MODELING ELECTRICITY MARKETS BY INTEGRATING RENEWABLE ENERGY

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YUSUF KENAN PAKYARDIM

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submitted by YUSUF KENAN PAKYARDIM in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics, the Graduate School of Social Sciences of Middle East Technical University by,

Prof. Dr. Sadettin KİRAZCI
Dean
Graduate School of Social Sciences

Prof. Dr. Şirin SARAÇOĞLU
Head of Department
Department of Economics

Assoc. Prof. Dr. Esma GAYGISIZ
Supervisor
Department of Economics

Examiner Committee Members:

Prof. Dr. Serhan DURAN (Head of the Examining Committee)
Middle East Technical University
Department of Industrial Engineering

Assoc. Prof. Dr. Esma GAYGISIZ (Supervisor)
Middle East Technical University
Department of Economics

Prof. Dr. Serkan KÜÇÜKŞENEL
Middle East Technical University
Department of Economics

Prof. Dr. Okan Örsan ÖZENER
Özyeğin University
Department of Industrial Engineering

Assoc. Prof. Dr. Ertan YAKICI
National Defense University
Naval Academy
I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: Yusuf Kenan PAKYARDIM

Signature:
Sustainability concerns arising from anthropogenic climate change necessitated fundamental changes in the electricity sector. As set out in the 2015 Paris Agreement, the key element to deal with the threat posed by climate change is to increase the shares of renewable sources in exchange for reducing the shares of fossil fuels. Nevertheless, the traditional electricity systems have not evolved in a way to accommodate large-scale renewable energy smoothly. The large-scale integration of renewable energy poses several challenges for almost all components of the electricity sector.

The energy transition towards low-carbon energy now faces a multifaceted implementation problem. The problems are economic efficiency and managerial problems rather than technical difficulties. This dissertation comprises three analytical essays on analysis of the challenges of renewable energy integration. The first essay studies the problems in the wholesale markets. Particularly, the distortion of equilibrium prices in wholesale market because of the negligibly small marginal cost of production of renewables, impacts of different renewable subsidization mechanisms, and impacts of the industrial organization of renewable energy
generation are analyzed in a Cournot-Nash competition framework. The second essay proposes a novel retail market model based on a Demand Response mechanism and market segmentation. The model includes dynamic programming involving the dynamic interaction of retailers and consumers decision making processes and addresses the problem due to the intermittency of renewable energy. The last essay analyzes and compares the day-ahead hourly demand forecasting performances of three different forecasting methods on Turkish electricity markets.

**Keywords:** Renewable Energy Integration, Demand Response, Wholesale Competition, Forecasting Hourly-load
ÖZ

ELEKTİRİK PİYASALARININ YENİLENEBİLİR ENERJİ İLE BÜTÜNLEŞİK MODELLENMESİ

PAKYARDIM, Yusuf Kenan
Doktora, İktisat Bölümü
Tez Yöneticisi: Doç. Dr. Esma GAYGISIZ

Eylül 2022, 160 sayfa

Antropojenik iklim değişikliğinden kaynaklanan sürdürülebilirlik endişeleri, elektrik sektöründe köklü değişiklikleri zorunlu kılmıştır. 2015 Paris Anlaşması'nda belirtildiği gibi, iklim değişikliğinin yarattığı tehditle başa çıkmanın kilit unsuru, fosil yakıtların paylarını azaltarak bunları yenilenebilir enerji kaynakları ile değiştirmektir. Bununla birlikte, geleneksel elektrik sistemleri, büyük ölçekli yenilenebilir enerjiyi sorunsuz bir şekilde kullanabilecek yapılar olarak gelişmemiştir. Yenilenebilir enerjinin büyük ölçekli entegrasyonu elektrik sektörünün neredeyse tüm bileşenleri için çeşitli zorluklar ortaya çıkarmaktadır.

Düşük-karbon enerji dönüşümü, bu noktada çok yönlü bir uygulama sorunuyla karşı karşıya kalmıştır. Bu sorunların büyük bölümü, teknik zorluklardan ziyade ekonomik verimlilik ve yönetim sorunlardır. Bu tez, yenilenebilir enerji entegrasyonu ile ilgili zorlukların ekonomik analizi üzerine makalelerden oluşmaktadır. İlk makale toptan satış pazarlarındaki sorunları inceledi. Özellikle, yenilenebilir enerjinin çok düşük marjinal üretim maliyeti ile rekabete katılmasıının piyasa denge fiyatları üzerindeki etkileri ile birlikte farklı yenilenebilir sübvansiyon mekanizmalarının ve...
yenilenebilir enerji üretiminin endüstriyel organizasyonunun bu dengeye etkileri Cournot-Nash çerçevesinde incelenmiştir. İkinci makale, Talep Tepki (Demand Response) yöntemi ve piyasa segmantasyonuna dayalı yeni market yapısı önermektedir ve bunun ile ilgili analizleri içermektedir. Model üretici ile tüketicinin karar verme süreçlerinin dinamik etkileşimini içeren dinamik programlama kullanmaktadır ve yenilenebilir enerjinin kontrol edilememesinden kaynaklanan sorunu ele almaktadır. Son makalede, üç farklı tahmin yönteminin Türkiye elektrik piyasalarındaki gün-öncesi saatlik talep tahmin performansları analiz edip birbirleri ile karşılaştırmaktadır.

**Anahtar Kelimeler:** Yenilenebilir Enerji, Demand Response, Toptan Piyasalarda Rekabet, Saatlik Talep Tahmini
to my family
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<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
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<td>AIC</td>
<td>Akaike Information Criteria</td>
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<td>AR</td>
<td>Autoregressive</td>
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<tr>
<td>ARIMA</td>
<td>Autoregressive Integrated Moving Average</td>
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<tr>
<td>BIC</td>
<td>Bayesian Information Criterion</td>
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<tr>
<td>CSP</td>
<td>Concentrated Solar Power</td>
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<td>CCGT</td>
<td>Combined Cycle Gas Turbine</td>
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<td>DLC</td>
<td>Direct Load Control</td>
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<td>DMS</td>
<td>Demand Side Management</td>
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<td>DR</td>
<td>Demand Response</td>
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<td>DSARIMA</td>
<td>Double Seasonal ARIMA</td>
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<td>DSES</td>
<td>Double Seasonal Exponential Smoothing</td>
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<td>EU</td>
<td>European Union</td>
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<tr>
<td>ETS</td>
<td>Exponential Smoothing</td>
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<td>EXIST</td>
<td>Energy Exchange Istanbul</td>
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<tr>
<td>ISO</td>
<td>Independent System Operator</td>
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<tr>
<td>LACE</td>
<td>Levelized Avoided Cost of Electricity</td>
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<tr>
<td>LCOE</td>
<td>Levelized Cost of Electricity</td>
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<td>MA</td>
<td>Moving Average</td>
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<td>MAE</td>
<td>Mean Absolute Error</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>MAPE</td>
<td>Mean Absolute Percentage Error</td>
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<td>MSE</td>
<td>Mean Squared Error</td>
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<td>MWh</td>
<td>Megawatt hours</td>
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<td>GDP</td>
<td>Gross Domestic Product</td>
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<td>GHG</td>
<td>Greenhouse Gases</td>
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<tr>
<td>GWh</td>
<td>Gigawatt hours</td>
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<td>MOE</td>
<td>Merit Order Effect</td>
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<td>MSTL</td>
<td>Multiple STL</td>
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<tr>
<td>PACF</td>
<td>Partial ACF</td>
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<td>PAR</td>
<td>Periodic Autoregressive Models</td>
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<tr>
<td>PHEV</td>
<td>Plug-in Hybrid Electric Vehicle</td>
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<td>PV</td>
<td>Photovoltaic</td>
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<tr>
<td>RCE</td>
<td>Representative Consumer of Economy</td>
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<td>RES</td>
<td>Renewable Energy Source</td>
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<tr>
<td>RMS</td>
<td>Root Mean Squared Error</td>
</tr>
<tr>
<td>RMSPE</td>
<td>Root Mean Squared Percentage Error</td>
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<td>RTP</td>
<td>Real Time Pricing</td>
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<tr>
<td>SARIMA</td>
<td>Seasonal ARIMA</td>
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<td>SFE</td>
<td>Supply Function Equilibria</td>
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<td>SMP</td>
<td>System Marginal Price</td>
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<td>S.O.</td>
<td>System Operator</td>
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<td>S.P.</td>
<td>Social Planner</td>
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<td>SPNE</td>
<td>Subgame Perfect Nash Equilibrium</td>
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<td>Seasonal and Trend decomposition using Loess</td>
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<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
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<tr>
<td>TBATS</td>
<td>Trigonometric terms, Box-Cox transformations, ARMA errors, Trend, and Seasonal</td>
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<td>TEK</td>
<td>Türkiye Elektrik Kurumu</td>
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<td>TOU</td>
<td>Time of Use</td>
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CHAPTER 1

INTRODUCTION

Electrical power industries are constantly evolving and changing around the world throughout their entire history. Previously, the evolution has been driven primarily by ever-increasing demand, technological advances on both the supply and demand sides, policies that facilitate increased access to electricity, and other regulatory policies. In this context, traditional electricity systems have developed as vertically integrated and predominantly state-owned entities that combine generation, transmission, and distribution functions altogether. Since the 1980s, many countries have started to restructuring their electricity systems to increase the efficiency of these monopolistic organizations. This restructuring mainly involves i) the separation of vertically integrated generation, transmission, and distribution functions, ii) facilitating competition between generators in the wholesale markets as well as between the retailers and related services, and iii) separating financial markets from the physical distribution of electricity. However, the serious sustainability threat posed by anthropogenic climate change caused further changes. Greenhouse gases emitted from fossil fuels, which are also the main source of traditional energy systems, are the main cause of climate change. As an important step to handle climate change, an international consensus was achieved in the 2015 Paris agreement between 196 parties (195 countries and E.U.). The goal of the agreement is to keep global warming to under 20 compared to the pre-industrial levels with a target of 1.50 (Bernardo et al., 2021). The key element to accomplishing the goal is to replace fossil fuels with zero-carbon renewable energy sources such as intermittent solar and wind power. Accordingly, countries have committed to reducing their emissions in proportion to their “nationally determined contributions” and initiated policies to achieve these targets. Though, policies designed to increase renewable energy have introduced a new set of problems for almost all components of the electricity sector, including network and system
operations, investment and generation, distribution, consumption, and related businesses. The starting point for transition into low-carbon energy efforts was to implement policies to subsidise renewable energy to make it economically viable, since expected returns from power sales were not enough to attract low-carbon investment. The share of renewable energy has increased considerably in several countries and will be in many. In addition, renewable energy investments are becoming more attractive than before in time because of the decrease in costs as a result of developments in technology and growing markets. Thus, countries begin to revisit their support programs. However, the issues created by low-carbon policies have grown even more with the increase in the share of renewable energy since traditional energy systems in their existing structures are not appropriate to integrate large-scale intermittent renewable energy efficiently. At this stage of the low-carbon energy transition, where shares of intermittent renewable energy are becoming considerable, fundamental changes are required regarding market structures and system operations as well as other related components. Intermittent renewable energy has two main characteristics that differ greatly from conventional sources, which are the root causes of numerous problems.

The first and most prevailing one is intermittency. Without the possibility of nation-scale storage, electricity must be consumed as soon as it is generated. In this respect, input and output to the electricity network must be balanced all the time. In traditional electricity systems, all the input-output adjustments are carried out on the supply side and the supply is constantly adjusted according to the demand at every point in time. Therefore, scheduling production in terms of timing and quantity is central to electrical system operations. However, intermittent renewable energy cannot be scheduled according to the needs. Abundant inflexible production from renewable sources in a period imposes restrictions on production from other sources in that period. One way to deal with this problem is to shift the flexibility lost on the supply side to the demand side. However, specially designed market mechanisms and market models are needed to alter the consumer’s consumption pattern and obtain flexibility on the demand side. The mechanism designed to change the consumption pattern of the end-users in response to a change in the price of electricity or in response to an incentive is referred to as Demand Response which is one of the main subjects of this thesis. Another and
theoretically more manageable problem of renewable integration is the almost negligible marginal cost of production from renewables. When renewable energy competes in the wholesale market at almost zero marginal production cost, the equilibrium price is distorted downwards, which is referred to as the Merit Order Effect of renewables. Together with intermittency, this effect creates several issues, such as inefficient wholesale prices, price volatilities, curtailments of new technology generation and under-investment problems, etc. In addition, subsidization of renewable energy elevates the problems even more. From this respect, the major issues related to integrating renewable energy into current electricity systems are mainly economic problems rather than technical difficulties. With this motivation, this thesis aims at the economic modeling of electricity markets and addresses the issues related to the integration of renewable energy. The organization of the thesis is:

Chapter two explains the fundamentals of power systems and current issues because of renewable energy integration. This section comprises two main parts. The first section presents the key features of all elements of the electricity value chain, from the energy source to the end-users. The second section explains how wholesale and retail energy markets work. This chapter is important to understand the special features of electrical systems, and current issues faced and also it is necessary for a better understanding of the rest of the thesis.

Chapter three is an essay on the impact of renewable integration on wholesale markets. Besides independent sales, power generation companies compete in the wholesale market regulated by a system operator. The system operator determines the equilibrium price according to the quantity-price bids of the suppliers. Renewable energy, unlike any other energy source, has an almost negligible marginal cost. The zero marginal cost of renewables affects equilibrium prices negatively. Also, the intermittent nature of these resources creates another constraint to consider. In addition, the way renewable energy is subsidized and the industrial organization of renewable energy also have impacts on equilibrium prices. This work analytically investigates the impacts of all these factors on equilibrium prices in a Cournot-Nash framework. The result indicates that equilibrium price and quantity of renewable energy are negatively related as expected. Strategic companies having also renewable
generation can make use of their diversified portfolio. Therefore, ownership impacts the equilibrium. The contributions of this chapter to the current literature are threefold. First, two different cost structures, linear and quadratic, are considered in the analysis and the results are compared. We show that the result may change significantly in some cases. Second, different from the current literature, a general form of heterogeneous ownership structure between strategic firms in terms of renewable ownership is employed in the model. Third, the impact of renewable energy subsidization methods is studied under heterogeneous ownership and different cost structures. The results provide useful insight for the policymakers.

Chapter four is an individual essay about the problem because of the intermittency of renewable. In this essay, a novel market mechanism is introduced and analyzed. In the model, electricity usage is segmented into two based on flexibility. The system operator provides a discounted price for the flexible usage in return consumers allow the system operator to manage the timing of some portion of the usage. The modeling is based on the multistage dynamic interaction of both supplier and consumer which addresses the intermittency of renewables. The result of the numerical study indicates that the model always improves efficiency when there is excess generation from renewables. The model can be evaluated under the direct control type Demand Response, but it is quite distinct from the models in current literature. Thus, the model itself is the main contribution of this chapter to the literature. Another contribution of this chapter is that consumer preferences are modeled based on a utility maximization problem. Most of the studies in the current literature use a simplified form of equations such as linear relations or make some assumption that a certain amount of demand is available for Demand Response.

Chapter five is another individual essay on forecasting hourly electricity demand for the next day. Forecasting the next day’s demand is an essential part for both suppliers and retailers for the planning and optimization. Especially when intermittent energy plays an important role, the number of parties who need accurate forecast would increase since consumers would like to know the potential demand when a Demand response offer is made to them. In the study, three different forecasting methods which are Double Seasonal Exponential Smoothing (DSES), TBATS, and Multiple STL
Decomposition (MSTL) are used and their performances are compared. The forecasting is carried out based on the consumption data of Turkey. The results show that MSTL outperforms the other two methods providing always better results over TBATS in all cases and better results over DES for most of the cases. The first contribution of this chapter to the current literature is that these techniques are applied and compared using Turkish data on an hourly basis. Another contribution is that MSTL is recently introduced in commercial statistical packages and up to our knowledge, it has almost no application when the study of this chapter was initiated.
CHAPTER 2

FUNDAMENTALS OF POWER SYSTEM ECONOMICS

In this chapter, special characteristics of electricity systems and electricity markets are explained in two main parts.

2.1 Electricity Value Chain

The value chain of modern electricity energy is composed of the source of the energy and the basic functions and components ranging from the generation of the energy to the end-user of the energy. These functions include generation, transmission, distribution, and consumption. Under each category, several governmental organizations and private firms undertake different responsibilities. Although these activities are highly interdependent and highly integrated due to both technical necessities and instantaneously perishable characteristics of the electricity, they are also very diverse in terms of their operational natures. Historically, due to the high investment requirements, technological limitations, interminable energy supply purposes, etc., most of the activities were carried out by the government agencies. For example, TEK was the state organization having the sole responsibility for the generation, transmission, and distribution of electricity in Turkey until 1994. Demand for electricity has grown rapidly over the last decades and these state-owned or natural monopolistic structures became insufficient to meet the increasing demand in terms of quantity, quality, and price (Tagare, 2011). In order the improve efficiency, many countries initiated reformation in their electricity sector around the 90s (Joskow, 2006). Liberalization and separation of some segments were among the first steps of these restructuring processes. Liberalization efforts and also advancements in technology have led the organizations and structure of the electricity sector to change over time (Jamash and Pollitt, 2005), (Sioshansi, 2013), (Sioshansi and Pfaffenberger, 2013).
2006). Liberalization efforts were initiated in the generation part. The majority of the countries have privatized and a competitive environment has been created in most of the countries. However, heavy regulations in the electricity markets and system operations still persist.

### 2.1.1 The Source

The source of electricity is one of the most relevant factors for sustainability, energy security, market structures, economic and environmental considerations as well as many others such as businesses, technology, etc. Modern electricity generation systems rely on many different sources for continuous operation. These sources are generally evaluated under three main categories: Fossil, Nuclear, and Renewables. Fossil sources primarily consist of coal, natural gas, and petroleum which originated from organic substances. Whereas main elements of renewables are hydropower, solar, wind, and biomass. In modern electricity systems, relying on a single source of energy is not adequate from both operational and economic perspectives. The portfolio of sources that are used to generate electricity is often referred to as the energy mix. Throughout the years, increasing demand for electricity, technological advancement, economic aspects, supply security considerations, and environmental concerns let diversification of the energy mix and also an upsurge in the amount utilized in each kind of source (Martchamadol and Kumar, 2013), (Augutis et al., 2015), (Roques et al., 2008), (Marrero, 2010), (Cucchiella et al., 2012), (Koch and Bassen, 2013).

![Electricity Production by Source, World](image)

**Figure 2.1:** Electricity Production by Source, World
Although slightly declined, fossil fuels still have the largest share among the others at present accounting for around 60% of the total electricity production worldwide. Figure 2.1 summarizes the evolution of the utilization quantity and type of the main electricity sources over the world during the last centuries. Each country has its own electricity source mix but fossil fuel constitute the backbone of the source portfolios for almost all countries around the world. See Figure 2.2 for examples.

![Energy Mix of States, 2019](image)

**Figure 2.2: Examples of the energy mixes**

Moreover, fossil fuels have been one of the major components contributing to the wealth of many countries and the GDPs of some countries as depicted in Figure 2.3 (Lange et al., 2018). However, due to GHG emission, fossil fuel is also a major threat to the environment and a triggering factor for anthropogenic climate change and global warming (Höök and Tang, 2013), (Johnsson et al., 2019). Greenhouse gas emissions associated with fossil fuel production and consumption account for around 70 percent of the total GHG emission from human activities (IEA, 2010 database). A significant reduction in the share of fossil fuels is necessary in order to mitigate the environmental effects of fossil fuel usage which will lead to significant increases in the share of renewable energy (Panwar et al., 2011), (European Commission. Directorate-General for Energy, 2012). In addition to the environmental consideration, the sustainability of fossil fuels in terms of availability is another matter. Fossil fuels as non-renewable sources, as the name suggests, will eventually deplete in the future (Shafiee and Topal, 2009). Although, fossil fuels are naturally generated by organic substances such as
decayed plants and animals, it took millions of years to be formed. However, people have been using such resources extensively for the last centuries. The reformation speed is extremely lower when compared to depletion speed such that fossil fuels are considered non-renewable.

![Figure 2.3: Fossil Fuel Rent as a percentage of GDP](image)

2.1.2 Generation

Electricity generation has many dimensions such as cost, required technology, capacity, availability, flexibility, etc. These dimensions are primarily related to the fuel used in the generation process and the type of the generating facility. Each country optimizes the utilization of generation plants according to available sources, associated costs, the flexibility of the generation, and the energy they need. Cost comprises the capital cost, the operating cost, and decommissioning (if any) cost. Capital cost is the cost spent on the construction of a power plant until it becomes operational. Operating costs include the maintenance & repair expenditures, personnel wages, fuel costs, etc. required for the continuous operation of the power plant. Cost-based comparison is a general method when considering the economic feasibility of power plant investments. Although not enough on their own, there are useful metrics generally used in assessing the investment decision. Levelized cost of electricity (LCOE) provides a useful metric to assess the overall economic competitiveness between the considered power generation investment options and Levelized avoided cost of electricity (LACE) is a metric for the value of these investment options to the system (EIA, 2022). LCOE
stands for the present value of the average cost of electricity production discounted over the lifecycle of the power plant. LCOE is calculated by dividing the sum of the present value of all costs associated with the power plant during the life cycle by the electrical energy generated:

\[
LCOE = \frac{\sum_{t=1}^{T} I_t + O_t}{\sum_{t=1}^{T} G_t (1 + r)^t}
\]  

(2.1)

where, \( I_t \) is the capital expenditure in year \( t \), \( O_t \) is the operational expenses including maintenance and fuel costs, \( G_t \) is generated electricity in year \( t \), \( T \) is the lifetime of the generation station and \( r \) is the discount factor. However, the calculation of LACE is more complicated and includes the cost that would occur in case of the unavailability of the option. LCOE, LACE, and their variants are mainly related to long-run competitiveness. When short-run competition is the case, another cost metric is used widely which is the marginal cost of production. The marginal cost of production is mostly related to the decision of production for daily market operations. However, there is an implicit relationship between the marginal cost of production and LCOE.

Generation technologies are specific to the type of fuel used and they are described below.

2.1.2.1 Fossil-Fuel Power Plants

Power plants running on fossil fuel “burn” the fossil fuel in order to produce thermal energy and then thermal energy is converted into mechanical energy in a prime mover. The prime movers drive the generators to generate electricity. Coal-fired power plants use steam turbines as prime movers. The coal is burnt in a boiler to generate heat to obtain steam. Steam, then, expands in a steam turbine and the steam turbine drives the electricity generator. These types of plants are usually established close to the location of the coal mines in order to avoid the transport cost. Coal power plants are generally less expensive to build and could operate consistently over a long period of time. Gas power plants use a similar principle to coal-fired power plants but instead of a steam turbine, the gas is directly fired in a gas combustion turbine. Some facilities also use
the hot exhaust gas from the gas turbine and direct it to an additional steam turbine to increase efficiency which is called a combined cycle gas turbine (CCGT). A gas-fired power station is very flexible in terms of installed capacity and operation. The capital cost for building a gas power plant is relatively inexpensive. However natural gas has the highest marginal production cost due to the high fuel cost.

2.1.2.2 Nuclear Power Plant

Nuclear power plants use a steam turbine as a prime mover which is similar to a coal-fired power plant. However, the thermal energy required to generate steam comes from the nuclear reaction, particularly from the fission reaction in which the nucleus of an atom splits into smaller nuclei. Nuclear power plants could provide electricity reliably over an extensively long period. The typical service time of a nuclear plant is more than 60 years. However, construction of such power plants requires extremely high, multi-billion USD, capital costs, and long construction time. The marginal cost of production is low compared to the other alternatives. On the other hand return from the investments is very slow and generally takes decades to cover the initial investment cost. The problem arising from financing such an investment is very complicated and brings unclear risks. Estimating the actual cost of such investment is very hard and uncertainties are very high due to the unpredictable future over the very long period of the project cycle. Especially for the liberalized generation market, investors must bear the risk from these uncertainties associated with construction, operation, the value of electricity in the future, etc. Transfer of these risks to third parties such as insurance usage or forward contract is very limited since typical forward contract options and has a limited duration. Thus, such investment is not favorable for private investors in liberalized electricity markets. The requirement for significant government subsidies and support is another issue. The huge investment paid by the taxpayers will be utilized by the future generation which is difficult to justify for the government. Further difficulties come from the security of nuclear power plants. Besides the loss of the capital invested in nuclear power plants, accidents or damages that occurred in the power plant may cause an irreversible adverse effect on the environment and the people. The fuel used in a nuclear power plant is highly radioactive which could contaminate the air, the water, and the soil. In addition to the possibility of operational
accidents due to technical problems which may be mitigated with precautions, natural disasters such as earthquakes and terrorist attacks are still potential. Compensation and recovery costs for big accidents might be devastating. For example, it has been estimated that the cost associated with compensation, decommissioning, and waste storage would be 187 billion USD for the Fukushima nuclear power plant disaster. Physiological and social consequences on the people especially on evacuees as well as loss of opportunity cost were not included in these estimates. After Fukushima, operations of other nuclear power plants (53 out of 54 nuclear power plants) were also ceased for several years due to social anxiety and political reasons.

2.1.2.3 Renewable Generation

Renewable electricity generation techniques are very heterogeneous due to the significant differences between the sources. Being one of the oldest sources of energy, hydropower has been utilized to generate electricity since the 1870s. Hydropower plants use the kinetic energy of running water to generate electricity. The water runs through a turbine and spins it, then, a generator coupled with the turbine produces the electricity. Large-scale plants use water flowing from the vast reservoir of water behind a hydroelectric dam whereas small-scale plants could be constructed on the running river (Run on River-RoR). Available sites for large-scale hydropower plants are subject to geographic limitations and most of the sites have already been in use. However, there is still potential for RoR hydroelectric power stations. Electricity generation from solar power employs two types of technologies: Photovoltaic (PV) which relies on photons from the sunlight and Concentrated Solar Power (CSP) which relies on the heat from the sunlight. Photovoltaic devices convert solar energy directly into electricity. The photons from sunlight fall on PV cells. The photons stimulate electrons in semiconductors inside the cell and generate an electrical charge. CSP technologies collect the sunlight and concentrate it on a receiver by mirrors and reflectors. Then, concentrated sunlight is converted into thermal energy in the receiver. Collected thermal energy is used to generate steam, as in the nuclear or coal-fired power station, to drive a steam turbine. In this technology, the concentration of sunlight at a specific point is required to obtain enough heat to generate steam. Wind power generation runs on the kinetic energy of the wind. Blowing wind rotates the blades of
wind turbines and rotating energy is converted into electricity by the generator. Output from the wind energy is proportional to the cube of the wind speed. Both solar power and wind power are completely dependent on environmental conditions. Integration of these sources at a large scale into the electricity network is challenging since it reduces the flexibility of the supply side

2.1.3 **Dispatchable vs Intermittent Generation**

The electricity generation power plants that can be controllable in terms of turning on, turning off, and adjusting their power output are classified as dispatchable generation. These kinds of power plants are “dispatched” on request according to the needs. The controllable nature of dispatchable power plants makes it possible for the system operators to adjust the supply of electricity according to stochastic and fluctuating demand patterns. Continuous adjustment of electricity generation according to the demand (load following) is necessary since input and output to the electricity network must be balanced continuously. Also, economic generation dispatch could be achieved by dispatching the generator according to the increasing order of their marginal costs. However, different kinds of power plants have different flexibilities and therefore have different dispatch characteristic. Dispatch times range from seconds to hours. Examples of fast dispatchable power plants are hydropower plants which could be dispatched as fast as 16 seconds and natural gas power plants which could be dispatched within minutes. Slow dispatchable power plants are coal-fired and nuclear power plants. It requires hours for such power plants to become fully operational from the cold state. Although theoretically dispatchable, these kinds of power plants are operated continuously and regarded as baseload power plants. On the other hand, intermittent generation is an uncontrollable generation and intermittent generators are neither available on-demand nor available continuously. Photovoltaic Solar Power and Wind Power are the two main intermittent energy sources. The timing and quantity of generation from Photovoltaic Solar Power and Wind Power are completely dependent on the weather conditions. Therefore, the control over such power generation is very limited. Integration of intermittent power sources into the electricity network is challenging due to their uncontrollable nature. Increasing shares of intermittent sources decrease the flexibility of the supply. For a small share of renewable energy,
inflexible generation can be integrated by adjusting other sources in the energy mix. However, when the volume of intermittent generation becomes considerable, it would be no longer possible to make such an adjustment.

