RADIATIVE TRANSITIONS OF NON-STRANGE DOUBLY HEAVY OCTET BARYONS TO DECUPLET BARYONS

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ABSTRACT

RADIATIVE TRANSITIONS OF NON-STRANGE DOUBLY HEAVY OCTET BARYONS TO DECUPLET BARYONS

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Electromagnetic form factors for transitions are studied through years using various methods. In this work, we calculated the electromagnetic form factors for the radiative decays of non-strange octet baryons to decuplet baryons in the QCD Sum Rules framework at $q^2 = 0$ up to twist-4. Calculation of the correlation function and the analysis of the results performed using Wolfram Mathematica 12.0.0 Software.

Keywords: QCD Sum Rules, Radiative Decays, Electromagnetic Form Factors, Electromagnetic Transitions

ÖZ

ACAYİP OLMAYAN İKİ AĞIR KUARK İÇEREN OKTET BARYONLARIN DEKUPLET BARYONLARA RADYATİF GEÇİŞLERİ

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Elektromanyetik geçişlerin form faktörleri yıllardır farklı yöntemler kullanılarak çalışılmıştır. Bu çalışmada garip kuark içermeyen oktet baryonların dekuplet baryonlara geçişlerindeki form faktörleri, KRD Toplam Kuralları kullanılarak $q^2 = 0$ için twist-4'e kadar hesaplanmıştır.Hesaplamalar ve sonuçların analizi için Wolfram Mathematica 12.0.0 yazılımı kullanılmıştır.

Anahtar Kelimeler: KRD Toplam Kuralları, Radyatif Bozunum, Elektromanyetik Form Faktörleri, Elektromanyetik Geçişler

To the dreams that I’ve been after

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QCD	Quantum Chromodynamics
QCDSR	Quantum Chromodynamics Sum Rules
QED	Quantum Electrodynamics
OPE	Operator Product Expansion
KRD	Kuantum Renk Dinamiği

CHAPTER 1

INTRODUCTION

1.1 Introduction

The standard model is an accepted explanation to dynamics of particle physics and it is a combination of different theories (i.e. QED, Weak Theory, QCD). There are different types of particles in the standard model; leptons, quarks, bosons.

Hadrons are particles that consist of quarks and gluons. There are different types of hadrons, i.e. conventional mesons (made of a quark-antiquark pair), conventional baryons (three quarks or three antiquarks), etc. [1]. The theory of the interactions between hadrons is called "Quantum Chromodynamics" (QCD).

Magnetic moment is an intrinsic property describing the electromagnetic interaction of a particle. It's related to the spin

$$\vec{\mu} = g\vec{S} \tag{1.1}$$

Table 1.1: Standard Model Particles

Leptons	Quarks	Bosons
e	u	γ
ν_e	d	$W^\pm, Z$
μ	c	g
ν_μ	s	H
τ	t	
ν_τ	b	

where $\vec{S}$ is spin and g is gyromagnetic constant.

The magnetic moments and transition magnetic moments are effective tools to investigate the internal structures of the baryons and have been studied in various methods (such as chiral perturbation theory, quark model, bag model, QCD sum rules) through the years [2–20]. In this work we studied the transition magnetic dipole moment (G_M) and the transition electric quadrupole moment (G_E) of radiative decays of doubly heavy octet non-strange baryons to decuplet baryons.

1.2 QCD

Table 1.2: Quarks

Quark	Mass	Electric Charge
up	$2.16 \times 10^{-3} GeV$	$+2/3$
down	$4.67 \times 10^{-3} GeV$	$-1/3$
charm	$1.27 GeV$	$+2/3$
strange	$93.4 \times 10^{-3} GeV$	$-1/3$
top	175 GeV	$+2/3$
bottom	4.18 GeV	$-1/3$

Quarks are spin-$1/2$ fermions. Hence due to the Pauli's exclusion principle more than one quark cannot occupy the same quantum state. But the observation of the Ω^- baryon was in contradiction with this principle [21]. Three spin-$1/2$ strange quarks combined to make the Ω^- baryon. To solve this problem, an additional SU(3) symmetry was introduced [22], [23], and it is called "color symmetry".

The color charge for quarks can take three different values: red, green and blue (antired, antigreen and antiblue for antiquarks). The interactions between the colored particles are mediated by the boson "gluon" which also has color charge.

1.2.1 Gluon

Gauge boson of the strong interactions is called gluon which has color charge and is massless. Since gluon has color charge, gluons have self interactions. Because of self interacting property of the gluon, despite being massless, the range of strong interactions are short. Gluon's color charge consists linear combinations of products of color and an anticolor. There are total 8 different gluons, since one of the linear combinations is a colorless combinations. One other important property of the QCD is asymptotic freedom. Color confinement is conjectured to be a property of strong interactions.

1.2.2 Color Confinement

Color confinement tells us that there are no color charged free particles in the nature. All color charged particles must form composite particles with other color charged particles to be in a colorless bound states (hadrons)[1].It is an observed property and it is been considered that QCD has color confinement.

1.2.3 Asymptotic Freedom

Strong force between two quarks is weak at small distances, they act as free particles. But for longer distances quark-gluon interactions becomes dominant, the force becomes stronger and they no longer act as free particles. In 1973, David J. Gross and Frank Wilczek [24] and Politzer [25] showed that QCD has this property, which was a viable candidate to explain the strong interactions.

1.2.4 QCD Lagrangian

The Lagrangian density for the QCD is given as [26]:

$$\mathcal{L}_{QCD} = \overline{\psi}^i_f \left(i\gamma^\mu \mathcal{D}^{ij}_\mu - m\delta^{ij} \right) \psi^j_f - \frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu} \tag{1.2}$$

[1] While in color theory RGB makes white, in QCD RGB,$\overline{RGB}$,$\frac{1}{\sqrt{3}}\left(R\overline{R} + G\overline{G} + B\overline{B}\right)$ are colorless combinations.

where a is gluon index (runs over from 1 to 8),i and j are color indices (runs from 1 to 3), f is the flavor index (runs u,d,s,c,b,t) and $G^a_{\mu\nu}$ is the gluon field-strength tensor which is given as:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu \tag{1.3}$$

where the A^a_μ is the gluon field and f^{abc} is structure constants of the SU(3) which the QCD is invariant under.

CHAPTER 2

METHOD

2.1 QCD Sum Rules

QCD sum rules(or SVZ sum rules) method was invented by M. A. Shifman, V. I. Zakharov and A. I. Vainshtein in 1979 [27] and it has been considered one of the most successful non-perturbative methods. To study the properties of hadrons, in this method, hadrons are represented via their interpolating currents.A suitably chosen correlation function is calculated in two different frameworks, hadronic and operator product expansion (OPE) sides, and matched to each other using their spectral representation.

In QCD sum rules, long distance and short distance quark-gluon interactions are separated from each other. Since in the short distances (low energy scale) quarks act as free particles, perturbative QCD can be used. In the long distance region quark-gluon interactions become dominant and this region is parametrized by non-vanishing vacuum expectation values (condensates) and light-cone distribution amplitudes. Light cone distribution amplitudes are defined through vacuum-hadron/photon matrix elements taken at light-like separations [28].

2.1.1 Correlation Function

As a simple application of QCDSR, in this chapter mass sum rules of a meson will be discussed in detail. In order to study the masses of hadrons, a suitable correlation function is,

$$\Pi(q) = \int d^4x e^{iqx} \langle\Omega| \mathcal{T}\{j(0)j^\dagger(x)\} |\Omega\rangle \tag{2.1}$$

where $|\Omega\rangle$ is hadronic vacuum,j is interpolating current for the hadron in study and $\mathcal{T}$ is the time ordering operator. The time ordering operator reorders the operators in a way that the operator with the earlier time acts on the ket first. To calculate the hadronic side we should expand the time ordered matrix element first. In order to expand the time ordering we can use the following;

$$\begin{aligned}\mathcal{T}\left(A(t_1)B(t_2)\right)|\psi\rangle &= \begin{cases}\eta B(t_2)A(t_1)\,|\psi\rangle & t_1 < t_2\\ A(t_1)B(t_2)\,|\psi\rangle & t_2 < t_1\end{cases}\\ &= \left[\theta(t_2 - t_1)B(t_2)A(t_1) + \eta\theta(t_1 - t_2)A(t_1)B(t_2)\right]|\psi\rangle\end{aligned} \tag{2.2}$$

where $\theta(t)$ is the heavyside step function, $\eta = 1$ for bosonic operators A and B and $\eta = -1$ for fermionic operators.

2.1.2 Hadronic Side

The time ordering operator in equation 2.1 can be written explicitly as:

$$\Pi(p^2) = i\int d^4x e^{ipx}\,\langle\Omega|\left(\theta(x^0)j^\dagger(x)j(0) + \theta(-x^0)j(0)j^\dagger(x)\right)|\Omega\rangle \tag{2.3}$$

where $j^\dagger(x)$ can be rewritten as $j^\dagger(x) = e^{i\hat{p}x}j^\dagger(0)e^{-i\hat{p}x}$.

To calculate matrix element $\langle\Omega|\left(\theta(x^0)j^\dagger(x)j(0) + \theta(-x^0)j(0)j^\dagger(x)\right)|\Omega\rangle$ the identity operator can be inserted between the interpolating currents.

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \sum_h\int d^4k\frac{1}{(2\pi)^3}\theta(k^0)\delta(k^2 - m_h^2)\,|h(k)\rangle\langle h(k)| + \text{continuum} \tag{2.4}$$

where the summation goes over all hadrons.

Hence, the correlation function becomes;

$$\begin{aligned}\Pi(p^2) =& i\int d^4x e^{ipx}\Big\{\langle\Omega|\,\theta(x^0)e^{i\hat{p}x}j^\dagger(0)e^{-i\hat{p}x}\mathbb{1}j(0)\,|\Omega\rangle\\ &+ \langle\Omega|\,\theta(-x^0)j(0)\mathbb{1}e^{i\hat{p}x}j^\dagger(0)e^{-i\hat{p}x}\,|\Omega\rangle\Big\}\end{aligned} \tag{2.5}$$

$$
\begin{aligned}
\Pi(p^2) =& i\sum_h \int d^4x \frac{d^4q}{(2\pi)^3}\delta(q^2-m_h^2)e^{ipx}\theta(q^0) \\
&\Big\{\theta(x^0)\left\langle\Omega\right| e^{i\hat{p}x}j^\dagger(0)e^{-i\hat{p}x}\left|h(q)\right\rangle\left\langle h(q)\right| j(0)\left|\Omega\right\rangle \\
&+\theta(-x^0)\left\langle\Omega\right| j(0)\left|h(q)\right\rangle\left\langle h(q)\right| e^{i\hat{p}x}j^\dagger(0)e^{-i\hat{p}x}\left|\Omega\right\rangle\Big\}
\end{aligned}
$$

For simplicity, in equation 2.6 onward, only the contributions from the single hadronic states are shown explicitly.

By applying the momentum operators to $|h(q)\rangle$ and $|\Omega\rangle$

$$
\begin{aligned}
e^{i\hat{p}x}\left|h(q)\right\rangle &= e^{iqx}\left|h(q)\right\rangle \\
e^{i\hat{p}x}\left|\Omega\right\rangle &= 1\left|\Omega\right\rangle
\end{aligned}
\tag{2.6}
$$

the correlation function becomes:

$$
\begin{aligned}
\Pi(p^2) =& i\sum_h \int d^4x \frac{d^4q}{(2\pi)^3}\delta(q^2-m_h^2)\theta(q^0) \\
&\Big\{e^{ipx-iqx}\theta(x^0)\left\langle\Omega\right| j^\dagger(0)\left|h(q)\right\rangle\left\langle h(q)\right| j(0)\left|\Omega\right\rangle \\
&+e^{ipx+iqx}\theta(-x^0)\left\langle\Omega\right| j(0)\left|h(q)\right\rangle\left\langle h(q)\right| j^\dagger(0)\left|\Omega\right\rangle\Big\}
\end{aligned}
\tag{2.7}
$$

where the dots at the end mean the continuum.

