational form from our treatment of the subject, with our good reason. Although the problems we elaborate in our framework lead to further important issues to be solved in non-atomic sentences, we argue that the most fundamental problems belonging to semantics and metaphysics stems from the atomic sentences. Moreover, a wide variety of problems in philosophy originates from atomic sentences having such a simple subject-predicate form but it is important to remark that this basic structure is the source of paramount puzzles and paradoxes. Hence, this point is not only limited to Frege's semantic theory but also many theories of philosophy of language perhaps beginning from Plato's theory. Our second methodological remark is about the choice of underlying logical language. Although we aim to avoid the baroque uses of logical formalism to explicate and provide our emendations to Frege's semantic theory, it is nevertheless impossible to eschew logical formalism. In our framework, we use λ-notation to express the functional expressions in the form of λ-abstraction. We use λ-abstraction for its perspicuity and clarity as a functional calculus in expressing Frege's semantic theory of *Thoughts*.[1] Our last remark is to stay as close as to Frege's intuitions for natural language semantics.

[1] λ-calculus, in a nutshell, is a simple formal notational system in symbolic logic for expressing functional abstraction and their applications to arguments. It is introduced by Alonzo Church, an admirer of Frege, as a universal model of computation in part of his project about foundations of mathematics. See Church (1932). λ-calculus consists of constructing lambda terms with bound variables and performing sets of operations and certain substitution rules. Nevertheless, we shall not dwell on the technical details of λ-calculus. See Barendregt (1985; 2013), J. Hindley and Seldin (2008), and Alama and Korbmacher (2018). In this thesis we prefer λ-abstraction for its simplicity. Our source of inspiration is Church's formulation of Frege's theory of sense and denotation. See Church (1946; 1974; 1993). λ-abstraction is a respectable formal tool for representation of the functional expressions of Frege, especially for his function-argument analysis. Moreover, there are applications in a wide range of semantic theories. For the important applications in natural language semantics which are partly based on Frege's compositionality principle see Tichý (1988), Partee et. al. (1990), Heim and Kratzer (1998), Duží et. al. (2010, p. 19).

Now let us present a brief outline of the thesis. Aside from the Introduction and Conclusion parts, this work is of six chapters. In the next chapter, Chapter 2, we present the historical background to Frege's *Thoughts* by investigating the theories of preceding philosophers in which they have attributed an abstract structured entity as the meanings and primary truth bearers of sentences in different names. We further investigate the nature and properties of these entities by focusing on their structure and unity. Then, we compare these theories with Frege's notion of *Thought*. In this chapter, we aim to establish a theoretical connection between Frege's account and its precursors which is occasionally ignored in the context of the history of philosophy.

In chapter 3, we aim to investigate the theoretical background of Frege's theory of *Thoughts* in the frame of his views on semantics and metaphysics of natural language. We investigate Frege's theory of *Thoughts* by providing an explicatory analysis of its semantic roles, structural properties, and nature. In this respect, we deploy two kinds of interconnected philosophical analyses to explicate the notion of Fregean *Thoughts*. We explicate meanings and truth conditions of sentences by a semantic analysis of atomic *Thoughts*, and we provide Frege's account for the structure and unity of atomic *Thoughts* by an ontological analysis. As a result, we aim to present the logico-semantic and ontological aspects of Frege's philosophy of language in detail by introducing his key notions mentioned above. In this chapter, we further briefly introduce fundamental problems concerning the structure and unity of atomic *Thoughts*.

In Chapter 4, we focus on analogous approaches to Fregean *Thoughts* which are collectively classified under the term *proposition*. The aim of this chapter is to argue that Fregean *Thoughts* are propositions. In this respect, we present a fundamental theory of propositions by analyzing their properties and nature. We also give some arguments for the existence of propositions and elaborate these arguments in the context of Frege's theory of *Thoughts*. In the final part of this chapter, we examine similar problems of propositions to that of *Thoughts* concerning the structure and unity of propositions.

In Chapter 5, we will provide a critical review of the contemporary accounts of propositions. We present classification of these accounts in two respects: First, with respect to their views concerning whether propositions are structured, and second whether they are reducible to other types of ontological entities, such as sets. We consider Russellian and neo-Russellian accounts of structured propositions, constituents of which are individuals, properties, and relations. We also focus on two important representatives of Russell's descendants, namely Jeffrey King and Jeff Speaks. Next, we consider an alternative account against the structured views, the Possible World Account of Propositions, according to which propositions expressed by sentences are either as sets of possible worlds, or characteristics of these functions, viz., functions from possible worlds to truth values. Then we consider non-reductionist or primitivist views according to which propositions cannot be reduced to other entities, but rather propositions are primitive and *sui generis* entities. In the following sections of this chapter, we focus on two main proponents for this view, namely Algebraic Accounts and Deflationary Views of propositions. The set of problems in Chapter 4 will be our criteria for comparing Frege's theory of *Thoughts* with contemporary theories of propositions in this chapter. Accordingly, for each type of account, we argue that Frege's theory of *Thoughts* is presumably the most adequate and satisfactory account for theorizing a comprehensive framework to explain the meaning and truth of natural language expressions.

In Chapter 6, we aim to explicate Frege's views concerning the structure and compositionality of atomic *Thoughts*. First, we present Frege's compositionality principles, namely the Function-Argument Compositionality Principle and the Part-Whole Compositionality Principle. Then, we focus on two problems regarding the structure of Fregean *Thoughts*. For the problem concerning the apparent tension between the Context Principle and the Compositionality Principles, we provide a reconciling position by holding both principles in our interpretation which is based on Frege's own writings. Frege's apparently conflicting theses regarding the analysis and decomposition of *Thoughts*, can be named the Dummett-Bell Problem. This problem poses a serious threat for the structure of *Thoughts* and their identity

with the corresponding sentence structure. We review the proposed solutions proposed by Dummett, Bell, Kemmerling, and Penco and then provide our respective criticism to the solutions in the literature. In the last part of this chapter, we provide our solution by an interpretive emendation following Hodes' notion of polymorphous structure of Fregean *Thoughts*. However, our position substantially differs from Hodes and we provide our criticisms of his account. We argue that our solution is similar to Frege's original position as we preserve both theses in his theory of Thoughts. We show in our framework that the *polymorphous* structure of *Thoughts* reveals each possible constituent by multiple decomposition and in the final analysis decomposition reveals the ultimate unique constituents of the same atomic *Thought*.

After considering the structure of atomic Fregean *Thoughts*, we turn to explain how their constituents are held together in Chapter 7. Frege establishes the unity of *Thoughts* as the functional composition of their corresponding saturated and unsaturated parts. Frege argues that the sense-denotation distinction applies to both saturated and unsaturated parts of the constituents of *Thoughts*. Frege explains the senses and denotations of saturated expressions and states that the same distinction applies to unsaturated expressions. Nevertheless, he hardly provides an explanation for how to conceive the senses and denotations of these expressions. Frege's lack of explanation results in the concept of *horse* paradox as we have presented above. We argue that the unity of atomic Fregean *Thoughts* cannot be established without solving the concept *horse* paradox. In this chapter, after explaining the paradox in detail, we survey the substantial solutions given by Geach, Dummett, Wiggins, Wright, Noonan, Hale, and MacBridge. Accordingly, we provide our critical review for each of these solutions by pointing out that although all of these solutions have certain merits to solve the paradox, they neither are adequate for explaining the unity of *Thoughts* nor preserve Frege's own semantic views. We provide our emendatory framework for senses and denotations of unsaturated expressions. We use the calculus of λ-abstraction to express the functional expressions as interconvertible λ-abstracts and then we explicate the senses and denotations of proper names, predicates, and sentences respectively. We give formal definitions of concepts

denoted by predicates, i.e., concept-functions, and senses expressed by predicates, i.e., sense-functions, in our framework. For the solution of the concept of *horse* paradox, we appeal to Russell's distinction between used and mentioned occurrences of concepts. In this sense, we consider the distinction between "relation in itself" and "relation actually relating" which is extended in the framework of the procedural semantics for Transparent Intensional Logic, according to which procedures are either *executed*, or else *displayed*. The functional application of "the concept *horse*" in the occurrence of a sentence "Bucephalus is a horse" corresponds to used or executed mode. However, we argue that its occurrence as a grammatical subject, as in the sentence "The concept *horse* is not a concept", corresponds to denotation of a concept in which it is *mentioned* (displayed), thus it denotes an unsaturated concept. As a result, we obtain the falsity of the paradoxical sentence (H) so that we affirm rather its negation (¬H), viz. "The concept *horse* is a concept." Thus, the paradoxical sentence (H) poses no longer a threat to Frege's semantic theory of *Thoughts*. We show that the structural composition of an atomic *Thought* consists in functional compositions between a function and its argument. To conclude, we explain that the unity of *Thoughts* results from the application of a function to an argument that are both constituents of the *Thoughts*.

CHAPTER 2

HISTORICAL ANTECEDENTS OF FREGEAN *THOUGHTS*

The purpose of this chapter is to investigate the historical precursors to Frege. There are some reasons for presenting this historical background. First, we provide evidence to establish the existence of propositions *qua* abstract entities in the history of philosophy. This would in turn give us certain reasons to understand Frege's idea of existence of *Thoughts qua* abstract entities. Second, it will ground the necessity of positing an abstract entity for two of its indispensable roles: (i) the semantic role as meanings of sentences, and (ii) the alethic role as being primary truth bearers of sentences. The latter is perhaps the most widely discussed role in the history of propositions. In connection with the truth-bearing properties of propositions, we will present its conceptual evolution parallel to the developments in logic. Third, we aim to present historical pretensions concerning the structure of propositions, namely constituents and composition, and their correspondence to the structure of linguistic expressions, i.e., grammatically complete and meaningful sentences. Most importantly, this will vindicate ontological grounds for the problem of the unity of propositions in the historical context. There is one more reason for this overview. Most propositionalist philosophers tend to ignore preceding historical views. In order not to repeat such neglect, we will show that Frege's account is not a standalone stance in the history of philosophy as it does have its roots in venerable line of thought in the history.

In the literature on propositions, historical treatments of the subject are either brief summaries of a long period of time, i.e., they cover the entire history of philosophy from antiquity to the present, or they are lengthy and detailed accounts

of relatively short periods of time, e.g., they focus on only medieval philosophy or early modern philosophy. All of these treatments have their merits and they are elegant treatments of the issue but they present no connection (with the exception of Church's work) to Frege. As for the subject matter of this thesis we shall focus on the historical antecedents of Fregean *Thoughts*. Just as Frege wrote about the influence of other philosophers whom he often quoted, we also would like to list the precursors to his thought and provide an overview of their views.

2.1. Ancient Conceptions of Propositions

Fregean *Thoughts qua* abstract entities have a long and significant conceptual history which can be traced back to ancient philosophy. In the first two subsections, we will focus on Plato's and Aristotle's attribution of truth and falsity to abstract entities. In the last section, we will consider the logico-semantic doctrines of the Stoic School which have striking similarities to the semantic theory of Frege.

2.1.1. Plato

In the historical context, attribution of truth-bearers to abstract entities other than linguistic expressions is said to be first found in the dialogues of Plato. Although it is a matter of debate whether Plato was the first philosopher to conceptualize propositions, it is certain that the arguments in his dialogues, especially about truth and falsity, can be called a proto theory of propositions. Furthermore, among his many other metaphysical views, Plato has presumably the strongest influence on Frege, in particular his realism is a profound source of inspiration regarding the nature of *Thoughts* with respect to their abstractness, objectivity, and mind-independence.

The question "What can properly be called true or false?" is one of the earliest questions to reflect on the nature of logic. The related problem of the possibility of existence of false judgments and beliefs had occupied many ancient philosophers, especially Sophists. In this respect, Plato was also the first to argue

about the objects of judgment and belief attitudes. Two of his dialogues, namely *Theaetetus* and *Sophist*, have central importance for our purposes.[2] The question about the nature of truth-bearers appears in an attempt to define the concept of *knowledge* in the *Theaetetus* in which the discussion is centered on the question of "How can there be a false judgment?" In the *Sophist*, the question addressed takes the form of "How can there be false saying and thought?" in which Plato makes a similar characterization between thoughts and speech.

In the *Theaetetus*, Plato focuses on the puzzle of how a false belief or judgment could have an object. Socrates presents the Sophist's argument against the possibility of a false judgment.

> […] a man who is judging some one thing is judging something which is. Then that means that a man who is judging something which is not is judging nothing. But a man who is judging nothing is not judging at all. And so it is not possible to judge what is not, whether about the things which are or just by itself. False judgment, then, is something different from judging things which are not. Then neither on this approach nor on the one we followed just now does false judgment exist in us. [3]

This argument, as presented, is grounded on the assertion that judgments are *about* something. When a judgment is about something which is true, this explanation appears to be correct. The true judgment that "Theaetetus sits" has an object, namely a *sitting Theaetetus*. However, the problem arises when we consider a false judgment. For example, the judgment that "Theaetetus flies" has no object, for there is no such thing as *flying Theaetetus*. Yet, a judgment cannot lack an object, otherwise it would not be a judgment at all. Hence, Sophists conclude there are no false judgments.

Prima facie Sophists' argument rests on a misleading analogy between perception and judgment. If we are to make a distinction between the act and the object of judgment, the problem vanishes. For one can attribute falsity to an act of judgment even when the judgment lacks an object. However, Plato did not solve the problem in the *Theaetetus*. Instead, Socrates and Theaetetus attempt to define

[2] *Theaetetus* (187a-202c) and *Sophist* (260c-264d).

[3] *Theaetetus* (188e-189a).

knowledge by true judgment with *logos*. [4] According to Socrates' Dream (201d-202c), the world is composed of complexes of primary elements, where the complexes, but not the elements, can be given account for and known.

> […] it is impossible that any of the primary things should be expressed in an account [*logos*]; it can only be named, for a name is all that it has. But with the things composed of these, it is another matter. Here, just in the same way as the elements themselves are woven together, so their names may be woven together and become to an account of something–an account of being essentially a complex of names. Thus the elements are unaccountable and unknowable, but they are perceivable, whereas the complexes are both knowable and expressible and can be the objects of true judgement. [5]

This passage declares not only the attribution of truth bearers to *logos*, but also the tentative distinction between structured complexes and their elements, names. Plato considers falsity (and similarly truth) as characterizing primarily *logos*. Therefore, Plato attributes truth and falsity to the *logos*. [6]

This foreshadows the account for truth bearers, which are also structured complexes in the *Sophist*, where the very same metaphor of *woven together* is mentioned.[7] "To dissociate each thing from everything else is to destroy totally everything there is to say. The weaving together of forms is what makes speech

[4] *Theaetetus* (201d). The Greek word "*logos*" is a verbal noun derived from the verb "*legein*" whose primary meanings are "speak", "tell", "say". Throughout this chapter, I use transliteration of the words belonging to the Greek terms.

[5] *Theaetetus* (202b).

[6] We should note a certain ambiguity of the word. According to Kneale and Kneale (1962, p. 18) the word *logos* is either used to mean plainly for sentence or used to mean the truth and falsity of thoughts and opinions. According to Nuchelmans (1973, p. 15) the word logos has following three senses: the token-sentence, the act of uttering that sentence, and the significance of the sentence. Due to this ambiguity, Nuchelmans (1973, p. 21) argues, perhaps in anachronistic terms, that statements can be tokens of inner speech. Brown (2020, p. 286) argues against the distinction between *name* and *logos* in *Theaetetus*, concluding that there is no explicit characterization of the role of truth bearers for the statement or proposition, though she attributes this to the *Sophist*.

[7] Brown (2020, p. 274 n. 17) points out verbal similarities between the *Theaetetus* and the *Sophist*. In corresponding passages of both of these dialogues, there is a contrast between naming and saying where "[t]he Theaetetus lacks the key distinction between *onoma* and *rhema*, while the Sophist's theory of true and false *logos* lacks any distinction between the knowable and the unknowable" Brown (2020, p. 277).

possible for us". [8] A similar puzzle in *Sophist* purports to show the impossibility of false (that which is not) belief and speech. [9] The Stranger goes on to argue against Sophists' argument and shows how one can think and say what is false. Plato explains our ability to say [*legein*] something by distinguishing between the parts of speech, names [*onoma*] and verbs [*rhema*]. Accordingly, verbs are the sort of indication that is applied to an action, and names are the kind of spoken sign that is applied to the things that perform those actions. [10]

Names "lion stag horse" and verbs "walks runs sleeps" do not make up speech, not until one combines or mixes them together.[11] So, one can manage to say something only by weaving names and verbs together as a unity, and this unity is what the word *logos* is used for to give an account for something true or false. [12] In other words, attributing structural unity to *logos* provides a solution to the Sophists' puzzle. As a result, the conclusion that there is no false saying does not follow, since the speech or thought has a part that corresponds to Theaetetus and it has a part that corresponds to flies. Each part of the speech corresponds to something that exists; therefore, it is possible to say something false even though the speech as a complex whole is about something that does not exist. Then, we can give an account for the possibility of falsity by saying that the parts do not weave together.

Moreover, Plato also makes a distinction between two types of speech. On the one hand, there is an *outer*, or as it is called vocal, speech which corresponds to sentences. On the other hand, there is an *inner*, or as it is called mental, speech which corresponds to thoughts. According to Plato's so-called realist conception, mental speech has priority over public speech. Thought then is defined as the soul in talk or

[8] *Sophist* (259e).

[9] *Sophist* (260a-e).

[10] *Sophist* (262a).

[11] *Sophist* (262b-c)

[12] *Sophist* (262c-d).

conversation with itself. [13] Thought is inner speech that occurs without the voice and judgment is achieved when the inner speech *affirms* or *denies* the same thing.

In its essence, Fregean *Thoughts* shares certain similarities concerning its structural unity to express the truth of sentences. More importantly, the nature of *Thoughts* is highly reminiscent of *logos* as abstract mind-independent entities which have a separate existence in the realm of Platonic forms. To conclude, Frege seems to have adopted almost every aspect of Plato's realism in this sense.

2.1.2. Aristotle

Aristotle's account of attribution of truth and falsity to entities can be found primarily in the *Categories*, *De Interpretatione* and *Metaphysics*. Aristotle's preliminary analysis of structured truth-bearers is introduced in the first book of the *Organon*, *Categories.* There, Aristotle introduces simple and complex forms of speech, a theory of predicables, and the categories of objects of thought. [14] The logical significance of the categorization of predicables is an initial attempt to give an account for the relation between subject and predicate by means of structural analysis of the grammatical forms.[15] Similar to Plato, neither names nor verbs (predicates) involve an affirmation, only combination of these terms results in positive and negative statements. [16]

In the *De Interpretatione*, Aristotle follows a similar distinction between the elements of speech, i.e., nouns and verbs, gives definitions of the terms "negation" and "affirmation", and then "statement" and "sentence" [*logoi*]. [17] Spoken words belong to the mental experience (affections in the soul, and later they are called

[13] See *Theaetetus* (189e-190a) and *Sophist* (263e).

[14] *Categories* (Chapters 1-4).

[15] See Bochenski (1961, pp. 51-52).

[16] *Categories*, (2^a4).

[17] Modern Library Edition of Aristotle translates the word *logos* as *propositions*.

thoughts), and written words are the symbols of spoken sounds. [18] Aristotle's account is tantamount to Plato's discussion of truth and falsity in the *Sophist*.

> Just as some thoughts in the soul are neither true nor false while some are necessarily one or the other, so also with spoken sounds. For falsity and truth have to do with combination and separation. Thus names and verbs by themselves—for instance 'man' or 'white' when nothing further is added—are like the thoughts that are without combination and separation; for so far they are neither true nor false. [19]

Hence, truth and falsity primarily belong to thoughts, and the truth or falsity of the speech is derivative in this sense. [20] Aristotle also adheres to the view that language is conventional, i.e., the spoken words may be different although the thoughts are the same for everyone. [21]

In the following part, Aristotle ascribes meaning to all sentences [*logos*]. Among these sentences Aristotle distinguishes *declarative* or *statement-making sentences*, which have truth and falsity in them, from *prayers* which are neither true nor false. The term for this certain class of statement-making declarative sentences, which have subject-predicate combinations, is *apophansis* or *apophantikos logos*. [22] There are two kinds of *apophantikos logos*, *kataphasis* involving an affirmation and *apophasis* involving negation or denying. [23] Therefore, it is the *apophantikos logos* which is true or false and corresponds to Fregean *Thoughts*. [24]

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[18] *De Interpretatione* ($16^{a}3$).

[19] *De Interpretatione* ($16^{a}9$).

[20] Kneale and Kneale (1962, p. 45).

[21] *De Interpretatione* ($16^{a}19$).

[22] *De Interpretatione* ($16^{b}33$-$17^{a}3$).

[23] *De Interpretatione* ($17^{a}25$).

[24] Kneale and Kneale, and Nuchelmans challenge this point. Kneale and Kneale (1962, pp. 49-51) discuss in the context of Aristotle's theory of meaning and truth in length and they point out several readings of Aristotle's *apophantikos logos* with contemporary usage of the word proposition. In the general sense, it refers to an indicative or declarative sentence, where the sentence means *type-sentence*. In some other sense, the term is used to mean the content which is asserted in the *making of a statement*. Then, they list Aristotle's certain mistakes due to this ambivalence (1962, pp. 51-54). According to Nuchelmans (1973, p. 44), Aristotle, in general, uses the word *apophantikos logos* to refer to a particular *apophansis*, which is a *token utterance* of a sentence (*logos*), thereby token

Notions of meaning and truth are essential to the explanation of *apophantikos logos,* although they demand an explanation, viz. how sentences can have meaning and how the terms truth and falsity are defined. [25] Unfortunately, Aristotle does not provide any definition for these terms for the rest of the *Organon*. Later in the *Metaphysics*, we can find the generic definition of true and false: "For it is false to say of that which is that it is not or of that which is not that it is, and it is true to say of that which is that it is or of that which is not that it is not". [26]

Aristotle's solution to the problem of existence of falsity in the sense of false judgments, has close resemblances with Plato. For instance, the nouns and verbs have significance, but they are not true or false. The structure of *apophantikos logos*, i.e., every thought or part of discourse which is to be true or false, must be composite. However, Aristotle recognizes certain difficulties in Plato's doctrine of forms. One major distinction between the noun and the verb is temporality, that the noun is without time but the verb "signifies time in addition". [27] Aristotle also rejects Plato's view that Forms, or universals, are fundamental. Aristotle instead gives metaphysical priority to particulars. Attributing a different ontological status to universals and particulars underlines the difference between subjects and predicates. Still, Aristotle's understanding of *logos* asserts that the subject-predicate composition ensures the unity.[28] The contrast between their conceptualizations of the matter results in Aristotle, but not Plato, considering sentences as entities on their own. As a result, the sentences are reduced to single entities which are true or

utterances, not sentence-types, are subject to truth and falsity, but in some other cases Aristotle extends the attribution of truth and falsity to *sentence-types*. McGrath and Frank (2020, sec. 1), following Nuchelmans, object that "it is unclear whether the resulting combination of thought elements is anything other than a token thought, as opposed to something which is the content of the token thought and which could be thought by others, could be denied, asserted, etc."

[25] See Kneale and Kneale (1962, p. 46).

[26] *Metaphysics* (Γ 7 1011^{b}26-27).

[27] *De Interpretatione* (16^{b}6). Kneale and Kneale (1962, p. 45) contend this as a serious step in history of logic that "to deny any strictly timeless predication on the ground that this was bound up with the Platonic metaphysics of the realm of timeless Forms, which he had abandoned."

[28] See Davidson (2005, pp. 92-93).

false. For instance, if we consider that "Theaetetus sits" and "Theaetetus flies", the former affirms the existence of both the object Theaetetus and the universal *sitting* as a single entity, and the latter denies them together.

Donald Davidson points out the connection between *intelligibility* of false statements and *the problem of predication* with reference to Plato and Aristotle. Davidson diagnoses one of the essential features for any sentence, i.e., how parts of the speech combine and produce something which is true or false. [29] For Davidson this aspect is the unity, and he concludes that the problem of predication is concerned with the problem of the unity of the proposition. He further distinguishes two aspects of the problem: The metaphysical aspect concerns how particulars are related to properties, and the semantical aspect concerns how subjects and predicates are related. [30]

Aristotle also mentions that contents of judgments are the fundamental bearers of truth and falsity, but there is a vague distinction between the act and the content of judgment. However, Aristotle did not treat these contents as abstract entities. As also noted by Nuchelmans, "it is the state of things in the world that determines the truth or falsity of the judgment; but the judgment itself, the actual bearer of truth and falsehood, is in the mind." [31]

In *Prior Analytics*, the word *protasis* is used for the bearer of truth and falsity, which is also translated as proposition. The *protasis* is an *apophansis* that is asserted, literally means the leading premise, to express the conclusion in a syllogism. Hence, it has a role for being a relation of entailment in a deductive argument. [32] Similar to *apophantikos logos*, *protasis* is composed of subject and predicate terms. In fact, this composition is essential to the assessment of validity of

[29] Davidson (2005, pp. 81-82).

[30] Davidson (2005, p. 83).

[31] Nuchelmans (1973, p. 25).

[32] *Prior Analytics* (1 24^{b}18).

deductions in syllogism. As we will see in the next section, the term *propositio* is derived from *protasis*. [33]

We can conclude this subsection as follows: Although Fregean *Thoughts* share roles of propositions present in Aristotle's works, Frege sharply deviates from Aristotle on two points. First, due to his fierce opposition to psychologism, Frege certainly denied propositions as mental entities. Second, with his *Begriffsschrift* he provided a brand-new logical framework, based on functional calculus, to replace the Aristotelian logical theory.

2.1.3. The Stoics

The logical doctrines of the Stoa School are found in the fragments and testimonies of its founder Zeno of Citium (BCE 335–264/3) and his successor Cleanthes of Assos (BCE 331–232), but they are primarily attributed the Chrysippus of Soli (BCE 280–207) who is the successor of Cleanthes. [34] The Stoics has conducted detailed studies of arguments, especially those involving conditionals and other complex forms, and the rules of inferences. Hence there is a tendency to characterize the Stoic logic as a type of propositional logic, distinguishing it from Aristotelian Term Logic.[35]

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[33] See Church (1956a, p. 356).

[34] *Cf.* Kneale and Kneale (1962, pp. 116-117) and Bobzien (2020). The chief sources for these doctrines belong to later period, but for the sake of following a general consensus we will follow Diogenes Lærtius *Lives of the Eminent Philosophers* Book 7, sections 55–83, and Sextus Empiricus's *Outline of Pyrrhonism Book* 2 and *Against the Mathematicians Book* 8. Both philosophers give the major credit to Chrysippus of Soli. However, it is important to note that there is a controversy about the credibility of these sources. Sextus Empiricus, being a skeptic, is opposed to the Stoics and reports these doctrines in order to refute them, and Diogenes Lærtius who irrationally admires Chrysippus of Soli, is opposed to those who are opposed to the Stoics, and also considered as not completely reliable source due to his gossipy prose.

[35] The significant difference is that in the Stoic logic not all valid arguments are syllogisms. Nevertheless, Kneale and Kneale (1962, p. 115) consider the doctrines of the Peripatetic School and the Stoics as complementary.

For the Stoics, the general word for logic is *dialectic*, and they divide it into two parts: a part corresponding to the things signified and a part corresponding to the things signifying the elements of speech. The Stoics divide the elements of speech into voice [*rhema*], speech [*lexeis*], and discourse [*logos*], i.e., a meaningful utterance. [36] Stoics named the things signified *lekton*, as a proper subject matter of logic. [37] Sextus Empiricus presents this account as follows:

> The Stoics say that three things are linked together, that which is signified, that which signifies, and the object; of these that which signifies is speech, as for example, 'Dion', that which is signified is the thing itself which is revealed by it and which we apprehend as subsisting with our thought […] Of these two are corporeal, that is, speech and the object, while one is incorporeal, that is the thing which is signified, i.e. the *lekton*, which is true or false.[38]

However, contrary to Sextus Empiricus reception, Kneale and Kneale argue that not all *lekta* are true or false.[39] They are divided among themselves. First division is between incomplete and complete *lekta*. Incomplete, or deficient, *lekta* comprises subjects and predicates, and they are incomplete in the sense that expression is unfinished. The complete *lekta*, on the other hand, are the meaning or the significance of whole sentences, and they are further divided into *axioma* and others [*pusma*], such as questions, commands, oaths, and wishes.[40] It is the *axioma* which is true or false, a complete entity declarative or assertoric by itself, and the meaning of declarative sentences. [41] Therefore, we can conclude that *axioma* fulfill similar propositional roles as Fregean *Thoughts*. The Stoics employed only *axioma* for

[36] Kneale and Kneale (1962, p. 139).

[37] The Greek word *lekton* is derived from the verb *legein* and literally means "what is said." *Cf.* Bochenski (1961, p. 110) and Kneale and Kneale (1962, p. 140). Note that there are some terminological differences between them, and we preferred the translation of the latter.

[38] *Adv. Math.* (2.11-12) cited from Kneale and Kneale (1962, p. 140).

[39] Kneale and Kneale (1962, pp. 141-142).

[40] Bochenski (1961, p. 112) who refers to Mates (1961).

[41] Kneale and Kneale (1962, p. 145) notes that Stoics regarded every *axioma* as either true or false: Cicero says that Chrysippus particularly stressed on this point.

deductions in their logic. Simple *axiomata* are augmented into complex *axioma* by logical connectives, to express new propositions.

The Stoics usually distinguish the material, or corporeal, aspects of words from the incorporeal aspects. They consider *lekta* as an incorporeal entity. [42] This attribution has an exceptional ontological status, since Stoics are generally accepted as materialists, as they claim that everything, including the soul, is corporeal. As a result, the Stoic conception of propositions has a problem, for what is real has a limited sense of being acted upon. *Axioma* being thought, said, and judged should correspond to the bodily entities. [43]

To conclude, the Stoic conception of propositions is generally considered as the closest ancestor of the modern concept of propositions in the sense of being truth bearers and having a significance. There are many similarities with the Stoic conception of *axioma* and Fregean *Thoughts*. To begin with, the Stoics aim to provide a semantic analysis of public language, similarly Frege also put his logical views in the use of semantic theory for natural languages. Next, Stoics embraced a materialistic metaphysical view, nevertheless for some reason they ascribed an immaterial stance to propositions. In this sense, *lekton* is not a quality of the mind which is similar to Frege's views about *Thoughts*. Lastly, terminologies of the Stoics and Frege share striking similarities. Susanne Bobzien documents these similarities. She claims that especially the works belonging to years between 1890 and 1925 (Frege's death), were adapted from Stoic logic without any reference. [44] Bobzien's

[42] Nuchelmans (1973, p. 86).

[43] See McGrath and Frank (2020, sec. 1).

[44] However, the title of Bobzien's article is in a more aggressive tone. Frege usually refers to the owners of ideas that he took up from. But the Stoics, or perhaps more precisely Carl Prantl, seems to be an exception. Bobzien (2021) condemns Frege borrowing the Stoic lexicon, mainly attributed to the fragments and testimonies of Chrysippus of Soli. She compares transliterated Greek phrases with their German counterparts: Of particular importance are (III.1) 'semantic content' (*lekta* / *Sinn*) of linguistic expressions; (III.1.1.) 'incomplete content' (*ellipe* / *ungesättigt*) and (III.1.2.) 'complete content' (*autoteles axioma* / *gesättigt*); and (III.1.2.1.) 'assertible contents' (*axiomata* / *Gedanken*). Note that terminological differences with the present thesis are due to Bobzien's translation of Carl Prantl's *History of Western Logic* (*Geschichte der Logik im Abendland*, 1855–1870) who sourced Diogenes Laertius' *Lives of the Eminent Philosophers*. Zalta (2022, part 3.3.) notes that the

study is concrete, however we should note that Frege's almost all contributions to logic, semantics, and metaphysics, differ radically from Stoics.

2.2. Medieval Conceptions of Propositions

Philosophers of the medieval period widely used the term *propositio* to attribute truth values, the semantic significance of sentences, contents of judgements and beliefs as abstract entities.[45] Following the so-called Boethian definition *oratio verum falsumve significans* (speech signifying what is true or false), most Medieval logicians used this term to refer to written, spoken, and mental types of sentences. Medieval philosophers also considered this use as a root for their sententialist views, which attributes truth values and significance to grammatical forms. [46] In this chapter, we shall only focus on the use of *propositio* in the abstract sense, which is the appropriate use concerning the roles and nature of Fregean *Thoughts.*

2.2.1. Boethius

Manlius Severinus Boethius (480–524) has a considerable importance for his writings which served as a primary source for most of the Scholastics logicians. Boethius is said to follow Lucius Apuleius of Madaura (*c.* 150) in his early writings who used the term *propositio* for asserted declarative sentences, which is the only

similarities and commonalities of elements have also been given by Bochenski (1961, p. 127); Mates (1961, pp. 19-26, 46-47); Kneale and Kneale (1962, p. 531), but in the final analysis, he finds "the overall effect is a kind of *res ipsa loquitur* [the thing speaks for itself]."

[45] The origin of the word *propositio* is attributed to the Roman philosopher Cicero. Although his contribution to the logic is not considered as authentic, he is well-credited for translating and inventing Latin equivalents for Greek term. Cicero has introduced the term as an equivalent for *protasis,* i.e., the leading premise of an argument. See Kneale and Kneale (1962, pp. 177-178).

[46] Church (1956a/2019, p. 356) makes a similar distinction between propositions in the traditional sense and propositions in the abstract sense. Propositions in the former sense are merely linguistic entities like sentences, whereas in the latter sense they are independent of any language or linguistic form.

kind of speech that truth and falsity can be attributed. This, on the whole, is the subject matter of logic. [47] Boethius had noticed certain differences and tensions between the Aristotelians and the Stoics and he favored the Aristotelians. In his commentary on the *De Interpretatione*, Boethius uses the word *oratio* as a substitute for *logos*. *Oratio* includes written words, which are the symbols of spoken words, and thoughts, which are composed of mental words. [48] Thus, there is a compositional order between three levels of discourse. This characterization in terms of its constitution is mainly borrowed from Aristotle. Boethius uses the word *propositio* to refer either spoken, written or mental sentences that are characterized in terms of their signification. However, it is the mental sentences [*propositiones mentales*] which are structured *complexion* of mental words that are the bearers of truth and falsity. [49] On the other hand, Nuchelmans points out that the word *sententia* is also used to express the significance of a *propositio*. In this sense, it is still not the linguistic expression of a thought. According to the so-called Boethian definition, a proposition is an expression signifying what is true or what is false. [50] Indeed, this definition becomes the generic definition of propositions, almost always quoted in this formulation.

2.2.2. Abelard

Pierre Abélard (1079–1142) defined *propositio* using exactly as Boethius. However, unlike Boethius, he held that when one speaks of a proposition as true, this implies either that it generates a true thought or that it corresponds to what is in fact the case. [51] Abelard favors the latter definition. Abelard has also made a

[47] See Nuchelmans (1973, p. 131).

[48] Kneale and Kneale (1962, p. 194).

[49] Nuchelmans (1996, p. 177).

[50] Kretzmann (1970, p. 771). Church (1956a/2019, p. 356) attributes this quotation to Peter of Spain (*c*. 1245).

[51] Kneale and Kneale (1962, p. 205).

distinction between mere predication and the act of assertion. Predication is the operation of linking the subject and the predicate either in an affirmative or a negative way, which is essentially realized by the copula. This structure is the common element in different speech acts; hence it is a manner of conceiving [*modus concipiendi*].[52] As such, Abelard is usually credited with the introduction of the word copula.[53] So, a *propositio* can signify only in a complex way, and this signification happens right after all of its parts have been uttered. Therefore, only complete utterances can express a proposition.

A *propositio* signifies in two senses. In the first sense, *propositio* signifies complex thoughts in mental language. However, a complex thought signified by a *propositio* is neither the fundamental bearer of truth and falsity nor stands in logical relations. In the second sense, a *propositio* signifies the way things stand. The roles of being the bearers of truth and falsity, in addition to modal properties such as necessity and possibility, and relata of logical relations are attributed to this sense. [54] Abelard held that written or spoken *propositiones* and their mental counterparts are true and false in a derivative manner. [55] Thus, *propositio* is the bearer of truth-values only derivatively, in virtue of signifying *dicta*. Accordingly, Abelard attributes this connection to *dictum* which is signified by *propositio*. On this basis, propositions are structured complexes, consisting of an act of combining the predicate with subjects. The arrangement of its elements, i.e., its subject and predicate components, determine truth-value. If those elements fit together, it is true, if it does not fit then false. Hence, *dictum*, "what is said" by the *propositio*, is the primary bearer of truth values, with a reference to the above given second definition: what is asserted to be the case.

[52] Nuchelmans (1996, p. 200).

[53] Kneale and Kneale (1964, p. 208).

[54] See Nuchelmans (1973, pp. 162-163).

[55] Nuchelmans (1996, pp. 200-201).

2.2.3. Medieval Logicians in the 13th and 14th Centuries

Among the medieval logicians, the writings of Boethius and Abelard have made a dominant impact throughout the following centuries. The essentials of the doctrines of Scholastic philosophers of 13th century, such as Albert the Great (1193–1280), William of Shyreswood (ob. 1249), and Peter of Spain have no independent arrangement, so their logical doctrines generally consist of commentaries on the writings of Aristotle and Boethius. [56] It is also hard to find a uniform terminology among medieval philosophers, except the Boethian definition. In addition to *dicta*, some medieval logicians use *enuncio*, and some others use *significatum* to designate what is true or false and what is signified by a *propositio*. [57] For instance, in the terminology of Peter of Spain, the term *oratio* is used to designate a *propositio* of the sort. *Oratio* is a *vox* of a certain kind which roughly corresponds to the form of words. Therefore, the difference in the *vox*, results in the difference in signification of what is expressed.

We can summarize the two types of logico-semantic theories developed about the nature of propositions during this time. The first kind, specially focusing on the semantic components, i.e., the significance of propositions, as discussed up to this point, is called *dictism*. In some respects, medieval logicians use this term for explaining the meaning of expressions. Abelard's view is considered as an important example of dictism. Following Abelard's account, philosophers have grounded their epistemological and metaphysical doctrines on the *significatum*, or *dictum* of the proposition. Proponents of the dictist tradition generally considered *significatum* (or sometimes *enuntiabile*) as meaning or sense of a *propositio* in their theories. Accordingly, what is asserted as true or false is not the *propositio*, but the *dictum*. [58] The second kind of theory is called *terminism* and the earliest version of such development is to be found in the writings of William of Shyreswood. As properly

[56] Bochenski (1961, p. 159).

[57] See Church (1956a/2019, p. 356) and Kretzmann (1970, pp. 772-773).

[58] Kretzmann (1970, pp. 772-773).

called *proprietates terminorum* aimed to give an account for the properties and structure of categorical propositions by an elaborate analysis of the terms making up the proposition, which is also known as term logic. [59]

In the 14th century, several accounts of propositions flourished among the English scholastics. The nominalist accounts of propositions by William of Ockham (ob. 1349/50) and John Buridan (ob. 1358) are such examples. In their accounts, mental propositions are formed internally before the corresponding grammatical proposition. Propositions in this sense refer to mental entities as complexes having corresponding analogous syncategorematic and categorematic term parts. [60] For Ockham, these parts are essential to the meaning of mental propositions, but they are devoid of grammatical properties, hence do not belong to any language. [61]

The heart of the matter at hand, for Frege's semantic theory, is Kretzmann's interesting discussion that both dictism and terminism should be regarded as complementary theories. [62] Arguably, he considers terminism as a theory of reference, and dictism as a theory of sense. Nevertheless, he remarks that medieval logicians and their followers did not recognize their complementary nature, as a result they considered these theories as separate developments. This is highly reminiscent of Frege's distinction between sense and denotation; still, it was Frege who gave an account of this complementary nature in his conception of *Thoughts*.

2.3. Modern Conception of Propositions

In the early modern period, it is somehow hard to locate the development of the propositions. The existence of propositions is generally accepted without an argument, although the views regarding its nature vary. The roles of propositions as meaning and truth bearers of sentences, and objects of judgment and belief are also

[59] Kneale and Kneale (1964, p. 247), Kretzmann (1970, p. 767).

[60] See Kretzmann (1970, pp. 780-781).

[61] Church (1956, p. 741).

[62] Kretzmann (1970, p. 768).

assumed with almost certainty. This period is often considered as a transitional period until the development of Frege's logic. In this section, we will survey the major figures, namely Descartes, Leibniz, and Kant, whose writings can be considered as a milestone for the development of contemporary conception of propositions in this period. In the last part of this section, we will focus on Bolzano, whose works are of particular importance for Frege.

2.3.1. Descartes and the *Port Royal Logic*

In his *Meditations on First Philosophy* (1642), Rene Descartes makes his famous division of thoughts into categories of ideas and mental activities. The ideas which belong to the category of pure intellection are distinguished from the mental activities which are the images of sensations and dreams. [63] In the Third Meditation, he ascribes truth and falsity to the acts of judgments in the exclusive and strict sense. Descartes holds that ideas are modes of thinking, and do not refer to extramental things. Hence, they cannot be the truth bearers. Instead, they refer to acts of judgments which in turn have agreement and conformance to things outside the mind. Acts of judgment comprise *intellect* and *will*. The former represents the content of judgment and the latter affirms or denies the content of judgments. For Descartes, propositions are the objects of judgment which consist of mainly eternal truths that are free creations of God. [64] As Nuchelmans puts it "what is judged […] is not the conformity of the idea with extramental thing, but only the agreement or disagreement of the ideas which, as subject and predicate, are the constituents of a propositional concept." [65] Descartes further discusses the falsity of propositions, although he considers falsity in a special material sense. Accordingly, he defines

[63] For the detail of Descartes' use of the word *idea* in the context of propositions, see Nuchelmans (1983, pp. 53-54).

[64] Descartes (1642, 1:145, 149, 151). We leave out the debate concerning the status of his eternal truths.

[65] Nuchelmans (1983, p. 50).

material falsity as ideas which represent non-things as things. [66] Some ideas are materially false since they are the object of false judgment, viz. the sensations belonging to secondary qualities, such as heat and cold, or color sensations, are materially false.

Descartes' conception of proposition has further influenced Antoine Arnauld and Pierre Nicole, the authors of the *Port Royal Logic*. The most evident Cartesian element in the *Port Royal Logic* conception of propositions is the adoption of Descartes' theory of ideas. Nonetheless, it differs from Descartes with respect to the constituents of propositions which are ideas.[67] Arnauld and Nicole define propositions to be the same as judgments. Accordingly, in an act of judgment, a person affirms an attribute of a subject, either by joining two ideas together, or denies that the subject has the relevant attribute, and separates the two ideas. [68] Within this framework, propositions are defined as judgments about things. They further argue about the structure of judgments, or propositions: A proposition contains three elements: (i) a subject-idea, (ii) an attribute idea, and (iii) the copula. The first two elements are called terms, and the third unifies these terms. [69] However, they have also noted that not all propositions have these three elements, as a result, the structure of propositions is certainly not isomorphic to the structure of the sentences that express them. They consider the example of a sentence lacking subject-copula-predicate structure, such as one-word sentences "Affirmo." This will be an important point in Frege's semantic theory as well, since he also considers the apparent distinction between the logical and linguistic forms.

[66] Descartes (1642, 7:43).

[67] See Nuchelmans (1983, pp. 73-75).

[68] Arnauld and Nicole (1662, p. 29).

[69] Nuchelmans (1983, pp. 73-75)

2.3.2. Leibniz

Another important figure in the Frege's theory of logic and semantics is Gottfried Wilhelm Leibniz. As we will see in the next chapter, Leibniz has inspired Frege in many aspects. First and foremost, Leibniz was interested in constructing a scientifically designed language that would provide a medium for human thought and reasoning in a clear and perspicuous way. The language of this medium had to contain a system of universal notation, *characteristica universalis*, to express the complex nature of human thought from primitive concepts and appropriate mechanical devices for applying and expressing formal notions such as predication, logical connections, and universality. His grand project was devising such logical calculus, *calculus ratiocinator*. In that regard, Leibniz was Frege's primary source of inspiration for devising the logical language in *Begriffsschrift*.

Church gives the best formulation of Leibniz's conception of proposition given in *Dialogus de Connexione inter Res et Verba*. The dialogue shows essential considerations on the duality of sentences as linguistic structures and propositions as abstract entities. [70] Dialogue starts with one of the participant's, call it A, assertion that truth and falsehood must be supposed as attached to things [*res*] and not thoughts [*cogitations*]. The immediate problem is, which circles back to the ancient puzzle, how can a thing be false? The respondent, B, argues for the point that one's thought about the thing must be false. But A replies by considering the point that one would still be in doubt whether something is true or false, which would eventually lead to the conclusion that both thought and its object are false. By reductio, then, Leibniz concludes that truth belongs to neither things nor thought, but rather it belongs to possible thoughts, *possible propositions* [*cogitato possibilis*]. Church remarks that

> The basis of truth is not in the notation, not in the symbols or characters themselves, but in something their use and interconnection which is not arbitrary, a certain relation (proportion) of the characters among themselves and between the characters and things, which is under transformation into a different language or

[70] See Church (1956a/2019, p. 358).

> notation either remains the same or is transformed into something suitably corresponding. [71]

Leibniz generally uses the word *propositio* and *enuncio* for written and spoken sentences and *possible propositions* as the eternal abstract objects of judgment.[72] Hence, *possible propositions* are the contents of thoughts and the fundamental bearers of truth-value. Nevertheless, their existence does not depend on any mind. Leibniz attributed all propositions a subject-predicate form, similarly to preceding traditions. However, he defines the truth of propositions in terms of conceptual containment, which has later inspired Kant's theory of judgment. Therefore, if the predicate concept is contained in the subject concept, then the proposition is true. Leibniz defends an objective conception of truth, independent of any forms. He exemplifies this theory by arguing that the geometry of the Greeks, Latins and Germans is the same. Similarly, there is no Greek, Latin, barbarian truth. Hence, synonymous expressions in different languages or forms express the same truth. [73] In this respect, Leibniz's notion of ideas is important since truth is a relation between ideas, and ideas are expressed by speech. In the mind of God, all necessary relations come together to form eternal truths. But ideas in the human mind are fallible. According to Leibniz, the human mind can demonstrate the truth of a necessary proposition, whereas the assessment of truth value to contingent propositions is infinite. As a result, it can only be grasped by God. [74]

According to Leibniz, constituents of propositions are concepts. Concepts are divided into three categories: (i) the concept of individuals such as "Aristotle", (ii) general concepts, such as "wisdom", and (iii) the proposition.[75] Simple concepts can form complex concepts. Since a proposition is also a concept, it can take part in other propositions. Therefore, complex propositions can be formed by simple

[71] Church (1956a/2019, p. 358).

[72] See Church (1956a/2019) and Mates (1989).

[73] Nuchelmans (1983, p. 222).

[74] Nuchelmans (1983, p. 216).

[75] Nuchelmans (1983, p. 227).

propositions, i.e., concepts.[76] The relationship between parts of propositions and their forming complex structures is closely connected to Frege's principles of compositionality, in terms of both part-whole relationship and functional relationship. Leibniz employs the notion of coincidence to express the identity of two different concepts. Two concepts or two propositions are called coincident if they are intersubstitutable in any proposition given that they preserve their truth values [*salva veritate*]. For instance, pairs of three kinds of concepts "Aristotle" and "the teacher of Alexander the Great"; "triangle" and "trilateral"; "Man is an animal" and "'Man is an animal' is true" are coincidental.[77] Since according to Leibniz these pairs are coextensional [*coextendi*], they can be substituted in place of each other without changing the truth values of the propositions. Yet, Leibniz also mentions some contexts in which substitutivity of coincident concepts fails. Leibniz gives the example of the proposition that "St. Peter in so far as he was the apostle who denied Christ sinned". If we substitute 'St. Peter' for coincident concept 'the apostle who denied Christ', then the resulting proposition "St. Peter in so far as he was St. Peter sinned" has a different truth value than the initial proposition.[78] In these contexts, coincident concepts are different since the mode or order of conceiving of the thing, but not the thing itself, is under discussion. The terms have *definite restrictions* in these contexts, and propositions having these terms are called *reduplicative propositions*. Reduplicative propositions have two functions: They introduce a *concomitance* which are analyzable into a conjunction of propositions, and they introduce a *cause*.[79] This analysis is considered as evidence for Leibniz's awareness of the distinction between extensional and intensional context.[80] This logico-

[76] Mates (1989, p. 58)

[77] Nuchelmans (1983, pp. 227-228).

[78] Nuchelmans (1983, p. 230) cites Leibniz (1960, p. 475) *Fragmente zur Logik* F. Schmidt (ed.), Berlin.

[79] Nuchelmans (1983, p. 228).

[80] See Nuchelmans (1983, pp. 230-231).

semantic analysis has a parallelism with Frege's distinction between *senses* as intensional entities, and *denotations* as extensional entities.

To conclude this section, we can see Leibniz has a remarkable influence on Frege's semantic theory for *Thoughts*. His refinements in the conceptual basis had a remarkable impact on descendant philosophical theories. A prominent effect of Leibniz on Frege's logico-semantic theory is presumably Frege's concern to preserve the substitutivity principle of coreferential terms by giving an account of meaning and truth by his famous distinction between *sense* and *denotation*.

2.3.3. Kant

Immanuel Kant uses the term judgment [*Urteil*] for the concept of proposition.[81] Therefore, we shall focus on his theory of judgment as a theory of propositions in the context of Fregean *Thoughts*. However, Kant's theory of judgment is very complex and has many diverse interpretations. In what follows, we shall only focus on his theory of judgment in the context of analytic and synthetic judgments. Then, we will discuss Kant's use of analytic and synthetic judgments concerning the meaning of sentences [*Satz*].

Kant has been considered as having a psychologistic account for semantic notions, and a confusing conceptualization of act and content of judgments. [82] Nuchelmans notes that there is no clear distinction between the act of judgment and the content of judgment.[83] However, Hanna interprets otherwise. For him, Kant makes a distinction between expression of judgment, the sentence, and the content (*Inhalt*) of judgment. [84] For Kant, it is the content of judgment which is the fundamental bearer of truth and falsity. Kant holds that judgments are complex cognitions that are conscious mental representations, yet they are objective.

[81] Church (1956a/2019, p. 741).

[82] See Nuchelmans (1983, p. 247) and Coffa (1991, pp. 18-20).

[83] See Nuchelmans (1983, pp. 246-247).

[84] See Hanna (2022, sec. 1).

Judgments refer to objects either directly through intuitions or indirectly by concepts. [85] Kant seems to adhere to a priority for the content of judgment over its cognitive contents, intuitions and concepts. In light of this, Kant has rejected earlier notions of judgment. Nevertheless, Kant distinguishes two perspectives, one from the formal point view, i.e., categorial Aristotelian logic, and the other from the transcendental point of view. [86] This divergence leads to different conceptions of judgment. Here we limit this survey to formal logic, since this conception is in accordance with the propositional roles and features of Fregean *Thoughts*. For Kant, a judgment is characterized by its structure, which consists of a thing as its subject and an attribute as its predicate and the copula that either affirms or denies. In this regard these logical functions establish the unity of judgment. Accordingly, judgments are the content of this structural unity and they mediate the formation of beliefs and other propositional attitudes. [87]

Both the conceptual structure of judgments and their truth bearer properties lead to Kant's central distinction between analytic and synthetic judgments that is given in the beginning of his *Critique of Pure Reason*:

> In all judgments in which the relation of a subject to the predicate is thought … this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, although it does indeed stand in connection with it. In the first case I call the judgment analytic, in the second synthetic.[88]

Kant gives "All bodies are extended" as an example of an analytic judgment. In this judgment, Kant regards the predicate "extended" belong to the subject "body". [89] On the other hand, in the judgment "All bodies are heavy", the predicate is distinct from the concept of body, so this structure results in a synthetic judgment.

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[85] *Cf.* Hanna (2022, sec. 1 and 2).

[86] Nuchelmans (1983, p. 246).

[87] See Hanna (2022, sec. 2.2).