2.1.4 Transmission and Distribution

After generation, the electricity is delivered to the end-users through a complex network which is called the electricity grid. This complex system is basically composed of two functional parts: the transmission system and the distribution system. The transmission system is responsible for the bulk transmission of the electricity from the location of generation such as power plants to the point close to the neighborhood where it will be used. The lines which convey the electricity are interconnected and configured as a network. The network structure allows the electricity to go through multiple paths from generation to distribution. Modern transmission networks are not only interconnected nationally but also internationally. The transmission loss is proportional to the square of the current Loss= I^2Rt where I is in ohm, R is the resistance coefficient and t is the time. In order to reduce the losses and improve the transmission efficiency, electricity is stepped up to a high voltage (low current) after generation through transformers before inflowing into the transmission lines. Afterward, it is stepped down to low voltage levels through transformers before entering into the distribution system (this is why two separate systems as transmission system and distribution system are needed).

Historically, the vast majority of transmission networks have been constructed by governments before the liberalization of electricity markets. In some countries such as Turkey, the transmission system is still a state-owned entity. However, several countries like Germany, the USA, etc. have privatized the transmission system operations such as maintenance, upgrade, and expansion of transmission networks.

A distribution system is a network that connects the transmission endpoint with the end-users such as buildings, facilities, etc. This network is responsible for the distribution of electricity within the neighborhood. The Distribution network is a local network and is separated from the transmission system for several reasons. The distribution system is operated at low voltage levels since it directly provides electricity to the end-users contrary to the transmission system which is operated at
high voltage levels due to efficiency concerns as explained above. Another main reason is that failures in the distribution system stay local and do not affect the entire network. In almost all liberalized electricity markets, the distribution system is operated by private companies. New end-user connection to the system, meter readings, and maintenance of the distribution network is the major duties performed by distribution companies.

2.1.5 Central vs Distributed Generation

Centralized generation is the traditional generation system in which electricity is generated at a large scale and away from the point of use. Then it is transferred and distributed to many end-users through networks. There are technical and strategical reasons why most of the architectures of modern electricity systems have evolved as centralized structures including economies of scale (investing in large power generators such as bigger turbine decreases the marginal production cost), efficiency (higher efficiency through high pressure and temperature which require large power plant), integration (electricity pool and grid structure, one compensate the other), environmental consideration (away from city centers). All these considerations and strategic policy drivers resulted in large-scale centralized power generation facilities which rely on integrated transmission and distribution systems.

Distributed generation, on the other hand, refers to the type of electricity generation at a point close to the user location and on a relatively small scale. Distributed generation includes a variety of technologies such as wind power, solar power, combined heat, and power, etc. A distributed generator may be connected to the distribution system and serve multiple end-users or operate as a part of a microgrid such as a college campus or a village and also may serve directly a single facility such as a hospital or industrial facility. Historically, distributed generation has been used as a backup system in case of any failure in the main system. However, liberalization of electricity markets and transition of the electricity systems towards more environment-friendly structures have given rise to the spreading of the usage of distributed generation (Pepermans et al., 2005). Liberalization of the electricity market and technological improvement makes it possible now to invest in distributed generation at various scales. Traditional distribution systems have been designed to distribute electricity
coming from the transmission-end flowing to the end-users which is a one-way flow. However, distributed generation requires two-way energy transfer at the distribution level. Moreover, the management of the distribution network and also the management of the demand become complicated due to the increasing usage of distributed generators.

2.1.6 Consumption

Modern economies rely on electricity as a main source of energy and there is a strong correlation between the economic output of countries and electricity usage. Electricity is the most versatile form of energy. It can be converted into many other forms like heat, light, motion, etc. with very high efficiency. Also, electricity is the most convenient form of energy since it does not possess any mass, can be easily controlled, and leaves no waste after usage. The daily life of people and most activities in modern economies require electricity for extensively diverse purposes. Electricity usage by sectors of the Turkish economy is shown in Figure 2.4. About half of the electrical energy is used by the industry. One-fourth of the energy is used by residential consumers. Although the load share of the transportation sector is pretty low compared to the other sectors, a dramatic increase will be likely in the near future in parallel with the increase in the share of PHEV. Across all sectors as well as within each sector, electricity usage patterns exhibit large differences in terms of their magnitude, timing, and flexibilities depending on the purpose of the usage. Therefore, the demand for electricity is very heterogeneous. For example, lighting needs are coupled with daylight availability in general and cannot be scheduled whereas the majority of the home appliances can be scheduled according to the needs. Likewise, some industrial processes require large power and draw a vast amount of electricity from the network when the process starts while charging a cell phone requires a little amount of power. Therefore, the aspects such as flexibility, volume, etc. are important parameters when modeling electricity load.
Tradition way of consuming electricity is that end-users utilize electricity whenever they want and necessary input-output adjustment of the network is done through tuning the supply. Nevertheless, large-scale integration of intermittent renewable energy sources reduces supply flexibility. The transition towards a low-carbon economy would change not only how electricity is generated but also the way it is consumed.

2.1.7 Electrification

There has been growing interest in electrification of other energy uses such as transportation, heating, industrial processes, etc. There are two main reasons behind this interest. First, electricity is the most convenient form of energy and technological advancement makes it possible to use electricity as an energy source in many applications. The other is that several analyses showed that electrification of fossil—fuel based applications such as transportation and residential heating is one of the key elements in reducing overall greenhouse gas emissions (Nadel, 2019). Therefore, electricity usage is likely to increase significantly in the near future.
2.2 Electricity Markets

2.2.1 Wholesale Markets

Liberalization of the electricity market has started on the generation side of the electricity value chain. Throughout the years, many countries have spent continuous effort in order to establish a competitive environment for the generation and supply of electricity. Many state-owned generation facilities have been privatized and new private companies have emerged. At present, in the majority of countries, electricity is generated in a competitive environment and traded in different forms of wholesale markets. The transition from a vertically integrated and monopolistic structure into a competitive environment was not an easy process and could not be done promptly due to the need for continuous electricity supply to the end-users and to ensure the necessary investment for the growing demand. The practices such as constituting the legal background first and letting the market respond accordingly are not reasonable since market interactions in a transition stage might result in very costly system failure and market breakdowns. Contrary to any other commodity markets, electricity markets must account for the special characteristic of electricity. These are mainly due to the two basic aspects of electricity: non-storability and the need for continuous input-output balancing in transition networks. California power crises and consequent shutdown of California power markets are good examples of this\(^1\). Therefore each transition process from monopolistic generation into a competitive structure has been planned in a step-by-step manner according to the properties of each country. Consequently, each country established its specific wholesale market considering its existing organization, resources, and legal situation. In addition, different countries may be at different stages of the liberalization process. Nevertheless, the competition

\(^1\) The crises was caused by market manipulation by the suppliers due to partial deregulation of the markets. Power generators deliberately pulled back the generation amount to create an artificial shortage resulting in demand-supply gap although available generation capacity was 45 GW while demand was 28 GW at that time. Therefore, due to the scarcity, electricity was traded at a rate up to 20 times higher than regular price in spot markets. However distribution companies had to sell electricity at a constant rate to the end users due to the price cap in the retail market. Thus, being unable to reflect high prices they paid in the spot market to their customers due to the price cap and retail companies faced big financial loses and some of them eventually went bankrupt. During this crises period of 2000-2001, there were several blackouts which caused economic and social loses.
for the wholesale electricity trade has fallen into two main categories: (i) One-sided Wholesale Markets in which power generators are independent and the rest of the system is either vertically integrated or unbundled but not competitive, and (ii) Competitive Wholesale Markets (Power Exchanges) which is a kind of decentralized market process. Due to the country-specific operational restrictions, historical market structure, and regulatory environment, there have been a variant of these market models

2.2.1.1 One-Sided Wholesale Markets (Centralized Trade)

At the beginning of the liberalization endeavors, switching from a monopolistic structure to a fully competitive and decentralized market model was seen as a huge step and a big deviation from the previously-existing structure. It was decided to make the trade-in centralized and controlled settings. Therefore, only the generation side was liberalized as a first step while the rest of the system remaining as vertically integrated. Thus, a one-sided competitive market was constructed. Generally, an agency that is referred to as System Operator is responsible for the operation of the market. Instead of a continuous interaction of suppliers and buyers to reach an equilibrium, System Operator systematically determines the equilibrium in a one-sided market. Generally, all generating firms must participate in one-sided markets. All the power-generating firms submit their price-quantity pairs to the System Operator. Next, System Operator aggregates these offers, finds the total quantities corresponding to each price, and sequences them in ascending order in price. Eventually, these price-aggregated supply pairs constitute the supply curve of the market. On the demand side, actual customers do not directly involve in the trade, and System Operator does not collect demand-price offers and construct the demand curve. Instead, System Operator estimates the total demand for the end users which is generally price-insensitive in the short run. This procedure is called a one-sided pool. (Total demand of the end-users is equal to the total demand of the buyer in the wholesale market since all generation firms and distribution companies participate in the centralized trade. The System Marginal Price that clears the market is determined by System Operator considering only the supply curve and the total demand. Then, the System Operator checks the transmission system feasibility and revises the dispatch decision if there would be any transmission
limitations. Finally, System Operators determine the equilibrium prices and which
generators would be dispatched for each time.

Another type of optimization is the consideration of transmission feasibility together
with the supply and the demand. In this case, each location is assigned a transmission
cost related to the transmission limitation to that location. Therefore, resulting prices
would be location specific which is referred to as Locational Marginal Price. Some
countries such as Italy use locational pricing. Thus, both equilibrium and which
generator would be dispatched are determined by the System Operator centrally.
Although trade is centrally managed in one-sided markets, bilateral trade and long-
term agreements are possible. Bilateral trade could be a physical contract as well as a
financial contract. Allowing additional bilateral trade is a deviation from the
centralization of the trade. Examples of markets using a one-sided structure are some
states of the USA, Brazil, etc. UK market previously relied on a one-sided market
structure but later switched to a Power exchange model.
Such markets are criticized by many economists for being an “approximation of a
market” rather than a true market due to the limited supplier-customer interaction

Figure 2.5: Operation of Centralized Power Markets
2.2.1.2 Competitive Wholesale Markets:

Competitive Wholesale Markets in which the trade occurs between sellers and various buyers such as distribution companies or large-scale end-users is another type of wholesale market model which is widely used by the majority of OECD countries and many other countries. This market model provides both decentralized and centralized trade opportunities. Bilateral trade, as the name suggests, occurs directly between the sellers and the buyers without the involvement of any other parties. In general, sellers are the generators and the buyers are the distribution companies and the large-scale end-users. However, there is no such limitation for the roles in this market model. A generation company could also be a buyer if there exists an opportunity to buy power cheaper than its marginal production cost and provide this power to its customer to fulfill its contractual obligation. Else, in case of a failure in production facilities, a generation company might purchase power from another source to fulfill its contractual obligation. Different types of contracts could be exercised between the supplier and the buyer. Especially, long-term contracts which cover the period of months to years are used for the base load of the distribution companies and large customers. The agreed price-quantity pair and the duration, conditions, and flexibilities of such long-term contracts are settled between two parties and kept private.

Although a certain amount of energy is traded between two contracting parties through bilateral trade, organized and controlled trade is still needed as the event time approaches since bilateral trade does not guarantee to satisfy the supply-demand balance and network constraints. Power Exchanges also include centralized day-ahead and intraday trade opportunities to ensure supply-demand equilibrium. Participation in these trades is voluntary and participants are diverse in terms of their roles and purposes. The system Operator collects quantity-price offers for the supply and constructs the supply curve. Similarly, the demand curve is constructed by collecting and aggregating quantity-price requests. Unlike the pool model, supply offers and demand requests may come from both generator companies and distribution companies as well as from third parties. Consider a generation company that already sold its entire production capacity through a bilateral contract, this generation company may participate in power exchange as a buyer requesting a certain amount at a price lower than its marginal cost. Table 2.1 provides an example of a typical bid in the
Turkish Power Exchange market (EXIST). Positive quantities are sell offers while negative quantities represent the demand offers. In this example, there are six price levels, four of which are for sell offers and the remaining two are for buy requests. When the price level drops below 270 TL, the generator company is no longer willing to sell any power instead the company would like to purchase power. It can be inferred that the marginal cost of generation for the company in the example is between 250TL and 270TL.

Table 2.1: Example bid in the wholesale competition

<table>
<thead>
<tr>
<th>Price (TL)</th>
<th>0</th>
<th>250</th>
<th>270</th>
<th>290</th>
<th>310</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity (MWh)</td>
<td>-700</td>
<td>-300</td>
<td>50</td>
<td>250</td>
<td>350</td>
<td>390</td>
</tr>
</tbody>
</table>

Several other examples could also be generated regarding the participation in the wholesale market in power exchanges. The operational flowchart of a power exchange market is displayed in Figure 2.6.
The equilibrium prices in power exchange are publicly visible to all participants as well as to non-participants. Although the negotiated prices in the bilateral contract are private information, the equilibrium prices in power exchanges are reference values for those negotiations in bilateral trade. In an ideal economy, the prices in both power exchange and prices in the bilateral contracts should be such that there would be no arbitrage opportunity. This market model facilitates supplier and buyer interaction together with the possibility of several types of trade opportunities that would not be possible otherwise. Large-scale companies and brokers use this trade opportunity as a hedging instrument. In this respect, these kinds of market models are more suitable than the pool model for the classical economic school of thought. Several countries all over the world employ this kind of market model. Examples are Turkey, Germany, France, the UK, Japan, some states of the USA, etc.

2.2.1.3 System Marginal Price

It is a general practice in power exchanges that generators whose bids are below the market-clearing price are paid with the market-clearing price although their bids i.e. their willingness to produce, might be lower. Similarly, the buyers whose offers are above the market-clearing price pay the market-clearing price. This uniform price is determined by the System Operator and is referred to as System Marginal Price. System Marginal Price is basically the price offer of the last amount of generation in the Merit Order Curve which clears the market. The main idea behind this uniform price system is to encourage generators to submit their marginal costs. In other auction mechanisms such as pay-as-bid, companies would try to estimate market-clearing prices rather than revealing their marginal costs. An example of the determination of system marginal price as an intersection of supply-demand curves constructed on supply-demand bids is shown in Figure 2.7.

2.2.1.4 Timeline for the Trade

The trade between parties could occur at different time frames. Bilateral trades typically cover long periods such as months and years. Thus, there is no time limitation neither when agreements are done or in the validity period for bilateral contracts.
Due to the stochastic nature of the demand and also the intermittency of some generators, buyers and sellers cannot estimate their required amount very well for future periods. There are also financial and technical reasons why sellers and buyers do not satisfy their entire needs through bilateral trade. Thus, bilateral trade only accounts for a certain portion of the trade between parties. The remaining portion is traded in the organized market such as power exchanges. When the event time approaches, the predictions about generation and the demand improve, thus organized trade should take place very close to the event time to improve network balancing accuracy. However, a spot market with immediate delivery is not possible due to technical limitations. Power exchanges typically operate according to the day-ahead principle (and real-time for some specific examples). In a day-ahead principal, every day up to a predetermined due time, sell and buy bids are collected for every 24 hours of the following day. Then System Marginal Price is constructed for every hour of the following day and the resulting price and matching quantities are announced to the market participants. Bilateral trade plus power exchange almost satisfies all the trade requirements of the parties. However, there might be still imbalances between the demand and supply due to stochastic parameters when the realization time approaches. The remaining part is traded in the intraday market which takes place up to minutes of the event time. In some markets, in contrast to the day-ahead operations, organized

Figure 2.7: Equilibrium price for 2 a.m. and 6 a.m. on 02.12.2020.
trade continues until 5-10 minutes of the event time which is referred to as real-time trade. Examples of organized trading markets are listed in the table

<table>
<thead>
<tr>
<th>Country</th>
<th>Market</th>
<th>Trading timeframe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey</td>
<td>EXIST</td>
<td>day-ahead</td>
</tr>
<tr>
<td>Germany</td>
<td>EPEX-D</td>
<td>day-ahead</td>
</tr>
<tr>
<td>France</td>
<td>EPEX-F</td>
<td>day-ahead</td>
</tr>
<tr>
<td>Italy</td>
<td>GME</td>
<td>day-ahead</td>
</tr>
<tr>
<td>Canada</td>
<td>OIESO</td>
<td>real-time</td>
</tr>
<tr>
<td>South Korea</td>
<td>KPX</td>
<td>day-ahead</td>
</tr>
<tr>
<td>Singapore</td>
<td>EMC</td>
<td>real-time</td>
</tr>
<tr>
<td>Russia</td>
<td>ATS</td>
<td>day-ahead</td>
</tr>
</tbody>
</table>

2.2.1.5 Investment and Missing Money

The rise of competition in wholesale markets causes a decrease in wholesale electricity prices. The fall in prices has been beneficial for some energy-intense industries by enhancing their competitiveness by reducing their costs and providing several trading and hedging opportunities. However, due to the decrease in prices, the wholesale electricity market has faced a serious problem: “missing money”. In general, missing money is referred to the situation in which it is not possible to obtain enough profit that is needed to provide the optimal generation portfolio either by maintaining the existing capacity and/or investing in building new capacity. Building a new power station requires a high capital cost that is supposed to be recovered by the future profits from the electricity sales. Reduced wholesale prices make this recovery period so long that investing in building such capacity is no longer feasible. The duration of the recovery period, that is the time until reaching the break-even investment point, is mainly related to long-run average cost and the rate of capacity utilization. However, competition in the wholesale market is mainly related to the marginal cost of production. A very old coal-fired power plant that already covered its investment cost
could survive by charging only its marginal production cost which is mainly the fuel cost. However, a newly-built power plant could not survive with this bidding strategy. In such a situation, there is not any financial incentive for the investors to build a new capacity. (Moreover, market-clearing offers always only have their marginal cost). Although eliminating the market power of the companies through constituting a competitive environment is desirable by policymakers, it brings about a new challenging situation to deal with.

The main reason for “missing money” is the heavy regulations in the electricity markets, especially the price caps. Imposing a price cap in the wholesale market is a rare application but a retail market price cap is a general regulation imposed by the majority of countries. In general, even if there is no price cap in the wholesale electricity market, the price cap in the retail market reduces the market power of the companies in the wholesale market. The price cap in the retail market implicitly imposes a limitation on wholesale prices since the only way for retail companies to survive is to keep the average electricity purchase price below the retail price cap.

The missing money problem has become more tricky due to the price impact of renewable energy in the wholesale market. Renewable energy takes part in the wholesale market with extremely small marginal costs and owners of those power plants greatly benefit from subsidization. The impact of renewable energy is twofold. First renewable energy reduces the equilibrium price in the wholesale market. Second, renewable energy decreases the frequency of the utilization of some power stations. When the production from renewables is high some of the conventional generating companies must pull back their production level either by completely switching off or reducing the output of some generators, which eventually increases the return on investment time and decreases the overall profitability. Consequently, the frequency of the utilization of installed capacity is reduced. This also means that the market-clearing generator is pulled back and the generator with the next lower offer (lower marginal cost) now becomes the market-clearing generator ultimately reducing the equilibrium price. In general, generators with newer technology and relatively less emissions are affected mainly since such kinds of generators have marginal costs around the marginal cost. However, high carbon generators such as coal-fired
generators would still work constantly, which contradicts the global goal of reduction in emissions.

2.2.2 Capacity Markets

Equilibrium prices in wholesale competition constitute price signals for long-term investment. When these prices are lower and more volatile, investment in new fossil fuel generation capacity became unattractive. However, those investments are required in the system when the supply from renewables is not sufficient. In some countries, in order to ensure the security and reliability of supply and make sure that sufficient investment for the future electricity supply is done, producers are paid under capacity payment to commit to production in the future. These kinds of markets are referred to as Capacity Markets. Capacity markets are criticized as being inconsistent with decarbonization policies since fossil fuel generation is subsidized as a result of renewable energy integration. In addition, the requirement for such subsidization is amplified when the share of renewable energy is increased.

2.2.3 Retail Markets

Small-scale end-users are not eligible to buy electricity from the wholesale market and do not have any interaction with generation companies. Instead, retailer companies that buy electricity from the supplier either through bilateral contracts or through auctions are responsible for selling electricity to the end-users. The ultimate goal of liberalization in the electricity sector is to create a completely competitive retail market. To this end, financial operations are separated from the physical distribution of electricity. Many OECD countries have initiated liberalization in the retail markets. In Turkey, 21 authorized retail companies are responsible for 21 separate regions. In fact, these companies are sister companies of 21 authorized distribution companies that were doing both distribution and retail marketing together previously. These companies are required to provide service to all end-users. In addition, there are also independent retail companies that provide services to the end-user that satisfy certain requirements.
Generally, the retail price is regulated by policymakers and a price cap is applied. However, wholesale prices are constantly fluctuating, which is putting a strain on retailers. Since they have to sell the electricity at a fixed price they buy from the wholesale market with a variable price. Therefore, their objective is to keep the quantity-weighted average cost below the retail price.
CHAPTER 3

MERIT ORDER EFFECT UNDER DIFFERENT RENEWABLE ENERGY SUBSIDIZATION PROGRAMS AND DIFFERENT OWNERSHIP STRUCTURES

3.1 Introduction

Beyond the technological feasibility, the integration of large-scale intermittent renewable energy presents economic efficiency problems to deal with (Henriot and Glachant, 2013). One of the toughest challenges associated with the increased use of renewable energy is posing on the wholesale market side of the electricity value chain. The equilibrium prices in the spot market are affected by large-scale renewable energy deployment in two ways. The first factor is the intermittency of the generation. Generation firms do not have any direct control over the amount and the timing of the generation for a specific renewable energy generation infrastructure. Therefore, competing firms in the wholesale market should consider this rigid production from renewables as a state variable in the optimization of its price-quantity offer while production amounts from conventional generators are still a decision variable. Another, definitely more prevailing, effect of increasing usage of Renewable energy on the wholesale market is the reduction in equilibrium prices due to the negligibly small marginal cost of production from renewables. The aggregate supply curve is generally constructed by collecting price-quantity offers and sequencing them in increasing order yielding the so-called “Merit Order” of the generators (Deane et al., 2015). Electricity from Renewable energy participates in short-run competition with almost zero marginal production cost and shifts the merit order curve rightward consequently resulting in lower equilibrium price (Figueiredo and da Silva, 2019).
This effect is referred to as the Merit Order Effect of renewable electricity generation.

![Merit Order Effect of Renewables](image)

**Figure 3.1**: Merit Order Effect of Renewables

A typical merit order curve with and without renewable energy production is illustrated in Figure 3.1. Although production from renewables has almost zero marginal cost, it requires larger capital cost resulting in a high Long-run average cost. “Levelized Cost of Electricity” for renewables is generally higher than the conventional generators, which makes renewable generation unattractive for investors. Aiming to increase the share of renewable energy, governments financially support renewable energy investment with certain support programs. The subsidies make it possible for the investor to cover the capital cost within a reduced timespan since the overall average payment per kWh through subsidization is generally higher than the average equilibrium price in the wholesale market. Another benefit of such a support system for the investor is that it eliminates the risks associated with price volatility in the wholesale market. A number of different support mechanisms with different implications are employed by various governments (Ragwitz and Steinhilber, 2014). Among all, one of the most widely used support mechanisms is the Feed-in Tariff program. In this support mechanism, the generator is paid a constant amount of money per kW of production regardless of the equilibrium price in the spot market. Moreover, priority dispatch of the renewables together with the feed-in tariff is also a common support practice (Antweiler and Muesgens, 2021). Therefore, in the planning stage, the amount of generation from renewables is directly deducted from the total demand.
and consequently does not enter into the wholesale competition. The resulting effect is displayed in Figure 3.2.

![Graph showing feed-in tariff effect](image)

**Figure 3.2:** Effect of feed-in tariff

The main feature of the feed-in tariff and its variants is that generation from renewables does not enter into the wholesale competition. Such kind of support mechanisms could be referred to as non-market support mechanisms since renewable energy is not traded in the market like the energy from conventional generation. Alternative to such kinds of non-market support mechanisms, there are also support mechanisms in which generation from renewable is traded the same way as energy from conventional generation is traded. For example, an investment reimbursement program in which a portion of the capital cost is paid to the investors. In such support mechanisms, renewable energy is not differentiated during the wholesale competition. Nevertheless, in both market-based and non-market support mechanisms, the equilibrium price is reduced due to the Merit Order Effect. In a perfectly competitive market where all companies bid their marginal cost, the effect of both support mechanisms visualized in Figure 3.1 and Figure 3.2 would be the same. However, in electricity wholesale markets in which the competition is characterized as imperfect, it is likely for a competitive company to follow a different strategy under different support schemes. Moreover, companies having more than one generation facility have the opportunity to further influence the equilibrium when they optimize the output of their entire portfolio rather than optimizing each generation facility independently. The key difference between these two groups of subsidization schemes which is relevant to
short-run wholesale competition is the way renewable energy participates in the trade. Thus, in order to investigate the implication of market-based and non-market support mechanisms, we consider the case where there is a feed-in tariff for renewables and the case where there is no discriminative price for renewables and a common price is determined through competition.

Another significant factor affecting the equilibrium price in the wholesale market is the ownership structure (industrial organization) of renewable energy generators. Under imperfect competition, a firm having both conventional generation and renewable generation could make use of the diversified portfolio in competition by optimizing its costly generation according to the available renewable generation. Hence, those companies may have the opportunity to mitigate Merit Order Effect by exercising their market power. On the other hand, companies generating only renewables should act as fringe companies (price takers) since they do not have any control over the output quantity, and so, are not able to strategically withhold generation to alter prices. Thus, ownership of this uncontrollable generation could also influence the strategic behavior of the firms.

This section analytically investigates the Merit order Effect and impact of subsidization mechanisms together with the ownership structure of the renewable energy sources on the equilibrium under both linear cost structure and quadratic one to account for a wide range of cases. We consider three types of ownership structure: companies having only conventional generators, companies having both conventional and renewable generators, and companies having only renewable generators. Moreover, we extend the analysis to include pre-committed bilateral contracts and how the volume of bilateral contracts moves with those parameters.

Our theoretical analyses show that the Merit Order Effect exists and the impact of the Merit Order Effect is always more powerful resulting in lower equilibrium prices in non-market support schemes such as feed-in tariffs than the market-based support systems. In addition, the share of renewables owned by strategic firms plays an important role in the mitigation of the Merit Order Effect. However, in the case of feed-in tariff sort of support programs, the impact of the ownership structure is eliminated. Heterogeneity within competitive firms has no impact on the equilibrium, the only important factor is the total potion of renewable sources owned by strategic
firms. The bilateral contract volume is negatively correlated with the amount of renewable energy and also with diversification.

3.2 Literature Review

There is a substantial and ever-growing body of literature related to the Merit Order Effect of renewables. Analysis of (Clò et al., 2015) provided empirical findings on the existence of the Merit order Effect in Italian Electricity Markets. They investigate the data for the period of 2005-2013 and find that a 1 GWh rise in an hourly average of daily RES generation decreased the wholesale electricity prices by 2.3 Eur or 4.2 Eur per MWh on average depending on the type of RES. In addition, volatility is increased due to RES penetration. Based on a time series regression and using German wholesale market data, (Cludius et al., 2014) show that spot prices decreased by 6 Eur per MWh in 2010, and with the growing share of RES this reduction reached 10 Eur per MWh in 2012. (Ciarrreta et al., 2014) find a similar implication for Spanish electricity markets for the period 2008-2012 where generation from renewables increased by 57%. (Figueiredo and da Silva, 2019) evaluate the Merit-Order Effect in the Iberian wholesale electricity market for the period of 2008 to 2017. In addition to the positive relation between Merit Order Effect and Renewable production, they find that volatility due to MOE reflects the intermittent behavior of uncontrollable RES generation. (Macedo et al., 2021) conducted a study on the Swedish electricity market using the data of the period 2016-2020 to explore the MOE on both mean price and volatility. An important feature of their work is that they use 24 separate models to represent each hour of a day aiming to investigate whether the hour of a day is a factor in the MOE of renewables. Their results indicate a significant MOE of renewable consistent with the literature but the magnitude of the MOE is not affected by the hour of the day. In addition to country-specific works, there are examples in the recent literature considering the global and cross-country effects of renewable integration. (Halttunen et al., 2020) compared 37 countries around the world and report that MOE is observed in almost all countries. Overall worldwide average MOE is estimated as 0.68±0.54 Eur per MWh corresponding to each percentage increase in intermittent RES. (Abrell and Kosch, 2022) showed that the cross-country Merit order Effect exists
in neighboring countries due to RES production in Germany and this effect reduces the profit of generators in those neighboring countries.