Here $\int d^4x$ can be splitted as $\int d^3x \int dx^0$ and by using the $\theta(x^0)$ functions the correlation function becomes;

$$
\begin{aligned}
\Pi(p^2) =& i\sum_h \int d^3x \frac{d^4q}{(2\pi)^3}\delta(q^2-m_h^2)\theta(q^0) \\
&\Big\{\int_0^\infty dx^0 e^{ipx-iqx}\left\langle\Omega\right| j^\dagger(0)\left|h(q)\right\rangle\left\langle h(q)\right| j(0)\left|\Omega\right\rangle \\
&+\int_{-\infty}^0 dx^0 e^{ipx+iqx}\left\langle\Omega\right| j(0)\left|h(q)\right\rangle\left\langle h(q)\right| j^\dagger(0)\left|\Omega\right\rangle\Big\}
\end{aligned}
\tag{2.8}
$$

$$
\begin{aligned}
&\int d^3x d^4q \delta(q^2 - m_h^2)\theta(q^0) \int_0^\infty dx^0 e^{i(p-q)x} \,|\, \langle \Omega | j(0) | h(q) \rangle |^2 \\
=& \int d^3x d^4q \delta(q^2 - m_h^2)\theta(q^0) \int_0^\infty dx^0 e^{i(p^0 - q^0 + i\epsilon)x^0} e^{i(\vec{p} - \vec{q})\cdot \vec{x}} \,|\, \langle \Omega | j(0) | h(q) \rangle |^2 \\
=& - \int d^4q \theta(q^0) (2\pi)^3 \delta(\vec{p} - \vec{q}) \frac{\delta((q^0)^2 - \vec{q}^2 - m_h^2)}{i(p^0 - q^0 + i\epsilon)} \left| \left\langle \Omega \middle| j(0) \middle| h(q^0, \vec{q}) \right\rangle \right|^2 \\
=& i \int d^4q \theta(q^0) \frac{(2\pi)^3}{p^0 - q^0 + i\epsilon} \delta((q^0)^2 - \vec{p}^2 - m_h^2) \left| \left\langle \Omega \middle| j(0) \middle| h(q^0, \vec{p}) \right\rangle \right|^2 \\
=& i \int d^4q \theta(q^0) \frac{(2\pi)^3}{p^0 - q^0 + i\epsilon} \frac{\delta(q^0 - \sqrt{\vec{p}^2 + m_h^2}}{2\sqrt{\vec{p}^2 + m_h^2}} \left| \left\langle \Omega \middle| j(0) \middle| h(q^0, \vec{p}) \right\rangle \right|^2 \\
=& i \frac{(2\pi)^3}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(\vec{p}), \vec{p}) \rangle |^2}{p^0 - \omega_h(\vec{p}) + i\epsilon}
\end{aligned}
\tag{2.9}
$$

where $\omega_h(\vec{p}) = \sqrt{\vec{p}^2 + m_h^2}$ and an $\epsilon = 0^+$ have been inserted to make the integral convergent in the upper limit of x^0 integral.

Same procedure holds for $\int_{-\infty}^{0} dx^0$ part too, with a result;

$$
\begin{aligned}
&\int d^3x d^4q \delta(q^2 - m_h^2)\theta(q^0) \int_{-\infty}^0 dx^0 e^{i(p+q)x} \,|\, \langle \Omega | j(0) | h(q) \rangle |^2 \\
=& - i \frac{(2\pi)^3}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(-\vec{p}), \vec{p}) \rangle |^2}{p^0 + \omega_h(\vec{p}) - i\epsilon}
\end{aligned}
\tag{2.10}
$$

Inserting equations 2.9 and 2.10 into equation 2.7 and taking the limit $\epsilon \to 0$, the correlation function becomes

$$
\begin{aligned}
\Pi(p^2) =& i \sum_h \left\{ \frac{1}{(2\pi)^3} i \frac{(2\pi)^3}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(\vec{p}), \vec{p}) \rangle |^2}{p^0 - \omega_h(\vec{p})} \right. \\
& \left. - i \frac{1}{(2\pi)^3} i \frac{(2\pi)^3}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(\vec{p}), -\vec{p}) \rangle |^2}{p^0 + \omega_h(\vec{p})} \right\} \\
=& - \sum_h \left\{ \frac{1}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(\vec{p}), \vec{p}) \rangle |^2}{p^0 - \omega_h(\vec{p})} \right. \\
& \left. + \frac{1}{2\omega_h(\vec{p})} \frac{|\, \langle \Omega | j(0) | h(\omega_h(\vec{p}), -\vec{p}) \rangle |^2}{p^0 + \omega_h(\vec{p})} \right\}
\end{aligned}
\tag{2.11}
$$

For $p^2 > 0$ we can find a reference frame where $\vec{p} = 0$. Hence in this frame

$$
\langle \Omega | j(0) | h(\omega_h(\vec{p}), -\vec{p}) \rangle = \langle \Omega | j(0) | h(\omega_h(\vec{p}, \vec{p}) \rangle \tag{2.12}
$$

$$\begin{aligned}
\Pi(p^2) &= \sum_h |\langle\Omega|j(0)|h(p)\rangle|^2 \frac{1}{2\omega_h(\vec{p})}\left\{\frac{1}{p^0+\omega_h(\vec{p})} - \frac{1}{p^0-\omega_h(\vec{p})}\right\} \\
&= \sum_h |\langle\Omega|j(0)|h(p)\rangle|^2 \frac{1}{2\omega_h(\vec{p})}\frac{-2\omega_h(\vec{p})}{(p^0)^2-\omega_h(\vec{p})^2} \\
&= -\sum_h \frac{|\langle\Omega|j(0)|h(p)\rangle|^2}{p^2-m_h^2}
\end{aligned} \tag{2.13}$$

Singling out the contribution of the hadron with the smallest mass, the correlation function can be written as

$$\Pi(p^2) = \frac{|\langle\Omega|j(0)|h_0(p)\rangle|^2}{m_{h_0^2}-p^2} + \text{higher states} + \text{ continuum} \tag{2.14}$$

where $|h_0(p)\rangle$ is lowest energy state that $j(0)$ can create and continuum stands for contributions from multi hadron states.

From the equation 2.13 it is known that the $\Pi(p^2)$ is analytic function. Now that we have $\Pi(p^2)$ for positive real values of p^2 the analytic continuity [29] can be used to get the $\Pi(p^2)$ for negative p^2 values. To get the $\Pi(p^2)$ for $p^2 < 0$ region the dispersion relation can be deployed. From above calculations it is seen that the correlation function has poles at positive p^2 values because of $\dfrac{1}{m_{h_0}^2-p^2}$, and it has a branch cut at the positive real axis after a threshold due to the continuum states.

To obtain the spectral representation, Cauchy Theorem can be used to write the correlation function as

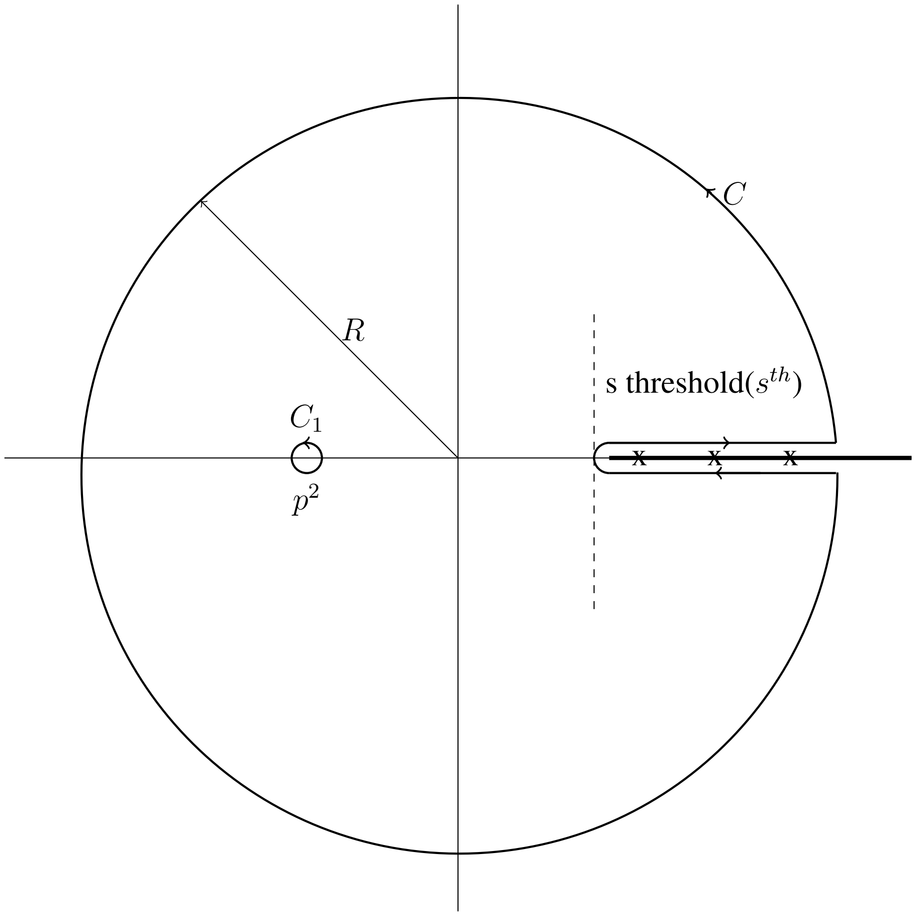

Figure 2.1: Correlation function on the s-plane. In the figure above, the thick line shows the branch cut and crosses show the poles.

$$\Pi(p^2) = \frac{1}{2\pi i} \oint_{C_1} ds \frac{\Pi(s)}{s - p^2} \tag{2.15}$$

The contour can be deformed without passing any branch cut or singularity, hence the above integral will be equal to

$$\Pi(p^2) = \frac{1}{2\pi i} \oint_{C} ds \frac{\Pi(s)}{s - p^2} \tag{2.16}$$

which can be split as

$$\begin{aligned} \Pi(p^2) = \frac{1}{2\pi i} \int_{|s|=R} ds \frac{\Pi(s)}{s - p^2} + \frac{1}{2\pi i} \int_{R-i\epsilon}^{s^{th}-i\epsilon} ds \frac{\Pi(s)}{s - p^2} \\ + \frac{1}{2\pi i} \int_{s^{th}+i\epsilon}^{R+i\epsilon} ds \frac{\Pi(s)}{s - p^2} + \frac{1}{2\pi i} \int_{s^{th}-i\epsilon}^{s^{th}+i\epsilon} ds \frac{\Pi(s)}{s - p^2} \end{aligned} \tag{2.17}$$

Combining the integrals along a line just above the real line, and just below the real line, the integral becomes:

$$\Pi(p^2) = \frac{1}{2\pi i}\int_{|s|=R} ds\frac{\Pi(s)}{s-p^2} + \frac{1}{2\pi i}\int_{s^{th}}^{\infty} ds\frac{\Pi(s+i\epsilon)-\Pi(s-i\epsilon)}{s-p^2} \tag{2.18}$$

According to the Schwarz reflection principle which states if a function $f(x)$ is analytic over a part of the real axis and $x \in \mathbb{R}$ then $f^*(x) = f(x^*)$ [30], in $\epsilon \to 0$ limit we have

$$\Pi(s+i\epsilon) - \Pi(s-i\epsilon) = 2i\,\mathrm{Im}\,\Pi(s) \tag{2.19}$$

and as $R \to \infty$ first term on the right hand side of the equation 2.18 becomes

$$\lim_{R\to\infty}\int_{|s|=R} ds\frac{\Pi(s)}{s-p^2} = \lim_{R\to\infty}\int_{|s|=R} ds\frac{\Pi(s)}{s\left(1-\frac{p^2}{s}\right)} = \int_{|s|=R} ds\frac{\Pi(s)}{s}\sum_{n}^{\infty}\left(\frac{p^2}{s}\right)^n \tag{2.20}$$

In equation 2.20, when $R \to \infty$, for a sufficiently large (but finite) n value

$$\int_{|s|=R} ds\frac{\Pi(s)}{s^n} \to 0 \tag{2.21}$$

hence, the equation 2.20 a polynomial in p^2. By using equations 2.19 and 2.20, equation 2.18 becomes

$$\Pi(p^2) = \frac{1}{\pi}\int_{s^{th}}^{\infty} \frac{\mathrm{Im}\,\Pi(s)}{s-p^2} + \text{polynomials in } p^2 \tag{2.22}$$

Hence, by using equations 2.15 and 2.22 the correlation function becomes

$$\Pi(p^2) = \frac{1}{2\pi i}\oint_{C_1} ds\,\frac{\Pi(s)}{s-p^2}\bigg|_{p^2<0} = \frac{1}{\pi}\int_{s^{th}}^{\infty}\frac{\mathrm{Im}\,\Pi(s)}{s-p^2} + \text{polynomials in } p^2 \tag{2.23}$$

2.1.3 Operator Product Expansion

To calculate the operator product expansion, explicit expressions of the interpolating currents are needed. For the sake of simplicity, an example current for a meson

$j_\mu(x) = q_1^i \gamma_\mu q_2^i$ where i is color index can be used. Hence the correlation function can be written as

$$\Pi_{\mu\nu}(p^2) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \left\{ \overline{q}_2^i(x) \gamma_\mu q_1^i(x) \overline{q}_1^j(0) \gamma_\nu q_2^j(0) \right\} |0\rangle \tag{2.24}$$

To calculate the equation 2.24, Wick's Theorem can be used to express the time ordering. Wick's Theorem states that one can expand the time ordering as a normal ordering and all possible contractions.

$$\begin{aligned} \Pi_{\mu\nu}(p^2) = i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_2^i(x) \gamma_\mu q_1^i(x) \overline{q}_1^j(0) \gamma_\nu q_2^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_2^i(x) \gamma_\mu q_1^i(x) \overline{q}_1^j(0) \gamma_\nu q_2^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_2^i(x) \gamma_\mu q_1^i(x) \overline{q}_1^j(0) \gamma_\nu q_2^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_2^i(x) \gamma_\mu q_1^i(x) \overline{q}_1^j(0) \gamma_\nu q_2^j(0) :\!| 0\rangle \end{aligned} \tag{2.25}$$

where $\langle 0 |\!: \cdots :\!| 0\rangle$ is normal ordering. In the normal ordering, the operators are ordered in a way that the creation operators are on the left and the annihilation operators are on the right side.