[88] Kant (1787, B11). Cited from Juhl and Loomis (2009, p. 6).

[89] Juhl and Loomis (2009, p. 6).

There are most certainly epistemic implications of the analytic-synthetic distinction. Analytic judgments do not extend one's knowledge, for they do not reveal anything about the conceptual content of the judgment. On the other hand, synthetic judgments extend knowledge.

Initially Kant considers that the containment criterion applies only to propositions which have a subject-predicate form, but he later lists twelve primitive judgment-types, including statements in hypothetical or conditional form. [90] Arguably, Kant also holds a second criterion of analyticity which is the principle of non-contradiction. He writes: "[i]f the judgment is analytic, whether it be negative or affirmative, its truth must always be able to be cognized sufficiently in accordance with the principle of contradiction." [91]

To conclude, Kant's notion of judgment has the fundamental roles of Fregean *Thoughts* in the present context. However, it marginally contrasts Frege's account, due to Kant's conception of judgment having an almost entirely psychological character. Frege, who strictly opposed use of psychological notions in his logic and semantics, in fact showed great effort to challenge Kant.

2.3.4. Bolzano

Bernard Bolzano (1781–1848) deserves consideration in two important aspects. First, he has strongly opposed the use of psychological notions in philosophy and logic. In particular, he provided his substantial account of propositions as completely detached from psychological references. Second, and more importantly for the present purposes of this thesis, he is the closest forerunner concerning the nature and structure of Fregean *Thoughts*.

Bolzano gives his account for propositions in order to ground his logical theory. [92] Bolzano's aim was to eliminate certain ambiguities in the Kantian

[90] Nuchelmans (1983, p. 246).

[91] Kant (1787, B190–1). Cited from Juhl and Loomis (2009, p. 8).

[92] Lapointe (2011, p. 8).

conception of judgment, especially mental and cognitive elements in representation. The root of the problem was distinguishing two senses of the term representation: the subjective and the objective. In the subjective sense, representation corresponds to mental states, as the psychologists call it, they are "representations in us". In the objective sense, however, there is an intersubjective content of the psychological representation, which Bolzano calls it *representation in itself* or *objective representations*. [93] Meaningful expressions can have more than one subjective representation, but they can have only one objective representation. For these concerns, in his *Theory of Science* (1848), Bolzano needed a more refined conception for propositions but never provides arguments for the existence of such entities. Instead, he merely attributed a separate realm of existence to logical entities such as truth values, properties, relations. This in turn requires him to posit purely logical objects, such as objective sentences or sentences in themselves, *Sätze an sich*.

Bolzano, without a doubt, attributed a structural character to *Sätze an sich.* It has an inherent and uniform unity, composed of objective ideas or ideas in themselves [*Vorstellungen an sich*] as their parts. [94] Bolzano earlier held a traditional subject-predicate-copula structure for propositions. Later, in his *Theory of Science*, he holds that all propositions are in the form of 'A has B'. [95] In that regard, *Sätze an sich* also have the subject-predicate form, but the copula 'has' is considered tenseless and free of any other contextual determiners. In fact, it merely indicates possession of an attribute. [96] This structure also provides an account for the relational claims, construed as statements about the collections or sets. The elements belonging to this collection have unity as former sub-collections have corresponding

[93] Bolzano considered a tacit isomorphism between the object and its mental counterparts in representation. See Coffa (1991, pp. 30-32).

[94] Rusnock and Šebestik (2022, sec. 3).

[95] See Bolzano (1848, p. 48). *Cf.* Lapointe (2011, p. 8).

[96] See Rusnock and Šebestik (2022, sec. 5).

attributes for the latter subcollections. [97] For Bolzano, fundamental bearers of truth and falsity are properties of *Sätze an sich.* He accordingly defines the truth of a proposition as: "a proposition is true when it states what belongs to its object." [98]

Bolzano's conception of *Sätze an sich* is similar to Frege's conception of *Thoughts* in many respects. Bolzano sharply distinguishes subjective content of linguistic expressions from their objective content. This is the predominant influence of Bolzano's views: sharply distinguishing logic from psychological laws of thought. In this respect, he asserts the existence of *Sätze an sich* as the meaning of sentences, bearers of truth and falsity. It is evident, at least from a Fregean perspective, that Bolzano's sentences in themselves, or *Sätze an sich, are what Frege called Thoughts*. Moreover, for the meanings of expressions, he further refined and used the word *Sinn*, with a similar connotation to that of Frege. [99]

> The purpose of analysis in [Bolzano's] theory is to reveal the "Sinn" of expressions (cf. [1842], §285, 67). Since propositions are the "Sinn" of sentences ([1842], §28, 12), analysis aims at making them explicit, which given the immutability requirement appears to require that we eliminate context-sensitive elements and replace them by non-context-sensitive ones. [100]

Bolzano maintains a structural composition for *Sinn* as abstract mind-independent constituents, i.e., representations in themselves. Bolzano was first among the philosophers of the 19th century to discredit the largely held view that logic can be derived from the laws of thought in the psychological sense and they belong to mental entities. Followers of Bolzano's path, Husserl and Frege further discredited the use of identification of logical laws with *psychological* laws in addition to their epistemic and ontological theories.

We shall end this chapter by a discussion of the quasi-conceptual character of the word "proposition". We have considered various lexical cognates of the term

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[97] Rusnock and Šebestik (2022) refers to Bolzano (1848, §135, no.15).

[98] Rusnock and Šebestik (2022) refers to Bolzano (1848, §28).

[99] See Kusch (1995) and Coffa (1991).

[100] Lapointe (2011, p. 29).

proposition as an abstract entity, from ancient to early modern period. For contemporary use corresponding to these terms, most languages employed the Latin derivatives of *propositio*, such as English and French words "proposition"; or Spanish word "proposición" ambiguously to use the connotations of the term either in the abstract sense or merely in the linguistic sense, i.e., sentence. However, German speaking philosophers did not follow this path. Bolzano and Frege coined German words '*Sätze an sich*', and '*Gedanke*' respectively to compensate for the abstract, objective and language-independent character. Both philosophers used the term *Satz* in a linguistic or grammatical sense. [101] Whatever the naming conventions, we always use the proposition in the logico-semantic character with an abstract ontological status, hereby endorsing a realist stance for the existence of this entity. Nevertheless, we shall give arguments for its existence on both ontological and metasemantical grounds.

[101] Church (1956a/2019, p. 358).

CHAPTER 3

AN EXPLICATION OF FREGE'S THEORY OF *THOUGHTS*

In this chapter, we will present Frege's theory of *Thoughts* in the frame of his views on philosophy of language. In the first section, we shall explicate Frege's philosophy of language by introducing his key semantic notions: (i) function – argument analysis, (ii) object – concept dichotomy, and (iii) sense – denotation distinction. In the second section, we will focus on Frege's theory of *Thoughts*, by providing an explicatory analysis of its roles, properties, and nature in Frege's semantic theory. The ultimate aim of this thesis is to argue that Frege's theory of *Thoughts* is the best fit for any adequate account for providing a comprehensive framework explaining the meaning and truth values of natural language expressions. In this respect, we aim to deploy two kinds of interconnected philosophical analyses to explicate the notion of Fregean *Thoughts*. The first kind of analysis concerns the semantic aspects that aims to explicate meanings and truth conditions of sentences, and the second type analysis concerns the ontological aspects which aims to provide an account for the structure and unity of *Thoughts*. In the third section, we will examine fundamental problems about the structure and unity of atomic *Thoughts*. Among other problems of Frege's theory of logic, semantics, and ontology, we shall set two problems of particular importance for the structure and unity of *Thoughts*. These problems are respectively conflicting theses regarding the decomposition of *Thoughts*, and the concept of *horse* paradox regarding the unity of *Thoughts*. Our conjecture in this thesis is that Frege's semantic theory is incomplete in the sense of being vulnerable to certain puzzles and paradoxes. Therefore, his semantic account must be supplemented to provide a satisfactory theory of *Thoughts* by certain

amendments in an interconnected framework of semantics and metaphysics. Before we begin, we shall remark three preliminary notes on terminology and methodology.

I. Thought and Thinking

Frege has explicitly and sharply distinguished thought [*der Gedanke*] as the semantic content of sentences denoting truth-values from acts of thinking [*das Denken*] which is a mental process. In order to avoid any confusion, we have italicized and capitalized the first letter of the word. In ordinary language, the word thought has psychological connotations. However, Frege is an exception: his *Thoughts* are purely objective. It is a matter of debate whether Fregean *Thoughts* are propositions in the contemporary sense. We will test this hypothesis in the following chapters.

II. Translations of Frege's Terms

We shall note some translation preferences of Frege's terms into English as a methodological remark for an ongoing matter of controversy. In the translations of works and studies about Frege, we can list the following German words and their translations in English:

Ausdruck	expression
Bedeutung	reference, denotation, nominatum, meaning, significance
Begriffswort	concept-word
Eigenname	(proper) name
Sinn	sense, meaning
Gedanke	thought, proposition
Satz	proposition, sentence, and also theorem

Some scholars left these terms untranslated.[102] Some scholars left *Sinn* and *Bedeutung* untranslated in particular, but for other German words they use English translations. There is still no consensus in the recent literature. In this thesis, due to complexities that might arise in the text, we shall avoid frequent use of German words, unless first introduced as an important notion in Frege's philosophy. Unless otherwise mentioned, we prefer following italicized English words for their German counterparts:

Bedeutung	*denotation*
Sinn	*sense*
Gedanke	*Thought*
Satz	*proposition* if it is a judgeable content or a semantic value of a declarative sentence, *sentence* if it is used as a declarative sentence as a linguistic entity.
Begriff	*concept*
Gegenstand	*object*

III. Works Cited in this Thesis

There is no single complete edition of Frege's works in English. Hence, throughout this thesis we have cited from the following editions of his works:

[FR] *The Frege Reader*, Michael Beaney (ed.), Blackwell Publishers: Oxford, 1997.

[CP] *Collected Papers on Mathematics, Logic and Philosophy*, Brian McGuinness (ed.), Basil Blackwell: New York, 1984.

[102] Beaney (1997, pp. 36-46) devotes a lengthy part for translating these terms in his edition of Frege's works. He mainly focuses on the proper terminological counterpart of '*Bedeutung*', but he left the word untranslated. See Geach and Black (1960, ix), Bell (1980), Klement (2002, p. 9 n. 7, 11), and Morris (2006, p. 31).

[PMC] *Philosophical and Mathematical Correspondence of Gottlob Frege*, Brian McGuinness (ed.), Hans Kaal (trans.), Basil Blackwell: Oxford, 1980.

[PW] *Posthumous Writings*, Hans Hermes, Friedrich Kambartel and Freidrich Kaulbach (eds.) Basil Blackwell: Oxford, 1979.

[TPW] *Translations from the Philosophical Writings of Gottlob Frege*, Peter Geach and Max Black (eds.), Basil Blackwell: Oxford, 1960.

3.1. An Overview of Frege's Semantic Theory of *Thoughts*

Frege's views on semantics and metaphysics of natural language and thoughts are raised in a book and a series of influential articles. Frege's views are constructed mainly in his *Begriffsschrift* (1879), "Function and Concept" (1891), "On *Sense* and *Denotation*" (1892a), "On Concept and Object" (1892b), and the works belonging to his later period, "The Thought" (1918a), "Negation" (1918b), and "Compound Thoughts" (1923). However, we shall note that these works belonging to his later period are generally considered to be based on his drafts of earlier works.[103] We shall vaguely follow the chronological order of his works in this section.

We shall begin our analysis by an explication of Frege's metaphysics of natural language semantics starting from his *Begriffsschrift.* Frege introduces a new language of notation to formulate a language of logic, which he named *concept-script*. It has certainly revolutionized a new era of logic and philosophy; indeed, many philosophers and logicians consider this renovation as a dawn of analytic philosophy. Frege's *Begriffsschrift* has the subtitle *a formula language of pure thought modeled upon the language of arithmetic*, which indicates a new method of

[103] Frege scholars argue that collective articles in his *Logical Investigations* are not completely new works, but they are nevertheless a collective that comprise his mature views on language and thought. Beaney (1997, p. 9), Klement (2002, p. 8), and Zalta (2022). Frege scholars mention this by pointing out certain passages from his *Nachlass* sharing which are almost identical, word by word, to works as early as 1890s.

analysis within a new form of language of logic which in turn shapes his approach to natural languages. [104]

Frege's philosophical motivations for creating such a perspicuous conceptual language of logic can be traced back to Leibniz's idea of *lingua characteristica* and *calculus ratiocinator*. Both Frege and Leibniz considered natural language as ill-suited for its vagueness and ambiguity to express logical relations and inferences of mathematics, and reasoning of thought.[105] Hence, Frege has considered his work as a steppingstone to his program which aims to show that mathematics, especially arithmetic, is a part of logic.[106] In *Begriffsschrift*, Frege provides a comprehensive formal theory of logic which has its unique significance in the following respects: (i) unifying and incorporating two traditional parts of logic, namely Aristotelian categorical logic and the Stoic propositional logic, (ii) an invention of a system of quantificational logic to express statements of generality and multiple generalities, and (iii) a higher order logic which allows quantification over functions as well as quantification over objects. Arguably, Frege's concept-script and related logico-semantic theory have initiated the logicistic program

[104] The earliest hint was given in the preface of the *Begriffsschrift*:

> I believe I can make the relationship of my *Begriffsschrift* to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for just such purposes, but precisely because of this is useless for all others. (Frege, 1879, V (*FR*, p. 49))

[105] Frege most probably borrowed the name "*Begriffsschrift*" from a paper on Leibniz by Prussian philosopher and philologist Adolf Trendelenburg (1802-1872). As a historical notice, some mathematicians and logicians inspired by Leibniz, had attempted to formulate such a language. Among them, most often Frege compares his approach with that of George Boole (1815-1864), however, he finds Boole's language imprecise and thus unacceptable, because (i) Boole's language is ambiguous for he uses the same signs used in mathematics, (ii) he dissociates propositional and categorical elements of logic thus handling them separately, and (iii) an inadequacy of dealing expressions and inferences with multiple generalities. *Cf.* Bynum (1972, pp. 15-20).

[106] Frege (1879, V (*FR* p. 50)).

according to which truths of arithmetic are deductively derivable from logical truths by a number of definitions and axioms. In *Die Grundlagen der Arithmetik* (1884) and *Grundgesetze der Arithmetik* (1893/1903), Frege took further the task of providing philosophical and logico-mathematical grounds respectively, as a complementation and supplementation to formal ground given in his *Begriffsschrift*.

Frege has introduced the symbol '⊢' which expresses the *content* [*Inhalt*] of judgment [*Urteil*]. This symbol is a complex symbol consisting of (i) a vertical stroke '|', a *judgment stroke*, and (ii) a horizontal stroke '—' which signifies that the group of symbols preceding it form a whole sentence expressed by a judgment. When the vertical stroke is added to the horizontal, it indicates a *conceptual content* or *cognitive assent*.[107] Accordingly, the conceptual content of declarative sentence is called a *judgeable content*. Sentences are true or false for they have judgeable content and the expression act of a judgment is called *assertion*. For Frege, two *propositions*[108]

At Maritsa, the Ottomans defeated the Serbians

and

At Maritsa, the Serbians are defeated by the Ottomans

shares the same conceptual content.[109] To complete the basic logical representations of propositions, Frege presents his symbolic notation by adding symbols for conditionality, negation, and identity sign to represent more complex propositions.[110] Frege also adds function signs to express relations, and generality

[107] See Szabó and Thomason (2019, p. 25).

[108] Here, the German word *Satz* is generally translated as proposition, so we shall follow this convention. The ontological assumption of this concept is in accordance with the present purposes of this thesis, but still in some other context it simply means sentence or statement. *Cf.* Klement (2002, p. 10 n. 8).

[109] Frege (1879 (*FR* p. 55)).

[110] Frege (1879 (*FR* p. 65)) explains that "The need for a symbol for identity of content rests on the following: the same content can be fully determined in different ways; but that, in a particular case, *the same content* is actually given by *two modes of determination* is the content of a *judgement*."

signs to express propositions involving quantifier phrases, and multiple generalities.[111]

In the following subsections, we shall focus on the core elements of Frege's semantic theory by considering three key notions in order to conceptualize Frege's semantic framework. These are (i) function – argument analysis, (ii) object – concept dichotomy, and (iii) sense – denotation distinction. As we shall see below, these distinctions are indeed interconnected, if not inextricably knotted.

3.1.1. The Function – Argument Analysis

Frege revolutionized logic by rejecting the subject-predicate analysis of the dominant Aristotelian theory of logical forms (syllogisms) and he argued for function-argument structure. For Frege, functions are similar to the mathematical notion of functions, but they are further applicable to natural language expressions in order to make his logical language suitable for purposes other than arithmetic. We shall limit our focus to Frege's application of function–argument structure to natural language sentences. Frege held that all terms and well-formed formulas are denoting expressions and distinguished simple expressions from complex expressions. Simple expressions consist of names of objects such as 'Chrysippus' and '4', and complex expressions consist of complex terms, such as descriptions like 'the student of Cleanthes' and sentences like 'Chrysippus is a philosopher'. Complex expressions are formed with the help of incomplete expressions which signify functions, such as 'the student of ξ' and 'ξ is a philosopher'. These expressions have the corresponding placeholder(s) as shown here by 'ξ'. These functions can be represented by the λ-calculus as 'λ*x* (the student of *x*)' and 'λ*x* (*x* is a philosopher)'.

[111] Frege's symbolic notation later refined his *Grundgesetze der Arithmetik* (1893/1903). Since Frege's two-dimensional notation is overtly confusing and not conventionally used, hence we shall not mention it here. See Beaney (1997, Appendix 2), Reck and Awodey (2004, pp. 26-34), Mendelsohn (2005, Appendix A and B), and Zalta (2022, sec. 2.2) for Frege's notation.

Frege holds that the content of complete expressions can be split up [112] into (i) a *constant component*, which represents the totality of relations, and (ii) a *variable* which *denotes* [*bedeutet*] an object that stands in these relations. These variables are replaceable by other expressions. Frege calls this constant component *function*, and the variables filling placeholders *arguments* of functions.[113] So, the following *proposition*

(1) Chrysippus is a philosopher

can be split up into the function 'λx (x is a philosopher)' and its argument 'Chrysippus'. The function part, which is specifically a concept, remains the same, and its argument can be replaced by some other name, e.g. 'Cleanthes', so that we have:

(2) Cleanthes is a philosopher

Similarly, a sentence involving a binary relation

(3) John wrote more books than Chrysippus

can be split up into its function '$\lambda x\ \lambda y$ (x wrote more books than y)' and its two arguments 'John' and 'Chrysippus'.

[112] Frege (1897 (*FR*, p. 66)) remarks that "the function-argument distinction has nothing to do with the conceptual content, but only with our way of grasping it.".

[113] "If, in an expression […], a simple or complex symbol occurs in one or more places, and we think of it as replaceable at all or some of its occurrences by another symbol, then we call the part of the expression that on this occasion appears invariant the function, and the replaceable part its argument" (Frege (1897 (*FR* p. 67)).

3.1.2. The Object – Concept Dichotomy

The second important notion in Frege's semantic theory is the dichotomy between objects and concepts. In his *Grundlagen,* Frege puts forward his commitment to three important principles:[114]

I. A sharp separation of the psychological from the logical, and the subjective from the objective.

Frege is famous for his stance against psychologism. This is indeed very explicit in all of his works. In the context of the theory of *Thoughts,* adherence to this principle commits Frege to the thesis that *Thoughts* can neither be present with anyone's mental image, nor truth of *Thoughts* are subjective. [115]

II. The so-called *context principle* that "the meaning of a word must be asked for in the context of a proposition, not in isolation."

Frege puts forward the context principle against the views holding the meaning of numbers independently of the contexts in which they appear in sentences. There is an important set of problems for the incompatibility of this principle with his compositionality principles which we shall turn to in Chapter 6.

III. The distinction between concept and object.

This principle is an inalienable, and central to Frege's ontology of meaning.

An Interlude: *Saturated vs. Unsaturated*

In "Function and Concept", Frege revisits his analysis of functions and rehearses some of his arguments about conceptual definitions of numbers. Then,

[114] Frege (1884 (*FR* p. 90)).

[115] *Cf.* Frege (1882? (*PW*, p. 175)) where he says "no psychological investigation can justify the laws of logic."

Frege gives his ontological division of two types of distinct entities, objects and functions. For Frege, objects are complete, self-subsistent entities, whereas functions are not. Accordingly, functions are "incomplete [*unvollständig*], in need of supplementation [*ergänzungsbedürftig*], or unsaturated [*ungesättigt*]."[116] Hence, Frege argues that the argument does not belong to a function, though it completes the function to make up a complete entity. The result of completing, or saturating, the function with the argument is called the value of a function [*Wertverläuf*] for an argument which is an object, and a concept is a function whose value is always a truth value.[117]

Frege considers two functions '$x\ (x - 4)$' and '$x^2 - 4x$' and when these functions are put in the form of an identity function, viz. '$x\ (x - 4) = x^2 - 4x$', Frege says that we have not put one function equal to the other, but rather values of one equal to the those of others. This function holds for any argument substituted for x.[118] For Frege, the value-ranges of identity functions, inequalities, and relational functions are truth-values. Frege treats truth values as objects, and he calls the former the True, and the latter the False. [119] Hence, given the function '$(\ )^2 = 4$' is completed with the argument '2', then '$2^2 = 4$' *denotes* the True, and when the function '$(\ )^2 = 4$' is completed with the argument '1', then $1^2 = 4$' *denotes* the False.

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[116] Frege (1891a (*FR* p. 133)). Earlier in a letter, Frege (1882 (*FR* p. 81)) says that

> A concept is unsaturated in that it requires something to fall under it; hence it cannot exist on its own. That an individual falls under it is a judgeable content, and here the concept appears as a predicate and is always predicative. In this case, where the subject is an individual, the relation of subject to predicate is not a third thing added to the two, but it belongs to the content of the predicate, which is what makes the predicate unsaturated.

[117] Frege (1891a (*FR* p. 134)). This corresponds to Frege's (1884) earlier identification of the difference between values of functions, which are objects thus saturated, and denotations of functions, which are concepts thus unsaturated.

[118] Frege (1891a (*FR* p. 135)) notes that "an equality between value-ranges is indemonstrable; it must be taken to be a fundamental law of logic." The slight difference is that in "ordinary mathematical terminology, the word 'function' certainly corresponds to what I have here called the value-range of a function. But function, in the sense of the word employed here, is the logically prior [notion]" (Frege 1891a (*FR* p. 135 n. 2)).

[119] Frege (1891a (*FR* p. 137)).

Next, Frege applies his theory to natural languages, and then provides a basic framework for the difference and first distinction between *sense* [*Sinn*] and *denotation* [*Bedeutung*].[120] Frege regarded mathematical expressions and their natural language correlates as notational variants. Thus, he transfers his theory concerning the function-argument analysis of the mathematics to analysis of the structure of the natural language. All in all, Frege aimed to maximize perspicuity, clarity, and precision in reasoning. He considers sentences as the linguistic form of equations.[121]

Arguably, in this work we find the earliest statement of his theory of *Thoughts* in the explicit sense. Frege argues that a statement contains a *Thought* as its sense, and this *Thought* is true or false. *Thoughts* are *senses* of sentences, and they denote truth-values. Thus, Frege generalizes his analysis to all statements in general which are *split up* into two parts: one complete in itself, and the other in need of supplementation, or *unsaturated*. If a sentence contains no empty place, and then its *denotation* is an object, more precisely a truth-value. Hence, Frege concludes that the two truth-values, i.e., the True and the False, are objects. For the definition of objects Frege makes the following remark:

> When we have thus admitted objects without restriction as arguments and values of functions, the question arises what it is that we are here calling an object. I regard

[120] We shall cite from Frege (1891a (*FR* p. 138)) in length:

> [F]rom identity of [*denotation*] there does not follow identity of the thought [expressed]. If we say 'The Evening Star is a planet with a shorter period of revolution than the Earth', the thought we express is other than in the sentence 'The Morning Star is a planet with a shorter period of revolution than the Earth'; for somebody who does not know that the Morning Star is the Evening Star might regard one as true and the other as false. And yet the [*denotation*] of both sentences must be the same; for it is just a matter of interchange of the words 'Evening Star' and 'Morning Star', which have the same [*denotation*], i.e. are proper names [*Eigenname*] of the same heavenly body. We must distinguish between *sense* and [*denotation*]. '2^4' and '4.4'certainly have the same [*denotation*], i.e. are proper names of the same number; but they have not the same *sense*; consequently, '$2^4 = 4^2$' and '$4.4 = 4^2$' have the same [*denotation*], but not the same *sense* (i.e., in this case: they do not contain the same *thought*).

In the footnote of the last sentence, Frege mentions his forthcoming "On *Sense* and *Denotation.*"

[121] Frege (1891a (*FR* p. 139)).

> a regular definition as impossible, since we have here something too simple to admit of logical analysis. It is only possible to indicate what is meant [*gemeint*]. Here I can only say briefly: an object is anything that is not a function, so that an expression for it does not contain any empty place. [122]

Frege exemplifies and extends applications of function–argument analysis by embedding the *Begriffsschrift* notation. We can give basic tenets of his logical theory. Frege calls first-level unary-functions a *concept* and first-level binary-functions *relations*. He further analyzes functions with multiple arguments, logical connectives, and generalities. Frege distinguishes kinds[123] and levels[124] of functions. First-level functions are functions whose arguments are objects and second-level functions have functions as their arguments. Similarly, he makes a distinction between first-level concepts and second-level concepts. Nevertheless, Frege seems to have a wide permissibility, since some functions can take first-level functions as their arguments and yield objects as values, and some functions take second-level functions. However, this will build up into a very complicated problem for the structure and unity of *Thoughts,* together with his strict and exclusive distinction between objects and concepts.

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[122] Frege (1891a (*FR* p. 140)). In Frege (1892b (*FR* p. 140 n. J)) he makes a remark "I call anything a proper name if it is a sign for an object."

[123] There are several kinds of functions in Frege's logical theory. The first one, the judgment stroke, expresses the *correctness of Thought* [*die Richtigkeit des Gedanken*] that what follows it stands for the True, i.e., the argument of this function is a true *proposition*. It maps the object to the True; otherwise, it maps all other objects to the False. The second, the conditional stroke, expresses a conditional function which maps a pair of objects to the False if its antecedent is the True and its consequent is the False, otherwise maps all pairs of objects to The True. The third one expresses a negation function which maps the True to the False, and vice versa. Frege did not use other the primitive connectives such as 'and', 'or', or 'if and only if' since they can be defined in terms of negations and conditionals. He did not give the reason for preference of primitive logical connectives either. Also, in the *Begriffsschrift* he did not use the word 'True' and 'False' but rather used 'affirmed' and 'denied.'

[124] Frege considers logical generalities as second-level concepts and they are expressed by the second-level functions which maps a first-level function Φ to the True, if Φ maps every object to the True; otherwise it maps Φ to the False. Frege's logic is second-order calculus, i.e., it allows quantification over functions as well as quantification over objects.

To sum up the points till now, we have seen Frege's function–argument analysis supplemented with the saturated-unsaturated distinction applying to simple and complex expressions, and his distinction between two ontological categories objects and concepts.

3.1.3. The Sense – Denotation Distinction

In his next paper, "On *Sense* and *Denotation*" Frege reveals important consequences of his semantic theory applying to all kinds of saturated linguistic expressions, altogether with his previous semantic notions that we have covered. Moreover, he also systematically analyses attitude ascriptions in intensional contexts, and applies his distinction to such expressions.

Frege grounds the core elements of his semantic theory on the distinction between *sense* and *denotation*. This distinction is the most famous and influential one in Frege's mature philosophy. Frege begins with an illuminating passage on identity. We can extract two identity puzzles. The first puzzle is about identity statements between coreferential terms, and the other is about sentences consisting subordinate clauses, i.e., sentences consisting of that-clauses and intensional attitude verbs. In each puzzle, Frege shows that one cannot account for the meaning of sentences solely on the basis of the *denotations* of the singular terms (proper names or descriptions) in a sentence.

Let us begin with the first puzzle which concerns a challenging question between identity statements. We shall consider the following sentences:

(1) Lewis Carroll is Lewis Carroll

(2) Lewis Carroll is Charles Lutwidge Dodgson

One can express these sentences in the identity forms of $a = a$ and $a = b$, respectively. Frege treats signs '*a*' and '*b*' as names, or descriptions that denote objects. Obviously, these two identity forms differ in cognitive value [*Erkenntniswert*], since the former of the form $a = a$ holds *a priori* and is analytic, whereas the latter

statements of the form $a = b$ "often contains very valuable extensions of our knowledge and cannot always be established *a priori*." [125] Following the Kantian distinction, we can say that (1) is known a priori whereas (2) is not. Indeed, the sentence (2) extends our knowledge, at least for people who did not know that Lewis Carroll and Charles Lutwidge Dodgson are one and the same person, whereas (1) expresses a trivial self-identity. Therefore, sentence (1) is true if and only if the object Lewis Carroll is the very person Lewis Carroll and (2) is true if and only if the object Lewis Carroll is the very person Charles Lutwidge Dodgson. However, the account of truth of these two sentences is not sufficient to explain their differences in cognitive value, since the truth of statement of the form '$a = a$' is trivial as in (1), whereas the truth of the statement of the form '$a = b$' is not, for only historical or literary inspection can reveal the truth of (2). Hence, the puzzle demands an explanation for the cognitive difference between these two sentences.

The second puzzle is about the attitude reports which involves certain attitude verbs following a subordinate clause.[126] Thus, we can begin by considering two attitude statements:

(3) Mehmet believes that Lewis Carroll is the author of *Alice in Wonderland*.

For instance, the attitude verb 'believes' relates the person Mehmet to the object of attitude expressed by the sentence that Lewis Carroll is the author of *Alice in Wonderland* in (3). The problem with the attitude reports is that if the proper name Charles Lutwidge Dodgson is substituted in place of Lewis Carroll, the following sentence

(4) Mehmet believes that Charles Lutwidge Dodgson is the author of *Alice in Wonderland*.

[125] Frege (1892a (*FR* p. 151)).

[126] Frege's presentation of this puzzle is a bit different. We shall analyze an overall presentation of this puzzle in order to explain this role of *Thoughts* in the following section. See Klement (2002, pp. 126-128) and Zalta (2022, sec. 3.1.).

must have the same truth value as (3). This is due to Leibniz's Substitution Principle, *substitutivity salva veritate,* according to which the substitution of the name *a* for the name *b* will not affect the truth values of the sentences. In other words, when two names denoting the same object, Lewis Carroll and Charles Lutwidge Dodgson, are substituted then the sentences containing these names must have the same truth value. However, as Frege noticed, this is not the case as in (3) and (4). While Mehmet believes that Lewis Carroll is the author of *Alice in Wonderland*, he may not believe that Charles Lutwidge Dodgson is the author of *Alice in Wonderland*. It turns out, then, (3) is true whereas (4) false. As a result, he concludes that a proper name cannot be substituted by another name of the same object. As with the sentence (3) and (4) in which there seems to be a difference in content.

Having considered these two puzzles, Frege argues that one cannot give account for the meaning and truth of these sentences solely on the basis of the denotations of the singular terms. Therefore, Frege makes a distinction between *sense* and *denotation* of singular terms and argues that this distinction is required to explain the meaning of the singular terms and sentences. Frege have proposed two distinct solutions to these puzzles.

We shall first consider the solution to the first puzzle. Earlier in the *Begriffsschrift*, the sentences (1) and (2) has the same cognitive value, due to Frege's two commitments: (i) The view that identity relates objects, and (ii) the Substitution Principle in the *Begriffsschrift* according to which if two coreferential singular terms are substituted, the resulting expressions must have the same cognitive value. In *Begriffsschrift* Frege regarded the puzzles as a reductio of (i), though he does not explain why he did choose to take identity as a relation between expressions. [127] In "On *Sense* and *Denotation*", however, he regards the puzzles as a reductio of (ii). By distinguishing *sense* and *denotation*, Frege's Substitution Principle is then split up into two distinct substitution principles, one for *denotations* which corresponds to truth values, and one for *senses* which corresponds to cognitive values. It is the cognitive value that is preserved under the substitution of singular terms having the

[127] *Cf*. Mendelsohn (2005, pp. 21-22).

same sense.[128] Thus, the *sense* of a singular term accounts for its cognitive significance "wherein the mode of presentation [*Art des Gegebenseins*] is contained" in the way by which one conceives of the *denotation.* [129] Therefore, the *sense* of a singular term determines its *denotatum*. [130] As a result, proper names "Lewis Carroll" and "Charles Lutwidge Dodgson" denote the same person but these names express different *senses*, presenting him in different ways.

For the solution of the second puzzle, Frege proposes that expressions following attitude verbs do not denote their ordinary *denotation*, instead they *denote* the *senses* they ordinarily express.[131] Frege distinguishes *denotations* of sentences between *customary* or *primary denotations* and *indirect denotations*, and similarly for the *senses* he distinguishes between their *customary senses* and *indirect senses*. In Frege's own analysis of the sentences containing subordinate clauses, i.e., the intensional attitude verbs followed by a that-clause such as 'believes that', express *denotation* function which maps their *denotations* indirectly to customary *senses* of the sentences.[132] Accordingly, the denotation of the sentences (3) and (4) have different indirect *denotations*, and they map the sentences followed 'believes that'

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[128] *Cf.* Klement (2002, pp. 14-22) and Mendelsohn (2005, pp. 30-33).

[129] Frege (1892a (*FR* p. 152)). From the following quote, Frege says that *senses* can be presented in different ways: "The sense of a proper name is grasped by everybody who is sufficiently familiar with the language or totality of designations to which it belongs; but this serves to illuminate only a single aspect of the denotation, supposing it to have one. Comprehensive knowledge of the *denotation* would require us to be able to say immediately whether any given sense attaches to it. To such knowledge we never attain." (Frege 1892a (*FR* p. 153))

[130] In a footnote Frege remarks that senses can also be a definite description, e.g., 'the author of Alice in the Wonderland' or 'the Lecturer in Mathematics at Christ Church' all of which denote the same object Charles Lutwidge Dodgson. Frege (1892a (*FR* p. 153 n. B)). However, this point is highly problematic and Kripke famously provided decisive counterarguments. Following Soames' (2002, pp. 18-19) these are classified as namely semantic, epistemic and modal arguments. See Kripke (1972; 2008), Soames (2002, Chs. 2 and 3; 2003, Ch. 7).

[131] See Frege (1892a (*FR* p. 154)).

[132] See Frege (1892a (*FR* pp. 162-164)). However, there some objection to Frege's hierarchical approach to senses and denotations. Davidson (1967) argues against Frege's approach that it would make language impossible to learn. But provided that one can understand senses, she can understand what an indirect sense corresponds. For important modifications of Frege's account see Dummett (1981a, Ch. 9), Beaney (1996, pp. 181-183), Mendelsohn (2005, p. 140).

to different *Thoughts*, since the customary *senses* of the proper names occurring in these sentences are different. In this respect, these sentences express different *Thoughts*. In other words, *indirect denotation* does not map the denotation function into truth values, but rather maps to their customary sense. On the other hand, the whole subordinate clause 'that Lewis Carroll is the author of *Alice in Wonderland*' *denotes* its customary *sense*, the *Thought*, which maps it to truth values. Therefore, Mehmet can rationally and consistently believe in the truth of the *Thought* expressed by sentence (3) and at the same in the falsehood of the *Thought* expressed by sentence (4).

Frege's proposed distinction between sense and denotation of linguistic expressions gives a solution to these puzzles. In a nutshell, *denotation* alone cannot capture cognitive value, and it is the *sense* of an expression which is intended to capture its cognitive value. Therefore, Frege concludes that *sense* is distinct from *denotation*.

3.2. Frege's Theory of Thoughts

Following the publication of the second volume of his *Grundgesetze*, Frege did not publish anything until 1918. Between 1918 and 1923, his publications mostly focus on refinements of his conception of *Thoughts.* He generally endorsed his earlier views on philosophy of language, but he elaborates and improves his theory on some points. In this section, we shall present an overview of his theory of thoughts. In the first part, we shall focus on the semantic roles, and in the second section we will focus on his general considerations concerning the nature of thoughts.

3.2.1. The Roles of Fregean Thoughts

In this section, we shall focus on the roles of *Thoughts* in his semantic theory. Frege's *sense-denotation* distinction reveals the roles of *Thoughts* as the meanings

of sentences, bearers of truth-values, the objects denoted by that-clauses and intensional attitudes.

A primary task of Frege's semantic theory is to explain the meaning of complex linguistic expressions. In this respect, he has extended the *sense-denotation* distinction from proper names to sentences. For Frege, the sense of a declarative [*Behauptungssätze*] sentence is the *Thought* it expresses, and its denotation is its truth-value. Frege has argued in length that *Thoughts* are complex entities. A *Thought* is the sense expressed by a sentence, and word(s) belonging to different syntactic categories, e.g., names, predicates etc., forms a sentence. In other words, there are corresponding word parts which build up the sentence. Since Frege has attributed senses corresponding to these sentence parts, it follows that the *sense* of a sentence is determined by the *senses* of its corresponding expressions in addition to certain modes of combination, e.g., arrangements and punctuations, of these expressions to constitute a meaningful sentence. This is famously known as Frege's compositionality principle for the *senses*. Therefore, Frege attributes a complex structure to *Thoughts*. Since *senses* account for the semantic values, or meanings, of expressions, as a result *Thoughts* account for the meanings of sentences. This shows the first important semantic role of *Thoughts* as the meanings of sentences. In this regard, the unity of *Thoughts* depends on its saturated nature.

Frege holds that *Thoughts* are the bearers of truth values. In fact, Frege's notion of truth-values has an indispensable importance in Frege's philosophy. First and foremost, he took denotations of *propositions* as truth-values even before his sense-denotation distinction. As we have previously considered, the function-argument analysis of his formal system depends on the truth-functional inferences of mathematical or linguistic expressions. Hence, he always thought that logic has a special concern regarding the truth: "All sciences have truth as their goal; but logic [is] also concerned with it in a quite different way: logic has much the same relation to truth as physics has to weight or heat. To discover the truths is the task of all science; it falls to logic to discern the laws of truth." [133] Therefore, we can assume

[133] Frege (1918a (*FR* p. 325)). Almost the same passage appears in Frege (1897 (*FR* pp. 227-228)).

that he has endorsed the same motivation in his semantic theory. Accordingly, Frege has argued that "it is striving for the truth that drives us always to advance from the sense to reference."[134] As a result, Frege conceives *denotations* of saturated expressions are objects, thus *denotations* of sentences are also objects, but of a special kind, namely the True and the False. Frege established a key relationship between *sense* and *denotation*; *senses* of linguistic expressions determine denotations of linguistic expressions, but not the other way around. Frege applied the implication of this principle to sentences, therefore the *denotation* of a sentence is determined by the *sense* of the sentence.

As we have considered above, Frege appealed to a composition principle for *sense* parts of a sentence. Earlier, Frege held a corresponding claim for *denotations* of sentences: "I have in fact transferred the relation between the parts and the whole of the sentence to its [*denotation*], by calling the [*denotation*] of a word part of the [*denotation*] of the sentence, if the word itself is part of the sentence." [135] However, Frege later argued that *denotations* of sentence parts are not parts of a *Thought* expressed by a sentence. Since Frege defines *Thought* as the *sense* of a sentence, it only contains the *sense* of the singular term as its part but not as its part of *denotation*. Otherwise, it would be tantamount to say that the *Thought* 'Ankara is a city in Anatolia' has the object 'Ankara' as its part. However, in that case the denotation of 'Ankara', i.e., the object, would be the part of the denotation of the *Thought* expressed by that sentence, i.e., the True.

Frege also considered expressions without *denotations* and held that all singular terms have *senses* but they may not *denote* an object. Hence, a sentence containing a non-denoting object lacks a truth value, nevertheless it has a *Thought*. [136] For instance, two sentences 'Tepegöz is a giant' and 'The largest prime number is odd' express (distinct) *Thoughts*, however they do not *denote* any truth-value, since there is no object corresponding to the proper name 'Tepegöz' nor to the

[134] Frege (1892a (*FR* p. 157)).

[135] Frege (1892a (*FR* p. 159)).

[136] Frege (1892a (*FR* p. 157)).

definite description ‘the largest prime number.’ Yet these sentences are meaningful, and they indeed express *Thoughts*: The former expresses a thought about a certain mode of presentation of an object called Tepegöz by its property of being a giant, and the latter expresses a thought about a descriptive phrase by the property of being odd.

In his works, Frege has always taken truth as a given primitive notion. In his later works, he focuses on the definition of truth, but in the final analysis he concluded that truth is not an explicable notion. Any formal definition of truth cannot be given and any attempt to reduce truth to simpler terms would result in a circle, since it would tantamount “to presuppose the very thing that is being defined.” [137] Frege argues that the word ‘true’ itself can be seen as a word for a property, however he held that this property must be indefinable.[138] For Frege, one cannot understand the definition of the True, unless the notion of truth is already grasped. Frege also argued that definition of truth with correspondence to reality is also circular. Of course, he does not discard extralinguistic reality for determining the actual truth-values of sentences, however, this is out of boundaries of any inquiry concerning logic and semantics. He once said: “What is a fact? A fact is a thought that is true.” [139]

Frege is generally interpreted as having a sort of redundancy view of truth for *Thoughts*. Earlier he simply says “by the truth value I understand the circumstance that it is true or false”[140] and he partly expounds his argument later by claiming that “we cannot recognize a property of a thing without at the same time

[137] Frege (1897 (*PW* pp. 128-129)).

[138] Frege (1918a (*FR* pp. 326-327)).

> [A]ny other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were true that the characteristics were present. So we should be going round in a circle. So it seems likely that the content of the word 'true' is *sui generis* and indefinable. (1918a (*FR* p. 327)).

[139] Frege (1918a (*FR* p. 342)).

[140] Frege (1892a (*FR* p. 163)).

finding the thought this thing has this property to be true. So with every property of a thing there is tied up a property of a *thought*, namely truth." [141] In this respect, to make a judgment is the same as acknowledging the truth of a *Thought*. Since, truth is essential property of *Thoughts*, predicating truth of a *thought* adds nothing new, thus one does not need to use the word 'true.' [142]

Frege attends a third semantic role of thoughts, as the objects of certain cognitive attitudes *denoted* by what Frege calls subordinate clauses. As we have considered in the previous section, the sense-denotation distinction has a certain modification when they are parts of direct or indirect speech. Frege considers such cases exceptional, but they are crucial to understand the mode of connection between expressions and their sense and denotation. [143] To conclude, Frege's distinction between sense and denotation gives his account for the semantic content of attitude ascriptions, and in this respect, he firmly establishes the connection between language and cognition in his semantic theory of *Thoughts*.

Frege was well aware that his distinction between sense and denotation is not exhaustive. Thus, he has recognized that certain aspects of meaning cannot be solely explained by this distinction. In his "On Sense and Reference" he

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[141] Frege (1918a (*FR* p. 328)).

[142] Frege (1918a (*FR* p. 330)). He gives numerous examples for this point in his works:

> The sentence 'The thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number'. (1892a (*FR* p. 158))
>
> that the sentence 'I smell the scent of violets' has just the same content as the sentence 'It is true that I smell the scent of violets'. So it seems, then, that nothing is added to the thought by my ascribing to it the property of truth." (1918a (*FR* p.328))

And also in his posthumous writings:

> in the two sentences 'Frederick the Great won the battle of Rossbach' and 'It is true that Frederick the Great won the battle of Rossbach', have […] the same thought in a different verbal form. (1897 (*PW* p. 141; *FR* p. 242))
>
> 'The thought that 3 > 2 is true' can be more simply said by the sentence '3 is greater than 2'. (1914 (*PW* p. 233))
>
> [T]he the sense of the word 'true' is such that it does not make any essential contribution to the thought. If I assert 'it is true that sea-water is salt', I assert the same thing as if I assert 'sea-water is salt'. (1915 (*PW* p. 251))

[143] Frege (1892a (*FR* pp. 161-164)).

distinguishes the sense and the *force* of a sentence. This distinction corresponds to the difference of the content of a sentence from the assertion of that content at the level of the utterance. An utterance of a sentence may lack its usual force, for example "in the mouth of an actor upon the stage." [144] In his late works of *Logical Investigations*, he further distinguishes *sense* between *force* and *tone* of an expression to give account for the additional aspects of meaning. The former notion accounts for the certain distinctions between modes of speech such as assertions, imperatives, questions etc. The latter accounts for the other aspects of meaning, such as differences in cases of the following sentences 'Chrysippus is a philosopher and he is wealthy' and 'Chrysippus is a philosopher but he is wealthy.' Sentences having such conjunctive phrases have the same sense and same truth conditions. Yet, Frege argued that they differ in their *tone* or *coloring*. Frege also argued that variations of tone of expressions may also be connected to the stylistic differences of word choices. [145]

3.2.2. Nature of Fregean Thoughts

In this part we shall focus on some properties concerning the nature of *Thoughts*. For our purposes, we shall mainly focus on the nature of thoughts in the context of objectivity, mind, and language independent abstract existence. We shall begin by considering the nature of *senses*. Frege, due to his fierce opposition to use of any psychologicist notion in logic and mathematics, distinguished *senses* from any subjective ideas [*Vorstellungen*] which an agent associates with a name. "When a person grasps [*fassen*] a *sense*, there must be something in his consciousness that is aimed at." [146] Nevertheless, we do not create *senses*, we can only stand in a certain

[144] Frege (1892a (*FR* p. 158)).

[145] However, we shall not take this aspect in Frege's semantic theory of *Thoughts* since Frege (1892a (*FR* p. 155)) did note that "Such colouring and shading are not objective, and must be evoked by each hearer or reader according to the hints of the poet or the speaker. Without some affinity in human ideas art would certainly be impossible; but it can never be exactly determined how far the intentions of the poet are realized."

[146] Frege (1918a (*FR* p. 342)).

relation to them, but that relation is different from both perceptions and ideas. Therefore, *senses* have strictly objective existence.

Frege attributed a *sui generis* ontological nature to *senses* which exists in a third realm [*dritte Reich*][147], apart from both the mental and physical entities. Since Frege attributed an objectivity criterion to *senses*, the same criterion applies also to *Thoughts*. [148] Frege ascribes an abstract existence for both *senses* and *Thoughts* taken for granted in his papers. For Frege, the existence of *Thoughts* is both language and mind independent. In other words, its existence does not depend on any psychological or linguistic behavior of agents. Moreover, explanations about *Thoughts* are not reducible to any description of them. *Thoughts* are distinct from the mental states and their physical realizations in the brain. As we have considered earlier, *Thoughts* are distinct from the act of thinking which Frege considers to be subjective. A Fregean *Thought* is considered to be objective in the sense of being an abstract entity qua the semantic content of a given sentence. Contrary to sensations or mental images which have a subjective character, the identity conditions of *Thoughts* make no essential reference to the identity of the person who has those thoughts. According to Frege, same *Thoughts* can be grasped and shared by different people "via use of language".[149] This feature is known as the *intersubjective* character of *Thoughts*. [150]

Thoughts have truth-values regardless of whether someone knows or believes, even thinks about them. Agents can grasp thoughts by their power of thinking, nevertheless their existence is *eternal* independent of their thinkers. *Thoughts* are not brought into existence by use of language; therefore, their existence cannot be annihilated. Thoughts are not created by thinking them, but

[147] "A third realm must be recognized." (Frege (1918a (*FR* p. 337)). Beaney (1997, pp. 30-32) presents an interesting discussion between the Stoic and Fregean separate existence of propositions. See also Dummett (1991a, pp. 249-262), and Burge (1992).

[148] See Frege (1892a (*FR* p. 156 and n. E)); esp. 1897 (*PW* pp. 133-149); 1918a (*FR* pp. 336-345)).

[149] Frege (1918a (*FR*, p. 343)). However, according to Currie (1980, pp. 238-241), the question of how mind grasps a thought is not clearly answered by Frege.

[150] *Cf.* Bell (1987, pp. 37-38).

rather they are discovered by the act of thinking. All in all, whether Frege's objectivity criterion is an obtainable standard is a matter of another debate, yet we can obviously argue that it is a high standard, imposing a consideration of some sort of ideal language free of polysemy, ambiguity and vagueness. [151]

[151] For further interpretations on nature of Frege's *sense* and *Thoughts* see Dummett (1981a, Chs. 6, 11, 14), Currie (1980), Bell (1987, pp. 36-50), Klement (2002, pp. 58, 63-64), Mendelsohn (2006, pp. 35-36), and Heck and May (2006).

CHAPTER 4

PROPOSITIONS AND *THOUGHTS*

The aim of this chapter is to argue that Fregean *Thoughts* are propositions. For this reason, we will focus on analogous approaches, often collectively classified under the term *proposition*. In the first part, we will give an account for a fundamental theory of propositions by analyzing their properties and nature. In the second part of this chapter, we will extract some arguments for the existence of propositions. In the final part, we will state problems concerning structure and unity for propositions. These problems will be our criteria for comparing Frege's theory of *Thoughts* with contemporary theories of propositions in the next chapter.

We shall begin this chapter with an important terminological clarification by defining the terms "sentence" and "proposition". A *sentence* is defined to be a complete string of words formed according to syntactic or grammatical rules of a given natural or artificial language.[152] A *proposition* is commonly defined by two characteristic features both of which comprise the term "sentence": First, propositions are what-is-said or expressed by sentences which characterize the meanings of sentences. In other words, propositions are what is stated, asserted, judged, believed, denied, etc. by sentences. Second, propositions are the primary bearers of truth-values. This feature characterizes the capability of a sentence to be true or false.

[152] We shall use the term in the sense of declarative closed sentence, excluding other kinds of expressions, such as questions and commands.

As we have considered in the second chapter, the term proposition is widely used in an ambiguous way. Naturally, this ambiguity is carried over to the early analytic philosophy of language. For Carnap, the term proposition is ambiguous between two different concepts: (i) a declarative sentence, and (ii) "that which is expressed (signified, formulated, represented, designated) by a (declarative) sentence."[153] According to the former, the following sentences express different propositions.

(1) Snow is white

(2) Kar beyazdır

(3) Schnee ist weiss

On the other hand, according to the latter definition, which Carnap favors, these sentences in English, Turkish, and German express the same proposition. Church adds a third connotation to Carnap's definition, which goes as follows:

> [T]he content of meaning of declarative sentence, i.e., a postulated abstract object common not only to different occurrences of the same declarative sentence but also to different sentences (whether of the same language or not) which are synonymous or, […] mean the same thing. [154]

Church remarks that propositions in this sense is neither a physical entity nor a linguistic entity, instead they are obtained by abstraction from language.[155] Sentences (1), (2), and (3) express the same proposition in this case, for they have the same meaning. Note that the notion of synonym, or sameness of meaning, is an antecedent notion for different sentences to express the same proposition, either in the same language, such as "Mehmet loves Ayşe" and "Ayşe is loved by Mehmet", or in different languages such as (2) and (3).[156] Note also that different occurrences

[153] Carnap (1942, pp. 18, and 52; 235).

[154] Church (1959/2019, p. 514).

[155] Church (1956a/2019, p. 356).

[156] A more elucidative sense synonymy can be defined as "for each meaningful expression there are correct answers to the question "What does it mean?", and that two expressions mean the same thing when the answer to this question is the same for both" Soames (2010, p. 1). As a grounding

of the same sentence, such as “today is Friday”, express the true proposition when the day is Friday, and for other days of the week it expresses a false proposition. [157]

4.1. What are Propositions?

In this part, we shall first introduce a standard theory of proposition. This theory includes the essential semantic roles and ontological features of propositions. Philosophers who accept propositions think that propositions have an indispensable importance in philosophy and logic, especially for explaining the interconnected notions of meaning and truth. Moreover, they take propositions to provide an analysis of intensional attitudes, such as knowledge, belief, desire etc. In the first subsection, we will consider roles of propositions in semantic theories by providing a minimal theory of propositions, and then we will consider some additional roles. In the last subsection, we will consider some features of propositions regarding their nature.