While the results of empirical studies are in consensus, theoretical analysis is required in order to generalize the results of these empirical studies and gain further insight into how the deployment of renewables affects the equilibrium. Theoretical analyses begin with modeling the competition in wholesale markets. Even though electricity markets have been liberalized and a competitive environment has been built in the majority of the countries, equilibrium prices are still far from the competitive prices and spot prices are generally higher than the marginal cost of generation (Borenstein et al., 2002), (Mansur, 2008). The situation is attributed to the market power of the companies. For example, (Wood and Blowers, 2018) state that the rise in electricity prices by 130% from 2015 to 2017 in the Australian market is partially related to the exercise of market power of the generation companies. The market power results from the fact that wholesale electricity markets generally consist of a limited number of electricity-generating firms (or a limited number of large firms regardless of the number of small ones). Price-quantity offers of those firms directly affect the equilibrium prices. Moreover, these firms strategically determine their price-quantity offers to maximize their profit and these offers do not necessarily have to reflect their marginal costs. (Twomey and Neuhoff, 2010) demonstrate that conventional energy generation companies can manipulate the equilibrium prices in a way that they increase the prices while selling electricity and reduce them when they buy power from the market. (McRae and Wolak, 2009) shows that companies place higher-priced bids for the periods in which price elasticity is lower. Therefore, the competition in the wholesale market is characterized as imperfect competition. Two major types of theoretical frameworks are widely used to model such oligopolistic competition in the wholesale market: Cournot-Nash Competition and Supply Function Equilibria. Cournot-Nash model is employed by several studies such as (Bushnell, 2007a), (Borenstein et al., 2002), (Neuhoff et al., 2005),(Sioshansi, 2014), (Ribó-Pérez et al., 2019). Based on a Cournot competition framework, (Borenstein et al., 2000) states that the price-cost premium is 16% in California power markets, and (Borenstein et al., 2002) found that more than 50% of the rise in electricity prices is due to market power of the companies. (Neuhoff et al., 2005) compare three different Cournot models developed by different research groups. Their results show that the assumptions about market design and how
a fringe company acts highly effect the Cournot equilibrium. There are also studies that verify the appropriateness of the usage of Cournot-Nash model to represent wholesale competition. Based on recently available bidding data of Nord Pool electricity market, (Lundin and Tangerås, 2020) shows that competition in Nordic day-ahead wholesale market is consistent with Cournot-Nash Competition context.

In Supply Function Equilibria modeling, firms submit their supply function considering the supply functions of their rivals, and the equilibrium price is determined by the system operator. Hence, this approach assumes that firms compete on both price and quantity rather than quantity only. The basic model was developed by (Klemperer and Meyer, 1989) and then adopted in the electricity market by (Green and Newbery, 1992).

Both models have their strengths and weakness. (Baldick et al., 2004) argue that SFE is more appropriate than the Cournot model to represent competition in the wholesale market. Cournot model is criticized for providing overestimated equilibrium prices and being more sensitive to demand elasticity. Nevertheless, the overestimation problem could be evaded by including forward contracts (Willems et al., 2009). On the other hand, Supply Function Equilibria is mathematically complicated and formulation does not accept any external parameters such as capacity restrictions, etc. Thus SFE is not suitable for a variety of cases. Furthermore, it is very hard to draw any conclusions from the mathematical results of SFE.

(Willems et al., 2009) compare these two popular oligopolistic competition models and test them using the data from the German wholesale electricity market. Their results indicate that both models perform reasonably well and they suggest using the Cournot model in order to study short-term competition in the wholesale electricity market since the Cournot model makes it possible to include additional constraints and provide analytical flexibility.

Our study is mainly related to the recent literature that studies the impacts of renewable deployment on the wholesale market in a theoretical framework (based on Cournot-Nash competition setup). One of the early theoretical works addressing renewable integration and market power is (Twomey and Neuhoff, 2010). They demonstrate that conventional generation firms strategically adjust their output according to RES
availability. Additionally, these firms are able to raise or suppress the prices along with their sell or purchase needs. (Ben-Moshe and Rubin, 2015) examine the oligopolistic competition considering diversified portfolio and indicate that ownership structure has an impact on MOE e.g., a strategic firm may increase its market power by investing in renewable energy. In addition to a diversified portfolio, (Acemoglu et al., 2017) extends the analysis to include forward contracts and incomplete information cases. They construct all the derivations assuming that cost of production is linear. Their results suggest that MoE is fully neutralized in the case of full diversification. However, this result is derived under the specific case of linear cost structure, which does not hold when the cost structure is quadratic (or in general when the cost structure is different than linear). To highlight the effect of cost structure, we consider both linear and quadratic cost structures in all the analyses. Additionally, in contrast to these works, our study considers a heterogeneous ownership structure for the competitive firms, i.e., we consider the general case of ownership such that some competitive firms also own some portion of the renewable generators while the rest of the competitive firms only generates conventional energy. Therefore, we consider three types of companies: having conventional only, having both conventional and renewable, and having renewables only.

Analysis of the impact of the support mechanism is another important feature of our work. (Rubin and Babcock, 2013) investigate the impact of the pricing method for wind energy and showed that an increase in wind energy capacity reduces the market power in feed-in tariff. (Brown and Eckert, 2020) study the impact of renewable support policies on firm behavior and the outcome of the competition under an oligopolistic market setting. Their basic model consists of two firms that first compete in procurement auction for a certain capacity of renewable production which is determined by a regulator and then compete in the wholesale market. Their result shows that the support mechanism has an impact on both renewable production auction and wholesale competition such that market power is reduced in the feed-in tariff support schema. Different than the current literature, our study comprises the analysis of the impact of different support mechanisms on the MOE and volume of bilateral contracts under heterogeneous ownership and under different cost structures.
Finally, analyses of bilateral contracts which constitute the major part of the trade-in liberalized electricity markets are grounded on growing literature initiated by (Allaz and Vila, 1993). We compared the impact of renewable energy on bilateral contract volume under different support programs.

3.3 Model

Our theoretical analysis begins with describing the overall specifications of the competition in the wholesale electricity market. In the following subsection, analysis concerning the impact of ownership structure and support scheme on the equilibrium is investigated for linear cost structure. Then, similar analyzes are done under competitive pricing for renewables and the result is compared with the previous ones. Finally, we introduce the bilateral contracts into the model to build up the complete formulation.

One of the demand functions mostly used in related literature is the linear function. A linear relationship is also a good approximation of the demand pattern in Turkish wholesale markets as shown in Figure 2.7. In this study, we also assume a linear demand function in the form of

\[ p = a - bD \]  
(3.1)

where \( D \) is total demand, \( a > 0 \) and \( b > 0 \) are demand parameters. In equilibrium total generation must be equalized to total demand due to electricity network constraints. Therefore, we assume that

\[ G = D. \]

Unlike long-run cost factors such as investment, the marginal production cost is primarily related to the fuel used and operating expenses. Even if the same technology and the same fuel are used, the marginal cost could vary according to the efficiency of the generator and the scale of the generation (Walheer, 2018). When a firm uses more than one production technology, it possesses a collection of assets with different variable costs depending on fuel type and operating expenses. Starting from the least costly generation, when these dissimilar generation capacities are collected in ascending order, the overall marginal cost structure of the entire portfolio would be a
stepwise increasing function in quantity. These cost structures are most closely represented by quadratic functions. Nevertheless, for the analysis, we consider both linear and quadratic cost function cases to examine the implications. A linear cost function is suitable when the generating firm relies on a single technology and the fuel cost is the main variable cost. On the other hand, a quadratic cost function is a more general and realistic one representing the portfolio of generators and also a combination of different generation technologies as well as different generation scales. Although the assumption that all the firms have a linear cost function may be very restrictive, it is still worth analyzing this case for specific applications or markets. For example, more than 80% of the total electricity is produced from nuclear energy in France. Nuclear energy requires huge investment costs however only the effective cost in the generation stage is the fuel and disposal cost. In this case, linear cost approximation for large-scale nuclear generations may be reasonable. However, countries such as Turkey and Germany as well as most of the countries over the world have a collection of different sources and also companies in those countries have a portfolio of generators. Therefore quadratic cost function is more realistic for these countries.

3.3.1 General Assumptions

The following problem setup and assumptions are common for the following analysis. The additional assumptions and constraints relevant to specific cases are listed in the related section.

- We consider three different types of electricity generation companies:
  - Type-1: Companies using only conventional technology for the generation,
  - Type-2: Companies using both renewable resources and conventional technology for the generation
  - Type-3: Companies using only renewable resources for the generation.

- The number of Type-1, Type-2, and Type-3 companies are $k$, $l$ and $m$ respectively.
• $K = \{1, \ldots, k\}$ is the set of the Type-1 companies, $L = \{1, \ldots, l\}$ is the set of the Type-2 companies, and $M = \{1, \ldots, m\}$ is the set of the Type-3.

• Type-1 and Type-2 companies are competitive companies, i.e., they can adjust their outputs to maximize profit, while Type-3 companies are price takers.

• $n = k + l$ is the total number of conventional electricity-producing companies. The set $N = K \cup L = \{1, \ldots, k, \ldots n\}$ is the set of conventional electricity generating companies.

• The total capacity of the renewable energy generated for the relevant time-slot is $R$ and it is constant.

• Shares of the renewable generators by the companies are such that:
  • $\lambda \in [0,1]$ of the total amount of renewable energy, $\lambda R$, is produced by Type-1 companies, and each Type-1 company $i \in \{1, \ldots, k\}$ has a share of $\frac{\lambda R}{k}$.
  • $[1-\lambda]R$ is the total amount of renewable energy generated by $m$ Type-3 companies and each Type-3 company $i \in \{1, \ldots, m\}$ has a share of $\frac{[1-\lambda]R}{m}$.

• $n$ firms provide electricity from conventional resources: each Type-1 company $i \in \{1, \ldots, k\}$ produces electricity $g_i$ from only conventional resources and each Type-2 company $j \in \{1, \ldots, l\}$ produces electricity $g_j$ from only conventional resources.

• The cost of producing each unit of electricity from conventional resources is $C(g_i)$ and
  $$\frac{dC(g_i)}{dg_i} > 0 \text{ for all } i \in \{1, \ldots, n\}.$$

• The cost of producing each unit of electricity from renewable resources is zero.
- The total supply of electricity (total amount of electricity generated conventionally by \( n \) firms plus the available renewable energy) is:

\[
G = R + \sum_{i=1}^{N} g_i
\]

- The total supply (generation) of electricity \( G \) is assumed to be equal to the demand for electricity \( D \), \( G = D \) resulting in the following inverse demand function:

\[
p = a - bG = a - b[g_1 + ... + g_i + ... + g_n + R]
\]
where \( a > 0 \), \( b > 0 \) and \( G \leq a/b \).

- The total amount of electricity produced from renewable resources is less than the total demand: \( R \leq D \) implying

\[
R \leq G \leq a/b
\]

Each competitive firm solves the following problem:

\[
\text{Maximize} \quad \Pi_i(g_1,\ldots,g_i,\ldots,g_n;R)
\]
where the profit function for each type is as follows:

Type-1 competitive firms having both conventional and renewable generations:

\[
\Pi^1_i(g_1,\ldots,g_i,\ldots,g_n;R) = p(G)g_i + p_{\text{new}} \frac{\lambda R}{k} - C(g_i) \quad \text{where} \quad i \in K \quad (3.2)
\]

Type-2 competitive firms having only conventional generation:

\[
\Pi^2_i(g_1,\ldots,g_i,\ldots,g_n;R) = p(G)g_i - C(g_i) \quad \text{where} \quad i \in L \quad (3.3)
\]

Type-3 firms having only renewable companies:
\[ \Pi_i^\lambda(g_1, \ldots, g_i, \ldots, g_n; R) = p_{\text{renew}} \frac{[1 - \lambda]R}{m} \quad \text{where} \quad i \in M \quad (3.4) \]

In this setup, renewable-only companies do not have any decision variables. Competitive firms’ strategies are the selection of conventional generation amounts in order to maximize their profits.

The price for electricity produced from renewable resources \( p_{\text{renew}} \) depends on the renewable support program. There are two cases to be considered in the following analysis:

**Case 1:** The electricity produced from renewable resources is subsidized by a feed-in tariff program and a constant predetermined feed-in tariff for each unit of renewable electricity is \( p^R \) for each kWh produced:

\[ p_{\text{renew}} = p^R \]

**Case 2:** There is not a predetermined price for the renewables and the price would be determined in the wholesale market with Cournot competition. The electricity is priced at the oligopolistic price:

\[ p_{\text{renew}} = p(G) \]

### 3.3.2 Linear Cost Function for the Conventional Generation

This section assumes that the cost function for production from the conventional resources is linear:

\[ C(g_i) = c g_i \quad \text{for all} \quad i \in N \quad \text{where} \quad c > 0 \]

The analysis for two kinds of the support program is carried out separately:

**Case 1.1:** Feed-in tariff for renewable electricity with a linear cost function

In this case profit functions for Type-1, Type-2 and Type-3 firms become:

\[ \Pi_i^\lambda(g_1, \ldots, g_i, \ldots, g_n; R) = \left( a - b \left( \sum_{N} g_i + R \right) \right) g_i + p^R \frac{\lambda R}{k} - c g_i \quad \text{where} \quad i \in K \quad (3.5) \]
\[ \Pi^2_i(g_1, ..., g_i, ..., g_n; R) = \left( a - b \left( \sum_{N} g_i + R \right) \right) g_i - c g_i \quad \text{where} \quad i \in L \] (3.6)

\[ \Pi^1_i(g_1, ..., g_i, ..., g_n; R) = p^r \frac{[1 - \lambda]R}{n} \quad \text{where} \quad i \in M \] (3.7)

respectively.

The best response of each competitive company satisfies the following first-order conditions:

\[ \frac{\partial \Pi^1_i(g; g_{-i})}{g_i} = 0 \quad i \in K \]

\[ \frac{\partial \Pi^2_i(g; g_{-i})}{g_i} = 0 \quad i \in L \]

Note that second-order conditions are also satisfied. The solution for the equilibrium provides the following lemma.

**Lemma 3.1:** In a wholesale competition with companies having linear cost function, when renewable energy is subsidized through a feed-in support program, there exists a pure strategy Cournot-Nash equilibrium solution such that regardless of its type, each competitive firm chooses the following amount of conventional generation:

\[ g_i(R) = \frac{a - c}{b(n+1)} - \frac{R}{(n+1)} \quad i \in N \] (3.8)

And equilibrium prices with the total amount of production are:

\[ G(R) = \frac{n}{b(n+1)} \left( (a - c) + \frac{bR}{n} \right) \] (3.9)
\[ p^{1.1}(R) = a - \frac{n}{(n+1)} \left( (a-c) + \frac{bR}{n} \right) \]  

(3.10)

Case 1.2: **Oligopolistic price for renewable electricity with a linear cost function**

Consider the same market organization as the previous one with exactly the same number and types of companies. However, in this case, renewable energy is not subsidized through a feed-in tariff. Instead, renewable energy is traded in the same way as conventional generation is traded. Therefore there will be a unique competitive price for electricity regardless of the source of the energy.

In this setup, the expression for the pre-determinate price of renewable energy \( p^R \) in the equation (3.5) and (3.7) is replaced by the competitive price

\[ p(G) = a - b \left( \sum_{i} g_i + R \right). \]

Following the same procedure as in the previous case, the equilibrium solution is provided in Lemma 3.2

**Lemma 3.2:** In a wholesale competition with companies having linear cost function, when the renewable energy is traded in the market, there exists a pure strategy Cournot-Nash equilibrium solution such that Type-1 firms choose the following amount of conventional generation:

\[ g_i^1(R, \lambda) = \frac{1}{b(n+1)} \left( (a-c) - \frac{k + \lambda(l+1)}{n} bR \right) \quad \forall i \in K \]  

(3.11)

and the Type-2 firms would choose the following amount of conventional generation:

\[ g_i^2(R, \lambda) = \frac{1}{b(n+1)} \left( (a-c) - (1-\lambda) bR \right) \quad \forall i \in M \]  

(3.12)

At equilibrium, total generation and the price would be:

\[ G(R, \lambda) = kg_1 + lg_2 + R = \frac{n}{b(n+1)} \left( a - c + \frac{(1-\lambda)}{n} R \right) \]  

(3.13)

\[ p^{1.2}(R, \lambda) = a - \frac{n}{(n+1)} \left( (a-c) + \frac{(1-\lambda)}{n} bR \right) \]  

(3.14)
3.3.3 Quadratic Cost Function for the Conventional Generation

The linear cost function assumption for the generation is a confining assumption that represents only a limited number of situations. The majority of companies in most of the market possess a collection of different generators relying on dissimilar technologies. In this case, a quadratic cost function is a more appropriate representation of the actual cost structure. For that reason, in this section, we will consider the quadratic cost function case.

Assume that the cost function for production from the conventional resources is quadratic:

\[ C(g_i) = \frac{1}{2}cg_i^2 \text{ for all } i \in N \text{ where } c > 0 \]

Case 2.1: Feed-in tariff for renewable electricity with a quadratic cost function

In this case profit functions for type-1, type-2, and type-3 firms become respectively:

\[
\Pi_i^1(g_1,\ldots,g_n;R) = \left(a - b\left(\sum_{i} g_i + R\right)\right)g_i + p^R \frac{\lambda R}{k} - \frac{1}{2}cg_i^2 \quad \text{where } i \in K
\]

\[
\Pi_i^2(g_1,\ldots,g_n;R) = \left(a - b\left(\sum_{i} g_i + R\right)\right)g_i - \frac{1}{2}cg_i^2 \quad \text{where } i \in L
\]

(3.15)

\[
\Pi_i^3(g_1,\ldots,g_n;R) = p^R \frac{[1-\lambda]R}{m} \quad \text{where } i \in M
\]

Similarly, firms' best responses satisfy the following first-order conditions:

\[
\frac{\partial \Pi_i^1(g_1,\ldots,g_n)}{g_i} = 0 \quad i \in K
\]

\[
\frac{\partial \Pi_i^2(g_1,\ldots,g_n)}{g_i} = 0 \quad i \in L
\]
The result of the solution for equilibrium, in this case, is summarized in Lemma 3.3.

**Lemma 3.3**: In a wholesale competition with companies having quadratic cost function, when renewable energy is subsidized through a feed-in support program, there exists a pure strategy Cournot-Nash equilibrium solution such that each competitive firm chooses the following amount of conventional generation

\[
g_i = \frac{1}{b(n+1)+c}(a-bR) \quad \forall i \in N
\]  

(3.16)

Equilibrium price and total production at equilibrium:

\[
G = \frac{n}{b(n+1)+c}(a-bR) + R
\]  

(3.17)

\[
p^{2e} = a - \frac{b}{b(n+1)+c}(na+(b+c)R)
\]  

(3.18)

**Case 2.2: Oligopolistic price for renewable electricity with a quadratic cost function**

Consider now, the feed-in tariff program in 2.a. has been abolished and authorities decided to implement another support program which is a non-market support program. Thus there is no predetermined price for renewables and the price would be determined in the wholesale market. Thus, replacing

\[
p^R = p(G) = a - b\left(\sum g_i + R\right)
\]

in (3.15). This modification yields the results in Lemma 3.4.

**Lemma 3.4**: In a wholesale competition with companies having quadratic cost function, when the renewable energy is traded in the market, there exists a pure strategy Cournot-Nash equilibrium solution such that each competitive firm chooses the following amount of conventional generation
\[ g_i = \frac{1}{b(n+1)+c} \left( a - \frac{(n+\lambda)}{n} b R \right) \quad \forall i \in N \] (3.19)

Equilibrium price and total production at equilibrium:

\[ G = \frac{1}{b(n+1)+c} \left( na + ((1-\lambda)b+c)R \right) \] (3.20)

\[ p^{2b} = a - \frac{b}{b(n+1)+c} \left( na + ((1-\lambda)b+c)R \right) \] (3.21)

Analysis of these results leads us to the following two propositions:

**Proposition 3.1**: (Impact of support scheme on the equilibrium).

i) Impact of Merit-Order Effect is always higher in feed-in tariff support schemes than in market-based support schemes resulting in lower equilibrium price \( P^{\text{feed-in}} \leq P^{\text{competitive}} \)

ii) The equilibrium price is a strictly decreasing function of \( R \) in a feed-in tariff support scheme and a non-increasing function of \( R \) in competitive pricing

**Proof of Proposition 3.1:**

The proof will be built on the result of Lemma-1 through Lemma-4. To prove that i) holds, we simply subtract the equilibrium price in the feed-in tariff system from the equilibrium price in the competitive market for both linear and quadratic cost cases and show that these differences are always non-negative.

For the linear cost case, from (3.10) and (3.14) we have

\[ p^{ib} - p^{ia} = a - \frac{n}{n+1} \left( a - c + \frac{(1-\lambda)}{n} b R \right) - a + \frac{n}{n+1} \left( a - c + \frac{b R}{n} \right) \]

\[ p^{ib} - p^{ia} = \frac{n}{n+1} \left( -\frac{(1-\lambda)}{n} b R \right) \] (3.22)

\[ p^{ib} - p^{ia} = \frac{\lambda n}{n+1} b R \]

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For the quadratic cost case, from (3.18) and (3.21) we have

\[
p^{2b} - p^{2a} = a - \frac{b}{b(n+1)+c} \left( na + \left((1 - \lambda)b + c\right) R \right) - a + \frac{b}{b(n+1)+c} \left( na + (b + c)R \right)
\]

\[
p^{2b} - p^{2a} = \frac{b}{b(n+1)+c} \left( na + (b + c) - na - \left((1 - \lambda)b + c\right) \right) R
\]

\[
p^{2b} - p^{2a} = \frac{\lambda b^2}{b(n+1)+c} R
\]

(3.23)

Both expressions in (3.22) and (3.23) are nonnegative for the possible parameters of demand and cost functions, showing that feed-in tariff always results in a lower equilibrium price.

ii) Directly follows from the partial derivatives of equilibrium prices. For the feed-in tariff partial derivatives of the prices in (3.10), (3.18) with respect to R are

\[
\frac{\partial p^{1a}}{\partial R} = -\frac{b}{n+1} \text{ and } \frac{\partial p^{2a}}{\partial R} = -\frac{b(b + c)}{b(n+1)+c}
\]

respectively. Both expressions are strictly negative for all possible system parameters, which means that the equilibrium prices (3.10) and (3.18) are strictly decreasing with R. For the competitive case, derivatives of the prices in (3.14) and (3.21) with respect to R are respectively:

\[
\frac{\partial p^{1b}}{\partial R} = -\frac{(1 - \lambda)b}{(n+1)} \text{ and } \frac{\partial p^{2b}}{\partial R} = -\frac{b(1 - \lambda)b + c}{b(n+1)+c}.
\]

One interesting result to point out in Proposition-1 is that although the equilibrium price decreases with increasing R in all renewable energy support mechanisms, the difference between the equilibrium prices in 1.24 and 1.25 are increasing with R. This means that when a considerable amount of renewable energy is available in the market, equilibrium price would be much lower in feed-in subsidization support mechanism.
Proposition 3.2: (Effect of Ownership Structure)

i) Diversified ownership mitigates the Merit Order Effect in competitive pricing. The more competitive firms’ share of R, the higher the equilibrium price. That is, the equilibrium price is increasing in share of the competitive firms λ.

ii) The feed-in tariff subsidization scheme eliminates the impact of the ownership structure. Thus, ownership structure does not have any impact on the equilibrium price when renewable energy is subsidized through a feed-in tariff.

Proof of Proposition 3.2: The proof will be built on the result of Lemma 1, Lemma 2, Lemma 3, and Lemma 4.

i) The proof directly follows from the partial derivatives of the competitive prices in (3.14) and (3.21) with respect to ownership parameter λ.

\[
\frac{\partial p^{1b}}{\partial \lambda} = \frac{bR}{n+1}, \quad \frac{\partial p^{2b}}{\partial \lambda} = \frac{b^2R}{b(n+1)+c} \tag{3.24}
\]

both derivatives are positive showing that equilibrium prices increase in λ.

ii) It is clear from (3.10) and (3.18) that equilibrium prices in feed-in tariff do not include the term λ.

3.3.4 Bilateral Contracts

Bilateral contracts in wholesale power markets are an essential part of the trade. Theoretically, a company increases its market share and profit by selling its part of the generation through bilateral contracts. However, due to the reduced volume traded in the spot market, the market power of the company also decreases. When all firms eventually commit to bilateral contracts at equilibrium, total profit would reduce due to the decreased market power. The only way to avoid such a situation is collusion and not to commit any bilateral contract which is not legal. Besides strategic consideration, there are also several other reasons why companies commit to bilateral trade. These
reasons may be technical limitations specific to the technology used in the generation. For example, consider a thermal generation facility that requires several hours to become fully operational and synchronized with the electricity network from the cold start. In this case, switching off and then switching on the generator is a very costly process. To avoid such a situation and associated costs, the owner may want to commit to a bilateral contract for a certain amount to make sure that this generator runs continuously. However, we do not include these technical restrictions in our model. We assume that generator companies can smoothly adjust their output. In order to analyze the bilateral contract in a basic environment, we assume the same market setting as in 2b in which companies compete for quantities at time $t$. In addition to this basic setting, we assume that companies have signed bilateral contracts before the competition period starts. Bilateral contracts are formulated as forward contracts which are committed before the day ahead competition period:

Consider a two-stage game for $n$ companies producing electricity by using renewable and conventional resources. They commit to independent bilateral contracts and also compete in an organized day-ahead market. Thus their obligation for electricity generation each time $t$ originates from two bases. One is from bilateral contract obligations and the other is due to day-ahead market obligations.

**Stage I:** Stage I covers the time period before the day-ahead competition for time $t$. In this stage, each company $i \in \{1, \ldots, n\} = N$ signs bilateral contracts with the customer $j$ for a certain amount of electricity $B_{i,j}$ to be delivered at the time $t$ at a price $p_i^B$, where either $B_{i,j} > 0$ or $B_{i,j} = 0$. The portfolio of the bilateral contracts of the company $i \in N$ is $B_{\text{port},i} = \{B_{i,1}, \ldots, B_{i,j}, \ldots, B_{i,J}\}$ and the volume of bilateral contracts for each company is the sum of the contracts in its portfolio $\sum_j B_{i,j} = B_i$.

**Stage II:** In a day-ahead competition, after observing $\{B_j\}_{j=1}^n$, each company $i \in \{1, \ldots, n\}$ chooses the generation amount $g_i$ for the time $t$ a la Cournot competition. Therefore, the strategy of each company $i \in N$ is the selection of bilateral contract volume $B_i$ for time $t$ and bid $g_i$ in the day-ahead market for a time $t$:  

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The analysis will be conducted for two separate cases:

**Case 3.1:** Competitive price for renewable energy.

**Case 3.2:** Feed-in tariff for renewable energy cases.

We use backward induction to solve the above two-stage problem and obtain the expression for bilateral contract volume.