$$\langle 0 |\!: a a^\dagger b b^\dagger :\!| 0\rangle = \eta \langle 0 | a^\dagger b^\dagger a b | 0\rangle \tag{2.26}$$

where $\eta = 1$ for bosonic operators and $\eta = -1$ for fermionic operators.

$$\begin{aligned} \Pi_{\mu\nu}(p^2) = i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_{2\alpha}^i(x) \left(\gamma_\mu\right)_{\alpha\beta} q_{1\beta}^i(x) \overline{q}_{1\gamma}^j(0) \left(\gamma_\nu\right)_{\gamma\delta} q_{2\delta}^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_{2\alpha}^i(x) \left(\gamma_\mu\right)_{\alpha\beta} q_{1\beta}^i(x) \overline{q}_{1\gamma}^j(0) \gamma_\nu q_{2\delta}^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_{2\alpha}^i(x) \left(\gamma_\mu\right)_{\alpha\beta} q_{1\beta}^i(x) \overline{q}_{1\gamma}^j(0) \gamma_\nu q_{2\delta}^j(0) :\!| 0\rangle \\ + i &\int d^4x e^{ipx} \langle 0 |\!: \overline{q}_{2\alpha}^i(x) \left(\gamma_\mu\right)_{\alpha\beta} q_{1\beta}^i(x) \overline{q}_{1\gamma}^j(0) \gamma_\nu q_{2\delta}^j(0) :\!| 0\rangle \end{aligned} \tag{2.27}$$

After the contractions, we simply rewrite our expression in terms of components of the matrices and replacing the contractions with the propagators.

$$\Pi_{\mu\nu}(p^2) = -i\int d^4x e^{ipx} S^{q_1 ij}_{\beta\gamma}(x)\,(\gamma_\mu)_{\alpha\beta}\,S^{q_2 ij}_{\delta\alpha}(-x)\,(\gamma_\nu)_{\gamma\delta}$$
$$-\,i\int d^4x e^{ipx}\,(\gamma_\mu)_{\alpha\beta}\,S^{q_2 ij}_{\delta\alpha}(-x)\,(\gamma_\nu)_{\gamma\delta}\,\langle 0|:\overline{q}^i_{1_\gamma}(0) q^j_{1_\beta}(x):|0\rangle$$
$$+\,i\int d^4x e^{ipx}\,(\gamma_\mu)_{\alpha\beta}\,S^{q_1 ij}_{\beta\gamma}(x)\,(\gamma_\nu)_{\gamma\delta}\,\langle 0|:\overline{q}^i_{2_\alpha}(x) q^j_{2_\delta}(0):|0\rangle$$
$$+\,i\int d^4x e^{ipx}\,\langle 0|:\overline{q}^i_{1_\gamma}(0)\,(\gamma_\mu)_{\alpha\beta}\,q^j_{1_\beta}(x)\overline{q}^i_{2_\alpha}(x)\,(\gamma_\nu)_{\gamma\delta}\,q^j_{2_\delta}(0):|0\rangle \tag{2.28}$$

where $S^{qij}_{\alpha\beta}$ is the $\alpha\beta$ component of the quark propagator and i, j are color indices.

The idea is to expand the above expression in terms of non-vanishing vacuum expectation values (condensates). In perturbative theory, when an annihilation operator acts on the vacuum, the result is 0. But, in the non-perturbative region of QCD there are non-vanishing matrix elements of normal ordered products. When an annihilation operator of a light quark acts on the vacuum, it gives non-zero result.

To calculate the matrix element $\langle 0|:\overline{q}^i_{1\beta}(x) q^j_{1\gamma}(0):|0\rangle$ one can expand $\overline{q}(x)$ around 0 and a useful gauge Fock-Schwinger gauge where $x^\eta A_\eta = 0$ can be chosen.

$$\overline{q}(x) = \overline{q}(0) + \overline{q}(0)\overleftarrow{\mathcal{D}}_\eta x^\eta \tag{2.29}$$

$$\langle 0|:\overline{q}^i_{1\beta}(x) q^j_{1\gamma}(0):|0\rangle = \langle 0|\overline{q}^i_{1\beta}(0)\left(1 + \overleftarrow{\mathcal{D}}_\eta x^\eta\right) q^j_{1\gamma}(0)|0\rangle \tag{2.30}$$

The first element of the equation 2.30 can be defined as

$$\langle 0|\overline{q}^i_{1\beta}(0) q^j_{1\gamma}(0)|0\rangle = A\delta^{ij}\delta_{\beta\gamma} \tag{2.31}$$

Since the vacuum is colorless, i and j should be equal, and due to parity and rotational symmetry, equation 2.31 should be proportional to the identity matrix in spinor space. Thus, equation 2.31 is the only possible form of the matrix element $\langle 0|\overline{q}^i_{1\beta}(0) q^j_{1\gamma}(0)|0\rangle$. A can be calculated by multiplying both sides with $\delta^{ij}\delta_{\beta\gamma}$.

$$\delta^{ij}\delta_{\beta\gamma}A\delta^{ij}\delta_{\beta\gamma} = \delta^{ij}\delta_{\beta\gamma}\,\langle 0|\overline{q}^i_{1\beta}(0) q^j_{1\gamma}(0)|0\rangle \tag{2.32}$$

$$4\mathrm{N_C}A = \langle 0|\overline{q}_1 q_1|0\rangle \equiv \langle \overline{q}q\rangle \tag{2.33}$$

In equation 2.33 $\delta^{ij}\delta^{ij} = \mathrm{N_C}$ was used since i and j are color indices and $\mathrm{N_C}$ is number of color and $\delta_{\beta\gamma}\delta_{\beta\gamma} = 4$.

The second element of the equation 2.30 can be written as

$$B\delta^{ij}\left(\gamma_\eta\right)_{\gamma\beta} = \langle 0|\overline{q}^i_{1\beta}(0)\overleftarrow{\mathcal{D}_\eta} q^j_{1\gamma}(0)|0\rangle \tag{2.34}$$

since it has a Lorentz index and two spinor indices. B can be calculated by multiplying both sides with $\left(\gamma^\eta\right)_{\beta\gamma}\delta^{ij}$

$$\left(\gamma^\eta\right)_{\beta\gamma}\delta^{ij}B\delta^{ij}\left(\gamma_\eta\right)_{\gamma\beta} = \left(\gamma^\eta\right)_{\beta\gamma}\delta^{ij}\langle 0|\overline{q}^i_{1\beta}(0)\overleftarrow{\mathcal{D}_\eta} q^j_{1\gamma}(0)|0\rangle \tag{2.35}$$

$$\mathrm{Tr}[\gamma^\eta\gamma_\eta]\mathrm{N_C}B = \langle 0|\overline{q}_1\overleftarrow{\not{\mathcal{D}}}q_1|0\rangle \tag{2.36}$$

With the help of the Dirac equation, $\left(\not{\mathcal{D}} - im\right)\overline{q} = 0$, B becomes

$$B = \frac{im}{16\mathrm{N_C}}\langle\overline{q}q\rangle \tag{2.37}$$

In equation 2.37 $Tr[\gamma^\mu\gamma_\mu] = 16$ was used. By inserting equations 2.33 and 2.37 into 2.30 we have;

$$\langle 0|:\overline{q}^i_{1\beta}(x)q^j_{1\gamma}(0):|0\rangle = \left(\frac{1}{4\mathrm{N_C}}\delta_{\gamma\beta}\delta^{ij} + \frac{im}{16\mathrm{N_C}}\left(\not{x}\right)_{\beta\gamma}\delta^{ij}\right)\langle\overline{q}q\rangle \tag{2.38}$$

List of non-zero condensates up to $d = 6$ is given below [28].

$$\begin{aligned}
\langle O_3\rangle &= \langle\overline{q}q\rangle \\
\langle O_4\rangle &= \langle G^a_{\mu\nu}G^{a\mu\nu}\rangle \\
\langle O_5\rangle &= \langle\overline{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q\rangle \\
\langle O^q_6\rangle &= \langle\left(\overline{q}\Gamma_r q\right)\left(\overline{q}\Gamma_s q\right)\rangle \\
\langle O^G_6\rangle &= \langle f_{abc}G^a_{\mu\nu}G^{b\nu}_\sigma G^{c\sigma\mu}\rangle
\end{aligned} \tag{2.39}$$

After expanding the equation 2.28 and Fourier transforming, the OPE part of the correlation function becomes

$$\Pi^{OPE}(p^2) = \sum_n \frac{C_n(p^2)}{\left(\sqrt{-p^2}\right)^n}\langle O_n\rangle \tag{2.40}$$

In equation 2.40 $C_n(p^2)$ are defined such that for all n they have the same mass dimension. Note that due to the $\frac{1}{\sqrt{-p^2}}$ contributions of higher dimensional operators are suppressed by higher powers of $\frac{1}{\sqrt{-p^2}}$.

2.1.4 Borel Transformation and Continuum Subtraction

Let $\rho^{cont}(s)$ denote the spectral density corresponding to the contributions of the higher states and continuum to the correlation function. Hence, equation 2.14 can be written as

$$\Pi(p^2) = \frac{|\langle\Omega|j(0)|h_0(p)\rangle|^2}{m_{h_0}^2 - p^2} + \int_{s^{th}}^{\infty} ds \frac{\rho^{cont}(s)}{s - p^2} \\ + \text{polynomials in } p^2 \tag{2.41}$$

To eliminate the polynomials in p^2 the Borel transformation can be applied:

$$\Pi(M^2) = \mathcal{B}_{M^2}\Pi(p^2) = \lim_{\substack{-p^2, n\to\infty \\ -p^2/n\to M^2}} \frac{(-p^2)^{n+1}}{n!} \left(\frac{d}{dp^2}\right)^n \Pi(p^2) \tag{2.42}$$

Examples of Borel transformation are present at the Appendix B.

With the Borel transformation, for a very small M^2 any state with a greater mass than the ground state will vanish.After the Borel transform, the correlation function 2.41 becomes

$$\Pi(M^2) = |\langle\Omega|j(0)|h_0(p)\rangle|^2 e^{-m_{h_0}^2/M^2} + \int_{s_0}^{\infty} ds \rho^{cont}(s) e^{-s/M^2} \tag{2.43}$$

At the same time, after applying the Borel transformation to the OPE side, OPE side of the correlation function becomes

$$\Pi^{OPE}(M^2) = \int_{s^{th}}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2} \tag{2.44}$$

Equating the OPE and hadronic representations of the Borel transformed correlation function the following equation is achieved.

$$\int_{s^{th}}^{\infty} ds\rho^{OPE}(s)e^{-s/M^2} = |\langle\Omega|j(0)|h_0(p)\rangle|^2 e^{-m_{h_0}^2/M^2} + \int_{s^{th}}^{\infty} ds\rho^{cont}(s)e^{-s/M^2} \tag{2.45}$$

To cancel the contributions from the higher states and the continuum to spectral density, continuum subtraction can be employed. Using the quark-hadron duality

$$\rho^{cont}(s) = \rho^{OPE}(s)\theta(s - s_0) \tag{2.46}$$

in equation 2.45, the QCD sum rules for the mass of the hadron h_0 can be obtained from equation 2.47

$$|\langle\Omega|j(0)|h_0(p)\rangle|^2 e^{-m_{h_0}^2/M^2} = \int_{s^{th}}^{s_0} ds\rho^{OPE}(s)e^{-s/M^2} \tag{2.47}$$

2.1.5 Magnetic Moments in QCD Sum Rules

In this thesis, the magnetic moments are studied using QCD sum rules. For this problem, the suitable correlation function is in the form:

$$\Pi(p) = i\int d^4x e^{ipx} \langle\gamma(q)|\,\mathcal{T}\left\{\overline{\eta}^{S(A)}(x)\eta_\mu(0)\right\}|0\rangle \tag{2.48}$$

where η_μ is the interpolating current of doubly heavy spin-$3/2$ baryons and $\eta^{S(A)}$ is the interpolating current of symmetric (anti-symmetric) double heavy spin-$1/2$ baryons.

To calculate the hadronic side of such a correlation function, same procedure will be used, inserting full set of hadronic states and deriving the dispersion relation. The explicit expression of the hadronic representation of this correlation function is given in the next section.

In this case the traditional OPE is not suitable.This is because, for large momentum or mass, there is a problem in power counting. The correlation function takes a form [31, 32]

$$F(Q^2) \sim \#\frac{1}{Q^2} + \#\frac{\langle g_s^2 G^2\rangle}{M^4} + \#Q^2\frac{\langle\overline{q}q\overline{q}q\rangle}{M^8} + ... \tag{2.49}$$

As seen in 2.49, higher order condensate terms have coefficients containing increasing Q^2. To overcome the problem, a method of partial summation of the OPE side and organizing the expansion in terms of increasing twists was introduced [33–35]. Twist is defined as the difference between the dimension and the spin of the local operator [28].