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presupposition, we shall take it for granted that words and sentences have meaning as a commonsensical fact. Hence, we consider the following sentences “Snow is white” and “Snow is black” as meaningful, the former expresses the truth, and the latter expresses the falsehood. On the other hand, sentences “Ecocily chumble phleeb” or “Dreams drink the book trigonometrically” are arguably meaningless and/or nonsensical, thus they are neither true nor false. These examples are sufficient to show that for a sentence to express a proposition, it must have a meaning and *only then* we can attribute the truth values.

[157] It is important to note that things get much more complicated when we consider the sentences containing context-sensitive or token-reflexive expressions, such as time, place, and person etc. Throughout this thesis, we presuppose a context-independent language, in which every term has the same semantic value, hence the same interpretation. Expressions only in a given context *C* can have a well-defined content, thus *Thoughts* have well-defined content in a given context *C*, thereby express a truth-value. In all of our examples, we shall assume that context is tacitly known. Davies (2006, p.19) notes that “there is an intuitive distinction to be drawn between the message that a speaker communicates and the meaning of the sentence that the speaker uses” and then he gives the following example: The utterance of the sentence “Coffee keeps me awake” can be used to report (i) to accept an offer for coffee; (ii) to decline the offer; or (iii) to state a mere fact that coffee prevents sleeping. We hold no stance concerning priority between semantics or pragmatics. However, in the framework of this thesis, we shall deal with only literal meanings of sentences. We also exclude the truth of pathological sentences such as *the Liar*.

4.1.1. Roles of Propositions

In this part, we shall present the key semantic roles of propositions, which is grounded in commonsensical ideas about how language functions. This will show that propositions share the same semantic roles as *Thoughts*. It will also ease the effective presentation of different theories of propositions, as we will explore in the subsequent chapter. According to the so-called *Classical* or *Naïve Theory of Propositions*, propositions are

(R1) The meanings of sentences

(R2) The primary bearers of truth-values

(R3) The objects of (intensional) attitudes, denoted by that-clauses

These roles present the basic definitional roles of propositions and are called *minimal* roles of propositions. The first two roles of proposition are perhaps the most important ones. Propositions are expressed by sentences, and they are the meanings and the primary bearers of truth-values. In this context, propositional-realist philosophers have attributed abstract entities for the meanings expressed by sentences. For the truth ascription clause for propositions, it exemplifies or instantiates the property expressed by 'true'. Propositions stand in a certain relation with objective reality, i.e., the way the world is and the way the things are, or simply what is the case. For the present purposes, we shall define the truth in its most basic sense as follows:

> Any sentence *p* is true if and only if the proposition expressed by *p* has the property expressed by 'true.' [158]

[158] There is a great diversity of conceptions of truth. However, the definition of truth is a long, if not the longest, debate among philosophers. We shall consider truth as a primitive notion without further metaphysical grounding. See Horwich (2010). True propositions are called *facts*. As we have considered in the previous chapter, Frege considered *facts* as true *Thoughts*. For a variety of views concerning the propositional truth and facts see Kirkham (1995, esp. pp. 54-58), Künne (2003, Chs. 2 and 5), Mulligan and Correia (2021).

The last minimal role of propositions is that they are the objects of attitude ascriptions and referents of that-clauses. An attitude ascription (or report) is in the form of '*a* $\mathcal{V}$s that *p*' where *a* is an agent, $\mathcal{V}$ is an intesional attitude verb, and *p* is a propositional variable. Note that propositional attitude ascriptions can be expressed in the absence of a that-clause, viz. "Mehmet believes what Ayşe said." Intensional transitive verbs contain knows, believes, hopes, desires, etc. and they relate attitudes to agents. The verbs relating propositions to attitudes of agents are called *propositional attitude verbs*.[159] This role is important to analyze and to give an account for the certain cognitive relations agents bear to proposition. In other words, if an agent has a propositional attitude towards the truth expressed by a proposition, viz. she rationally and sincerely accepts it, and if her attitude is consistent with the falsity expressed by a different sentence, then these two sentences express *different* propositions. For instance, "Mehmet believes that the Morning Star is a planet" expresses a belief relation which relates Mehmet to the thought that the Morning Star is a planet. Similarly, "Mehmet believes that the Morning Star is a supernova" expresses an attitude relation to the proposition that the Morning Star is a supernova. Although what Mehmet believes is a false proposition, the proposition that Mehmet believes that the Morning Star is a supernova is true, on the condition that our agent Mehmet sincerely and consistently believes that the Morning Star is a supernova.

We shall end this section by making a comparative review of the propositional roles in Frege's theory of *Thoughts*. For Frege, the meaning of a declarative sentence is the *sense* expressed by that sentence which he named *Thought*. The *denotation* of a *Thought* is a truth value, either the True or the False. Hence, *Thoughts* are the primary bearers of truth values. Finally, *Thoughts* are objects of propositional attitudes, which Frege named as denotations of subordinate clauses. However, for Frege, expressions in these "indirect" or "opaque" contexts or "*oratio obliqua*" have secondary *references* which denote their ordinary *senses*.

[159] This term is coined by Russell. Note that some propositionalists define propositional attitude ascriptions as cognitive mental states directed at a proposition. We shall also note that not all attitude ascriptions are propositional.

We conclude that *Thoughts* have all the necessary roles of propositions in a generally conceived theory of propositions.

4.1.2. Additional Roles of Propositions

Some philosophers have attached additional roles to propositions for their theoretical needs. In addition to above mentioned minimal roles, a number of different roles have been associated with the propositions. These additional roles are as follows:

(AR1) The bearers of modal properties, such as necessity, and contingency

According to this role, propositions expressed by sentences are bearers of modal properties such as necessity, contingency, and actuality in virtue of expressing a proposition in a possible world. A theory of proposition endorsing these modal properties are classified as Possible World conception of propositions, which we will consider in the next chapter.

(AR2) The relata of logical relations, such as entailment and validity of arguments

This role is sometimes taken implicit in the role of being truth-bearers. However, there are some controversies concerning this role. [160]

[160] Dyke (2012, p. 128) and McGrath and Frank (2020) take (AR2) as an additional role. On the other hand, Merricks (2015) rejects this role. By citing Church (1956b/2019), Merrick defines logical validity in terms of logical form, and defines modal validity in terms of necessity, i.e., "an argument is *modally valid* just in case, necessarily, if its premises are true, then its conclusion is true" (p. 1). According to Merricks, there are logically valid arguments only if there are modally valid arguments. In other words, logically valid arguments are also modally valid, but its converse does not hold. Hence, only the premises and conclusions of modally valid arguments are propositions (p. 18). He also provides an argument for the existence of propositions in which he argues in length that the existence of propositions can be deduced from the existence of modally valid arguments (pp. 34-35).

(AR3) The informational content of sentences[161]

(AR4) The objects of acts of assertion, and other cognitive act or event types[162]

(AR5) Common ground in a conversation [163]

An important question is whether a single theory of proposition can account for all these roles. For this question, we can present two important problems that rise from multiplying the roles of propositions. First and foremost, it increases the semantic burden of a theory of propositions. In this respect, keeping the roles in the minimal quantity is indeed advantageous for a Fregean theory since it has the minimal set of semantic commitments. Second, some philosophers have argued that there are certain tensions, even inconsistencies, between these roles.[164] Hence, they claim we have another reason for construing an adequate and satisfactory theory of *Thoughts* in its minimal condition. In our opinion this point has a particular credibility, however we have to note that there is no particular theory of proposition which holds all these roles. Usually, these roles are considered as extensions to satisfy additional analytical and explanatory needs of their theories. Nevertheless, investigating these roles will help us understand points of divergences between different accounts of propositions some of which we will focus in the next chapter.

[161] There is an ambiguity about what the term informational content purports to designate. King (2007, p. 1) mentions this as information encoding role, according to which different sentences can encode the same information. The encoded information along with the state of the world determine the truth values. King (2018, p. 307) considers this role as commonality between interlinguistic synonymy and differentiates it from intralinguistic synonymy. Soames (2010, p. 4) mentions this role in the pragmatic sense. Dyke (2012, pp. 128-129) classifies it as a complement of (R1).

[162] Soames (2010, 2014a, 2014b, 2014c) and Hanks (2011, 2015) include this role for the needs of their so-called *cognitive-realist* accounts of propositions.

[163] Stalnaker (1999) refers to this role to characterize the context of utterance to provide the common conversational ground. See King (2018, pp. 307-308).

[164] See Cartwright (1962), Iacona (2003), and Dyke (2012) for detailed considerations of this point.

4.1.3. Nature of Propositions

There are different characteristic features of propositions because there are many diverse theories concerning their nature. These theories have valuable insights and intuitions about the nature of meaning and truth. Realist philosophers accept propositions as language independent abstract entities in a broad sense, however they significantly differ on their views concerning the nature of propositions. Philosophers tend to have conflicting views about following features:

a. Propositions are structured / unstructured entities
b. Propositions are primitive / non-primitive entities
c. Propositions are representational / non-representational entities
d. Propositions have truth conditions essentially / derivatively
e. Propositions exist eternally / necessarily / contingently

Arguably, these divergent views on the nature of propositions are due to underlying ontological commitments of their philosophical considerations. For the discussion surrounding the nature of propositions is beyond the scope of this thesis, it will be sufficient to note that we merely adopt Frege's ontological commitments as we have previously presented.

4.2. Arguments for Propositions

In this part, we shall present some arguments for the existence of propositions. Propositionalists often take the existence of propositions for granted as an ontological presupposition without an argument. Remarkably, few philosophers have presented general arguments for their existence. In this section we shall consider two classes of arguments, arguments from their roles in semantic theories and ontological arguments.

4.2.1. Argument from Roles of Propositions in Semantic Theories

The most common argument for the existence of propositions rests on the roles of propositions in philosophical theories. Many propositional-realist philosophers often argue that the existence of propositions is required to play minimal roles. [165] Therefore, the roles of propositions give the reason to assert the existence of propositions. From this perspective, propositions exist as entities for analyzing and explaining the notions of meaning and truth in the philosophy of language. [166]

There are some objections to appealing to propositional roles for establishing the existence of propositions. For some philosophers there are certain doubts concerning the existence of such entities fulfilling these roles. According to these philosophers, positing an abstract entity for meanings of sentences, bearers of truth values, and objects of attitudes is a metaphysical extravaganza. We shall consider one dominant view against the existence of propositions namely, s*ententialism.* Regarding the ontological status of meaning, sententialism in its broadly construed form does not attribute propositions to the meanings of sentences. This view attributes primary bearers of truth values to sentences by distinguishing them between tokens and types. A sentence token is generally conceived as a physical entity, a series of marks on a paper or sound waves in the air. Sentence type, on the other hand, is conceived either as a pattern which different tokens exemplify or an

[165] See King (2007; 2014a; 2018; 2019), Speaks (2014a), Soames (2010; 2014a; 2014b), Hanks (2015).

[166] The appeal to the roles of propositions is not limited to semantic theories. Indeed, the concept of propositions is used in a wide variety of philosophical theories. In epistemology, for instance, propositions are used to distinguish different types of knowledge, namely acquaintance knowledge, know-how knowledge, and propositional knowledge. Propositional knowledge is about the knowledge gained by the sentences such as "Ankara is a city". In philosophy of mind, propositions are used to explain the contents of mental representations. In this regard, cognitive-realist philosophers appeal to propositions explaining mental representations and relation between contents of thoughts and perceptual experiences. See Pitt (2020). Last but not least, propositions play important roles in value theories, such as ethics and aesthetics. See McGrath and Frank (2020, sec. 10.1)

equivalence class between different sentence tokens. In this context, Sententialists generally attribute meanings to sentence types.[167] Sententialism is often criticized by propositionalists for the inadequacy of its explanatory use. First, if sentence types are defined as identities between their tokens, then two tokens of the same sentence type, e.g., "I am here now", never express the same meaning. Second, different sentence tokens can express the same meaning, such as active-passive use. Third, only if the differences between sentence tokens are to be explained in terms of sentence types as being abstract entities, then this view can be reconciled with propositionalism in terms of meaning and truth bearing roles. For the last point, we can argue that perhaps the origin of the problem is due to the idiosyncrasy of naming the abstract notion, for apparently there are certain variations in the terminology which could hide the identity between the concept of proposition and sentence type.[168] This issue does not present an issue for us as Frege's semantic theory has a particular and definite use of the word *Thought*, and it is elegant in the sense of being uninhabited by ambiguity or vagueness.

Apart from sententialist worries, one particularly important argument against the propositions as meanings of sentences is given by Donald Davidson. Davidson investigated the notion of meaning from a non-propositionalist perspective and argued that one cannot appeal to undefined sematic notions, such as meanings as entities. In an oft-cited quote, Davidson puts forward the following remark:

> Paradoxically, the one thing meanings do not seem to do is oil the wheels of a theory of meaning – at least as long as we require of such a theory that it non-trivially give the meaning of every sentence in the language. My objection to meanings in the theory of meaning is not that they are abstract or that their identity conditions are obscure, but that they have no demonstrated use.[169]

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[167] For a variety of views concerning sentence tokens and types see Haack (1978) and Kirkham (1995),

[168] For instance, Susan Haack (1978, p. 76) uses the word statement as the content, or "what is said when a declarative sentence is uttered or inscribe". She (1978, pp. 76-77) uses the term proposition to explain what is common to a set of synonymous declarative sentences.

[169] Davidson (1967, pp. 21-22).

In a nutshell, for Davidson, the existence of propositions cannot construct a theory of meaning. Accordingly, he aimed to construe an extensional theory of meaning based on Tarskian truth theories. Davidson argued that a Tarskian theory of truth, which takes the concept of truth as a primitive notion, can account for the meanings of expressions in natural languages. For him "a theory of meaning for a language *L* shows 'how the meanings of sentences depend upon the meanings of words' if it contains a (recursive) definition of truth-in-*L*."[170] Davidson's theory of meaning pairs sentences with the truth conditions, i.e., schematic form of *T*-sentences, generated from the truth theory.

(T) *s* is *true* if and only if *p*

where '*s*' is replaced by a structural-descriptive name of a sentence in an object language and '*p*' is replaced by a sentence of a metalanguage providing the truth-condition under which *s* is true. Davidson's meaning theory then generates theorems which couples true sentences in object language with true sentences in metalanguage. In this respect, there is no appeal to the identity of meaning, otherwise it would be circular since sentences in the object language are coupled with their synonymous sentences in the metalanguage. Davidson also holds a compositionality principle according to which meanings of sentences can be constructed from meaning of their parts to satisfy understandability and learnability of languages.[171]

We can give three general responses to Davidson. First, one can argue that Davidson's schematic form of *T*-sentences does not give the meaning of sentences, rather it gives *only* the truth conditions. [172] Hence, it does not result in the knowledge of what an expression means. Davidson has argued that knowledge of theorems of the interpretive truth theory would be sufficient to understand the meaning of

[170] Davidson (1967, p. 23).

[171] See Davidson (1967; 1984).

[172] See Davidson (1967, p. 309).

expressions in the object language.[173] However, even when one has the knowledge of all theorems of the theory, the problem still persists when one considers (heterophonic) *T*-sentences of the form " 'Kar beyazdır' is true in Turkish if and only if snow is white." If one does not already know what the object language sentence means, they cannot understand the Turkish sentence. Therefore, knowledge of the theorems of the truth theory is not sufficient to understand the sentence of the object language.[174] Second point is related to the problem of extensionality. As argued by Foster, one can derive true sentences of the form "'Snow is white' is true if and only if grass is green", whereas this sentence does not specify the meaning of the object language sentence. Davidson later put a restriction that "a theory of truth will yield interpretations only if its T-sentences state truth conditions that may be treated as 'giving the meaning' of object language sentences."[175] However, it seems that Davidson is appealing to the very notion of meaning to treat meaning. Furthermore, Davidson's theory can generate non-interpretive true T-sentences of the form "'Snow is green' is true if and only if snow is white and all consistent, axiomatizable, first-order theories of arithmetic are incomplete."[176] This results in the problematic consequence that there are more than one T-sentence for a sentence in the object language. Third, Davidson's account does not provide a satisfactory explanation for how to analyze attitude ascriptions. Davidson states that the sentences expressing attitude ascriptions of the form "*a* believes that *p*" should be analyzed as "*a* believes that**.** *p*'. Davidson argues that that-clause should be considered as demonstrative construction, where '*p*' after '**.**' refers the metalanguage sentence. However, it is not clear that how *a* can stand in belief relation to metalanguage sentence '*p*', especially when the metalanguage is different from the

[173] Davidson (1973, p. 150).

[174] Davidson (1976) provided certain replies, added a further criterion for the learnability of language.

[175] Davidson (1976). For an important discussion of problematic consequences of Davidson's truth conditional approach and related points, see Foster (1976), Soames (2003, ch. 12). For further criticism of Davidson's account see Soames (2010, Chapter 3) and Speaks (2014a, pp. 19-24).

[176] See Soames (2003, pp. 304-305).

object language that agent expresses her attitude. Davidson responds that *a* can stand in a belief relation to some sentence token which *samesays* the utterance of *p*.[177] But then the notion of samesaying seems to be a very similar notion to synonym.

We shall conclude this section by stating few remarks about the Davidson's approach. Davidson's view shares a commonality with Frege's theory of *Thought*. Both theories consider meanings of sentences in a compositional structure, however Davidson disallows the distinction between *sense* and *denotation*. For Davidson, meanings as entities do not prove useful to explain meaning and truth. In the final analysis, we can conclude that although truth conditional approaches provide necessary conditions for a meaning theory, they are not sufficient to explicate the notion of meaning. As a result, to provide a satisfactory account of meanings of sentences, the theory must elucidate the very notion of meaning, as Frege construed *Thoughts* as *senses* in terms of their contributions to semantic roles in sentences and their parts.

4.2.2. An Ontological Argument for the Existence of Propositions

In the previous subsection we have seen the existence of propositions investigated in light of their roles in semantic theories. Another important concern of the propositionalist philosophers is to give ontological arguments for the existence of propositions. Thus, these philosophers construe ontological arguments to give a satisfactory account for the existence of propositions as abstract entities. These arguments are also directed against certain tenets of sententialism and nominalism. In this subsection, we shall consider two ontological arguments for the existence of propositions.

The basic argument for grounding the existence of propositions appeals to the *commonsensical commonalities* between different utterances of sentences.[178]

[177] See Davidson (1984, pp. 52-54).

[178] Arguably, the best presentation is McGrath and Frank (2020, sec. 4.1) who attribute the argument to Cartwright (1962) and Soames (1999, pp. 15-16).

This argument is derived from the so-called *one-over-many argument* for the existence of universals. For instance, take three entities that have something in common. Snow, a tulip, and a book have something in common, being white. Then, these three things bear a common relation to a single entity, namely the universal *whiteness*. Similarly, different utterances of synonymous sentences and their translations to other languages share a single entity which is common to all these different utterances. Therefore, given these commonalities, propositions exist.

This argument depends on the realist view concerning the ontological status of propositions which holds that abstract entities, such as universals and propositions, have an independent existence. However, there are different sorts of philosophical views challenging this point. A prominent view in this ontological bent is Nominalism.[179] Nominalism denies the existence of universals and, more generally, of abstract entities and take abstract entities exist nominally, i.e., only in their names. A nominalist response can be presented in two general points. First, there is no *single* entity which many things have in common. Therefore, one can argue against the existence of propositions by stating that commonalities do not necessarily require relation to a single abstract entity. In other words, nominalists might reject the appeal to the *single* commonality, since there might be other commonalities related to different utterances of sentences. Second point is that in order to explain the commonality relation between these objects, one must first give an account for the relation of commonality itself. According to propositional realists, the commonality relation is the property of synonymy between different utterances of sentences. Nominalists then can argue that *synonymy* itself is arguably insufficient to explain that these sentences are commonly related to a proposition, unless this very notion is satisfactorily explained. [180]

[179] Of course, nominalism is not the *only* view against propositional realism. Philosophical views on the ontological status of universals indeed go back to ancient times, and there are other metaphysical views for the problem of universals, such as conceptualism and fictionalism.

[180] Quine (1960, pp. 200–209) has an important skeptical position on the existence of propositions as abstract entities. See McGrath and Frank (2020, sec. 8), and Carrara and Sacchi (2006, pp. 6-8) for Quine's (eliminativistic) stance on propositions.

There is a *metalinguistic* variant of the Basic Ontological Argument which conducts the commonalities in the apparent inferential validity of the argument schemas involving that-clauses, i.e., from 'that *p*' one can infer 'the propositions that *p*', and similarly propositional attitude ascriptions, e.g., from "Mehmet believes that Ankara is beautiful" one can infer "Mehmet believes the proposition that Ankara is beautiful." Proponents of propositionalism use this argument schema to assert the existence of propositions.[181] However, we can challenge intuitive validity of the metalinguistic argument on two points. First, we can argue that the argument schema seems to be constructed on the commitment of prior existence of the truth bearing roles of propositions. In other words, if one assumes the existence of propositions fitting the role of referents of that-clauses and objects of attitudes, then these inference schemas will be valid and sound. Second, this argument has arguably a linguistic character which depends on the referential use of that-clauses in natural languages, such as English. But due its language dependent character, it is questionable whether this would prove the existence of propositions in *all* natural languages. We can argue that providing such an account demands an immense empirical investigation and it is a matter of debate that such a complete investigation is achievable. Even if it could be consistently proven in some language, then this argument schema would only prove the existence of propositions in that language. This would be a serious threat to both abstractness and language independence of propositions. Third, validity and soundness of these inferences depend on the linguistic rules of a given language. But it has been argued the there are certain counterexamples to these inferences.[182] Taking all these into consideration, the

[181] Schiffer (1996, p. 150) grounds the existence of propositions as a term of art for the referents of that-clauses in order to secure the validity of such inferences. Hence, that-clauses must be bearers of truth values. Bealer (1998) also provides a similar argument for the existence of propositions for logical valid operations.

[182] This challenge is presented by construing invalid or unsound English argument schemas, as follows:

> Mehmet fears that it will snow in Ankara. (True)
>
> Mehmet fears the propositions that it will snow in Ankara. (False)

metalinguistic variant of the ontological argument is not adequate and satisfactory for establishing the existence of propositions.

Another ontological argument for the existence of propositions is given by Speaks. [183] This argument purports to show that in order to give an account of semantics of natural language sentences and the truth of sentences one must appeal to the existence of propositions. The argument starts with a class of true natural language sentences (which Speaks calls *apple* sentences) as "Mehmet ate something", "The thing Mehmet ate is a delicious apple", "What Mehmet ate is what Ayşe gave him." These sentences jointly entail the following logical form: "$\exists x$ (Mehmet ate $x \wedge x$ is a delicious apple $\wedge$ Ayşe gave x to Mehmet)". Speaks considers that the truth of the logical form does not depend on particular subject matter of these sentences. Accordingly, he carries this point to what he calls *proposition sentences* "Mehmet said something", "What Mehmet said was true", "What Mehmet said is what Ayşe believed." The corresponding logical form of these sentences jointly entail the following logical form "$\exists x$ (Mehmet said $x \wedge x$ is true $\wedge$ Ayşe believed x)". Therefore, Speaks concludes that the logical forms assigned to the

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or

Mehmet remembers that Ankara is beautiful. (False)

Mehmet remembers the proposition that Ankara is beautiful. (True)

These problematic instances shows that certain class of attitude verbs seems to invalidate substitution schema. Some philosophers generalize this point to argue that propositions cannot be the objects of attitudes. King (2002, pp. 345-346) offers a metalinguistic solution by pointing out the linguistic character of these verbs. He diagnoses these failures as syntactic nature of languages, hence certain classes of verbs result in substitution failures. King claims that in addition to general syntactic category of verbs, the syntactic properties of the complement noun phrase determine the verb meaning when taking that complement as the object of attitude. Higginbotham (1991) defends a similar language-based conception of meaning concerning the special case of belief, but he eventually rejects that objects of these attitudes are propositions. Moltmann (2003, 2013a, 2013b) and Ludwig (2013) use similar points to argue against the treatment of attitudes in the propositional context. The former characterizes *attitudinal objects* to replace propositions the objects of such attitudes, and the latter characterizes *higher-order attitude attributions*. See McGrath and Frank (2020, sec. 5).

[183] Speaks (2014a, pp. 9-11). In fact, he presents this argument as a counterargument for semantic theories which reject the existence of propositions.

proposition sentences explain what they entail. Then, Speaks deduces by logical inference the *existence of propositions*. The reason is that the validity of the logical form of these inferences lead to the conclusion that there are things which are said and believed, which are the bearers of truth values. Thus, there are propositions.

In this section, we have considered some arguments for the existence of propositions. In the framework of his semantic theory of *Thoughts*, Frege hardly provides an explicit ontological argument for the existence of propositions. Rather, he takes the existence of *Thoughts* implicitly in his Platonic realist framework. On the other hand, Frege would not need to argue for the intuitive validity of arguments appealing to the roles of propositions to ground the existence of propositions, since he already explains these roles beforehand by defining *sense* and *denotation* of a sentence and its parts. In this regard, he grants prior existence to these ontological entities when explaining the semantic roles of *Thoughts*. Moreover, Frege would definitely reject any sort of metalinguistic argument at first hand, since it would imply the language dependence of propositions.

4.3. Problems of Propositions

In the first section, we examined what roles were assigned to propositions. Then, we have given some arguments for their existence, and then evaluated these arguments. We shall now turn to important problems concerning the structure and unity of propositions in this section. Moreover, the problems concerning structure and unity will be a guideway to a measure for testing the theories of propositions which are alternative to *Thoughts*. Therefore, this section will provide a basis for arguing the strength of Frege's theory by comparing and critically evaluating these theories in the next chapter.

We shall consider the problems as criterion of adequacy for diverse theories of propositions. In this respect, a theory of propositions can be considered as an adequate and complete account when they can provide satisfactory answers to the problems regarding structure and unity of propositions. In the first part, we shall consider fundamental problems related to structure of propositions, and in the

second part, we shall consider problems related to the unity of propositions. Nevertheless, as we have mentioned before, in the case of the paradox of concept *horse*, problems concerning the structure and unity are arguably interconnected, in particular they provide related explanations for both semantic and metaphysical aspects of propositions.

4.3.1. Problems Concerning Structure of Propositions

Most propositionalist theories of meaning have considered propositions as structured entities. Since Frege and Russell, theories of propositions have endorsed a theoretical agenda for explaining structure and constituency of propositions. Proponents of structured propositions hold that propositions are complex entities having constituents as their parts. On the other hand, proponents of unstructured view hold that propositions do not have such parts. Defenders of structured propositions endorse a semantic *principle of compositionality* according to which propositions are composed of the semantic values of its constituent parts that the sentence expresses.[184] In addition to the compositionality principle, structured propositionalists have also endorsed theses concerning the metaphysics of propositional constituency.

There are three fundamental questions concerning the structure of propositions. The first question concerns the relationship between propositions and their constituents. In this regard, the main problem for structured propositions is to explain the ontological relation between propositions and their constituents which corresponds to the metaphysical aspect of the unity problem. Therefore, an adequate

[184] As a matter of fact, this principle, in both semantic and ontological senses, has been first given by Frege (1892a; 1923) and called 'Frege's Principle.' The principle of compositionality is one of the most important principles in semantics and there are hugely diverse views on it. In its most common sense, the compositionality principle states that the meanings of complex linguistic expressions – whether they are propositions or sentences or something else– are determined by the meanings of its constituents and its structure. Compositionality is generally considered as a basic requirement to learnability and understandability of languages, given finite capacities of the user of language, and given there are infinitely many sentences. *Cf.* Pelletier (1994), Pagin and Westerståhl (2010), and Szabo (2022).

theory of proposition must explain the relation of ontological constituency to the unity. Naturally, both semantic and metaphysical aspects of the unity problem are intertwined as in the case of Frege's concept of *horse* paradox. We shall focus on the semantic aspect of propositional unity in the next section. For this part, the metaphysical aspect of the unity problem is the constituency problem in a mereological sense, although this problem is not limited to structured propositions. The second question concerns the decomposition of propositions, whether there is a unique analysis or not. If propositions are decomposable into their constituent parts in a unique analysis, then how two different sentences with different constituents can result in the sameness of meaning and truth conditions? If, on the other hand, propositions are multiply analyzable, then how different analyses can give an account for the identity of constituent parts when re-composed into one and the same proposition? The third question is related to the individuation conditions of propositions. According to a general tenet of structured propositions, propositions are individuated by the identity and the order to a degree of so-called fineness of grain of their constituent parts. As a result, we can differentiate two characteristics of individuation. The first characteristic is identity which is related to the question whether the identity of constituent parts will entail the sameness of semantic content. If propositions are coarsely individuated, viz., only by truth conditions, then individuation would not explain the difference between the semantic contents of expressions, e.g., between the proposition that the Morning Star is the Morning Star, and the proposition that the Morning Star is the Evening Star. Fine-grained approaches aim to individuate propositions in that they give an account for the differences in meaning which is also an important concern in the case of propositional attitude ascriptions as in Frege's identity puzzle in the attitude contexts, viz., how to differentiate the propositions consisting of identical constituent parts in the sense of having co-referential terms. The second question is related to the order, or arrangement, of constituent parts of propositions. A satisfactory account of proposition should explain intuitive identity of the content between the proposition that Mehmet loves Ayşe and the proposition that Ayşe is loved by Mehmet. In this case, however, the complication comes from the opposite

direction, viz., excessively fine individuations of propositions which might result in different meanings due to the order of arrangement of their parts. We shall argue that Fregean *Thoughts* are optimally tuned considering the questions of individuation conditions of propositions.

4.3.2. Problems Concerning the Unity of Propositions

Propositions are the meanings of sentences. A natural question to ask is "what is the difference between a sentence and a mere list (or string) of words?" From a linguistic perspective this is the problem of unity of sentences. An answer to this question, whether adequate or not, rests on the sentence formation rules, i.e., syntax. On the other hand, the meanings of sentences are not inherent to sentences themselves as a linguistic or grammatical unity. In the beginning of this chapter, we have argued that meanings of sentences depend on the syntactic unity, however this unity is not inherent to the linguistic strings of words. Therefore, the linguistic unity is a necessary but not a sufficient condition for the semantic unity of propositions. In its general formulation the unity problem seeks to find an answer to the following question: if propositions are collections of their parts, then what distinguishes a proposition from a mere collection of its constituents? We have diagnosed two aspects of the unity problem, namely metaphysical and semantic aspects. In the previous subsection, we have focused on the metaphysical aspect, and we shall focus on the semantic aspect in this part.

The unity problem challenged philosophers since ancient times. The propositionalist philosophers consider the unity problem as a genuine problem threatening the minimal roles and structure of propositions. [185] Frege and Russell

[185] Russell (1903, sec. 55) was first to name the problem. The unity of propositions turns out to be very hard to explain, or even to properly formulate due to its intertwined nature when we consider both aspects of the problem. We briefly give the most-referred formulation of the problem as "What holds constituents of propositions together?" García-Carpintero and Jespersen (2019, p. 1210). For Gaskin (2008, p. 18), the problem is explaining "what distinguishes propositions from mere aggregates, and enables them to be true or false." On the other hand, Collins (2011) prioritizes the problem of unity of sentences over the unity of propositions. For him unity of propositions is not a semantic problem but rather a metaphysical problem, and he further argues that this problem can

explicitly considered the unity problem in their semantic theories. In Chapter 7, we will consider both aspects of the unity problem in the context of Frege's theory of *Thoughts* which eventually lead to the concept of *horse* paradox. Following Davidson, we can see the logico-semantic aspect of the unity problem has an interconnected nature with the *problem of predication*. [186]

> Clearly, what the problem of predication is concerned with is none other than an example of what is often called the unity of the proposition. Sentences express propositions, which is why the unity of the proposition guarantees the unity of the sentence […] Bearing in mind the distinction between meaning and reference, we can also speak of the truth value of a sentence as a sign of the unity of a sentence: only whole sentences have a truth value. [187]

For Davidson, primary concern of this problem is explaining the semantic role of predicates and the nature of predication. Conceived in this way, a solution to the problem of predication will be a solution to the unity problem. Or equivalently, if a theory of proposition can explain the unity problem on a par with accounting the nature of predication, then it will be a solution to the predication problem. We shall agree on this point that although both problems may appear in different forms, a proposed solution to the one of them will directly correspond to the other. Therefore, the question of unity is a logico-semantic question in the form of the problem of predication.

However, not all philosophers agree that the unity problem is a problem of predication. Soames and King prioritize the *problem of representation* which concerns explaining how propositions can be representational, thus have truth-

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only be solved if the unity is restricted to an empirical conception of linguistic meaning. For reception of the problem in the early analytic tradition, see Gibson (2004). See *Synthesé* (2019) Journal special issue on the unity of the proposition for a wide variety of solutions from different perspectives. Nevertheless, no solution has been given in the Fregean framework.

[186] Several philosophers have pointed out this connection. Jespersen (2012, p. 236) formulates the problem as "how an individual *a* and a property *F* combine into the propositions *P* that *a* is an *F*."

[187] Davidson (2005, p. 87). Nevertheless, in the final analysis Davidson rejects any theories which attribute predication to properties, universals, and similar entities. Instead, he argues that Tarski's theory of truth provides a solution.

conditions.[188] In this respect, the unity question is essentially a question "What makes propositions representational?" The unity question in this characterization seeks a theoretical framework to give an account for the representational features of propositions by focusing on truth-bearing properties of propositions and further give an account for false propositions.

In fact, both the predication and the representational aspects of the unity problem is explained in Frege's semantic theory. We shall argue that an emendation of the function-argument structure of Fregean *Thoughts* establishes the metaphysical unity as well as the semantic unity. Nevertheless, the problem at hand is the concept of *horse* paradox. In Frege's semantic theory, some sentences construed by the concept words as their singular terms result in a paradox. The paradox arises when we consider the atomic *Thought* expressed by the following sentence

(H) The concept *horse* is a not concept.

Frege treats the phrase 'the concept *horse*' as a singular term, thus it refers to an object. However, since an object cannot be denoted by concept-words, then the negation of (H) must be true:

(¬H) The concept *horse* is a concept.

Then, the paradox threatens one of Frege's key distinctions between objects and concepts. [189] For this reason, if we are able to solve this paradox by strengthening the semantic and ontological framework of Frege, we will also be able to solve the problems this paradox creates such as the unity problem and in relation to it, the problems of predication and representation. In Chapter 7 we shall present our solution to the paradox. Thus, we explain structure and unity in this context. By

[188] Soames (2010, p. 32) puts that "The real problem posed by [Frege and Russell] confused discussions of the unity of the proposition is that their conception of propositions makes it impossible to answer the question "*What makes propositions representational, and hence capable of interpreting sentences by providing their meanings?*" King (2007; 2014a) also points out a similar point "How does a structured complex have truth conditions?"

[189] "Quite so; the three words 'the concept "horse"' do designate an object, but on that very account they do not designate a concept, as I am using the word." Frege (1982b (*FR* p. 184)).

doing so, we aim to show the clear advantages of Frege's theory over the other important theories of proposition that we will investigate in the next chapter. Having considered these problems in a general framework of propositions, we shall devote our next chapter to an analysis of contemporary accounts of propositions.

CHAPTER 5

CLASSIFICATION OF CONTEMPORARY VIEWS OF PROPOSITIONS: SOME RECENT ACCOUNTS OF PROPOSITIONS

In the literature, there are two main varieties of structured propositions: Fregean and Russellian. According to a broadly conceived Fregean view, Fregean *Thoughts* are classified as structured propositions due to his adherence to compositionality principles with an assumption that the structure of propositions more or less mirrors the structure of sentences that express them. According to a broadly held Russellian variety of structured view, the constituents of propositions are individuals, properties, and relations. For Russell, these constituents are held together by the contribution of verbs. In this chapter, we will focus on Russell's view, and some representatives of Russell's descendant views, which are classified as neo-Russellian propositions. An alternative account against the structured views is the Possible World Account of Propositions, according to which propositions expressed by sentences are either as sets of possible worlds, or characteristics of these functions, viz., functions from possible worlds to truth values.

Propositions are also classified on whether they can be reduced to other types of ontological entities. The so-called *reductionist* views hold that propositions can be identified with other types of entities, such as sets, linguistic trees, cognitive act or event types, etc. [190] On the other hand, the so-called *non-reductionist* or

[190] Scott Soames and Peter Hanks are prominent representatives of the cognitive accounts of propositions; however these cognitive-act theories lie outside of our framework since they take propositions as cognitive entities. We shall only briefly summarize both accounts. According to Soames' account (2010, Ch. 6; 2014b pp. 95-112; 2014c, pp. 234-241; 2015) propositions are

primitivist views hold that propositions cannot be explanatorily reduced to other entities, rather propositions are primitive and *sui generis* entities. In the subsequent section of this chapter, we will focus on two main categories for this view, namely Algebraic Accounts and Deflationary Views of propositions. We exclude the views in which the underlying ontological view marginally deviates from realist account of propositions.

Above mentioned categories of propositions will guide our classification of contemporary approaches to propositions. In this regard, we will consider the most prominent representatives of these views, namely

i) Structured reductionist accounts: Russellian and neo-Russellian Propositions
ii) Structured primitive accounts: Zalta
iii) Unstructured reductionist accounts: Possible World Account of Propositions
iv) Unstructured primitivist accounts: Bealer, Deflationary Accounts

5.1. Russell's Account of Propositions

In this section, we shall focus on Bertrand Russell's account of propositions. Russell has many views on many areas of philosophy, his semantic theory of propositions is no exception. In this respect, Russell's account provides one of the earliest systematic theories of propositions along with Frege.

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inherently event types which agents entertain by cognitive faculty of thought. Hence agents entertain the proposition that Mehmet is wise by predicating the minimal event-type *wisdom* to Mehmet. In Hanks' (2011; 2015, chs. 3 and 5) account propositions are cognitive act types, and similarly the proposition that Mehmet is wise is the act-type of predicating *wisdom* to Mehmet. Both accounts argue that semantic contents of propositions are derived from either the tokens of events or acts. Hanks argues that the difference of his account from Soames' is that his act of predication is *assertive* and it does not involve the entertaining the thought that Mehmet is wise. Moreover, he rejects those individuals and properties are constituents of propositions. See Soames (2019) and Hanks (2019) for recent developments.

Russell's philosophy of language shares certain similarities with Frege. He considers the logical analysis as a key element of his philosophical method to solve problems concerning the nature of language. However, Russell differs from Frege in three main respects. First, Russell's ontological presuppositions are different from Frege. In this respect, Russell's peculiarity lies in the rejection of Frege's distinction between *sense* and *denotation*. Second, Russell considers an acquaintance-based theory in which individuals are direct constituents of propositions. On the other hand, Frege has employed senses, but not individuals, as the constituents of propositions. Third, if we consider Frege as the first philosopher to explain the unity of propositions, Russell is the first to state the problem concerning the unity. It is important to remark that Russell has given this problem as an important constraint for any account of propositions to explain in order to be a complete and satisfactory.

Throughout his career, Russell held a wide variety of views concerning the nature of propositions, although later he rejects most of these views. In this respect, we shall focus on Russell's most widely referred account in his *Principles of Mathematics* (1903). [191] We shall begin with Russell's initial remark:

> Whatever may be an object of thought, or may occur in any true or false proposition or can be counted as *one*, I call a term […] A man, a moment, a number, a class, a relation, a chimaera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false. [192]

Russell uses the word 'term' in a non-linguistic and non-psychological sense similar to Frege.[193] For Russell, terms are the meanings of linguistic expressions, the entities indicated by words and the constituents of propositions. [194] In other words, terms are the semantic values of words of which the sentences expressing propositions are composed. Russell distinguishes two kinds of terms as propositional constituents:

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[191] Russell also provides an extensive treatment of Frege's logical and semantic views in the appendix. See Russell (1903, Appendix A).

[192] Russell (1903, pp. 44-45).

[193] "[E]very term is immutable and indestructible. What a term is, it is, and no change can be conceived in it which would not destroy its identity and make it another term" Russell (1903, p. 45).

[194] Russell (1903, p. 48).

things and concepts. Things are indicated by proper names, such as 'Chrysippus', '4', and also non-existents such as the "pseudo-existents of a novel." [195] Concepts are indicated by all other words and are further divided into predicates, or class concepts, which are indicated by adjectives; and relations which are indicated by verbs. [196] For Russell, things are not logical subjects of predicates, instead they can *only* be logical subjects of propositions, whereas *everything*, including concepts, can be the logical subject of a proposition. In this sense, we can see that Russell's distinction between things and concepts is different from Frege's distinction between object and concept.

Russell considers propositions as structured complexes.[197] However, Russell differs from Frege on what binds these constituents together. As we have considered in the previous chapter, Frege has introduced the saturated-unsaturated distinction to bind the constituents of propositions, whereas Russell employed the notion of predication characterized by propositional functions which are functions from individuals to propositions. In this regard, it is the propositional contributions of verbs that hold together the constituents of propositions. As a result, Russellian propositions as structured entities consist of things and concepts as their parts.

Russell has employed the propositional analysis to examine the structure of atomic propositions, including relational propositions, into *subject* and *assertion*. [198]

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[195] Russell (1903, p. 46).

[196] Russell (1903, p. 45).

[197] According to Russell, the thought about an individual indeed has that individual as an immediate constituent. In one of his famous correspondences with Frege, Russell puts forward that:

> I believe that in spite of all its snowfields Mont Blanc itself is a component part of what is actually asserted in the proposition 'Mont Blanc is more than 4,000 metres high'. (1904 (*PMC* p. 169))

Russell explains the epistemological motivation behind this view as Mont Blanc itself must literally be a constituent of the proposition for "[i]f we do not admit this, then we get the conclusion that we know nothing at all about Mont Blanc." (1904 (*PMC* p. 169))

[198] Russell (1903, p. 45). Russell's notion of assertion has a particular similarity to Frege's concept-words; an assertion is simply what is left of a proposition when one of its subject terms is removed. "Thus, we shall say that 'Socrates is human' is a proposition having only one term; of the remaining

Atomic Russellian propositions are constructed by a subject-predicate analysis, such as

(1) Chrysippus is human.

where the concept denoted by "human" is a propositional predicate. The equivalent proposition can be constructed in a different way in which the proposition is *about humanity* [199]

(2) Chrysippus exemplifies humanity.

On the other hand, propositions containing more than one term, constructed by relational verbs, are multiply analyzable. An example for this type of proposition is

(3) Chrysippus differs from Cleanthes.

The relational proposition (3) can be analyzed in two ways: (i) 'Chrysippus' as subject and 'differs from Cleanthes' as assertion, or (ii) 'Cleanthes' as subject and 'Chrysippus differs from' as assertion. For Russell, a relational proposition in the form of '*a R b'* is divided into its constituents *a*, *R*, and *b*. However, Russell states that there can be no *complete* structural analysis of relational propositions. Russell makes this remark to avoid *Bradley's Regress* which argues against theoretical intelligibility of the reality of relations. [200] According to the regress problem, the proposition of the form '*a R b*', when explained by another relation *R* ' as '*R* ' (*a R b*)' then this relation has to be explained by another relation *R* " to infinity, resulting in the conclusion that relations and qualities are not truths about reality. [201] Although Russell's account avoids *Bradley's Regress,* another important problem arises.

components of the proposition, one is the verb, the other is a predicate" Russell (1903, p. 45). See Soames (2010, pp. 23-24).

[199] Bradley (1893, pp. 25-26) has argued that "the arrangement of given facts into relations and qualities may be necessary in practice, but it is theoretically unintelligible."

[200] Russell (1903, p. 46).

[201] See Russell (1903, pp. 99-101). Following Russell's remark on Bradley's Regress, there are further views concerning the types of regresses about relations: infinity, dependence, and constitution. *Cf.* Linsky (1992, p. 247), Gaskin (2008), Jubien (2001) and Eklund (2019, pp.1226-1227).

Russell himself was well aware of this problem and in fact he pointed for the first time in his famous passage in *Principles of Mathematics*:

> Consider, for example, the proposition "A differs from B." The constituents of this proposition, if we analyze it, appear to be only A, difference, B. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates A and B, whereas the difference after analysis is a notion which has no connection with A and B. [my emphasis] It may be said that we ought, in the analysis, to mention the relations which difference has to A and B, relations which are expressed by is and from when we say A is different from B. These relations consist in the fact that A is referent and B relatum with respect to difference. But A, referent, difference, relatum, B, is still merely a list of terms, not a proposition. *A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition*. The verb, when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of the distinction.[202]

Russell, at first, tries to provide a solution by employing propositional contribution of a verb by stressing its character holding the constituents of the proposition together: "Owing to the way in which the verb actually relates the terms of a proposition, every proposition has a unity which renders it distinct from the sum of its constituents."[203] However, Russell's initial reasoning leads to another problem: "There appears to be an ultimate notion of assertion, given by the verb, which is lost as soon as we substitute a verbal noun, and is lost when the proposition in question is made the subject of some proposition" [204] thereby still posing the problem of unity. Nevertheless, Russell does not provide a solution. He merely stresses the notion of assertion and concludes that propositional analysis into subject-predication form of relational verbs should be incomplete: "a subject and a verb, if simply juxtaposed, do not, it is true, constitute a proposition." [205]

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[202] Russell (1903, pp. 49-50).

[203] Russell (1903, p. 53).

[204] Russell (1903, p. 48).

[205] Russell (1903, pp. 83-84).

As a result, the propositional analysis cannot merely consist of Chrysippus, and Cleanthes, and the property *differs from* since mere agglomeration or collection of subject and assertion is not a unified entity. Unless the unity of propositions is not explained satisfactorily, it surely threatens the minimal roles of Russellian Propositions having semantic roles as meanings of sentences, the bearers of truth values and the objects of attitude ascriptions. Nevertheless, Russell does not explain what exactly contributes holding together of the constituent of propositions into unity.

The unity problem concerning the structure of Russellian propositions creates two inherently connected problems. The first problem is the representation of the constituents. Two propositions "Chrysippus is a student of Cleanthes" and "Cleanthes is a student of Chrysippus" have the same constituents according to Russell, however meanings and truth conditions of both propositions are different. Thus, they are distinct representations held together with the same contribution of the propositional verb. Therefore, Russell's theory of propositions does not account for the distinct representations of propositions having the same constituents. The second problem concerning Russell's account is to explain false propositions. Russell argues that true and false propositions are alike in being entities, nevertheless true propositions have some additional quality that false propositions lack. [206] Russell has attempted to capture this quality by *assertion,* which only true propositions have. The root of the problem is Russell's attribution of truth to the relation of verb, i.e., all true propositions are facts. In this regard, the question is how to relate something false to truth. Considering the false proposition that 'Cleanthes is a student of Chrysippus' how is it possible that Cleanthes can stand in the relation of being student to Chrysippus. Since Cleanthes cannot stand in such a relation, then there can be no false proposition to express this relation. This case is even more problematic for propositional attitude ascriptions. When we consider falsity of the following propositions 'Mehmet believes that Cleanthes is a student of Chrysippus.' Since for Russell such relation cannot be constructed, then all

[206] See King (2007; 2018), Soames (2010a; 2010b, 2014a).

propositions must refer to facts, and there can be no false propositions. Russell was also aware of this point and later rejected his account in *Principles of Mathematics* altogether.

We have considered Russell's account of propositions and the problems that lead him to reject his account of propositions. Russell, later disowns his earlier view on propositions in place of multiple relations theory of judgment. [207]

5.2. The Possible World Account of Propositions

According to the possible-world account of propositions, propositions are characterized as sets of possible worlds. In this section, we shall briefly consider two representatives of this account, David Lewis and Robert Stalnaker. Then we shall present two crucial problems of the Possible World Account of Propositions.

The possible world account of propositions, as its name suggests, is derived from the notion of possible worlds which is used to give an account of modal concepts such as necessity, possibility and actuality. The theoretical framework for this account, which is known as possible world semantics, has been developed parallel to developments in semantic applications of modal logic, in particular, Carnap's intensional framework, Hintikka's set-theoretical framework for the logic of knowledge and belief in the context of modality, Kripke's revolution in semantics for modal logic, and Richard Montague's further advancements in the applications of intensional semantics to natural languages in the 1970s.[208]

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[207] Russell has not considered his Multiple Relation Theory of Judgment (MRTJ) as a theory of proposition, but rather a theory judgment, in which the propositional attitudes, such as belief, demonstrates a multiple relation between the subject and the constituents.

> The theory of judgment which I am advocating is, that judgment is not a dual relation of the mind to a single objective, but a multiple relation of the mind to the various other terms with which the judgment is concerned. Thus if I judge that A loves B, that is not a relation of me to "A's love for B", but a relation between me and A and love and B. Russell (1910, p. 180).

However, Russell's characterization of judgment does not fulfil the minimal propositional roles, thus we shall not analyse his later account. See Lebens (2017), McGrath and Frank (2020) for attempts to reconcile (MRTJ) with Russell's preceding account on propositions.

[208] See Carnap (1946), Hintikka (1962), Kripke (1963), and Montague (1974).

In *Meaning and Necessity* (1946), Carnap develops methods for semantical meaning analysis by distinguishing linguistic expressions between extension and intension. Arguably, this distinction is comparable to Frege's distinction between *sense* and *denotation*. We can consider the notion of extension as a generalization of the notion of denotation, i.e., what the term applies to. In extensional semantics, the extension of a name is the object that it designates; the extension of a predicate is the set of things that predicate applies to; and the extension of a sentence is its truth value. However, extensional theories of meaning are insufficient to explicate the notion of meaning, due to coreferential expressions that differ in meanings, such as Quine's famous example 'renate' and 'cordate.' In need of a stronger semantics to explain the meaning in non-extensional context which includes modality, e.g., necessity and possibility, and the notion of intensional contexts has been suggested by Carnap. Carnapian intensions are functions from possible worlds to truth-values and they determine extensions depending on possible circumstances of evaluation. In this sense, an intension is a function from a possible world to an extension at that world. Possible World Account of Propositions take intensions as a common term for meaning and explicate propositions within possible-world intensions. In this sense, the possible-world account of propositions is suggested as an alternative approach to Frege's and Russell's views.

Two followers of Carnap's view further developed the possible world account of propositions. According to the first type of account developed by David Lewis, propositions are simply sets of possible worlds. [209] Lewis presents his account as follows:

> I identify propositions with certain properties –namely, with those that are instantiated only by entire possible worlds. Then if properties generally are the sets of their instances, a proposition is a set of possible worlds. A proposition is said to hold at a world, or to be true at a world. [210]

[209] See Lewis (1970).

[210] Lewis (1986, pp. 53-54).

Lewis concludes that propositions satisfying such properties in possible worlds are members of these possible worlds. For instance, the proposition that 'Chrysippus is a philosopher' is true with respect to a possible world w_1, but it can be false in another possible world w_2. If there is at least one world in which it is true, and if there is another world in which it is false, then it is called a *contingent proposition*. If a proposition, such as 'Chrysippus is Chrysippus' is true in all possible worlds in which it is expressed, then it is called a *necessary proposition*. In this regard, contingency and necessity are constructed by existential and universal quantification over possible worlds.

According to the second type of possible world account presented by Robert Stalnaker, propositions are functions from worlds to truth-values. [211] Stalnaker conceives propositions as a relational analysis of property attributions, i.e., truth and falsity, to propositions. Stalnaker takes possible worlds as properties, or "ways a world might be." [212] In this regard, Stalnaker's account differs from Lewis that a proposition bears its truth values only relative to a possible world.

> A possible world is the kind of thing that is, or can be, instantiated or exemplified. An actualist needs the distinction between existing and being exemplified in order to be able explain the sense in which a merely possible world exists (a property the world might have had exists) and the sense in which it does not (no world that is that way exists). But second –and this is the point I want to emphasize – if possible worlds are properties, they are not *representations*– not mental or linguistic entities.[213]

For Stalnaker, propositions are ordered pairs of truth-values with respect to possible worlds. For example, the proposition expressed by the sentence 'Chrysippus is a philosopher' can be identified by as the ordered pairs relative to possible worlds w_1 and w_2 as $\langle w_1, T\rangle$, and $\langle w_2, F\rangle$, respectively. Accordingly, Stalnaker defines a function from sets of possible worlds *W* to truth values *V*, i.e., either true (denoted by T) or false (denoted by F) to represent propositions by set-theoretical

[211] See Stalnaker (1999; 2003; 2012).