**Solution for Case 3.1**

We start with the competitive price case. Considering the bilateral contracts from Stage I and assuming the generation amounts $g_i$ of its rivals in Stage II, each company $i \in \{1, \ldots, n\}$ has the following profit function:

$$
\Pi_i(g_1, \ldots, g_i, \ldots, g_n; B_1, \ldots, B_i, \ldots, B_n; p_i^g) = \left( p(G)\left(g_i - B_i + \frac{\lambda R}{n}\right) + p_i^g B_i - \frac{1}{2} c g_i^2 \right) \forall i \in N
$$

(3.25)

**Stage II Solution:**

In this stage, each company takes the volume of bilateral contracts $\{B_i\}_{i=1}^n$ as given from Stage I, and also given the total equilibrium quantity of generation

$$
G = \sum_N g_j + R
$$

(3.26)

and associated equilibrium price

$$
p(G) = a - bG
$$

(3.27)

The objective of each company is to maximize its profit. Thus the best response of the company $i$ solves the following maximization problem:
\[
\max_{\pi_i} = p(G) \left( g_i - B_i + \frac{\lambda R}{n} \right) + p_i^n B_i - \frac{1}{2} c g_i^2 \quad \forall i \in N
\]

\[
s.t.
\]

\[
p(G) = a - bG
\]

\[
G = \sum_N g_j + R
\]

Which can be expressed as:

\[
\arg \max_{g_i} \Pi_i = \left\{ a - b \left( g_i + \sum_{j \neq i} g_j + R \right) \left( g_i - B_i + \frac{\lambda R}{n} \right) + p_i^n B_i - \frac{1}{2} c g_i^2 \right\} \forall i \in N
\]

(3.29)

The company \( i \)'s best response is characterized by the first order condition of the above problem:

\[
\frac{\partial \Pi_i}{\partial g_i} \left( g_i^*; g_{-i}, B \right) = 0
\]

\[
a - bg_i - b \sum_{j \neq i} g_j - bR - bg_i + bB_i - b \frac{\lambda R}{n} + c g_i = 0
\]

(3.30)

All the firms \( i \in N \) solve the same problem considering their bilateral obligations and the decision of the rivals. Their resulting best responses are governed by the \( n \) equations in the form of (3.30). Cournot-Nash equilibrium in this setting can be calculated as a fixed point by the intersection of the best responses governed by \( n \) system of equations. The solution yields the following Best Response equation:

\[
g_i = \frac{1}{b(n+1) + c} \left( a + \left( \frac{b}{b+c} \right) \left( bn + c \right) B_i - b \sum_{j \neq i} B_j \right) - \left( \frac{n+\lambda}{n} \right) bR \quad \forall i \in N
\]

(3.31)
Inserting the best response expression in (3.31) into (3.26) and (3.27), the results regarding Stage-II are summarized in the following lemma:

**Lemma 3.5:** Given the set of bilateral contracts, there exists a pure strategy Cournot Nash equilibrium in Stage II such that each firm selects the following amount of generation:

\[
g_i(B) = \frac{1}{b(n+1)+c} \left( a + \frac{b}{b+c} \left( (bn+c)B_j - b \sum_{j \neq i} B_j \right) - \frac{(n+\lambda)}{n} B_i \right) \quad \forall i \in N
\]

(3.32)

with the equilibrium price:

\[
p = a - \frac{b}{b(n+1)+c} \left( na + b \sum_{i} B_i + ((1-\lambda)b + c)R \right)
\]

(3.33)

and the total production:

\[
G(B) = \sum_{i} g_i(B_i) + R
\]

\[
G(B) = \frac{1}{b(n+1)+c} \left( na + b \sum_{i} B_i + ((1-\lambda)b + c)R \right)
\]

(3.34)

**Stage I Solution:**

In this stage, each company selects the volume of bilateral contract volume to maximize: (3.25):

\[
\Pi_i = p(G(B)) \left( g_i - B_i + \frac{\lambda R}{n} \right) + p^g_i B_i - \frac{1}{2} c g_i^2 \quad \forall i \in N
\]

(3.35)

In order to eliminate any arbitrage opportunity, bilateral contract prices in the contracting stage can be assumed to be equal to the equilibrium spot price (Bushnell, 2007b). Thus:

\[
E[p^g_i] = p(G(B)) \quad \forall i \in N
\]

(3.36)

So, the profit expression in the contract stage turns out to be:
Each company assumes that any commitment to a bilateral contract volume $B$ in Stage I would yield the equilibrium values in Lemma 3.5 in Stage II. Therefore, equilibrium can be calculated by using the anticipated results of Stage II inserted in Stage I profit expression. Using the anticipated equilibrium condition in Stage II, i.e., best repose correspondences in (3.32), equilibrium aggregated demand in (3.38), and the equilibrium price in (3.39), Best Response optimal bilateral contract volumes for $t$, solves the following maximization problem:

$$B_i \in \arg \max \Pi_i = p \left( G \left( B_i \right) \right) \left( g_i \left( B_i \right) + \frac{\lambda R}{n} \right) - \frac{1}{2} c \left( g_i \left( B_i \right) \right)^2 \forall i \in N$$

Subject to

$$g_i \left( B \right) = \frac{1}{b(n+1)+c} \left( a + \frac{b}{b+c} \left( bn + c \right) B_i - b \sum_{j \neq i} B_j \right) - \frac{\left( n + \lambda \right) bR}{n}$$

$$p \left( G \left( B \right) \right) = a - \frac{b}{b(n+1)+c} \left( na + b \sum_{i} B_i + \left( (1-\lambda)b + c \right) R \right)$$

which can be reduced to the following unconstrained optimization problem:

$$B_i \in \arg \max \Pi_i = p \left( g_i + \frac{\lambda R}{n} \right) - \frac{1}{2} c \left( g_i \right)^2 \forall i \in N$$

where

$$g_i = \frac{1}{b(n+1)+c} \left( a + \frac{b}{b+c} \left( bn + c \right) B_i - b \sum_{j \neq i} B_j \right) - \frac{\left( n + \lambda \right) bR}{n}$$

and

$$p = a - \frac{b}{b(n+1)+c} \left( na + b \sum_{i} B_i + \left( (1-\lambda)b + c \right) R \right)$$

First Order Condition for the above problem is:
\[
\frac{\partial \Pi_i}{\partial B_i} = \frac{\partial \Pi_i}{\partial B_i} p \left( g_i + \frac{\lambda R}{n} \right) + p \frac{\partial \Pi_i}{\partial B_i} g_i - c g_i \frac{\partial \Pi_i}{\partial B_i}
\]

Inserting the partial derivatives:

\[
\frac{\partial \Pi_i}{\partial B_i} = -b^2 \left( g_i + \frac{\lambda R}{n} \right) + p \frac{b(bn+c)}{(b(n+1)+c)(b+c)} - c g_i \frac{b(bn+c)}{(b(n+1)+c)(b+c)} = 0
\]

where

\[
g_i = \frac{1}{b(n+1)+c} \left( a + \frac{b}{b+c} \left( (bn+c) B_i - b \sum_{j \neq i} B_j \right) - \frac{(n+\lambda)}{n} b R \right)
\]

and

\[
p = a - \frac{b}{b(n+1)+c} \left( na + b \sum_{j \neq i} B_j + (1-\lambda) b + c \right) R
\]

Since the firms are symmetric, the contract volumes for each firm must be identical at equilibrium: \( B_i = B_j = B \quad \forall i, j \in N \). Performing the necessary calculations, the results for optimal contract volume are summarized in Lemma 3.6.

**Lemma 3.6:** There exists a pure strategy Subgame Perfect Cournot-Nash equilibrium bilateral contract volume which is identical for each firm given by:

\[
B = \frac{(n-1)}{b(b+c)+(bn+c)^2} \left[ ba - \left[ b - \frac{(bn+c)}{n} \lambda \right] R \right]
\]  

(3.42)

**Proposition 3.3:** When the price of renewable energy is determined through competition, there exists a unique Subgame Perfect Nash Equilibrium of the game such that each firm \( i \in N \) chooses the following strategy:

\[
s_i^{3.1}(B_i, g_i) = \left\{ \begin{array}{c}
B_i^{3.1} = \frac{(n-1)}{b(b+c)+(bn+c)^2} \left[ ba - \left[ b - \frac{(bn+c)}{n} \lambda \right] R \right] \\
g_i^{3.1}(B) = \frac{1}{b(n+1)+c} \left( a + \frac{b}{b+c} \left( (bn+c) B_i - b \sum_{j \neq i} B_j \right) - b R \right)
\end{array} \right\} \quad \forall i \in N
\]
Solution for Case 3.2

In this case, renewable energy is subsidized through a feed-in tariff $p^R$. The profit function takes the following form:

$$\Pi_i(g_1, \ldots, g_n; B_1, \ldots, B_n; R; p^R_i) = p(G)(g_i - B_i) + p^R_i \frac{\lambda R}{n} + p^R_i B_i - \frac{1}{2} c g_i^2 \quad \forall i \in N$$
ollowing the same solution concept, the results for this case are summarized in Proposition 3.4: When renewable energy is subsidized with a feed-in tariff, there exists a unique Subgame Perfect Nash Equilibrium of the game such that each firm $i \in N$ chooses the following strategy:

$$s_i^{3.2}(B_i, g_i) = \begin{cases} B_i^{3.2} = \frac{b(n-1)}{b(b+c)+(bn+c)^2} [a - R] \\ g_i^{3.2}(B) = \frac{1}{b(n+1)+c} \left[ a + \frac{b}{b+c} \left( (bn+c) B_i - b \sum_{j \in i} B_j \right) - bR \right] \end{cases} \quad \forall i \in N$$

The comparison of the result of both cases provides the following proposition:

Proposition 3.5:

The equilibrium bilateral contract volume $B$ is such that:

i) The equilibrium contract volume $B$ is a strictly decreasing function of $R$ in the feed-in tariff support program. However, it depends on demand and cost parameters $\{a, b, c, n\}$ and ownership fraction $\lambda$ in the case of competitive price.

ii) The equilibrium contract volume is strictly increasing with the ownership fraction $\lambda$ of the competitive firms.

iii) Feed-in tariff support programs always result in a lower bilateral contract volume $B$.

Proof of Proposition 3.5:

i) For the feed-in tariff case $\frac{\partial B^{3.2}}{\partial R} = -\frac{b(n-1)}{b(b+c)+(bn+c)^2} < 0$, thus the
ii) contract volume decreases in \( R \). For the competitive price case

\[
\frac{\partial B_{i}^{3.1}}{\partial R} = -\frac{b(n-1)}{b(b+c)+(bn+c)^2}\left[b - \left(\frac{bn+c}{n}\right)\lambda\right], \quad \text{when} \quad \lambda < \frac{bn}{bn+c},
\]

the contract volume decreases in \( R \) and the converse when \( \lambda > \frac{bn}{bn+c} \).

iii) When every other parameter is fixed in the system,

\[
\frac{\partial B_{i}^{3.1}}{\partial \lambda} = \frac{(n-1)R}{b(b+c)+(bn+c)^2}\left(\frac{bn+c}{n}\right) > 0 \quad \forall R > 0
\]

which shows that contract volume is increasing in \( \lambda \).

iv) A representative inverse demand function is constructed in order to obtain similar equilibrium prices in the Turkish Wholesale Market, “Energy Exchange Istanbul” (EXIST). For this reason, the demand function \( p=1100-0.35*G \) where \( a=1100 \) and \( b=0.35 \) are selected. We assume that there are 8 competitive generation firms. For the cost parameter, we anticipate 35% marginal profit at equilibrium. Therefore \( c=0.65 \) is selected for the quadratic cost case and \( c=180 \) is selected for the linear cost curve. With these parameters, equilibrium prices for 1a, 1b, 2a, and 2b when \( R=0 \) are 289.5 TL, 289.5 TL, 282.2 TL, and 282.2 TL.
respectively, which is fairly close to the actual equilibrium price of 280 TL at that time point.

The result of Proposition-1 is illustrated in Figure 3.3 and Figure 3.4. In both cases, namely in both linear cost and quadratic cost cases, equilibrium price decreases with available renewable energy at that time, reflecting the Merit-order effect. The main result of Proposition-1 is about the impact of the subsidization mechanism for renewable in equilibrium price. The results indicate that the Merit order Effect is more powerful in the feed-in tariff support scheme. As a result, for a given market setting, the equilibrium price is lower when renewable energy is subsidized through a feed-in tariff. Figure 3.5 and Figure 3.6 confirm the finding of Proposition-1. In both linear and quadratic cost cases, the equilibrium price for the feed-in tariff indicated by the blue line is always below the equilibrium price for the competitive tariff which is indicated by the orange lines. In both linear and quadratic cost situations, the difference between equilibrium prices for alternative support mechanisms is increasing with R. This result implies that when the share of renewable energy is increased to considerable amounts and this increase is supported through a feed-in tariff program, prices in spot market would eventually decrease dramatically.

![Merit Order Effect (Linear Cost)](image)

**Figure 3.3:** Equilibrium price as a function of R, linear cost case

The ownership structure also plays an important role in the equilibrium price. However, Proposition 2 suggests the effect of ownership is eliminated in feed-in tariff support programs. Nevertheless, when the prices are competitive, Merit order Effect
is mitigated by the increasing shares of competitive firms. Figure 3.5 shows that, while every other parameter in the economy is fixed, the equilibrium price increases with increasing $\lambda$.

**Figure 3.4:** Equilibrium price as a function of $R$, quadratic cost case

**Figure 3.5:** Effect of $\lambda$ on equilibrium price in a fixed market setup
How equilibrium price moves according to the amount of available renewable energy under different ownership percentages is illustrated in Figure 3.6. As expected, the equilibrium price decreases slower when the share of competitive firms is higher.

![Effect of Ownership](image)

**Figure 3.6:** Equilibrium Price vs Available Renewable Energy under different Ownership Structures

### 3.4 Conclusion

The ever-increasing sustainability concerns necessitated the development of policies to combat climate change. Energy systems with their distinct characteristics have been greatly influenced by the policies imposed. Due to these policies, many issues emerged that needed to be studied by different disciplines. In this study, we examine the impact of intermittent renewable energy integration on the wholesale market. Specifically, the problem due to negligibly small marginal production cost is studied in various cases. In addition, we try to understand whether the way how renewable energy is subsidized and ownership of renewable energy play any role in the equilibrium. Different than the current literature, we consider heterogeneous ownership structures with both quadratic and linear cost structures and compare the results for both market-based and non-market renewable energy support structures. Our results indicate that renewable energy negatively affects equilibrium prices in all cases. Furthermore, this negative effect is exacerbated when renewable energy is supported by a non-market support
mechanism such as a feed-in tariff. Ownership also plays an important role in equilibrium. Since the competition is characterized as imperfect, strategic players can take advantage of diversified portfolios and mitigate the adverse effect. However, feed-in-type support programs eliminate the effect of ownership. Another important set of findings is related to the volume of bilateral contracts. In the linear cost case, the volume of bilateral contracts is negatively related to the quantity of renewable energy and the ownership fraction improves this negative relation. However, the situation in the quadratic cost case is complicated and the relation between bilateral contact volume and quantity of renewable energy depends on demand parameters and ownership fraction. For a certain set of parameters, bilateral contract volume and renewable energy quantity are negatively related when the ownership fraction is close to 0. The negative relation improves with the increasing ownership fraction and depending on the system parameter, may reach a break-even point after which the bilateral contract volume becomes positively related to the available renewable energy.

Various policy implications can be drawn from these results. First, in order to ease adverse effects, policymakers should revisit their subsidization programs and implement market-based mechanisms instead of non-market mechanisms. Furthermore, strategic companies should be encouraged to add renewable energy into their portfolios.
CHAPTER 4

A NOVAL DEMAND RESPONSE MODEL AND ITS ANALYSIS

4.1 Introduction

Electricity has a unique characteristic among all the other consumable goods. Due to its nature and the current technological limits, electricity cannot be stored in large quantities feasibly and should be consumed instantaneously when generated. This characteristic exposes one of the most challenging limitations in both infrastructure design in the technical domain and economic design of the electricity markets. Another limitation is network feasibility constraints which require input to and output from the transmission network must be balanced with very tight limits at all times. Within these restrictions, the operations of conventional electricity markets have been established on the principle that electricity generation is adjusted continuously according to the corresponding demand. Thus, only the supply side of the system is active in the traditional adjustment mechanism (Hu et al., 2013). On the other hand, demand for electricity fluctuates through the day along with daily routines like appliance usage, transportation, production processes, lighting needs, heating and cooling requirements, etc., and, in most markets, makes a peak or peaks at certain hours. Unfortunately, the consumption side is not able to observe the efficiency signals of the supply side and does not have any incentive to adjust its consumption accordingly (Kirschen, 2003). The practice is not compatible with the competitive market idea where prices adjust according to the scarcity of the product or service (Zarnikau, 2008). Throughout the years, the need for elasticity on the demand side and interest in altering the consumption patterns of the users by suppliers and system operators to improve the efficiency of electricity markets and electricity systems have been growing. The desire to influence the consumption decision of the customer brought about the idea of Demand Side Management (DMS). DMS is a very broad concept incorporating several
long-term and short-term activities designed to alter electricity consumption patterns. Demand Response (DR) is a subcategory of DMS related to short-term market operations. DR is a tariff scheme, a program, or an incentive mechanism established to influence the end-user customers’ consumption patterns in response to the changes in the price of the electricity (US Dept. Energy, 2006).

The early objective intended by Demand Response methods was to eliminate the inefficiencies due to peak load and promote the system safety of the transmission networks. All infrastructure and investment should be settled according to the peak demand to prevent blackouts since blackouts cause huge economic costs in addition to undesirable discomfort (Shuai et al., 2018). Moreover, there should be some reserve margin for generation in case of unexpected demand increases and production uncertainties due to system unavailability. Together with this reserve capacity, installed generation capacity would be more than 100% of the probable peak demand for most of the markets. Unfortunately, the system should be run under its capacity during off-peak times. This peak load capacity stays idle during off-peak periods resulting in a loss of opportunity cost and a reduction in system efficiency. When average electricity usage is compared with installed capacity, it can be inferred that utilization of installed production capacity could be as low as 55% (Strbac, 2008).

From the system safety point of view, peak demand always poses technical threats to the system by ramping of generation, thermal loads, stress on transmission lines, etc. Hence, changing the demand profile in order to decrease the peak load and distribute the load as evenly as possible has been one of the main objectives of Demand Response. However, increasing utilization of renewable energy, decentralization of generation, and participation of small-scale producers bring another dimension to the problem: supply uncertainty.

Low carbon policies such as the EU’s objective to reduce greenhouse gas emissions by at least 80% below 1990 levels by 2050 necessitates a significant increase in the share of renewables (da Graça Carvalho, 2012). Besides technological feasibility, efficient use of higher renewable energy requires the implementation of new market models facilitating the flexible demand profiles to account for the supply uncertainty. Generation amount from renewables and timing of generation is almost completely dependent on the weather condition. If the production from renewables is high when
the demand is low, the unused amount would be wasted due to the instantaneously perishable characteristic of the electricity. However, a few hours later other conventional sources with marginal production costs associated with greenhouse gas emissions would be used in order to meet the demand when renewable production decreases below the demand at that time. Consequently, continuous adjustment of the supply according to the demand is no longer an effective practice when higher renewable energy utilization is envisioned since supply from renewables is rigid. Alternatively, shifting the demand from one point in time to another by Demand Response following the available generation schedule has great potential. There are several other consequences of this supply-demand mismatch. Oversupply of renewables causes a decrease in prices in the wholesale market and even the occurrence of negative prices which make it hard to cover operation costs and affect the investment decision in the long run (Cramton and Stoft, 2006), (Joskow, 2008). This paradigm change due to the increasing usage of renewable energy has changed the focus of demand response practices. Therefore, within the context of renewable energy utilization, the primary goal of demand-side management become changing the demand profile in order to make it compatible with the inflexible production profile rather than preventing the peak demand only.

Although Demand Response is a promising concept to obtain flexibility on the demand side, most of the traditional DR methods which are designed to mitigate the adverse effect of peak demand fail to address the problem arising from supply uncertainty and do not handle the challenges of increasing renewable utilization. For example, “Critical Peak Pricing” where prices are high for certain times when peak load occurs is only effective to shave the “peak” load. “Time-Of-Use (TOU)” where there is a set of pre-determined tariffs for certain periods is one of the oldest programs that has many practical applications. TOU rates are not flexible enough in the short run to influence consumer demand dynamically to account for the supply uncertainty (Borenstein, 2005). Among many others, Real-Time Pricing is theoretically a very efficient dynamic pricing practice since it reflects the actual cost of supply by continuously updating the price. However, real-time pricing brings maximum uncertainty and risk for the customer. Moreover, real-time pricing requires a very high communication rate and customer involvement which is not possible often (Dutta and Mitra, 2017), (Dütschke and Paetz, 2013). Another drawback of Real-Time pricing is that the
customers may not be dynamic enough to adapt to the price signals and respond to the price change rapidly. Another important aspect of Demand Response programs is how flexibility is obtained. Price-based DR programs generally use price signals to alter customer consumption patterns. These kinds of programs aim to influence customer behavior indirectly. However, when the demand is controlled directly, the System Operator obtains superior certainty compared to the indirect control through price signals. (Callaway and Hiskens, 2010). Therefore, instead of manipulating the demand through price signals, the system operator may have some control over the timing of the electricity usage of the customer as a more effective way to address supply uncertainty. It has been discussed that Direct Load Control could produce more reliable demand flexibility (Stenner et al., 2017), (He et al., 2013). Besides, load shifts should be explicitly included in the effective Demand Response Program.

Based on these arguments, we propose a Demand Response model in which customers voluntarily let the System Operator decide the timing of some amount of electricity usage in order to get an incentive in the form of a discounted price. In this model, electricity usage is segmented into two types according to time flexibility. Before each optimization period, discounted prices are offered for flexible usage for which the System Operator decides the exact usage time within predefined time boundaries. The customer selects the amount for flexible and ordinary usage according to price offers. Therefore, customers get benefits in terms of incentives in return provide time-flexibility to the system operator in such a way that the system operator makes sure that generated renewable energy is consumed efficiently while maximizing profit. The model could be categorized as Direct Load Control demand response practice, but it also includes dynamic pricing since the discounted price is determined dynamically at the beginning of the optimization period.

This novel market model addresses some of the major problems related to the efficient usage of renewable energy and provides more reliable and practical solutions compared to traditional price-signal-based demand response practices available in the literature. From the customer perspective, the risks associated with volatile prices are eliminated since the prices for ordinary usage and flexible usage are determined before the event time. The total incentive is proportional to the volume of flexible usage and the customer knows the outcome before committing with certainty. The arguments
regarding bill stability which is one of the main concerns related to dynamic pricing (Borenstein, 2009) disappear in this model. Moreover, customer involvement requires less effort. A well-established communication system is required but this is a “must” for any other dynamic pricing technique. In other dynamic pricing practices like real-time pricing, customer needs to search for the optimal price continuously which requires continuous effort from the customers. From a system operator perspective, having the right to control the scheduling of the demand rather than influencing the customer through price signals provides a great advantage in better utilization of renewable energy since the System Operator obtains more reliable and adjustable demand flexibility with a greater amount of certainty. Our base model can be extended to include constraints about the duration of DR event and maximum shift periods could be predefined.

4.2 Related Literature

Demand Side Management has always been regarded as a tool with high potential for eliminating inefficiencies in the electricity sector. The policies and regulations implemented for reducing greenhouse gas emissions and promoting sustainability have boosted this potential since decreased flexibility due to uncontrollable renewables on the supply side could be compensated on the demand side through Demand Side Management (Miscoel et al., 2021). Accordingly, the attention on Demand Side Management, particularly on Demand Response, has been growing for the last decade. In parallel, the literature on Demand Side Management has increased steeply from around 130 publications in 2009 to more than 1800 yearly articles in 2020 (Morales-España et al., 2021). Several articles are reviewing these publications in the literature focusing on certain aspects.

Benefits and Challenges of Demand Side Management have been discussed by many authors (O’Connell et al., 2014),(US Dept. Energy, 2006), (Conchado and Linares, 2012). Among several others, benefits from three main perspectives stand out: financial, operational, and better renewable energy utilization perspectives. Both supplier and end-user sides could obtain financial benefits through DR. Supply-side face competitive market conditions and prices in the spot market are frequently volatile (De Jonghe et al., 2008). One of the main reasons for this volatility is inelastic demand
due to flat retail prices since the demand side is irresponsible to the supply side's efficient signals. Load that can be shiftable or curtailable can act as an additional source of supply for demand-supply balancing and reduce the market power. Therefore, with smoothed net demand profiles, price spikes and inefficient prices due to excess supply or supply shortages occur less frequently (Bergaentzlé et al., 2014). Moreover, smooth generation from thermal sources promotes generation efficiency and reduces fuel costs (Müller and Möst, 2018). Therefore, wholesale electricity prices, as well as the price volatilities, would be reduced as a result of DR (Asadinejad and Tomsovic, 2017). This reduction would be eventually reflected in the bills of the end-users. The end-user may also benefit from the direct incentive provided by DR practices (Gottwald et al., 2016). Flexibility on the demand side allows network operators to manage network constraints more efficiently (Affonso et al., 2005) (Zibelman and Krapels, 2008). Efficient management of the lines also promotes a reduction in line losses (Shaw et al., 2009). Another significant potential operational benefit is that the costly investment required for peak-load capacity, especially in such an uncertain future, can be avoided (IRENA, 2019) (Veldman et al., 2013) (Blokhuis et al., 2011). All these potential benefits of DR are even greater when it comes to renewable energy integration since all these problems and complications amplify in the case of renewable energy integration (Simshauser, 2019). Shares of total renewable energy can be increased by compensating for the loss of flexibility on the supply side and preventing curtailment through Demand Response (Gils, 2014).

DR Modeling in the literature varies significantly depending on the type of DR strategy in consideration, scale, the proposed problem setup, etc. In the literature, there is no common Demand Response modeling framework that can be applied in general and on which a consensus has been achieved. A large group of literature attempts to quantify demand response potential based on available resources or rational economic behavior of the users. Some of these even do not include any analytical formulation that relates price or incentive to demand response behavior, instead, rely on assumptions such that a certain fraction out of total demand response potentials was available on hand. For example, (Märkle-Huß et al., 2018) first calculates overall demand response potential based on available shiftable sources such as appliances usage, heating devices, etc., and then analyzed the effect of demand shifts on the wholesale prices assuming they would use 1% and 10% of the available demand
response potential. These kinds of analyses are useful in understanding the potential value of the Demand Response sources and enable the researchers to make a comparison of the potential value with the cost of implementation. A major group of authors utilizes the price elasticity of demand to represent Demand Response potentials (O’Connell et al., 2015). Many of those models such as (Heydarian-Forushani et al., 2020) and (Allcott, 2011) rely on a reduction or increase in the demand at a certain point in time and do not explicitly represent a demand shift from one point in time to another. Therefore, those models fail to address demand recovery. However, proper DR modeling should include demand recovery of the affected demand within a reasonable timeframe, i.e., a change in demand at one point in time should be compensated by a change in the reverse direction at a certain point in time (Zerrahn and Schill, 2015). It is important that proper modeling should include time-related constraints ensuring that the net load shift within the optimization period should be zero. Otherwise, the results obtained from the model would be ambiguous as users are not expected to forego electricity usage for certain applications in the short term because it is costly at one point. Some authors such as (Asensio et al., 2017) and (De Jonghe et al., 2012) include cross-price elasticities for different time slots to account for demand shifts between time slots. However, cross-price elasticities do not ensure that the demand shift balance is satisfied (Zerrahn and Schill, 2015).

Another main group of modeling considers DR in capacity planning and includes DR sources in Unit Commitment problems as a source of negative generation (McPherson and Stoll, 2020). The objective of the Unit Commitment Model or Production Cost Model is to find out the least costly generation plan to meet the demand considering various generation sources with different variable costs (Hummon et al., 2013). Therefore, including DR in these models facilitates understanding the potential benefit of DR on system capacity planning and cost reduction. Nevertheless, the majority of these models do not provide the mechanisms for how Demand Response is obtained on the demand side.

Optimization methods are also closely related to the strategy of Demand Response and the definition of the problem. The objective of a Demand Response program could be minimizing production cost, maximizing social welfare, maximizing economic benefit, maximizing renewable energy utilization, etc (Vardakas et al., 2014).
(Mohsenian-Rad and Leon-Garcia, 2010) employed Linear Optimization to find the optimal consumption of different appliances in a real-time pricing Demand Response setting. The inclusion of binary decision variables such as on-off status leads to Mixed Integer Programming as in (Kriett and Salani, 2012) and (Nan et al., 2018). Examples of Game-Theory based optimization can be found in (Feng et al., 2020) and (Li et al., 2021). More complicated optimization methods are utilized such as Nonlinear models in (Leithon et al., 2018) and Stochastic models in (Chen et al., 2012). Dynamic Optimization considering the interaction of supply and demand is found in (Jiang and Low, 2011).

Our modeling differs from current literature in that we consider the dynamic interaction of the supply and demand sides together in a comprehensive way. In addition, we derived consumer preferences between inflexible and flexible usage for a given incentive based on the strategic decision process of the customer rather than assuming linear relations or simplified forms of demand response relations. To the best of our knowledge, such a kind of customer modeling in the direct load control context is missing in the literature. Period-by-period energy balance constraints and time-related load shift constraints are two fundamental constraints that ensure that demand recovery and demand matching requirements are satisfied. We also explicitly model load shifts including these constraints. Our resulting mathematical problem is a nonlinear dynamic optimization.

4.3 The Model

The market we consider consists of a System Operator who provides electricity and a representative customer who is the only consumer of electricity. The System Operator has two kinds of electricity generation sources: one is from renewable energy generators and the other is from a portfolio of conventional (nonrenewable) generators. The two kinds of sources differ from each other in terms of controllability of timing and quantity: the timing and quantity of the generation from renewables completely depend on uncontrollable external factors (particularly on weather conditions), whereas the quantity and timing of the conventional generation are controllable. The marginal cost of the production from renewables is zero but the marginal cost of production from conventional resources is equal to the market price of the electricity
The system operator first uses the costless renewables to meet the demand and then dispatches the conventional if needed. In order to better utilize renewables and increase revenue by decreasing the cost originating from conventional generation, the system operator wants to shift some of the demand from the point where there is a lesser amount of renewable generation to the point where renewable generation is abundant. To do so, the SO implements a Demand Response mechanism. In this mechanism, S.O. segments the electricity usage into two according to time flexibility and offers two options to the customer. The first option is ordinary (inflexible) usage the customer uses the electricity whenever she desires at the market price \( p \). The other option is flexible consumption such that the customer lets the system operator selects the usage time within \( \pm n \) of the desired time at a discounted price \( p_{\text{flex}} \). The representative customer’s optimal decisions determine the level of consumption in each category.