After using the Wick's theorem, the matrix element of the form

$$\langle\gamma(q)|\overline{u}(x)\gamma^{\mu}\gamma^{5}u(0)|0\rangle \tag{2.50}$$

will be needed. To calculate the matrix element, the operator is expanded around $x=0$ as in the traditional OPE:

$$\overline{u}(x)\gamma^{\mu}\gamma^{5}u(0)=\sum_{n}\frac{1}{n!}\overline{u}(0)\left(\overleftarrow{D}\cdot x\right)^{n}\gamma^{\mu}\gamma^{5}u(0) \tag{2.51}$$

The matrix element of an arbitrary operator in this sum is,

$$\begin{aligned}\langle\gamma(q)|\overline{u}\mathcal{D}_{[\alpha_1}\mathcal{D}_{\alpha_2}...\mathcal{D}_{\alpha_n]}\gamma^{\mu}\gamma^{5}u|0\rangle &= (-i)^n q_{\alpha_1}q_{\alpha_2}...q_{\alpha_n}q^{\mu}M_{0n}\\ &+(-i)^n g_{[\alpha_1\alpha_2}q_{\alpha_3}...q_{\alpha_n]}q^{\mu}M_{1n}\\ &+(-i)^n g_{\mu[\alpha_1}q_{\alpha_2}...q_{\alpha_n]}M_{2n}\\ &+...\end{aligned} \tag{2.52}$$

In the above expression, ... stands for terms with more than one $g_{\alpha_i\alpha_j}$, [...] are used to denote symmetrization of the indices between the brackets. To obtain the matrix element $\langle\gamma(q)|\overline{u}\gamma^{\mu}\gamma^{5}u|0\rangle$, equation 2.52 should be multiplied by $x_{\alpha_1}x_{\alpha_2}...x_{\alpha_n}$. This product can be simplified by noting that:

- For term with no metric, where all of the lorentz indices are on the momenta $q_{\alpha_1}x^{\alpha_1}q_{\alpha_2}x^{\alpha_2}...q_{\alpha_n}x^{\alpha_n}=(qx)^n$
- For terms that contain a single metric:
  - If free index μ is on a momentum $x^{\alpha_1}x^{\alpha_2}g_{\alpha_1\alpha_2}q_{\alpha_3}...q^{\mu}=x^2q^{\mu}\left(qx\right)^{n-2}$
  - If free index μ is on the metric, terms are in the form $x_{\mu}\left(qx\right)^{n-1}$

- Terms containing more metric terms will introduce additional factors of x^2

Hence:

$$
\begin{aligned}
\langle\gamma(p)|\overline{u}(x)\gamma^\mu\gamma^5 u(0)|0\rangle = &\sum_{n=0}^{\infty} q^\mu(-i)^n(qx)^n\frac{M_{0n}}{n!} \\
&+\sum_{n=2}^{\infty}(-i)^n x^2 q^\mu (qx)^{n-2}\frac{M_{1n}}{n!} \\
&+\sum_{n=2}^{\infty}(-i)^n x_\mu (qx)^{n-1}\frac{M_{2n}}{n!} \\
&+ ...
\end{aligned}
\tag{2.53}
$$

Defining $M_{mr} = \displaystyle\int_0^1 duu^r DA_m(u)$, after simplifications equation 2.53 takes the form

$$
\begin{aligned}
\langle\gamma(q)|\overline{u}(x)\gamma^\mu\gamma^5 u(0)|0\rangle = &\, q^\mu\int_0^1 du e^{iuqx}\left(\varphi(u) - \frac{x^2}{(qx)^2}DA_1(u)\right) \\
&- \frac{x^\mu}{(qx)}\int_0^1 du e^{iuqx}DA_2(u) \\
&+ ...
\end{aligned}
\tag{2.54}
$$

where $\varphi(u)$ sums the contribution from twist-2 while $DA_1(u)$ and $DA_2(u)$ sums contributions from twist-4. It might seem that unknown functions are needed but in this study only the value of such functions for $u = 0.5$ are needed. The matrix elements and distribution amplitudes are studied in detail in [36].

In the calculations, photon distribution amplitudes given in Appendix C will be used.

CHAPTER 3

CALCULATIONS

In this study, the radiative transitions of octet baryons to decuplet baryons will be studied. In order to use QCD sum rules to study hadronic properties, the interpolating currents should be chosen first.

Currents for the baryons consist of a diquark system and a quark. Total spin of the diquark system can be 1 or 0. And with the addition of the single quarks spin we can have spin-1/2 (decuplet) and spin-3/2 (octet) baryons. For $Q = Q'$ there is only symmetric current in exchange of the heavy quarks, but for the case of $Q \neq Q'$ there is also an anti-symmetric current alongside of the symmetric one.

Spin-1/2 doubly heavy baryon with spin-1 diquark have symmetric current [37] :

$$\eta^S(Q,Q^{'},q) = \frac{1}{\sqrt{2}}\varepsilon^{abc}\Big\{\Big(Q^{aT}Cq^b\Big)\gamma_5 Q^{'c} + \Big(Q^{'aT}Cq^b\Big)\gamma_5 Q^c + \beta\Big(Q^{aT}C\gamma_5 q^b\Big)Q^{'C} + \beta\Big(Q^{'aT}C\gamma_5 q^b\Big)Q^c\Big\} \tag{3.1}$$

For spin-0 diquark case, there is anti-symmetric current:

$$\eta^A(Q,Q^{'},q) = \frac{1}{\sqrt{6}}\varepsilon^{abc}\Big\{2\Big(Q^{aT}CQ^{'b}\Big)\gamma_5 q^c + \Big(Q^{aT}Cq^b\Big)\gamma_5 Q^{'c} - \Big(Q^{'aT}Cq^b\Big)\gamma_5 Q^c + 2\beta\Big(Q^{aT}C\gamma_5 Q^{'b}\Big)q^c + \beta\Big(Q^{aT}C\gamma_5 q^b\Big)Q^{'c} - \beta\Big(Q^{'aT}C\gamma_5 q^b\Big)Q^c\Big\} \tag{3.2}$$

where a, b and c are color indices, T is the transpose sign and the β is an auxiliary parameter.

The doubly heavy spin-3/2 baryons have the following current [38]:

$$\eta_\mu = \frac{1}{\sqrt{3}}\varepsilon^{abc}\Big\{\Big(q^{aT}C\gamma_\mu Q^b\Big)Q^{'c} + \Big(q^{aT}C\gamma_\mu Q^{'b}\Big)Q^c + \Big(Q^{aT}C\gamma_\mu Q^{'b}\Big)q^c\Big\} \quad (3.3)$$

Since this current is completely symmetric under the exchange of the flavors, there is no anti-symmetric current.

Correlation function for the transition under study is;

$$\Pi^{S(A)}_{\mu\nu}(p) = (i)^2\int d^4x d^4y e^{ipx}e^{iqy}\langle 0|\,\mathcal{T}\left\{J^{em}(y)_\nu \overline{\eta}^{S(A)}(x)\eta_\mu(0)\right\}|0\rangle \quad (3.4)$$

where $\overline{\eta} = \eta^\dagger\gamma^0$ and $J^{em}(y)_\nu$ is the electromagnetic current that creates the photon. The correlation function can be rewritten in the following form by multiplying the 3.4 with ε^ν.

$$\Pi^{S(A)}_{\mu}(p) = i\int d^4x e^{ipx}\langle\gamma(q)|\,\mathcal{T}\left\{\overline{\eta}^{S(A)}(x)\eta_\mu(0)\right\}|0\rangle \quad (3.5)$$

Using various gamma matrix identities given in the appendix A to obtain the conjugate currents as,

$$\begin{aligned}\overline{\eta}^S =& \frac{1}{\sqrt{2}}\varepsilon^{abc}\Big\{\overline{Q'}^c\gamma_5\Big(\overline{q}^bC\overline{Q}^{aT}\Big) + \overline{Q}^c\gamma_5\Big(\overline{q}^bC\overline{Q'}^{aT}\Big)\\ &+ \beta\overline{Q'}^c\Big(\overline{q}^bC\gamma_5\overline{Q}^{aT}\Big) + \beta\overline{Q}^c\Big(\overline{q}^bC\gamma_5\overline{Q'}^{aT}\Big)\Big\}\end{aligned} \quad (3.6)$$

and

$$\begin{aligned}\overline{\eta}^A =& \frac{1}{\sqrt{6}}\varepsilon^{abc}\Big\{\overline{q}^c\gamma^5\Big(\overline{Q'}^bC\overline{Q}^{aT}\Big) + \overline{Q'}^c\gamma^5\Big(\overline{q}^bC\overline{Q}^{aT}\Big)\\ &- \overline{Q}^c\gamma^5\Big(\overline{q}^bC\overline{Q'}^{aT}\Big) + 2\beta\overline{q}^c\Big(\overline{Q'}^bC\gamma^5\overline{Q}^{aT}\Big)\\ &+ \beta\overline{Q'}^c\Big(\overline{q}^bC\gamma^5\overline{Q}^{aT}\Big) - \beta\overline{Q}^c\Big(\overline{q}^bC\gamma^5\overline{Q'}^{aT}\Big)\Big\}\end{aligned} \quad (3.7)$$

3.1 Hadronic Side

The hadronic side is given as [39]

$$\Pi_\mu(p,q) = \frac{\langle\Omega|\,\overline{J_{\mathcal{O}}}\,\left|\frac{1}{2}\,(p)\right\rangle}{p^2 - m_{\mathcal{O}}^2}\left\langle\frac{1}{2}\middle|\frac{3}{2}\right\rangle_\gamma \frac{\left\langle\frac{3}{2}\,(p+q)\right|J_{\mathcal{D}\mu}\,|\Omega\rangle}{(p+q)^2 - m_{\mathcal{D}}^2} \tag{3.8}$$

where $\left|\frac{1}{2}\right\rangle$ $\left(\left|\frac{3}{2}\right\rangle\right)$ and $m_{\mathcal{O}}(m_{\mathcal{D}})$ denote the octet(decuplet) baryon and its mass and the contribution of higher states and continuum are not explicitly shown. The matrix elements appearing in equation 3.8 are defined as

$$\left\langle\Omega\middle|\overline{J_{\mathcal{O}}}\middle|\frac{1}{2}\,(p,s)\right\rangle = \lambda_{\mathcal{O}}u(p,s) \tag{3.9}$$
$$\left\langle\frac{3}{2},(p+q,s')\middle|J_{\mathcal{D}\mu}\middle|\Omega\right\rangle = \lambda_{\mathcal{D}}u_\mu(p+q,s')$$

where u is Dirac spinor which describes spin-1/2 particle and u_μ is a Rarita-Schwinger spinor which describes a spin-3/2 particle [40] and $s(s')$ is the spin vector of the octet(decuplet) baryon. Summation over spins for the Rarita-Schwinger spinor is given as [37]:

$$\sum_s u_\mu(q,s)\overline{u}_\nu(q,s) = (\not{q}+m)\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{3m^2} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m}\right) \tag{3.10}$$

and the spin sum for Dirac spinors is given as [41]

$$\sum_s u(p,s)\overline{u}(p,s) = \not{p} + m \tag{3.11}$$

The matrix element $\left\langle\frac{1}{2}\middle|\frac{3}{2}\right\rangle_\gamma$ is parametrized in terms of form factors [42, 43] as:

$$\left\langle\frac{1}{2}\middle|\frac{3}{2}\right\rangle_\gamma = eu(p)\{G_1(q_\alpha\not{\varepsilon} - \varepsilon_\alpha\not{q})\gamma^5 \tag{3.12}$$
$$+ G_2((\mathcal{P}\varepsilon)q_\alpha - (\mathcal{P}q)\varepsilon_\alpha)\gamma^5$$
$$+ G_3((q\varepsilon)q_\alpha - q^2\varepsilon_\alpha)\gamma^5\}u^\alpha(p+q)$$

where $\mathcal{P} = \frac{1}{2}(p+(p+q))$ and ε is the polarization vector of the photon. Since the photon is real in our case, $q^2 = 0$ and $\varepsilon q = 0$. Hence G_3 gives no contribution to the process.