[212] Stalnaker (2012).

[213] Stalnaker (2012, pp. 8-9).

constructions. In this sense, Stalnaker reduces propositions to functions from worlds to truth-values. Stalnaker lists several axioms for this theory of propositions, however it is beyond our scope to give a detailed analysis of Stalnaker's account. [214]

As we have considered in the previous section, a major advantage of the possible world semantics is to explain the modal properties of propositions. Both accounts represent the propositions with set theoretical structure. Nevertheless, they assign no inherent structure to propositions, which distinguishes them from the structured accounts of Frege and Russell. [215] Possible world accounts do not distinguish constituents of propositions from propositions themselves. They do not even consider the propositional constituency by set-membership relation. [216] In this sense, the Possible World Conceptions of Propositions are not threatened by the unity question. Nevertheless, such non-reductionism still faces two important problems. [217]

One problem concerns the individuation conditions of propositions. [218] According to the possible world conception of propositions, two sentences having the same truth value at the same possible world express the same proposition. This brings out the concern for the coarse-grained individuation of conditions of propositions, since it leads to the problematic consequence that all true propositions in the same possible world express the same meaning and the same holds for all false propositions. For instance, propositions expressed by the sentences 'Chrysippus is a philosopher' and 'Cleanthes is a philosopher' have the same truth values in some possible world, say the actual world. However, they attribute the property of being philosopher to different individuals, thus they express different propositions. Moreover, when we consider the sentences expressing necessarily true

[214] Stalnaker (2012, pp. 24-29).

[215] See King (2007; 2018), Soames (2010; 2014a; 2014b), Keller (2019).

[216] See Soames (2010) and Keller (2013).

[217] We shall leave out the inherent problems concerning the ontology of the possible worlds from our discussion.

[218] See Soames (2014a) and King (2018).

propositions, such as "All triangles are trilateral" and "All bachelors are unmarried", it results that all true propositions express the *same* true proposition, since their truth values do not change from possible world to possible world. This reduces all necessarily true propositions to one necessary truth and all necessary falsehoods to one falsehood. These sentences nevertheless express different meanings. This point can even be carried further to argue the unintuitive consequences concerning propositional attitude ascriptions. For instance, consider an attitude ascription relating Mehmet to the knowledge of the true proposition that "Snow is white". Then by the same reasoning, it relates Mehmet to the knowledge of all other consistent set of propositions in the same possible world, such as "Mehmet knows that all renates are cordates", yet Mehmet may be ignorant of the truth of the proposition that all renates are cordates. As a result, a general concern for the possible world conception of propositions is that they individuate propositions in a coarse-grained manner. [219] When Frege distinguishes his conception of *sense* and *denotation*, he satisfactorily provides an explanation for the difference in semantic values of all these sentences.

The second problem is problem of representation. In the literature, this problem is presented in a way analogous to Benacerraf problem concerning the representations of numbers in set theoretical constructions. One similar problem arises for the Possible World account of propositions. The problem is concerned with which of the following set theoretical constructions adequately represent the proposition that A differs from B and give an account for their meaning and truth-conditions. [220]

<A, B, difference>

< <A, B>, difference >

< <B, A>, difference >

[219] See Soames (2010; 2014b), Speaks (2014a), King (2018) and McGrath and Frank (2020).

[220] Jubien (2001, pp. 50-54) and Soames (2010b, 2014a) argue against the theories of propositions, according to which propositions are sets of possible worlds or sets of Russellian propositional constituents, by showing their inability to explain how propositions come to have truth-conditions. *Cf.* Keller (2013), McGrath and Frank (2020).

< difference, <A, B> >

< difference, <B, A> >

or,

{ { {A}, {A, B}}, {{ {A}, {A, B}}, difference }}

{ {difference}, {difference, { {A}, {A, B}}}}

{ difference, {difference, { B, {B, A}}}}

The question is which of these candidates can be used as a model for construing the proposition that 'A differs from B'. The difficulty in so construed set theoretical representation of the propositions that A differs from B is that there are too many equal candidates of propositional representation, all of which can equally represent the same proposition. There is no procedure of deciding which construction is preferable to others, therefore these set theoretical representations of propositions are arbitrary, yet they are not identical. Thus, one can conclude that in Possible World Semantic Account of Propositions there is nothing inherent to above given set theoretical representations which provides a satisfactory representation of proposition.

5.3. Neo-Russellian Accounts of Propositions

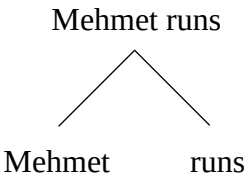
In this section, we shall consider the contemporary neo-Russellian accounts of structured propositions presented by Jeffrey King and Jeff Speaks. Both philosophers consider the problems that arise for Russell's theory of propositions. They also argue against the possible world account of propositions because of the problems concerning coarse-grained individuation conditions and representation. They are further against other primitivist accounts as they consider these theories mysterious and they rule them out as inadequate to explain the constituency of propositions. We consider these accounts for they provide an alternative approach for explaining the relationship between constituents and propositions as a structured unity. They provide some original approaches that deserve mention in an up-to-date

survey of propositions as an addendum for the divergent approaches from Frege, Russell and the other approaches considered in this chapter.

5.3.1. King's Account of Propositions

Jeffrey King in his account of propositions focuses on the representational character of propositions and names his theory Naturalized Propositions. King conceives propositions as complex metalinguistic facts. [221] He assumes a broadly Russellian account of propositions according to which propositions are abstract structured entities and their constituents are individuals, properties and relations. [222] For instance, the proposition "Mehmet runs" has the individual Mehmet and the property of *running* as its constituents, and the proposition "Chrysippus adores Cleanthes" has the individuals Chrysippus and Cleanthes, and the adoring relation.

King represents atomic propositions by syntactic trees



In this representation, lexical items stand in to form sentential relations between the individual Mehmet and the property of running. King calls this, the syntactic relation *R.* In this way, speakers of language, say English, interpret *R* by ascribing the semantic value of "runs" to the semantic value of "Mehmet". According to King, this relation results in the fact that speakers of English take the proposition "Mehmet runs" true if and only if the individual possesses the property of running. [223]

[221] See King (2007, ch. 2; 2014b, pp. 49–59).

[222] (King 2014b, p. 48).

[223] King (2014b, p. 49-52).

King appeals to the fact that speakers of English interpret the syntactic relation *R* by saying that *R* encodes ascription in English. King explains this point as follows:

> *There is a context c such that ___ is the semantic value in c of a lexical item e of some language L and ___ is the semantic value in c of a lexical item e' of L such that e occursat the left terminal node of the sentential relation R that in L encodes ascription and e' occurs at R's right terminal node.* [224]

According to King, the proposition "Mehmet runs" has its semantic content by adjoining two features of propositions[225]: (i) the proposition is identical to the fact that *R* (Mehmet, running) and (ii) speakers of English interpret the syntactic relation *R* as encoding ascription. In this regard, King explains the unity of proposition by appealing to the unity of sentence. Accordingly, King attributes the unity of propositions to propositional relation. Therefore, he claims that propositions represent truth conditions as speakers interpret the propositional relation ascribing the properties to individuals.

King argues that the notion of propositions has representational structure by their natures and that there can be no further explanation that is credible. In this regard, he merely considers propositions as linguistic facts about natural languages. In addition, King puts forward that any explanation for the propositional representations has to appeal to the representational powers of thinking agents. [226] For the speakers of different languages similarly interpret the syntactic concatenation. He appeals to the biologically endowed faculty of language, as the speakers of different languages can also employ the syntactic relation to interpret propositions.

We shall now consider three important criticisms to King's account of propositions. Firstly, it is a concern that King only examines the syntactic structure of English, yet he makes claims about the inherent structure of propositions in

[224] King (2014b, p. 50).

[225] King (2007, pp. 59-64; 2014b, pp. 52-55). *Cf.* King (2018).

[226] See King (2007; 2014b; 2018).

general. Furthermore, King fails to explain how his account would apply to possible artificial languages.[227] Since he grounds his account on metalinguistic facts, it is not perfectly clear how he can account for the difference between propositions as abstract entities and sentences as abstract entities. The second problem arises from the fact that reducing propositions to facts may lead to complications similar to the problem of false propositions in Russell. At first, it seems that properties, such as running, need not be instantiated by individuals. Then, the question arises whether these propositions can be represented or have unity.[228] Moreover, this becomes an even bigger issue when propositional attitude ascriptions are considered as false propositions can be the object of such attitudes. Nevertheless, King accommodates false propositions but at the cost of making propositions language dependent.

5.3.2. Speaks's Account of Propositions

Jeff Speaks argues that in order to account for the relationship between propositions and their constituent one should take propositions as properties.[229] In a nutshell, Jeff Speaks speaks of propositions as monadic properties. The overall advantage of considering propositions as properties is ontological parsimony: if propositions are properties of anything, then they are properties of everything.

> If we think of the semantic content of ["Mehmet talks"] as a property, one natural view is that the property is the property of being such that [Mehmet] talks. On this kind of view, what is contributed by the syntax of a simple predication—the semantic significance (in English) of this bit of syntax, in King's terms –is something like the three-place relation corresponding to the open sentence "__ is such that __ instantiates __." In the case of the sentence ["Mehmet talks"] the contents of the name and predicate fill in the second two slots to deliver the monadic

[227] See Soames (2014c), Speaks (2014c), and Hanks (2015).

[228] See Soames (2014b), Speaks (2014c).

[229] See Speaks (2014a).

> property expressed by "__ is such that Mehmet instantiates the property of talking."[230]

Thus, the proposition expressed by the sentence "Mehmet talks" is the property of *being such that Mehmet instantiates talking*. This monadic property can be shown by the syntactic string of this open sentence:

> ___*is such that* ___ *instantiates*___

This monadic property expresses the following proposition:

> Mehmet talks

The semantic content of this sentence is Mehmet and the property of talking. For the truth conditions of propositions, Speaks states that propositions are true if and only if they are *instantiated*. He also adds possible world considerations; "propositions are true with respect to a world *w* if and only if, were *w* actual, that property would be instantiated, or equivalently, the proposition would be true." [231] Accordingly, for the entailment relations between propositions, Speaks summons coinstantiation relation, namely, "one proposition *p* would entail another proposition *q* if and only if any world in which *p* is instantiated is also a world in which *q* is instantiated." [232] Having considered propositions as properties, he makes the following strong claim: Once a proposition is instantiated, everything instantiates it.

Interestingly, Speaks denies that propositions have representational properties.[233] Indeed, propositions do not bear any representational properties since their intentionality, or aboutness, need not to be explained for they are just monadic properties, they are not *about* anything. Yet he is not successful in explaining, how we can account for the unity of propositions if they are not representational

[230] Speaks (2014b, p. 75). *Cf.* King (2007; 2018; 2019).

[231] Speaks (2014b, p. 76).

[232] Speaks (2014b, p. 77).

[233] See Speaks (2014b; 2014c).

entities.[234] However, for the propositional attitude ascriptions, Russell's later theory seems to inspire Speaks. Speaks argues that to believe the proposition "Mehmet talks" is to merely bear an attitude toward the property *being such that Mehmet talks*. In particular, he claims it is to believe something is such that Mehmet talks. There is also no room for neither necessarily false nor false propositions in Speaks' account. Moreover, the ontological structure of Speak's account, i.e., taking propositions as monadic properties, comes with a cost that Speaks initially aims to discard. The cost is the existence of necessarily false sentences. [235] If one considers the existence of uninstantiated properties, no proposition can express necessary false sentences, if propositions are properties which are true if and only if they are instantiated. Another related objection can be made against Speaks' universal generalization claim that if a proposition is instantiated, everything instantiates it. This account is highly counter intuitive and is not well grounded. Speaks claims that this brings the advantage of ontological parsimony, yet the far bigger problems this view brings may outweigh this advantage.

5.4. The Algebraic Accounts of Propositions

5.4.1. Bealer's Account of Algebraic Propositions

The approaches we have considered in the previous section suggested propositions are sets of possible worlds. George Bealer considers the account of intensional entities so construed uncompelling on metaphysical grounds for explaining the notion of propositions, due to its inadequacy for solving certain problems of intensionality. [236] Bealer argues for a reductive approach similar to a possible world approach to propositions, in the sense that it reduces propositions to extensional entities. He argues against what he calls extensional reductionism by

[234] See King (2014c; 2018).

[235] See Soames (2014c), and King (2014c; 2018).

[236] Bealer (1998).

arguing that intensions cannot be constructed as extensions.[237] He considers problematic cases concerning modality on the one hand, and the problems concerning the substitution problems of propositional attitudes on the other. He rules out the possible world conception of propositions as inadequate, and he outlines his non-reductionist algebraic approach which avoids these problems.

In *Quality and Concept* (1982), Bealer takes intensional objects, such as properties, relations and propositions (PRPs), as *sui generis* abstract entities and as Platonic modes of presentation.[238] PRPs are not reducible, and they are ever-present features of the world. Thus, Bealer's theory of intensional objects, i.e., PRPs, is divided into two types. One type consists of qualities, connections, and conditions. According to this type, qualities and connections are unified to produce logical operations which are the sort of things that can be said to obtain. The other type consists of concepts and thoughts, which are in relation to one's thinking about the world, although they do not belong to the world. [239]

Bealer then provides his account for representing intensionality by constructing PRPs on models in algebraic structures.[240] Bealer develops a theory in

[237] Bealer (1998).

[238] See Bealer (1982).

[239] Bealer (1998, pp. 10-11).

[240] Basically, intensional models structures are constructed on M = <D, τ, K>. In this triple, D is a domain D which is union of disjoint subdomains, D_{-1}, D_0, D_1, D_2, … D_n. The elements of D are primitive and irreducible. The subdomain D_{-1} consists of extensional entities (particulars); D_0 consists of propositions; D_1 consists of properties; D_2 consists of binary relations-in-intensions; and D_n consists of n-ary relations-in-intensions. τ is a set of fundamental logical operations, such as negation, conjunction, singular predication ($pred_x$), and existential generalizations, and further operations. K is a set of possible extensionalization functions and *G* is the actual extensionalization function. Then, Bealer defines an arbitrary externalization function H such that H ϵ K assigns appropriate extensions to the elements of D. For each proposition $x \epsilon D_0$ it assing truth-values, i.e. H(x) = T or H(x) = F; for each property $x \epsilon D_1$ $H(x) \subseteq D$, for each n-ary relation $x \epsilon D_n$ $H(x) \subseteq D_1 \times \ldots \times D_n$, denoting nth cartesian product of D. In the case of particulars $x \epsilon D_{-1}$ it assigns the particular itself, i.e. H(x) = x. Finally, extensionalization functions are constructed with respect to the logical operations in the following way: for all x,y ε D_0, and H ε K, H(conj(x,y)) = T iff H(x) = T and H(y) = T. The model M is intensional that there are elements in some $Di \subset D$, $i \geq 0$, that can have the same possible extension but can be distinct. For instance, for some x and y in $Di \subset D$, $i \geq 0$, and for some H ε K,

algebraic intensional framework involving the two conceptions of identity: course-grained and fine-grained. The former conception is presented to give account for the modal contexts in which qualities, connections, and conditions are identical if and only if they are necessarily equivalent. The latter conception of identity consists of the concepts (properties and relations) and thoughts are presented to give account for propositional attitudes. In this sense, propositions are treated as 0-place intensional objects, properties are treated as 1-place intensional objects; and relations are treated as *n*-place intensional objects, where $n \geq 2$. Bealer notes that necessary equivalence among PRPs is a necessary but not sufficient condition for identity, since there are distinct, necessarily equivalent propositions. Bealer gives a simple example of double negation: 'neg(neg(p)) ≠ p' Since two propositions 'neg(neg(p))' and 'p' have different constituents, the algebraic account of propositions provides a more fine-grained approach than the possible worlds accounts. For propositions in context of propositional attitudes, Bealer invokes non-Platonic modes of presentations. [241] Bealer gives three kinds that qualify as non-Platonic modes of presentation: (i) The name itself, as a fine-grained entity, whose existence is an empirical fact,[242] (ii) the naming practices associated with a name that provides access to objects, and (iii) historical naming trees as the causal naming chain. Bealer notes that whichever alternative is chosen it will not affect non-Platonic modes of presentation since "there is a natural one-one mapping from living names onto naming practices and a natural one-one mapping from living names practices onto naming trees." [243] As a result, by invoking these kinds of non-Platonic modes of presentation, Bealer is able to distinguish the proposition that 'Hesperus

H(x) = H(y), but x ≠ y. See Bealer (1993, pp. 25-26; 1998, p. 11) and Parsons (2016, Sec.7.2 pp. 97-102).

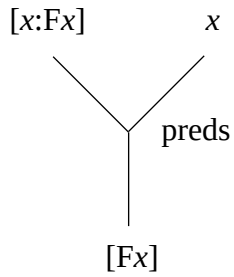
[241] See Bealer (1998, pp. 16-19).

[242] Bealer (1998, p. 16). He further claims that this conception combines with Kripke's (1972) notion of rigid-designation of proper names.

[243] Bealer (1998, p. 17).

is a planet' from the proposition that 'Phosphorus is a planet' in the propositional attitudes contexts.

It is important to note that, the fineness of the conception of identity should not be confused with the notion of fineness of grain in propositional structure, since according to Bealer propositions are unanalyzable abstract Platonic entities. Hence, they are not structured. As his theory suggests, Bealer is a proponent of Primitive Entity Theory. For Bealer, propositions are primitive and simple, i.e., they have parts neither in set-theoretical nor mereological sense. Despite this, Bealer has a notion of propositional constituency, shown in the following a composition tree [244]



The tree diagram shows that composition results from the application of logical operations on members of domain of PRPs. The elements of subdomain propositions have a unique decomposition tree which shows the unique decomposition of proposition: the property '[*x*:F*x*]' and the individual '*x*' are constituents of the proposition '[F*x*]' which is the result of an application of 'preds' to the property '[*x*:F*x*]' and the individual '*x*'.

5.4.2. Zalta's Account of Algebraic Propositions

We shall consider another kind of algebraic approach in which propositions have parts obtained by complex algebraic structures. A proponent of this view is Edward Zalta. In *Intensional Logic and the Metaphysics of Intentionality*, Zalta argues for a Meinongian account for non-existent objects to replace the possible-

[244] Bealer (1993, p. 30).

worlds approach. [245] Zalta indeed provides an extensively detailed axiomatized theory of abstract objects such as abstract individuals, abstract properties and relations, abstract propositions, and non-existing abstract objects of every kind.

For the present purposes of this thesis, we shall not consider formal details of Zalta's approach, but we shall give a brief informal overview. For Zalta, propositions are zero-place relations which are *primitive entities* in the sense that they are not defined in terms of other entities. Predicates, on the other hand, are one-place relations. To obtain complex relations from one-place relations, Zalta puts a comprehension schema for relations and logical functions that obtains all complex n-place relations. [246] He names these logical functions as predicate functors that maps two categories of things, either relations and objects, or relations and relations. The key predicate functor is called *PLUG* which yields propositions. For instance, applying the *PLUG* functor to the predicate *is a philosopher* and Chrysippus yields the proposition that Chrysippus is a philosopher. [247] In order to explain the structure of propositions in a fine-grained sense than possible world semantics approach, Zalta states that it must give an account for the distinctness of necessarily equivalent propositions, otherwise the theory of propositions would not be fine-grained enough to accurately represent belief. [248]

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[245] Zalta (1988). See also Zalta (1983).

[246] See Zalta (1988, p. 46).

[247] Zalta's logical functions have also the following group of logical functions: NEG, COND, UNIV_i, $\text{REFL}_{i,j}$, $\text{CONV}_{i,j}$, VACi, NEC, WAS and WILL. New propositions can be obtained by applications of *PLUG* functor and negation functor to yield the proposition expressed by the sentence 'Chrysippus is not a philosopher.' Accordingly, applications of *PLUG* functor and universal generalization function to the property of *being philosopher* yields the proposition expressed by the sentence 'Everything is philosopher.' In this way repeated applications of functors to entities yield complex propositions which is the characteristic of Zalta's algebraic approach. See Zalta (1988, esp. pp. 46-51; 58-61, and the Appendix containing his formal intensional logic). *Cf.* Parsons (2016, pp. 115-119).

[248] Indeed, his theory distinguishes propositions expressed by sentences, for example, 'All brothers are male siblings' and 'All bachelors are unmarried', as follows: The former results from application of COND and REFL to the properties of *being a brother* and *being a male sibling*, respectively. Then applying UNIV1 to this output, the latter results from the same applications of the former functors to the properties of *being a bachelor* and *being unmarried*. See Zalta (1988, p. 57).

We have considered Bealer's and Zalta's algebraic accounts of propositions. Both Zalta's and Bealer's approach has an advantage of explaining propositions in terms of algebraic models. They overcome the representation problem since their views account for the propositional representation in the algebraic framework. Furthermore, both accounts individuate propositions in a fine-grained sense to account for the differences among necessary equivalent propositions. Although algebraic accounts have the advantage of explaining the truth values, they are not completely satisfactory when explaining the sematic and cognitive values of sentences.

Bealer has an advantage of distinguishing the necessary propositions compared to possible world account. However, modal notions such as the notion of possibility is not well-defined by Bealer. Moreover, the temporal and deontic notions are not explained. As a result, possible world semantics has an advantage of treating further aspects of modality in possible worlds. [249] The second problem with Bealer's account is his separate treatment between fine and coarse-grained individuation conditions of identity concerning the different identity puzzles. Frege's *sense-denotation* distinction has an advantage of treating both types of puzzles in the same framework. Bealer also makes a distinction between Platonic and non-Platonic modes of representation, but the question that how the differences in these representation modes corresponds to differences in cognitive values of complex expressions is unanswered. Since Bealer is committed to an unstructured view, he simply cannot explain the differences in terms of having different semantic values of expressions, such as Frege's senses. Third, although Bealer's account is immune to the unity problem, by explaining both predication and representation in an algebraic framework, he has a disadvantage of being not able to give an account for how the same proposition can be expressed by different sentences. [250] Bealer's explanation of propositional constituency by composition trees is not sufficient to

[249] See Parsons (2016, pp. 104-106) for problems of defining such notions in Bealer's algebraic framework.

[250] See King (2019, sec. 3.3).

express the identity of propositional content among different sentences. Given that the decomposition tree of a sentence expresses a unique proposition, this would result in the untenable implication that synonymous sentences express different propositions.

One advantage of Zalta's approach over Bealer's is that the semantic values of expressions are recoverable from propositions expressed by sentences. On the other hand, Zalta's approach to structure of propositional constituency is fundamentally Fregean. However, he does not appeal to sense-denotation distinction similarly to Bealer. Zalta also does not employ a distinction between intension and extension of linguistic expressions. In particular, he does not assign any intension to terms, except non-existent objects that are denoted in intensional contexts. Zalta suggests these abstract entities are not intensions, however it is not clear how they can have certain roles of intensions, especially for the problems associated with intensional contexts. [251] It has been also argued that Zalta's account does not adequately explain propositional unity to represent truth and falsity.[252] Zalta appears to implement truth bearing properties to the structure of propositions. However, his account is not completely satisfactory to explain the unity. He merely says that the structure of propositions is somehow inherently arranged to express truth as well as falsity and there is nothing else that makes a proposition true or false. [253] The constituents of propositions in Zalta's account are objects and properties built into complex logico-semantic structures. As a result, Zalta faces a similar problem with Russell's view of proposition.

5.5. Deflationary Approaches to Propositions

In the preceding sections of this chapter, we have considered Russellian, neo-Russellian, set theoretical, algebraic and structured approaches to propositions.

[251] See Parsons (2016, p. 113).

[252] *Cf.* King (2019, sec. 3.3.).

[253] See Zalta (1988, p. 56).

In this section, we shall focus on philosophical views of some discontented philosophers about propositional structure. These theories are collectively classified as deflationary approaches to propositions. According to this variety of propositional realism, propositions are unstructured and primitive entities which should neither be analyzed in terms of other entities nor have structural parts and constituents. Propositions are simply *sui generis* entities. First, we will consider Stephen Schiffer's Theory of Pleonastic Propositions, then we will consider Trenton Merricks' and Lorianne Keller's approaches.

5.5.1. Schiffer's Theory of Pleonastic Propositions

In his book *The Things We Mean* (2003), Stephen Schiffer construes propositions as pleonastic entities.[254] Schiffer notes that his use of word "pleonastic entities" appeals neither to pleonasms nor redundancy. In Schiffer's ontological framework, a pleonastic entity generated by certain process transformations, which he calls something-from-nothing transformation. [255] In a nutshell, a type of entity *F* can be derived from a sentence about that entity in which there is no reference to that entity *F*. In this regard, pleonastic entity is an entity that falls under a pleonastic concept: A concept *C* is pleonastic if and only if it implies true something-from-nothing transformations. [256] For Schiffer, if this condition holds, then the entity *F* exists with its relevant features. For instance, from the sentence 'Chrysippus is a philosopher' one can infer 'the property of being philosopher is exemplified by Chrysippus, exists' by something-from-nothing transformations. Moreover, propositions as pleonastic entities are also generated by the same transformation

[254] The Merriam-Webster Dictionary defines "pleonasm" as "the use of more words than those necessary to denote mere sense."

[255] "'Pleonastic' entities are entities whose existence is secured by something-from-nothing transformations (I call these things 'pleonastic' entities because something-from-nothing transformations often take us to pleonastic equivalents of the statements from which they are inferred)." (Schiffer, 2003, p. 51)

[256] Schiffer (2003, p. 61).

procedure from sentences which has no reference to a propositional entity. Such as from 'Chrysippus is a philosopher' to 'the proposition that Chrysippus is a philosopher exists and is true'. [257]

Schiffer's argument for pleonastic propositions is in fact a part of his argument for pleonastic entities such as fictional entities, events, and properties. Further aspects of Schiffer's theory are related to his theory of meaning, nevertheless his theory of pleonastic entities is overtly complex and complicated. We shall not dwell further into details but focus on Schiffer's account of propositions. According to Schiffer, propositions merely exist as objects. Obviously, they are not concrete objects, since mind has no access to objects of external world by sense perceptions. The knowledge of existence of propositions can be apprehended by thinking about them by linguistic or conceptual practice. [258] Therefore, from these examples, the something-from-nothing transformation grants permit to the introduction of properties and propositions. In the course his philosophical attitude, Schiffer carefully refers to propositions as abstract objects, for naming them so may result in the perplexing inference that "our minds can reach beyond the physical world to make contact with denizens of a Platonic universe."[259]

Schiffer's account presents an interesting deflationary approach to propositions. However, perhaps the most critical point in his account is that too many propositions can be created from his something-from-nothing transformations. Thus, Schiffer's account has the burden of explanation for what gives the justification for the existence of pleonastic entities. Since Schiffer aims to deflate certain aspects of propositions such as their nature or being, this approach can be considered deflationary. However, if every property instantiates corresponding propositions, then this account is in fact inflationary. Another

[257] *Cf.* Carrara and Sacchi (2006), and McGrath and Frank (2020, sec.7.3).

[258] See Schiffer (2012).

[259] Schiffer (2003, pp. 66-67).

concern for this account is the explanation of truth conditions. [260] Although his commitment to something-from-nothing transformations make it possible to derive propositions, it is unable to account for their truth values. Inevitably, a question concerning the possibility of a deflationist explanation about the truth conditions of proposition rises. If propositions are *sui generis* entities, how can the property of bearing truth values be related to the actual, concrete conditions that the proposition is meant to represent?

5.5.2. Merricks's Accounts of Propositions

Trenton Merricks defends a deflationary view of propositions in his book *Propositions* (2015). Merricks holds that propositions are unanalyzable *sui generis* entities. "A proposition is a necessary existent that essentially represents things as being a certain way." [261] He argues that representational feature of propositions is primitive and cannot be explained.[262] For Merricks, since propositions are primitive entities, one cannot proclaim any explanation concerning their truth bearing properties. [263] Moreover, propositions do not have a structure for they have neither constituents nor parts. Thus, there cannot be any genuine distinction between simple and complex propositions or any relation between any proposition.[264] Nevertheless, Merricks argues that propositions are primary truth bearers, the objects of belief and abstract objects by their nature. As we have considered in Chapter 4, Merricks argues for the necessary existence of propositions which appeals to the notion of modal validity, i.e., the existence of propositions are necessary for they are the

[260] See McGrath and Frank (2020, sec. 7.3).

[261] Merricks (2015, p. 191).

[262] Merricks (2015, pp. 194-196 and 207-210).

[263] Merricks (2015, p. 195).

[264] Merricks (2015, pp. 45-47, 78 and esp. 205-207).

premises and conclusions of modally valid arguments. As a result, the existence of modal valid arguments *a fortiori* guarantees the existence of propositions. [265]

5.5.3. Keller's Accounts of Propositions

Lorraine Juliano Keller argues against the structural approaches in the context of propositional constituency. Similar to general tenets of other deflationary accounts, propositions are *sui generis* abstract entities, Keller criticizes several approaches to metaphysics of structural constituency but in the final analysis she endorses a propositional primitivist view, according to which "constituency has not been explained, but is taken as brute. This strategy deprives [structured approaches] of the dialectical advantage of having fewer primitive predicates." [266] She considers Fregean, Russellian, set theoretic, mereological, and hylomorphic accounts of structured propositions and rejects them. [267] Keller claims constituency cannot be analyzed in terms of parthood, membership and so on. [268] These accounts have certain ontological commitments all of which are vulnerable to objections about the structure of propositions. We have considered all of these problems, except for the problems concerning mereological and hylomorphic approaches. Nevertheless, these set of problems have something in common: structured propositions do not manage to provide a complete and satisfactory account of fine-grained structure. Accordingly, Keller uses this reasoning to argue that propositions are not reducible, nor they can be explained by entities in another ontological category. [269] Thus, her main argument appeals to *reductio* of structural approaches. Keller endorses a deflationary approach as the sole option concerning the nature and structure of propositions by construing constituency as a primitive *sui generis* relation.

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[265] Merricks (2015, pp. 18-22).

[266] Keller (2013, p. 668).

[267] See Keller (2013; 2019).

[268] Keller (2013).

[269] See Keller (2013).

We shall end this section by a general review of Merrick's and Keller's deflationary approaches. Although these approaches are useful for their minimal roles, they are inadequate and insufficient to explain semantic roles of propositions. Propositions, as meanings of sentences, has perhaps the most important role of explaining the meaning of sentences and truth-values. Nevertheless, deflationary approaches do not provide any account for the explanation of semantic content. They simply appeal to their so-called *sui generis* existence for the semantic roles. However, an appeal to such explanation do not give any further explication. As a result, proponents of structured account find deflationary accounts as mysterious.[270] However, these philosophers have advocated a return to a basic conception of propositions on which they are simply mind and language independent abstract objects. They have truth conditions by their very nature, so they represent the world as being a certain way. In this view, they deny that there can be any explanation of how or why propositions have truth conditions other than the explanation already given above, yet it is not a proper explanation. Therefore, Frege's account proves to be more explanatory and overall more satisfying to give an elaborate account than deflationary approaches.

[270] *Cf.* Soames (2010, ch. 5), King (2018, pp. 329-330; 2019, sec. 3.1) and McGrath and Frank (2020, sec.7.3).

CHAPTER 6

THE STRUCTURE OF ATOMIC FREGEAN *THOUGHTS*

The aim of this chapter is to elucidate Frege's mature views concerning the structure and compositionality of *Thoughts*. In the first part, we will present Frege's two main principles for compositionality of *Thoughts*. The first is the Function-Argument Compositionality Principle and the second is Part – Whole Compositionality Principle. We will explicate these principles with regard to the structure of *Thoughts*. In the second part, we will focus on problems regarding the structural analysis of Fregean Thoughts. The first problem is about the apparent tension between Frege's Context Principle and Compositionality Principles. Although this tension signals certain problems in Frege's semantic theory of *Thoughts*, we will argue for a reconciling position by holding both principles in an interpretation of a plausible account for structural composition of *Thoughts*. The second problem concerns Frege's commitment to two conflicting theses about the structural analysis of *Thoughts*. These theses are Unique Analysis of *Thoughts* and Multiple Decomposition of Fregean *Thoughts*. Nevertheless, the apparent conflict between these theses poses a serious problem for the structure of *Thoughts* and their identity with the corresponding sentence structure. In the third part, we shall provide a critical review of proposed solutions concerning this problem in the literature. In the last part, we shall provide our solution in light of the second problem by providing an interpretive emendation, for Frege holds both theses. We shall provide our solution similar to Frege's original position by implementing both theses in his theory of Thoughts. We believe our solution offers the closest position to that of Frege's, as the textual evidence suggests. We will argue for the *polymorphous*

structure of *Thoughts* to defend a consistent and satisfactory account for explaining the compositionality of Fregean *Thoughts*.

6.1. Frege's Compositionality Principles

Frege's distinction between *sense* and *denotation* results in two different principles of compositionality. We shall begin with a Compositionality Principle for *sense*

($Comp_{\mathfrak{s}}$) The *sense* of a complex expression is composed of the *senses* of its constituent parts.

In addition to the compositionality of *senses*, Frege had endorsed a Compositionality Principle for *denotation*:

($Comp_{\mathfrak{d}}$) The *denotation* of a complex expression is composed of the *denotations* of its constituent parts.

For Frege, sentences are complex linguistic expressions, and their *senses* are *Thoughts*. Thus, we can construe the corresponding Compositionality Principle for *Thoughts*

($Comp_{\tau}$) *Thoughts* are composed of the *senses* of its constituent parts.

Hence, Frege conceives of *Thoughts* as structured complexes of *senses*. Frege had held a Compositionality Principle for *denotations* of sentences, however he later rejected it.[271]

[271] Historically, Frege's compositionality principle for *denotation* precedes the compositionality principle for *senses*. In *Begriffsschrift*, Frege held a functional compositionality principle for *denotations*, i.e., the *denotation* of a complex is a function of the *denotations* of its parts. He held same principle in *Grundgesetze* and applied it to numerous examples in mathematics. After his distinction between *sense* and *denotation* of sentences, Frege (1892a (*FR* p. 159)) initially applied compositionality principle for *senses* then he "transfers" the principle for *denotations*: "I have in fact transferred the relation between the parts and the whole of the sentence to its *denotation*, by calling the *denotation* of a word part of the *denotation* of the sentence, if the word itself is a part of the sentence." (Frege 1892a (*FR* p. 159)) However, in his later writings, he disowned this view. See Frege (1903 (*CP* pp. 281-282); 1919 (*PW* pp. 255-256)). Nevertheless, he did not drop the part-whole

In the following subsections, we shall explicate two models of compositionality: the Function-Argument Compositionality Principle ($Comp_{FA}$) and the Part-Whole Compositionality Principle ($Comp_{PW}$).

6.1.1. The Function–Argument Compositionality ($Comp_{FA}$) Principle

As we have argued in Chapter 3, Frege puts forward the function–argument analysis for expressions. In his *Begriffsschrift*, he held a Function–Argument Compositionality Principle only for *denotation* functions. Accordingly, we can construe the following corresponding principle[272]:

(Comp. $FA_{\mathfrak{d}}$) For any function-argument expression $\alpha = \Phi(\zeta_1,\ldots,\zeta_n)$

$$[\![\alpha]\!]_{\mathfrak{d}} = [\![\Phi]\!]_{\mathfrak{d}}([\![\zeta_1]\!]_{\mathfrak{d}},\ldots,[\![\zeta_n]\!]_{\mathfrak{d}})$$

where $[\![\alpha]\!]_{\mathfrak{d}}$ is the *denotation function* of complex expression α, and 'Φ' is the *n*-place functional expression with arguments $\zeta_1,\ldots,\zeta_n$.

This principle states that the *denotation* of function-argument expressions is the *denotation* of its corresponding functional expression and argument parts. This principle provides a unique connection between the *denotation* of the characteristics of a complex function and the *denotation* of its parts. By (Comp. $FA_{\mathfrak{d}}$), the *denotation*

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relation between thoughts and its parts, however he (1919 (*PW* p. 255)) only remarked that "things are different in the domain of *denotation*." *Cf.* Beaney (1997) and Klement (2002, p. 68). Heck and May (2011, p. 128) also cites Carnap's notes on Frege's lectures that "The *denotation* of the parts of a sentence are not parts of the *denotation* of the sentence. However: The sense of a part of the sentence is part of the sense of the sentence." (Reck and Awodey, 2004, p. 87).

[272] See Pickel (2021, p. 6918). Carnap (1947, pp. 120–121) was first to formulate the function-argument compositionality as Frege's Principles:

> (28-6) First principle. If A_j and A_k have the same nominatum [*denotation*], then … A_j… and …A_k… have the same [*denotation*]. In other words, the [sense] of the whole expression is a function of the [*denotation*] of the names occurring in it.
>
> (28-7) Second principle. If A_j and A_k have the same sense, then …A_j… and …A_k…have the same sense. In other words, the sense of the whole expression is a function of the senses of the names occurring in it.

of the complex expression is thus shown to be a function of the *denotation* of its parts. [273]

After his sense-denotation distinction, we can explicate Frege's Function–Argument Compositionality Principle for *senses* as follows[274]:

(Comp. $FA_{\mathfrak{s}}$) For any function-argument expression $\alpha = \Phi(\zeta_1,\ldots,\zeta_n)$

$$[\![\alpha]\!]_{\mathfrak{s}} = [\![\Phi]\!]_{\mathfrak{s}}([\![\zeta_1]\!]_{\mathfrak{s}},\ldots,[\![\zeta_n]\!]_{\mathfrak{s}})$$

where $[\![\alpha]\!]_{\mathfrak{s}}$ is the *sense function* of complex expression α, and 'Φ' is the *n*-place functional expression with arguments $\zeta_1,\ldots,\zeta_n$.

This principle states that the *sense* of a function-argument expression is a function of the *sense* of its parts.

The reasoning for the function-argument compositionality of *Thoughts* is grounded in Frege's sense-*denotation* distinction supplemented with saturated-unsaturated distinction. When explaining the function–argument analysis, Frege applies sense-*denotation* distinction to both saturated and unsaturated expressions.[275] Frege has considered singular terms, as well as sentences, as saturated expressions. Accordingly, saturatedness of singular terms and sentences is reflected in their *senses* and *denotation* being saturated. On the other hand, functional expressions, such as concept-words and relations, are unsaturated, thus their *senses* and *denotation* are also unsaturated. [276]

[273] In *Grundgesetze*, Frege further explained the general function-argument analysis is the context of functional compositionality: "Any symbol or word can indeed be regarded as consisting of parts; but we do not deny its simplicity unless, given the general rules of grammar, or of the symbolism, the [*denotation*] of the whole would follow from the [*denotations*] of the parts, and these parts occur also in other combinations and are treated as independent signs with a [*denotation*] of their own." (1903a (*FR* p. 269).

[274] See Pickel (2021, p. 6919).

[275] See Frege (1893c (*PW* pp. 118-119); 1891b, (*PMC* pp. 63)). *Cf.* Klement (2002, p. 65).

[276] Frege (1892a (*FR* p. 139); 1892b (*FR* p. 174)).

A *Thought* is the *sense* expressed by a declarative sentence. Thus, we can construe the Function-Argument Compositionality Principle for *Thoughts* as follows:

(Comp. $\mathrm{FA}_{\mathcal{T}}$) $\qquad \mathcal{T} = [\![\sigma]\!]_{\mathfrak{s}} = [\![\Phi]\!]_{\mathfrak{s}} ([\![\zeta_1]\!]_{\mathfrak{s}},\ldots, [\![\zeta_n]\!]_{\mathfrak{s}})$

where $[\![\sigma]\!]_{\mathfrak{s}}$ is the *sense function* of a sentence σ, and 'Φ' is the *n*-place functional expression with arguments $\zeta_1,\ldots, \zeta_n$.

According to (Comp. $\mathrm{FA}_{\mathcal{T}}$), a *Thought* expressed by a sentence is the functional composition of the *senses* of its singular term parts and the *senses* of functional or predicate parts.

6.1.2. The Part-Whole Compositionality (*Comp*$_{PW}$) Principle

Frege's second and perhaps the most cited compositionality principle is the Part-Whole Compositionality Principle (*Comp*$_{PW}$). This principle, without an exception, is referred to Frege's following oft-cited quotation:

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even a thought grasped by a human being for the very first time can be put into a form of words which will be understood by someone to whom the thought is entirely new. This would be impossible, were we not able to distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence serves as an image of the structure of the thought. To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly even bothered by the hitches which occur from time to time.
>
> If, then, we look upon thoughts as composed of simple parts, and take these, in turn, to correspond to the simple parts of sentences, we can understand how a few parts of sentences can go to make up a great multitude of sentences, to which, in t urn, there correspond a great multitude of thoughts. [277]

[277] Frege (1923 (*CP* p. 390)).

Frege in his later writings often emphasizes this mereological aspect of the compositionality of *Thoughts*. His general view has been to build a correspondence between parts of sentences and parts of *Thoughts* on the one hand, and a correspondence between sentences as a whole and *Thoughts* as a whole on the other. His provision was to reflect the logico-semantic structure on the ontological structure of *Thoughts*. Moreover, in these writings, Frege also elaborated further aspects of semantic meaning other than the sense-denotation distinction. One of the reasons for such a principle is considering the apprehension of new linguistic constructions as an ability.[278] Nevertheless, it seems hardly possible to explicate the formal structure of this principle, since Frege only talked in metaphors and hints about it.

6.2. The Context Principle and the Compositionality Principles

Having considered Frege's Compositionality Principles, we can turn to the tension between the Compositionality Principles and the Context Principle. According to Frege's Context Principle (hereafter (CP)), words have meaning only in the context of a sentence.[279] (CP) is often considered Frege's one of the most important principles along with his Compositionality Principles.[280]

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[278] *Cf*. Beaney (1997, pp. 319-320), Dummett (1981b, Ch. 15), Heim and Kratzer (1998, p. 2).

[279] Interestingly, Frege's Context Principle is also known as 'Frege's Principle'. See Pelletier (2001) and Janssen (2012, pp. 19-20).

[280] Frege's (CP) has an enormous impact on the posterior semantic theories in the name of contextualism. The tension between these principles has certain reflections on the contemporary semantic theories. Different views prioritize meaning by distinguishing the priority between word meaning and sentence meaning. There are two general classifications of approaches: the contextualist approaches and the compositional approaches. Both approaches ground their theoretical frameworks to Frege's corresponding principles. The contextualist approaches prioritize sentence meaning over word meaning. They hold that contextual factors primarily determine sentence meanings, and word meanings depend on sentence meaning relative to contexts. For example, radical contextualists argue that all lexical items are context dependent, and they challenge the compositional approaches. See Searle (1980) and Recanati (2004). The compositional approaches prioritize the contribution of word meaning to sentence meaning by giving an account for the determination conditions of word meaning to explain the sentence meaning. There are diverse views on conditions and degrees of determination for such contributions. In formal semantics, compositional approaches primarily study complex

We shall begin with Frege's formulation of the (CP). Frege first formulated this principle in his *Grundlagen*, as the second of three fundamental principles:

> The meaning of a word must be asked for in the context of a proposition, not in isolation. (Frege, 1884, X (*FR* p. 90)).

Frege also mentions (CP) thrice in the text:[281]

> That no idea can be formed of the content of a word is therefore no reason for denying it any meaning or for excluding it from use. The appearance to the contrary doubtless arises because we consider the words in isolation and in asking for their meaning look only for an idea. A word for which we lack a corresponding mental picture thus appears to have no content. But one must always keep in mind a complete proposition. Only in a proposition do the words really have a meaning. The mental pictures that may pass before us need not correspond to the logical components of the judgement. It is enough if the proposition as a whole has a sense; its parts thereby also obtain their content. Frege (1884, §60 (*FR*) p. 108)).

> How, then, is a number to be given to us, if we cannot have any idea or intuition of it? Only in the context of a proposition do words mean something. It will therefore depend on defining the sense of a proposition in which a number word occurs. As it stands, this still leaves much undetermined. But we have already established that number words are to be understood as standing for independent objects. Frege (1884, §62 (*FR* p. 109)).

> Numbers thus appeared as reidentifiable objects, though not as physical or even merely spatial ones, nor as ones which we can picture through the power of imagination. We then laid down the principle that the meaning of a word is to be denned not in isolation, but in the context of a proposition; only by adhering to this, I believe, can the physical conception of number be avoided, without falling into a psychological one. Frege (1884, §106 (*FR* p. 127)).

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semantic structures, such as phrases and sentences, by recursively constructing or combining the meanings of words and terms as the fundamental constituents of such structures. For example, lexical semantics attribute priority to word meaning over sentence meaning by focusing on the internal structure of words, and relations within the vocabulary of natural languages. See Pelletier (2001), Fodor (2001), Pagin and Pelletier (2007), Szabo (2010). There are also hybrid approaches which aim to reconciliate these opposite ends. See Fodor and Lepore (2001), Lasersohn (2012), and Szabo (2022).

[281] Dummett (1991a, p. 111) famously remarked the paragraphs mentioning it (§§60, 62) as "arguably the most pregnant philosophical paragraph ever written."

In the first appearance of (CP), Frege seems to give a semantic thesis in simple and explicit terms.[282] As a result, (CP) has been often considered as Frege's commitment to the view that sentences are the primary medium of meaning and words have their meanings derivatively in the context. In this regard, it can be argued that Frege's reception of (CP) commits us to the view that *Thoughts* have a priority over the *senses* of its constituent words.

In his *Grundlagen*, Frege's main concern was to define numbers. As we can see in the text, the following mentions of (CP) gradually present it as a specific principle concerning meanings of numbers. In fact, (CP) has an indispensable role in Frege's formulation of numbers as objects, since their meaning is inherently connected to their conceptualization of entities in sentences.[283] Frege uses (CP) to argue against empiricist, psychologistic, and formalist conceptions of numbers. Nevertheless, Frege did not explicitly mention (CP) in his later works.[284]

On the other hand, Compositionality Principles have been considered as Frege's commitment to the view that the *sense* of a sentence is determined by the

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[282] *Cf*. Dummett (1981a, pp. 192-196, 495-505; 1981b, Ch.19; 1991b, Chs. 15-18), Currie (1980, pp. 234-248), Beaney (1996, pp. 234-245; 1997, pp. 15-20), Klement (2002, pp. 76-82), Ricketts (2010, Ch. 6), Heck and May (2011, pp. 142-144), Szabó and Thomason (2019, pp. 60-62), Szabo (2021, sec. 1.6.4.).

[283] Frege (1884) held that statements about numbers are higher-level statements about concepts. In a nutshell, Frege appeals to value-ranges of concepts to define numbers as second-level statements about concepts. For instance, in the proposition "There are two apples in my pocket" there is a statement which has the first-level concept *being apple* which falls under the second-level concept *being a concept under which two objects fall*. In this respect, Frege does not appeal to the concept *two* to define numbers. Frege then uses the notion of equinumerosity by defining 'the number of the concept *F*' as the extension consisting of all the concepts that are equinumerous with *F* (1884, §68). In λ-notation, let '[$\lambda x.\ \phi$]' be the name of complex concepts *being such that* ϕ, and '*#F*' to represent 'the number of *F*' where '#' is a primitive operator. Frege defines number zero as the number of the concept *being not self-identical* (1884, §74) which can be represented as '[$\lambda x.\ x \neq x$]'. Then, the number zero is defined in term of the extension of all the concepts equinumerous to the concept *not being self-identical*. Accordingly, the number zero can be defined as '$\#(\lambda x.\ x \neq x)$'. Remaining natural numbers are expressed by Frege's *successor* (or *ancestral*) relation (1897, Part III, *Satz* 76; 1884, §79; 1893/1903 I §43-45) as successors of the number zero (1884, §83). *Cf.* Zalta (2021; 2022, esp. secs. 2.5 and 2.6). For λ-definitions of numbers see Church (1932), Barendregt (1984, Ch. 6), Alama and Korbmacher (2018, sec. 9).

[284] See Beaney (1997, pp. 15-17), Klement (2002, pp. 76-78).

senses of its constituent expressions. In other words, the *senses* of constituent parts of a sentence, i.e., words, has priority over the sentence itself. From the above consideration, (CP) sets a constrain on the compositionality principles. As result, both principles are regarded as in tension for they prioritize meanings of different entities. On the one end of this tension, the constituents of sentences have their meaning only in whole sentences, whereas on the opposite end sentences have their meaning in virtue of their components.

There are two main aspects of this problem. The first aspect is related to the problem of explanatory priority between sentence meaning and word meaning. We shall call this aspect *the priority of explanation problem*. From the perspective of Compositionality Principles, the *senses* of singular terms can be elucidated or analyzed prior to the meaning of the sentence. On the other hand, (CP) attributes an explanatory priority to sentence meaning over word meaning. The second aspect of this tension is *the problem of ontological priority*. According to this aspect, there are two apparently inconsistent principles concerning the ontological dependency between *senses* of words and sentences. We shall consider both aspects of this tension in the discourse about the structure of Fregean *Thoughts*. We shall argue, at least for Frege's semantic theory, that the so-called tension between the Compositionality Principles and (CP) does not result in inconsistency regarding the structure of *Thoughts*.

We shall begin by focusing on the first aspect. In his early his works, Frege held that judgments are explanatorily prior to their constituents [285] and his views concerning the function-argument structure and saturated-unsaturated distinction has been put forward to argue for the explanatory priority of (CP). It is obvious that Frege has aimed to exclude all context dependent factors in his logical theory. We see that Frege held this approach for a particular reason, namely, to define numbers in a logical precision. However, we argue that there are no context dependent cases

[285] *Cf*. Frege (1879, §9 (*FR* pp. 65-68)). Later, when Frege elaborates his views concerning truth *still* in the discourse of his *Begriffsschrift* he says "I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analyzing the thought." (Frege, 1919 (*PW* p. 253; *FR* p. 362)).

for words of logic and arithmetic in his *Grundgesetze*.[286] As a result, Frege's (CP) at best should be considered as a supplementary principle against the formalist, empiricist, and psychologist definition of arithmetical notions, numbers in particular.

Nevertheless, some interpreters have taken (CP) as a general semantic principle concerning the structure of *Thoughts*. In this regard, if we consider the problem of explanatory priority between the semantic values of sentences and words, (CP) will be important for the cases in which the same word has different meanings in different sentences. For example, the word 'bank' has two different meanings in the sentences 'Mehmet lives along the bank of the Maritsa' and 'Mehmet deposited all his money in the bank of Maritsa.' In the former sentence, the word 'bank' *denotes* a rising ground bordering a river, whereas in the latter it *denotes* a financial institution. However, Frege does not mention such cases in the discourse of (CP). Moreover, the meanings of words not only depend on their occurrences in the sentence but also depend on the circumstance of the utterance. Indeed, it is this aspect of context dependency, i.e., the circumstances depending on indexicals and demonstratives, of which Frege has focused on his late works. In these works, Frege always gives explanatory priority to such context dependent words and phrases to account for the semantic values of sentences in the compositional structure.[287] If words had their meanings *only* in the context of a whole sentence, this would bear even worse results such as semantic incompleteness of *senses* of singular terms. Frege would never endorse such a view in his theory of *Thoughts*. For the explanatory priority aspect of this problem, we conclude that there can be no priority between the *senses* of proper names and sentences, thus it poses no challenge for the structure of *Thoughts*.

We shall now turn to the second aspect of the tension which we called *the problem of ontological priority*. On the logical level, Frege held that denotations of

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[286] See Frege especially (1903, II, §§56- 57 (*FR* pp. 259-270)). *Cf*. Resnik (1980, pp. 161-171), Dummett (1981b; 1995), Wright (1983, Ch. 1), Ruffino (1991), and Linnebo (2019).

[287] *Cf*. Frege (1918a; 1923).

concept words are determined by their values-ranges. However, in his later works he clearly endorsed the priority of compositionality principle that the sense of a sentence is determined by the senses of their constituent parts. [288] As we have argued, Frege's sense–denotation and object–concept distinctions are indeed ontologically prior notions for they are the building blocks of *Thoughts*. In this regard, the structure of *Thoughts* is determined by the *senses* of the constituents of sentences, not the other way around.