- The planning period is \( \{1, \ldots, T\} \), where \( T \in \mathbb{N} \) and \( 1 < T < \infty \).
- Nature reveals each period \( t \)'s state:
  \[
  \{s_1, \ldots, s_t, \ldots, s_T\} = \{s_t\}^T_{t=1} \text{ where } s_t \in \Omega \text{ and } t \in \{1, \ldots, T\}.
  \]
  The state of the Nature includes all the short-term information such as weather conditions, the day of the week, etc. which are the determinates of the next day's consumption and renewable generation levels.
- For all levels of the electricity produced from nonrenewable resources \( q_c \in (0, \infty) \), the unit cost of production is \( c_c \in (0, \infty) \).
- For all levels of the electricity produced from renewable resources \( q_R \in (0, \infty) \), the unit cost of production is \( c_R = 0 \).
- The price of the electricity for ordinary usage is determined by policymakers as \( p \in \mathbb{R}^+ \) and the discounted price for flexible usage is determined by the S.O. \( p_{\text{flex}} \in [0, p] \).
4.3.1 Demand Side:

The consumer has an upcoming planning period of \( \{1,...,t,...,T\} \). The consumer consumption plan in the very short-run is only affected by the state of the planning period \( \{s_t\}_{t=1}^T \) where \( s_t \in \Omega \) since all the other factors affecting the consumption decision can be assumed to be stationary for such short-term planning. Moreover, state of the nature is revealed by Nature to the RCE well before the upcoming period. Thus, given \( \{s_t\}_{t=1}^T \) where \( s_t \in \Omega \) for the upcoming period, the RCE’s set of ex-ante consumption levels is:

\[
\bar{D}_{1:T} = \{\bar{D}_1(s_1),...,\bar{D}_t(s_t),...,\bar{D}_T(s_T)\}
\]

However, the consumer can allow System Operator to shift some of its consumption \( \bar{D}_t(s_t) \) from \( t \in \{1,...,T\} \) to \( \tau \in \{1,...,t-1,t+1,...,T\} \) in exchange for a discounted price, which is referred to as flexible consumption. The portion of the demand that is committed to flexible usage for price \( p_{\text{flex}} \) at time \( t \in \{1,...,T\} \) is denoted with \( d_{\text{flex},t}(s_t) \). In the same way, the remaining demand at the time \( t \in \{1,...,T\} \) that the consumer does not want to shift over time is referred to as inflexible consumption and it is denoted with \( d_{\text{inflex},t}(s_t) \).

We assume that wealth increase due to the incentive does not induce any increase in consumption levels. The sum of the flexible and inflexible demand consumption levels obey the ex-ante consumption plan at each \( t \in \{1,...,T\} \).

\[
d_{\text{inflex},t}(s_t) + d_{\text{flex},t}(s_t) = \bar{D}_t(s_t) \quad (4.1)
\]

Assume that for all \( t \in \{1,...,T\} \), the unit price of the inflexible consumption is \( p \) and that of flexible consumption is \( p_{\text{flex}} \). Then, the cost of time \( t \in \{1,...,T\} \) consumption is:

\[
p d_{\text{inflex},t}(s_t) + p_{\text{flex}} d_{\text{flex},t}(s_t) = p d_{\text{inflex},t}(s_t) + p_{\text{flex}} \left[ \bar{D}_t(s_t) d_{\text{flex},t}(s_t) \right]
\]

The nominal gain from the flexible consumption is:
\[ p \left[ d_{\text{inflex},t}(s_t) + d_{\text{flex},t}(s_t) \right] - p d_{\text{inflex},t}(s_t) = d_{\text{inflex},t}(s_t) + \left[ p - p_{\text{flex}} \right] = M_{\text{i}}(s_t) \]

Taking \( p \) as a numeraire, the consumer’s real wealth gain from the flexible consumption can be written as:

\[ \frac{M_{\text{i}}(s_t)}{p_t} = \mu_t(s_t) = d_{\text{flex},t}(s_t) \left( 1 - \frac{p_{\text{flex},t}}{p_t} \right) \] (4.2)

The consumer derives utility from inflexible consumption \( d_{\text{inflex}} \), flexible consumption \( d_{\text{flex}} \), and the real wealth gain obtained from committing to flexible consumption, \( \mu \).

\[ U(d_{\text{inflex}}, d_{\text{flex}}, \mu) = \delta_{\text{inflex}} U(d_{\text{inflex}}) + \delta_{\text{flex}} U(d_{\text{flex}}) + \delta_{\mu} U(\mu) \] (4.3)

with weights \( \delta_{\text{inflex}} > 0, \delta_{\text{flex}} > 0, \delta_{\mu} > 0 \)

This utility function can be written in terms of demand and prices as:

\[ U(d_{\text{inflex}}, d_{\text{flex}}, \mu) = \delta_{\text{inflex}} U(d_{\text{inflex}}) + \delta_{\text{flex}} U(d_{\text{flex}}) + \delta_{\mu} U \left( d_{\text{flex}} \left[ 1 - \frac{p_{\text{flex},t}}{p_t} \right] \right) \] (4.4)

From constraint in (4.1), we have

\[ d_{\text{inflex},t} = D_t(s_t) - d_{\text{flex},t} \]

Then

\[ U(d_{\text{inflex}}, d_{\text{flex}}, \mu) = \delta_{\text{inflex}} U(D_t(s_t) - d_{\text{flex},t}) + \delta_{\text{flex}} U(d_{\text{flex}}) + \delta_{\mu} U \left( d_{\text{flex}} \left[ 1 - \frac{p_{\text{flex},t}}{p_t} \right] \right) \] (4.5)

Let:

\[ U(x) = \frac{x^{1-\rho}}{1-\rho} \]

where \( \rho \in (0,1) \). So:

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\[ U(d_{\text{flex}}) = \frac{1}{1-\rho} \left[ \delta_{\text{inflex}} \left( \bar{D}_{t} - d_{\text{flex},t} \right)^{-\rho} + \delta_{\text{flex}} + \delta_{\mu} \left[ 1 - \frac{p_{\text{flex},t}}{p_t} \right]^{-\rho} \right] d_{\text{flex},t} \]  

(4.6)

Since \( U(d_{\text{flex}}) \) is concave in \( d_{\text{flex}} \), the first order condition

\[ \frac{\partial U(d_{\text{flex}})}{\partial d_{\text{flex}}} = 0 \]

Provides the utility-maximizing \( d_{\text{flex}} \) and hence utility maximizing \( d_{\text{flex},t}, d_{\text{inflex},t}, \) and \( \mu_t \) turns out to be:

\[ \hat{d}_{\text{flex},t} \left( p_{\text{flex}}, p, s_t \right) = \alpha \left( \frac{p_{\text{flex}}}{p} \right) \bar{D}_{t} \left( s_t \right) \]

\[ \hat{d}_{\text{inflex},t} \left( p_{\text{flex}}, p, s_t \right) = \left[ 1 - \alpha \left( \frac{p_{\text{flex}}}{p} \right) \right] \bar{D}_{t} \left( s_t \right) \]  \hspace{1cm} (4.7)

\[ \hat{\mu}_t \left( p_{\text{flex}}, p, s_t \right) = \left[ 1 - \frac{p_{\text{flex}}}{p} \right] \alpha \left( \frac{p_{\text{flex}}}{p} \right) \bar{D}_{t} \left( s_t \right) = \left[ 1 - \frac{p_{\text{flex}}}{p} \right] d_{\text{inflex},t} \left( p_{\text{flex}}, p, s_t \right) \]

where

\[ \alpha \left( \frac{p_{\text{flex}}}{p} \right) = \left[ 1 + \delta_{\text{inflex}}^{\frac{1}{\rho}} \delta_{\text{flex}} + \delta_{\mu} \left[ 1 - \frac{p_{\text{flex}}}{p} \right]^{-\rho} \right]^{-1} \]

4.3.2 Supply Side:

The System Operator does not have any direct control over the consumer decision process however, it can influence the output by setting the discounted price level. The amount of flexible usage that the System Operator needs is the key factor in the optimization. The System Operator’s decision-making process proceeds in a multi-stage context. Firstly, it decides the unit price of the time-flexible usage of electricity, \( p_{\text{flex}} \). Then, this price determines the time flexible and inflexible demands for each \( t \in \{1, \ldots, T\} \) by influencing the RCE’s optimal choices. Next, in order to minimize
generation from conventional resources and maximize the utilization of renewables, the SO shifts time flexible demands among periods and then determines the amounts of electricity to be provided from nonrenewable resources to match total generation with the total demand.

4.3.3 Complete Model:

The whole model can be formulated as the following multi-stage problem:

**Stage 0:**  
*Nature:* Nature reveals each period $t$’s state $\{s_t, \ldots, s_T\} = \{s_t\}_{t=1}^T$ where $s_t \in \Omega$ and $t \in \{1, \ldots, T\}$

*Electricity Providers from Renewables:* Each period $t$’s state $s_t \in \Omega$ determines the amount of electricity to be generated from the available renewable resources:

$$\left\{ q_{R,t}(s_t) \right\}_{t=1}^T = \left[ q_{R,1}(s_t), \ldots, q_{R,t}(s_t), \ldots, q_{R,T}(s_T) \right],$$

where $s_t \in \Omega$ and $t \in \{1, \ldots, T\}$

*Representative Consumer:* The representative consumer of electricity (RCE) first observes $\left\{s_t\right\}_{t=1}^T$, where $s_t \in \Omega$ and prepares a state and time-dependent ex-ante consumption plan for $\{1, \ldots, T\}$ in terms of electricity usage:

$$\left\{D_t(s_t)\right\}_{t=1}^T = \{D_t(s_t), \ldots, D_T(s_T)\}, \ s_t \in \Omega$$

The RCE cannot observe $\left\{q_{R,t}(s_t)\right\}_{t=1}^T$ and its ex-ante consumption plan $\left\{D_t(s_t)\right\}_{t=1}^T$ is exogenously determined.

**Stage 1:** SO observes $\left\{q_{R,t}(s_t)\right\}_{t=1}^T$ of Stage 0. It takes the unit price of electricity generated from non-renewable resources $p \in (0, \infty)$ as given.
Given $s_t \in \Omega$ for all $t \in \{1, \ldots, T\}$, the S.O. sets the price for flexible usage $p_{\text{flex}} \in (0, p)$ to be valid for all $t \in \{1, \ldots, T\}$. Once it announces the price of flexible usage, it correctly anticipates the optimal choices to be made thereafter its decisions.

**Stage 2:** The representative consumer of electricity observes the SO’s stage 1 price announcements for the flexible electricity usage $p_{\text{flex}}$ and inflexible usage $p$ over the period $\{1, \ldots, T\}$. Given $s_t \in \Omega$ for all $t \in \{1, \ldots, T\}$ and Stage 0’s $\{D_t(s_t)\}_{t=1}^{T}$, depending on $p_{\text{flex}}$, the RCE decides how much of the electricity $D_t(s_t)$ to be provided as a flexible usage for each $t \in \{1, \ldots, T\}$, $d_{\text{flex},t}$ and hence how much of it to be inflexible, $d_{\text{inflex},t} = D_t - d_{\text{flex},t}$:

$$\left\{d_{\text{flex},t}(s, p, p_{\text{flex}}), d_{\text{inflex},t}(s, p, p_{\text{flex}}) = D_t(s_t) - d_{\text{flex},t}(s, p, p_{\text{flex}})\right\}_{t=1}^{T}$$

**Stage 3:** Given Stage 0’s supply of electricity from renewable resources, $\{q_{\text{renew},t}(s_t)\}_{t=1}^{T}$, and Stage 2’s optimal allocations between flexible and inflexible demand levels $\{d_{\text{flex},t}(s, p, p_{\text{flex}}), d_{\text{inflex},t}(s, p, p_{\text{flex}})\}_{t=1}^{T}$, the SO shifts flexible demands between different periods over the horizon $\{1, \ldots, T\}$: $f_{t,\tau}$ is the portion of flexible demand $d_{\text{flex},t}$ at the time $t$ shifted to the time $\tau$, where $t \in \{1, \ldots, T\}$, $\tau \in \{t-n, \ldots, t+n\}$ and $0 \leq \tau \leq T$. The SO’s aim is to utilize renewables efficiently and reduce the generation from non-renewable resources. The determination of optimal demand shifts also provides the required minimum generation from conventional resources for each $t \in \{1, \ldots, T\}$.

In this setup, we assume that the SO has perfect foresight and can solve the optimality problems following its decisions and use them in its preceding decision-making processes. The logic as applied in the backward induction method is used in solving dynamic programming problems.
Solution of the Multi-Stage Problem:

Using the backward induction method, starting from the last decision stage, Stage 3, we solve this multi-stage decision-making problem through the first decision stage, Stage 1.

Stage 3 solution for the SO

At Stage 3, the SO takes the following as given:

- \( \{s_t\}_{t=1}^{T} \) where \( s_t \in \Omega \), Stage 0’s supplies of electricity from renewable resources \( \{q_{R,t}(s_t)\}_{t=1}^{T} \), Stage 0’s ex-ante electricity consumption plans
- \( \{D_t(s_t)\}_{t=1}^{T} \), Stage 1’s price levels \( (p_{flex}, p) \), Stage 2’s adjusted flexible and inflexible electricity demand levels

\[
\{d_{flex,t}(s, p, p_{flex}), d_{inflex,t}(s, p, p_{flex}) = D_t(s_t) - d_{flex,t}(s, p, p_{flex})\}_{t=1}^{T}
\]

Then, it shifts portions of each \( t \in \{1, \ldots, T\} \) flexible demand \( d_{flex,t} \) among periods \( \{1, \ldots, T\} \):

\[
\{f_{t_{-n}, \ldots, f_{t-1}, f_{t_1}, f_{t_1+1}, \ldots, f_{t_{+n}}} = \{f_{t, \tau}\}_{\tau=-n}^{+n}
\]

where \( f_{t, \tau} \) is the portion of \( t \in \{1, \ldots, T\} \) flexible demand shifted to a period \( \tau \in \{1, \ldots, T\} \), such that for each \( \tau \in \{1, \ldots, T\} \):

\[
\sum_{\tau=-n}^{+n} f_{t, \tau} = d_{flex,t} \quad \forall t \in \{1, \ldots, T\} \tag{4.8}
\]

This constraint can be referred to as the energy shift constraint. The total load that is shifted from one point of time to the other periods is equal to the available flexible demand at that specific period. The equality constraint makes sure that demand shifted from one point of time must be served within a certain time interval. Therefore demand recovery which is one of the problematic aspects of many applications is satisfied in this model.
The main purpose of demand shifts is to reduce costly production from conventional resources. Thus, the decision about flexible demand shifts \( \{ f_{t,T} \}_{t=1}^{T} \) over the horizon \( \{1,...,T\} \) is closely related to the generation from conventional sources \( \{ q_{t,T} \}_{t=1}^{T} \). Therefore, in Stage 3, SO also decides on electricity generation from conventional sources. Note that for each period \( t \in \{1,...,T\} \), the SO first uses the available renewable resources \( q_{t,T} \) and then generates from conventional sources \( q_{t,t} \) to meet the demand. Thus total generation in each period is \( q_{t,T} + q_{t,t} \). On the other hand demand in each period \( t \in \{1,...,T\} \) is composed of inflexible demand \( d_{inflex,t} \) in addition to the total flexible demand shifted from other periods to period \( t \sum_{t'=t-h}^{t} f_{t',t} \). The total demand that must be served at the time \( t \in \{1,...,T\} \) is then \( d_{inflex,t} + \sum_{t'=t-h}^{t} f_{t',t} \). Total generation must not be less than the total demand for each \( t \in \{1,...,T\} \), which lead us to the following period-by-period energy balance constrained:

\[
d_{inflex,t} + \sum_{t'=t-h}^{t} f_{t',t} \leq q_{R,t} + q_{t,t} \quad \forall t \in \{1,...,T\} \tag{4.9}
\]

SO’s profit for each \( t \in \{1,...,T\} \) is composed of three parts: revenue from inflexible demand \( p d_{inflex,t} \), revenue from total flexible demand that is served at time \( t \)

\[
p_{flex} \sum_{t'=t-h}^{t} f_{t',t} \), minus the cost of conventional generation \( p q_{t,t} \) at time \( t \). Thus

\[
\pi_{SO,t} \left( q_{c,t}, \{ f_{t,t} \}_{t=1+1}^{T} \right) = p d_{inflex,t} + p_{flex} \sum_{t'=t-h}^{t} f_{t',t} - p q_{t,t} \quad \forall t \in \{1,...,T\} \tag{4.10}
\]

Summing up the revenues over the entire planning horizon \( \{1,...,T\} \) to obtain the Total Revenue:
\[
\Pi_{SO} \left( \{q_{c,t}\}_1^T, \{\{f_{R,t}\}_{\tau=t-n}^{\tau=t+n}\}_1^T \right) = \sum_1^T \Pi_{SO,t} \left( q_{R,t}, \{f_{R,t}\}_{\tau=t-n}^{\tau=t+n} \right) \\
= \sum_1^T \left( p_{d_{inflex,t}} + p_{d_{flex}} \sum_{\tau=t-n}^{\tau=t+n} f_{R,t} - p_{c,t} \right)
\]

(4.11)

Therefore, given \( \{p, p_{flex}\}, \{q_{R,t}\}_1^T \) and \( \{d_{flex,t}, d_{inflex,t}\}_{\tau=t-n}^{\tau=t+n} \), SO solves the following revenue maximization problem:

**SO’s profit maximization problem at Stage 3:**

\[
\text{Maximize} \quad \Pi_{SO} = \sum_1^T \left( p_{d_{inflex,t}} + p_{d_{flex}} \sum_{\tau=t-n}^{\tau=t+n} f_{R,t} - p_{c,t} \right)
\]

subject to

\[
d_{inflex,t} + \sum_{\tau=t-n}^{\tau=t+n} f_{R,t} \leq q_{R,t} + q_{c,t} \quad \forall t \in \{1, \ldots, T\}
\]

(4.13)

\[
\sum_{\tau=t-n}^{\tau=t+n} f_{R,t} = d_{flex,t} \quad \forall t \in \{1, \ldots, T\}
\]

(4.14)

\[
q_{c,t}, f_{t,i,j} \geq 0 \quad \forall t, i, j \in \{1, \ldots, T\}
\]

(4.15)

**Stage 2 solution for the RCE**

Stage 2 does not have any input from Stage 3 results. Stage 2 solution is given above in the “demand side” section. The result of Stage 2 is passed to Stage 1 as the following constraint:

\[
d_{flex, t}, d_{inflex, t} \in \arg\max_{B, p} \left( U (d_{inflex, t}, d_{flex, t}, \mu) : s.t. d_{inflex, t} + d_{flex, t} = D, p_{d_{flex, t}} + M = p_{flex, d_{flex, t}} \right)
\]

77
which is amended to the Stage 3 results

**Stage 1 solution for the SO**

In this stage, the System Operator determines the profit-maximizing $p_{flex}$ considering Stage 2 and Stage 3 equilibrium solutions. Thus, anticipating the Stage 2 and Stage 3 outcomes, the system operator solves the following maximization problem at this stage:

\[
\begin{align*}
\text{Maximize} & \quad \sum_{t=1}^{T} \left( pd_{inflex,t} + p_{flex} \sum_{\tau=t-n}^{\tau=t+n} f_{\tau,t} - pq_{c,t} \right) \\
\text{subject to} & \quad d_{inflex,t} + \sum_{\tau=t-n}^{\tau=t+n} f_{\tau,t} \leq q_{R,t} + q_{c,t} \quad \forall t \in \{1, ..., T\} \\
& \quad \sum_{\tau=t-n}^{\tau=t+n} f_{\tau,t} = d_{flex,t} \quad \forall t \in \{1, ..., T\} \\
& \quad q_{c,t}, f_{i,j} \geq 0 \quad \forall t, i, j \in \{1, ..., T\} \\
\end{align*}
\]

\[
d_{flex}, d_{inflex} \in \arg\max_{D, p} \left\{ U(d_{inflex}, d_{flex}, \mu) : \text{s.t.} d_{inflex,t} + d_{flex,t} = D_t, pd_{flex,t} + M = p_{flex}d_{flex,t} \right\}
\]

The maximization operator outside the parenthesis can be taken inside to obtain the following objective function:

\[
\begin{align*}
\text{Maximize} & \quad \sum_{t=1}^{T} \left( pd_{inflex,t} + p_{flex} \sum_{\tau=t-n}^{\tau=t+n} f_{\tau,t} - pq_{c,t} \right) \\
\end{align*}
\]

The resulting problem will be a bi-level optimization problem such that the upper-level problem consists of the S.O.’s profit maximization over the decision set of $p_{flex}, \{q_{R,t} \}^T, \{f_{\tau,t} \}^T$ and the lower-level problem is the utility maximization problem of the consumer over the choice of $d_{flex}, d_{inflex}$. Note that in this specific case, the problem
can also be converted into a single-level non-linear optimization problem by replacing the lower-level problem with the value functions of \(d_{\text{flex}}, d_{\text{inflex}}\). However, to account for a variety of cases, we prefer to formulate the resulting problem as a bi-level optimization problem. The resulting bi-level optimization problem is given

\[
\begin{align*}
\text{Maximize} & \quad \Pi_{SO} = \sum_{t=1}^{T} \left( pd_{\text{inflex},t} + p_{\text{flex}} \sum_{\tau=1-n}^{\tau+n} f_{\tau,t} - pq_{c,t} \right) \\
\text{subject to} & \quad d_{\text{inflex},t} + \sum_{\tau=1-n}^{\tau+n} f_{\tau,t} \leq q_{R,t} + q_{c,t} \quad \forall t \in \{1,\ldots,T\} \\
& \quad \sum_{\tau=1-n}^{\tau+n} f_{i,\tau} = d_{\text{flex},t} \quad \forall t \in \{1,\ldots,T\} \\
& \quad q_{c,t}, f_{i,j} \geq 0 \quad \forall t, i, j \in \{1,\ldots,T\} \\
\end{align*}
\]

\[
d_{\text{flex}}, d_{\text{inflex}} \in \arg\max_{D, p} \left\{ U(d_{\text{flex}}, d_{\text{flex}}, \mu) : \text{s.t.} \ d_{\text{inflex},t} + d_{\text{flex},t} = D_t, \ pd_{\text{flex},t} + M = p_{\text{flex}}d_{\text{flex},t} \right\}
\]

(4.16)

4.3.3.1 Existence of the solution:

The following definitions are useful to show the existence of the solution:

1. Constraint set of the problem:

\[
\Omega = \left\{ (p_{\text{flex}}, f_{i,j}, q_{c}, d_{\text{inflex}}, d_{\text{flex}}) : \text{4.1, 4.13, 4.14 and 4.15 hold} \right\}
\]

(4.17)

The constraint set of the problem corresponds to the combination of all possible choice sets of the System Operator and the Customer

2. Feasible set for the customer for each \(p_{\text{flex}}\):

The feasible set of the customer is governed by the constraint in (4.1)
\[ \Upsilon(p_{\text{flex}}) = \{(d_{\text{inflex}}, d_{\text{flex}}) : d_{\text{inflex},t} + d_{\text{flex},t} = D_t \} \]  

(4.18)

In fact, in this specific bi-level problem, the choice of S.O does not have any effect on the feasible set of the customer. Since ex-ante Demand is assumed to be fixed, all \((d_{\text{inflex}}, d_{\text{flex}})\) combinations such that \(d_{\text{inflex}} + d_{\text{flex}} = D\) are feasible for the customer.

3. Projection of constraint set \(\Omega\) onto S.O. problem:

\[ \Psi = \left\{ (p_{\text{flex}}, f_{i,t}, q_{c,t}) : \exists (d_{\text{inflex}}, d_{\text{flex}}), d_{\text{inflex}} + \sum_{t} f_{i,t} \leq q_{R,t} + q_{c,t}, \sum_{t} f_{i,t} = d_{\text{flex}}, q_{c,t}, f_{i,t} \geq 0 \right\} \]

(4.19)

This set is the subset of the constraint set \(\Omega\) and refers to the combination of possible choice sets of the S.O. defined for all possible \((d_{\text{inflex}}, d_{\text{flex}})\).

4. The Customer rational reaction set for each \(p_{\text{flex}} \in \Psi\):

The customer observes the \(p_{\text{flex}}\) choice of S.O. and responds to maximize her utility. Thus, the set is the collection of the optimal \((d_{\text{inflex}}, d_{\text{flex}})\) values out of the customer optimization problem against the selection of each possible \(p_{\text{flex}} \in \Psi\) by S.O.

\[ \Lambda(p_f) = \arg\max_{\hat{D}_{p_f}} \left\{ \tilde{U}(d_{\text{inflex}}, d_{\text{flex}}, \mu) : s.t. d_{\text{inflex},t} + d_{\text{flex},t} = D_t, p_f d_{\text{flex},t} + \mathcal{M} = p_{\text{flex}} d_{\text{flex},t} \right\} \]

(4.20)

It is also important to note that customer rational reaction corresponds to a single solution for each \(p_f\) since the customer has a strictly concave utility function.

5. Inducible Region:

The S.O. problem depends on upper-level constraints some of which are conditional on the lower-level decision. The S.O. does not have any direct control but can influence the lower-level decision. Since the lower-level problem has a unique solution for each \(p_{\text{flex}}\), the S.O.’s choice of each \(p_{\text{flex}}\) results in a specific lower-level decision.
and thus results in a specific choice set for the S.O. Therefore, each \( p_{\text{flex}} \) induces a specific constraint set for the upper-level problem.

The inducible region is the union of the feasible sets for the S.O. for each optimal solution \( d_{\text{inflex}}, d_{\text{flex}} \) of the Customer against each \( p_{\text{flex}} \).

\[
\Gamma = \left\{ \left( p_{\text{flex}}, f_{i,x}, q_{c}, d_{\text{inflex}}, d_{\text{flex}} \right) : \left( p_{\text{flex}}, f_{i,x}, q_{c}, d_{\text{inflex}}, d_{\text{flex}} \right) \in \Omega; \ d_{\text{inflex}}, d_{\text{flex}} \in \Lambda(p_{\text{flex}}) \right\}
\]

(4.21)

In fact, the inducible region refers to the choice set of the upper-level problem which is constructed on the optimal solution of the lower-level problem. Therefore, the complete problem of the S.O. turns out to be a selection out of the inducible region.

**Lemma 1:** For a nonempty constraint set \( \Omega \), the inducible region \( \Gamma \) is closed.

**Proof of Lemma 1:** For better readability and saving notation, let \( x = \left\{ p_{\text{flex}}, f_{i,j}, q_{c} \right\} \ t, i, j \in \{1, ..., T\} \) be the upper level and \( y = \left\{ d_{\text{inflex}}, d_{\text{flex}} \right\} \ t \in \{1, ..., T\} \) be the lower level choice vectors respectively. Since \( \Omega \) is nonempty, by definition 3, there exists at least one \( x^* \in \Psi \). By definition 2 in the equation (4.18), the feasible set of the customer is nonempty, thus \( Y(x^*) \neq \phi \). Since the lower level problem is compact we have \( \Lambda(x^*) \neq \phi \) and hence there exists \( y_0 \in \Lambda(x^*) \). Therefore \( (x^*, y_0) \in \Gamma \), which shows that the inducible region \( \Gamma \) is nonempty. Consider a sequence \( \left\{ (x^n, y^n) \right\}_{n=1}^{\infty} \subseteq \Gamma \) converging to \( (x^*, y^*) \), definition 4 implies that \( y^* \in \Lambda(x^*) \). Therefore, \( \Gamma \) is closed.

**Corollary 1:** For a nonempty constraint set \( \Omega \), the optimal solution for the System Operator problem exists.

**Proof of Corollary 1:** From Lemma-1, \( \Gamma \) is closed. \( \Gamma \) is also a subset of the constraint set \( \Omega \) which is bounded, thus it is compact. By the Weierstrass theorem, the optimal solution exists.

**Proposition-1 (Pareto Improvement):** If the optimal solution to the above problem \( p_{\text{flex}} \) is such that \( p_{\text{flex}} \neq p \), the solution is Pareto efficient.
**Proof of proposition-1:** The proof is straightforward. The customer is a utility maximizer. They select $d_{\text{flex},t}$ and $d_{\text{inflex},t}$ such that

$$\delta_{\text{flex}}U(d_{\text{flex}}) + \delta_{\text{inflex}}U(d_{\text{inflex}}) + \delta_{M}U\left(\frac{M}{p}\right) \geq U(\bar{D})$$

Therefore customers are better off.

The System Operator chooses $p_f \neq p$ if only $\text{Revenue}(p_f) > \text{Revenue}(p)$, therefore, the producer is better off when $p_f \neq p$.

**Definition-1:**

i) Set of the periods with excess demand $K = \{k : q_{R,k} < \bar{D}_t\}$: Collection of periods where demand exceeds available renewable energy at that period.

ii) Set of the period with excess renewable energy generation $L = \{l : q_{R,l} > \bar{D}_t\}$: Collection of periods where available renewable energy exceeds the demand at that period.

**Theorem-1:** For non-empty $K$ and $L$, there always exists an optimal price for flexible usage $p_{\text{flex}}$ which is strictly less than the market price $p$, i.e. $p_{\text{flex}} < p$, such that the model provides an improvement in the profit and utilization of renewable energy while the customer is not being worse off. That is if there exists at least one period with excess renewable energy production and there exists at least one time period with excess demand, the model increase the profit of the System Operator and renewable energy utilization for a price $p_{\text{flex}} < p$.