Using 3.9, 3.10 and 3.11, equation 3.8 becomes

$$
\begin{aligned}
\Pi_\mu(p,q) = e&\frac{\lambda_{\mathcal{O}}}{p^2-m_{\mathcal{O}}^2}\frac{\lambda_{\mathcal{D}}}{(p+q)^2-m_{\mathcal{D}}^2} \\
&\left\{(-\not{p}+m_{\mathcal{O}})G_1\left[\not{\varepsilon}\gamma_5(\not{q}+m_{\mathcal{D}})\left(q_\mu-\frac{1}{3}\not{q}\gamma_\mu-\frac{1}{3m_{\mathcal{D}}}\not{q}q_\mu\right)\right.\right. \\
&\left.-\not{q}\gamma_5(\not{p}+\not{q}+m_{\mathcal{D}})\left(\varepsilon_\mu-\frac{1}{3}\not{\varepsilon}\gamma_\mu-\frac{1}{3m_{\mathcal{D}}}\not{\varepsilon}q_\mu\right)\right] \\
&+(-\not{p}+m_{\mathcal{O}})G_2\gamma_5(\not{p}+\not{q}+m_{\mathcal{D}})\left[(\mathcal{P}\varepsilon)\left(q_\mu-\frac{1}{3}\not{q}\gamma_\mu-\frac{1}{3m_{\mathcal{D}}}\not{q}q_\mu\right)\right. \\
&\left.\left.-(\mathcal{P}q)\left(\varepsilon_\mu-\frac{1}{3}\not{\varepsilon}\gamma_\mu-\frac{1}{3m_{\mathcal{D}}}\not{\varepsilon}q_\mu\right)\right]\right\}
\end{aligned}
\tag{3.13}
$$

where $q^2 = 0$ and $\varepsilon q = 0$ is used to simplify the final expression. In equation 3.13, there are several structures but not all of them are independent of others. To overcome this problem, an ordering can be used. A suitable ordering is $\not{\varepsilon}\not{q}\not{p}\gamma^\mu$ [39]. After ordering the structures and using $\mathcal{P} = \frac{1}{2}(p+(p+q))$ the correlation function becomes

$$
\begin{aligned}
\Pi_\mu(p,q) =& e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m_{\mathcal{O}}^2}\frac{1}{(p+q)^2-m_{\mathcal{D}}^2}\Bigg\{ \\
&[\varepsilon_\mu(pq)-(\varepsilon p)q_\mu]\left\{-2G_1m_{\mathcal{D}}-G_2m_{\mathcal{D}}m_{\mathcal{O}}+G_2(p+q)^2\right. \\
&\left.+\left[2G_1+G_2\left(m_{\mathcal{O}}-m_{\mathcal{D}}\right)\right]\not{p}+m_{\mathcal{O}}G_2\not{q}-G_2\not{q}\not{p}\right\}\gamma_5 \\
&+\left[q_\mu\not{\varepsilon}-\varepsilon_\mu\not{q}\right]\left\{G_1\left(p^2+m_{\mathcal{D}}m_{\mathcal{O}}\right)-G_1\left(m_{\mathcal{D}}+m_{\mathcal{O}}\right)\not{p}\right\}\gamma_5 \\
&+2G_1[\not{\varepsilon}(pq)-\not{q}(\varepsilon p)]q_\mu\gamma_5 \\
&-G_1\not{\varepsilon}q\{m+\not{p}\}q_\mu\gamma_5 \\
&+\text{ other structures with }\gamma^\mu\text{ at the end} \\
&+\text{structures which are proportional to }(p+q)^\mu\Bigg\}
\end{aligned}
\tag{3.14}
$$

The reason why the structures ending with γ_μ or structures proportional to $(p+q)_\mu$ are not used is that the current for spin-$3/2$ state has a non-zero overlap with spin-$1/2$ state. Since $\eta_\mu^{3/2}$ has one Lorentz index, the overlap can be written as

$$
\left\langle 0\left|\eta_\mu^{3/2}\right|\frac{1}{2}\right\rangle = \left(A\gamma_\mu+Bp_\mu\right)\gamma_5u(p) \tag{3.15}
$$

With the given ordering,since spin-1/2 particles only contribute to the structures with $(p+q)_\mu$ or γ_μ at the end, they do not contribute to other structures [39].

To determine the G_1 and G_2, we need two different structures. A suitable set is $\not{\varepsilon}\not{p}\gamma_5 q_\mu$ and $\not{q}\not{p}\gamma_5(\varepsilon p)q_\mu$. Showing only these structures, the correlation function can be written as:

$$\Pi_\mu(p,q) = e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m_{\mathcal{O}}^2}\frac{1}{(p+q)^2-m_{\mathcal{D}}^2} \left\{-(m_{\mathcal{O}}+m_{\mathcal{D}})G_1\not{\varepsilon}\not{p}\gamma_5 q_\mu + G_2\not{q}\not{p}\gamma_5(\varepsilon p)q_\mu\right\} \tag{3.16}$$

3.2 Operator Product Expansion

Inserting equations 3.6, 3.7 and 3.3 to equation 3.5 the operator product expansion can be calculated. Instead of calculating the Wilson coefficient for different vacuum expectation values, one can insert all the condensate contributions coming from the other normal ordering terms to the propagator and only have fully contracted term with modified propagators. We can show the process as;

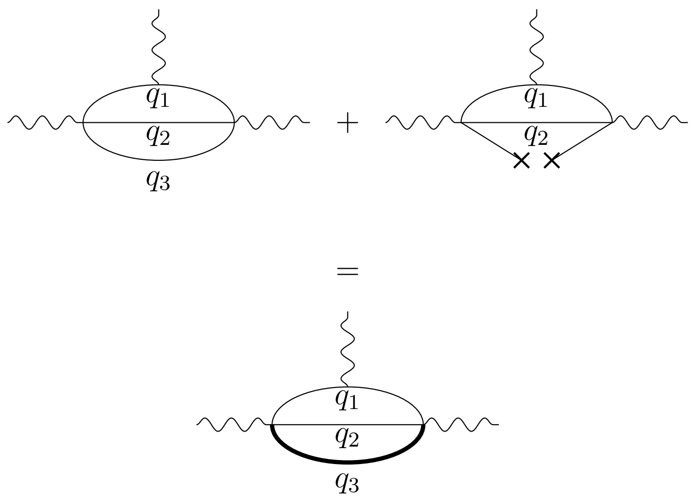

Figure 3.1: Feynman diagram of the process

where the thick line is the full quark propagators and there are three different cases for photon emitted from three different quarks.

As an example, consider the first term of equation 3.6:

$$\overline{Q'}^c\gamma_5\left(\overline{q}^bC\overline{Q}^{aT}\right) = \left(\overline{Q'}^c\right)_\alpha\left(\gamma_5\right)_{\alpha\beta}\left(\left(\overline{q}^b\right)_\gamma\left(C\right)_{\gamma\delta}\left(\overline{Q}^a\right)_\delta\right) \tag{3.17}$$

where transpose sign fell off since elements of matrices are used and summation over matrix indices (α, β etc.) is assumed.

By using the definitions for the currents now we can write the first element of the $\Pi^S(p)$, rest of the calculation for symmetric and anti-symmetric currents are given in Appendix-C:

$$\begin{aligned}\Pi^S(p) =& i\int d^4xe^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\left\langle\gamma(q)\right| \\ &\mathcal{T}\Bigg\{\left(\overline{Q'}^c\right)_\alpha\left(\gamma_5\right)_{\alpha\beta}\left(\left(\overline{q}^b\right)_\gamma\left(C\right)_{\gamma\delta}\left(\overline{Q}^a\right)_\delta\right) \\ &\left(\left(q^{a'}\right)_{\alpha'}\left(C\right)_{\alpha'\beta'}\left(\gamma_\mu\right)_{\beta'\gamma'}\left(Q^{b'}\right)_{\gamma'}\right)\left(Q'^{c'}\right)_{\delta'}\Bigg\}\left|0\right\rangle\end{aligned} \tag{3.18}$$

Using the Wick's theorem the correlation function becomes:

$$\begin{aligned}&\Pi^S(p) = i\int d^4xe^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'} \\ &\left\langle\gamma(q)\right|\left(S_{Q'}^{cc'}(x)\right)_{\delta'\alpha}\left(S_q^{ba'}(x)\right)_{\alpha'\gamma}\left(S_Q^{ab'}(x)\right)_{\gamma'\delta}\left(\gamma_5\right)_{\alpha\beta}\left(C\right)_{\gamma\delta}\left(C\right)_{\alpha'\beta'}\left(\gamma_\mu\right)_{\beta'\gamma'}\left|0\right\rangle \\ &= i\int d^4xe^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'} \\ &\left\langle\gamma(q)\right|\left(S_{Q'}^{cc'}(x)\right)_{\delta'\alpha}\left(\gamma_5\right)_{\alpha\beta} \\ &\left[\left(\gamma_\mu\right)_{\beta'\gamma'}\left(S_Q^{ab'}(x)\right)_{\gamma'\delta}\left(C^T\right)_{\delta\gamma}\left(S_q^{ba'T}(x)\right)_{\gamma\alpha'}\left(C\right)_{\alpha'\beta'}\right]\left|0\right\rangle \\ &= -i\int d^4xe^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\left\langle\gamma(q)\right|S_{Q'}^{cc'}(x)\gamma_5\,\mathrm{Tr}\left[\gamma_\mu S_Q^{ab'}(x)\widetilde{S}_q^{ba'}(x)\right]\left|0\right\rangle \\ &= -i\int d^4xe^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\left\langle\gamma(q)\right|S_{Q'}^{cc'}(x)\gamma_5Tr\left[\gamma_\mu S_Q^{ab'}(x)\widetilde{S}_q^{ba'}(x)\right]\left|0\right\rangle \\ &= i\int d^4xe^{ipx}\frac{6}{\sqrt{6}}\left\langle\gamma(q)\right|S_{Q'}(x)\gamma_5\,\mathrm{Tr}\left[\gamma_\mu S_Q(x)\widetilde{S}_q(x)\right]\left|0\right\rangle\end{aligned} \tag{3.19}$$

The correlation function gets contribution from perturbative and non-perturbative parts. Calculations of these parts are given in the Appendix-C.

Free propagators for light(q) and heavy(Q) quarks are given below.

$$
\begin{aligned}
S_q^{abfree}(x) =& \frac{i\not{x}}{2\pi^2 x^4}\delta^{ab} - \frac{m_q}{4\pi^2 x^2}\delta^{ab} \\
S_Q^{abfree}(x) =& \frac{m_Q^2}{4\pi^2}\frac{K_1\big(m_Q\sqrt{-x^2}\big)}{\sqrt{-x^2}}\delta^{ab} - i\frac{m_Q^2\not{x}}{4\pi^2 x^2}K_2\big(m_Q\sqrt{-x^2}\big)\delta^{ab}
\end{aligned}
\tag{3.20}
$$

where K_i are modified Bessel functions.

Full propagators are given as [39]:

$$
\begin{aligned}
S_Q^{ab}(x) = & S_Q^{abfree}(x) - ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv\Big[\frac{\not{k}+m_Q}{\big(m_Q^2-k^2\big)^2}G_{\mu\nu}^{ab}(vx)\sigma^{\mu\nu} \\
& + \frac{1}{m_Q^2-k^2}vx_\mu G_{\mu\nu}^{ab}\gamma^\nu\Big] \\
S_q^{ab}(x) = & S_q^{abfree}(x) - \frac{m_q}{4\pi^2x^2}\delta^{ab} - \frac{\langle\overline{q}q\rangle}{12}\big(1-i\frac{m_q}{4}\not{x}\big)\delta^{ab} \\
& - \frac{x^2}{192}m_0^2\langle\overline{q}q\rangle\big(1-i\frac{m_q}{6}\not{x}\big)\delta^{ab} \\
& - ig_s\int_0^1 du\Big[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}^{ab}(ux)\sigma^{\mu\nu} - ux_\mu G_{\mu\nu}^{ab}(ux)\gamma^\nu\frac{i}{4\pi^2x^2} \\
& - i\frac{m_q}{32\pi^2}G_{\mu\nu}^{ab}\sigma^{\mu\nu}\Big(\ln\Big(\frac{-x^2\Lambda^2}{4}\Big)+2\gamma_E\Big)\Big]
\end{aligned}
\tag{3.21}
$$

where a, b are color indices, Λ is the scale parameter which separates the long distance and short distance parts of the QCD and has a dimension of mass [44], g_s is coupling constant, γ_E is the Euler's constant, $\langle\overline{q}q\rangle$ is the quark condensate and $G_{\mu\nu}$ is the gluon field strength tensor.

After replacing the propagators for perturbative and non-perturbative parts and writing the expressions for propagators, one can start calculations. A sample calculation is given at Appendix-D.

3.3 Multipole Moments

Taking the Borel transformation of the equation 3.16 and using the results of the OPE part we have

$$
\begin{aligned}
-e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(m_{\mathcal{O}}+m_{\mathcal{D}}\right)G_1 e^{-(m_{\mathcal{O}}^2+m_{\mathcal{D}}^2)/2M^2} &= \Pi_1(M^2) \\
e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}G_2 e^{-(m_{\mathcal{O}}^2+m_{\mathcal{D}}^2)/2M^2} &= \Pi_2(M^2)
\end{aligned}
\tag{3.22}
$$

where $\lambda_{\mathcal{O}}$ $(\lambda_{\mathcal{D}})$ is the residue for the octet(decuplet) baryon and $m_{\mathcal{O}}$ $(m_{\mathcal{D}})$ is the mass of the octet(decuplet) baryon.

Once $\Pi_{1,2}$ are calculated, the form factors G_1 and G_2 can be obtained from the equation 3.22 as:

$$\begin{aligned} G_1 &= -\frac{\Pi_1(M^2)}{e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(m_{\mathcal{O}}+m_{\mathcal{D}}\right)} e^{(m_{\mathcal{O}}^2+m_{\mathcal{D}}^2)/2M^2} \\ G_2 &= \frac{\Pi_2(M^2)}{e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}} e^{(m_{\mathcal{O}}^2+m_{\mathcal{D}}^2)/2M^2} \end{aligned} \tag{3.23}$$

Values of $\dfrac{(m_{\mathcal{O}}^2+m_{\mathcal{D}}^2)}{2}$ can also be determined from the equation 3.22. The natural logarithms of $\Pi_{1,2}$ can be written as:

$$\log\left(\Pi_{1,2}\left(M^2\right)\right) = C_{1,2} - \frac{m_{\mathcal{O}}^2+m_{\mathcal{D}}^2}{2}\frac{1}{M^2} \tag{3.24}$$

where $C_{1,2}$ are independent of M^2 and depends on the values e, $\lambda_{\mathcal{O}}$, $\lambda_{\mathcal{D}}$, G_1, G_2, $m_{\mathcal{O}}$ and $m_{\mathcal{D}}$ which are constants. As can be seen from the equation 3.24, logarithm of the $\Pi_{1,2}$ is a linear function of $\dfrac{1}{M^2}$ with the slope given by $-\dfrac{m_{\mathcal{O}}^2+m_{\mathcal{D}}^2}{2}$.