To certain extends, the tension between the Compositionality Principles and (CP) has its origins in interpreting (CP) in the setting of sense–denotation distinction. Some interpreters have applied this distinction to (CP), they split up the (CP) and obtained two corresponding principles by arguing that (CP) is chronologically prior. [289] However, an appeal to chronological priority results in an unjustified and un-Fregean commitment to the theory of *Thoughts*. We can give two reasons for this point. First, the German word '*Bedeutung*' is translated as 'meaning' in above cited passages from *Grundlagen*. [290] However, the term '*Bedeutung*' does

[288] This was in fact considered as early as 1892, see Frege (1893/1903 §§28-32). But also explicitly stated later in 1914 as a necessary and indispensable account for understanding the complex linguistic expressions. See Frege (1914, (*FR* pp. 319-320); 1923 (*PW* p.390)). *Cf.* Beaney (1997, p. 18)

[289] We must note that Frege split up the content, not the context. We shall anyway mention this un-Fregan approach: (Beaney, 1997, pp. 16-17).

*$(CP)_{Sense}$ The *senses* of parts of a sentence are determined by the sense of the sentence.

In the order of explanation for expressions to have a *sense*, Frege starts with proper names and then concerns sentences. Moreover, when accounting for how the *senses* of complex expressions are understood the priority of determination still on the compositionality principles.

*$(CP)_{Denotation}$ The denotations of parts of a sentence are determined by the denotation of the sentence.

This modified principle is even more implausible at least as far as use of ordinary language is concerned for it will result in the unsound claim that denotations of a name is somehow determined by the truth-value of a sentence. Moreover, in Frege's truth-functional logical theory, function-argument structure requires strictly that the *denotations* of complex expressions are constructed by their constituent expression by the primitive logical connectives.

[290] Szabo (2022) remarks a very important point for the translation. He translates the last sentence of above given citation from Frege (1884, § 60) as "it is enough if the sentence as whole has meaning; thereby also its parts obtain their meanings." He further points out that "Frege indeed seems to have

not correspond to cognitive meaning in Frege's later semantic theory, rather only to truth values of sentences. Second reason is that Frege in his later works never even mentions splitting up (CP). Thus, as it has always been the case for our explication of Frege's own views about the semantics of natural language, we discard such unjustified modification as inadmissible.

As a result, we conclude that there is no tension between the Compositionality Principles and (CP). Moreover, neither explanatory priority nor ontological priority poses a difficulty to the structure of Fregean *Thoughts*. Conceiving (CP) in further aspects is indeed an anachronistic misinterpretation of Frege's theory of *Thoughts*. Frege in his later works primarily focuses on the understanding and grasping of sentences that we never heard before. [291] When we consider language learning versus Semantic theory for a language, we *learn* natural languages, including the language of arithmetic, by learning the meanings of a selection of sentences. Then, by Frege's (CP) we extract from our knowledge of the meaning of these sentences the meanings of the constituent words. Moreover, by applying semantic theory for these languages, our input consists of the previously learned meanings of words in L – which are always *finite* in number. By Frege's Compositionality Principles we acquire a semantic-theoretic knowledge for understanding an infinite number of sentences constating of these words. Thus, his Compositionality Principle perfectly fits in the structural explanation of *Thoughts*.[292]

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a sufficient condition but not a necessary condition." *Cf.* Beaney (1997, pp. 15-20) and Klement (2002, p. 77).

[291] "The world of thoughts has a model in the world of sentences, expressions, words, signs. To the structure of the thought corresponds the compounding of words into a sentence." (Frege, 1918a (*CP* p. 378)). *Cf.* Frege (1914 (*PMC* p. 79); 1923).

[292] We shall mention two attempts of reconciliation of these principles. (i) Dummett (1981b, ch. 15) has argued for a reconciling position by accepting both principles as central to Frege's semantic theory: "in the order of explanation the sense of a sentence is primary, but in the order of recognition the sense of a word is primary." (1981a, p. 4; *cf.* 1981b, p. 374). (ii) Szabo (2021, secs. 1.6. and 3.2.) puts forward an alternative construction of Frege's both principles in a way to reconcile the generality of priority determination. In this regard, he argues that "we should drop the talk of words and sentences, and talk instead about complex expressions and their constituents" and he puts forward the following combined principle:

6.3. Problem of Analysis and Decomposition of Fregean Thoughts: The Dummett-Bell Problem

In this section, we shall focus on the second problem concerning the structure of *Thoughts*. This problem was first diagnosed by Michael Dummett and later readressed and improved by David Bell. [293] This problem is related to two different models of analysis or decomposition concerning *Thoughts.* However, when these two theses are considered together, they lead to an inconsistency. [294] Hereafter, we shall name the problem as the Dummett-Bell Problem by giving respective credit to both philosophers. This problem is a consequence of Frege's holding two different theses concerning the structure of *Thoughts*. We shall first state these theses by expounding their foundations and roots in Frege's writings.

First Thesis: Unique Analysis of Fregean Thoughts (UAT)

Frege's first thesis (hereafter (UAT)) of theory of *Thoughts* states that every *Thought* is isomorphic with the (unambiguous) sentence which expresses it. This thesis clarifies the relation between a sentence and its corresponding *sense*, i.e., *Thought*. This thesis is essential for the expressibility of *Thoughts*. Without this thesis, *Thoughts* would be merely explained as *senses* of assertoric sentences. This

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(SzC) The meaning of an expression is determined by the meanings of all complex expressions in which it occurs as a constituent.

He considers Frege's compositionality principles as a bottom-up meaning-determination, and (CP) as top-down meaning-determination: "As long as it is not understood as a causal or explanatory relation determination can be symmetric, so any version of [compositionality principle] is compatible with the corresponding version of [(SzC)]." Szabo mentions the *reverse compositionality* principle according to which the meaning of a complex expression determines the structure of the expression and the meanings of its constituents. He lists Fodor and Lepore (2001) and Pagin (2003) as proponents of the reverse compositionality principle, and lists Patterson (2005), Robbins (2005), Johnson (2006) as opponents.

[293] Dummett (1981a, pp. 27-30, 60-62; 1981b, Ch. 15; 1991a, pp.301-302), and Bell (1987, 1996).

[294] *Cf*. Levine (2002, pp. 195-197), Klement (2002, pp. 84-88), Heck and May (2011, pp. 127-129).

thesis is also essential for the intersubjectivity of *Thoughts*. Without this thesis, assertoric sentences would not communicate (convey) *Thoughts.* Dummett names this thesis simply as 'analysis'. [295] Bell's formulation of this thesis is related with the identification of the structure of *Thought* with the structure of the sentence which expresses that *Thought*. [296] We can explicate (UAT) as follows:

(UAT) A thought $\mathcal{T}$ is isomorphic with a sentence *S* which expresses it.

There are numerous textual evidence in Frege's writings for the isomorphism between sentences and *Thoughts*. [297] We shall now define the term *isomorphism*. We shall use '≈' to denote isomorphism.[298]

(Isomorphism) Two structures Σ and Σ′ are called *isomorphic* (Σ ≈ Σ′) if and only if, the constituents of Σ are c_1, …, c_i,…$_j$, those of Σ′ are $c_1^{'}$, …, $c_i^{'}$,…$_j$, the relations between c_i and c_j in Σ and $c_i^{'}$ and $c_j^{'}$ in Σ′ are respectively $R_{i,j}^{k}$ and $R_{i,j}^{'k}$, and the following conditions are satisfied:

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[295] This is known as first pair of contradictory theses (the A theses) that Dummett (1981a, pp. 261-262) attributes to Frege.

A1 A thought may be analysed in different ways

A2 A thought is not built up out of its constituent parts; the constituents of the thought are arrived at by analysis of it.

Dummett later formulates this thesis as follows: Each propositional content admits of a unique ultimate analysis into simple constituents.

[296] Bell's (1987, p. 41) formulation is as follows: "Thesis 1: A thought is isomorphic with the sentence whose sense it is." See also Bell (1996, p. 584).

[297] Bell (1996) mainly refers to Frege (1923 (*CP* p. 390)). This thesis is supported on the one hand by a number of Frege's views concerning sentence structure, and on the other hand by a number of views concerning the nature of *senses*. *Cf.* Frege (1884; 1891a; 1892a; 1892b; 1918a).

[298] Note that the notion of isomorphism is applicable to (i) A and B are both sentences, (ii) A and B are both *Thoughts*, and (iii) One of A, and B, is a sentence, the other is *Thought*.

i. There is one-to-one correspondence (bijection) between constituents of Σ and Σ′,

ii. $c_i R^k_{i,j} c_j \rightarrow c'_i R'^k_{i,j} c'_j$

Dummett argues that Frege's commitment to ($Comp_{PW}$) commits Frege to acceptance of (UAT), otherwise as Dummett argues "[it] means nothing if it does not mean that a grasp of the thought depends on a grasp of that constituent sense."[299]

Second Thesis: Multiple Analysis of Fregean Thoughts (MAT)

Frege's second thesis (hereafter (MAT)) of theory of thoughts is attributed to Frege's famous statement "[A] thought can be split up in many ways, so that now one thing, now another appears as subject or predicate. […] But [it must never be forgotten] that different sentences may express the same *Thought*." [300]

(MAT) Two structurally different (i.e., non-isomorphic) sentences can express the same *thought* $\mathcal{T}$.

For Dummett, this thesis is required for the objectivity criterion of thoughts in order to distinguish the common pattern between sentences and their corresponding thought. [301] However, he argues that Frege's function-argument model of analysis is in accordance with (MAT) but not with (UAT). His argument

[299] Dummett (1991b, p. 192). According to Dummett's formulation of the Part – Whole Compositionality Principle, the *Thought* expressed by a sentence is a whole whose parts are *senses* expressed by the words in that sentence.

[300] Frege (1892b, (*FR* p. 188)). Almost same sentence is repeated in Frege (1897 (*FR* pp. 243-244): "Let us never forget that two different sentences can express the same thought."

[301] Dummett's (1981b, pp. 296-297) second pair of theses are

B1 The senses of the parts of a sentence are parts of the thought expressed by the whole.

B2 A thought is built up out of its constituents, which correspond, by and large, to the parts of a sentence expressing it.

See also Dummett (1991a, p. 302).

is that since sentences can be decomposed in a variety of different ways, (MAT) implies that each content admits distinct decompositions, and none of these decompositions is intrinsically prior to the others. However, Dummett denies that Frege should be interpreted as accepting ($Comp_{FA}$), for he holds that it conflicts with ($Comp_{PW}$). [302] Moreover, he supplies his reasoning by firmly committing Frege to ($Comp_{PW}$). [303] In this regard, if there is compatibility between these principles of compositionality, then it must be coherent to hold that a function can be part of a *Thought*. Nevertheless, Dummett rejects this view on two grounds. First, given the assumption that ($Comp_{PW}$) implies (UAT), then Frege cannot endorse both Compositionality Principles, since it will contradict with (UAT).[304] Dummett's second ground does not depend on the incompatibility of compositionality principles. Instead, it is implied in general considerations regarding functions. Accordingly, he argues that one cannot hold that the value of a function for its argument has that function and argument among its parts. We see that Dummett distinguishes two theses as *analysis* and *decomposition*. Dummett concludes that there is no inconsistency in Frege's acceptance of both theses, since analysis conciliates (UAT) and ($Comp_{PW}$), and decomposition conciliates (MAT) and ($Comp_{FA}$). [305]

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[302] Dummett (1981b, p. 482). See also Baker and Hacker (1984, p. 331). However, Geach (1975, p. 149) argues otherwise. Geach accepts ($Comp_{FA}$) and denies ($Comp_{PW}$) by warning that the latter is a bad metaphor, should be charitably expounded but not imitated, for construing *thoughts*.

[303] See Dummett (1981b, pp. 296-297; 1991a, p. 291).

[304] Dummett (1981b, p. 251; 1991a, p.87).

[305] Levine (2002) presents an interesting interpretation of Dummett concerning ($Comp_{FA}$) and ($Comp_{PW}$), and (UAT) and (MAT). Levine (p. 201) argues that both Russell and Frege are committed to ($Comp_{PW}$), although they had different views on propositional constituency. Russell is committed to atomistic mereology, thereby held (UAT) to avoid the Bradley's Regress. Levine (pp. 201-202) holds that Frege can be interpreted as to accept ($Comp_{PW}$), without committing himself to (UAT), but in this case Frege should reject his views on grasping *Thoughts*. Levine (p. 204) does not give further reason but refers to Hodes (1982) which we shall argue below. Levine (pp. 206-207) continues that Russell accepts the view of *decomposition* which Dummett has ascribed to Frege. Thus, for Levine, Russell's first 'method of analysis' corresponds to Dummett's 'analysis'; and Russell's 'analysis by propositional functions' corresponds to Dummett's 'decomposition.' However, contrary

In our formulation of (UAT), it corresponds to what Dummett names *analysis*, and (MAT) corresponds to what Dummett names *decomposition*. However, in this thesis we shall use these two terms interchangeably for simplicity. Bell considers (MAT) as a consequence of ($Comp_{FA}$).[306] Actually, Levine dates back this interpretation to Geach who argues that the second thesis is in complementation of the function–argument analysis of Frege and it fits better in the frame of (MAT) than the ($Comp_{FA}$). [307] Thus, agreeing with both Geach and Bell, we shall accept that ($Comp_{FA}$) immediately commits Frege to acceptance of (MAT).

6.3.1. The Dummett-Bell Problem

We shall begin with Bell's three interpretations of (MAT).[308] First interpretation is that "[Frege's] remark is true in the weakest possible sense, i.e., insofar as different sentence tokens of the same type can express the same thought […] this claim is so weak as to be unobjectionable."[309] Second interpretation considers two sentences of different types that have the same linguistic structure or logical form and their difference is due solely to tone or coloring. For example, consider the list of following pairs of sentences, all of which express the same thought.

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to Dummett's interpretation, Levine argues that such distinctions cannot be drawn to interpret Frege's Compositionality Principles, thereby incorporating both principles into Frege's theory of *Thoughts.*

[306] Bell (1987, p. 41) "Thesis 3: Every unambiguous sentence has a unique function-argument analysis." See Bell (1987, pp. 41-43; 1996, p. 584).

[307] The reason is that "what corresponds in the realm of sense to an incomplete expression [such as a predicate] ... [is] a function with senses as its values and senses as its arguments." (Geach, 1975, p. 150). Cited from Levine (2002, p. 198)).

[308] See Bell (1987, p. 44).

[309] Bell (1987, p. 44).

1A: A [310]

1B: A ∧ A

1C: A [311]

1D: ¬¬ A

2A: A and B [312]

2B: A, but B

3A: Alfred has not come. [313]

3B: Alfred has not come yet.

4A: Frederick the Great won the battle of Rossbach. [314]

4B: It is true that Frederick the Great won the battle of Rossbach.

5A: Caesar conquered Gaul. [315]

5B: Gaul is conquered by Caesar.

Bell concludes that both interpretations are not problematic for (UAT) and (MAT). However, the third interpretation is in conflict with (UAT), and this is the heart of the problem. For Bell, this interpretation is illustrated by the following pairs of sentences which according to Frege, express the same thought.[316]

6A: Line *a* is parallel to line *b*. [317] *a* // *b*

6B: The direction of *a* = the direction of *b*. *dir* (*a*) = *dir* (*b*)

[310] Frege (1923 (*CP* p. 393, n. 21; p. 404)).

[311] Frege (1919b (*FR* p. 360)) and Frege (1923 (*CP* p. 399)).

[312] Frege (1918a (*FR* p. 331)). See also Frege (1923 (*CP* p. 393)).

[313] Frege (1918a (*FR* p. 331)).

[314] "There are not two different acts of judgement, but only one." Frege (1897 (*FR* p. 242)).

[315] According to Frege, active and passive forms of a sentence definitely express the same thought. "The grammatical categories of subject and predicate have no significance for logic" Frege (1897 (*FR* p. 242)). *Cf*. Frege (1893b (*FR* p. 188); 1918 (*FR* p. 331)).

[316] Bell (1987, pp. 45-46; 1993, p. 587) also gives sentence pairs of the first-order and higher-order quantificational forms. We shall consider only atomic sentence pairs.

[317] Frege (1884 (*FR* p. 111)).

We shall interpret the above pairs of sentences in the light of the definition of isomorphism. The sentence pairs 6A and 6B are not isomorphic, yet Frege conceived these sentences as having the same conceptual content. [318] The example is sufficient to formulate tension between these theses. Thus, we are now justified in claiming that there are pairs of non-isomorphic sentences expressing the same thought. This result implies (MAT). [319] Agreeing with Bell, it is undeniable that Frege held both theses, and each thesis has sufficient intuitive plausibility.

6.3.2. A Critical Review of the Proposals of Solutions

In this part, we shall survey recent proposals of solutions to the Dummett-Bell Problem. We can list four possible analytic strategies for a plausible solution. First strategy is to reject both (UAT) and (MAT). However, this strategy is implausible for there are abundant textual evidence from Frege writings that he continuously emphasizes both theses. Hence, there is no plausible interpretation for simultaneously rejecting both theses from Frege's theory of thoughts. The second strategy is to accept (MAT) and reject (UAT). However, (UAT) has indeed a very central and fundamental importance in Frege's philosophy including his theory of *Thoughts*. Thus, we shall discard the first two strategies as they are inapplicable in

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[318] See Frege (1884 (*FR* pp. 111-112)).

[319] We can briefly show below the formal inconsistency between (UAT) and (MAT). Let $\mathcal{T}$ be a thought expressed by non-isomorphic pairs of sentences S_1 and S_2 in virtue of (MAT).

1.	$\mathcal{T}$ is an (unambiguous) thought	by (MAT)
2.	(i) $\mathcal{T}$ is expressed by sentences S_1 and S_2, and (ii) S_1 and S_2 have different structures	by (MAT)
3.	$\mathcal{T}$ is isomorphic to S_1 ($T \approx S_1$)	2 (i), by (UAT)
4.	$\mathcal{T}$ is isomorphic to S_2 ($T \approx S_1$)	2 (i), by (UAT)
5.	S_1 and S_2 are non-isomorphic $\neg (S_1 \approx S_2)$	2 (ii)
6.	$\mathcal{T} \approx S_1 \,\&\, \mathcal{T} \approx S_2 \,\&\, \neg (S_1 \approx S_2)$	3, 4, 5
7.	$\mathcal{T} \approx S_1 \,\&\, \mathcal{T} \approx S_2$	6
8.	$S_1 \approx S_2$	7, by transitivity of isomorphism
9.	$\neg (S_1 \approx S_2)$	6

Q.E.D.

Thus (UAT) and (MAT) are contradictory to each other.

Frege's semantic theory. Furthermore, there are no proponents for these two strategies. In the following parts we shall analyze the rest of the possible strategies.

6.3.2.1. Michael Dummett and the Principle K

We shall begin with considering the third possible strategy, i.e. accept (UAT) and reject (MAT). This is Dummett's solution. He endorses the view that the (UAT) is central to Fregean doctrine of *Thoughts*. [320] Contrary to Bell, Dummett offers the following argument for the falsity of (MAT). (MAT) is incompatible with the compositionality of *Thoughts*, since in the absence of this concept, one cannot grasp the thought expressed by a sentence without grasping its constituent senses. Therefore, (MAT) is incompatible with the so-called Principle K endorsed by Dummett[321]:

(Principle K) If one sentence involves a concept that another sentence does not involve, the two sentences cannot express the same thought or have the same content. Dummett's argument essentially depends on two claims which are expressed as follows: [322]

(i) (MAT) is incompatible with Principle K,

and

(ii) Principle K is itself 'compelling'.

Dummett applies Principle K to above mentioned pairs of sentences. Indeed, Principle K entails that 6A cannot be synonymous with 6B, and similarly for 8A with 8B, and 9A with 9B. Dummett defends Principle K and its application to all pairs.

[320] Dummett (1989, p. 1) renames (MAT) as Thesis T.

[321] Dummett (1989, p. 1).

[322] See Bell (1996, p. 588).

> [Frege] was not in a position to repudiate Principle K; for it is implicit in his frequently repeated thesis that the sense of a part of a sentence is a part of the thought expressed by the sentence as a whole: if that does not mean that a grasp of the sense expressed by the constituent part is a necessary condition for grasping the thought, it means nothing at all. Nor could he deny that Principle K applies to pairs: for he expressly contends in Section 64 of [Grundlagen] that by means of the transition from the first member of the pair to the second "we attain a new concept" –that of a direction or of a number. [323]

Dummett explains the reason for this problem in Frege's theory as follows: The synonymy of the first and second members of each of the sentence pairs is guaranteed by Criterion R, which is Frege's criterion for synonymy.

> (Criterion R) Anyone who grasps the thought expressed by each of a given pair of synonymous sentences must immediately recognize one as true if he recognizes the other as true.

According to Dummett, Criterion R is neither a necessary nor sufficient condition for synonymy. [324] If two sentences do not satisfy Criterion R, then they cannot be synonymous. However, two sentences can satisfy Criterion R but they still may not be synonymous. Now, we can summarize Dummett's argument by stating that Principle K implies (UAT). He states that Principle K and Thesis (II) are directly incompatible. [325] He holds Principle K and rejects (MAT). The isomorphic sentence pairs violate Principle K, hence non-isomorphic sentence pairs *a fortiori* violate Principle K.

6.3.2.2. Bell's Reply to Dummett and His Revised Solution

Bell's initial solution in his first paper where he posited the problem, is to discard the requirement that there be "a univocal notion of thought" but he gives no

[323] Dummett (1989, p. 4).

[324] Dummett (1989, pp. 5-7).

[325] Dummett (1989, pp. 10-11).

reason for this solution.[326] He merely refers to his book [327] in which he was "distinguishing two quite different notions of thought (or sense)." [328]

In his second paper, Bell replies to Dummett and he concludes that both theses are indispensable for Frege's theory of *Thoughts*. As a result, he keeps both theses but modifies them – so that they refer to two different kinds of *Thoughts* – although he admits that this solution still poses serious problems for Frege's theory.

Bell agrees with Dummett in acceptance of (UAT) and gives two important intuitions for acceptance of this thesis. The first intuition is related to isomorphism: "If a given sentence is an exact expression of a certain thought, then something in the thought must be performing a role comparable to each essential role performed by an element in the sentence."[329] Hence, isomorphism between sentences and thoughts are inevitable. The second intuition is related to communicability of *Thought*s. The possibility of communication would be unintelligible if thought and language were not internally related one to the other.

According to Bell, (MAT) is related to the nature of *Thoughts* rather than the *senses* of linguistic expressions. "[(MAT)] is confirmed by the phenomenological evidence; it is required if we are to make sense of concept formation by 'transformation'; and it avoids the mistake of taking function/argument analysis to reveal intrinsic structure." [330] Contrary to Dummett, Bell also accepts this thesis. According to Bell, the possibility of transformation seems to require the truth of (MAT). Bell objects to Dummett's proposal, for Principle K is "strategically questionable". [331] As a result, he also rejects that Principle K is implied by (UAT).

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[326] Bell (1987, p.46).

[327] Bell (1979).

[328] Bell (1987, p.46 n.6).

[329] Bell (1996, p. 585).

[330] Bell (1996, p. 596).

[331] See Bell (1996, p. 588).

Bell concludes that both theses are indispensable for Frege's theory of thoughts. As a result, Bell keeps both theses by introducing two different notions of thoughts: (i) structured thought satisfying (UAT) and (ii) unstructured thought satisfying (MAT).

> The resolution I propose requires us to give up neither thesis; but it does require us to construe Thesis [A] as a claim about the senses of sentences and the nature of linguistic understanding, while Thesis [B] will concern the nature of thoughts and their relation to the language in which we express them. Construed in this way, both theses turn out to be true. [332]

Bell admits "the consequence that thoughts no longer have a determinate, intrinsic structure."[333]

We shall now focus on two views against unique composition of thoughts, viz. that of amorphous thoughts and that of polymorphous thoughts.

6.3.2.3. Amorphous Thoughts

The first view against the unique composition of *Thoughts* is called *the amorphousness doctrine*. Kemmerling is a proponent of this view. Kemmerling's main thesis is that "[f]or Frege, a thought proper is an amorphous entity, but one which can be decomposed, into more than one way, into parts." [334] On the one hand, he argues for thoughts as intrinsically unstructured entities.[335] On the other hand, he admits that Frege accepted the view that *Thoughts* can be conceived as structured, and interestingly he exposes some textual evidences.[336] Kemmerling's handling of this dichotomy is simple: Fregean *Thoughts* are *amorphous*. In order to show that

[332] Bell (1996, p. 594).

[333] Bell (1996, p. 596).

[334] Kemmerling (2010, p. 165).

[335] Kemmerling (2010, p. 166).

[336] See Kemmerling (2010, pp. 182-186).

thoughts are amorphous entities, he starts with defining the following two identity criterions for thoughts: [337]

(Equipollence Criterion) Sentences express the same *Thought* if and only if acceptance of one sentence commits us to acceptance of the other (provided that they are both understood).

(Thought-Parts Criterion) Two sentences express the same *Thought* only if their corresponding components express the same *sense*.

Contrary to Dummett, Kemmerling argues that the Equipollence Criterion is a necessary and sufficient criterion for identity of thoughts. Kemmerling also argues that the Thought-Parts Criterion is only a necessary criterion for thought identity. We see that the Equipollence Criterion and the Thought-Parts Criterion are respectively analogous to Dummett's Criterion R and Principle K mentioned above. The following problem emerges from these two criteria which leads to a problem similar to Bell's.

7A Socrates is wise. [338]

7B Wisdom characterizes Socrates.

Kemmerling gives the following four claims related with the example 7A and B which is attributed to Frege: [339]

(i) An unambiguous sentence expresses exactly one *Thought*.

(ii) There are non-synonymous univocal sentences which express the same *Thoughts*.

(iii) *Thoughts* consist of parts.

[337] Kemmerling (2010, p. 169)

[338] See Ramsey (1931, p. 116). Ramsey formulates this statement as 'Wisdom is a characteristic of Socrates.'

[339] Kemmerling (2010, p. 173).

(iv) There are different correct methods of decomposition of the *Thoughts*.

Kemmerling uses the above stated sentence pair 10A and 10B to show inconsistency of the set of four claims listed below and analyses the instances of each claim respectively. Accordingly, he considers rejection of each claim, then attributes a special doctrine to each one. Kemmerling holds both theses and solves the Dummett-Bell problem by rejecting the unique structure of thoughts.

According to the first claim, the above stated unambiguous sentence pairs 10A and 10B express *exactly one* thought. This claim is definitely correct. The rejection of the first claim is called *the indeterminacy doctrine*. [340] The indeterminacy doctrine would lead to the absurd result that every sentence expresses thought only *relative* to decomposition. Since there can be infinitely many different models of correct decomposition, an unambiguous sentence may express infinitely many different thoughts. However, Kemmerling finds this claim unacceptable. The second claim states there are different and non-synonymous sentences which express the same thought. we have added the qualification "synonymous", because otherwise the second claim would be *trivially* true. According to the second claim the above stated sentences 7A and 7B express the same *thought*. Indeed, this claim is in accordance with the equipollence criterion. The rejection of the second claim is called *the super-determinacy doctrine*. [341] To conclude, rejection of the first or second claim is both unreasonable and lacks textual support from Frege's writings. Therefore, both doctrines are rejected. The third claim states that *Thoughts* consist of parts. According to the third claim the *Thought* expressed by the sentences 10A and 10B consist of parts. The rejection of the third claim is called *the amorphousness doctrine*. Kemmerling finds the rejection of the third claim reasonable.

[340] Kemmerling (2010, p. 174). Of course, in this context we leave out pragmatic factors.

[341] According to this doctrine, except for strictly synonymous ones, two sentences could not express the same thought. However, Frege frequently gives non-synonymous sentence pairs that express the same Thought, such as pairs of sentences given above. See Kemmerling (2010, p. 175).

> All correct methods of decomposition assign the same thought to a sentence. The thought expressed does not consist of thought-parts, yet it is decomposable into thought-parts. [342]

> Correspondingly […] decomposing a thought into a completing and unsaturated part, however it is done correctly, does not affect the identity of thought. [343]

The fourth claim states that there are different methods of decompositions of the sentences 7A and 7B all of which are acceptable. Indeed, multiple decomposition of the same thought was exemplified above. The same case is also valid for the sentences 7A and 7B. To conclude, Kemmerling adopts the amorphousness doctrine against unique composition of *Thoughts*. Hence, Fregean doctrine of *Thoughts* can endorse both amorphousness doctrine and different correct methods of decompositions without any problem.

6.3.2.4. Polymorphous Thoughts

The second view against the unique composition of *Thoughts* is called *polymorphous structure of Thoughts*. In other words, a sentence can express one and the same thought by different ways of decomposing it to its elements. Harold Hodes is one of the proponents of this view. In his article, he construes a similarity between Carnapian intensional isomorphism and Fregean isomorphism between sentence and *Thought*.[344] Hodes argues that "Fregean thoughts are compositionally polymorphous, that a single thought may be built up in different ways out of different constituent senses." [345] Hodes' view is based on the following reasoning: If the compositional polymorphism of *Thoughts* is rejected, then this rejection will lead to both inescapable proliferation of different *Thoughts* and certain ambiguities in sentences expressing the same thoughts. Hodes focuses on different analyses of

[342] Kemmerling (2010, p. 178).

[343] Kemmerling (2010, p. 180).

[344] See Hodes (1982, p. 161).

[345] Hodes (1982, p. 162).

the same sentence in which each has different saturated-unsaturated *sense* parts. If there is no polymorphous composition, then each of these *sense* parts of the very same sentence will constitute different thoughts.[346] Hodes considers this point to argue against the unique structure of *Thoughts*. Hodes argues that if two different analyses can correspond to a unique and single analysis of the same thought, and if we accept that all the analyses are equally plausible, then this will lead to polymorphous structure of *Thoughts*. [347] Hodes also remarks that in Frege's writings there is no textual evidence to provide a decision mechanism to select the unique analysis of sentences. If one cannot decide which analysis to be the unique composition of *Thoughts*, Hodes concludes that we are enforced to accept that *Thoughts* are structurally polymorphous. [348] In the following section, we will exemplify possible corresponding analyses of Frege's atomic *Thoughts* and also establish the unique composition of atomic *Thoughts*, contrary to Hodes.

6.3.2.5. Penco and Different Conceptions of Sense

Carlo Penco gives a systematic analysis of the inconsistencies between (UAT) and (MAT). According to Penco, these theses cannot consistently support the Fregean notion of *Thought*.[349] To begin with, Penco does not agree with Dummett in his solution to the problem which was to reject (MAT). He does not agree with Bell's solution either, because "[Bell's solution] cuts too neatly between thought and sense, transforming thought into an unstructured element of the mind and sense into linguistic meaning." [350] His solution is that Frege accepts both theses but as referring to two different conceptions of *senses* of sentences. The first

[346] *Cf.* Frege (1891a (*FR* p. 140)).

[347] Hodes (1982, p. 166).

[348] Hodes (1982, pp. 167-168) Hodes concludes that construing thoughts structurally polymorphous is indeed essential for Frege's project about formal languages, such as foundations of arithmetic, in particular for equinumerosity. See Frege (1884, (*FR* pp. 116-117)).

[349] Penco (2003, p. 87).

[350] Penco (2003, p. 80).

conception of *sense* constitutes the truth conditions, and the second conception of *sense* constitutes cognitive significance of the sentences. [351] For example, the sentences "Lewis Carroll is Lewis Carroll" and "Lewis Carroll is Charles Lutwidge Dodgson" have the same *sense* according to the first conception, whereas they have different *senses* according to the second conception. Indeed, both sentences have the same truth condition, but they have different cognitive significance. Since Fregean *Thoughts* are the *senses* of sentences, each different conception of *sense* leads to a different thesis about Frege's theory of *Thoughts*. According to Penco, the first conception of *sense* leads to (MAT), and the second conception of *sense* leads to (UAT). [352] All of the sentence pairs have the same truth condition; they express the same *Thought* with respect to first conception of *sense*. Nevertheless, they express different *Thoughts* with respect to second conception. As a result, the same notion of *Thoughts* cannot support both theses. Therefore, according to Penco, Frege has two different theses about *Thoughts* and both of them are inseparable from Frege's theory.

6.4. The Structure of Atomic Fregean *Thoughts*

In the previous section, we have considered the Dummett-Bell Problem and analyzed some proposals for solutions. In this section, we shall begin with a critical evaluation of the proposed solutions and then present our view concerning the structure of *Thoughts*.

We argue that none of the solutions provide an authentic account for Frege's theory of *Thoughts*. The general consensus between the aforementioned philosophers is the agreement upon the importance of (UAT), however they differ on their respective views about (MAT). Although such approaches to modifying (MAT) is intuitively plausible and a promising way of solving the Dummett-Bell problem, we are not in total agreement with these modifications, altering Frege's

[351] Penco (2003, pp. 87-88). *Cf.* Penco (2013).

[352] Penco (2003, p. 88).

original views regarding the structure of *Thoughts*. In order to stay as true as possible to Frege's writings, we shall present our own account without discarding or modifying both (UAT) and (MAT). Without (MAT) we can make little sense of the transformations used by Frege to introduce specific concepts, such as the concepts of abstract objects. Moreover, (MAT) is important for the communication of *Thoughts* in terms of their expressibility by different sentences.

Among the applicable strategies considered in the last section, we first considered Dummett's solution in which he endorsed the view that (UAT) is central to Frege's theory of *Thoughts*. Since the isomorphic sentence pairs, 1A-B, 2A-B, 3A-B, 4A-B, 5A-B, and 6A-B violate Principle K, Dummett rejects (MAT) on the grounds that (MAT) is incompatible with Principle K which is an indispensable principle of Frege's Theory of *Thoughts*. Dummett diagnosed the origin of the problem in Frege's criterion for synonymy, the Criterion R. Thus, according to Dummett, the lesson to be learned is that there needs to be a more satisfactory principle in Frege's account that satisfies both necessary and sufficient conditions of synonymy. Then, the question becomes whether such a principle of synonymy is definable in Frege. As we have argued in Chapter 4, we hold that the notion of synonymy, i.e., sameness of meaning, has to be a prior principle due to our ontological commitments. Therefore, the answer to the question regarding the definability of synonymy in Frege's semantic theory would lead to an impasse. In addition, Principle K that Dummett attributes to Frege in Frege's works to establish the conceptual relations between notions in formal languages, such as arithmetic. It is perfectly reasonable that it would not apply to natural languages in the same manner as Dummett anticipates. The strict nature of this principle would make it almost impossible to find sentences that express the same *Thoughts*. Hence, Principle K itself is not as compelling when applied to natural languages as opposed to formal languages.

Bell also argues against Dummett's Principle K and concludes that both theses are indispensable for Frege's theory of *Thoughts*. Bell's solution to the problem is to reject the determinate and intrinsic structure of *Thoughts*. Bell keeps both theses by introducing two different notions of thoughts. On the one hand, a

structured notion of Thoughts satisfying (UAT), on the other hand an unstructured notion of Thoughts satisfying (MAT). We see that this construction is not compatible with Frege's commitment to the Compositionality Principles regarding *Thoughts* because of their determinate and intrinsic structure. Rejecting the view that *Thoughts* have intrinsic structure would lead to an unattainable view for Frege that states *Thoughts* are not composed of sense parts. Even in Bell's examples, one can see that there are corresponding sense parts of the sentences decomposable by either function-argument or part-whole principles of compositionality. To conclude, we reject Bell's view on the grounds that his view seems un-Fregean at least for the structure of *Thoughts*.

Penco's view is similar to Bell's, but he diagnoses the roots of the Dummett-Bell Problem in Frege's conception of *sense*. Penco states that there are two different conceptions of *sense* in Frege. As a result, the same notion of *Thought* cannot support both (UAT) and (MAT). Penco interprets Frege's semantic theory to have two conceptions of *senses* with different concerns. The first conception of *sense* constitutes the truth conditions, and the second conception of *sense* constitutes cognitive significance of the sentences. The former leads to (MAT), and the latter leads to (UAT). Penco's first conception has its origins in Frege's early writings in which Frege considered meanings of linguistic expressions as *denotations,* i.e., truth values. Later, Frege's ontology has divided the meaning of linguistic expressions into the realm of sense, and the realm of denotation. This is indeed one of the central aims of Frege's theory to build a bridge between these two different realms. In this regard, the *senses* expressed by sentences determine their truth values. However, the correspondence between Penco's first conception of *sense* and (MAT) results in a coarse-grained individuation condition for *Thoughts,* since different sentences expressing different *Thoughts* can have the same truth values. The components of sense that Penco distinguishes are in fact taken together in Frege's mature writings. Therefore, we conclude that these different notions of sense cannot be attributed to Frege's theory of *Thoughts*.

Kemmerling argues for the inconsistency of both theses and solves the Dummett-Bell Problem by attributing amorphous structure to *Thoughts*. We agree

with Kemmerling concerning the accountability and correctness of the claims (i), (ii), and (iv). Contra Kemmerling, we do not agree with the rejection of (iii) which states that *Thoughts* consist of parts. As a matter of fact, rejection of (iii) is incompatible with endorsement of (iv) which shows Frege's strict commitment to the compositionality principles. The conclusion Kemmerling reaches by holding both theses is that the structure of Thoughts is amorphous. Nonetheless, he fails to give a comprehensive and convincing account of what is inherent to an amorphous structure, if there is such a structure at all.

In order to provide a satisfactory account for the structure of Fregean *Thoughts*, we have to give an independent explanation for the identity of sense parts of different sentences.

It appears that all of the positions we have considered to solve the problem at hand fail in one way or another to provide this satisfactory account. We aim to solve the Dummett-Bell Problem with a similar approach to that of Hodes. In what follows the following chapter we argue for the polymorphous structure of *Thoughts*. While doing so, we shall explicate our differences with the position defended by Hodes.

What the Dummett-Bell problem shows is that the same atomic *Thought* cannot be analyzed by multiple structures and it poses a serious threat to Frege's thesis if *Thoughts* do not have a determinate inner structure. However, contrary to this point, Frege puts forward that the same atomic *Thought* can be decomposed in different ways:

> If several proper names occur in a sentence, the corresponding *Thought* can be analyzed into a complete part and an unsaturated part in different ways. The *sense* of each of these proper names can be set up as the complete part over against the rest of the *Thought* as the unsaturated part.[353]

[353] Frege (1897 (*PW* p. 192); *cf.* 1903 (*CP* p. 281)). Frege further considers natural language sentences containing more complex constituents, such as logical connectives, quantifiers, and second-level concepts and argues that these sentences have similar decompositional structures:

> Language has means of presenting now one, now another part of the *Thought* as the subject; one of the most familiar is the distinction of active and passive forms. It is thus not impossible that one way of analysing a given *Thought* should make it appear as a singular judgment; another, as a particular judgment; and a third, as a universal judgment. It need

Moreover, if we carefully examine Frege, we can see that he exemplifies sentences having more than one object combined with a relation. Let us now analyze Frege's sentence 'Jupiter is larger than Mars' which he gives after the above quotation. (*PW* 192). There are three different analyses, all of which obeys the saturated-unsaturated distinction:

(i) The saturated sense of 'Jupiter' and the unsaturated sense of 'ξ is larger than Mars'

(ii) The unsaturated sense of 'Jupiter is larger than ζ' and the saturated sense of 'Mars'

(iii) The saturated sense of 'Jupiter', the saturated sense of 'Mars', and the unsaturated sense of 'ξ is larger than ζ'.

The analyses (i) – (iii) may seem different at first sight. However, in our view, what these different analyses show is that there are three different possible ways of analyzing or decomposing the structure of a *Thought*. Accordingly, we can notice that if the analyses (i) and (ii) are carried one step further, then we obtain the same *sense* constituents just as the analysis (iii). Since all of the *sense* constituents – for both saturated and unsaturated parts– of the last analysis are simple, i.e., they cannot be decomposed any further, then we reach the ultimate *sense* constituents by alternative analyses of the same atomic *Thought*. Therefore, different analyses of the same *Thought* express the same ultimate *sense* parts which in turn commits us to the view that Fregean *Thoughts* have polymorphous structure.

As we have considered throughout this thesis, Frege makes similar remarks in various discussions. Frege indeed aimed to analyze and identify the basic elements or constituents to explicate relationships between saturated and

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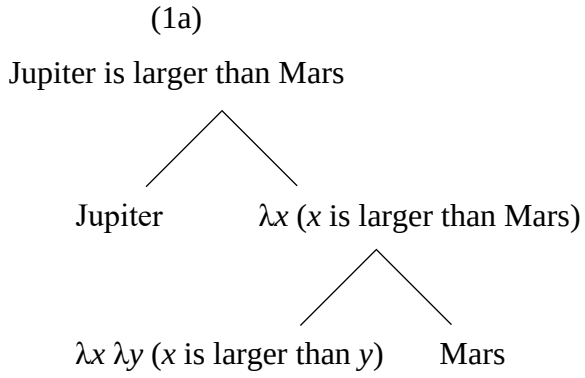
> not then surprise us that the same sentence may be conceived as saying something about an object; only we must observe that what is being said is different. (Frege, 1892b (*CP* pp. 188-189))

Another example Frege gives of a proposition that can be decomposed in different ways is, "Christ converted some people to his teachings." (Frege, 1906a (*PW* p. 187)). Nevertheless, since the framework of our thesis is limited to atomic *Thoughts*. It must be noted that the number of different analyses increases greatly for the level of complexity escalates.

unsaturated parts of the *Thought* structure. In this respect, accepting the polymorphous structure of *Thoughts* will become even more reasonable if we consider semantic variations in natural languages expressing the same *Thought*. For this reason, it is an undeniable fact that appealing to the sameness of meaning, i.e., synonymy, provides a strong motivation to explicate the commonalities between the sentences having the same meaning.

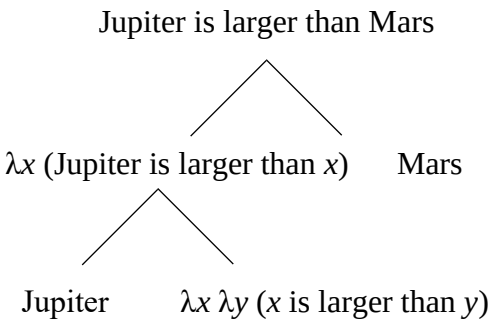
We shall now consider the representation of the polymorphous structure of *Thoughts*. We shall explicate the compositional structure of atomic *Thoughts* by considering Frege's view that the unsaturated, i.e., predicative or functional, component of a sentence is obtained when one or more saturated constituents have been removed. Following Hodes[354], we shall employ λ-conventions as a device for predicate abstractions in order to represent the function-argument structure of *Thoughts*. We can construe decomposition trees showing the polymorphous structures of the *Thought* expressed by the following sentence:

(1) Jupiter is larger than Mars.

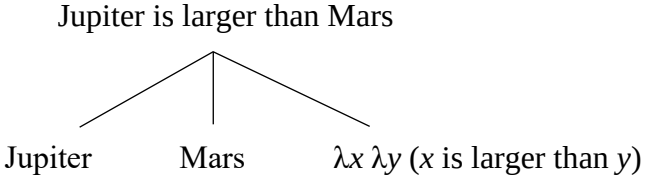


[354] *Cf.* Hodes (1982, pp. 163-165).

(1b)



(1c)



Note that the last line of each decomposition tree expresses the same ultimate saturated and unsaturated *Thought* parts. Naturally, there are further similar constructions by *alternative* sentence forms such as 'Mars is smaller than Jupiter' expressing the same *Thought*. These alternative sentences can be construed by constructions similar to (1), nevertheless all of them express the same atomic *Thought* according to Frege.

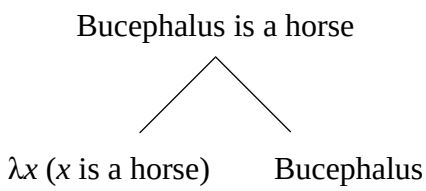
At first sight, there is a problem with the possibility of constructing indefinite number of different constructions of the *same Thought*. The question that demands an answer is, exactly how many polymorphous structures can be constructed? It is quite hard, if not impossible, to answer this question. In fact, we hold that no given answer can be complete because of the variations of sentential expressions in natural languages are infinitely many. Moreover, assuming that natural languages are also evolving, their complexity increases, leaving the question unanswered.

Now we shall consider the apparently simple atomic *Thought* expressed by the sentence

(2) Bucephalus is a horse

We can construe its structure analogous to (1) by the following construction trees:

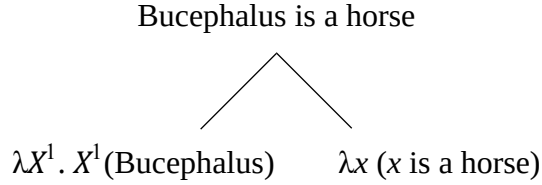
(2a)



(2a) construes an analysis of the sentence (2) into the saturated proper name part 'Bucephalus' and the unsaturated predicative part 'λ*x* (*x* is a horse).'

Hodes further gives two distinct[355] analyses of (2) in the context of his polymorphous structure of *Thoughts* by corresponding tree constructions (2b) and (2c):

(2b)



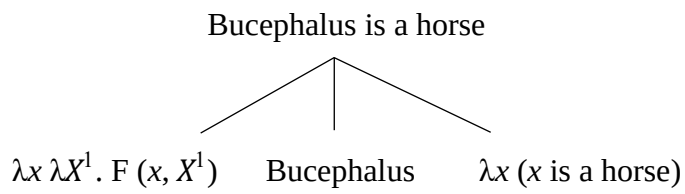
(2b) construes an analysis of the sentence (2) involving a second-order concept '$\lambda X^1 . X^1$(Bucephalus)' to the first-order concept 'λ*x* (*x* is a horse)'. Hodes justifies this construction by Frege's commitment "to a curious reduplication of simple concept-object predications into all levels of the type hierarchy."[356] In this respect, the sentence (2) is analyzed by considering its saturated part Bucephalus as the quantifier after removing 'λx (x is a horse)' from the sentence 'Bucephalus is a horse.'

[355] *Cf.* Hodes (1982, pp. 168-170).

[356] Hodes (1981, p. 167).

Now simple as it may seem, the atomic *Thought* expressed by the sentence 'Bucephalus is a horse' is analyzed into distinct constructions in the type hierarchy.[357]

(2c)



(2c) construes the concept *being a horse* as the binary relational function '$\lambda x\, \lambda X^1 .\, F\, (x, X^1)$'. [358] In other words, it relates all objects falling under the class of all horse objects to the object 'Bucephalus' and the function 'λx (x is a horse).' [359]

We shall argue against Hodes on three points. To begin with, Hodes introduces and appeals to λ-notation to represent predicate abstraction. He states that "[Frege's own notation] can represent all needed distinctions within Frege's realm of sense. The distinctions in the realm of [denotation], for which we'll need predicate abstraction, do not correspond to analogous distinctions in the realm of sense." [360] However, this is not correct. First, Frege has never explicitly given such an account for predicate abstraction in the realm of sense. Hodes gives no justification for this point. Second, in our view, the use of predicate abstraction in the realm of denotation is not applicable simply because there is nothing more than truth values in the realm

[357] See Hodes (1982, pp. 169-170). Hodes justifies the accountability of these analyses to Frege's paper "Concept and Object".

> Second-level concepts, which concepts fall under, are essentially different from first-level concepts, which objects fall under. The relation of an object to a first-level concept that it fails under is different from the (admittedly similar) relation of a first-level to a second-level concept. (Frege, 1892b (*FR* p. 189)).

[358] The type of the relation under this construction is ⟨0, 1⟩. See Hodes (1982, p. 169).

[359] Hodes (1982, p. 169).

[360] Hodes (1982, p. 163).

of denotation. On the contrary, predicate abstractions must *only* be applied in the realm of *sense* since *Thoughts are the senses expressed by sentences*. Moreover, Hodes himself applies predicate abstractions ambiguously between the realm of *senses* and *denotations* in the text.

The second point is related to the number of polymorphous structures permissible in Frege's theory of *Thoughts*. [361] Hodes favors polymorphous composition of sentences; however, this enforces us to admit that there are infinitely many decompositions since there would be infinitely many possible *sense* parts in one simple atomic *Thought*. Hodes seems to admit infinity of different *Thought* compositions but unless the individuation conditions of *sense* parts are satisfactorily explicated, it seriously threatens (UAT). One can consider a type hierarchy between concepts. Hodes indeed appeals to the 'shadow' metaphor to argue that the object 'casts a shadow' in each level above the level one.

> [H]ere we depart from the Fregean letter, senses do not present unique entities. Rather, they primarily present one entity, secondarily another entity of different type, and so forth. So the sense expressed by 'Socrates' primarily presents a person, secondarily an entity of level two, etc. The tree of senses determined by the thought expressed by 'Socrates is wise' may be unique; there is no unique corresponding tree of referents, but rather an ordered infinitude of such trees. This cost may seem slight. [362]

However, the cost is not slight but substantial. Frege cannot be attributed such levels of presentation of proper names. For atomic *thoughts* having first-level objects and first-level functions, this approach is indeed problematic, since it will create an unnecessary infinite hierarchy for simple proper names and concepts. Perhaps, the most defective consequence of such an infinite hierarchy approach is Bradley's Regress Problem which poses a serious threat to explain *Thought* unity, just as it was the case in Russell's conception of propositions. If there is no relation sustained between these different types of entities then, since the displayed syntactic forms

[361] Hodes (1982, pp. 167-168).

[362] Hodes (1982, p. 168).

will be distinct in each decomposition, this will eventually result in infinitely many ambiguous forms corresponding to the same *Thought*. [363]

The third point is even more problematic which poses the most serious threat concerning the unity of *Thoughts*. Hodes himself attributes (2b) and (2c) to Frege as distinct decompositions but rejects the legitimacy of (2c). Nevertheless, Hodes is unaware that both decompositions collapse into the paradox of concept *horse*. If Frege considers 'the concept *horse'* denotes objects, then he is committed to the falsity of the claim that 'The concept horse is a concept.' This paradox threatens many aspects of Frege's philosophy. One point to mention for the structure of *Thoughts* is that this paradox eradicates the possibility of conciliating (UAT) and (MAT), in addition to Frege's key semantic and ontological distinctions between sense and denotation, and concepts and objects. In our view, both (2b) and (2c) cannot be attributed to Frege. (Hodes argues that they are Fregean but (2c) fails nonetheless). [364]. We cannot attribute (2b) and (2c) to Frege, for there can be no syntactic correspondence between the levels of saturated objects and unsaturated concepts. It is not possible to explain such correspondence between meanings of two unsaturated functions, i.e., '$\lambda X^1 . X^1$ (Bucephalus)' and 'λx (x is a horse)' of different levels.

In the next chapter, we will investigate the unity of Fregean *Thoughts* and provide our solution to the concept *horse* paradox.

[363] *Cf.* Hodes (1982).

[364] See Hodes (1982, pp. 169-170).

CHAPTER 7

THE UNITY OF ATOMIC FREGEAN *THOUGHTS* AND THE CONCEPT *HORSE* PARADOX

As we have argued in the preceding chapters, Frege puts forward an account of both the structure of atomic *Thoughts* and how its constituents are held together. In the previous chapter, we have presented our view by defending the polymorphous structure of atomic Fregean *Thoughts*. In the first section of this chapter, we will focus on the unity of *Thoughts*. Accordingly, we will consider the relationship between constituents of atomic *Thoughts* and their arrangement to provide the unity of this structure. In the second section, we shall focus on the most important problem concerning the unity of *Thoughts*, namely the concept *horse* paradox. In the third section, we will survey some important solutions to the paradox. In the fourth section, we will revisit Frege's theory of *Thought* by providing our emendations to solve the paradox and defend a satisfactory account for both structure and unity of atomic *Thoughts*.