**Proof of Theorem-1:** The proof of the theorem is presented in Appendix.

**4.3.4 The Social Planner Perspective**

The base model has been configured from the producer's perspective to utilize renewable energy more efficiently through revenue maximization. However, the overall cost structure of the implementation of renewable energy, related support mechanisms, and also how this cost is reflected to the end users are quite complicated. Thus, in some cases, the objective might be to maximize renewable energy utilization regardless of the marginal revenue improvement. The model could be modified to
fulfill this objective. In this case, we introduce a Social Planner who is responsible for price settings for flexible usage $p_{\text{flex}}$. The S.P.’s objective is to maximize renewable energy utilization and set $p_{\text{flex}}$ accordingly. In general retail electricity prices are heavily regulated. Since the marginal cost is generally lower than the average cost, marginal cost pricing is not sustainable. Therefore, another pricing strategy which is referred to as Ramsey pricing or second best pricing is used. This pricing strategy leaves a certain amount of revenue to the suppliers to make sure that the average cost of generation is covered in the long run. In our modeling, we also use a restriction for the revenue in S.P. problem such that S.O. collects at least the same amount of revenue if the model were not used. Let the revenue that the S.O. could make without the model is referred to as nominal revenue. So, in the optimization, the S.P. include the restriction that the revenue of the S.O. will be greater or equal to the nominal revenue.

The setting of the game defined in Section 2 is modified as follows:

**Stage 0:** Nature reveals $\{s_1, ..., s_t, ..., s_T\} = \{s_t\}_{t=1}^T$ where $s_t \in \Omega$ and $t \in \{1, ..., T\}$

**Stage 1:** Social Planner chooses $p_{\text{flex}}$ to maximize renewable utilization while keeping producers not worse-off.

**Stage 2:** the Consumer chooses $d_{\text{flex}}$ and $d_{\text{inflex}}$ out of Utility max Problem

**Stage 3:** System Operator decides the Revenue maximizing demand shifts $f_{i,j}$ and conventional generation amounts $q_{c,d}$ according to the available $d_{\text{flex}}$ and $d_{\text{inflex}}$ stage-2 and anticipated Production profile for renewables.

**The Solution to the SP’s Problem**

Stage-3 and Stage-2 solution is the same as in the previous case. Backward induction from Stage-3 to Stage-2:

Let $\Delta$ be the Stage-3 and Stage-2 combined problem in backward induction

$\forall t \in (1...T)$
\[
\text{Maximize} \quad \sum_{t=1}^{T} \left( p_{\text{inf},t} + p_{\text{flex}} \sum_{\tau=t-n}^{t} f_{\tau,t} - p_{t,i,j} \right)
\]

subject to
\[
d_{\text{inf},t} + \sum_{\tau=t-n}^{t} f_{\tau,t} \leq q_{R,t} + q_{e,t} \quad \forall t \in \{1, \ldots, T\}
\]
\[
\sum_{\tau=t-n}^{t} f_{\tau,t} = d_{\text{flex},t} \quad \forall t \in \{1, \ldots, T\}
\]
\[
q_{e,t}, f_{i,j} \geq 0 \quad \forall t, i, j \in \{1, \ldots, T\}
\]
\[
d_{\text{flex}}, d_{\text{inf}} \in \arg\max_{D, p} \left( U(d_{\text{inf}}, d_{\text{flex}}, \mu) : s.t. d_{\text{inf},t} + d_{\text{flex},t} = D_t, p_{\text{flex}} \right)
\]

(4.22)

**Stage-1:**

Different from the base case, we need to define the nominal revenue for the producers. Nominal profit of the producer:

\[
P_N = p \sum_{T} \text{Max}(q_{R,t}, D_t)
\]

Thus the problem of S.P.

\[
\text{Min} \quad \sum_{T} q_{e,t}
\]

subject to
\[
P \geq P_N
\]
\[
q_{e,t} \in \Delta \quad \forall t \in \{1, \ldots, T\}
\]
4.4 Numerical Study

In this section, we will provide numerical examples based on actual demand data and actual renewable energy production profiles. We consider daily optimization periods and hourly data for numerical studies.

4.4.1 Data and Materials

Demand Data

We use realized actual demand data from California ISO in order to attain more realistic results. Since consumption patterns change from week-day to weekend-day and also from season to season, we consider four categories just for diversity: summer week-day, summer weekend-day, winter week-day, and winter weekend-day. A random day which is 15.01.2020 is selected as a representative demand profile for each category from 2020 consumption data. (note that the number of categories and number of demand profiles for each category may be increased, however, we believe that the examples provided in this study are enough to demonstrate the output of the model).

Production data

For renewable energy production profiles, we use exactly the same day corresponding to selected demand dates for each category.

Scheduling of renewable generation such as PV and Wind is completely dependent on the weather condition. This means that for a specific location, installed capacity does not have any considerable effect on the profile of renewable generation for a definite technology. Thus, when the installed capacity is increased by a certain factor, the output would increase proportionally with the same production schedule profile. Although we use the realized renewable energy production profile, based on this idea, we developed two kinds of production scenarios having different renewable energy generation quantities for each category. In addition to renewables, the portfolio of generators consists of several sources like natural gas, coal, nuclear, etc., and imports.
Figure 4.1: Realized Demand and Renewable generation data

However, just for simplicity, we refer to all these resources as “conventional generation”. We also assume that there is a perfectly competitive market for the conventional generation which allows us to assume that marginal production cost is equal to market price $p$. Figure 4.1 exhibits realized the demand and renewable generation on 15.01.2020 which is an example for the winter weekday category.

4.4.2 Scenarios

Scenario-1: This scenario assumes that the installed capacity of renewable is increased such that the quantity of total daily renewable energy production matches with the total daily demand realized on that date. Nevertheless, due to the difference between the demand profile and production profile, additional conventional generation needs to be dispatched for the hours when the generation amount falls below the demand, and over-generated power is wasted for the hours when the generation quantity exceeds the demand. This scenario highlights the inefficiency of the current market structure under intensive renewable energy penetration. Although the total daily generation of renewables is sufficient to satisfy total daily demand, due to the scheduling difference additional production is needed while renewable production cannot be fully utilized.

Scenario-2: In this scenario, installed capacity is increased such that total daily renewable energy production corresponds to 50% of total daily demand. In addition, there is a must-run fixed conventional production facility which accounts for only 25%
of the total daily demand. Additional conventional generation is dispatched whenever needed in order to maintain the balance between demand and supply.

Example scenarios generated with 15.01.2020 data are presented in Figure 4.2. From the customer's point of view, the degree of commitment to flexible usage depends on the utility weights $\delta$ and risk awareness $\rho$ parameters. These parameters are completely case-specific and should be carefully tuned according to the characteristics of the market. Nevertheless, for numerical study, we consider two different sets of parameters: one for the case where the customer is relatively less responsive to the discounted price which we refer to as the pessimistic case, and one for the case where the customer is relatively more responsive the discounted price which we refer to as optimistic case.

Figure 4.2: Scenarios generated with sample day 15.01.2020 data

We investigate both scenarios under both optimistic and pessimistic customer parameter sets separately. The parameters and corresponding alfa values vs $p_{flex}$ are depicted in Figure 4.3:
In the pessimistic case, both the level and rate of commitment are lower than that of the optimistic case. On the contrary, the Consumer commits to flexible usage easier and more frequent.

### 4.4.3 Methodology

The upper-level Producer revenue maximization problem is conditional on the optimal values of the lower-level customer utility maximization problem. Due to the non-convex nature of the bi-level optimization problems, the solution to both of the problems at the same time is not possible with industrial optimization tools. Finding the solution to such bi-level problems is complicated and several algorithms such as penalty function methods, single-level reduction methods, decent methods, nested methods, etc. have been proposed in the literature(ref). The success and efficiency of each method are related to the structure of the problem. In this work, we will use a nested algorithm in which we solve the customer problem for each possible $p_{flex}$ value and use corresponding optimal values in the constraint set of the S.O. problem. In our problem, the System Operator problem is linked to the customer problem through $p_f$, which also makes the S.O. problem non-linear. When $p_f$ is set exogenously and a
corresponding optimal solution for the customer is obtained, the S.O. problem turns out to be a linear program. Although the number of iterations is increased in nested methods, computation effort is significantly reduced in each iteration.

We do the related calculations in three steps. In the first step, 5-min data is converted to hourly data, total demand, total renewable energy production, ex-ante revenue, ex-ante renewable energy utilization, renewable energy generation scenarios, set of $\alpha$ values are calculated and Customer problem is solved for different $p_f$ values to obtain corresponding $d_{\text{flex}}$ and $d_{\text{inflex}}$ values. In the second step, for each $p_{\text{flex}}$, $d_{\text{flex}}$ and $d_{\text{inflex}}$ which are obtained in the first step, optimal demand shifts $f_{i,j}$, conventional energy dispatch quantities $q_{c,i}$, and revenues are calculated. In the post-processing stage $p_{\text{flex}}$ values that generate maximum revenue is selected as the optimal solution. Also, ex-post revenues, ex-post renewable energy utilization, improvement in revenue, and improvement in energy utilization are calculated. The process and the tools are summarized in Figure 4.4.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Data import, data initialization, ex-ante values, scenarios, customer problem opt.</td>
<td>System operator problem opt., demand shifts, conventional generation, revenue</td>
<td>Data export, ex-post values, visualization</td>
</tr>
<tr>
<td>Python Script</td>
<td>IBM ILOG Cplex</td>
<td>Python Script, MS Excel</td>
</tr>
</tbody>
</table>

**Figure 4.4:** Computational Methodology

### 4.4.4 Results

This section discusses the main results and findings on several aspects of the model. The plots of total revenue vs $p_f$ for all cases are provided in Figure 4.5.

- In all cases, total revenue first increases with a decrease in $p_f$, reaches a maximum point, and then decreases. This result is consistent with Theorem-1. There is always positive revenue when there is any excess renewable energy.
However, the rate of marginal revenue gain diminishes due to the decrease in revenue as a result of decreasing $p_f$.

- In optimistic cases, the rate of revenue increase is higher compared to the pessimistic counterparts since a higher volume of flexible demand could be obtained by a relatively smaller discount. Therefore, the optimal point could be reached rapidly.

![Figure 4.5: Total Revenue vs Discounted Price $p_{flex}$](image)

Total Conventional Generation vs $p_{flex}$ is shown in Figure 4.6. The main finding can be summarized as:

- Required Total Conventional generation decreases with decreasing $p_{flex}$.
  Therefore, Renewable energy utilization is increasing with decreasing $p_{flex}$.

- The rate of Renewable energy utilization is higher in optimistic cases.

- In Scenario-2 optimistic case, the required total generation decreases rapidly, and then further decrease in $p_{flex}$ does not improve the renewable utilization since all the excess renewable generation is used at optimal points. Therefore
the point where all the excess generation is utilized constitutes the saturation point for the solution.

Figure 4.6: Total Conventional Generation vs Discounted Price $p_{flex}$

**Impact on demand re-distribution:**

The problem arising from the mismatch between the demand schedule and generation schedule is mitigated with the application of the model. The model provides the decision maker with the ability to control the scheduling of the flexible demand. Accordingly, the decision maker shifts the flexible demands from low generation time slots to high generation time slots. The degree of the control is related to the existing flexible demand which is proportional to $(1-\alpha)$ according to the equation (4.7). Optimal solutions for S.P. and S.O. in each case are provided in Table 4.1
Table 4.1: Optimal Solutions for both SO's and SP's problems in both cases

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Without model</th>
<th>S.O. Optimal Solution</th>
<th>S.P. Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td>1</td>
<td>4923200</td>
<td>1987118</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>5365900</td>
<td>1389431</td>
</tr>
<tr>
<td></td>
<td>0.68</td>
<td>4925400</td>
<td>931322</td>
</tr>
<tr>
<td></td>
<td>0.68</td>
<td>4923200</td>
<td>1581891</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>1</td>
<td>4746800</td>
<td>2163471</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>5111500</td>
<td>1727573</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>5111500</td>
<td>1727573</td>
</tr>
</tbody>
</table>

Demand redistribution as a result of optimal demand shift solution of the decision maker is the major output of the model. Related demand re-distributions of both S.O and S.P. for each scenario are depicted in Figure 4.7:

- As expected, in optimistic cases where the customer is more eager to commit to flexible usage, renewable energy utilization is higher due to more flexible demand availability.
- In most cases, S.P.’s optimal solution yields more renewable energy utilization. In those cases, there is some potential for further renewable energy utilization than S.O.’s optimal point.
- In Scenario-2 Optimistic case, S.O. and S.P. optimal points coincide, for the excess renewable production region, and all the excess generation is utilized. This is because of the fact that excess generation is less and as a result required flexible demand is lower. Thus, with the optimistic customer profile, required flexible demand could be reached rapidly.
- The commitment rate of the consumer which is proportional to $\alpha$ is very important for the success of the implementation.
Figure 4.7: Demand Re-distribution as a result of model application

4.5 Conclusion

In this study, a new demand response model is introduced and analyzed. The model segments the electricity usage into two according to flexibility. The consumer transfer authority for the exact timing of flexible usage to the S.O. in return for invective in the form of a discounted price. Discounted price is determined dynamically before every planning period according to the need for the amount of flexibility. The model uses both direct load control principles and dynamic price signals together. Hence, the model has combined benefits from both direct load control and dynamic planning. This new model requires less consumer involvement relative to the other dynamic pricing models. The model ensures bill stability and demand recovery in all cases. Moreover, a more reliable source of flexibility can be obtained through a predetermined direct load control mechanism.
The analysis shows that the suggested model always provides overall improvements in both revenue and renewable energy utilization in the case of excess renewable energy production. The result of the consumer’s problem shows that the consumer commits a certain portion of demand to flexible usage proportional to $\alpha$ which is composed of utility weights, relative prices, and risk aversion parameters. When these parameters are favorable as in the optimistic scenarios, analysis shows that desired volume of flexible usage can be achieved rapidly with relatively less discount. Therefore, promoting an environment in which the consumer can use flexible use easily and with relatively little risk is key to the model’s success.

We also modified the model to include Social Planner as a responsible agent to determine the discounted price. Numerical analysis showed that the optimal solution to the Social Planner’s problem provides greater renewable energy utilization in most cases. However, when flexible demand parameters are favorable and required flexibility is relatively low, both S.O and S.P reach the same optimal solution with a little discount since benefits from demand shifts saturate quickly in such cases.
CHAPTER 5

FORECASTING HOURLY ELECTRICITY DEMAND

5.1 Introduction

Forecasting the demand for electricity has always been of crucial importance for the operation of electricity systems and electricity markets as well as the planning of the power systems. In addition, the liberalization of electricity markets has drastically increased the need for a reliable forecast of the demand for policymakers, system operators, generation companies, distribution companies, and market participants since most of their decisions are based on \textit{a priori} information obtained through forecasting.

In general, electricity demand forecasting is divided into three categories depending on the length of the forecasting horizon. Long Term Demand Forecast covers the forecasting horizon from several months ahead to years ahead. This type of forecast is used for the design and development of transmission & distribution networks, capacity planning, investment decisions, and investment scheduling for new power plants, etc. Underestimation of long-term demand will result in supply shortage and troubles in satisfying the demand in the future. On the other hand, overestimating will result in inefficiency problems in both power systems and power markets in addition to the waste of capital due to overinvestment. Mid-term forecasting is related to the forecasting horizon spanning from several days ahead to months ahead. Mid-term forecast results are useful data that is used in capacity planning, risk management, maintenance scheduling of the power plant and transmission lines, etc. Short-term load forecasting corresponds to the forecasting horizon from a few minutes to a few days ago. Overall, most of the effort is dedicated to short-term load forecasting for several reasons. Input and output to the power network must be balanced with very tight boundaries and short-term forecasts are key inputs to the balancing process. In most
electricity systems, the input-output matching process is initiated in a day-ahead market. The system operator first forecasts the demand for the following day and then collects supply offers for each 24-hour for the next day and sequences them in ascending order in price to obtain the supply curves. The crossing point of the supply curve with the demand line constitutes the market price for the specific hour of the next day in the day-ahead market. The major part of demand-supply matching is performed in the day-ahead market and input and output adjustment for the remaining imbalances typically continues until the power is physically delivered to the end-user. Electricity systems rely on different sources to generate electricity. These sources have different flexibilities and different dispatch costs. Generally, the marginal cost of production is higher for the flexible sources and lower for the less flexible sources. Thus, a last-minute dispatch order decision is more costly than the dispatch order determined in the day-ahead markets, which implies that the cost-effective dispatch order necessitates a proper day-ahead forecast. The Short-term forecast is also essential for the market participants. In liberalized power markets, generation companies, distribution companies, and third parties such as energy brokers enter into price and quantity competition. Both distribution companies and generation companies as well as third parties should have a reliable forecast in order to place a proper bid and reduce associated risks. Therefore, short-term load forecasting is essential and integral element of the power system and power market operations, which is performed on a continuous basis. In addition, liberalization of the power markets is still under progression for most of the countries in the World. Also, policies regarding climate change and policies promoting renewable energy will affect the way people use electricity. Novel market models associated with further liberalization of electricity markets and increased shares of renewable energy will increase the number of parties involved in power markets. Furthermore, forecasting the short demand will be more challenging due to the complexities introduced by the policies whereas the need for decision support systems such as demand forecasting will raise.

In this study, we compare the performance of three univariate time series methods based on the aggregate electricity consumption data of Turkey. Particularly, the day-ahead forecast performances of alternative methods are evaluated. Dynamics of the short-term load forecasting are quite different from long-term forecasting (Rob Hyndman- density forecasting). Almost all of the factors affecting electricity
consumption in the long run such as GDP, population growth, electrification, etc. are stationary and only the weather is effective in the short run. The weather effect can be captured up to a certain duration by univariate time series since the effect of weather is smooth and also with some lag. In addition, weather information dramatically changes from region to region all over Turkey at a specific point in time. Hence, when working with aggregate data of the country, the inclusion of weather variables smoothly into the analysis is virtually impossible since there is a vector of weather data indexed by regions corresponding to a single aggregated load value. Therefore, in such a context, univariate time series are effective methods to forecast short-term electricity load. The univariate methods considered in this study are Double Seasonal Exponential Smoothing (DSES), TBATS, and a decomposition technique (MSTL) combined with a Simple Exponential Smoothing method (MSTL+ETS). DSES has been studied in the literature since first introduced in 2003 whereas TBATS is a relatively new method. Among the alternatives, MSTL is the most recent method. MSTL is the multiple seasonal adjustments of STL decomposition which has been designed to deal with multiple seasonal data and the algorithm for automated MSTL is recently developed (Bandara et al., 2021). Up to our knowledge, the application of these methods to the hourly load data of Turkey is not available.

The result of the analysis shows that MSTL+ETS outperforms DSES and TBATS. Although DSES has comparable results with MSTL+ETS, TBATS is outperformed in all the cases.

5.2 Literature Review and Theoretical Background

Hourly electricity consumption data has many characteristics such as complex seasonality with high frequency and special day variation etc. Daily human activities have a major impact on the characteristics of the data in hourly resolution. Within a day, electricity consumption varies from a minimum point (baseload) to a maximum point (peak load). The variation in electricity load depends on many factors such as economic activities, daylight availability, weather conditions, etc. Generally, the minimum load is observed before sunrise when both economic and social activities are at the minimum level. Then, it steadily increases to a peak point. There may be two peak points one being local, in some regions, or only one depending on the conditions.
The pattern of daily variation constitutes daily cycling. The cycling pattern differs for each day of a week although it is alike for similar days, which implies weakly cycling behavior. The specific characteristics of the data complicate modeling and forecasting processes. Numerous methods have been proposed in the literature for modeling and forecasting short-term electricity consumption (Hong and others, 2014) (Abu-El-Magd and Sinha, 1982). In general, these methods can be grouped as time series models, decomposition techniques, multiple regression, and Artificial Intelligence.

**ARIMA Models**

Time series models include autoregressive methods and exponential smoothing. Autoregressive models compose of Autoregressive (AR) and Moving Average (MA) parts in their general structure (Box et al., 2015). An autoregressive process is the weighted linear combination of the past observations plus an error term. Pure AR\(_p\) process is given by:

\[
y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \cdots + \varphi_p y_{t-p} + \epsilon_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \epsilon_t
\]

\(5.1\)

\[
\epsilon_t = \left(1 - \sum_{i=1}^{p} \varphi_i L^i \right) y_t
\]

\(5.2\)

where \( p \) is the order of autoregression and \( L \) is the backshift operator. On the other hand, MA\(_q\) process is the weighted linear combination of past error terms plus the current error term given by:

\[
y_t = \psi_1 \epsilon_{t-1} + \psi_2 \epsilon_{t-2} + \cdots + \psi_q \epsilon_{t-q} + \epsilon_t = \sum_{i=1}^{q} \psi_i \epsilon_{t-i} + \epsilon_t
\]

\(5.3\)

\[
y_t = \left(1 - \sum_{i=1}^{q} \psi_i L^i \right) \epsilon_t
\]

\(5.4\)

where \( q \) is the order of the MA process. Standard ARMA\(_p,q\) process is the combination of AR and MA processes given by:

\[
\left(1 - \sum_{i=1}^{p} \varphi_i L^i \right) y_t = \left(1 - \sum_{i=1}^{q} \psi_i L^i \right) \epsilon_t
\]

\(5.5\)
Standard ARMA\((p,q)\) family models have restrictive assumptions for instance time series should be stationary. Non-stationarity due to non-stable variance may be eliminated (or reduced) by transformations such as log transformation or Box-Cox transformation. In order to eliminate non-stationarity due to non-stable mean (non-stationarity due to trend), differencing can be applied to the original data. Such a differencing procedure can also be included in the model. Finally, the generalization of autoregressive models is referred to as Autoregressive Integrated Moving Average ARIMA\((p,d,q)\) given by:

\[
(1 - \sum_{i=1}^{p} \phi_i L^i)(1 - L)^d y_i = (1 - \sum_{i=1}^{q} \psi_i L^i) \epsilon_i
\]

where \(d\) is the number of differencing needed.

**Seasonal ARIMA**

Hourly electricity data is always non-stationary due to complex seasonality (Hyndman and Athanasopoulos, 2018). Non-stationarity in hourly electricity data may not be eliminated by simple differencing and subsequently classical autoregressive models in their basic structures may not be appropriate for hourly data with high-frequency seasonal components. Instead, Seasonal ARIMA models are employed in the form of ARIMA\((p,q,d)(P,Q,D)\), where \(P,Q,D\) is the seasonal length (Box et al., 2015). In this formulation, the seasonal term is multiplicative which implies that

\[
\left(1 - \sum_{i=1}^{p} \phi_i L^i \right) \left(1 - \sum_{i=1}^{q} \psi_i L^i \right) \left(1 - \sum_{j=1}^{P} \phi_j L^{jS} \right) \left(1 - \sum_{j=1}^{Q} \psi_j L^{jS} \right) \left(1 - \sum_{j=1}^{D} \theta_j L^{jS} \right) (y_i - \mu) = \left(1 - \sum_{i=1}^{p} \phi_i L^i \right) \left(1 - \sum_{i=1}^{q} \psi_i L^i \right) \epsilon_i
\]

After multiplication:

\[
\left(1 - \sum_{i=1}^{p} \phi_i L^i - \sum_{j=1}^{p} \phi_j L^{jS} + \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_k \phi_m L^{k+mS} \right) \left(1 - \sum_{i=1}^{q} \psi_i L^i - \sum_{j=1}^{Q} \psi_j L^{jS} + \sum_{k=1}^{Q} \sum_{m=1}^{Q} \psi_k \psi_m L^{k+mS} \right) \left(1 - \sum_{j=1}^{D} \theta_j L^{jS} \right) (y_i - \mu) = \left(1 - \sum_{i=1}^{p} \phi_i L^i - \sum_{j=1}^{Q} \psi_j L^{jS} + \sum_{k=1}^{Q} \sum_{m=1}^{Q} \psi_k \psi_m L^{k+mS} \right) \epsilon_i
\]
Performing the lag operations and rearranging the terms to obtain the open form of the equation:

\[
y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{p} \phi_j y_{t-j} - \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_{km} y_{t-k-m} - \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} - \sum_{j=1}^{Q} \theta_j \varepsilon_{t-j} + \sum_{k=1}^{q} \sum_{m=1}^{Q} \psi_{km} \varepsilon_{t-k-m} + \varepsilon_t
\]  

(5.9)

It is also possible to include double seasonality in ARIMA models in the form of \(ARIMA(p,q,d)(P^1,Q^1,D^1)(P^2,Q^2,D^2)\), where \(s_1\) and \(s_2\) are the seasonal lengths and \(P^1,Q^1,D^1\) and \(P^2,Q^2,D^2\) are the seasonal orders for the first and second seasonality respectively. Representation of the DSARIMA in closed form is:

\[
\left(1 - \sum_{j=1}^{p} \phi_j L^j \right) \left(1 - \sum_{j=1}^{p} \phi_j L^{j_1} \right) \left(1 - \sum_{j=1}^{p} \gamma_j L^{j_2} \right) (y_t - \mu) = \left(1 - \sum_{j=1}^{q} \psi_j L^j \right) \left(1 - \sum_{j=1}^{q} \theta_j L^{j_1} \right) \left(1 - \sum_{j=1}^{q} \lambda_j L^{j_2} \right) \varepsilon_t
\]

(5.10)

After performing multiplication and backshift operations and then rearranging the terms we have the following open form for DSARIMA:

\[
y_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \sum_{j=1}^{p} \phi_j y_{t-j_1} + \sum_{j=1}^{p} \gamma_j y_{t-j_2} \\
- \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_{km} y_{t-k-m} - \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_{km} y_{t-k-mz} - \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_{km} y_{t-k-mz} - \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_{km} y_{t-k-mz_2} \\
+ \sum_{j=1}^{p} \sum_{k=1}^{p} \sum_{m=1}^{p} \phi_j \gamma_m y_{t-j-kmz} - \sum_{j=1}^{q} \psi_j \varepsilon_{t-j} - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j_1} - \sum_{j=1}^{q} \lambda_j \varepsilon_{t-j_2} \\
- \sum_{j=1}^{q} \psi_j \varepsilon_{t-j} - \sum_{j=1}^{q} \theta_j \varepsilon_{t-j_1} - \sum_{j=1}^{q} \lambda_j \varepsilon_{t-j_2} + \sum_{k=1}^{q} \sum_{m=1}^{Q} \psi_k \theta_m \varepsilon_{t-k-m} + \sum_{k=1}^{q} \sum_{m=1}^{Q} \psi_k \theta_m \varepsilon_{t-k-mz} + \sum_{k=1}^{q} \sum_{m=1}^{Q} \psi_k \theta_m \varepsilon_{t-k-mz_2} \\
+ \sum_{k=1}^{Q} \sum_{m=1}^{Q} \theta_k \lambda_m \varepsilon_{t-k-mz_2} - \sum_{j=1}^{q} \sum_{k=1}^{q} \sum_{m=1}^{Q} \psi_j \theta_k \lambda_m \varepsilon_{t-j-kmz} + \varepsilon_t
\]

(5.11)
Double Seasonal version of Seasonal ARIMA is studied in hourly electricity demand forecasting and sometimes used as a benchmark in the literature for example (Soares and Medeiros, 2008) (Taylor et al., 2006) (Darbellay and Slama, 2000). However, implementation of double seasonal ARIMA models is quite tricky as could be inferred from the open form of DSARIMA in (5.11). Up to now, there is no build-in function or package in the standard statistical programs which can accommodate only one seasonality in ARIMA modeling. Moreover, interpretation of MA terms is difficult in ARIMA models.

**Periodic Autoregressive Models**

In addition to Seasonal ARIMA, there are alternative ways to include seasonal behavior in autoregressive models. Due to its cycling behavior, the seasonality can be represented by Fourier terms which consist of sinusoidal expression

\[ f_i = \alpha_h \sin \left( \frac{2\pi h t}{s} \right) + \beta_h \cos \left( \frac{2\pi h t}{s} \right), \]

where \( s \) is the seasonal length and \( h \) is the harmonic (Young et al., 1999). Fourier terms can be added to the model as an external variable in the form of

\[ y_t \sim ARIMAX(p, d, q, EX) \]

where

\[ EX = \sum_{i=1}^{m_{day}} f_i^{\text{day}} + \sum_{i=1}^{m_{week}} f_i^{\text{week}}. \]

Periodic autoregressive regression is another method that incorporates Fourier terms in the model to account for seasonality (Taylor et al., 2006), (Franses and Paap, 2004). In Periodic autoregressive models, coefficients are multiplied by a Fourier expression to reflect the seasonal adjustment. The periodic autoregressive models are represented by:

\[ y_t = \theta_0(t) + \theta_1(t)y_{t-1} + \theta_{s_1}(t)y_{t-s_1} + \theta_{s_2}(t)y_{t-s_2} + \epsilon_t \]

where
\[ \theta_p(t) = \omega_p \]
\[ + \sum_{i=1}^{k} \left( \alpha_{pi} \sin \left( \frac{2i\pi d(t)}{s_1} \right) + \beta_{pi} \cos \left( \frac{2i\pi d(t)}{s_1} \right) + \delta_{pi} \sin \left( \frac{2i\pi w(t)}{s_2} \right) + \gamma_{pi} \cos \left( \frac{2i\pi w(t)}{s_2} \right) \right) \]

The key characteristic of this representation is that many aspects of cycling behavior are represented by Fourier terms rather than including several seasonal lags.