Residues can be calculated using the mass sum rules for the baryons. Their explicit expressions are given in Appendix-E. Since the sum rules gives the values of λ^2, the sign of λ can not be determined. This ambiguity leads to an ambiguity in the signs of G_1 and G_2. Note that although the overall signs of G_1 and G_2 are not determined, there is no ambiguity in the relative signs of G_1 and G_2.

The relation between the form factors $G_{1,2}$ and the multipole moments is [20]

$$\begin{aligned} G_M &= \left[\frac{3m_{\mathcal{D}}+m_{\mathcal{O}}}{m_{\mathcal{D}}}G_1 + \left(m_{\mathcal{D}}-m_{\mathcal{O}}\right)G_2\right]\frac{m_{\mathcal{O}}}{3} \\ G_E &= \left(m_{\mathcal{D}}-m_{\mathcal{O}}\right)\left[\frac{G_1}{m_{\mathcal{D}}}+G_2\right]\frac{m_{\mathcal{O}}}{3} \end{aligned} \tag{3.25}$$

where G_M is magnetic dipole moment and G_E is electric quadrupole moment. Note that due to the ambiguity in the overall signs of the G_1 and G_2, overall signs of G_M and G_E are also ambiguous. Hence, only absolute values of G_M and G_E will be given.

To be able to use the expressions in equation 3.25 masses of the octet and decuplet baryons are also needed. Except than the Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons, the masses of the double heavy baryons are not known experimentally. The masses predicted by various approaches [37,38,45] and experimentally measured masses [46,47] are given in table 3.1.

Table 3.1: Baryon masses

Baryon	QCDSR [38]	QCDSR [37]	Lattice [45]	Exp [46,47]
Ξ_{bc}	$6.72GeV$	-	$6.78GeV$	-
Ξ'_{bc}	$6.79GeV$	-	$6.84GeV$	-
Ξ_{bb}	$9.96GeV$	-	$10.1GeV$	-
Ξ_{cc}	$3.72GeV$	-	-	$3.52GeV$
Ξ^*_{bc}	-	$7.25GeV$	$6.83GeV$	-
Ξ^*_{cc}	-	$3.69GeV$	-	-
Ξ^*_{bb}	-	$10.4GeV$	$10.1GeV$	-

Since, masses calculated in different works are similar, the masses from [37] and [38] can be chosen.

CHAPTER 4

RESULTS

Expressions for the correlation functions of the transitions are given as

$$
\begin{aligned}
\Pi_{\Xi_{bc}^{+}\to\Xi_{bc}^{*+}} &= \not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(b,c,u) + \not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(b,c,u) \\
\Pi_{\Xi_{bc}^{0}\to\Xi_{bc}^{*0}} &= \not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(b,c,d) + \not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(b,c,d) \\
\Pi_{\Xi_{bc}^{\prime +}\to\Xi_{bc}^{*+}} &= \not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^A(b,c,u) + \not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^A(b,c,u) \\
\Pi_{\Xi_{bc}^{\prime 0}\to\Xi_{bc}^{*0}} &= \not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^A(b,c,d) + \not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^A(b,c,d) \\
\Pi_{\Xi_{cc}^{++}\to\Xi_{cc}^{*++}} &= 2\not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(c,c,u) + 2\not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(c,c,u) \\
\Pi_{\Xi_{cc}^{+}\to\Xi_{cc}^{*+}} &= 2\not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(c,c,d) + 2\not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(c,c,d) \\
\Pi_{\Xi_{bb}^{0}\to\Xi_{bb}^{*0}} &= 2\not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(b,b,u) + 2\not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(b,b,u) \\
\Pi_{\Xi_{bb}^{-}\to\Xi_{bb}^{*-}} &= 2\not{\varepsilon}\not{p}\gamma_5 q_\mu \Pi_1^S(b,b,d) + 2\not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu \Pi_2^S(b,b,d)
\end{aligned}
\tag{4.1}
$$

where $\Pi_{1(2)}^{S(A)}(Q,Q',q)$ is symmetric (antisymmetic) correlation function and the index $1(2)$ denotes the coefficient of the $\not{\varepsilon}\not{p}\gamma_5 q_\mu$ $\left(\not{q}\not{p}\gamma_5\left(\varepsilon p\right) q_\mu\right)$ structure. Explicit expressions for the correlation functions in terms of the distribution amplitudes and condensates are given in Appendix F.

In the numerical calculations, following values of the parameters appearing in the correlation functions are used;

$$\begin{aligned}
\langle \overline{u}u \rangle &= \langle \overline{d}d \rangle = -(0.243)^3 \, GeV^3 \\
\langle \frac{1}{\pi^2} g_s^2 G^2 \rangle &= 0.048 \, GeV^4 \\
m_u &= m_d = 0 \\
m_c &= 1.4 \, GeV \\
m_b &= 4.8 \, GeV \\
f_{3\gamma} &= -0.0039 \, GeV^2 \\
\chi &= 3.15 \, GeV^{-2}
\end{aligned} \tag{4.2}$$

Results should be independent of β since it is not a physical parameter. In subfigures (c) and (d) of the figures G.1-G.16 the β dependencies of G_M and G_E form factors are shown. It is seen that all the form factors are stable with respect to variation of β for $|\beta| > 3$. The parameter s_0 has physical interpretation as the threshold that continuum contributions starts. It is usually determined as $s_0 = (\text{mass of the hadron} + 0.5GeV)^2$. To determine the dependence of the predictions on s_0, it is varied by a few GeV^2 in the literature. Using this approach, s_0 is determined to be $s_0 = 57 \pm 1GeV^2$ for baryons containing bottom and charm quark, $s_0 = 20 \pm 1GeV^2$ for double charmed baryons and $s_0 = 121 \pm 2GeV^2$ for the double bottomed baryons. M^2 is not a physical parameter, hence a range of M^2 should be chosen such that physical predictions are independent of M^2. In the subplots (a) of the figures G.1-G.16, M^2 dependence of the form factors G_M and G_E are shown. As can be seen in the subplots (a) of the figures G.1-G.8, for the bottom-charm baryons, G_M and G_E are independent of M^2 for $6GeV^2 \leqslant M^2 \leqslant 9GeV^2$. For the double charmed baryons, from the figures G.9-G.12 it can be seen G_M and G_E are independent of M^2 for $3GeV^2 \leqslant M^2 \leqslant 6GeV^2$. And finally, for double bottomed baryons, from the figures G.13-G.16 it can be seen G_M and G_E are independent of M^2 for $9GeV^2 \leqslant M^2 \leqslant 12GeV^2$. The stable regions of M^2, β and s_0 are summarized in table 4.1. Within these ranges the predicted values for $|G_M|$ and $|G_E|$ are given in table 4.2 and table 4.3.

In the figures G.1-G.16 the plots are shown for various values of s_0. As seen in these figures, 2% change on the s_0 results in 14% change in the both G_M and G_E

Table 4.1: Stable Regions for M^2, β and s_0

Transition	M^2	β	s_0
$\Xi_{bc}^{+} \to \Xi_{bc}^{*+}$	$6-9GeV^2$	$\|\beta\|>3$	$57\pm1GeV^2$
$\Xi_{bc}^{\prime+} \to \Xi_{bc}^{*+}$	$6-9GeV^2$	$\|\beta\|>3$	$57\pm1GeV^2$
$\Xi_{bc}^{0} \to \Xi_{bc}^{*0}$	$6-9GeV^2$	$\|\beta\|>3$	$57\pm1GeV^2$
$\Xi_{bc}^{\prime0} \to \Xi_{bc}^{*0}$	$6-9GeV^2$	$\|\beta\|>3$	$57\pm1GeV^2$
$\Xi_{cc}^{++} \to \Xi_{cc}^{*++}$	$3-6GeV^2$	$\|\beta\|>3$	$20\pm1GeV^2$
$\Xi_{cc}^{+} \to \Xi_{cc}^{*+}$	$3-6GeV^2$	$\|\beta\|>3$	$20\pm1GeV^2$
$\Xi_{bb}^{0} \to \Xi_{bb}^{*0}$	$9-12GeV^2$	$\|\beta\|>3$	$121\pm2GeV^2$
$\Xi_{bb}^{-} \to \Xi_{bb}^{*-}$	$9-12GeV^2$	$\|\beta\|>3$	$121\pm2GeV^2$

for the transition $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}$, 20% change in the G_M and 17% change in the G_E for the transition $\Xi_{bc}^{\prime+} \to \Xi_{bc}^{*+}$, 15% change in the both G_M and G_E for the transition $\Xi_{bc}^{0} \to \Xi_{bc}^{*0}$, 20% change in the G_M and 18% change in the G_E for the transition $\Xi_{bc}^{\prime0} \to \Xi_{bc}^{*0}$, 17% change in the both G_M and G_E for the transition $\Xi_{cc}^{++} \to \Xi_{cc}^{*++}$, 20% change in the both G_M and G_E for the transition $\Xi_{cc}^{+} \to \Xi_{cc}^{*+}$, 18% change in the both G_M and G_E for the transition $\Xi_{bb}^{0} \to \Xi_{bb}^{*0}$ and finally 17% change in the both G_M and G_E for the transition $\Xi_{bb}^{-} \to \Xi_{bb}^{*-}$.

To analyze the convergence of the twist expansions, in the subplot (b)[1] of the figures G.1-G.16 contributions from different twists and perturbative part are shown. In all the cases twists higher than 2 are negligible. Hence the twist expansion is strongly convergent. For the transitions $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}$, $\Xi_{cc}^{++} \to \Xi_{cc}^{*++}$, $\Xi_{bb}^{-} \to \Xi_{bb}^{*-}$ and $\Xi_{bb}^{0} \to \Xi_{bb}^{*0}$ main contributions come from the twist-2 term, which depends on χ and the electrical charge of the light quark while for the transitions $\Xi_{bc}^{\prime+} \to \Xi_{bc}^{*+}$, $\Xi_{bc}^{0} \to \Xi_{bc}^{*0}$, $\Xi_{bc}^{\prime0} \to \Xi_{bc}^{*0}$ and $\Xi_{cc}^{+} \to \Xi_{cc}^{*++}$ dominant contribution is the perturbative contributions which depend on electrical charges of both light and heavy quark charges for the transitions with symmetric current and electrical charges of the heavy quarks for the transitions with anti-symmetric currents.

The errors in the predictions of the form factors G_M and G_E are determined by

[1] Note that in subplots (b), to be able to show the relative signs of the different contributions, the values are plotted, not their absolute values.

analysing the maximum and minimum values of the form factors within the determined ranges. Results for G_M and G_E are shown in table 4.2 and table 4.3 respectively.

Note that, another error source is the value of the χ, magnetic susceptibility of the quark condensate, which presents in twist-2 contributions as shown in Appendix C. In different works [36, 48–51], the value of χ ranges between $3GeV^2$ and $9GeV^2$ with different signs. Since the definitions used in this study are taken from [36], $\chi = 3.15GeV^{-2}$ [36] is used in this study. The errors shown in tables 4.2 and 4.3 do not contain the errors due to the value of χ.

Table 4.2: Transition magnetic moments in nuclear magneton

Transition	$\lvert G_M \rvert$	QM [16]	χ PT [16]	Bag Model [11]
$\Xi_{bc}^{+} \to \Xi_{bc}^{*+}$	0.400 ± 0.057	-1.61	-2.56	0.695
$\Xi_{bc}^{\prime +} \to \Xi_{bc}^{*+}$	0.045 ± 0.009	-0.36	-0.36	0.672
$\Xi_{bc}^{0} \to \Xi_{bc}^{*0}$	0.238 ± 0.036	1.02	1.03	-0.747
$\Xi_{bc}^{\prime 0} \to \Xi_{bc}^{*0}$	0.048 ± 0.010	-0.36	-0.36	0.070
$\Xi_{cc}^{++} \to \Xi_{cc}^{*++}$	1.089 ± 0.187	-1.4	-2.35	-0.787
$\Xi_{cc}^{+} \to \Xi_{cc}^{*+}$	0.910 ± 0.186	1.23	1.55	0.945
$\Xi_{bb}^{0} \to \Xi_{bb}^{*0}$	0.715 ± 0.130	-1.82	-2.77	-1.039
$\Xi_{bb}^{-} \to \Xi_{bb}^{*-}$	0.333 ± 0.058	0.81	1.13	0.428

Table 4.3: Transition electric quadrupole moments in nuclear magneton

Transition	$\lvert G_E \rvert$
$\Xi_{bc}^{+} \to \Xi_{bc}^{*+}$	$(8.183 \pm 1.146) \times 10^{-3}$
$\Xi_{bc}^{\prime +} \to \Xi_{bc}^{*+}$	$(1.043 \pm 0.184) \times 10^{-3}$
$\Xi_{bc}^{0} \to \Xi_{bc}^{*0}$	$(4.943 \pm 0.751) \times 10^{-3}$
$\Xi_{bc}^{\prime 0} \to \Xi_{bc}^{*0}$	$(1.191 \pm 0.223) \times 10^{-3}$
$\Xi_{cc}^{++} \to \Xi_{cc}^{*++}$	$(2.371 \pm 0.403) \times 10^{-3}$
$\Xi_{cc}^{+} \to \Xi_{cc}^{*+}$	$(2.123 \pm 0.439) \times 10^{-3}$
$\Xi_{bb}^{0} \to \Xi_{bb}^{*0}$	$(8.338 \pm 1.519) \times 10^{-3}$
$\Xi_{bb}^{-} \to \Xi_{bb}^{*-}$	$(3.843 \pm 0.669) \times 10^{-3}$

Once the values of G_M and G_E are determined, the decay width can also be calculated using [39]:

$$\Gamma = 3\frac{\alpha}{32}\frac{\left(m_{\mathcal{D}}^2 - m_{\mathcal{O}}^2\right)^3}{m_{\mathcal{D}}^3 m_{\mathcal{O}}^2}\left(G_M^2 + 3G_E^2\right) \tag{4.3}$$

where α is the fine structure constant with the value $\alpha = \dfrac{1}{137.0}$ [44], and the masses $m_{\mathcal{O}}$ and $m_{\mathcal{D}}$ are the masses shown in table 3.1.