7.1. The Unity of Atomic *Thoughts*

According to Frege, a mere list of words does not constitute a unity by themselves to form a complete sentence to express a *Thought*. Frege has a sophisticated account of how the constituents of *Thoughts* are held together in order to provide the unity of *Thoughts*. Frege has argued that the saturated-unsaturated distinction applies to both senses and denotations of linguistic expressions. Then, by

applying compositionality principles, Frege establishes the unity of sentences from the corresponding senses and denotations of its parts.

For the unity of sense of the complete declarative sentence, Frege puts forward that these parts are held together in virtue of having at least one of the sense parts that is unsaturated. The unsaturated sense part of a *Thought* corresponds to the predicative part. Accordingly, the sense of a proper name, which has a saturated sense, binds to complete the unsaturated sense of predicates. As a result, a saturated expression is obtained which establishes the unity of a sentence expressing the *Thought*. Let us exemplify the formation of the structural unity of the atomic *Thought* expressed by the sentence "Bucephalus is horse". The saturated sense of the proper name "Bucephalus" binds the unsaturated sense of the predicative part, i.e., the concept-word, "is a horse" thereby forming the unity. Similarly, when we consider the atomic *Thought* expressed by the sentence "Jupiter is larger than Mars", the unsaturated senses of the relation "is larger than" is completed with the saturated senses of "Jupiter" and "Mars". As a result, we obtain the unity of structural composition of *Thoughts* by the procedure of binding the saturated senses to unsaturated sense parts.

We can consider a similar procedure for the unity of denotations of sentences which express truth values. The saturated proper names that denote objects bind to complete the unsaturated part, i.e., concepts that denote functions, thereby establishing the unity of denotation of sentence to express truth values. The difference here is that the Function-Argument Compositionality Principle can be applied for the structural unity of denotations, since Frege has rejected the Part-Whole Compositionality Principle for denotations of sentences. As an example, we can consider the unity of the denotation of "Bucephalus is a horse". According to Frege, the denotations of unsaturated expressions are functions. Hence, the unsaturated predicative part "is a horse" takes denotation of the proper name "Bucephalus" as its argument thereby forms a unity of complex expression which yields the truth-value, the True.

We can see the key importance of the union of asymmetrical elements that provides the unity of the *Thoughts*. Since a sentence is not a collection of proper

names and predicates, a sequence of singular terms and predicates forms the unity of sentences expressing *Thoughts* only when the unsaturated part of the expression is completed by the saturated parts. Frege masterfully describes the senses and denotations of saturated expressions in detail. However, Frege hardly provides any detail concerning the nature of unsaturated expressions. Still, he argues that they are unsaturated in two realms, senses and denotations. Frege explains the unsaturated expressions in the realm of denotation on two levels: At the level of concepts, the denotations of unsaturated expressions are predicates as predicates are used to denote concepts. At the level of predicates, the unsaturated expressions denote functions as functions are used to denote predicates. Nevertheless, Frege does not give an explanation of unsaturated expressions in the realm of sense at all. He only says that unsaturated expressions have unsaturated entities for their *senses*. Frege considers this account in numerous places for the structural composition of *Thoughts*.[365] However, Frege does not further explain what corresponds to the *senses* of these expressions. Thus, there is an explanatory gap when explaining the nature of *sense* functions by Frege's appeal to the saturated-unsaturated distinction. The lack of explanation for the senses of concept-words has resulted in many divergent views concerning how to interpret these expressions. The general convention for this point in the literature is established on Alonzo Church's understanding of *senses* of unsaturated expressions as *sense-functions*. However, there are some conflicting views on the notion of *senses* of functions.[366] We shall address this problem and its implications concerning the unity of atomic *Thoughts*. We shall present our view in the subsequent sections of this chapter by explicating the notion of sense-functions with our emendation on this point, thereby accounting for the unity of *Thoughts*.

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[365] See Frege (1884 §32; 1897 (*PW* p. 151); 1906a (*FR* pp. 187, 191-192); 1906b (*PW* p. 201); 1914 (*PW* pp. 225 and 243); 1918b (*CP* p. 378 and also p. 386); 1919 (*PW* pp. 254-255); 1923 (*CP* pp. 390-391). See also *PMC* (pp. 79-80, 98, 142, 149).

[366] See Church (1946; 1974; 1993), Jackson (1965, pp. 84 - 87), Tichý (1988, pp. 79-82, 98-105), Geach (1976, pp. 440-445), Parsons (1981, pp. 37-57), Baker and Hacker (1984, pp. 324-326), Diller (1993b, pp. 71-79), Klement (2002, pp. 73-76 and Ch. 4; 2010, pp.172-177), Duží et. al. (2010, Ch.2 esp. pp. 148-152). See also Dummett (1981a; 1981b, 1991a), Oliver (2010), Heck and May (2011; 2013).

Frege emphasizes the dichotomy between object and concept. He argues that they are two distinct kinds of ontological categories. Accordingly, he states that "the fundamental difference of objects from concepts that an object can never occur predicatively or unsaturatedly; and that logically, a concept can never substitute of an object." [367] Similarly, the arguments of functions have to be saturated, which means that they cannot denote anything other than objects. However, when we consider the sentence "The concept *horse* is a concept" then the saturated part of this sentence must be "the concept *horse*" as it is the argument of the predicate "is a concept". It is clear that this creates a substantial problem. Frege clearly distinguishes concepts and objects and further argues for the impossibility of using objects in place of concepts and vice versa. Yet, this example clearly demonstrates that we are left with the conclusion that "the concept horse" is not a concept but an object. This paradox is referred to as the concept *horse* paradox and it poses serious questions not only for the concept-object dichotomy but also for the unity of *Thoughts*. As we have discussed, for the unity of *Thoughts* to be achieved, the expression needs both saturated and unsaturated parts. If we are not able to definitively claim whether "the concept horse" is a concept or an object, then the unity is inevitably compromised. We shall investigate this paradox in the following sections.

7.2. The Concept *Horse* Paradox

The concept *horse* paradox (hereafter (HP)) is initially diagnosed by Benno Kerry, a contemporary of Frege. [368] Kerry argues against Frege's distinction between concept and object. According to Kerry, the expression 'the concept *horse*' is a proper name so it must stand for an object. Although, the exact point of Kerry's position against Frege is not clear since it is related to many issues on Frege's

[367] Frege (1903 (*CP* pp. 281-282)).

[368] This paper is written in response to a series of Kerry's articles, two of which particularly criticizes Frege's views. See Beaney (1997, p. 181) and Mendelsohn (2005, pp. 73-74).

semantic theory. Nevertheless, Kerry points out a major flaw in Frege's semantic theory, perhaps comparable to Russell's paradox. Kerry concludes that the distinction between concept and object is not mutually exclusive. As a response to Kerry, Frege admits that 'the concept *horse*' is a proper name, so it must stand for an object, and he rejects Kerry's assumption that the concept *horse* is a concept. Therefore, Frege accepts the truth of the paradoxical statement

(H) The concept *horse* is not a concept.

Frege devotes his paper "On Concept and Object" predominantly to explain this puzzle and argues about it on several points. As a solution, Frege argues that the concept *horse* is not a concept and asks his readers to simply accept the "awkward" consequences of his theory.

> It must indeed be recognized that here we are confronted by an awkwardness of language, which I admit cannot be avoided, if we say that the concept *horse* is not a concept, [Frege's footnote: A similar thing happens when we say as regards the sentence 'This rose is red': the grammatical predicate 'is red' belongs to the subject 'this rose'. Here the words 'The grammatical predicate "is red"' are not a grammatical predicate but a subject. By the very act of explicitly calling it a predicate, we deprive it of this property.] whereas, e.g., the city of Berlin is a city and the volcano Vesuvius is a volcano. Language is here in a predicament that justifies the departure from custom. [369]

Frege puts forward that

> [T]he behaviour of the concept is essentially predicative, even where something is being asserted about it; consequently it can be replaced there only by another concept, never by an object.[370]

His famous remarks were "meet him half-way" and "not begrudge a pinch of salt [*granum salis*]."

> I admit that there is a quite peculiar obstacle in the way of an understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept.

[369] Frege (1892b (*FR* p. 185)).

[370] Frege (1892b (*FR* p. 189)).

> I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way – who does not begrudge a pinch of salt. [371]

However, we shall disagree with Frege's reception of the problem. The paradoxical sentence is quite meaningful and understandable; thus attribution of mere awkwardness to natural languages cannot amount to a solution. In fact, Frege does use concept-words to talk about concepts in his semantic theory.

The paradox arises with respect to the following theses that Frege states about concepts.

(i) The Object - Concept Distinction

Frege considers a fundamental distinction between two types of entities: objects and concepts. He argues that anything that is an object cannot be a concept, and anything that is not a function is an object. Indeed, Frege takes the distinction between objects and concepts to be "a distinction of the highest importance." [372] This distinction is both exhaustive, i.e., one thing is either an object or a concept, and exclusive, i.e., nothing is both object and concept.

(ii) The Saturated - Unsaturated Distinction

Frege attributes both sense and denotations to categories of linguistic expressions. Accordingly, proper names, definite descriptions, and sentences have saturated senses and they denote saturated entities, i.e., objects. On the other hand, concept-words have unsaturated senses and denotations. As a result, an unsaturated expression 'ξ is a concept' can only be saturated by objects, but then the concept *horse* turns out to be an object, not a concept.

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[371] Frege (1892b (*FR* p. 192)).

[372] Frege (1892b (*FR* p. 192), *cf*. p. 183).

(iii) Frege's *Denotation* Principle

Singular terms denote objects, and predicates denote concepts. For Frege, besides proper names and sentences, singular terms also comprise expressions containing the definite article 'the', and they denote objects. In other words, expressions containing definite articles count as an object. [373] Accordingly, if an expression *denotes* something, it denotes an object, then if 'the concept horse' denotes something, then it should denote the concept *h*orse. It follows that the concept *horse* is an object. But again whatever is an object is not a concept. Therefore, the concept *horse* is not a concept. Perhaps most importantly, it has been a problematic case what functional expressions *denote* since the paradox also applies to functions generally. A dyadic predicate, which Frege calls relations, such as 'the relation of an object to the concept it falls under' denotes an object, hence *the relation of an object to the concept it falls under* is not a relation, but an object.

7.3. The Proposed Solutions to the Concept *Horse* Paradox

7.3.1. Geach's Solution

One of the earliest solutions to (HP) is given by Peter Geach.[374] He begins with maintaining Frege's categorial distinction between concept and object. He considers this distinction to be the feature of both linguistic expressions and the reality of language. Therefore, he argues that no solution in terms of semantic ascent, viz., transforming ontological confrontations into semantic confrontations, can provide a solution to the paradox. Geach focuses on Frege's *denotations* principle for functional expressions, and distinguishes two types:[375]

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[373] Frege (1884, §51, see also §57 and the footnotes to §§66 and 67).

[374] Geach (1972, p. 55) focuses on some problems of Frege's semantic theory in the light of Wittgenstein's philosophy. He argues that essential aspects of Wittgenstein's distinction between *saying* and *showing* are present in Frege's writings.

[375] Geach (1972, p. 56).

i) The *denotation* [376] of the predicate ‘ξ loves Mehmet’

ii) The *denotation* of the functor ‘the square of ξ’

Geach considers Frege’s reasoning on this point that predicates *denote* concepts, and functors *denote* functions. However, one can neither attain the first type that the predicate is a concept; nor attain the second type that the predicate is a function. The reason is that the expression in the former clause is not a predicate, and similarly the expression in the latter clause is not a function. Geach concludes that since the grammatical structure of both expressions are definite descriptions for Frege, they must *denote* objects. [377]

Geach examines a special line of reasoning in “On Concept and Object” where Frege distinguished special classes of entities between concepts and functions. Geach then considers the English phrase ‘what ξ stands for’ would be a Fregean *denotation*. [378] He argues that there is a difference between proper names and predicates, namely proper names stand for *whatever* the expression stands for. [379] For instance, ‘that function of 2 which ‘‘the square of’ stands for’ is a long-winged saying of the functor ‘the square of 2’. However, thinking functor *denotation* in this way would be nonsensical, since the expression corresponding to (ii) would be tantamount to saying ‘that function which ‘the square of’ stands for’ cannot be used for a proper name. Geach holds that the same reasoning is also valid for (i). Then, he considers the following sentence

[376] Geach uses the word ‘*Bedeutung*’ for ‘*denotation*’.

[377] Geach (1972, p. 56) says “*Eigenname* [proper name] in Frege’s sense”.

[378] Geach (1972, p. 56).

[379] For this case, Geach (1972, pp. 56-57) considers two cases inserting an English expression inside blank between two parantheses of “what ( ) stands for”. He argues that expressions stands for whatever an English expression or its translation to other languages stands for. In the former case ‘what ‘the capital of Türkiye’ stands for’ is a long substitute for the name of the capital; whereas in the latter case the expression ‘She is what ‘öke’ stands for in Turkish’ will be *long-winged* saying of ‘She is a genius.’

(1) There is a difference between what ‘Ayşe’ stands for and what the predicate ‘ξ loves Mehmet’ stands for

If any (English) expression standing between quotes in

what ‘ξ’ stands for

is to be substituted *salva veritate*, then it would result in the following nonsensical expression:

(2) There is a difference between Ayşe and loves Mehmet

For Geach, this inference is the result of inequity between proper names, predicates and functors that stand alone without their arguments. Accordingly, he applies the same line of reasoning to (H). Geach concludes that (H) is meaningless and cannot even be formulated in Frege’s *Begriffsschrift*. Nevertheless, he argues that sentences of the similar form (1) are of didactic use, especially to understand these languages. Nevertheless, such languages may result in nonsensical consequences when we express (H).

> A solution that is often offered to the difficulty just mentioned is that it comes about from trying to discuss in the object language what ought properly to be discussed in a metalanguage […] Frege’s trouble would be diagnosed as his having thought of all signs as names – a diagnosis confirmed by his use of the term [function name] for functors. If all signs have to be names, each one standing for something, then signs that do not name or stand for objects will have to be assigned some strange non-objects, such as concepts and functions, as their [denotations]. What he ought to have done is to distinguish the different mode of significance of signs; instead he misconstrued these distinctions as difference of ontological category between things names.[380]

To summarize Geach in the context of the paradox, first he considers that ‘the concept *horse*’ is not a proper name, but a predicate. Hence ‘the concept horse’ cannot function as a singular term as the proper name ‘Bucephalus’ does. For Geach, unsaturated expressions, i.e., predicates, cannot be identified separately. “What signifies a function is not the presence of in a formula of a given piece of type, but

[380] Geach (1972, p. 58).

[…] the occurrence of a predicate must be recognized from the occurrence of a pattern, not from the occurrence of a quotable part of a sentence." [381] The second and perhaps more important for Geach is that any attempt to denote functions by use of singular terms will result in nonsense. Since a predicate expression about a concept cannot be eliminated by a singular term, then the sentence would be deemed nonsense. [382] Geach further discusses Frege's logico-semantic theory in the *Tractarian* framework and in the final analysis, Geach argues Wittgenstein's point that "the failure of these utterances to be genuinely propositional could be demonstrated by what [Witttgenstein] calls correct philosophical method." [383]

We shall now present our critical review of Geach's view. First, Geach diagnoses an important problem concerning Frege's *denotation* principle. Geach has argued that the surface form of the expression 'what "ξ is the capital of Türkiye" stands for' looks like a singular term, but this expression is a predicate thus it cannot be used to denote objects. Construed in this way, his diagnosis is correct. Therefore, we have to construct our proposal in a manner to emend the denotation principle in order to block substitution failures.[384] Furthermore, the emendation of this principle is of paramount importance to *denote* singular terms with functional expressions, as Frege explicitly remarks. For the second point, Geach's appeal to the notion of nonsense to express sentences involving concept words, does not prove useful as a

[381] Geach (1972, p. 60).

[382] See Geach (1972, pp. 59-60).

[383] Geach (1972, p. 69). Wittgenstein (1922, 4.126) says the following about in the context of formal concepts:

> When something falls under a formal concept as one of its objects, this cannot be expressed by means of a proposition. Instead, it is shown in the very sign for this object. (A name shows that it signifies an object, a sign for a number that it signifies a number, etc.)
>
> Formal concepts cannot, in fact, be represented by means of a function, as concepts proper can.

See Geach (1972, pp. 64-70). *Cf.* Mendelsohn (2005, pp. 82-83).

[384] See Diller (1993a, pp. 347-348).

solution to the paradox. [385] Additionally, if Frege's view is nonsense, then it would be nonsense to talk about it or work on it. Yet people continue to read and write on this issue which shows that people indeed gather some sense from what Geach considers to be nonsense. Although, it is sometimes unescapable to speak nonsense when we express a philosophical theory accounting for the relationship between language and reality, Geach seems to miss the point that reducing the concept-object distinction to unsayables will not succeed in resolving the paradox. More than the didactic use of (H), we use and understand what Frege explains in his philosophy when he uses terms such as concepts, objects, denotation. As a result, we find Geach untenable on this point.

7.3.2. Dummett's Solution

According to Michael Dummett, the construction of the paradox is the negation of (H), viz., the sentence 'The concept horse is not a concept'. He argues that the sentence cannot be constructed in this way. Similar to Geach, Dummett holds that 'the concept *horse*' is not a proper name. Dummett begins with Frege's solution in "On Concept and Object" that 'the predicate "ξ is a horse" ' is not a predicate. [386] However, this would also be paradoxical in nature, comparable to saying that 'the city of Ankara' is not a city. Dummett then offers an interpretation of Frege's pointing out the "Comments on *Sinn* and *Bedeutung*." [387]

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[385] *Cf.* Diller (1993a, p. 351), and Hale and Wright (2012, pp. 103-104).

[386] Dummett (1981a, p. 212)

[387] This is generally considered as Frege's another solution to paradox:

> Now we have seen that the relation of equality between objects cannot be conceived as holding between concepts too, but there is a corresponding relation for concepts. It follows that the word 'the same' that is used to designate the former relation between objects cannot properly be used to designate the latter as well. If we try to use it to do this, the only recourse we really have is to say, 'The concept *Φ* is the same as the concept *X*' and in saying this we have of course named a relation between objects, [Frege's footnote: These objects have the names 'the concept *Φ*' and 'the concept *X*'] where what is intended is a relation between concepts. We have the same case if we say, 'The [*Bedeutung*] of the concept-word *A* is the same as that of the concept-word *B*'. Indeed we should really outlaw the expression 'the

Dummett argues that the expression 'ξ is a concept' is a *pseudo-predicate*.[388] This expression should be treated as a second-level predicate, thus it would not properly construe an expression of the form 'what the predicate "ξ is a horse" stands for' as a first-level predicate. He discusses this issue similarly to Geach concerning the *denotations* and substitutions of singular terms and predicative expressions. A first-level predicate is formed from a sentence by omitting one or more occurrences of a proper name. Predicates consist of the copula and an adjective phrase with an indefinite article, and predicative expressions are formed by merely dropping the copula. [389] However, Dummett argues that second-level predicates cannot be construed in this way, since incompatible requirements are supposed in these constructions. [390] Dummett considers a construction representing a second-level predicate which has a corresponding empty place filled with a predicative expression that is a second-level predicate:

> … is something which everything either is or is not

The symbolic form of this construction is '$\forall a\ (\Phi(a) \vee \neg\ \Phi(a))$'. The result of completing this predicate with a denoting singular term will always be true. Hence, Dummett says that this will replace the incorrect formulation 'the concept *horse* is

> [*Bedeutung*] of the concept-word *A*' because the definite article before '[*Bedeutung*]' points to an object and belies the predicative nature of a concept. It would be better to confine ourselves to 'what the concept-word *A* stands for [*bedeutet*]', for this at any rate is to be used predicatively: 'Jesus is, what the concept-word "man" stands for [*bedeutet*] is the sense of 'Jesus is a man'. (Frege, 1892c (*FR* p. 177))

Dummett interprets this line of reasoning as follows: What-clauses can be used (i) to *denote* concepts, and (ii) predicatively. For (i) the expression of the form 'what "ξ is a horse" refers to' can be used in place of the expression 'the concept *horse'*. For (ii) the expression of the form 'what "ξ is a horse" refers to' can be used to form sentences "Bucephalus is 'what "ξ is a horse" refers to'" to express the *Thought* that Bucephalus is a horse.

[388] Dummett (1981a, p. 213).

[389] Dummett (1981a, pp. 214-215).

[390] See Dummett (1981a, pp. 216-217).

a concept' with the correct construction: 'the concept *horse* is something which everything either is or is not.' [391]

In the literature, Dummett's solution has been severely criticized. [392] First weakness of Dummett's proposal that appeals to the notion of pseudo-predicates is that it gives a hardly acceptable presentation of functions in Frege's semantic theory. [393] For instance, consider Frege's claim that "everything which is not an object is a function". This claim involves the predicate expression 'ξ is a function'. Dummett banishes such predicates which are of key importance from Frege's theory of *Thoughts* in addition to their essential role to establish the unity of *Thoughts*. [394] Moreover, Priest argues that a further problem arises in this approach when we construe unsaturated predicates in Dummett's framework as '$\forall\Phi$ [$\forall a$ ($\Phi(a) \vee \neg \Phi(a)) \rightarrow \Phi$ is unsaturated]'. The problem with this approach is that 'Φ is unsaturated' is a first level predicate which applies only to objects. [395]

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[391] Dummett (1981a, pp. 216-217). Here, "something" expresses a second-level generality, and "everything" expresses a first-level generality. As a result, pseudo-predicates such as 'ξ is a concept' or 'ξ is a function' should be dismissed, and the paradoxical expression cannot be generated.

[392] See Diller (1993a, pp. 348-350), Priest (1995, pp. 201-203), Wright (1998, pp. 247-251), Mendelsohn (2005, pp. 81-83), and Hale and Wright (2012, pp. 95-98).

[393] *Cf.* Diller (1993a, pp. 348-350) and Hale (2010, p. 143).

[394] Hale and Wright (2012, pp. 95-98) dub Dummett's approach *eliminative paraphrase*. They (2012, p. 95) consider a general method of eliminative paraphrase as needed for paraphrasing out all uses of sortal terms such as concepts, relations, and general statements involving these terms. Hale and Wright stress that apparent uses of first-order quantifications over these sortal terms are needed in order not to generate the paradox. However, difficulties may arise when formulating the Frege's semantic theory without talking in general terms such as concepts, relations, functions etc. and presumably Frege's theory could not even be stated, but the theoretical aspect of Frege's theory must be expressible. Hale and Wright (2012, pp. 95-96) consider simple cases for eliminative paraphrasing, such as statements involving "ostensibly singular reference to the concepts" as in the cases of paraphrasing "the relation of marriage is reflexive" as "one may be married to one another without latter being married to the former". However, for Hale and Wright, in the case of the first-level sortal predicates, this is not so simple. They argue that Dummett should have taken these predicates as 'what "ξ is horse" stands for is something which everything either is or is not' in which "something" expresses a second-level generality, and "everything" expresses a first-level generality.

[395] See Priest (1995, p. 201).

The second point of weakness is that Dummett's construction of predicates violates Frege's *denotation* principle, especially for predicate denotation. Frege explicitly says that concept-words denote concepts: "To every concept word, or proper name, there corresponds as a rule a sense and a [denotation], as I use these words."[396] However, Dummett's proposal is disqualified for the rule of *denotation*, for it is especially inadequate to specify predicate *denotation*.[397] Dummett's solution is inadequate for denotation of predicates, since Dummett holds that 'ξ' and "what ξ denotes" should co-denote in general. Dummett does not give any reason to hold that "what 'is a horse' denotes" is not a singular term but co-denotes with "is a horse". Because it would require, certainly, a further step to conclude that "what 'is a horse' refers to" is a singular term after all.[398]

The third weakness is that Dummett's proposal disobeys Frege's dictum that an expression cannot belong to more than one syntactic category. Frege puts forward that proper names denote objects, and the concept-words denote predicates, then one cannot make the same claim about objects and concepts. The problem that emerges is that Frege is unable to make generalizations about concepts and objects even to say that they are different.[399] We see that Dummett's proposal is construed on the apparent endorsement of a generalized law of excluded middle which Frege assumes

[396] Frege (1892c (*FR* p.173)).

[397] "Even if what "is a horse" refers to is a concept is stipulated introduction of the term, "the concept horse", or "the concept referred to by 'is a horse'" can reinstantiate the paradox." (Wright, 1998, p. 248)

[398] See Wright (1998, p. 251). Wright (1998, p. 250) further argues against Dummett's view concerning the syntactically irrelevance of the copula. Dummett (1981a, p. 214) reads Frege as "the copula is a mere grammatical device, with no content." For Wright, this approach is ad hoc and hard to maintain. The reason is that "for Frege it is essential that incomplete expressions are characterised in the first place as those which result from deletion of one or more occurrences of singular terms from a sentence." (Wright,1998, p. 250). Therefore, Wright rules out Dummett's syntactic irrelevance of the copula thesis.

[399] See Diller (1993a, pp. 356-358) and Priest (1995, pp. 201-202).

without a doubt. However, this approach is problematic since it also excludes vague concepts or some failures of the bivalence rule. [400]

7.3.3. Wiggins' Solution

David Wiggins argues that if Frege's thesis that predicates *denotes* anything is rejected, then the paradox is blocked. His fundamental idea is that incomplete expressions *denote* incomplete entities, therefore predicates do not denote anything at all. [401] Wiggins states that "if someone then asks what the reference of 'is a man' is in 'Jesus is a man' the new answer is that it has no [denotatum]." [402] However, Wiggins does not reject that the copula is a predicate; he only rejects that the copula does not denote anything. [403] Wiggins considers rather a nominalist reading about that predicate. Accordingly, Wiggins divides the sentence 'Jesus is a man' into three components and considers the following analysis as the proper analysis of every subject-predicate sentence [404]:

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[400] Wright (1998, p. 247) suggests an alternative approach for construing Dummett's second-level predicative expressions as "... is something which nothing is or *something* could be". Assuming that the interpretation of second level predicates only get Dummett's intended results if its "is or is not" component is construed as involving the copula rather than the "is" of identity, for otherwise what we have a predicate which is distinctive of objects. However, Hale and Wright (2012, pp. 96-97) argue that new and more serious problems arise when construing the simple statement "First-level predicates stand for concepts." When they paraphrase this statement out by partial rendering as "$\forall x$ (x is a first level predicate $\rightarrow$ x stands for a concept)" the consequent has the structure of "$\exists x$ (x stands for y & y is a concept)." It still has Dummett's pseudo-predicate "is a concept" thus paraphrasing it results in "$\exists x$ (x stands for y & y is something everything is either is or is not)." This is problematic since the bound variable y is an individual variable, thus an object. So paraphrasing attempt is still ill-formed. (Hale and Wright, 2012, p. 98). See also Priest (1995, pp. 201-202).

[401] Wiggins (1984, p. 319) provides a redescription of the analysis of the paradox in Categorial Grammar by referring to Evans (1982, Ch. 1). Wiggins starts with Fregean primitive categories viz. S (Sentence) and N (*Singular* term), adds a new primitive category B (*Begriffswort* or concept-word). He interprets Frege's theory as predicates belonging to the category S/N, but in his variant theory predicates belong to the category B (the category of a bare predicate).

[402] Wiggins (1984, p. 319).

[403] Wiggins (1984, p. 318).

[404] Wiggins (1984, p. 320). "If the copula takes an expression that stands for a concept to give us a properly unsaturated predicative phrase that can be completed by a name to give a sentence, and if

Jesus + (is + manhood)

where ‘Jesus’ denotes Jesus; ‘the concept *man*’ denotes *man*; and the copula ‘is’ attributes the property that Jesus falls under *man*. Hence, Wiggins takes the expression ‘manhood’ as an alternative form of, i.e., synonymous with, the *nominalization* ‘being a man’. However, ‘man’ and ‘manhood’ do not stand for the same thing.[405] According to Wiggins, the expression ‘manhood’ presupposes the unsaturated expression ‘ξ is a man’ which in turn presupposes the Fregean concept *man*. Wiggins draws the conclusion that properties are best understood by reference to nominalizations. [406]

Wiggins argues that the same point can be pursued in solution of the concept *horse* paradox, since in this framework the semantic value of ‘ξ is a horse’ is a function in extension. Accordingly, he attributes this role to the copula. [407] In this regard the role of the copula is to bind expressions having concepts as their semantic value to form complex expressions which have semantic value as functions from objects to truth-values. In other words, there is a distinction between semantic value and denotation of expressions for Wiggins: “The substantial reference of the predicate and everything else we need in order to understand second-level quantification is already secure, in the shape of the concept.” [408]

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concepts are indistinguishable from properties, then how is it that the copula cannot combine with the property-name 'manhood' to give us an unsaturated phrase equivalent in meaning to 'ξ is a man'?” (Wiggins, 1984, p. 320).

[405] Wiggins (1984, p. 320) puts forward as follows: “[W]hat now needs to be remarked is, first, that we also have the true identity: ‘The property of manhood is the property of being a man’; and secondly, that the one term we cannot licitly form as a name of this property is: ‘The property of man’. Still less can we affirm the identity: ‘The property of man is the property of manhood.’

[406] See Wiggins (1984, p. 321).

[407] Wiggins (1984, pp. 323-324) states that the following two diverse roles are combined in the copula: (i) the role of correlating objects with satisfaction conditions for truth or falsehood. In other words, this has the semantic role of proving biconditionals of the form “[‘Socrates’ + copula + ‘sit’] is true if and only if Socrates sits.” (ii) The role of standing for forms or characters or traits or universals, such as Socrates *falls under the concept that* 'sit' *stands for* if and only if it sits. See Trueman (2021, pp. 107-110).

[408] Wiggins (1984, p. 319).

In this respect, we shall also briefly consider an analogous solution by Strawson. [409] For Strawson, similar to Wiggins, the semantic function of copula decomposes the sentence "Jesus is a man" into three: [410]

i) An expression *specifying* to the individual Jesus (particular specification)

ii) An expression *specifying* to the concept *man* (concept specification)

iii) A combination or mode of combination of (i) and (ii) in the sentence as a whole to yield truth or falsity. (Propositional combination)

Whereas Wiggins attributes the role of the propositional unity to copula, Strawson does not assign any unity to the 'Jesus', 'man' and the copula. For Strawson, it is rather the propositional combination which holds together the constituents of propositions to yield truth-values. Moreover, contra Wiggins, in Strawson's framework the copula plus concept word complex has two roles: (i) They are both used to denote the concept *man*, and (ii) establish the propositional unity.[411]

We shall now present our critical evaluation of Wiggins' and Strawson's proposed solutions. First point is that Wiggins has a divergent view from Frege's realist attitude for concept-words due to Wiggins' nominalist reading, nevertheless Frege is strictly against such reading of the copula. The second important drawback of these approaches poses a serious problem threatening the unique decomposition of Fregean *Thoughts* since they reject the predicate denotation. Wiggins, nevertheless, makes "the distinction between semantic value and the special case of full reference […] The substantial reference of the predicate and everything else we need in order to understand second-level quantification is already secure, in the

[409] See Strawson (1974/2004, Ch. 1).

[410] Strawson (1974/2004, p. 17). *Cf.* Trueman (2021, p. 108).

[411] For predicate composition Strawson (1974/2004, p. 30) offers the logical formalism of '*ass* (*i c*)' where '*i*' represents particular specification, '*c*' represents concept specification, and '*ass* ( )' represents the function of propositional combination. Strawson (1974/2004, p. 26) notes that this does not mean that '*ass* ( )' itself represents concept-specifying expression. Similar to Wiggins, Strawson does not reject that it is a predicate, but rather rejects that '*ass* ( )' denotes anything. See Strawson (1974/2004, pp. 29-31).

shape of the concept." [412] Wright argues against Wiggins' point by stating that it is still problematic for the construal of higher-order quantification as these contexts which seem to require an ontology of properties. [413] According to Wright, the semantic role of the copula cannot *denote* a relation, i.e., the relation of *subsumption*, in Wiggins' solution. The reason is that it would reintroduce the problem (what Wright calls Russell's problem) of inadequacy in explaining the difference between "Bucephalus is a horse" and "Bucephalus the relation of subsumption a horse". [414] Therefore, the unity of Fregean *Thoughts* is not established in this Wiggins' framework. We can see that Wiggins' solution to the paradox treats concepts and relations as the denotations of *proper parts* of incomplete expressions, rather than of incomplete expressions themselves.[415]

Moreover, we can argue against both Wiggins and Strawson from a Fregean point of view: Frege cannot be interpreted as the tripartite decomposition of the sentence "Jesus is a man", for he explicitly remarks that there are only two significant parts in sentences, viz. the proper name part "Jesus" and the predicative part "ξ is man". [416] It is important to note that there are also cases in which the semantic function of the copula is not explicitly marked at all, e.g., 'Mehmet runs' and the arithmetical expression two plus two equals four. In this regard, both Strawson's and Wiggin's views are divergent variants of Frege's theory of *Thoughts*. As a result, both solutions fall short to be a satisfactory solution in the Fregean spirit.

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[412] Wiggins (1984, p. 319).

[413] Wright (1998, p. 251).

[414] Wright (1998, p. 252). See Trueman (2021, pp. 109-110).

[415] Wright (1984, p. 253) argues that an important problem arises for this approach in Frege's semantic theory: Wiggins' approach allows expressions belonging to syntactically different categories such as 'the concept *horse*', 'the denotatum of "*horse*" ', and "horse" might co-refer. However, as we shall see below, this is not in accordance with Wright's consideration of what he calls the Reference Principle (RP), since these expressions are not substitutable *salva congruitate*. See Wright (1998, pp. 252-253).

[416] See Frege (1892b (*FR* p. 182)). *Cf.* Trueman (2021, p. 109).

7.3.4. Wright's Solution

Crispin Wright begins with considering six alternative readings of Frege's predicate *denotation* and finds them unacceptable[417]:

(R_{A1}) "is a horse" denotes is a horse;

(R_{A2}) "is a horse" denotes the concept horse;

(R_{A3}) "is a horse" denotes being a horse;

(R_{A4}) "is a horse" denotes what anything is that is a horse;

(R_{A5}) "is a horse" denotes is what anything is that is a horse.

(R_{A6}) "is a horse" may truly be applied to an object just in case that object is a horse.

Accordingly, Wright considers that (R_{A1}) and (R_{A5}) are ill-formed; (R_{A2}) and (R_{A3}) are inconsistent with Frege's denotation principle since both "the concept horse" and "being a horse" stands for a singular term, thus the paradox persists; (R_{A4}) is rejected since "what anything is that is a horse" is a complete expression, thereby cannot substituted *salva congruitate* with "is a horse" but with "a horse" (he

[417] See Wright (1998, pp. 243-244). For type-categorization of *denotations* of predicates see Diller (1993a, pp. 355-357). Diller's proposed solution is indeed very complicated for our purposes since it involves typing linguistic expressions as well as partitions and stratifications of levels of syntactic and ontological categories of Frege's semantic theory. Arguably this approach seems to solve the paradox, nevertheless it is questionable whether such tools are present in Frege's logico-semantic theory. Here we shall briefly mention Diller's solution. Diller (1993a, pp. 347-351) begins his paper by rejecting the solutions of Geach and Dummett. Diller aims to construct Frege's semantic theory of *Thoughts* in which the paradox generating phrase 'the concept *horse*' is a singular term, thus it *denotes* an object. In this respect, Diller's (1993a, p. 360) solution involves division Frege's unsaturated part of language into (i) a *rich* part which contains predicates, relational expressions, functional expressions and quantifiers belonging to various levels, logical connectives and so on; and (ii) an *austere* part which contains the corresponding singular terms as well as at least one unsaturated expression. (Diller considers that there is indeed *only* one unsaturated expression in the austere part, namely "ξ falls under ζ".) Diller then attributes a functional character to expressions in the austere part and he (1993a, p. 362) further introduces the notion *stratification* – which he derives from Curry and Feys (1958, *Combinatory Logic*) – for the syntactic categories of expressions. Diller (1993a, p. 363) further introduces ontological stratification for denotations of expressions. As a result, Diller solves the paradox by putting a stratified expression in the place of the gaps in 'what … stands for' which is substitutable *salva congruitate* everywhere but can only substituted *salva veritate*, viz. in place of the un-stratified expressions. See Diller (1993a, pp. 363-364).

compares "what anything is that is red" and "red"); and finally (R_{A6}) involves no explicit recourse to the idea of *denotation* and does not assign a *Bedeutung* to predicates. [418]

Wright sets five constraints[419] for the dissolution of the paradox according to which the solution to the paradox should (i) allow non-substitutional interpretation of higher-order quantification, and be consistent with Frege's concept-object distinction; (ii) avoid the unity problem, i.e., Russell's Problem, thus give an account for the difference between "Bucephalus is a horse" and "Bucephalus the concept horse" – however, it is interesting to see that Wright does not further elaborate the implication of the paradox which targets the unity of Fregean *Thoughts*; (iii) be in accordance with the Reference Principle[420], or (RP) for short, according to which the sameness of *reference* should ensure sameness of semantic role, thereby allow co-denoting expressions should be cross-substitutable *salva veritate* in extensional contexts, and *salva congruitate* in all contexts. The solution should further (iv) provide the semantics of individual predicates and other unsaturated expressions "in a fashion that involves no coyness about the type of semantic relations involved" [421]; and (v) preserve the other elements in Frege's semantic theory, primarily Frege's principle that *sense* determines *denotation*.

As a solution to the paradox, Wright argues that the third constraint dictates no account of the semantics of predicates. Therefore, other four constraints can construe Fregean semantics in which predicates *refers* to concepts and singular

[418] See Wright (1998, p. 244).

[419] Wright (1998, pp. 244-245). Wright (1998, p. 245) says that "Frege's semantics of predication satisfies the first three of these at the cost of failing the fourth and fifth" but then Wright asks, "Are the constraints co-satisfiable at all?" Witfully Wright says that respecting three out of five constraints was Frege 's original score. *Cf.* Hale and Wright (2012, pp. 89-94) and Trueman (2021, pp. 99-100).

[420] We shall remark that we translate the German word 'Bedeutung' as 'denotation'. Here we shall make an exception. We use Wright's naming the *Reference Principle* when we mention his talk of Frege's *denotation* principle.

[421] Wright (1998, p. 245).

terms *refers* to objects. [422] As an initial step for his solution Wright argues that drop the first constraint. [423] The same relation to predicates and other unsaturated expressions as objects bear on the singular terms which *denote* them. [424] For the second constraint, Wright replies with a reflection that "is a horse" and "the concept horse" differ in the sense that the former *ascribes* the concept horse, whereas the latter *refers* to concept horse. Then, "Bucephalus the concept horse" can be considered as a mere list of words, since it does not have a unity. The second constraint is further in accordance with the fourth constraint that "is a horse" *ascribes* the concept horse, without a problem for *referring* to a singular term. For the third and fifth constraints Wright thinks that his proposal is consistent with (RP). The reason is that two co-denoting expressions "is a horse" and "a horse" are inter-substitutable *salva veritate* in extensional contexts, and *salva congruitate* in general contexts. As a result, for the latter there is no problem for any singular term standing for both concepts and objects. Wright shows that

1* "The ascriptum of 'is a horse'" is a singular term;
hence 2* Its reference, if any, is to an object.
3 * The reference of "The ascriptum of 'is a horse'" is the ascriptum of "is a horse"
hence 4* The ascriptum of "is a horse" is an object.

The reasoning is still sound. But 4* is not a paradox since the truth of 4* can be acknowledged without appealing '"The ascriptum of 'is a horse'" does not refer

[422] This constraint has been respected by Wiggins in which predicates denotes nothing, whereas Wright argues that Wiggins' proposal violates either the fifth constraint, i.e. "the concept horse" still does not refer to what intuitively it ought to refer; or the third constraint, i.e., "is a horse" and "a horse" are co-referring expression. As we have argued above, Dummett's solution conflicts with Frege's denotation principle and also with Wright's (RP), and moreover it respects neither the third nor the fifth constraints. See Hale and Wright (2012, esp. pp. 93 and 105) for different formulations of (RP).

[423] Wright (1998, pp. 258-259).

[424] See Trueman (2015; 2021, p. 99).

> to what intuitively it ought to refer. For that entity *is* an object, qua referent of a singular term, *and a concept too,* qua ascriptum of a predicate.[425]

To summarize Wright, he diagnoses the paradox in (RP), and he solves the paradox by distinguishing it from the Ascription Principle (AP).

(RP) Co-referring expressions are everywhere intersubstitutable *salva congruitate*.

(AP) Co-ascribing predicates are everywhere intersubstitutable *salva congruitate*,

We can see that Wright does not merely reject (RP), but instead he overrides (RP) by distinguishing two kinds of *denotations*: (i) singular term, or object, *denotations*; and (ii) predicate-*denotations* which he calls *ascription*. In this reading of (RP), singular terms and predicates *cannot* denote the same entity, since predicates simply cannot refer. According to (AP), singular terms and predicates cannot ascribe the same entity, since singular terms cannot ascribe anything. But it can be seen that neither of these revised principles entails that no singular term *denotes* something which predicates ascribe. [426] Therefore, 'x is a horse' and 'the concept horse' are not co-denoting expressions. In this regard, 'the concept *horse*' denotes what 'x is a horse' ascribes. [427] We shall propose our solution without bifurcating Frege's original denotation principle in our emendatory framework thereby preserving authenticity of Frege's semantic theory, and we shall leave our further discussion of Wright's proposal to section 7.5.5.

[425] Wright (1998, pp. 259-260).

[426] See Trueman (2021, p. 100).

[427] Hale and Wright (2012, pp. 106-107).

7.3.5. Noonan's Solution

Harold Noonan departs from Frege's solution in "On Concept and Object" where Frege maintains that "the concept *horse*" is not a concept, but an object. [428] Noonan interprets Frege's response to Kerry consisting of two claims[429]: (i) The distinction between objects and other entities; (ii) nothing can be both a concept and an object. In this sense the former claim roughly corresponds to the traditional distinction between particulars and universals. The latter claim is undeniable in Frege's ontology. However, as we have argued above, an important aspect of Frege's paradox is indeed contrary to such distinction since universals can be predicated of other particulars, and they can be subjects of predications. [430] It is indeed an aspect of the paradox that threatens the logico-semantic unity of Fregean *Thoughts*.

Noonan rejects (RP) – (AP) dichotomy and also Wright's modification of Frege's principle of *denotation*: "[(RP)] was used to infer that predicates and singular terms do not co-refer, but since predicates do not refer at all, it is powerless in this role" [431]. Noonan states that Wright's argument falls short for rejecting the division between concept and object. Arguably, Noonan argues that this should be taken as to claim that the paradox "grounded on something other than the nature of language; rather, it is precisely because the distinction between saturated and unsaturated expressions is the necessary origin of our grasp of the distinction between objects and functions that the Fregean paradox is unavoidable." [432] Arguably, Noonan endorses Geach's point of view and offers a similar solution.[433]

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[428] Noonan (2006, p. 155).

[429] Noonan (2006, p. 161).

[430] Noonan (2006, p. 155) states that "in Frege's mature ontology there is no place for universals" *Cf.* Hale and Wright (2012, p. 87 n. 11).

[431] Noonan (2006, p. 167).

[432] Noonan (2006, p. 171).

[433] Noonan (2006, p. 165). See Geach (1976, pp. 56-57), and above section 7.3.1.

In a nutshell, Noonan interprets Frege as requiring Wittgenstein's distinction between what can be *said*, and what can only be *shown*. As a result, the saying – showing distinction provides the only way out for the paradox, although he concludes that the paradox is not reducible to nonsense thus cannot be avoidable.[434]

7.3.6. Hale's Solution

Bob Hale aims to defend a Fregean-realist approach to ontology in his paper. Hale acknowledges that his aim is to construe an ontology of categories by adopting a broadly Fregean approach. [435] For Hale, Frege's most important ontological commitment is establishing the distinction between objects and properties.[436] Accordingly, Hale lists the following four constraints and states that taken together they form a set of inconsistent quadruple: [437]

(i) Objects are what actual or possible non-empty singular terms stand for

(ii) No property is an object

(iii) Some expressions of the form 'the property of being *F*' are non-empty singular terms

(iv) If an expression of the form 'the property of being *F*' stands for anything, it stands for a property.

Denying (iv) is not an option for Hale. He holds (i) but it conflicts with (ii) and (iii) and he suggests Frege proposes a solution to the paradox by denying (iii).

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[434] Noonan (2006, pp. 165 and 171). *Cf.* Hale and Wright (2012, p. 100).

[435] Hale (2010, p. 403). Hale's ontology encompasses a wide range of categories. He uses the term 'object' to cover 'particulars' or 'individuals'. Contrary to Noonan, Hale does not distinguish universals from properties and relations. Not surprisingly the title of Hale's paper talks for itself. He argues that these categories do not exhaust the list of existing objects. Thus, besides facts and states of affairs, Hale (2010, pp. 402-403) also recognizes events and processes as separate and independent categories. He (2010, pp. 404-407) further provides an elegant Quine-Frege comparison.

[436] Hale (2010, p. 412). Hale (n.27) uses the word 'property' in place of Frege's term 'concept'.

[437] Hale (2010, p. 412). Hale (2010, p. 417 n.39) also remarks that "things", but not "objects", must be completely disjoint categories.

Hale is in agreement with Wright that the paradox results from Frege's *denotation* principle. But against Wright, Hale does not think that (RP) is "sacrosanct."[438] Hale further argues against Wright, who denies (ii), that there is one type of *reference* for all different types of expressions, viz., proper names, concept words, and sentences. Thus, for Hale, two expressions 'ξ is a horse' and 'the concept *horse*' denote the very same entity and also in the same sense of 'reference'. As a result, Hale provides a solution to the paradox by rejecting (RP). Hale argues that referring to concept words with predicates is indeed more fundamental than referring to these entities with singular terms. [439] Hence, he distinguishes modes of denotations, or reference in his terminology, between *primary* and *secondary* as his solution to (HP). [440] Hale puts forward the following motivation as a rejection of (i) and endorsement of the primary-secondary distinction:

> Of course, simply rejecting [(i)] is tantamount to scrapping the Fregean approach altogether. But, at least if we accept –pace Dummett-Frege and Wright – that entities may be referred to by expressions belonging to different logical types, there is a simple and plausible modification of [(i)] which avoids our problem whilst preserving the essential ideas of the Fregean approach. Where there are expressions of different logical types having reference to entities of a given kind, we distinguish between primary and secondary, or derivative, modes of reference to them. For example, while we can refer to properties by means of (complex) singular terms (such as terms of the form: the property of being something that φs), the basic mode of reference is by means of the incomplete predicate 'φ(…)'. [441]

According to Hale, the distinction between primary and secondary denotations preserves (ii), i.e., Frege's distinction between objects and concepts. Hale modifies definitions for object as anything that is *primarily* referred to by singular terms, and

[438] See Hale (2010, p. 413). *Cf.* Trueman (2015; 2021, p. 101).

[439] Hale (2010, pp. 414-415).

[440] See Hale and Wright (2012, pp. 126-128) where he argues that terms denote properties only in a derivative sense. *Cf.* Hale (2013, pp. 21-34) and (2020, pp. 73-103).

[441] Hale (2010, p. 415).

he defines concepts (properties) as anything that is primarily referred to by a predicate. [442] Thus, Hale replaces (i) by

(i') Objects are what are primarily referred to by actual or possible non-empty singular terms

and puts forward the counterpart of (i)

(properties^n) n^{th} level properties are what are primarily referred to by actual or possible predicates of level n

> Our revised explanations allow that entities of a given category may be referred to by expressions other than those of the type in terms of which the category is defined. Thus a first-level property such as that of being a horse may be referred to by singular term (such as the one just used), but because that it not the primary mode of reference to properties, it does not make objects of them, and so precipitates no conflict with [(ii)] (or, of course, [(iii)] or [(iv)]). [443]

Hale suggests that an advantage of his solution compared to Wright's is that his distinction allows for the ascription of denotation to expressions of different syntactic categories. [444]

Now let us briefly evaluate what Hale proposes. Hale rejects (RP) on the grounds that there is no explicit argument in any of Frege's writings to back it up. Unfortunately, however, Hale leaves his rebuttal there, not providing additional support for the argument. Moreover, the literature is in almost a consensus that the principle is indeed a Fregean one. The principle is an intuitively advantageous one, ripe with argumentative virtue. Thus, although Hale's maneuver has some merit due to its demarcation between primary and secondary references, it ultimately falls short of presenting a solution to Frege's semantic theory of *Thoughts*. We can see that introducing the word primary in this sense seems to redesign Frege's definitions of object and concept. For Hale, secondary references are derivative modes of

[442] See Hale (2010, p. 416), Hale and Wright (2012, p. 117).

[443] Hale (2010, p. 416).

[444] See Hale (2010, p. 416). *Cf.* Trueman (2015; 2021, p. 102).

denotations. As a result, the distinction between primary and secondary references (RP), which generates the concept *horse* paradox as one of the basic assumptions. Hale's argument is hardly cogent because it is based on a very broad account of Frege's ontology, bordering on what one may call "unfaithful to the original".

7.3.7. MacBride's Solution

Fraser MacBride begins with considering some natural-language counterexamples to (RP) in which the principle fails in substitutions of "me" in place of "I" in the sentence "I am Oğuz." [445] He characterizes this failure in terms of what he calls *impure reference*. Thus, although 'me' and 'I' are co-denoting, the resulting expression "me am Oğuz" is not a grammatically correct sentence. [446] MacBride then takes this as an independent motivation for restricting (RP). His point of departure is that a similar case holds for predicates. For Frege, predicates have the role of being *denotations* of concepts. However, (HP) shows that predicates also share this role with corresponding singular terms. As a result, their nature prevents them from expressing a well-formed *Thought*. [447] We can see that this point shares exactly the same concern with the unity problem.

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[445] See MacBridge (2011, pp. 298-301). *Cf.* Black (1954, pp. 235-236), Wright (1998), and Oliver (2005, pp. 182-184), and Trueman (2015; 2021, p. 104) for further considerations.

[446] Although these singular terms have the same denotation, differences in their syntactic nature set constrains on their type of roles in well-formed grammatically correct sentences. MacBridge explains the difference between 'I' and 'me' by stating that

> pronouns embody *descriptive* content about whether a referent of a given occurrence of a pronoun is the Agent or the Patient of the action expressed by the corresponding active verb. It is this extra content they carry to which the contexts in which pronouns occur are sensitive and which make some contexts liable, others resistant, to their substitution. (2011, p. 305)

According to this approach, when we use the word 'I' in a sentence, it not only denotes the person themselves but also indicates that they are the Agents of the event described by the sentence. Thus, when we say "I phoned Mehmet', we point out the fact that I am the agent who did the phoning. By contrast, when we use 'me' to say 'Mehmet phoned me', we point out the fact that I am the Patient who was phoned. (We have used Trueman's example. See Trueman (2021, pp. 104-106).)

[447] See MacBridge (2011, pp. 303-306).

According to MacBridge, predicates cannot stand for properties[448] as singular terms stand for objects. Predicates cannot be *purely* referential expressions "[o]therwise it appears problematic how an *n*-place predicate and *n* singular terms could ever be used to say something, true or false, rather than merely enumerate (rather as a list does) what these words pick out, an *n*-ary property and some objects"[449] For him, a *purely referring expression* is an expression which only refers (does nothing else), "regardless of whether names or other singular phrases." [450] Predicates are *impurely referring expressions*. An impurely referring expression not only *refers*, but does something more, according to MacBridge:

> [T]hey also contribute in their own distinctive way towards the representation of how the properties or relations they signify are exhibited. Whereas a monadic predicate such as 'ξ flies' comes equipped with a rule for interpreting the representational significance of flanking it with a singleton occurrence of a name, a dyadic predicate such as 'ξ kissed ζ' comes with an order sensitive rule for interpreting the significance of flanking it with occurrences of right and left-flanking names etc. By contrast singular phrases don't come equipped with any such rule. Singular phrases can't be substituted for predicates because they're incapable of discharging the further semantic function predicates perform, viz. representing how the objects picked out by flanking names exhibit the properties or relations predicates signify. But this doesn't prevent singular phrases from picking out what predicates signify. [451]

As a result, MacBridge proposes his solution to (HP) by restricting (RP) to purely referring expressions, viz.,

(RP$_2$) If α and β are (i) co-referential expressions and (ii) purely referential then (iii) α and β are substitutable in extensional contexts *salva veritate* and everywhere else *salva congruitate*. [452]

[448] MacBridge also uses the term 'property' for 'concept'.