**Model Selection and Diagnostics in Autoregressive Models**

In order to implement ARMA family methods, a proper model should be selected, i.e., proper orders of \( d, p, \) and \( q \) and for Integration, AR and MA processes respectively should be identified. Autocorrelation Functions (ACF) and Partial Autocorrelation Functions (PACF) provide an initial idea about up to how many lags can play an explanatory role in a stationary time series. However analytical metrics developed for model selection provide more reliable results. Standard significance tests for model selection have some shortcomings such as when the sample size is increasing, the likelihood of rejecting simple models is increased radically and thus favoring over-parameterization (Kuha, 2004). To overcome this, criteria penalizing the over parametrization such as Akaike Information Criteria (AIC) and Schwarz’s Bayesian Information Criterion (BIC) are employed.

AIC which has roots in information theory is one of the widely used metrics to assess the quality of the model. In general, statistical models that are used to represent the data generation process for a given data are practically never exact. Therefore, some information is lost when trying to fit a model over a given data. AIC provides a relative estimate about the lost information such that the quality of the model could be assessed with this estimate:

\[ AIC = 2k - 2\ln(\mathcal{L}) \]  

(5.13)

where \( k \) is the number of parameters in the model and \( \mathcal{L} \) is the log-likelihood function. Since it is an estimate for the lost information, the model with a lower AIC score is better in quality. AIC is an asymptotically efficient estimator (Flynn et al., 2013).
BIC is another penalized likelihood estimate. However, BIC is an estimate of the probability that the model is the true model:

\[ BIC = k \ln(n) - 2 \ln(\mathcal{L}) \]  

(5.14)

where \( n \) is the sample size. BIC penalizes the number of parameters more than AIC. Thus, the model suggested by BIC tends to have fewer parameters than or the same number of parameters as the model suggested by AIC.

Although AIC and BIC have similar penalized likelihood estimates, they have different implications. AIC is a better metric when the purpose is forecasting and BIC is better when the purpose is approximating the true model (Chakrabarti and Ghosh, 2011). Thus, AIC is suggested by many authors since there is almost no exact model in reality (Hyndman and Athanasopoulos, 2018).

**Exponential Smoothing**

Exponential smoothing is used extensively in forecasting univariate time series due to its robustness and accuracy. Just like most time series methods such as ARIMA family models, exponential smoothing relies on the weighted sum of the past observations. However, the idea behind exponential smoothing is that more recent observations get higher weights than the weights of older observations (Hyndman and Athanasopoulos, 2018). It may be the case for many of the applications that the predictive value of the information that the recent observations carry may be more important than the value of the information that older observations have. The simplest form of exponential smoothing is given as:

\[ \hat{y}_{t+1} = \alpha y_t + (1 - \alpha) \hat{y}_{t-1} \]

(5.15)

Replacing \( \hat{y}_{t+1} = \alpha y_t + (1 - \alpha) \hat{y}_{t-1} \) in (5.15) for the past estimates:

\[ \hat{y}_{t+1} = \alpha y_t + \alpha (1 - \alpha) y_{t-1} + \alpha (1 - \alpha)^2 y_{t-2} + \cdots + \alpha (1 - \alpha)^i y_{t-i} + \cdots \]

(5.16)

Thus, the weight attached to the past observation decreases exponentially as the observation time gets older. The exponential smoothing method can be customized to
handle time series with different structures and can be applied to each component, i.e., Level, Trend, and Seasonal of a time series if relevant. The Standard Holt-Winters method was developed to deal with the time series with level, trend, and seasonal components (Winters, 1960). However, standard Hold-Winters is not suitable for handling time series with complex seasonality such as electricity load data which has more than one seasonal cycle. (Taylor, 2003) modified the standard Holt-Winters method to accommodate multiple seasonal cycles. The methodology is outlined below:

**Level:**
\[ l_t = \alpha \left( \frac{y_t}{d_{t-1}w_{t-1}} \right) + (1 - \alpha)(l_{t-1} + T_{t-1}) \] (5.17)

**Trend:**
\[ T_t = \beta (l_t - l_{t-1}) + (1 - \beta)T_{t-1} \] (5.18)

**Seasonality 1:**
\[ d_t = \delta \left( \frac{y_t}{l_t w_{t-1}} \right) + (1 + \delta)d_{t-1} \] (5.19)

**Seasonality 2:**
\[ w_t = \omega \left( \frac{y_t}{l_t d_{t-1}} \right) + (1 + \omega)w_{t-1} \] (5.20)

**Forecast:**
\[ y_{t+k} = (l_t + kT_t)d_{t-s_k}w_{t-s_k} \] (5.21)

where equation (5.17), (5.18), (5.19) and (5.20) are smoothing expressions and \( \alpha, \beta, \delta \) and \( \omega \) are the smoothing parameter for the local level, trend, first and second seasonalities respectively, while equation (5.21) is the expression for \( k \) step forecast. Note that in the equation(5.21), the forecast expression is adjusted by the product of two seasonal components \( d_{t-s_k}w_{t-s_k} \) which implies multiplicative seasonality. An additive version of this formulation could also be generated, nevertheless, multiplicative seasonality is more suitable when the electricity data is concerned. Moreover, previous studies on exponential smoothing with multiplicative seasonality indicate that residuals are correlated and AR(1) model can be used for the adjustment (Chatfield, 1978) (Taylor, 2003).

Taylor (2003, 2006) reported that the modified version outperforms double seasonal ARIMA in short-term load forecasting. There is extensive literature on the application
of double seasonal exponential smoothing to short-term load forecasting including the variants of the method such as the triple seasonal adjustment in (Taylor, 2010) and a variant enabling the more frequent update of the inner cycle in (Gould et al., 2008), application to specific data such as in (Bernardi and Petrella, 2015) and (Taylor and McSharry, 2007) as well as studies using exponential smoothing in comparison purposes such as (Souza et al., 2007) (Taylor et al., 2006), (Taylor, 2012). Exponential smoothing is relatively easy to implement with a few model parameters and provides decent performance. Another advantage of exponential smoothing is that exponential smoothing does not require a model specification procedure. These attractive features of the method led us to choose it as the benchmark model.

**TBATS**

(De Livera et al., 2011) introduced TBATS in order to overcome restrictions of exponential smoothing to deal with broader ranges or time series with complex seasonality. TBATS stands for Trigonometric terms, Box-Cox transformations, ARMA errors, Trend, and Seasonality. In addition to exponential smoothing, the method incorporates Fourier terms to represent seasonality, which accepts also non-integer seasonal lengths. Fourier terms are powerful instruments for modeling any type of periodic data. Box-Cox transformation is used for heterogeneity and ARMA errors capture the short-term dynamics. The model can be represented as (De Livera, Hyndman, & Snyder (2011)):

\[
y_t^{(\lambda)} = \begin{cases} 
\frac{y_t^\lambda - 1}{\lambda}, & \lambda \neq 0 \\
\log y_t, & \lambda = 0 
\end{cases}
\]  

(5.22)

\[
y_t^{(\lambda)} = l_{t-1} + \phi b_{t-1} + \sum_{i=1}^T s_{i-m_t} + d_t
\]  

(5.23)

\[
l_t = l_{t-1} + \phi b_{t-1} + \alpha d_t
\]  

(5.24)

\[
b_t = (1-\phi)b_t + \phi b_{t-1} + \beta d_t
\]  

(5.25)

\[
d_t = \sum_{i=1}^p \phi_i d_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t
\]  

(5.26)
\begin{align*}
    s_{t}^{(i)} &= \sum_{j=1}^{(k_{i})} s_{j,t}^{(i)} \\
    s_{j,t}^{(i)} &= s_{j,t}^{(i)} \cos \left( \frac{2\pi j}{m_{i}} \right) + s_{j,t}^{(i)} \sin \left( \frac{2\pi j}{m_{i}} \right) + \gamma_{i}^{(i)} d_{i} \\
    s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \left( \frac{2\pi j}{m_{i}} \right) + s_{j,t-1}^{(i)} \cos \left( \frac{2\pi j}{m_{i}} \right) + \gamma_{i}^{(i)} d_{i}
\end{align*} \quad (5.27)

where

- $y_{t}^{(i)}$ in (5.22) is Box-Cox transformed time series,
- $l_{t}$ in (5.24) local level smoothing
- $b_{t}$ in (5.25) is trend component
- $d_{t}$ in (5.26) is the ARMA structure for the errors

and, the model parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Number of Seasonal cycles</td>
</tr>
<tr>
<td>$m_{i}$</td>
<td>Length of the season $i$</td>
</tr>
<tr>
<td>$k_{i}$</td>
<td>Number of harmonics in season $i$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Smoothing parameter for the level</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Smoothing parameter for the trend</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Damping parameter for the trend</td>
</tr>
<tr>
<td>$\varphi_{i}, \theta_{i}$</td>
<td>Coefficients of ARMA($p,q$) process</td>
</tr>
<tr>
<td>$\gamma_{1}^{(i)}, \gamma_{2}^{(i)}$</td>
<td>Smoothing parameter for seasonal terms</td>
</tr>
</tbody>
</table>

In general, the TBATS model is designated as

$$TBATS \left( \lambda, \phi, p, q, \{m_{1}, k_{1}\}, \ldots, \{m_{T}, k_{T}\} \right).$$

**Decomposition**

Decomposition techniques together with simple forecasting methods are also used in short-term electricity forecasting (Wang et al., 2012), (Shao et al., 2017). The concept behind decomposition is that cycling and trend components are isolated from the data
so that remaining residuals become more proper to be modeled. According to the purpose and methodology to be followed, decomposition may be carried out in several ways. For example, (Goh and Choi, 1984) decomposed electricity consumption data into the hour of a day, the day of a week, and the week of year components and modeled those components individually. A more general approach is to decompose time series $y(t)$ into seasonal $s(t)$, trend $l(t)$, and irregular $e(t)$ components such that

$$y(t) = s(t) + l(t) + e(t).$$

The seasonal and trend component is modeled with a decomposition method and the remaining deseasonalized data is modeled by a simple time series method. For such an application, in addition to classical decomposition, SEATS (Seasonal Extraction in ARIMA Time Series), X11, and STL decomposition techniques can be used. STL—“Seasonal and Trend decomposition using Loess” decomposes the data into the seasonal, trend, and remainder components using Loess regression (Cleveland et al., 1990). STL decomposition has many advantages in modeling time series with complex seasonality such as robustness to outliers and accommodating almost any type of seasonality. Loess is a non-parametric method that depends on local weights to form a smooth curve fitted to the data points. Thus STL does not require model specification and parameter estimation. Since it is a non-parametric method, complex seasonality which is difficult to model parametrically can be modeled with STL decomposition. There is no restriction on the type of seasonality and any kind of seasonal component can be modeled with STL. STL decomposition consists of two recursive processes as an inner loop which is nested in an outer loop. Seasonal and trend components are smoothed and updated in each pass through the inner loop. In the outer loop, robustness weights are calculated following the inner loop. Each inner loop consists of six steps:

1. **Detrending:** Detrended series are obtained by subtracting the trend component from the series $y_t - T^k_t$ where $T^k_t$ is the trend component calculated at $k^{th}$ pass and $T^0_t = 0$.
2. **Cycle-Subseries Smoothing:** Subseries of values at each position of the seasonal cycle are smoothed by LOESS. The procedure provides the temporary seasonal series $C_{t}^{k+1}$.

3. **Low-pass Filtering of Smoothed Cycle-Subseries:** A low pass filter consisting of two Moving Average filters and a LOESS procedure. The result is referred to as $L_{t}^{k+1}$

4. **Detrending of smoothed Cycle-Subseries:** to obtain the seasonal component, $L_{t}^{k+1}$ is subtracted from the temporary seasonal series to isolate low-frequency data from the seasonal component. Thus $S_{t}^{k+1} = C_{t}^{k+1} - L_{t}^{k+1}$

5. **Deseasonalizing:** which is simply subtracting out the seasonal component $y_{t} - S_{t}^{k+1}$

6. **Trend Smoothing:** In order to obtain the trend component $T_{t}^{k+1}$, LOESS is applied to deseasonalized series from the previous step.

The outer loop checks the remaining after seasonal and trend components are isolated and assign robustness weights to each point. Let $R_{t} = y_{t} - T_{t} - S_{t}$ be the remainder, the robustness weights indicate the degree of the extremity of $R_{t}$. Then, these weights are used in the next inner loop pass in steps 2 and 6.

(Theodosiou, 2011) investigated the performance of the STL decomposition technique and compared the result with traditional methods like ARIMA and Exponential Smoothing. He used a set of monthly and quarterly data and reported that STL provides consistently well forecasts for a diverse set of data with different structures. Standard STL produces a single seasonal component, however, STL can also be used to decompose data with multiple seasonality into multiple cycling components (Ollech 2018). An algorithm for automated STL with multiple seasonality is recently developed by (Bandara et al., 2021). The model with multiple seasonal adjustments is referred to as MSTL. Their procedure first determines whether the time series contains multiple seasonality or not. Then, the STL procedure is applied iteratively to remove multiple seasonality starting from the lower seasonal length. The application of MSTL combined with a simple time series method in forecasting day-ahead electricity load is almost missing in the literature.
Artificial Intelligence

Another group of widely used methods in electricity load forecasting is Artificial Intelligence methods. Among many others, Artificial Neural Networks (Kouhi and Keynia, 2013), Fuzzy Logic (Pandian et al., 2006), Support Vector Mechanism (Chen et al., 2004), Gradient Boosting (Taieb and Hyndman, 2014) are some examples. In addition, Artificial Intelligence is also used in Hybrid models such as ARMA SVM (Nie et al., 2012).

Application to Specific Data

The performance of univariate methods in short-term load forecasting depends on several factors. These factors are usually due to the structure of the methods, ie how the method handles seasonality and the characteristics of the relevant data. For example, methods such as dynamic harmonic regression, which represents seasonal components in fixed terms, may yield poor results in periods when seasonality patterns change. Also, the nature of the data is an important factor, and applying the methods to different datasets may yield different model specifications and different results. Thus, our case is a specific one concentrating on the short-term load profile of Turkey. There are mainly examples of artificial intelligence in the literature on short-term load forecasting of Turkey's electricity consumption such as in (Topalli and Erkmen, 2003), (Bilgic et al., 2010), and (Akay and Atak, 2007). (Yukseltan et al., 2017) employs regression methods whose periodic variations are captured with external variables.

5.2.1 Performance Evaluation

Measuring the performance of the forecast method is one of the critical tasks in evaluating the validity of the method used and in comparison with the alternatives. Nevertheless, there is not a single performance measure that is prevailing in every case. There are several performance indicators used to measure forecast accuracy in the literature. These performance indicators can be evaluated under two groups as scale-dependent metrics and percentage-based metrics (Hyndman and Koehler, 2006). Scale-dependent metrics provide error measures that depend on the scale of the data. The most commonly used scale-dependent measures are:
**Mean Squared Error-MSE:**  
*MSE* is defined as the mean of the squared forecast errors:

\[
MSE = \frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{y}_t)^2 .
\]  
(5.28)

Since each error term is squared, the impact of the error on MSE grows with the increasing deviation from the original data. Thus, only a few large deviations may result in a poor *MSE* score even if the rest of the forecasted values are pretty good (Chatfield and others, 1988). Another issue with *MSE* is that due to the squaring, the error metric is not on the same scale as the data.

**Root Mean Squared Error-RMSE:**  
*RMS* is described as the square root of the average squared error. RMSE brings back MSE into the same scale as the data.

\[
RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{y}_t)^2}
\]  
(5.29)

Some authors like (Armstrong, 2001) strongly suggest not to use “squared error” metrics for both comparison and validation of forecast methods since these methods are very sensitive to outliers and may provide misleading interpretations (Chatfield and others, 1988).

**Mean Absolute Error-MAE:**  
*MAE* is the mean of the deviations of forecasted values from the original data. All the errors are treated with the same weight.

\[
MAE = \frac{1}{n} \sum_{t=1}^{n} |y_t - \hat{y}_t| 
\]  
(5.30)

*MAE* is useful when the volume of deviation from the original data is important. When comparing forecast methods with different data sets, scale-based metrics may provide misleading interpretations since the metric value is correlated with the level of the data (Armstrong, 2001). When comparing across data sets as well as across alternative methods, percentage-based metrics provide more reliable results. The most commonly used percentage-based metrics are:
**Mean Absolute Percentage Error-MAPE:** MAPE measures the accuracy of the forecast by taking the average of percentage deviations. The percentage deviation is calculated for each forecasted value separately and then their simple average is taken.

\[
MAPE = \frac{1}{N} \sum_{i=1}^{N} \frac{|y_i - \hat{y}_i|}{y_i}
\]  

(5.31)

MAPE is one of the most commonly used and recommended error measures in the literature (Bowerman et al., 2005). Feasibility and consistency in risk minimization of MAPE usage are shown in (De Myttenaere et al., 2016). Scale independence and easy interpretability of MAPE make it popular in industrial applications in addition to academic studies (Byrne, 2012). However, MAPE has some disadvantages when the level of the actual data is very small, especially when the actual data is close to zero (Kim and Kim, 2016). The reason for this disadvantage is that MAPE expression in (5.31) has the actual value in the denominator and a very small actual value may result in large MAPE values. However, hourly electricity data fluctuates between a base point and peak point which are quite far from the origin. Therefore, MAPE is one of the most convenient performance metrics for forecasting hourly electricity consumption data.

Scale-dependent metrics can also be converted into percentage-based metrics. However, the interpretation of the metric completely changes.

**Root Mean Squared Percentage Error-RMSPE:** This metric is a variant RMSE in such a way that an error in RMSE is replaced by a percentage error

\[
RMSPE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i - \hat{y}_i}{y_i} \right)^2}
\]

5.3 Data and Methodology

5.3.1 Data

Hourly electricity consumption data from 16.09.2019 to 09.12.2019 is used for empirical analysis. The data is extracted from EXIST Transparency platform database(https://seffaflik.epias.com.tr/transparency/tuketim/gerceklesen-
The data set consists of 2016 hourly consumption observations. The set of the last 368 observations which corresponds to the last two weeks' hourly data is allocated as test data and the remaining data consisting of 1680 observations are assigned as training data to optimize the model parameters.

The complete data is depicted in Figure 5.1.


**Figure 5.1:** Electricity Consumption in Turkey from 16.09.2019 to 08.12.2019


It can be seen from the figure that consumption is always well above the x-axis and fluctuates within certain limits. The minimum level of demand over a period of time is referred to as base load. Therefore, the base load also reflects the minimum quantity of generation required for all time points in that period. The descriptive statistics of the data are provided in Table 5.1.

**Table 5.1:** Descriptive Statistics of the Consumption Data

<table>
<thead>
<tr>
<th>Consumption(MWh)</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Median</th>
<th>S.D.</th>
<th>1st Qu.</th>
<th>3rd Qu.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>23269</td>
<td>41933</td>
<td>31863</td>
<td>32348</td>
<td>3981.7</td>
<td>28246</td>
<td>35066</td>
</tr>
</tbody>
</table>

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In general, electricity consumption data exhibits daily and weekly cycling behavior due to sunlight availability, daily routines such as working hours, weekly routines such as weekend holidays, etc. Turkish electricity consumption data is not an exception to this. The Daily and weekly cycles are visible in Figure 5.2 which presents a closer view of the date for two weeks. Since the data is hourly, a period of 24 is set for daily seasonality and 24*7=168 is set for the weekly seasonality.

![Figure 5.2: Weekly and daily cycling of the data](https://seffaflik.epias.com.tr/transparency/tuketim/gerceklesen-tuketim/gercek-zamanli-tuketim.xhtml)

There are some specific characteristics of daily cycles. Average hourly consumption data for each day of a week is presented in Figure 5.3. The line represents the average hourly consumption whereas the blue shade corresponds to 95% confidence intervals for data. 95% confidence intervals are generated using the sample variance of the load in each hour on every weekday. First of all, all daily cycles are not identical and there are certain differences between weekday and weekend days. In general, the cycling behaviors on weekdays are similar. Electricity consumption decreases from midnight towards the morning and eventually reaches the lowest point somewhere between 4:00 and 5:00. From this point, it
gradually increases until noon-time. Then, there is a local drop in consumption between 12:00 and 13:00.

**Figure 5.3:** Average hourly consumption profiles for different days
The main reason for this specific drop is the lunch break during which electricity consumption of the industry and service sector decreases. Next, electricity consumption ramps up to reach the daily peak around 19:00. On Friday, the period of drop in noontime is longer than the periods on other weekdays which is due to Friday prey.

Although the cycling pattern on Saturday is similar to that of weekdays, the level of consumption during working hours is less. The situation is mainly due to the fact that Saturday is a free day for only part of the industry and service sector. Sunday is totally different from the other days. The lowest consumption on Sunday occurs around 7:00 and there isn’t any definite noon peak. Also, the level of consumption is considerably lower than the other days during day time.

On Friday, the period of drop in noontime is longer than the periods on other weekdays which is due to Friday prey. Although the cycling pattern on Saturday is similar to that of weekdays, the level of consumption during working hours is less. The situation is mainly due to the fact that Saturday is a free day for only part of the industry and service sector. Sunday is totally different from the other days.

5.3.2 Methodology

In this study, three alternative univariate time series models, DSES, TBATS, and MSTL+ETS are used. First, the model specifications that best fit the data are provided, and forecasts out of these models are evaluated.

DSES

Double Seasonal Exponential Smoothing outlined in (5.17)-(5.21) is used in the analysis. Different from many other forecasting methods, double seasonal exponential smoothing does not involve a model specification step. However, in order to implement the method, starting values of the levels and smoothing parameters must be specified. There are some alternative ways to estimate the initial values such as using the simple average of the first few data (Williams and Miller, Taylor 2003). However, estimating all the parameters simultaneously from the data provides more reliable results compared to the other alternatives. Starting values and smoothing parameters
can be obtained by minimizing the sum of squared errors of one step ahead forecast. Let \( \varepsilon_t = y_t - \hat{y}_{t|t-1} \) be the residual from the forecast for time \( t \), the sum of squared errors for the data set consisting of \( N \) observation is given by

\[
SSE = \sum_{t=1}^{N} \varepsilon_t^2 = \sum_{t=1}^{N} (y_t - \hat{y}_{t|t-1})^2
\]  

(5.32)

Unlike the stochastic processes such as autoregressive models, no specific formula provides SSE in exponential smoothing. Parameters that minimize SSE in (5.32) can only be obtained through non-linear optimization methods.

**TBATS**

TBATS model is designated as TBATS(\( \lambda, \phi, p, q, \{ m_1, k_1 \}, \ldots, \{ m_T, k_T \} \)). The estimation and model selection process for TBATS module is explained in (De Livera et al., 2011) in detail. TBATS estimation procedure is constructed on the reduced form of the conditional likelihood function which is derived in Section 3 of (De Livera et al., 2011):

\[
\mathcal{L}(\Omega) = n \log \left( SSE^* \right) - 2(\omega - 1) \sum_{i=1}^{n} \log y_i
\]  

(5.33)

where \( \Omega \) is the vector of parameters and \( SSE^* \) is the optimized sum of squared errors.

There may be special TBATS model formulations for example with or without Box-Cox transformation, with or without ARMA errors, etc. The final form of the model is selected by AIC; the model with minimum AIC is selected among the alternatives. For the inclusion of ARMA errors, a two-step procedure is followed. First, an appropriate model without ARMA error is fitted and an ARMA model is applied to the residuals to find the optimum \( p \) and \( q \) values. Then, in the second step, the TBATS model with \( ARMA(p,q) \) error is fit. However, in this case, all the parameters including \( p \) and \( q \) are estimated simultaneously. The final decision to keep ARMA error is based on the AIC; ARMA error is kept if AIC is improved upon inclusion.

**MSTL + ETS Model**

In this study, the MSTL+ETS model is used. Seasonal and trend components are first
modeled with the MSTL decomposition technique. Then exponential smoothing (ETS) is applied to model the subsequent residuals. In general, exponential smoothing is referred to as ETS(Error, Trend, Seasonal) and there are several alternative exponential smoothing model formulations. Those alternatives arise from the possible combination of error, trends, and seasonal components. Possible candidate components are as follows:

- **Error:** Additive (A), Multiplicative (M),
- **Trend:** None (N), Additive (A), Additive damped (Ad),
- **Seasonal:** None (N), Additive (A), Multiplicative (M).

For example, ETS(A,N,M) corresponds to an exponential smoothing model with additive error, without trend component and multiplicative seasonality. In total, there may be 18 possible ETS models. Nevertheless, since trend and seasonal components are modeled with STL decomposition in our case, possible candidate models for the residuals are without trend and seasonal components. Therefore, the simple ETS formulation of ETS(A,N,N) and ETS(M,N,N) are the only two feasible candidate models that are suitable for detrended and deseasonalized data. The final selection of the model between two feasible alternatives will be based on information criteria AIC.

### 5.4 Empirical Results

Empirical analyses are carried out in order to assess the individual and combined performances of the methods presented in the previous section. First, the model parameter that best fits the data is calculated for all methods and then forecasts are performed with specified models. Particularly, the day-ahead forecast of the next day’s hourly demand for the entire 24 hours is performed. That is, after fitting the model using training data, the forecast for the next 24 hours is performed and compared with the test data to assess the performance. Then the training set is updated and the forecast for the following 24 hours is performed. The procedure is repeated for 14 days in a moving window forecast fashion. The procedure is visualized in Figure 5.4. Thereby, forecast results for the three models are obtained for the test period. First, the individual performances of the methods are analyzed and then compared with each other.
In our comparison, MAPE (Mean Absolute Percentage Error) is used as a performance metric as it is an appropriate and commonly used error metric in forecasting electricity load.

\[
MAPE = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{y_t - \hat{y}_t}{y_t} \right|
\]  

**DSES**

The model parameter estimated using the training set is given in Table 5.2. Note that these parameters are estimated through a non-linear optimization technique, not based on a likelihood function. Thus the forecast out of this model provides point estimates rather than forecast intervals.

**Table 5.2: Optimized parameters of Double Seasonal Holt-Winters Model**

<table>
<thead>
<tr>
<th>Double Seasonal Exponential Smoothing AR(1) adjusted</th>
<th>(\alpha)</th>
<th>(\lambda)</th>
<th>(\delta)</th>
<th>(\omega)</th>
<th>(\phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Level) (Trend) (Seasonal-1) (Seasonal-2) (AR(1) error)</td>
<td>0.0117</td>
<td>0.0579</td>
<td>0.2189</td>
<td>0.2281</td>
<td>0.8937</td>
</tr>
</tbody>
</table>

Forecasts are performed for each day of the test series. A forecast for one day is presented in Figure 5.5 as an example.
TBATS

TBATS Model that best fit to the data is TBATS (0.127, {0.5}, -, {<24,11>, <168,6>})). The daily seasonality is represented by 11 harmonics while the weekly seasonality is represented by 6 harmonics. There is no AR part in the residuals and MA part has five legs. Also damping for the trend is not applied in the final model. An example forecast of TBATS for one day is presented in Figure 5.6

Figure 5.5: Forecast of DSES for one day

Figure 5.6: TBATS Forecast example for one day
MSTL+ETS

A segment corresponding to the last five weeks from MSTL decomposition of the data is depicted in Figure 5.7. The plot on the uppermost panel of the figure belongs to the original data. The panel under the original data is the trend component followed by components for two seasonal cycles of 24-hour and 168-hour periods. The component for the residual is on the bottom panel of the figure. As a second step, simple exponential smoothing is fitted to the residuals.

Between two candidate ETS models of ETS(A,N,N) and ETS(M,N,N), ETS(M,N,N) provides slightly better AIC value. Therefore, ETS(M,N,N) is selected to model the residuals.