Table 4.4: Decay widths in keV

Transition	Γ	χ PT [16]	Bag Model [11]
$\Xi_{bc}^{*+} \to \Xi_{bc}^{+}$	0.753 ± 0.208	26.2	0.533
$\Xi_{bc}^{*+} \to \Xi_{bc}^{\prime +}$	0.006 ± 0.002	0.52	0.031
$\Xi_{bc}^{*0} \to \Xi_{bc}^{0}$	0.265 ± 0.079	7.19	0.612
$\Xi_{bc}^{*0} \to \Xi_{bc}^{\prime 0}$	0.007 ± 0.003	0.52	0.000
$\Xi_{bb}^{*0} \to \Xi_{bb}^{0}$	0.664 ± 0.234	31.1	0.126
$\Xi_{bb}^{*-} \to \Xi_{bb}^{-}$	0.144 ± 0.049	5.17	0.022

4.1 Conclusion

To conclude, we have calculated the G_M, G_E and Γ values for the radiative transitions of non-strange doubly heavy baryons using the QCD Sum Rules method. Our results for G_M are significantly different from chiral perturbation theory and the quark model, but close to the bag model except for the transitions $\Xi_{bc}^{*+} \to \Xi_{bc}^{\prime +}$ and $\Xi_{bc}^{*0} \to \Xi_{bc}^{0}$, and similarly for the Γ except $\Xi_{bc}^{*+} \to \Xi_{bc}^{\prime +}$, $\Xi_{bc}^{*0} \to \Xi_{bc}^{0}$ and $\Xi_{bb}^{*-} \to \Xi_{bb}^{-}$ transitions. Results for decay widths are given in table 4.4. For the transitions $\Xi_{cc}^{*++} \to \Xi_{cc}^{++}$ and $\Xi_{cc}^{*+} \to \Xi_{cc}^{+}$, Γ are not presented since it is proportional to $\left(m_{\mathcal{O}}^2 - m_{\mathcal{D}}^2\right)^3$ and $m_{\mathcal{O}}$ and $m_{\mathcal{D}}$ for those transitions are nearly equal to each other as shown in the table 3.1.

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Appendix A

IDENTITIES

A.1 Identities for Structures with Gamma Matrices

$$\left(\gamma_0\right)^2 = \mathbb{1} \tag{A.1}$$

$$\left\{\gamma_\mu, \gamma_\nu\right\} = 2g_{\mu\nu} \tag{A.2}$$

$$\left\{\gamma_5, \gamma_\mu\right\} = 0 \tag{A.3}$$

$$\left(\gamma_5\right)^2 = \mathbb{1} \tag{A.4}$$

$$\left(\gamma_5\right)^T = \gamma_5 \tag{A.5}$$

$$C^T = -C \tag{A.6}$$

$$C^2 = -\mathbb{1} \tag{A.7}$$

$$Tr\left[\gamma_\mu\right] = 0 \tag{A.8}$$

$$Tr\left[\gamma_5\right] = 0 \tag{A.9}$$

$$Tr\left[\text{odd \# of gamma matrices}\right] = 0 \tag{A.10}$$

$$C\gamma_\mu C = -\gamma_\mu^T \tag{A.11}$$

$$C\gamma_5 C = -\gamma_5^T = -\gamma_5 \tag{A.12}$$

$$
\begin{aligned}
CS_Q^{abT}(x)\gamma_\mu^T C &= -CS_Q^{abT}(x)CC\gamma_\mu^T C \\
&= -\widetilde{S}_Q^{ab}(x)\big(\gamma_\mu^T\big)^T = -\widetilde{S}_Q^{ab}(x)\gamma_\mu
\end{aligned}
\tag{A.13}
$$

To get the above equation, $-C^2 = \mathbb{1}$ was inserted between the $S_Q^{abT}(x)$ and γ_μ^T and the equation A.11 is used

$$
\begin{aligned}
C\gamma_5 S_Q^{abT}(x)C &= -C\gamma_5 CCS_Q^{abT}(x)C \\
&= \gamma_5\widetilde{S}_Q^{ab}(x)
\end{aligned}
\tag{A.14}
$$

Where $-C^2 = \mathbb{1}$ was inserted and equation A.12 was used.

$$
\begin{aligned}
C\gamma_5 S_Q^{abT}(x)\gamma_\mu^T C &= C\gamma_5 CCS_Q^{abT}(x)CC\gamma_\mu^T C \\
&= -\gamma_5\widetilde{S}_Q^{ab}(x)\gamma_\mu
\end{aligned}
\tag{A.15}
$$

Again $-C^2 = \mathbb{1}$ was inserted and equations A.12 and A.11 were used.

A.2 Other Identities

$$
\begin{aligned}
\epsilon_{abc}\epsilon_{a'b'c'} &= \begin{vmatrix} \delta_{aa'} & \delta_{ab'} & \delta_{ac'} \\ \delta_{ba'} & \delta_{bb'} & \delta_{bc'} \\ \delta_{ca'} & \delta_{cb'} & \delta_{cc'} \end{vmatrix} \\
&= \delta_{aa'}\big(\delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'}\big) \\
&- \delta_{ab'}\big(\delta_{ba'}\delta_{cc'} - \delta_{bc'}\delta_{ca'}\big) \\
&+ \delta_{ac'}\big(\delta_{ba'}\delta_{cb'} - \delta_{bb'}\delta_{ca'}\big)
\end{aligned}
\tag{A.16}
$$

$$
\epsilon_{abc}\epsilon_{ab'c'} = \delta_{bb'}\delta_{cc'} - \delta_{bc'}\delta_{cb'} \tag{A.17}
$$

$$
\epsilon_{abc}\epsilon_{abc'} = 2\delta_{cc'} \tag{A.18}
$$

$$
\epsilon_{abc}\epsilon_{abc} = 6 \tag{A.19}
$$

Appendix B

BOREL TRANSFORMATION

The Borel transformation is defined as

$$\Pi(M^2) = \mathcal{B}_{M^2}\Pi(p^2) = \lim_{\substack{-p^2, n\to\infty \\ -p^2/n\to M^2}} \frac{(-p^2)^{n+1}}{n!}\left(\frac{d}{dp^2}\right)^n \Pi(p^2) \tag{B.1}$$

And Borel transformations of several expressions given as

$$\mathcal{B}\left[\frac{1}{(m^2-p^2)}\right] = e^{-m^2/M^2} \tag{B.2}$$

$$\mathcal{B}\left[e^{-p^2 t}\right] = \delta\left(\frac{1}{M^2} - t\right) \tag{B.3}$$

$$\mathcal{B}\left[\left(p^2\right)^n\right] = 0 \tag{B.4}$$

$$\mathcal{B}\left[\frac{1}{\left(p^2\right)^n}\right] = \frac{(-1)^n}{(n-1)!(M^2)^{n-1}} \tag{B.5}$$

Appendix C

CALCULATION OF THE OPE SIDE

In this study, OPE side was calculated as following using the explicit expressions of the interpolating currents.

$$
\begin{aligned}
\Pi^S(p) =& i\int d^4x e^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\langle\gamma(q)|\mathcal{T}\Bigg\{\Bigg[(\overline{Q'}^c)_\alpha(\gamma_5)_{\alpha\beta}\Big((\overline{q}^b)_\gamma(C)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)\\
&+(\overline{Q}^c)_\alpha(\gamma_5)_{\alpha\beta}\Big((\overline{q}^b)_\gamma(C)_{\gamma\delta}(\overline{Q'}^T)_\delta\Big)\\
&+\beta(\overline{Q'}^c)_\alpha\Big((\overline{q}^b)_\beta(C)_{\beta\gamma}(\gamma_5)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)\\
&+\beta(\overline{Q}^c)_\alpha\Big((\overline{q}^b)_\beta(C)_{\beta\gamma}(\gamma_5)_{\gamma\delta}(\overline{Q'}^a)_\delta\Big)\Bigg]\Big(x\Big)\\
&\Bigg[\Big((q^{a'})_{\alpha'}(C)_{\alpha'\beta'}(\gamma_\mu)_{\beta'\gamma'}(Q^{b'})_{\gamma'}\Big)(Q'^{c'})_{\delta'}\\
&+\Big((q^{a'})_{\alpha'}(C)_{\alpha'\beta'}(\gamma_\mu)_{\beta'\gamma'}(Q'^{b'})_{\gamma'}\Big)(Q^{c'})_{\delta'}\\
&+\Big((Q^{a'})_{\alpha'}(C)_{\alpha'\beta'}(\gamma_\mu)_{\beta'\gamma'}(Q'^{b'})_{\gamma'}\Big)(q^{c'})_{\delta'}\Bigg]\Big(0\Big)\Bigg\}|0\rangle
\end{aligned} \tag{C.1}
$$

$$
\begin{aligned}
\Pi^A(p) =& i\int d^4x e^{ipx}\frac{1}{3\sqrt{2}}\varepsilon^{abc}\varepsilon^{a'b'c'}\langle\gamma(q)|\mathcal{T}\Bigg\{\\
&\Bigg[2(\overline{q}^c)_\alpha(\gamma_5)_{\alpha\beta}\Big((\overline{Q'}^b)_\gamma(C)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)\\
&+(\overline{Q'}^c)_\alpha(\gamma_5)_{\alpha\beta}\Big((\overline{q}^b)_\gamma(C)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)\\
&-(\overline{Q}^c)_\alpha(\gamma_5)_{\alpha\beta}\Big((\overline{q}^b)_\gamma(C)_{\gamma\delta}(\overline{Q'}^a)_\delta\Big)\\
&+2\beta(\overline{q}^c)_\alpha\Big((\overline{Q'}^b)_\beta(C)_{\beta\gamma}(\gamma_5)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)\\
&+\beta(\overline{Q'}^c)_\alpha\Big((\overline{q}^b)_\beta(C)_{\beta\gamma}(\gamma_5)_{\gamma\delta}(\overline{Q}^a)_\delta\Big)
\end{aligned} \tag{C.2}
$$

$$
\begin{aligned}
&- \beta\big(\overline{Q}^c\big)_\alpha\Big(\big(\overline{q}^b\big)_\beta\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(\overline{Q'}^a\big)_\delta\Big)\Big]\Big(x\Big) \\
&\Big[\Big(\big(q^{a'}\big)_{\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(Q^{b'}\big)_{\gamma'}\Big)\big(Q'^{c'}\big)_{\delta'} \\
&+\Big(\big(q^{a'}\big)_{\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(Q'^{b'}\big)_{\gamma'}\Big)\big(Q^{c'}\big)_{\delta'} \\
&+\Big(\big(Q^{a'}\big)_{\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(Q'^{b'}\big)_{\gamma'}\Big)\big(q^{c'}\big)_{\delta'}\Big]\Big(0\Big)\Big\}|0\rangle
\end{aligned}
$$

Using the Wick theorem and contracting the quark fields the correlation function becomes:

$$
\begin{aligned}
\Pi^S(p) =& i\int d^4x e^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\,\langle\gamma(q)|\Big\{ \\
&\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+\beta\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&\Big\}|0\rangle
\end{aligned} \tag{C.3}
$$

$$
\begin{aligned}
\Pi^A(p) = i&\int d^4x e^{ipx}\frac{1}{3\sqrt{2}}\varepsilon^{abc}\varepsilon^{a'b'c'}\,\langle\gamma(q)|\Big\{ \\
&2\big(S_{Q'}^{bc'}(x)\big)_{\delta'\gamma}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+2\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\gamma}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \\
&+2\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\gamma}\big(S_q^{cc'}(x)\big)_{\delta'\alpha}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}
\end{aligned}
$$

$$
\begin{aligned}
&+\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}\big(C\big)_{\gamma\delta}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+2\beta\big(S_{Q'}^{bc'}(x)\big)_{\delta'\beta}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'} \qquad \text{(C.4)}\\
&+2\beta\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\beta}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+2\beta\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\beta}\big(S_q^{cc'}(x)\big)_{\delta'\alpha}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+\beta\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+\beta\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&+\beta\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(S_Q^{aa'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\beta\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\beta\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&-\beta\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\gamma\delta}\big(C\big)_{\beta\gamma}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\\
&\Big\}\,|0\rangle
\end{aligned}
$$