[449] MacBridge (2011, p. 297).

[450] MacBridge (2011, p. 308).

[451] MacBridge (2011, pp. 308-309).

[452] See MacBridge (2011, p. 306).

In other words, if two expressions purely refer to the same thing, then they are everywhere intersubstitutable *salva congruitate*.

MacBridge argues that Frege was wrong in the first place in being committed to (RP). If the range of this principle is restricted to purely referential expressions, only then it becomes an adequate principle. [453] The reason is that predicates are devices of impure reference, and by (RP_2) proper names and other singular phrases are not allowed to be picked out from what predicates refer. As a result, one shall be in a problematic position to specify the denotation of the predicate such as "ξ is a horse", and similarly for the following expressions "the property of being a horse", "the denotation of "ξ is a horse"", or even "the concept *horse*". Accordingly, "there's no need for us to deny that the concept horse is a concept either." [454]

We see that MacBridge focuses on (RP) which is similar to Hale and Wright. MacBridge introduces different denotation types for different types of expressions that is also similar to Hale. However, contrary to Wright's (RP) – (AP) distinction, and Hale's primary-secondary distinction, MacBridge introduces another distinction. [455] As Hale and Wright rightfully observes, Frege ascribed denotations to all types of expressions, including co-extensive predicates. In this sense MacBridge's proposal is not in accordance with Frege's metaphysical commitments and his approach to ontology of expressions of all logical types. [456]

We shall consider an important question on whether predicates are impurely referential. MacBridge strictly defends that they are, for otherwise there would be

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[453] See MacBridge (2011, p. 309). *Cf.* Trueman (2021, pp. 106-107).

[454] MacBridge (2011, p. 309). See Trueman (2021, pp. 106-107).

[455] Hale and Wright in their postscript, analyze MacBridge's solution. They accept that it is different from Wright's (1998) and Hale's (2010). Hale and Wright observe that MacBridge's revised reference principle (RP_2) restricts (RP) only to singular terms which they consider as a narrow subclass of ontological category of entities. See Hale and Wright (2012, pp. 130-131) and Trueman (2021, p. 104).

[456] Hale and Wright (2012, p. 131) add: "Indeed, on MacBridge's view, it appears that the distinction between singular term and a predicate is merely a *grammatical* distinction, rather than one of ontological significance all fours with, say, the distinction between "Brutus" and "Brutum" in Latin."

no explanation for the difference between a sentence and a mere list. [457] 'Bucephalus is a horse', for instance, is a whole sentence not a mere list. Therefore, the predicate 'is a horse' not only denotes the property horse, but also applies the predicate to Bucephalus. [458] We definitely agree with this diagnosis since for our present purposes the solution of the *horse* paradox first and foremost should establish the unity of *Thoughts*. Nevertheless, bifurcating Frege's notion denotation between pure and impure reference is not in Fregean spirit. The reason that MacBridge's approach falls short, is that if predicates are impurely referential expressions, then impure reference principle would not allow co-denoting singular terms and predicates. Thus, against MacBridge, we shall consider whether there is any explicit textual evidence for the assumption that predicates denote and *only* denote. If predicates, similar to MacBridge's solution, does something else then it may not allow co-referring singular terms and predicates, since given a distinction between singular term-denotation and predicate denotation, then it would no longer be possible that a singular term and a predicate can co-denote.

7.4. An Emendation of Frege: Our Framework

We shall begin this section with a classification of expressions, denotations, and senses on the basis of Frege's saturated-unsaturated distinction.

[457] See MacBridge (2011, pp. 309-310).

[458] *Cf.* Trueman (2021, pp. 104-108).

7.4.1. A Classification of Linguistic and Extra-Linguistic Entities

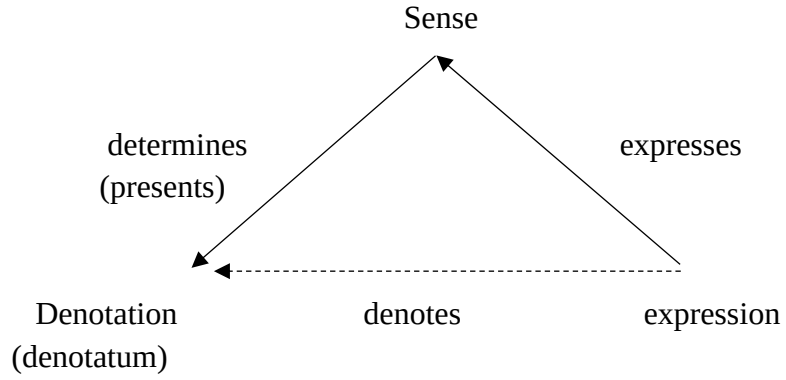
I. Saturated (Complete)

(a) *Expressions*	(b) *Denotations*	(c) *Senses*
1. Names	Objects	Senses of names:
2. Sentences	Truth-Values	Modes of presentation

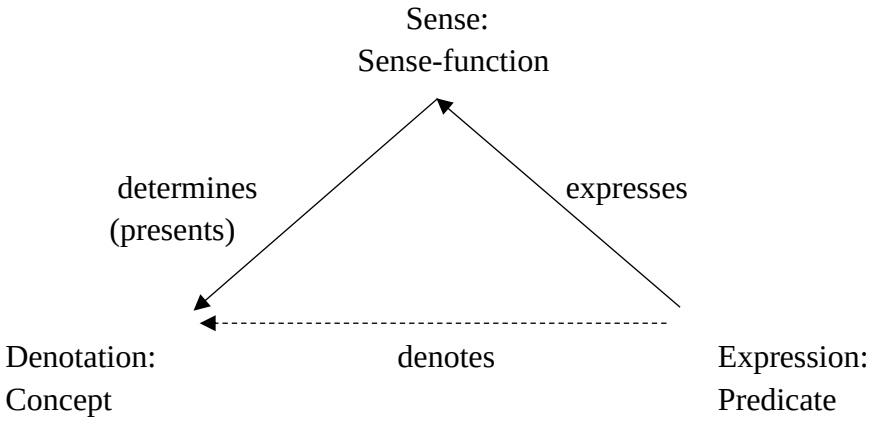
II. Unsaturated (Incomplete)

(a) *Expressions*	(b) *Denotations*	(c) *Senses*
1. Predicates	1. Concepts: Truth-valued functions of objects	Sense-Function
2. Truth-functional connectives	2. Truth-valued functions of truth-values	
3. Functional expressions (In general)	3. Functions	

Tichý provides the following schema summarizing the Fregean classification[459]:



An instance of Tichý's schema for the case of predicates can be shown as follows:



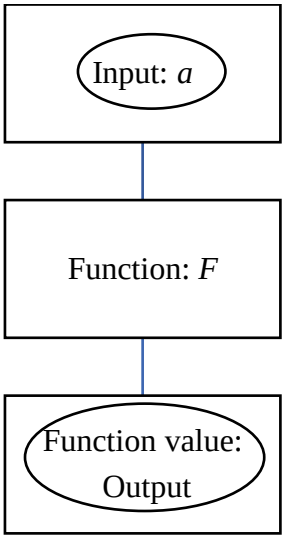
The sense expressed by an expression *presents* the denotation of this expression. [460]

We shall now show a flowchart for Fregean functions. A function (in intension) *F* is a *procedure* construed as an Input-Output device where the inputs consist of the arguments and the outputs consist of the values of the function *F*.

[459] Tichý (1988, p. 98). *Cf.* Duží et. al. (2010, p. 19).

[460] *Cf.* Klement (2002).

A Flowchart for function *F*



All functions in Frege's semantic theory are functions-in-intension (in Church's sense). [461] The corresponding function-in-extension is merely the value-range of the functions.[462]

We shall now provide the infrastructure of our solution of the unity problem and that of the concept *horse* paradox.

7.4.2. Language, Metalanguage and Meta-metalanguage

Frege's semantic theory is formulated in metalanguage *ML* for an object language *L*.[463] On the other hand, one need a meta-metalanguage *MML* to talk about Frege's semantic theory. Object language *L* contains exclusively atomic sentences of the forms '*a* is *F*' and '*aRb*' whose predicates are respectively of the form 'ξ is *F*' and 'ξ*R*ζ'. Atomic sentences are exemplified by:

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[461] See Church (1941/2019, p. 201). *Cf.* Tichý (1988, Ch. 2) and Klement (2002, pp. 96-101).

[462] See Frege (1891a (*FR* p. 135 and n. E3)).

[463] Frege (*PW* pp. 260-261) indeed distinguishes object language (*Darlegungssprache* [explained language]) from metalanguage (*Hilfssprache* [helping language]). See Klement (2002, pp. 27-28).

(1) Venus is a planet

(2) The morning star is a planet

(3) Caesar conquered Gaul

(4) Jupiter is larger than Mars

The vocabulary of *L* consists of names on the one hand, and monadic and dyadic predicates on the other hand. Names denote exclusively objects, monadic predicates denote concepts, and dyadic predicates denote dyadic concepts (relations) in Frege's system. Concerning the relationship between linguistic expressions and their denotations Frege says: "[T]aking 'subject' and 'predicate' in the linguistic sense: a concept is the [denotation] of a predicate; an object is something that can never be the [denotation] of a predicate, but can be the [denotation] of a subject."[464]

We assume that the object language *L* is included in the metalanguage *ML* which is regimented in accordance with Fregean semantic theory. In particular, predicates in *L* such as 'is a planet' and 'is larger than' are regimented respectively as 'ξ is a planet' and ''ξ is larger than ζ', We shall use the following regimented form for predicates in *L* on the one hand, and for denotations and senses (in *ML*) on the other hand.

(a) Predicates: '[λ*x*.*x* is *F*]' stands for 'ξ is *F*'.

Example: '[λ*x*.*x* is a planet]' stands for 'ξ is a planet'.

We use here Church's *calculus of* λ*-conversion*.[465] '[λ*x*.*x* is *F*]' is a λ-abstract. Given that '*N*' is a name in *L*,

(1) [λ*x*.*x* is *F*] (*N*) *cnv N* is *F*.

The sign '*cnv*' is read as 'interconvertible' express the relation of λ*-convertibility*. This relation is defined in our case by the following three rules:

[464] Frege (1892b (*FR* p. 198)).

[465] Church (1941/2019, pp. 208-209).

I. [λ*x*.*x* is *F*] *cnv* [λ*y*.*y* is *F*]

II. [λ*x*.*x* is *F*] (*N*) *cnv N* is *F*

III. *N* is *F cnv* [λ*x*.*x* is *F*] (*N*)

(b) Denotations

1. ⟦*N*⟧ = The denotation of name '*N*'	an object
2. ⟦λ*x*. *x* is *F*⟧ = The denotation of predicate '[λ*x*.*x* is *F*]'	a concept
3. ⟦*N* is *F* ⟧ = The denotation of a sentence '*N* is *F*'	truth-values.

Then the following holds:

⟦λ*x*. *x* is *F*⟧ (⟦*N*⟧) = ⟦*N* is *F* ⟧

(c) Senses

1. ⟨*N*⟩ = The sense of name '*N*'	a mode of presentation
2. ⟨λ*x*.*x* is *F*⟩ = The sense of predicate '[λ*x*.*x* is *F*]'	a sense-function
3. ⟨*N* is *F*⟩ = The sense of a sentence '*N* is *F*'	a *Thought*

Then the following holds:

⟨λ*x*.*x* is *F*⟩ (⟨*N*⟩) = ⟨*N* is *F*⟩

7.4.3. Denotation and Sense

Origin of the notation '⟦λ*x*. *x* is *F*⟧' in Frege

As mentioned by Klement, "in a letter to Russell (*PMC* pp. 161-2), Frege considers a notation in many ways similar to Church's lambda notation … such as 'ἐ (ε > 7)' … to mark that the result is a function, not a value-range. However, he

finds it unworkable, because it obscures the unsaturatedness of functions." [466] But on that very account Frege's use of the *open* expression 'ξ >7' "obscures" the very fact that 'ξ >7' stands for a function *qua* extralinguistic entity. Obviously, our own notation

$$[\![\lambda x.\ x > 7]\!]$$

is a mere variant of the notation

$$\grave{\varepsilon}\ (\varepsilon > 7)$$

envisaged by Frege himself (in his letter to Russell).

7.4.3.1. Senses Expressed by Names

Let *N* be a variable ranging over the names of object language *L*. We assume that any name of *L* is unambiguous and non-context-sensitive so that, whether empty or not, expresses exactly one sense. The sense expressed by a name *N* is said to be the *sense* of *N*. The sense of names are themselves (saturated) objects.

Let us call the objects which are *not* themselves senses of any name *individual objects*, or *individuals* for short. We assume that every individual is presented by one or more senses. We assume that for every individual *a* and every sense *s* presenting *a*, the object language *L* contains exactly one name *N* such that *N* denotes *a* and expresses *s*. As an example of an individual consider the planet Venus. (It is indeed an individual since it cannot constitute the sense of any name.) The individual Venus is presented by various senses including, among others, the senses of 'The evening star' and 'The morning star', respectively.

[466] Klement (2002, p. 105).

7.4.3.2. Concepts Denoted by Predicates

A monadic predicate of the form 'is *F*' is expressed by Frege as 'ξ is *F*' in object language *L*. We shall use, occasionally, in place of Frege's notation 'ξ is *F*', the λ-abstract '[λ*x*.*x* is *F*]' construed as a functor operating on a name to form a sentence.

Definition 1 The *concept-function*, or *concept* for short, denoted by predicate '[λ*x*.*x* is *F*]' is the function ⟦λ*x*. *x* is *F*⟧ whose arguments are objects and whose values are the truth-values viz., the True and the False; and which also satisfies the following conditions:

(i) For every object *a* in the universe of discourse of *L*,

$$⟦\lambda x.\ x \text{ is } F⟧(a) = \begin{cases} \text{the True, if } a \text{ is } F \\ \text{the False, if otherwise} \end{cases}$$

(ii) ⟦λ*x*. *x* is *F*⟧ = ⟦λ*x*. *x* is *G*⟧ if and only if [λ*x*.*x* is *F*] *cnv* [λ*x*.*x* is *G*]

where '*x*' and 'y' range over the elements of the universe of discourse of object-language *L*, and '*cnv*' is read as "*interconvertible*."[467]

We assume that for every predicate '[λ*x*.*x* is *F*]', there is exactly one function satisfying this condition. In order to avoid circularity, we presuppose the possibility of pretheoretic and intuitive knowledge of whether it is the case that *a* is *F*.

Definition 2 The *value-range* of ⟦λ*x*. *x* is *F*⟧ = (<*y*, *t*>: ⟦λ*x*. *x* is *F*⟧ (*y*) = *t*) where '*t*' is a variable ranging over the truth-values.

Definition 3 Object *a falls under* concept *F* if and only if ⟦λ*x*. *x* is *F*⟧ (*a*) = the True

[467] See Church (1941/2019, p. 209).

The following holds:

(1) $[\![\lambda x.\ x \text{ is } F]\!](a) \in \{\text{the True, the False}\}$

For example

(2) $[\![\lambda x.\ x \text{ is a planet}]\!](\text{The evening star}) = \text{the True}$

Following Klement, we consider the following coextensional predicates:

(3) ξ has a heart

(4) ξ has a kidney

Klement claims that "for Frege [expressions (3) and (4)] denote the same concept since they are coextensional." [468] However, Frege remarks in his "Function and Concept" that by asserting the equality of the value-ranges of functions '$x^2 - 4x$' and '$x\ (x - 4)$' "we have not put one function equal to the other, but only the values of one equal to those of the other." [469] Indeed, Frege's sharp distinction in note E3 between functions and their value-ranges implies that Fregean functions are functions *in intension*. [470] Hence, it is natural to hold that the above-mentioned predicates (3) and (4) denote, after all, *different* concepts.

7.4.3.3. Senses Expressed by Predicates: Sense-Functions

Definition 1 The sense expressed by predicate '[$\lambda x.x$ is F]' is the function (called *sense-function*), in symbols '$\langle \lambda x.x$ is $F \rangle$', whose arguments are senses presenting individuals and whose values are thoughts, which satisfies the following conditions:

[468] Klement (2002, p. 66).

[469] Frege (1891a (*FR* p. 135)).

[470] See Frege (1891a (*FR* p. 135)).

(i) For every individual belonging to the universe of discourse L and every sense s presenting this individual:

$\langle \lambda x.x \text{ is } F \rangle (s)$ = The thought expressed by the sentence $\ulcorner N_s \text{ is } F \urcorner$

where 'N_s' is a name in L which expresses the sense s.

(ii) $\langle \lambda x.x \text{ is } F \rangle = \langle \lambda x.x \text{ is } G \rangle$ if $[\lambda x.x \text{ is } F]$ *cnv* $[\lambda x.x \text{ is } G]$

(iii) $\langle \lambda x.x \text{ is } F \rangle = \langle \lambda x.x \text{ is } G \rangle$ iff $\forall s \, (\langle \lambda x.x \text{ is } F \rangle (s) = \langle \lambda x.x \text{ is } G \rangle (s))$

For example: $\langle \lambda x.x \text{ is a planet} \rangle (\langle \text{The evening star} \rangle) = \langle \text{The evening star is a planet.} \rangle$

The relationship between the sense of a predicate and a concept denoted by the predicate is as follows:

For every name N, ($[\![\lambda x. \, x \text{ is } F]\!] \, ([\![N]\!]$ = the True) if and only if $\langle \lambda x.x \text{ is } F \rangle (\langle N \rangle)$ is a true *Thought*.

It is important to remark that the thought which is the value of the function $\langle \lambda x.x \text{ is } F \rangle$ for the argument s, contains both the argument s and the function $\langle \lambda x.x \text{ is } F \rangle$ as constituents. Indeed, the *unity of the Thought* results from the application of a function to an argument which are both constituents of the *Thought*.[471]

7.5. The Concept *Horse* Paradox: Our Solution

7.5.1. Frege's Concept *Horse* Paradox

For the solution of the paradox, we shall begin by using sense and denotation functions of unsaturated expressions as interconvertible λ-abstracts in our emendatory framework. Next, we shall appeal to the distinction between used (executed) and mentioned (displayed) occurrences of concepts. We shall argue that

[471] See Levine (2002).

the functional application of "the concept *horse*" in the occurrence of a sentence "Bucephalus is a horse" corresponds to used or executed mode, whereas its occurrence as a grammatical subject in the paradoxical sentence "The concept *horse* is not a concept" corresponds to *mentioned* (displayed) mode. As a result, we shall show that "the concept *horse*" denotes an unsaturated concept and hence we obtain the truth of statement "The concept *horse* is a concept" which resolves the paradox.

7.5.2. Used (executed) versus Mentioned (displayed) Occurrences of Concepts

Russell distinguishes a "relation in itself" from a "relation actually relating".[472] He exemplifies the distinction by means of the proposition

(1) A differs from B

and the analysis of (1) in terms of

(2) A, difference, B

The occurrence of the relation *difference* in (1) is *actually relating* A and B, but its occurrence in (2) is a *relation in itself* "which has no connection with A and B".[473] Russell's distinction concerning relations-in-intension reappears in an extended form in *Transparent Intensional Logic*, or TIL for short.[474] In TIL, relations and functions in intension are construed as *procedures*. Procedures are either *used* (executed), or else *mentioned* (displayed). For example, the occurrence of the concept ⟦λ*x*. *x* is a horse⟧ in

(3) ⟦λ*x*. *x* is a horse⟧(Bucephalus) = the True

[472] Russell (1903, §54).

[473] Russell (1903, p. 50).

[474] *Cf.* Tichý (1988) and Duží et. al. (2010).

is *used* (executed). On the other hand, the occurrence in

(4) The predicate 'ξ is a horse' denotes the concept $[\![\lambda x.\ x \text{ is a horse}]\!]$

of the same concept is *mentioned* (displayed). In Russell's terminology concept $[\![\lambda x.\ x \text{ is a horse}]\!]$ occurs in (4) as *concept in itself* with no connection with its arguments. [475]

We claim that a grammatical subject can denote a concept occurring in a context in which it is *mentioned* (displayed). For example, the occurrence of $[\![\lambda x.\ x \text{ is a horse}]\!]$ in

(5) $[\![\lambda x.\ x \text{ is a horse}]\!]$ is a horse

is *mentioned* (displayed).

7.5.3. Three Analyses of the Concept *Horse* Paradox

Recall, the paradoxical sentence (H)

(H) The concept *horse* is not a concept

We can analyze (H) in the following different ways:

Analysis I. 'The concept *horse*' is a definite description. Then

The concept *horse* = ιx (x is a concept $\wedge$ 'ξ is a horse' denotes x)

Given Frege's criterion to the effect that the singular definite article indicates an object and that no object is a concept, 'the concept *horse*' is an *improper description*

[475] See Russell (1903, §§54-55).

denoting – *qua* Fregean description – an arbitrarily fixed *object*. It follows, then, that (H) is trivially true. [476]

Analysis II. The concept *horse* is reduced to an object which must "go proxy for it." [477] Most often the object which must go proxy for a concept taken to be its value-range. Then it follows that (H) is again true.

Analysis III. As we have argued above, Wright construes 'the concept *horse*' as denoting the *property* "*ascribed*" by the predicate 'ξ is a horse'. [478] He takes properties to be a special kind of objects, and consequently as being saturated and identifies then the Fregean concepts with the properties in question. We argue that our solution differs from Wright's in two main respects. First, we do not appeal to any distinction between predicate denotations as Wright has distinguished (RP) and (AP). Second, we do not treat properties as particular kinds of objects. Consequently, we see that Wright excludes unsaturated entities, hence functions in intension, from Frege's ontology. Nevertheless, Wright provides a solution to the concept *horse* paradox. Indeed, in Wright's framework 'the property ascribed by 'ξ is a horse' is synonymous with the 'the concept ascribed to 'ξ is a horse''. Given then that 'the concept *horse*' is construed as 'the property ascribed by 'ξ is a horse'', then the sentence

(¬H) The concept *horse* is a concept

is true in Wright's construal of Fregean semantics.

[476] See Frege (1892b (*FR* p. 184)).

[477] Frege (1892b (*FR* p. 185)).

[478] Wright (1998, pp. 258-259).

7.5.4. Untying the Gordian Knot

'The concept *horse*' is short for 'the concept denoted by 'ξ is a horse'' or, simply, the denotation of ⟦λ*x*. *x* is a horse⟧ of 'ξ is a horse'. Then we can say that the paradoxical *horse*-sentence for (H) is equivalent to the following sentence (H)*

(H)* ⟦λ*x*. *x* is a horse⟧ is not a concept.

We present the solution of the paradox as follows:

(1) 'The concept *horse*' is not a definite description.

We begin by explicating the denotation of the concept-word by denotation function of the predicate "ξ is a horse" as follows:

(2) The concept *horse* = ⟦λ*x*. *x* is a horse⟧

Following the above given distinction between used (executed) and mentioned (displayed) distinction, we see that

(3) ⟦λ*x*. *x* is a horse⟧ is mentioned (displayed) in

(i) ⟦λ*x*. *x* is a horse⟧ is a concept

But it is used (executed) in

(ii) ⟦λ*x*. *x* is a horse⟧ (Bucephalus)

Given the fundamental role of unsaturated concepts in Frege's semantic theory, we should naturally introduce the language of theory, i.e., the metalanguage *ML*, and then

(4) '⟦λ*x*. *x* is a horse⟧' belongs to the regimented part of the metalanguage

and

(5) 'The concept *horse*' belongs to the unregimented part of the metalanguage

(6) ‘The concept *horse*’ is never the grammatical predicate of any sentence

(7) ‘The concept *horse*’ is the grammatical subject of some sentence, the concept *horse* itself is a concept which is mentioned (displayed).

We obtain then,

(¬H) The concept *horse* is a concept

Therefore, the paradox is solved. Note that ‘the concept *horse*’ denotes an unsaturated concept, hence an unsaturated property.

7.5.5. Hale and Wright’s Reference Principle and the Concept *Horse* Paradox

In Section 7.3.4. we have considered Wright’s solution to the paradox which makes a distinction between (RP) and (AP). Hale and Wright further attributes two distinct constraints to the Reference Principle as follows. According to *Single Relation* principle “some *one reference relation* uniformly connects expressions of each syntactic type with the kinds of entity that provide their respective semantic values.” [479] According to *Type-Kind Uniqueness* principle “*syntactic types* of expressions correlate one-to-one with the *kinds of entity* among which their tokens are eligible to refer.” [480] We shall now complete our critical review to both Wright (1998) and Hale and Wright (2012).

Consider an object language *L* whose sentences are atomic of the form ‘*a* is *F*’. An example of such sentences is

(1) Bucephalus is a horse

[479] Hale and Wright (2012, p. 114).

[480] Hale and Wright (2012, p. 114).

Let *ML* be a metalanguage for *L* which is regimented with a Fregean semantics. Applying *the denotation relation* involved in Hale and Wright's *Single Relation* principle to (1), we obtain the following sentences in *ML*:

(2) The name 'Bucephalus' denotes the object Bucephalus

(3) The predicate 'ξ is a horse' denotes the concept ⟦λ*x*. *x* is a horse⟧

(4) The sentence 'Bucephalus is a horse' denotes the True

Notice that the following holds:

(5) 'Bucephalus is a horse' denotes ⟦λ*x*. *x* is a horse⟧ (Bucephalus)

Let 'Den' stands for 'the denotatum of'. Then

(6) Den ('*a* is *F*') = Den ('ξ is *F*') (Den ('*a*'))

holds generally. Since Den ('ξ is *F*') is a concept, it follows that

(7) Den ('ξ is *F*') (Den ('*a*')) ∈ {the True, the False}

Hale and Wright's *Type-Kind Uniqueness* principle, in conjunction with the *Single Relation* principle, implies that expressions of different syntactic types cannot denote the same thing. [481] Consequently, the predicative expression '… is a horse' and the singular term 'the concept *horse*' cannot co-denote. Our solution is based on interpreting both expressions as standing concept '⟦λ*x*. *x* is a horse⟧' in the regimented metalanguage *ML*. In this way we can preserve both principles introduced by Hale and Wright, which are essential to a Fregean semantic theory. In other words, our solution does not bifurcate Frege's denotation principle, hence it preserves authenticity of Frege's semantic theory.

[481] Hale and Wright (2012, pp. 105-106).

7.6. The Unity of Atomic Fregean Thoughts

In first part of this chapter, we have first presented Frege's account for the unity of *Thoughts* according to the which the functional composition of saturated and unsaturated expressions holds the constituents of an atomic *Thoughts* together. Then in the second section, we have investigated the paradox in detail. We have argued that the paradox compromises the ontological distinction between objects and concepts, and we were left with the conclusion that "the concept horse" is not a concept. We argued that the paradox not only poses serious problems for the concept-object dichotomy but also for the unity of *Thoughts*. Without solving the concept *horse* paradox, we cannot establish the unity. In this respect, we have argued that the solution of the concept *horse* paradox is a necessary condition of the unity of thoughts. We have surveyed the substantial solutions in the literature in the third section of this chapter. These solutions have divergent approaches to provide a solution to the paradox. However, even the most promising solutions are not concerned with the explanation of the unity of *Thoughts*. Moreover, these solutions generally trace the roots of the paradox in *only* Frege's denotation principle. They aimed to overcome the paradox by dividing Frege's denotation principle to block the cross-categorial denotations between saturated expressions, i.e., objects, and unsaturated expressions, i.e. concept-words. However, bifurcating the denotation principle poses a serious threat to Frege's semantic theory since it is indeed the composition of these asymmetrical elements that establishes the unity. Hence, we claim that these solutions cannot establish the unity of atomic *Thoughts*. As we have argued in this chapter, a satisfactory solution to the paradox should explain the senses and denotations of functional or predicative expressions, of which Frege has not provided an account. Therefore, we take our point of departure from our explicatory emendation of Frege's semantic theory in the fourth section.

We begin with a schematic classification of linguistic and extralinguistic entities. Following Church, we have construed all functions in Frege's semantic theory as functions-in-intension. We use the regimented form of predicates in an

object language and their denotations and senses in the metalanguage. We have used λ-calculus to express predicates as '[λ*x*.*x* is *F*]' which stand for 'ξ is *F*'. Accordingly, for each category of linguistic entities, viz. names, predicates, and sentences we have defined corresponding functions in the realm of denotation. We have also defined functions in the realm of sense for the corresponding categories of these linguistic entities. Then we have provided our emendation for both concepts denoted by predicates as concept-functions, and senses expressed by predicates as sense-functions. We have provided our explicatory definitions of these types of functions in Frege's semantic theory.

In the fifth part, we have provided our solution to the concept of *horse* paradox. We begin with Russell's distinction between used and mentioned occurrences of concepts, and an extended framework given by TIL which construes relations and functions in intension as *procedures*. In this respect, functions are either *used* (executed) or *mentioned* (displayed). Then, we have provided our analysis of the paradox in this emended framework. We have argued that 'the concept *horse*' is short for 'the concept denoted by 'ξ is a horse''. We have expressed the denotation function of this expression as ⟦λ*x*. *x* is a horse⟧ and we have expressed the paradoxical sentence (H) in our framework as its equivalent form '⟦λ*x*. *x* is a horse⟧ is not a concept'. As result, we claim that 'the concept *horse* = ⟦λ*x*. *x* is a horse⟧'. We note that the unsaturated expression '⟦λ*x*. *x* is a horse⟧' occurs in both of the above given sentences as a *mentioned* (displayed) concept. Hence, we only obtain the truth of negation of the paradoxical sentence, viz. 'The concept *horse* is a concept', in which 'the concept *horse*' denotes an unsaturated concept. As a result, the paradox is solved.

CHAPTER 8

CONCLUSION

In this thesis, we have focused on Frege's semantic theory of *Thoughts* which is grounded on his key semantic and ontological notions, namely the function-argument analysis, the object-concept dichotomy, the sense-denotation distinction together with the saturated- unsaturated division. Over the course of this thesis, we have explicated the structure and unity of atomic Fregean *Thoughts*. We have presented our emendation to Frege's Theory of *Thoughts* to achieve two primary goals. First, we have argued that Frege's theory of *Thoughts* is the most adequate semantic theory for explaining the meaning and truth of complex natural language expressions. For this purpose, we have investigated the theories of preceding philosophers as the historical background to Frege's *Thoughts*. Then we have presented our explicatory analysis of Frege's own theoretical framework in the context of his semantic theory of *Thoughts*. We have delineated the analogous approaches by realist philosophers which have theorized under the name "proposition". Then, we have focused on the contemporary views on propositions, and we have provided a critical review of these theories by comparing these theories with Frege's theory of *Thoughts*.

For the second goal of this thesis, we have considered an important set of problems concerning the structure and unity of Fregean *Thoughts*. For problems regarding the structure of atomic Fregean *Thoughts,* we have pondered a problem concerning the apparent tension between the Context Principle and the Compositionality Principles, and a problem about Frege's holding two incompatible theses, which results in an inconsistency between the unique analysis of *Thoughts*

and multiple decompositions of *Thoughts*. We have argued that the former problem poses no tension for the compositional structure of *Thoughts*, since he has never endorsed the Context Principle in his theory of *Thoughts*. It seems that this principle is limited to his *Grundlagen*. In our solution of the second problem, we have presented our account by arguing in favor of the polymorphous structure of atomic *Thoughts* according to which each possible constituent of a structure of *Thought* can be shown by multiple decomposition and they reveal the ultimate unique constituents of the same atomic *Thought*. We have shown that both of these theses can be held in Frege's semantic theory, hence preserving Frege's original semantic views.

For the problems concerning the unity of atomic Fregean *Thoughts*, we have focused on the concept *horse* paradox. Frege had considered the unity of *Thoughts* as the functional composition of their corresponding saturated and unsaturated parts. However, he has not provided any explanation of how to conceive the senses and denotations of unsaturated, i.e., functional expressions which denotes concepts. We have considered various solutions, nevertheless we have shown the inadequacy of these solutions in establishing the unity of atomic *Thoughts*. For the solution of the paradox, we begin by providing our emendatory framework to explicate both senses and denotations of unsaturated expressions. We have explicated the senses and denotations of proper names, predicates, and sentences as interconvertible λ-abstracts. Next, we have presented formal definitions of senses and denotations of unsaturated expressions in our framework. Then, we appealed to the distinction between used (executed) and mentioned (displayed) occurrences of concepts. We have argued that the functional application of "the concept *horse*" in the occurrence of a sentence "Bucephalus is a horse" corresponds to used or executed mode, whereas its occurrence as a grammatical subject in the paradoxical sentence corresponds to denotation of a concept in the context in which it is *mentioned* (displayed). Hence, we have shown in the context of the paradoxical sentence, "the concept *horse*" denotes an unsaturated concept and we demonstrated the truth of the statement "The concept *horse* is a concept," which resolved the paradox. As a result,

we have explained the structural unity of an atomic *Thought* established by the application of a function to an argument which are respective saturated and unsaturated constituents of the *Thought* without any paradoxical consequences.

In the beginning of this thesis, our initial conjecture was that Frege's semantic theory is incomplete in the sense of being vulnerable to the problems concerning their structure and unity of atomic *Thoughts*. We believe that our explicatory framework supplements his semantic theory at least for the problems considered in this thesis, hence they no longer pose any threat to the structure and unity of atomic *Thoughts*. However, we are well aware of the fact that Frege's semantic theory is not limited to atomic thoughts. As a connection to possible future studies, we aim to carry the achievement of this thesis to explicate and provide solutions to the problems concerning the nature and structure of complex *Thoughts*. Nevertheless, such a project requires an even more rigorous consideration for higher-order concepts. Hopefully, we have at least taken the first steps to progress further.

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APPENDICES

A. CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Akçelik, Oğuz

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RESEARCH INTERESTS

Philosophical Logic • Philosophy of Language • Gottlob Frege • Paradoxes • Formal Epistemology • Automated Reasoning • Computational Ontology and Knowledge Representation • Critical Thinking and Argumentation Theory

EDUCATION

Degree	Institution	Year of Graduation
PhD	METU Department of Philosophy	2022
MA	METU Department of Philosophy	2014
BSc	METU Department of Biological Sciences, Biology	2012
High School	Ankara Atatürk Anadolu High School	2007

WORK EXPERIENCE

Year	Place	Enrollment
2014 – 2022	METU Department of Philosophy	Research Assistant
2014 – 2020	Laboratory for Computational Ontology (LCO) METU-TAF Modeling and Simulation R&D Center	Senior Researcher
2013 – 2014	Reo-Tek METU Technopolis, Ankara	Chief Content Editor

THESES

Ph.D. Thesis: “The Structure and Unity of Atomic Fregean Thoughts: An Explication and Emendation” METU Department of Philosophy, August 2022. Ankara, Turkey. Supervisor: Prof. Dr. Teo Grünberg.

MA Thesis: “A Solution to the Knowability Paradox and the Paradox of Idealization in Modal Epistemic Languages” METU Department of Philosophy, September 2014. Ankara, Turkey. Supervisor: Prof. Dr. Teo Grünberg.

ACADEMIC AWARDS

METU Academic Performance Award

The most successful student in the Ph.D. Program of Philosophy in the academic year 2014–2015 with cum.GPA. 4,00/4,00.

METU Best Thesis Award

METU Best Thesis Award Winner (Philosophy) in the 2013–2014 Academic Year.

METU Academic Performance Award

The most successful student in the M.A. Program of Philosophy in the academic year 2012–2013 with cum.GPA. 3,93/4,00.

METU Undergraduate Honor Roll in the academic year 2011–2012.

PUBLICATIONS

Journal Articles

1. Grünberg, T., Grünberg, D., Akçelik, O. “On the Fundamental Role of ‘Means That’ in Semantic Theorizing” *Journal of Logic, Language and Information* [AHCI], [SCI-expanded]. (Forthcoming).

2. Akçelik, O., “Nesne Ontolojisi, Varlıksal Öndayanaklar ve Bağımsız Mantıklar” [“Ontology of Objects, Existential Assumptions and Free Logics”] *Felsefe Arkivi*, 51, 1-16 (2019). [Philosopher’s Index] DOI :10.26650/arcp2019-5101 https://dergipark.org.tr/en/download/article-file/948994

Chapters in Books

1. Akçelik, O., "The Modal-Epistemic Square of Opposition and its Applications" in *Istanbul International Congress on Philosophy Proceedings Book, Volume 5: Contemporary Logical Discussions*, Nazlı İnönü (ed.) Mantık Derneği Yayınları, Istanbul, December 2018. ISBN: 978-605-80953-0-4
2. Akçelik, O., "Semantic Completeness Proofs in Predicate Logic and their Metalogical Interpretations" in *VII. Mantık Çalıştayı Kitabı* Kamer, Vedat & Ural, Şafak (eds.) Mantık Derneği Yayınları, Istanbul, December 2017. ISBN: 978-605-66311-2-2
3. Akçelik, O., "Formal Ontology and Description Logics" in *VI. Mantık Çalıştayı Kitabı* Kamer, Vedat & Ural, Şafak (eds.) Mantık Derneği Yayınları, Istanbul, December 2016. ISBN: 978-605-663111-5

Translations

1. Fisher, Alec *Gerçek Argümanların Mantığı* [Logic of Real Arguments, Cambridge University Press], (with Cenk Özdağ) İmge Kitabevi Yayınları, Ankara. (2018) ISBN: 978-975-533-888-0
2. Blackmore, Susan *Bilinç Çok Kısa Bir Başlangıç* [Consicousness Oxford A Very Short Introductions, Oxford University Press] Istanbul Kultur University, Istanbul. (2019) ISBN: 978-605-4763-74-0
3. Tanner, Michael *Nietzsche Çok Kısa Bir Başlangıç* [Nietzsche Oxford A Very Short Introductions, Oxford University Press] Istanbul Kultur University, Istanbul. (2021) ISBN: 978-605-4763-82-5

Conference Presentations

1. *Istanbul International Congress on Philosophy*, 2018, Istanbul University, Faculty of Letters Oğuz Akçelik, "The Modal-Epistemic Square of Opposition and its Applications"
2. *Logic Colloquium* 2017, August 14-20 Stockholm University, Sweden.
Oğuz Akçelik & Aziz Fevzi Zambak, "A Decision Procedure Model for Finding the Missing remise in Automated Reasoning"
https://www.math-stockholm.se/polopoly_fs/1.745278!/LC2017book.pdf
3. *Ninth Annual Cambridge Graduate Conference on the Philosophy of Mathematics and Logic*, 16th - 17th January 2016, St John's College, Cambridge, UK Oğuz Akçelik "A Solution to the Paradox of Idealization in Modal Epistemic Languages" Respondent: Dr. Arif Ahmed (Cambridge)
http://www.phil.cam.ac.uk/events/camb-grad-conf-2016

4. *British Logic Colloquium* 2015, 2-4 September, Isaac Newton Institute for Mathematical Sciences, Cambridge, UK. Oğuz Akçelik & Aliyar Özercan "In Pursuit of the Missing Premise in First Order Logic"
http://www.newton.ac.uk/seminar/20150901140015001

ACADEMIC TRAININGS

1. 2nd Urbino Summer School in Epistemology, Università degli Studi di Urbino Carlo Bo, Urbino, Italy. 21-24 August 2018 https://www.uniurb.it/academic-programs/1755283
2. The Third Nordic Logic Summer School 2017, Stockholm University, Sweden. 7-11 August 2017 https://www.math-stockholm.se/en/konferenser-och-akti/logic-in-stockholm-2/nls-summer-school-in
3. 5th Universal Logic School at UNILOG 2015, Istanbul University, 20 - 24 June 2015 https://www.uni-log.org/start5.html
4. The Second Hamburg Summerschool: 2013 Kit Fine on Truthmaker Semantics 22 - 26 of July, 2013 Department of Philosophy, University of Hamburg https://hamburgersommerkurs.wordpress.com/past-courses/about-2/

ORGANIZATIONS

1. Organization Committee Co-Chair, 4th METU Graduate Philosophy Students Congress, 11 May 2019, Ankara/Turkey
2. Organization Committee Member, 3rd National Applied Ethics Congress, METU, 28-29-30 November 2018, Ankara/Turkey
3. Organization Committee Co-Chair, 3rd METU Graduate Philosophy Students Congress, 14 December 2018, Ankara/Turkey
4. Organization Committee Member, 2nd METU Graduate Philosophy Students Congress, 7-8 December 2017, Ankara/Turkey
5. Organization Committee Chair, Teo Grünberg Lectures in Philosophy, Celebrating his 90th Birthday, 24 November 2017, Ankara/Turkey
6. Organization Committee Member 1st METU Graduate Philosophy Students Congress, 11-12 June 2016, Ankara/Turkey

TEACHING EXPERIENCE (METU)

As Primary Instructor

Phil 507 Philosophical Logic I (Elective Course) (Fall 2020, Spring 2020, Fall 2021, Spring 2021, Fall 2022)

Phil 203 Modern Logic I (Must Course) (Fall 2017, Fall 2018, Fall 2019)
Phil 350 Argumentation and Rhetoric in Philosophy (with Dr. Aziz F. Zambak and Cenk Özdağ) (Elective Course) (Fall 2017)
Phil 103 Introduction to Symbolic Logic (Must Course) (Fall 2016, Fall 2017, Fall 2018, Fall 2019)

As Teaching Assistant and occasional lecturer

Phil 551 Advanced Logic (Must Course) (Spring 2014, Spring 2015, Fall 2018)
Phil 442 Contemporary Philosophy II (Must Course) (Spring 2017, Spring 2018, Spring 2019)
Phil 203 Modern Logic I (Must Course) (Fall 2020, Fall 2021, Fall 2022)
Phil 235 Introduction to Deductive Logic (Spring 2016, Spring 2017)
Phil 108 Introduction to Philosophy II (Must Course) (for Psychology Majors) (Spring 2014, Spring 2015)
Phil 302 Systematic Philosophy (Elective Course) (Spring 2016, Spring 2017)

Talks and Seminars

1. Aziz F. Zambak, Oğuz Akçelik & Cenk Özdağ, Critical Thinking – Reasoning and Decision Making 12 October – 10 November 2019, organised by ders101.com and Turkish-American Association. [Lecture series in Turkish, invited]
2. Oğuz Akçelik, Informal Logic, Boğaziçi University, 7 August 2019. organised by Turkish Logic Society [Logic Summer School for Undergraduates, Lecture in Turkish, Invited] https://mantik.org.tr/2019/05/23/mantik-yaz-okulu-2019/
3. Oğuz Akçelik & Cenk Özdağ, Critical Thinking and Analysis, ODTÜ Ege Mezunlar Derneği, 9-10 March 2019, İzmir.
4. Oğuz Akçelik & Cenk Özdağ, "Introduction to Critical Thinking", Ankara Özel Tevfik Fikret Okulları, 2 December 2018. [Seminar for high school students in Turkish. Invited]
5. Oğuz Akçelik, Metalogic, Boğaziçi University, 17 July 2018. organised by Turkish Logic Society [Logic Summer School for Undergraduates, Lecture in Turkish, Invited] https://mantik.org.tr/2018/04/10/mantik-yaz-okulu-2018/
6. Aziz F. Zambak, Oğuz Akçelik & Cenk Özdağ, Critical Thinking and Analysis, ODTÜ Ege Mezunlar Derneği, 14-15 April 2018, İzmir. [Talk in Turkish, Invited]

MEMBERSHIPS OF SOCIETIES

Mantık Derneği [Turkish Logic Society]
http://mantik.org.tr/

The Aristotelian Society
https://www.aristoteliansociety.org.uk/

The Association for Symbolic Logic
https://www.aslonline.org/index.htm

The Scandinavian Logic Society
http://scandinavianlogic.org/

FOREIGN LANGUAGES

English, German

B. TURKISH SUMMARY / TÜRKÇE ÖZET

1. Giriş

Bu tezin konusu Gottlob Frege'nin *Düşünceler* kuramıdır. Frege'ye göre *Düşünce* olarak adlandırılan şey, zihinsel olan düşünme edimlerinden keskin bir şekilde ayrılan soyut yapılı varlıklardır. *Düşünceler,* doğruluk değerlerini ifade eden cümlelerin anlamları olarak kavramsallaştırılmaktadır. Bu tezde bu ayrımı ifade etmek için bu sözcük italik ve ilk harfi büyük olarak yazılmıştır. Bu bakımdan Frege'nin *Düşünce* terimi, onun dil felsefesinde vazgeçilmez bir öneme sahiptir. Ne var ki, Frege'nin anlambilim kuramında bu kavramın yapısı ve birliği ile ilgili bazı felsefi problemler ortaya çıkmaktadır. Bu tezde, atomsal Fregeci Düşüncelerin yapısının ve birliğinin bir açımlamasını ve güçlendirmesini sağlamayı amaçlıyoruz.

Frege'nin *Düşünceler* kuramı üç anahtar kavram üzerine inşa edilmiştir. Bu kavramlar fonksiyon-argüman çözümlemesi, nesne-kavram ayrımı ve duyum-gönderge ayrımıdır. Frege mantıkta bir devrim yaratmış ve doğal dil ifadelerinin çözümlemesi için oldukça işlevsel bir yöntem ortaya koymuştur. Bu bakımdan birinci kavram olan fonksiyon-argüman çözümlemesi, *Düşüncelerin* mantıksal-anlamsal bileşimini ortaya çıkartır. Frege tüm terimlerin ve iyi biçimlendirilmiş formüllerin birer göndergeyi ifade ettiğini savunur ve yalın ifadeleri karmaşık ifadelerden ayırt eder. Frege'ye göre karmaşık ifadeler olarak sınıflandırılan cümleler, *doygun olmayan* veya eksik ifadelerin, yani fonksiyonların veya yüklem ifadelerinin, bu ifadeleri tamamlayan tekil terimlere karşılık gelen *doygun* veya eksiksiz ifadelerle olan bileşimiyle oluşturulur. Frege ikinci olarak varlıkların ontolojik sınıflandırmasını, tüketici bir ayrımı ifade eden *nesne* ve *kavram* olarak dizgesel bir biçimde ortaya koymuştur. Bu ayrımda, tüm doygun varlıklar birer nesnedir ve tüm doygun olmayan varlıklar ise birer kavram olarak

adlandırılmaktadır. Son olarak en önemlisi Frege özdeşlik ifadeleri arasındaki bilişsel farkı açıklamak için ifadelerin duyumu ve göndergesi arasındaki ayrımı ortaya koymuştur. Tekil terimlerin duyumları nesnelerinin sunum biçimleridir; göndergeleri ise nesnelerin kendileridir. Frege bu ayrımı cümlelere de uygulamıştır. Bu bakımdan *Düşünceler* cümlelerin anlamını ifade ederken, göndergeleri doğru ya da yanlış doğruluk değerleridir. Buna göre, Fregeci atomsal *Düşünceler*, cümlelerin anlamları ve doğruluk değerlerinin birincil taşıyıcıları olmalarının yanı sıra bilgi, inanç, arzu vb. önermesel tutumların nesneleridir. Frege, *Düşüncelerin* yapısını tekil terimlerinin ve yüklemsel ifadelerin bileşimi olarak sunar. Bu bakımdan tekil terimlerin doygun duyumları, işlevsel ifadelerin doygun olmayan duyumlarını tamamlar, dolayısıyla *Düşüncelerin* birliğini sağlayan, yani bileşenleri bir arada tutan şey, doygun ve doygun olmayan parçaların bileşimidir.

Bu tezde, iki temel amaca ulaşmak için Frege'nin *Düşünce* kuramını açımlayacağız. İlk olarak bu kuramın doğal dil ifadelerinin anlam ve doğruluğunu açıklamak için en yetkin anlambilimsel kuram olduğunu savunacağız. Ne var ki, bu görüşü savunurken Fregeci *Düşüncelerin* yapısı ve birliği hakkında çeşitli felsefi sorunlarla karşılaşacağız ki bu da tezimizin ikinci temel amacını oluşturacaktır. Bu bakımdan, Frege'nin mantık ve varlık kuramına dair diğer problemlerin yanı sıra *Düşüncelerin* yapısı ve birliği ile ilgili belirli önemli problemleri inceleyeceğiz.

Fregeci atomsal *Düşüncelerin* yapısı ile ilgili sorunlar için iki önemli problem üzerinde durulacaktır. İlk problem, Bağlam İlkesi ile Bileşim İlkeleri arasındaki gerilimdir. Atomsal *Düşüncelerin* yapısıyla ilgili ikinci sorun, Frege'nin *Düşüncelerin* çözümlemesi ve ayrıştırılmasına ilişkin görünüşte çelişkili tezlerinden kaynaklanmaktadır. Frege, ilk bakışta birbiriyle uyuşmayan iki teze sahip olduğu için eleştirilmiştir. İlk tez, bir *Düşüncenin* onu ifade eden cümle ile eşbiçimli (izomorfik) olduğunu belirtir. Öte yandan, ikinci tez, yapısal olarak farklı, yani eşbiçimli olmayan iki cümlenin aynı atomsal *Düşünceyi* ifade edebileceğini belirtir. Sorun burada atomik Düşüncelerin bileşimsel yapısı için ciddi bir tehdit oluşturmaktadır, çünkü bir yanda *Düşüncelerin* benzersiz çözümlemesi diğer yanda ise *Düşüncelerin* çoklu ayrışması arasında bir tutarsızlığı imlemektedir.

Fregeci Atomsal *Düşüncelerin* birliği ile ilgili sorunların en önemlisi *at* kavramı paradoksudur. Bu paradoks, aşağıdaki cümle ile ifade edilen atomsal *Düşünceyi* ele aldığımızda ortaya çıkar:

(H) *At* kavramı bir kavram değildir.

Frege, 'at kavramı' ifadesini tekil bir terim olarak ele alır, dolayısıyla bu kavram bir nesneye gönderimde bulunur. Ancak Frege'ye göre kavram-sözcükleri bir nesneyi ifade edemez çünkü doygun olmayan nesneler ancak doygun olmayan nesnelere gönderimde bulunabilir. Dolayısıyla (H) ifadesinin olumsuzlaması doğru olmak zorundadır:

(¬H) *At* kavramı bir kavramdır.

O halde paradoks, '*at* kavramının' bir kavram olmadığı, bun yerinde bir nesne olduğu sonucunu göstermektedir. Yukarıda tartıştığımız gibi, *Düşüncelerin* birliği doygun ve doygun olmayan ifadelerin işlevsel bileşimi üzerine kuramlaştırılmıştır. Bununla birlikte, paradoks cümlesi, "*at* kavramı"nın hem doygun hem de doygun olmayan bir ifade olduğunu göstermektedir. Bu bakımdan paradoks, Frege için birbirilerini dışladığı varsayılan nesneler ve kavramlar arasındaki ontolojik ayrıma karşı bir sorun ortaya koyar. Diğer bir deyişle, bu ifade aynı anda hem doygun bir duyuma sahip ve göndergesi nesne olan tekil terimi hem de doygun olmayan bir duyuma sahip ve göndergesi kavram olan yüklemi veya fonksiyonu belirtemez. Sonuçta bu paradoks kaçınılmaz olarak *Düşüncelerin* birliğini tehdit eder. Bu nedenle *at* kavramı paradoksunu atomsal *Düşüncelerin* birliği için en önemli sorun olarak görüyoruz.

İlksel kestirimiz, Frege'nin anlambilimsel *Düşünceler* kuramının, belirli meselelere ve paradokslara karşı savunmasız olması bakımında eksik olduğudur. Bu nedenden ötürü atomsal *Düşüncelerin* yapısı ve birliği ile ilgili sorunların, Frege'nin anlambilimsel çerçevesinde belirli ayrıntılandırma ve eklemelerin zorunluluğunu gösterdiğini görüyoruz. Bu tezde, bu problemleri başarılı bir şekilde çözmek için Frege'nin anlambilimsel kuramında yaptığımız düzeltmeleri ve güçlendirmeleri sunacağız. Yine de, Frege'nin dil felsefesi hakkında sezgilerinin doğasının

özgünlüğünü korumak için *Düşünceler* kuramında mümkün olan en az değişikleri yapacak biçimde açımlamayı ve güçlendirmeyi amaçlıyoruz. Sonuç olarak, doğal dilin birbirine bağlı anlambilimi ve metafiziği çerçevesinde yaptığımız değişikliklerle Fregeci tinde tatmin edici bir *Düşünceler* kuramı ortaya koymayı amaçlıyoruz.