The resulting model turns out to be:

\[
\text{Model: MSTL+ETS(M,N,N)}
\]

Smoothing parameters for the simple ETS(M,N,N) is \( \alpha = 0.8753 \)

![Figure 5.7: Multiple STL decomposition of the data](image)

The forecast of MSTL+ETS method for one day is presented in Figure 5.8 as an example
In this section, individual performances of the forecast methods are compared. The comparison is carried out based on MAPE values. Day-ahead forecast MAPE values for each model for 7 consecutive days are presented in Figure 5.9.
MAPE of forecast from TBATS is always higher than MAPE of STL+ETS in all 7 days and higher than MAPE of DSHW in most of the days. DSHW and STL+ETS have comparable MAPE values but STE+ETS provided a slightly better result. The 7-day-average day ahead MAPEs are presented in Table 5.3

<table>
<thead>
<tr>
<th></th>
<th>DSHW</th>
<th>MSTL+EST</th>
<th>TBATS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average MAPE</td>
<td>1.234779</td>
<td>1.024888</td>
<td>1.948151</td>
</tr>
</tbody>
</table>

MSTL+ETS has the lowest average MAPE value. STL+ETS outperforms TBATS both on average and on each day. Although STL+ETS has marginally lower MAPE than DSHW on average, there are specific days in which DSHW performs better.

5.5 Conclusion

Over the last decades, there has been a growing need for a reliable electricity demand forecast. Especially, short-term load forecasting become a very important component of daily system and market operations. Various stakeholders of electricity systems and markets need such forecasts for a variety of reasons. Moreover, the required level of accuracy, complexity, resolution and forecast horizon, etc. may be rather different from application to application. In addition, electricity load depends on many factors such as weather conditions and exhibits cycling behaviors. The complexity of the problem leads to a large number of methods for forecasting hourly load. Univariate methods provide promising results for the aggregated load data. Yet, there is no single method that is superior in every case. In addition, the performance of a method may vary for different data sets.

In this study, we compare the performance of three alternative univariate forecasting methods using hourly load data of Turkey. Among alternatives, MSTL decomposition method provides better results than DSHW in forecasting most of the days and strictly dominates TBATS in each case. In this kind of performance comparison, methodology and problem setup have a considerable effect on the results. In this work, each time
the forecast for 24 hours of the next day is performed, the average MAPE of the 24 hours forecast is calculated. This procedure is repeated for each of the 7 days in the test data. The motivation behind this method is that in day-ahead markets studied in Chapter 3 and also proposed Demand Response method in Chapter 4 both require accurate forecasts for 24 hours of the next day.
CHAPTER 6

CONCLUSION

Electricity markets face challenges from transitioning to low carbon generation. Particularly, integrating large-scale renewable energy requires various changes in the existing systems. To facilitate a smooth and sustainable transition, carefully designed market mechanisms incorporating both technical and economic aspects, as well as the policies to implement and support those mechanisms are required. Because of the complexity of the systems and multidimensional challenges, a variety of fields may contribute to dealing with these challenges. However, among many others, economic analyzes are of great importance since the current transition is not technology bounded but rather a network, resource, markets, and operations-oriented. This dissertation aims to contribute to the current literature on electricity markets and the integration of renewable energy by addressing several topics.

Chapter 2 provides the basics of the electricity value chain and electricity market. Moreover, this chapter discusses the current challenges in the electricity markets.

In Chapter 3, inefficient equilibrium prices in the wholesale electricity market are studied. Due to the intermittency and almost zero marginal cost of renewable energy, equilibrium prices are affected negatively. Improper price signals threaten supply security by discouraging new investment. In this study, specifically, the impact of different support mechanisms, ownership structures, and cost structures are investigated. The results indicate that non-market support mechanisms result in lower equilibrium prices. Moreover, firms’ behavior is affected by ownership structure.

Chapter 4 proposes a novel Demand Response method. Demand Response methods have great potential in mitigating the adverse effect because of the intermittency of renewable resources by transferring flexibility from the supply side to the demand side. The suggested model has numerous advantages compared to the alternatives in the
literature. It is relatively easy to implement by consumers and provides greater flexibility to the system operators. The model is also flexible to be modified to include a variety of cases.

Chapter 5 models and compares three alternative univariate forecasting methods by using the electricity load data of Turkey. Specifically, Load forecast performances for the entire 24 hours of the next day are compared. MSTL which is relatively new provides better results compared to TBATS and DSES. TBATS demonstrated the worst overall performance on average and on each time. Although MSTL outperforms DSES on average, there are some days on which DSES performs better. Thus DSES is still comparable results with MSTL.


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Cramton, P.C., Stoft, S., 2006. The convergence of market designs for adequate generating capacity with special attention to the CAISO’s resource adequacy problem.


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APPENDICES

A. PROOF OF THEOREM 1

Without loss of generality assume that there are two time period \( i \) and \( j \) such that 
\[ q_{R,i} > \overline{D}_i \] and \( q_{R,j} < \overline{D}_j \) and also \( q_{R,k} = \overline{D}_k \quad \forall \in T \) where \( k \neq i, j \). That is there is one time period with excess renewable generation and there is one period with excess demand whereas demand and renewable generation for other time periods are exactly matching. Without the model implementation, additional conventional generation of 
\[ q_{c,i} = \overline{D}_j - q_{R,j} \] for time \( j \) is required to meet the demand while the amount of \( q_{R,j} - \overline{D}_j \) is unused in time \( i \). Therefore without implementation of the model, the revenue would be 
\[ B = \sum_D p \sum_D \left( \overline{D}_i - p q_{c,i} - p(\overline{D}_j - q_{R,j}) \right). \]

Let this revenue be the base revenue
\[
B = p \sum_D \overline{D}_i - p \max \left\{ (\overline{D}_j - q_{R,j}), 0 \right\}.
\]

The question is that is there any \( p_{\text{flex}} < p \) that improve the baseline revenue?

Assume that system operator offers \( p_{\text{flex}} = p - \tau, \quad \tau \in \mathbb{R}^+ \). In this case, some of the demand in period \( j \) where there is excess demand could be shifted to other periods (consecutively up to time \( j \) where there is excess generation with zero marginal cost). Thus, required conventional generation would be reduced. In return, there is a revenue loss due to lower price for flexible usage. Let \( q^*_{c,j} \) be the required conventional generation when \( p_{\text{flex}} \) is offered. The revenue as a result of the model is:
\[
\sum_D \left[ pd_{\text{inflex},i} + \sum_{t} p_{\text{flex},i,t} - pq^*_{c,j} \right].
\]

Rearranging as following:
Here the required generation for time \( j \) is the excess demand \( \overline{D}_j - q_{R,j} \) minus the total of the demand shifted to other periods \( \sum_{j-n}^{j+n} f_{i,j} \) from time \( j \). Thus,

\[ q_{c,j}^* = \overline{D}_j - q_{R,j} - \sum_{j-n}^{j+n} f_{i,j} \text{ or } 0 \text{ if } \overline{D}_j - q_{R,j} < 0. \]

So the revenue is:

\[ p \sum_{t} \overline{D}_t - p \sum_{t} \left( d_{flex,t} - \tau \sum_{t} \left( d_{flex,t} \right) - \tau \sum_{t} \left( d_{flex,t} \right) - p q_{c,j}^* \right). \]

Let this additional part be \( A(\tau) = -\tau \sum_{t} \left( d_{flex,t} \right) + \tau \sum_{t} \left( d_{flex,t} \right) + p d_{flex,j}. \) This additional part is the revenue from reduction in conventional generation minus the cost of selling of flexible at a discount. If there exist a \( \tau \) such that \( -\tau \sum_{t} \left( d_{flex,t} \right) + p d_{flex,j} > 0 \) then \( P_{flex} < p \) is the provides optimal solution.
\[ A(\tau) = -\tau \sum_{\tau} \left( \left[ 1 + \delta_{\text{inflex}}^{p} \left[ \delta_{\text{flex}}^{p} + \delta_{\mu} \left[ 1 - \frac{p_{\text{flex}}}{p} \right]^{1-\rho} \right]^{1-\rho} \right]^{-1} \right) + \right. \\
\left. p \left( \left[ 1 + \delta_{\text{inflex}}^{p} \left[ \delta_{\text{flex}}^{p} + \delta_{\mu} \left[ 1 - \frac{p_{\text{flex}}}{p} \right]^{1-\rho} \right]^{1-\rho} \right]^{-1} \right) \right) \\
\]

Although the value of \( A(\tau) \) depend on the parameters, it is guaranteed that there always be a positive additional revenue for some values of \( \tau \) since \( A(\tau) \) is always positive since \( \lim_{\tau \to \infty} A(\tau) > 0 \).
B. CURRICULUM VITAE

Surname, Name: PAKYARDIM Yusuf Kenan

EDUCATION

<table>
<thead>
<tr>
<th>Degree</th>
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<td>2011</td>
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WORK EXPERIENCE

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<tr>
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<tbody>
<tr>
<td>2012-Present</td>
<td>ASELSAN A.Ş.</td>
<td>Project Manager</td>
</tr>
<tr>
<td>2007-2012</td>
<td>ASELSAN A.Ş.</td>
<td>System Engineer</td>
</tr>
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</table>

FOREIGN LANGUAGES

Advanced English, Basic German
C. TURKISH SUMMARY / TÜRKÇE ÖZET


Bir diğer özellik ise yenilenebilir enerjinin marjinal üretim maliyetinin yaklaşık sıfır olmasıdır. Toptan piyasalarında denge fiyatları günlük olarak organize bir şekilde düzenlenenen rekabet ile belirlenmektedir. Yenilenebilir enerji bu rekabete sıfır marjinal üretim maliyeti ile katıldığından dolayı denge fiyatlarını aşağı yönlü bozmaktadır.

İkinci bölümde elektrik sistemleri ve piyasaları ile ilgili temel bilgiler verilmektedir. Bu alanda çalışma yapabilmek için ve tezin geri kalan kısımlarını anlayabilmek için bu bölüm önem arz etmektedir.


Dördüncü bölüm yenilenebilir enerjinin kontrol edilemiyor olmasından kaynaklanan sorunu adresleyen bağımsız bir makaledir. Bu çalışmada özgün bir piyasa modeli önerilmiş ve analiz edilmiştir. Modelde elektrik kullanımı esnekliklerine göre iki farklı kategorije ayrılmıştır. Sistem Operatörü esnek kullanım için indirimli fiyat önermektedir ve bunun karşılığında kullanıcı esnek kullanımın kesin zamanlanmasını Sistem Operatörüne bırakmaktadır. Modellene arz ve talep tarzının çok aşamalı dinamik etkileşimine dayanmaktadır ve esnek kullanım imkânı sağlanarak


Büyük ölçekli yenilenebilir enerji entegrasyonunun, üstesinden gelmesi gereken ekonomik verimlilik problemleri ortaya çıkarmaktadır. (Henriot and Glachant, 2013). Bu problemlerin en zor olanlarından biri de toptan piyasalarda ortaya çıkmaktadır. İlk olarak, toptan piyasa rekabetinde konvansiyonel üretim miktarı bir karar değişkeni olan, yenilenebilir enerji üretim sabit durum değişkeni olarak hesaba katılmaktadır. Bir diğer ve çok daha büyük zorluk getiren faktör ise yenilenebilir enerjinin neredeyse sıfır olan marjinal üretim maliyetinden dolayı toptan piyasalarda oluşan denge fiyatlarının aşağı yönlü çekilmesidir. Genel olarak toplam arz eğrisi, fiyat-miktar tekliflerinin toplanması ve “Merit Order” diye adlandırılan, artan bir sıraya konulması ile elde edilir (Deane et al., 2015). Yenilenebilir enerji, sıfır olan marjinal üretim maliyetinden dolayı toplam arz eğrisini sağ yönlü kaydırarak denge fiyatlarının düşmesini sağlamaktadır (Figueiredo and da Silva, 2019). Yenilenebilir kaynakların “Merit Order” Etkisi olarak adlandırılan bu durum Figure 3.1 de gösterilmiştir. Yenilenebilir enerjinin marjinal üretim maliyeti her ne kadar sıfıra yakın olsa da yatırım maliyetleri ve dolayısıyla uzun dönemli ortalama üretim maliyetleri diğer
bulunmaz. Dolayısıyla, yenilenebilir enerjinin firmanın üretim portföyünde olup olmaması durumunun da denge fiyatları üzerinde etkisi olmaktadır.


Literatürde “Merit Order” etkisinin varlığına yönelik farklı ülkelerden veriler kullanılarak yapılan birçok çalışma bulunmaktadır. Yenilenebilir enerjinin toptan piyasalarda denge fiyatlarını düşürdüğüne yönelik çalışmalar; İtalya için (Clò et al., 2015), Almanya için (Cludius et al., 2014) ve İspanya için (Ciarreta et al., 2014) tarafından yapılan çalışmalar örneği gösterilebilir. Amaç, yaptıkları çalışmaların sonuçlarını genelleyebilmek için teorik çalışmaların da yapılmasını gerektirmektedir. Bu alanda yapılacak teorik çalışmaların temelini rekabetin modellenmesi oluşturacaktır. Toptan elektrik piyasalarındaki rekabet “eksik rekabet” olarak nitelendirilmekte ve denge fiyatlarının genel olarak rekabetçi fiyat yüksek mertebelede oluşmaktadır (Borenstein et al., 2002), (Mansur, 2008). Bu çerçevede firmaların denge fiyatlarını artırabilme için üretim miktarlarını azaltabilirlerdi ortaya konmuştur (Wood and Blowers, 2018), (Twomey and Neuhoff, 2010). (McRae and Wolak, 2009) fiyat esnekliğinin düşük olduğu periyotlarda firmaların yüksek teklifler verdiğini göstermiştir. Bütün bu ve benzeri çalışmaları toptan elektrik piyasalarındaki rekabetin


Yenilenebilir enerji destek mekanizmalarının denge fiyatları üzerindeki etkisinin incelenmesi bu çalışmının bir başka özelliği olup, Yenilenebilir enerjinin desteklenme mekanizmasının denge fiyatlarını üzerine etkisinin olduğu literatürde birkaç çalışma belirtilmiştir (Brown and Eckert, 2020). Fakat bu konuda kapsamlı bir teorik çalışma güncel literatürde bulunmamaktadır. Bu çalışmada güncel literatürde bir başka katkı
olarak heterojen firma yapılarında ve farklı maliyet fonksiyonları durumlarında yenilenebilir enerji destek mekanizmalarının etkileri incelenmiştir.


Tip-1: Hem konvansiyonel hem de yenilenebilir enerji kaynaklarını kullanan firmalar. Bu firmaların setidir.

Tip-2: Sadece konvansiyonel enerji kaynaklarını kullanan firmalar. Bu firmaların setidir.

Tip-3: Sadece konvansiyonel enerji kaynaklarını kullanan firmalar. Bu firmaların setidir.


Önerme 3.2: i) çeşitlendirilmiş üretim portföyü, yani konvansiyonel kaynaklara sahip olan firmaların aynı zamanda yenilenebilir kaynaklara sahip olması, “Merit Order” etkisini azaltmaktadır. ii) piyasa temelli olmayan destek programları madde i) de verilen etkiyi ortadan kaldırılmaktadır. Dolayısıyla sabit alım politikası uygulandığı zaman yenilenebilir enerjinin endüstriyel organizasyonunun etkisi kalmamaktadır.


durumunda ikili kontrat hacmi Tip-1 firmanın sahip olduğu yenilenebilir enerji oranı ile tamamen doğru orantılıdır. iii) piyasa temelli olmayan destek programları sürekli daha düşük ikili kontrat hacmi sağlamaktadır.

Teorik analizler ile elde edilen sonuçları görselleştirmek için sayısal örnekler verilerek grafikler oluşturulmuştur. Sistem parametreleri, çalışmanın yapıldığı dönemde Türkiye piyasalarında gerçekleşen denge fiyatlarını sağlayacak şekilde seçilmiştir. Sayısal örnekler ile elde edilen sonuçlar Şekil 3.3 ile Şekil 3.6 arasında görselleştirilmiştir

Uzun yıllar boyunca sistem operatörleri ve üreticiler tarafından son kullanıcıının kullanım profilinde esneklik sağlamak ve kullanım zamanlamasını üzerinde etkili olmak adına bir niyet bulunmaktadır. Tüketim tarafının kararlarını etkileme isteği Talep Taraflı Yönetimi (Demand Side Management) fikrini ortaya çıkarmıştır. Uzun ve kısa vadeli birçok aktiviteyi barındıran Talep Taraflı Yönetiminin günlük piyasa işlemleriyle ilgili bir alt başlığı is Talep Tepki (Demand Response) programlardır. Genel anlamda Talep Tepki programları, değişen fiyatlar doğrultusunda son kullanımın kullanıma etkili olarak girerek fikrini ortaya çıkarmıştır. Bu mekanizmaların ilk ortaya çıkış amaçları talep tarafından dalgalanmaları azaltmak, zirve noktası olan tüketimin bir kısmını daha düşük zirve noktasına kaydırarak sistem güvenliğini sağlamak vb. gibi olmuştur. Sistema normal işleyişinde kesintisiz bir şekilde enerji sağlayabilmek için en düşük kapasite ve altyapı gereksinimini talep tarafında oluşabilecek en yüksek zirve noktasından daha fazla olmalıdır. Fakat talep çoğu zaman zirve noktasının altında gerçekleşmektedir ve sistemler çoğu zaman düşük kapasite ile çalışmak zorunda kalmaktadır. Sistem güvenlik marjları gibi diğer faktörler de hesaba katıldığı zaman sistemlerin bazı durumlarda kapasitesinin büyük bir kısmının kullanılmadığı ortaya çıkmaktadır (Strbac, 2008). Bu nedenle sistem verimliliğinin artırılması için talep tarafından esnekliğin sağlanması önemli görülmüştür. Fakat yenilenebilir enerjinin artan kullanımı, talep tarafından esnekliğin yalnızca önemli olmaktan çıkıp belirli ölçüde zorunlu kılınmuştur. Yenilenebilir enerjinin zamanlamasının ve miktarının kontrol edilemiyor olması, arz tarafından belirsizlik getirmiştir ve arz tarafının esnekliğini kısıtlamıştır. Düşük karbon politikaları ile belirlenen hedeflere ulaşmak yenilenebilir


Talep Tepki programları tüketim tarafında esnekliğin doğrudan ve dolaylı olarak elde edildiği ile ilgilidir. Dolaylı yöntemlerde son kullanıcıya fiyat sinyallerini gözlemleyip kullanım profilini değiştirmesi beklenir. Doğrudan kontrolde ise kullanım zamanlaması bir fayda karşılığında sistem operatörüne devredilir.

Bu çalışmada doğrudan kontrol prensibine dayalı yeni bir piyasa modeli önerilmektedir. Model son kullanıcıların, kullanımlarının bir kısmının kontrolünü sağlanan teşvik karşılığına Sistem Operatörüne devretmesini içermektedir. Bu modelde elektrik kullanımını esnek kullanım ve sabit kullanım olarak iki tipe ayrılmıştır.
Her planlama periyodunun başlangıcında, sistem Operatörü tarafindan esnek kullanım için indirimli bir fiyat son kullanıcısıya önerilmektedir. Son kullanıcı önerilen fiyat göre toplam tüketimin ne kadarını esnek kullanıma ayıracağını karar vermektedir. Böylelikle son kullanıcı indirimli fiyat formunda bir teşvik elde etmektedir ve Sistem Operatörü talep zamanlamasını konusunda bir esneklik sağlamaktadır ve Sistem Operatörü maliyeti sıfır olan yenilenebilir enerjiyi daha etkin kullanarak gelirini artırmaktadır.


Talep Tarafi Yönetimi ile ilgili hızla büyüyen bir literatür bulunmaktadır. Bu alanda yapılan Yayınlar 2009 yılında yıllık 130 mertebesindeyken 2020 yılına gelindiğinde yıllık 1800 yayın mertebesine çıkmıştır. Talep Tarafi Yönetiminin faydaları ve zorlukları ilgili farklı yönlere odaklara çeşitli araştırma makaleleri bulunmaktadır (O’Connell et al., 2014), (Conchado and Linares, 2012). Diğerlerine ek olarak bu faydaları genel olarak finansal, operasyonel ve yenilenebilir enerjiyi daha etkin kullanma şeklinde üç ana grupta toplamak mümkündür (De Jonghe et al., 2008), (Müller and Möst, 2018), (Gottwalt et al., 2016), (Simshauser, 2019).


Çalışmalarda kullanılan optimizasyon metotları da oldukça çeşitlilik göstermektedir. Lineer ve lineer olmayan programlama, tam sayılı programlama, stokastik modelleme, oyun teorisi, dinamik modelleme vb. optimizasyon yöntemleri kullanılan örnekler literatürde bulunmaktadır.

Bu çalışmada, mevcut literatürden farklı olarak, arz ve talep tarafının dinamik olarak etkileşimi kapsamlı bir şekilde modellenmiştir. Ayrıca, talep tarafının esnek ve sabit kullanım arasındaki seçimi güncel literatürdeki lineer varsayımın aksine fayda maksimizasyonu problemi olarak modellenmiştir. Bu tür modellene olduğu literatürde bildiğim ölçüde bulunmamaktadır. Ayrıca modellenede kullanılan koşullar ve yöntem sayesinde talebin korunması (demand recovery) kesin olarak sağlanmaktadır.

karar mekanizmasi olarak dinamik modellemeye eklenmiştir. Her periyod için Doğanın durumu $s_i \in \Omega$ olarak verilmiştir.


Aşama 0: Doğa’nın durumu ortaya çıkar. Tüketicinin gelecek dönem için tüketim planını belirlemen ve sabitlenir. Gelecek dönem için yenilenebilir enerji miktarı üretimi belirlenir.

Aşama 1: Sistem Operatörü ve tüketici gözlemek. İhtiyaç duyacağı esnek kullanım elde edebilmek için esnek kullanım fiyatını belirler.

Aşama 2: Tüketici ‘i gözlemek ve kullanım planının ne kadarını esnek kullanma geçireceğine kadar verir.

Aşama 3: Sistem Operatörü Aşama 0 da belirlenen yenilenebilir enerji üretim miktarları, Aşama 1 de sunulan fiyatları ve Aşama 2 de belirlenen sabit ve senek kullanım planlarını gözlemleyerek konvansiyonel kaynaklardan yapacağı üretim en aza indirmek için her bir periyottaki esnek kullanımının bir kısmını $t$ zamanından $\tau$
zamanına kaydırır. Böylelikle her periyot için ne kadar konvansiyonel kaynak kullanılabileceğini de karar verilmiştir olsun.


Nihai olarak elde edilen iki seviyeli optimizasyon probleminin sonucunun var olduğu 4.3.3.1 bölümünde gösterilmiştir. Öncelikli olarak alt seviye problemin olması en iyi değerleri ile şekillenen üst seviye problemin kısıt seti olan indirgenebilir bölge tanımlanmıştır. Bu bölgenin kapalı ve kompakt olduğu gösterilmiştir ve Weierstrass teoremine göre optimum çözümün var olduğu gösterilmiştir. Fala yenilenebilir enerji üretime olduğu dönemlerde önerilen bu modeli uygulamanın mutlak fayda saylayacağı Teorem-1 de ispatlanmıştır.


Her iki model için de elde edilen sonuçları görselleştirmek için gerçek verilere ve üretim profillerine dayalı uygulama örneği sunulmuştur. Tüketimi verisi olarak ve yenilenebilir enerji üretim profili olarak Şekil 4.1 de verilen Kaliforniya bölgesinde 2020 yılında gerçekleşen veriler kullanılmıştır. Bu verilere dayalı olarak Şekil 4.2 de verilen iki farklı senaryo oluşturulmuştur. Birinci senaryoda bir günlük yenilenebilir
enerji üretiminin o günkü toplam tüketimi karşılayacak kadar olduğu varsayılmıştır fakat yenilenebilir enerjinin üretim zamanlamasının talep edilen tüketim zamanlamasından farklı olması dolayısıyla ortaya çıkan sorun vurgulanmıştır. İkinci senaryoda ise toplam yenilenebilir enerji üretiminin toplam talebin yarısını karşıladığı varsayılmıştır. Tüketici tarafı için ise esnek kullanıma daha çabuk adaptasyon iyi ve daha dirençli olan tüketim olmak üzere iki farklı profil tanımlanmıştır. Şekil 4.3’de iki farklı tüketici profili için sunulan indirimli fiyat karşılığı esnek kullanım katsayıları gösterilmiştir.


Bu çalışmada tek değişkenli üç farklı talep yöntemi Türkiye’nin toplam elektrik talebi verileri kullanılarak modellenmiştir ve performansları karşılaştırılmıştır. Bu yöntemler: Çift Mevsimli Üstel Yumuşatma (Double Seasonal Exponential Smoothing-DSES), TABTS ve Çoklu STL Ayrıştırma (Multiple STL-MSTL) yöntemleridir. Spesifik olarak, bu üç alternatif yöntemin bir gün sonrasının 24 saatlik elektrik tüketim tahmin performansları analiz edilmiş ve karşılaştırılmıştır. 

gelmemektedir. Bu nedenle saatlik elektrik talebi tahminin yapan yöntemler, verinin bu karakteristik özelliklerini de modele dâhil eden yöntemler olması gerekmektedir.


Üstel Yumuşatmanın kısıtları ortadan kaldırmak ve daha geniş kapsamlı hale getirmek için TBATS (Trigonometric terms, Box-Cox transformations, ARMA errors, Trend, and Seasonality) yöntemi geliştirilmiştir (De Livera et al., 2011). Üstel yumuşatmaya ek olarak mevsimselliği temsil etmesi için Fourier terimleri kullanılmaktadır. Aynı zamanda sonuçlarda iyileşme sağlanamalarına göre veriye Box-Cox dönüşümü ve hata terimlerini ARMA ile modelle uygulaması de yapılmaktadır. Sonuçların iyileştirme kriteri olarak da AIC metriği kullanılmaktadır. Model
Kısa dönemde elektrik ihtiyacını tahminlerinde bileşenlerine ayırma (decomposition) yöntemi de kullanlabilmektedir (Wang et al., 2012), (Shao et al., 2017). Bu yöntemle ana fikir veriyi mevsimselliğin ve trend gibi etkilerden arındırıp geri kalan kısmı standart yöntemler ile modellemektedir. Bir zaman serisi, mevsimsel, trend ve düzensiz kısımlar olarak şeklinde üç bileşene ayrıştırılmaktadır ve bileşenleri ayrıştırma yöntemi ile temsil edilirken, bileşeni standart yöntemler ile modellenmektedir. SEATS, X11 ve STL gibi birçok ayrıştırma tekniği kullanabilir. Bu çalışmada STL (Seasonal and Trend decomposition using Loess) tekniğinin çoklu mevsimselliğ içerecek şekilde uyarlanmış versiyonu olan MSTL (Multiple STL) kullanılmıştır. STL tekniğinin parametrik olmaması, her türlü mevsimsel bileşenin modellenebiliyor olması, hatalı verilere karşı gürbüz olması gibi birçok avantajı bulunmaktadır.


Bu çalışmada incelenen yöntemlerin model parametrelerini belirlemek için kendine özgü yöntemler kullanılmıştır. DSES yöntemi bir istatistik yöntem olmadığı için olabilirlik fonksiyonu tanımlanamamaktadır. Bu nedenle model parametrelerini belirlemek için hataların karelerinin toplamı, lineer olmayan optimizasyon yöntemleri ile elde edilmiştir. TBATS’ın model parametrelerinin belirlenmesi için ise (De Livera et al., 2011)’nin türettği olabilirlik fonksiyonu kullanılmıştır ve AIC ye göre model seçimi yapılmıştır. MSTL ile bileşenlerine ayrılan verinin geri kalan kısmını modellemek için kullanılabilecek basit Üstel yumuşatma, mevsimsellik ve trend ortadan kalktığı için iki aday model kalmıştır. Model seçimi AIC ye bağlı olarak yapılmıştır.

Tahmin sırasında izlenen metodoloji, verileri kullanarak model seçimi ve parametrelerini belirlemek ve model ile bir gün sonrasının 24 saat için tüketim tahminini yapmak şeklinde olmuştur. Sonrasında veri seti güncellenerek bir sonraki günün 24 saat için tahmin yapılmıştır ve 7 gün için veriler elde edilmiştir. İzlenen yöntem, Şekil 5.4 de gösterilmiştir. DSES, TBATS ve MSTL ile yapılan günlük tahminlere örnekler sırası ile Şekil 5.6, Şekil 5.7 ve Şekil 5.’de gösterilmiştir. Bir haftalık tahmin periyodu sonunda elde edilen ve performans metriği olarak kullanılan MAPE değerleri Şekil 5.9 da verilmiştir. Buna göre MSTL, TBATS’ den bütün günler için, DSES’den ise birçok gün için daha iyi sonuç vermiştir. Bütün periyod için MAPE ortalamaları Tablo 5.3 de sunulmuştur. Sonuçlara göre MSTL en iyi performansı göstermiştir.
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YAZARIN / AUTHOR

Soyadı / Surname : PAKYARDIM
Adı / Name : Yusuf Kenan
Bölümü / Department : İktisat / Economics

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