Şimdi kısaca bu tezde bağlı kalacağımız yöntemimiz hakkında bazı ön açıklamalara değinelim. İlk nokta, yalnızca atomsal cümleleri bu tezin çerçevesine dahil ettiğimizdir. Diğer bir deyişle, bileşik ve nicel yapılardaki cümleleri ele aldığımız konunun kapsamı dışında tutacağız. Çerçevemizde detaylandırdığımız problemler, atomsal olmayan cümlelerde çözülmesi gereken daha önemli sorunlara yol açsa da, anlambilim ve metafiziğe ait en temel problemlerin atomsal cümlelerden kaynaklandığını görüyoruz. Felsefedeki pek çok problem, oldukça basit özne-yüklem biçimselliğine sahip olan atomsal cümlelerden kaynaklanmaktadır, ancak bu temel yapının en büyük sorunların ve paradoksların kaynağı olduğunu belirtmek önemlidir. İkinci yöntemsel noktamız, alt yapıyı oluşturan mantıksal dilin seçimi hakkındadır. Frege'nin anlambilimsel kuramını açımlamak ve güçlendirmelerimizi sağlamak için her ne kadar mantıksal biçimciliğin çetrefilli dilinden kaçınmayı amaçlasak da, biçimcilikten bütünüyle kaçınmak imkânsızdır. Çerçevemizde, fonksiyonel ifadeleri λ-soyutlama biçiminde ifade etmek için λ-notasyonunu kullanıyoruz. Frege'nin anlambilimsel *Düşünceler* kuramını ortaya koymak için işlevsel bir hesaplama aracı olarak anlaşılırlığı ve netliği için λ-soyutlamayı kullanıyoruz. Belirtmek istediğimiz son nokta, Frege'nin doğal dillerin anlambilimine ilişkin sezgilerine olabildiğince sadık kalmaktır.

Artık yukarıdaki iki temel amacımız ışığında tezimizin kapsamlı özetini sunabiliriz. Tez, giriş ve sonuç bölümleri dışında tez altı bölümden oluşmaktadır. 2. ve 5. Bölümler arasındaki tezimiz ilk amacımız doğrultusunda Frege'nin *Düşünceler* kuramının tarihsel, kuramsal ve kavramsal yapısı üzerine biçimlendirilmiştir. Tezin 6. ve 7. Bölümlerinde ise Fregeci atomsal *Düşüncelerin* yapısı ve birliğine odaklanarak karışılacağımız sorunları çözüm getirerek tutarlı ve sağlam bir açımlama ve güçlendirme sunacağız.

2. Fregeci *Düşüncelerin* Tarihsel Öncülleri

Bu bölümde, Frege'yi önceleyen filozofların cümlelerin anlamları ve birincil doğruluk taşıyıcıları olarak soyut yapılı bir varlık atfettikleri önerme kavramının farklı adlandırmalardaki kuramlarını inceleyerek, Fregeci *Düşüncelerin* tarihsel arka planını sunuyoruz. Bu düşünürlerin ortaya koyduğu felsefi görüşler çerçevesinde önermelerin yapılarına ve birliğine odaklanarak bu kavramsal varlıkların doğasını ve özelliklerini derinlemesine araştırıyoruz. Ardından, bu teorileri Frege'nin *Düşünce* kavramıyla karşılaştırarak felsefe tarihi bağlamında genellikle göz ardı edilen Frege'nin kuramı ile öncülleri arasında kuramsal bir bağ kurmayı amaçlıyoruz.

Bu tarihsel arka planı ele almamızın nedenlerini şöyle sıralayabiliriz. (i) Soyut varlıklar olarak önermelerin varlığını göstermek için felsefe tarihinden kanıtlar sunmak, önermelerin temel görevleri, yani cümlelerin anlamlarını üstlenen anlambilimsel görevi ve cümlelerin birincil doğruluk taşıyıcıları olma görevi için soyut bir varlık öne sürme zorunluluğunu temellendirmek. (ii) Önermelerin doğruluk taşıyıcısı olma özellikleriyle bağlantılı olarak, mantıktaki gelişmelere paralel olarak kavramsal evrimini sunmak. (iii) Önermelerin yapısı, yani bileşenleri ve bileşimi ile bunların dilbilimsel ifadelerin yapısına, yani dilbilgisel olarak tam ve anlamlı cümlelere uygunluğuna ilişkin tarihsel iddiaları sunmak. En önemlisi de bu bölüm tarihsel bağlamda önermelerin birliği sorununun ontolojik temellerini ortaya çıkaracaktır. Bu bölümün ek bir nedeni daha var. Çoğu önermeci filozof, kendilerinden önceki görüşleri görmezden gelme eğilimindedir. Böyle bir ihmali tekrar etmemek amacıyla, Frege'nin kuramının köklerinin geçmişteki düşünce çizgisinden filizlendiğini ve böylelikle felsefe tarihinde bağımsız bir duruş olmadığını göstereceğiz. Gelgelelim, önermeler yazınında konuya ilişkin tarihsel yaklaşımlar ya uzun bir dönemin kısa özetleridir, yani antik çağdan günümüze tüm felsefe tarihini kapsarlar. Öte taraftan bazı diğer yaklaşımlar görece kısa zaman dilimlerinin uzun ve ayrıntılı incelemeleridir (örneğin bu türden yaklaşımlar yalnızca ortaçağ felsefesine veya erken dönem modern felsefeye odaklanırlar). Tüm

bu incelemelerin kendine özgü yanları vardır, ancak Church'ün çalışması dışında hiçbiri Frege ile herhangi bir bağlantı kurmamaktadır. Bu tezin kapsamında Fregeci *Düşüncelerin* tarihsel öncüllerine odaklanacağız. Frege'nin sık sık alıntıladığı diğer filozofların etkisi hakkında yazdığı gibi, biz de onun düşüncesinin öncüllerini listelemek ve onların görüşlerinin özetini sunmayı amaçlıyoruz.

Birinci alt bölümde antik dönem önerme kavramını inceliyoruz. Dilsel ifadeler dışında soyut varlıklara doğruluk taşıyıcılığının atfedilmesinin ilk kuramsallaştırma örneği olarak Platon'un diyalogları gösterilebilir. Her ne kadar Platon'un önermeleri kavramsallaştıran ilk filozof olup olmadığı bir tartışma konusu olsa da, diyaloglarındaki özellikle doğruluk ve yanlışlık hakkındaki argümanların, önermelerin bir ön kuramı olarak adlandırılabileceği kesindir. Ayrıca, diğer metafizik görüşleri arasında, Platon, muhtemelen Frege üzerinde en baskın etkiye sahiptir. Özellikle önermelerin gerçekçiliği, soyutluğu, nesnelliği, dil ve zihinden bağımsızlığı söz konusu olduğunda Platon günümüz gerçekçi önerme kuramları için en güçlü ilham kaynağıdır.

"Doğru ya da yanlış olarak adlandırılabilecek olan şey nedir?" sorusu doğal dillerin doğası hakkında sorulan ilk sorulardan biridir. Yanlış yargıların ve inançların var olması sorunu, başta Sofistler olmak üzere birçok filozofun zihnini meşgul etmiştir. Bu bağlamda, yargının nesneleri ve inanç tutumları hakkında ilk tartışan filozof Platon'dur ve diyaloglarından ikisi, yani *Theaetetus* ve *Sophist,* konumuz için merkezi öneme sahiptir. Doğruluk taşıyıcılarının doğasına ilişkin soru, felsefi tartışmanın "Bir yargı nasıl yanlış olabilir?" sorusuna odaklandığı *Theaetetus* diyaloğunda bilgi kavramını tanımlama çabasında ortaya çıkar. Platon'un düşünceler ve konuşma arasında benzer bir tanımlama yaptığı diyaloğu *Sophist*'te ele alınan şey ise "Yanlış bir söylem ve düşünce nasıl mümkün olabilir?" sorusudur. Platon her iki diyaloğunda da doğruluk değeri sahibi olma özelliğini *logos* kavramına yükler. Ayrıca bu kavramı öğeleri adlar olan yapısal bir birleşim olarak ele alır. Platon bu yapıyı birlikte dokunulmuş olma benzetmesiyle açıklamaktadır. "Arslan geyik at" adları ve "yürümek koşmak uyumak" eylemleri birer cümle ifade etmemektedir. Bu bakımdan önermelerin birliği sorusu ancak Platon'un ifade ettiği

şekilde bu adların ve eylemlerin belirli bir şekilde dokunmasıyla ortaya konabilir ki ancak böylelikle anlamlı bir bütün oluşturarak doğru ya da yanlış birer doğruluk değerine sahip olabilsinler. Özünde Fregeci *Düşünceler* Platon'un *logos* kavramıyla yapısal birlik ve doğası üzerine özellikler bakımında önemli benzerlikler göstermektedir ki neredeyse Platon'un gerçekçiliğinin hemen hemen bütün yanlarını barındırmaktadır.

Aristoteles de söylemin öğeleri konusunda benzer bir ayrımı benimsemiştir. Yüklemlenebilir ifadelerin temel bir kuramını ortaya koymuş ve nesne ile yüklem arasında ilişkilendirmeyi kurarak cümlelerin mantıksal yapısını ortaya koymuştur. Aristoteles'e göre, doğruluk ve yanlışlık öncelikle düşüncelere ait bir niteliktir ve bir konuşmanın doğruluğu veya yanlışlığı bu bakımdan türevseldir. Aristoteles ayrıca dilin uzlaşımsal olduğu görüşüne de bağlıdır, yani düşünceler herkes için aynı olsa da konuşulan kelimeler farklı olabilir. Aristoteles tüm cümlelere bir anlam yükler. Bu cümleler içinde doğruluk ve yanlışlık bulunan bildirimsel veya açıklama yapan cümleleri, ne doğru ne de yanlış olan sorgulamalardan ayırır. Özne-yüklem yapısına sahip bu belirli bildirimsel cümle sınıfının adı, *apophansis* veya *apophantikos logos*'tur.

Stoacılar, özellikle koşullu ve diğer karmaşık formları ve çıkarım kurallarını içeren argümanlar hakkında ayrıntılı çalışmalar yürütmüştür. Dolayısıyla, Stoa mantığı, Aristotelesçi Terim Mantığı'ndan ayırarak bir tür önerme mantığı olarak sınıflandırılır. Stoacılar konuşmanın öğelerini ses, konuşma ve söylem, yani anlamlı söz olarak ayırırlar. Stoacılar, mantığın uygun bir konusu olarak anlam ifade eden cümleleri *lekton* olarak adlandırmışlardır.

Ortaçağ filozofları, doğruluk değerlerini, cümlelerin anlamsal önemini, yargıların içeriğini ve inançları soyut varlıklar olarak atfetmek için *propositio* terimini yaygın olarak kullandılar. Boethius'un *oratio verum falsumve significans* olarak adlandırılan tanımı takiben, çoğu Ortaçağ mantıkçısı bu terimi yazılı, sözlü ve zihinsel cümle türlerine atıfta bulunmak için kullandı. Ortaçağ filozofları da bu kullanımı, dilbilgisel biçimlere doğruluk değerleri ve anlam atfeden cümleci görüşlerinin bir kökü olarak gördüler.

Erken modern dönemde, önermelerin nitelikleri ve doğasına ilişkin görüşler değişiklik gösterse de, varlıkları genellikle tartışmasız kabul edilir. Bu bölümde, eserleri bu dönemde çağdaş önerme anlayışının gelişimi için bir mihenk taşı olarak kabul edilebilecek başlıca filozofları, yani Descartes, Leibniz ve Kant'ı inceleyeceğiz. Bu bölümün son kısmında ise çalışmaları Frege için ayrı bir önem taşıyan Bolzano'ya odaklanacağız ve önerme sözcüğünün kavramsal karakterinin tartışmasıyla bitireceğiz. Önerme terimine karşılık gelen kullanımlar için çoğu dil, Latince *propositio* sözcüğünün türevlerini benimsemiştir. Ancak Almanca konuşan filozoflar bu kullanımı izlememiştir. Bolzano ve Frege, soyut, nesnel ve dilden bağımsız karakteri ifade etmek için sırasıyla '*Sätze an sich*' ve '*Gedanke*' sözcüklerini kullanmışlardır. Adlandırma uzlaşımları ne olursa olsun, mantıksal-anlamsal karakterdeki önerme sözcüğü her zaman soyut varlığı ifade eder.

3. Frege'nin *Düşünceler* Kuramının Bir Açımlaması

Bu bölümde, Frege'nin *Düşünceler* kuramının arka planını, doğal dilin anlambilimi ve metafiziği üzerine görüşleri çerçevesinde araştırmayı amaçlıyoruz. Bu bağlamda, Fregeci *Düşünceler* kavramını açıklamak için birbirine ilintili iki tür felsefi çözümleme uyguluyoruz. İlk olarak atomsal *Düşüncelerin* anlambilimsel çözümlemesiyle cümlelerin anlamlarını ve doğruluk koşullarını açıklıyoruz. İkinci olarak atomsal *Düşüncelerin* yapısı ve birliğine ilişkin açımlamaları ontolojik bir çözümlemeyle ortaya koyuyoruz. Sonuç olarak, Frege'nin yukarıda bahsedilen anahtar kavramlarını tanıtarak onun dil felsefesinin mantıksal-anlamsal ve varlıksal yönlerini ayrıntılı bir şekilde sunmayı amaçlıyoruz. Bu bölümde son olarak, atomsal *Düşüncelerin* yapısı ve birliği ile ilgili temel sorunları tartışıyoruz.

Çözümleyici açımlamamıza Frege'nin *Begriffsschrift* eserinden başlayacağız. Frege, kavram yazısı adını verdiği yeni bir mantık dili ortaya koymak için yeni bir biçimsel dil sunar. Bu yeni yöntem aynı zamanda Frege'nin doğal dillere olan yaklaşımını baskın bir biçimde belirler. Frege'nin açık ve yalın bir kavramsal mantık dili yaratmaya yönelik felsefi güdülenimi Leibniz'in *lingua*

characteristica ve *calculus ratiocinator* fikirlerinden köken almaktadır. Hem Frege hem de Leibniz doğal dilin mantıksal ilişkilerini, matematik çıkarımlarını ve akıl yürütmeleri ifade etmeyi amaçlamaktadır.

Frege, kendi zamanına kadar baskın görüş olan Aristotelesçi tasımlar kuramının savunageldiği özne-yüklem çözümlemesini reddederek mantıkta devrim yaratmış ve bu kuram yerine fonksiyon-argüman yapısını savunmuştur. Fonksiyonlar matematikteki fonksiyonlar kavramına benzer ve doğal dil ifadelerine de uygulanabilir. Bu tezde odak noktamızı, Frege'nin fonksiyon-argüman yapısının doğal dil cümlelerine uygulamasıyla sınırlayacağız.

Frege, eksiksiz ifadelerin içeriğinin ilişkilerin bütününü temsil eden sabit bir bileşene ve bu ilişkilerde yer alan nesnelere gönderimde bulunan bir değişkenle bölünebileceğini ileri sürer. Bu değişkenler diğer başka ifadelerle değiştirilebilir. Frege, bu sabit bileşeni fonksiyon olarak adlandırır. Değişkenler de fonksiyon içinde yer alan argümanların yerini doldurur. Diğer bir değişle,

Chrysippus bir filozoftur

önermesi ‘λ*x* (*x* bir filozoftur)’ soyutlamasının ifade ettiği fonksiyona ve bu fonksiyonun argümanı olan ‘Chrysippus’ terimine ayrışır. Önermenin fonksiyon kısmı bir kavramı ifade eder ve önermenin argümanı değiştiğinde sabit kalır. Bu bakımdan argümanların değişmesinin önermenin anlamını ve doğruluk değerini değiştireceği açıktır.

Frege’nin anlambilim kuramındaki ikinci önemli kavram, nesneler ve kavramlar arasındaki ayrımdır. Frege bu ontolojik ayrımı iki farklı türden varlıklar yani nesneler ve fonksiyonlar üzerinde ortaya koyar. Frege için nesneler eksiksiz, kendi kendine var olan varlıklardır, gelgelelim fonksiyonlar böyle değildir. Bu ayrıma göre, fonksiyonlar “eksik, takviyeye ihtiyaç duyan veya doymamış” olarak nitelenir. Fonksiyonu argümanla tamamlamanın veya doyurmanın sonucuna bu fonksiyonun değeri denir. Ardından, Frege bu kuramını doğal dil cümlelerine uygulayarak anlam içeriğini duyum [*Sinn*] ve gönderge [*Bedeutung*] olarak ayırarak temel bir çerçeve sunar. Frege, bir ifadenin anlamının bir *Düşünce* içerdiğini ve bu

Düşüncenin doğru ya da yanlış olduğunu iddia eder. Bu bakımdan *Düşünceler* cümlelerin duyumlarıdır ve göndergeleri de birer doğruluk değeridir. Frege, Doğru ve Yanlış olan iki doğruluk değerinin birer nesne olduğu iddia eder.

Frege en önemli eserlerinden biri olan "Duyum ve Gönderge Üzerine" makalesinde daha önce ele aldığımız kavramlarıyla birlikte, her türlü doygun dilsel ifadeye uygulanan anlambilim kuramının önemli sonuçlarını ortaya koymaktadır. Ayrıca, içlemsel bağlamlardaki önermesel tutumların dizgesel çözümlemesini sunar ve bu ayrımını bu türden ifadelerin geçtiği cümlelere uygular. Frege, *Düşünceler* kuramının temel unsurlarını duyum ve gönderge arasındaki ayrım üzerine temellendirir. Bu ayrım, Frege'nin olgun felsefesindeki en ünlü ve etkili ayrımdır. Frege, özdeşlik önermeleri üzerine iki türden özdeşlik bulmacası ortaya koyar. İlk bulmaca, eşgönderimsel terimler arasındaki özdeşlik ifadeleri hakkında, ikincisi ise yan tümcelerden oluşan cümlelerdeki önermesel tutum ifadelerinin anlamı ve doğruluğu hakkındadır. Her ikisinde çözüm olarak, Frege, bir cümledeki tekil terimlerin (özel adların veya betimlemelerin) yalnızca gönderimde bulundukları şeylere dayanarak cümlelerin anlamının açıklanamayacağını gösterir. Böylelikle dilsel ifadelerin duyumları ve göndergeleri arasında ortaya konulan bu ayrım özetle, göndergenin tek başına bilişsel değeri yakalayamayacağını gösterir. Dolaysıyla Frege, anlamın göndergeden farklı olduğu sonucuna varır.

Frege'nin kuramının birincil görevi, karmaşık dilsel ifadelerin anlamını açıklamaktır. Bu bakımdan duyum-gönderge ayrımını özel adlardan cümlelere kadar genişletmiştir. Frege'ye göre, *Düşünceler* cümleler tarafından ifade edilen duyumlardır. O halde bir bildirim cümlesinin anlamı, ifade ettiği *Düşüncedir* ve göndergesi ise doğruluk değeridir. Frege, eserlerinde doğruluk kavramını her zaman verili bir ilkel kavram olarak kabul etmiştir. Daha sonraki çalışmalarında "doğru" sözcüğünün tanımına odaklanır, ancak son tahlilde açıklanabilir bir kavram olmadığı sonucuna varır.

Son olarak Frege'nin *Düşünce* kavramının doğasına değinelim. Frege, mantık ve matematikte herhangi bir psikolojik kavramın kullanılmasına şiddetle karşı çıkması nedeniyle, *Düşünceleri* herhangi bir öznel fikirden ayırır. Frege

eserlerinde hem duyumlara hem de *Düşüncelere* soyut bir varlık kategorisi atfeder. Dahası, *Düşüncelere* hem zihinsel hem de fiziksel varlıklardan ayrı olarak üçüncü bir âlemde var olan kendine özgü bir varlıksal doğa atfetmiştir. Frege duyulara bir nesnellik ölçütü atfettiği için, aynı ölçüt Düşünceler için de geçerlidir. Buna göre düşüncelerin varlığı hem dilden hem de zihinden bağımsızdır. Düşünceler, zihinsel durumlardan ve beyindeki fiziksel gerçekleşmelerinden bağımsızdır. Daha önce ele aldığımız gibi, *Düşünceler*, Frege'nin öznel olduğunu düşündüğü düşünme eyleminden de farklıdır. Düşünceler, birinin bilip bilmediğine ya da inanıp inanmadığına, hatta onlar hakkında düşünüp düşünmediğine bakılmaksızın, doğruluk değerlerine sahiptir. Kişiler, *Düşünceleri* düşünme yetileriyle ile kavrayabilirler, gelgelelim varlıkları düşünürlerinden bağımsız olarak ebedidir. Düşünceler dilin kullanımıyla var olmaz; dolayısıyla varlıkları da yok edilemez. Düşünceler düşünme eylemiyle yaratılmaz, daha ziyade keşfedilirler. Sonuç olarak, Frege'nin nesnellik ölçütlerinin gerçekten elde edilebilir bir ölçüt olup olmadığı başka bir tartışma konusudur, yine de bunun yüksek bir standart olduğunu, çok anlamlılık, anlamda belirsizlik ve muğlaklıktan arınmış bir tür ideal dil düşüncesini ifade ettiğini söyleyebiliriz.

4. Önermeler ve *Düşünceler*

Bu bölümde, Fregeci *Düşüncelere* benzer yaklaşımlar olan ve topluca önerme terimi altında sınıflandırılan temel bir kurama odaklanıyoruz. Bu bölümün amacı, Fregeci *Düşüncelerin* önerme olduğunu göstermektir. Bu bağlamda, önermelerin doğasını ve özelliklerini çözümleyerek bir önerme kuramı sunuyoruz. Ayrıca, önermelerin varlığına dair bazı argümanları ele alarak bu argümanları Frege'nin *Düşünceler* kuramı bağlamında detaylandırıyoruz. Bu bölümün son kısmında, önermelerin yapısı ve birliği ile ilgili benzer sorunları inceliyoruz.

Bu bölüme, "cümle" ve "önerme" terimlerini tanımlayarak önemli bir terminolojik açıklama ile başlayacağız. Bu bağlamda cümle, belirli bir doğal veya yapay dilin dilbilgisi kurallarına göre oluşturulmuş eksiksiz bir kelime dizisi olarak

tanımlanır. Bir önerme genellikle, her ikisi de "cümle" terimini içeren iki karakteristik özellik ile tanımlanır: İlk olarak, önerme, cümlelerin anlamlarını karakterize eden ifadedir. Başka bir deyişle, önermeler cümlelerle belirtilen, iddia edilen, inanılan, reddedilen vb. ifadelerdir. İkinci olarak, önermeler doğruluk değerlerinin birincil taşıyıcılarıdır. Bu özellik, bir cümlenin doğru veya yanlış olma özelliğini karakterize eder.

İkinci bölümde ele aldığımız üzere, önerme terimi çok anlamlı bir şekilde kullanılmaktadır. Carnap'a göre, önerme terimi iki farklı kavramı tanımlar: bir bildirim cümlesi ve bir (bildirimsel) cümle ile ifade edilen (gösterilen, temsil edilen) şey. İlkine göre, "Snow is White", "Kar beyazdır", "Schnee ist weiss" cümleleri farklı önermeleri ifade eder. Öte yandan Carnap'ın tercih ettiği ikinci tanıma göre İngilizce, Türkçe ve Almanca bu cümleler aynı önermeyi ifade etmektedir. Church, bu anlamda önermelerin ne fiziksel ne de dilsel bir varlık olduğunu, dilden soyutlama yoluyla elde edildiğini belirtir. Bu durumda yukarıdaki her üç cümle de aynı önermeyi ifade eder, çünkü aynı anlama ve doğruluk koşuluna sahiptir. Eşanlamlılık veya anlam aynılığı kavramının, aynı önermeyi ifade etmek için farklı cümlelerin, "Mehmet Ayşe'yi seviyor" ve "Ayşe, Mehmet tarafından seviliyor" gibi aynı dilde farklı biçimlerde ifade edilmesi için öncül bir kavram olduğuna dikkat etmeliyiz. Buna ek olarak "Bugün Cuma" gibi aynı cümle farklı bağlamlarda, örneğin bahsi geçen gün Cuma günü olduğunda doğru önermeyi, haftanın diğer günleri için ise yanlış bir önermeyi ifade eder.

Bu bölümün devamında, önermelerin anahtar anlambilimsel görevlerini sunacağız. Bu, önermelerin *Düşünceler* ile aynı görevleri paylaştığını gösterecektir. Aynı zamanda, sonraki bölümde inceleyeceğimiz gibi, farklı önerme görüşlerinin etkili sunumunu da kolaylaştıracaktır. Ele aldığımız yalın Klasik Önermeler Kuramı'na göre, önermeler

(R1) Cümlelerin anlamları

(R2) Doğruluk değerlerinin birincil taşıyıcıları

(R3) Bu cümlelerle ifade edilen (içlemsel) tutumların nesneleridir.

Bazı filozoflar kendi dil kuramsal ihtiyaçları için önermelere ek görevler yüklemişlerdir. Bunlar aşağıdaki gibidir:

(AR1) Zorunluluk ve olanaklılık gibi kipsel özelliklerin taşıyıcıları

(AR2) Argümanların mantıksal gerektirimi ve geçerliliği gibi ilişkilerin nesneleri

(AR3) Cümlelerin bilgi içeriği

(AR4) İddia edimlerin nesneleri ve diğer bilişsel edim veya olay türleri

(AR5) Bir konuşmada ortak zemin

Önemli bir soru, tek bir önerme kuramının tüm bu görevleri açıklayıp açıklayamayacağıdır ki önermelerin görevlerini çoğaltmaktan doğan bir takım önemli sorunlar ortaya çıkmaktadır.

Bu bölümün devamında önermelerin varlığı için bazı önemli argümanlar sunuyoruz. Önermeci filozoflar genellikle önermelerin varlığını bir argüman olmaksızın ontolojik bir önkabula dayandırırlar. Dikkat çekici bir şekilde, çok az filozof onların varlığına ilişkin genel argümanlar sunmuştur. Bu bölümde, önermelerin anlam kuramlarındaki görevlerine dayanan argümanlar ve ontolojik argümanlar olmak üzere iki argüman türünü ele alıyoruz. Önermelerin varlığına ilişkin en yaygın sunulan argüman, felsefe kuramlarında önermelerin görevlerine dayanır. Pek çok önermesel-gerçekçi filozof önermelerin varlığını yalın görevlerine dayandırarak gerekçelendirme sunar. Bu açıdan önermeler dil felsefesinde anlam ve doğruluk kavramlarını çözümlemek ve açıklamak için kullanılan soyut varlıklar olarak kabul edilir. Gelgelelim, önermelerin varlığını gerekçelendirmek için önermelerin görevlerine dayanan bazı karşıt görüşler vardır. Bazı filozoflara göre, bu rolleri yerine getiren bu tür şeylerin varlığına dair şüpheler vardır. Bu filozoflara göre, cümlelerin anlamları, doğruluk değerlerinin taşıyıcıları ve tutumların nesneleri için soyut bir varlık önermek gereksiz bir metafizik çoğaltımcılıktır. Bu karşıt görüşler Donald Davidson'un itirazı, cümlecilik ve adcılıktır. Önermeci filozofların bir diğer önemli amacı, önermelerin varlığına ilişkin ontolojik argümanlar

vermektir. Böylece bu filozoflar, soyut varlıklar olarak önermelerin varlığına tatmin edici bir açıklama vermek için ontolojik argümanları yorumlarlar. Bu alt bölümde ayrıca cümlenin farklı ifadeleri arasındaki sezgisel ortaklıklara başvurarak önermelerin varlığına ilişkin ortaya konulan iki ontolojik argüman ele alınmaktadır. İlki, birliğin çokluk üzerine argümanı ve onun üstdilsel çeşitlemesi, ikinci de Jeff Speaks'in önermelerin varlık üzerine nicelenmesi argümanıdır.

Bu bölümün son alt bölümünde önerme kuramları için yeterlilik ölçütü olarak ele alacağımız, önermelerin yapısı ve birliği ile ilgili problemlere değineceğiz. Birinci alt bölümde, önermelerin yapısına ilişkin temel sorunları ele alacağız, ikinci bölümde ise önermelerin birliğine ilişkin sorunları ele alacağız. Bu problemler önermelerin hem anlambilimsel hem de varlıksal özellikleri ve doğaları için güçlendirici açıklamalara zemin sağlayacaktır. Ardından bu problemler ışığında 6. ve 7. bölümlerde tezimizin ikinci temel amacı doğrultusunda Frege'nin *Düşünceler* kuramını inceleyeceğiz.

5. Önerme Kavramına dair Çağdaş Görüşlerin Sınıflandırılması: Güncel Önerme Kuramları

Önermeler yazınında yapısal önermelerin iki ana türü vardır. Genel olarak Fregeci görüşe göre *Düşünceler*, önermelerin yapısının onları ifade eden cümlelerin yapısını yansıttığı varsayımıyla bileşimsellik ilkelerine bağlılığı nedeniyle yapısal önermeler olarak sınıflandırılır. Yaygın olarak kabul edilen Russellcı yapısal görüşe göre, önermelerin bileşenleri bireyler, özellikler ve ilişkilerdir. Russell'a göre bu bileşenler fiillerin önermeler içindeki bağlayıcı katkısıyla bir arada tutulur. Yapısal önerme görüşüne karşıt bir görüş, cümlelerin ifade ettiği önermelerin ya olanaklı dünyalar kümeleri olarak ya da olanaklı dünyalardan doğruluk değerlerine tanımlanan fonksiyonlar aracılığıyla ifade edilen Önermelerin Olanaklı Dünyalar Kuramıdır. Önermeler ayrıca diğer varlık türlerine indirgenip indirgenemeyeceklerine göre sınıflandırılır. İndirgemeci görüşlere göre, önermelerin kümeler, dilsel ağaçlar, bilişsel edimler veya olay türleri vb. gibi diğer

varlık türleri ile tanımlanabileceğini savunur. Öte yandan, indirgemeci olmayan veya ilkelci görüşler olarak adlandırılan görüşlere göre önermeler açıklayıcı diğer varlıklara indirgenemezler, bunun yerine önermeler ilkel ve kendine özgü varlıklardır. Bu bölümde, bu görüşü temsil eden Cebirsel Yapılar ve Önermelerin İndirimci Görüşleri olmak üzere iki ana kategoriye odaklanıyoruz. Ardından tüm bu kuramları Frege'nin *Düşünceler* kuramıyla karşılaştırarak içerdikleri önemli problemler dolayısıyla doğal dil ifadelerinin anlam ve doğruluğunu açıklama konusunda yetersiz olduklarını göstererek Frege'nin kuramının bu bağlamda en tutarlı ve yetkin kuram olduğunu gösteriyoruz.

Yukarıda belirtilen önerme kategorileri, önermelere yönelik çağdaş yaklaşımları sınıflandırmamıza rehberlik edecektir. Bu bağlamda, bu görüşlerin en önde gelen temsilcilerini incelemekteyiz.

i) Yapısal indirgemeci görüşler: Russellcı ve neo-Russellcı Önermeler

ii) Yapısal ilkelci görüşler: Zalta

iii) Yapısal olmayan indirgemeci görüşler: Önermelerin Olanaklı Dünyalar Kuramı

iv) Yapısal olmayan ilkelci görüşler: Bealer, İndirimci Kuramlar

Bu bölümde, yukarıda temel sınıflandırmayı iki açıdan ele alıyoruz. İlk olarak, önermelerin yapıya sahip olmasına ilişkin görüşlerine göre önermelerin bileşenlerinin bireyler, özellikler ve ilişkiler olan yapısal Russellcı ve yeni-Russellcı kuramları ele alıyoruz. Bu bakımdan Russell'dan esinlenen kuramlar yapısal indirgemeci görüşler olarak sınıflandırılmaktadır. Ardından, yeni-Russellcı görüşün iki önemli temsilcisine, yani Jeffrey King ve Jeff Speaks'e odaklanıyoruz. King'e göre önermeler üst-dilsel olguları ifade ederken, Speaks'e göre önermeler birer tekil özelliklerdir. İkinci olarak yapısal ilkelci görüşler olarak sınıflandırılan Edward Zalta'nın önermeler görüşüne odaklanıyoruz. Zalta'ya göre önermeler yapısal parçalara sahip olan birer cebirsel yapıdır. Üçüncü olarak yapısal olmayan indirgemeci görüşler bağlamında, önermelerin kümeler gibi diğer ontolojik varlık

türlerine indirgenebilir olduğu savunan Önermelerin Olanaklı Dünyalar Kuramı'nı ele alıyoruz. Bu görüşün iki savunucusu olan David Lewis ve Robert Stalnaker'ın görüşlerine odaklanıyoruz. Lewis'in görüşüne göre cümlelerle ifade edilen önermeler birer olanaklı dünyalar kümesidir. Stalnaker'e göre ise önermeler olanaklı dünyalardan doğruluk değerlerine tanımlanan fonksiyonlar olarak tanımlanır. Dördüncü olarak, önermelerin başka varlıklara indirgenemeyeceğini, aksine önermelerin ilkel ve kendine özgü varlıklar olduğunu söyleyen indirgemeci olmayan veya ilkelci görüşleri ele alıyoruz. Bu bölümün alt bölümlerinde bölümlerinde, bu görüşün iki ana savunucusuna odaklanıyoruz. Bu görüşlerden ilki Bealer'ın Cebirsel Önermeler Kuramıdır. İkinci ana görüş Stephen Schiffer'in Pleonastik Önermeler Kuramı ve İndirimci Kuramlar olarak sınıflandırılan Trenton Merricks ile Lorraine Juliano Keller'in İndirimci Önermeler kuramlarıdır. Bir önceki bölümde ele aldığımız önemli felsefi problemler bu bölümde Frege'nin *Düşünceler* kuramını çağdaş önerme teorileriyle karşılaştırmak için ölçütümüz olacaktır. Buna göre, yukarıda belirttiğimiz her bir önerme görüşü türü için, Frege'nin *Düşünceler* kuramının, doğal dil cümlelerinin anlamını ve doğruluğunu açıklamak için kapsamlı bir çerçeve sunma ve kuramsallaştırma konusunda en uygun ve tatmin edici açıklama olduğunu savunuyoruz.

6. Atomsal Fregeci *Düşüncelerin* Yapısı

Bu bölümde, atomsal *Düşüncelerin* yapısı ve bileşimine ilişkin görüşleri açıklamayı amaçlıyoruz. İlk olarak, Frege'nin Bileşimsellik İlkelerini, yani Fonksiyon-Argüman Birleşimsellik İlkesi ve Parça-Bütün Bileşimsellik İlkesini sunuyoruz. Ardından, *Düşüncelerin* yapısıyla ilgili iki soruna odaklanıyoruz. Bağlam İlkesi ile Bileşimsellik İlkeleri arasındaki görünür gerilime ilişkin sorun bunların ilkidir. Bu sorunun çözümü için, Frege'nin kendi yazılarına dayanan yorumumuzda her iki ilkeyi de tutarak uzlaştırıcı bir yorumlama sağlıyoruz. Düşüncelerin yapısı ile ilgili ikinci sorun, Frege'nin Düşüncelerin analizi ve ayrıştırılmasıyla ilgili Dummett-Bell Problemi olarak adlandırılan görünüşte

çelişkili olan tezleridir. Bu sorun, *Düşüncelerin* yapısı ve karşılık gelen cümle yapısıyla özdeşliği için ciddi bir tehdit oluşturmaktadır. Dummett, Bell, Kemmerling ve Penco tarafından önerilen çözümleri gözden geçirerek literatürdeki çözümlere ilgili eleştirilerimizi sunuyoruz. Bu bölümün son kısmında, Hodes'un Fregeci *Düşüncelerin* çokbiçimli yapısı kavramını izleyen yorumlayıcı bir güçlendirmeyle çözümümüzü sunuyoruz. Ancak bizim konumumuz Hodes'tan önemli ölçüde farklıdır, bu bağlamda onun kuramına yönelik eleştirilerimizi de sunuyoruz. Her iki tezi de *Düşünceler* kuramında koruduğumuz için çözümümüzün Frege'nin özgün konumu olduğunu savunuyoruz. Çerçevemizde, *Düşüncelerin* çok biçimli yapısının, çoklu ayrıştırma yoluyla olası her bir bileşeni ortaya çıkardığını ve son çözümlemede ayrışmanın, aynı atomik Düşüncenin nihai benzersiz bileşenlerini ortaya çıkardığını gösteriyoruz. Yaklaşımımızı "Jüpiter, Mars'tan büyüktür" önermesini kullanarak özetleyebiliriz. Bu önermenin hepsi doygun-doygun olmayan ayrımına uyan üç farklı çözümlemesi vardır. Bu çözümlemeler şu şekildedir:

(i) Doygun 'Jüpiter' duyumu ve 'ξ, Mars'tan daha büyüktür' doygun olmayan duyumu

(ii) Doygun olmayan 'Jüpiter ζ'dan daha büyüktür' duyumu ve doygun 'Mars' duyumu

(iii) 'Jüpiter' doygun duyumu, 'Mars' doygun duyumu ve 'ξ, ζ'dan büyüktür' doygun olmayan duyumu

Bize göre bu farklı çözümlemelerin gösterdiği şey, bir *Düşüncenin* yapısını çözümlemenin veya ayrıştırmanın üç farklı olanaklı yolu olduğudur. Görülmektedir ki (i) – (iii) çözümleri her ne kadar ilk bakışta farklı görünse de, (i) ve (ii) çözümlemeleri bir adım daha ileri götürülürse, (iii) ile aynı duyum bileşenlerini elde edileceği kolaylıkla görülecektir. Son çözümlemenin tüm duyum bileşenleri – hem doygun hem de doygun olmayan parçaları için – basit olduğundan, daha fazla ayrıştırılamazlar. O halde, aynı atomsal *Düşüncenin* alternatif çözümlemeleriyle

nihai duyum bileşenlerine ulaşırız. Bu da, Fregeci *Düşüncelerin* çok biçimli bir yapıya sahip olduğu görüşünü kanıtlar.

7. Atomsal Fregeci *Düşüncelerin* Birliği ve *At* Kavramı Paradoksu

Atomsal *Düşüncelerin* yapısını inceledikten sonra, 7. Bölümde bileşenlerinin nasıl bir arada tutulduğunu açıklamaya odaklanıyoruz. Frege, *Düşüncelerin* birliğini, bunlara karşılık gelen doygun ve doygun olmayan kısımlarının işlevsel bileşimi olarak kurar. Frege, duyum-gönderge ayrımının *Düşüncelerin* bileşenlerinin hem doygun hem de doygun olmayan kısımları için geçerli olduğunu savunur. Buna göre, doygun ifadelerin duyum ve göndergelerini açıklar ve aynı ayrımın doygun olmayan ifadeler için de geçerli olduğunu belirtir. Bununla birlikte, bu ifadelerin duyum ve göndergelerini nasıl kavranacağımıza dair pek bir açıklama getirmez. Frege'nin bu noktadaki kuramsal açıklama eksikliği, yukarıda sunduğumuz *at* kavramı paradoksu ile sonuçlanır. Biz de paradoksun ortaya çıkardığı bu çok önemli çatışma nedeniyle Fregeci atomsal *Düşüncelerin* birliğinin, *at* kavramı paradoksu çözülmeden kurulamayacağını savunuyoruz. Bu bölümde, paradoksu ayrıntılı olarak açıkladıktan sonra, Geach, Dummett, Wiggins, Wright, Noonan, Hale ve MacBridge tarafından verilen önemli çözüm önerilerini inceliyoruz. Buna göre, bahsi geçen çözümlerin her biri için sunduğumuz eleştirel inceleme sonucunda, tüm bu çözümlerin paradoksu çözmek için belirli yetkinliklere sahip olmalarına rağmen, *Düşüncelerin* birliğini açıklamak için yeterli olmadıklarını ve Frege'nin kendi özgün anlambilimsel görüşlerini korumadıklarını görüyoruz.

Bu bakımdan, Frege'nin özgün görüşleri ışığında öncelikle doygun olmayan ifadelerin duyumları ve göndergeleri için düzeltici çerçevemizi sunuyoruz. Fonksiyonel ifadeleri birbirine dönüştürülebilir λ-soyutları olarak ifade etmek için λ-hesabını kullanarak sırasıyla özel isimlerin, yüklemlerin ve cümlelerin duyumlarını ve göndergelerini tanımlıyoruz.

Tezde sunduğumuz çerçevede yüklemlerle ifade edilen kavramların, yani kavram fonksiyonlarının ve yüklemlerle ifade edilen duyumların, yani duyum

fonksiyonlarının biçimsel tanımlarını veriyoruz. Buna göre '[λ*x*. *x F*'dir]' funktoru doğal dil yüklemi olan '*ξ F*'dir' yüklemini ifade eder. Örneğin, bir λ-soyutlaması olan '[λ*x*.*x* gezegendir]' funktoru '*ξ* gezegendir' ifadesine karşılık gelir. '*N*' nesne dili olan *L* dilinde adları ifade etmek üzere, dönüşüm yasaları şu üç kural üzerinde tanımlanır:

I. [λ*x*.*x F*'dir] *cnv* [λ*y*.*y* is *F*'dir]
II. [λ*x*.*x F*'dir] (*N*) *cnv N, F*'dir
III. *N, F*'dir *cnv* [λ*x*.*x F*'dir] (*N*)

Gönderge fonksiyonlarını şu şekilde ifade edilebilir:

1. ⟦*N* ⟧ = '*N*' adının göndergesini, yani bir nesneyi
2. ⟦λ*x*. *x F*'dir⟧ = '[λ*x*. *F*'dir]' yükleminin göndergesini, yani bir kavramı
3. ⟦*N F*'dir ⟧ = '*N, F*'dir' cümlesinin göndergesini, yani doğruluk değerlerini

ifade eder. O halde, şu ifadeyi elde ederiz.

⟦λ*x*. *x F*'dir⟧ (⟦*N* ⟧) = ⟦*N*, *F*'dir ⟧

Duyum fonksiyonları şu şekilde ifade edilebilir:

1. ⟨*N*⟩ = '*N*' adının duyumu, yani sunum biçimini
2. ⟨λ*x*.*x F*'dir ⟩ = '[λ*x*.*x* is *F*]' yükleminin duyumunu, yani duyum-fonksiyonunu
3. ⟨*N, F*'dir ⟩ = '*N, F*'dir' cümlesinin duyumunu, yani bir *Düşünceyi*

ifade eder. O halde, şu ifadeyi elde ederiz.

⟨λ*x*.*x F*'dir ⟩ (⟨*N*⟩) = ⟨*N*, *F*'dir ⟩

At kavramı paradoksunun çözümü için Russell'ın kavramların kullanılan ve bahsedilen arasında yaptığı ayrımına başvuruyoruz. Bu anlamda, Şeffaf İçlemsel Mantık için Süreçsel Anlambilim çerçevesinde genişletilerek "Bucephalus bir attır" cümlesinin oluşumunda "*at* kavramının" işlevsel uygulaması, kullanılan veya

yürütülen moda karşılık gelir. Ancak "*At* kavramı bir kavram değildir" cümlesinde olduğu gibi dilbilgisel bir özne olarak ifade edilmesinde, bahsedilen bir kavrama tekabül ettiğini, dolayısıyla doygun olmayan bir kavrama gönderimde bulunduğunu iddia ediyoruz. Sonuç olarak, paradoksal cümle olan (H) yanlışlığını elde ederek, böylece onun olumsuzlaması (¬H) cümlesini, yani "*At* kavramı bir kavramdır." cümlesini elde etmiş oluyoruz.

Öyleyse şimdi özetle çözümüzün ana hatlarını gösterelim. Öncelikle '*at* kavramı' ifadesi ''ξ bir attır' yükleminin göndergesi olan şey' ya da basitçe, ⟦λ*x*. *x* bir attır⟧ fonksiyonunun göndergesidir. Bu durumda paradoksal at cümlesi sunduğumuz güçlendirici açımlamada aşağıdaki ifadeye eşdeğerdir.

(H)* ⟦λ*x*. *x* bir attır⟧ bir kavram değildir.

1. *At* kavramı = ⟦λ*x*. *x* bir attır⟧
2. ⟦λ*x*. *x* bir attır⟧, '⟦λ*x*. *x* bir attır⟧ bir kavramdır' cümlesinde bahsedilen (gösterilen) bir fonksiyondur. Ancak aynı fonksiyon '⟦λ*x*. *x* bir attır⟧ (Bucephalus)' cümlesinde kullanılan (uygulanan) ifadeye karşılık gelmektedir.
3. '⟦λ*x*. *x* bir attır⟧' üstdilin düzelenmiş kısmına aittir.
4. '*At* kavramı' üstdilin düzelenmemiş kısmına aittir.
5. '*At* kavramı' asla bir cümlenin dilbilgisel yüklemi olmaz.
6. '*At* kavramı' bir cümlenin dilbilgisel öznesi olabilir, öyle ki *at* kavramının kendisi bahsedilen (gösterilen) bir kavramdır.

Böylelikle aşağıdaki cümlenin doğruluğunu elde etmiş oluruz ve paradoks çözülür.

(¬H) *At* kavramı bir kavram değildir.

Paradoksal cümle olan (H) artık Frege'nin anlambilimsel *Düşünceler* kuramı için bir tehdit oluşturmaz. Atomsal bir *Düşüncenin* yapısal bileşiminin, bir fonksiyon ile

onun argümanının işlevsel bileşimlerden oluştuğunu gösteriyoruz. Sonuç olarak, *Düşüncelerin* birliğinin, her ikisi de *Düşüncelerin* birer bileşeni olan bir argümana bir işlevin uygulanmasından ortaya çıktığını açıklıyoruz.

8. Sonuç

Bu tezde, Frege'nin temel anlamsal ve ontolojik kavramlarından fonksiyon-argüman analizine, nesne-kavram ikiliğine, duyum-gönderge ayrımına dayanan anlambilimsel *Düşünceler* kuramına odaklandık. Bu tez boyunca, Fregeci atomsal *Düşüncelerin* yapısını ve birliğini açıkladık. Tezin başında belirttiğimiz iki temel amaca ulaşmak için Frege'nin *Düşünce* kuramına güçlendirmemizi sunduk. İlk olarak, Frege'nin *Düşünceler* teorisinin, karmaşık doğal dil ifadelerinin anlamını ve doğruluğunu açıklamak için en uygun anlambilimsel kuram olduğunu savunduk. Bu amaçla, *Düşüncelerin* tarihsel arka planı olarak Frege'den önceki filozofların teorilerini inceledik. Ardından, Frege'nin kendi kuramsal çerçevesine ilişkin açımlayıcı çözümlememizi sunduk. Gerçekçi filozofların kuramsallaştırdıkları benzer yaklaşımları önerme kavramı altında betimledik. Daha sonra önermeler üzerine çağdaş görüşlere odaklandık ve bu filozofların görüşlerini Frege'nin *Düşünceler* kuramı ile karşılaştırarak eleştirel bir incelemesini belirttik.

Bu tezin ikinci amacına ulaşmak için, Fregeci *Düşüncelerin* yapısı ve birliği ile ilgili önemli bir dizi problemi ele aldık. Atomsal *Düşüncelerin* yapısıyla ilgili problemler için, Bağlam İlkesi ve Birleşim İlkeleri arasındaki gerilim ve Frege'de *Düşüncelerin* benzersiz analizi ve *Düşüncelerin* çoklu ayrışmaları ile ilgili problemleri ele aldık. Frege, *Düşünce* kuramında Bağlam İlkesi'ni Grundlagen eseriyle sınırlı olarak ele aldığı için *Düşüncelerin* yapısı için bir gerilim oluşturmadığını savunduk. İkinci problemin çözümünü atomsal *Düşüncelerin* çoklu yapısı lehine tartışarak sunduk. Görüşümüze göre bir *Düşünce* yapısının her olanaklı bileşeni çoklu ayrıştırma ile gösterilebilir ve bunlar, düşüncenin son çözümlemede benzersiz bileşenlerini ortaya çıkarır. Böylelikle aynı atomsal *Düşünce* hem çoklu ayrışımla hem de benzersiz çözümlemeyle yapısını oluşturan öğelere ayrıştırılabilir.

Bu tezlerin her ikisinin de Frege'nin anlambilimsel kuramında tutarlı bir şekilde bulunabileceğini, dolayısıyla Frege'nin özgün görüşlerini koruduğumuzu gösterdik.

İkinci olarak, atomsal *Düşüncelerin* birliği ile ilgili problemler söz konusu olduğunda *at* kavramı paradoksuna odaklandık. Frege, *Düşüncelerin* birliğini onlara karşılık gelen doygun ve doymamış parçaların fonksiyonel bileşimi olarak ortaya koymuştu. Ancak doymamış, yani kavramları ifade eden fonksiyonel ifadelerin duyumlarının ve göndergelerinin nasıl anlaşılacağına dair herhangi bir açıklama yapmamıştır. Bu konuda çeşitli çözüm önerilerini inceledik, gelgelelim tüm bu çözümlerin atomsal *Düşüncelerin* birliğini oluşturmadaki yetersizliklerini gösterdik. Paradoksun çözümü için, doymamış ifadelerin hem duyumlarını hem de göndergelerini açıklamak için güçlendirici çerçevemizi sağladık. Özel adların, yüklemlerin ve cümlelerin duyumlarını ve göndergelerini dönüştürülebilir λ-soyutlamaları olarak açıkladık. Daha sonra, çerçevemizde duyumların biçimsel tanımlarını ve doymamış ifadelerin anlamlarını sunduk. Ardından, kavramların kullanılan (uygulanan) ve bahsedilen (gösterilen) oluşumları arasındaki ayrımını ortaya koyduk. "Bucephalus bir attır" cümlesinin yapısında "*at* kavramının" işlevsel uygulamasının kullanılan veya uygulanan kipe karşılık geldiğini, paradoksal cümlede dilbilgisel bir özne olarak geçtiği bağlamda ise bu kavramın bahsedilen (gösterilen) ifadeye karşılık geldiğini savunduk. Dolayısıyla, "*at* kavramı"nın doymamış bir kavramı ifade ettiğini paradoksal cümle bağlamında gösterdik ve paradoksu çözen "*At* kavramı bir kavramdır" ifadesinin doğruluğunu ortaya koyduk. Sonuç olarak, herhangi bir paradoksal sonuç olmaksızın, *Düşüncenin* ilgili doygun ve doymamış bileşenleri olan bir argümana, fonksiyonun uygulanmasıyla kurulan atomsal *Düşüncenin* yapısal birliğini açıkladık.

Bu tezin başlangıcındaki kestirimimiz, Frege'nin anlambilimsel kuramının atomsal *Düşüncelerinin* yapısı ve birliği ile ilgili sorunlara karşı savunmasız olma anlamında eksik olduğuydu. Sunmuş olduğumuz açımlayıcı çerçevemizin, Frege'nin kuramını en azından bu tezde ele alınan felsefi sorunlar bağlamında sağlam ve tutarlı temeller üzerine inşa ettiğine inanıyoruz. Böylelikle, ele aldığımız sorunlar artık atomsal *Düşüncelerin* yapısı ve birliği için herhangi bir tehdit

oluşturamazlar. Gelgelelim Frege'nin anlambilim kuramının sadece atomsal düşüncelerle sınırlı olmadığının farkındayız. Gelecekteki çalışmalarımızla bir köprü olacak şekilde, bu tezin başarısını, daha karmaşık *Düşüncelerin* doğası ve yapısı ile ilgili sorunları açıklamaya ve çözümler sunma noktasına taşımayı hedefliyoruz. Dolayısıyla, böyle bir proje üst düzey kavramlar için daha da kapsamlı bir inceleme ve yorumlama gerektirecektir. Umarız ki en azından yeni bir sürece başlamak için ilk adımları atmışızdır.

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