Since the elements of the expression are matrix elements instead of matrices, their order can be changed freely. By changing the order and using $A_{\alpha\beta} = A^T_{\beta\alpha}$ identity the correlation function becomes:

$$
\begin{aligned}
&\Pi^S(p) = i\int d^4x e^{ipx}\frac{1}{\sqrt{6}}\varepsilon^{abc}\varepsilon^{a'b'c'}\,\langle\gamma(q)|\,\Big\{\\
&\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\Big[\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\Big]\\
&+\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}\\
&+\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(C\big)_{\gamma\delta}\big(S_Q^{aa'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}\\
&+\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}\\
&+\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\Big[\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta}\Big]\\
&+\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(C\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu^T\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta} \qquad \text{(C.5)}
\end{aligned}
$$

$$+\beta\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\Big[\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{ab'}(x)\big)_{\gamma'\delta}\big(\gamma_5^T\big)_{\delta\gamma}\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'T}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\Big]$$
$$+\beta\big(S_{Q'}^{cc'}(x)\big)_{\delta'\delta}\big(\gamma_5^T\big)_{\delta\gamma}\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'T}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}$$
$$+\beta\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_Q^{aa'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}$$
$$+\beta\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5^T\big)_{\delta\gamma}\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'T}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}$$
$$+\beta\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\Big[\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'T}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{ab'}(x)\big)_{\gamma'\delta'}\big(\gamma_5^T\big)_{\delta\gamma}\Big]$$
$$+\beta\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu^T\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}$$
$$\Big\}\,|0\rangle$$

$$\Pi^A(p)=i\int d^4x e^{ipx}\frac{1}{3\sqrt{2}}\varepsilon^{abc}\varepsilon^{a'b'c'}\,\langle\gamma(q)|\Big\{$$
$$2\big(S_{Q'}^{bc'}(x)\big)_{\delta'\gamma}\big(C\big)_{\gamma\delta}\big(S_Q^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu^T\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}$$
$$+2\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(C^T\big)_{\delta\gamma}S_{Q'}^{bb'T}(x)\big)_{\gamma\gamma'}\big(\gamma_\mu^T\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}$$
$$+2\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}\Big[\big(C\big)_{\gamma\delta}\big(S_Q^{aa'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\gamma}\Big]$$
$$+\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\Big[\big(C\big)_{\gamma\delta}\big(S_Q^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu^T\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\Big]$$
$$+\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma\alpha'}\big(\gamma_5\big)_{\alpha\beta}$$
$$+\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(C\big)_{\gamma\delta}\big(S_Q^{aa'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}$$
$$-S_{Q'}^{ac'}(x)\big(C^T\big)_{\delta\gamma}\big(S_q^{ba'T}(x)\big)_{\gamma\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}\big(\gamma_5\big)_{\alpha\beta}$$
$$-\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\big(\gamma_5\big)_{\alpha\beta}\Big[\big(C\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ba'}(x)\big)_{\alpha'\gamma}\Big] \tag{C.6}$$
$$-\big(S_q^{bc'}(x)\big)_{\delta'\gamma}\big(C\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}\big(\gamma_5\big)_{\alpha\beta}$$
$$+2\beta\big(S_{Q'}^{bc'}(x)\big)_{\delta'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_Q^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}$$
$$+2\beta\big(S_Q^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\delta\gamma}\big(C^T\big)_{\delta\gamma}\big(S_{Q'}^{bb'T}(x)\big)_{\beta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ca'}(x)\big)_{\alpha'\alpha}$$
$$+2\beta\big(S_q^{cc'}(x)\big)_{\delta'\alpha}\Big[\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{bb'}(x)\big)_{\gamma'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_Q^{ab'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\Big]$$
$$+\beta\big(S_{Q'}^{cc'}(x)\big)_{\delta'\alpha}\Big[\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_Q^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}\Big]$$
$$+\beta\big(S_Q^{ac'}(x)\big)_{\alpha'\delta}\big(\gamma_5\big)_{\delta\gamma}\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}$$
$$+\beta\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_Q^{aa'T}(x)\big)_{\delta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_{Q'}^{cb'}(x)\big)_{\gamma'\alpha}$$
$$-\beta\big(S_{Q'}^{ac'}(x)\big)_{\delta'\delta}\big(\gamma_5\big)_{\delta\gamma}\big(C^T\big)_{\gamma\beta}\big(S_q^{ba'T}(x)\big)_{\beta\alpha'}\big(C\big)_{\alpha'\beta'}\big(\gamma_\mu\big)_{\beta'\gamma'}\big(S_Q^{cb'}(x)\big)_{\gamma'\alpha}$$
$$-\beta\big(S_Q^{cc'}(x)\big)_{\delta'\alpha}\Big[\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_q^{ba'}(x)\big)_{\alpha'\beta}$$
$$-\beta\big(S_q^{bc'}(x)\big)_{\delta'\beta}\big(C\big)_{\beta\gamma}\big(\gamma_5\big)_{\gamma\delta}\big(S_{Q'}^{ab'T}(x)\big)_{\delta\gamma'}\big(\gamma_\mu\big)_{\gamma'\beta'}\big(C^T\big)_{\beta'\alpha'}\big(S_Q^{ca'}(x)\big)_{\alpha'\alpha}$$

$$\Big\} |0\rangle$$

Using the identity

$$A_{\alpha\beta}B_{\beta\gamma}C_{\gamma\alpha} = Tr\big[ABC\big]$$

and gamma matrix identities and replacing $CS^TC = \tilde{S}$ the correlation function becomes:

$$\begin{aligned}
\Pi^S(p) = i \int d^4x e^{ipx} \frac{1}{\sqrt{6}} \varepsilon^{abc}\varepsilon^{a'b'c'} \langle\gamma(q)| \Big\{ \\
&- S_{Q'}^{cc'}(x)\gamma_5 \,\mathrm{Tr}\left[\gamma_\mu S_Q^{ab'}(x)\widetilde{S}_q^{ba'}(x)\right] \\
&- S_Q^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&+ S_q^{bc'}(x)\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&- S_{Q'}^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x)\gamma_5 \\
&- S_Q^{cc'}(x)\gamma_5 \,\mathrm{Tr}\left[\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{ab'}(x)\right] \\
&- S_q^{bc'}(x)CS_{Q'}^{ab'T}(x)\gamma_\mu^T CS_Q^{ca'}(x)\gamma_5 \\
&- \beta S_{Q'}^{cc'}(x)\,\mathrm{Tr}\left[\gamma_\mu S_Q^{ab'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\right] \\
&- \beta S_Q^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x) \\
&+ \beta S_q^{bc'}(x)C\gamma_5 S_Q^{aa'T}(x)C\gamma_\mu S_{Q'}^{cb'}(x) \\
&- \beta S_{Q'}^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x) \\
&- \beta S_Q^{cc'}(x)\,\mathrm{Tr}\left[\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{ab'}(x)\gamma_5\right] \\
&- \beta S_q^{bc'}(x)C\gamma_5 S_{Q'}^{ab'T}(x)\gamma_\mu^T CS_Q^{ca'}(x)\Big\} |0\rangle
\end{aligned} \tag{C.7}$$

With the help of A.14, A.13 and A.15 the above equations become:

$$\begin{aligned}
\Pi^S(p) = i \int d^4x e^{ipx} \frac{1}{\sqrt{6}} \varepsilon^{abc}\varepsilon^{a'b'c'} \langle\gamma(q)| \Big\{ \\
&- S_{Q'}^{cc'}(x)\gamma_5 Tr\Big[\gamma_\mu S_Q^{ab'}(x)\widetilde{S}_q^{ba'}(x)\Big] \\
&- S_Q^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&+ S_q^{bc'}(x)\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&- S_{Q'}^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x)\gamma_5 \\
&- S_Q^{cc'}(x)\gamma_5 Tr\Big[\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{ab'}(x)\Big]
\end{aligned}$$

$$
\begin{aligned}
&+ S_q^{bc'}(x)\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_Q^{ca'}(x)\gamma_5 \\
&- \beta S_{Q'}^{cc'}(x)Tr\Big[\gamma_\mu S_Q^{ab'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\Big] \\
&- \beta S_Q^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x) \\
&+ \beta S_q^{bc'}(x)\gamma_5\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{cb'}(x) \\
&- \beta S_{Q'}^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x) \\
&- \beta S_Q^{cc'}(x)Tr\Big[\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{ab'}(x)\gamma_5\Big] \\
&- \beta S_q^{bc'}(x)\gamma_5\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_Q^{ca'}(x)\Big\}\,|0\rangle
\end{aligned}
\tag{C.8}
$$

$$
\begin{aligned}
&\Pi^A(p) = i\int d^4x e^{ipx}\frac{1}{3\sqrt{2}}\varepsilon^{abc}\varepsilon^{a'b'c'}\,\langle\gamma(q)|\Big\{ \\
&\quad 2S_{Q'}^{bc'}(x)\widetilde{S}_Q^{ab'}(x)\gamma_\mu S_q^{ca'}(x)\gamma_5 \\
&-2S_Q^{ac'}(x)\widetilde{S}_{Q'}^{bb'}(x)\gamma_\mu S_q^{ca'}(x)\gamma_5 \\
&+2S_q^{cc'}(x)\gamma_5\,\mathrm{Tr}\Big[\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{bb'}(x)\Big] \\
&+S_{Q'}^{cc'}(x)\gamma_5\,\mathrm{Tr}\Big[\widetilde{S}_Q^{ab'}(x)\gamma_\mu S_q^{ba'}(x)\Big] \\
&-S_Q^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&+S_q^{bc'}(x)\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{cb'}(x)\gamma_5 \\
&+S_{Q'}^{ac'}(x)\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x)\gamma_5 \\
&-S_Q^{cc'}(x)\gamma_5\,\mathrm{Tr}\Big[\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_q^{ba'}(x)\Big] \\
&-S_q^{bc'}(x)\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_Q^{ca'}(x)\gamma_5 \\
&+2\beta S_{Q'}^{bc'}(x)\gamma_5\widetilde{S}_Q^{ab'}(x)\gamma_\mu S_q^{ca'}(x) \\
&+2\beta S_Q^{ac'}(x)\gamma_5\widetilde{S}_{Q'}^{bb'}(x)\gamma_\mu S_q^{ca'}(x) \\
&+2\beta S_q^{cc'}(x)\,\mathrm{Tr}\Big[\gamma_\mu S_{Q'}^{bb'}(x)\gamma_5\widetilde{S}_Q^{aa'}(x)\Big] \\
&+\beta S_{Q'}^{cc'}(x)\,\mathrm{Tr}\Big[\gamma_5\widetilde{S}_Q^{ab'}(x)\gamma_\mu S_q^{ba'}(x)\Big] \\
&-\beta S_Q^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_{Q'}^{cb'}(x) \\
&+\beta S_q^{bc'}(x)\gamma_5\widetilde{S}_Q^{aa'}(x)\gamma_\mu S_{Q'}^{cb'}(x) \\
&+\beta S_{Q'}^{ac'}(x)\gamma_5\widetilde{S}_q^{ba'}(x)\gamma_\mu S_Q^{cb'}(x) \\
&-\beta S_Q^{cc'}(x)\,\mathrm{Tr}\Big[\gamma_5\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_q^{ba'}(x)\Big] \\
&-\beta S_q^{bc'}(x)\gamma_5\widetilde{S}_{Q'}^{ab'}(x)\gamma_\mu S_Q^{ca'}(x)\Big\}\,|0\rangle
\end{aligned}
\tag{C.9}
$$

With the help of A.16 and rewriting $S_Q^{ab}(x) = \delta^{ab} S_Q(x)$ the correlation function becomes:

$$
\begin{aligned}
\Pi^S(p) = i \int d^4x e^{ipx} \frac{6}{\sqrt{6}} \langle \gamma(q)| \Big\{ & \\
& S_{Q'}(x)\gamma_5 \,\mathrm{Tr}\left[\gamma_\mu S_Q(x)\widetilde{S}_q(x)\right] \\
& - S_Q(x)\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& - S_q(x)\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& - S_{Q'}(x)\widetilde{S}_q(x)\gamma_\mu S_Q(x)\gamma_5 \\
& + S_Q(x)\gamma_5 \,\mathrm{Tr}\left[\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x)\right] \\
& + S_q(x)\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\gamma_5 \\
& + \beta S_{Q'}(x) \,\mathrm{Tr}\left[\gamma_\mu S_Q(x)\gamma_5\widetilde{S}_q(x)\right] \\
& - \beta S_Q(x)\gamma_5\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x) \\
& - \beta S_q(x)\gamma_5\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x) \\
& - \beta S_{Q'}(x)\gamma_5\widetilde{S}_q(x)\gamma_\mu S_Q(x) \\
& + \beta S_Q(x) \,\mathrm{Tr}\left[\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x)\gamma_5\right] \\
& + \beta S_q(x)\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\Big\} |0\rangle
\end{aligned}
\tag{C.10}
$$

$$
\begin{aligned}
\Pi^A(p) = i \int d^4x e^{ipx} \frac{6}{3\sqrt{2}} \langle \gamma(q)| \Big\{ & \\
& 2S_{Q'}(x)\widetilde{S}_Q(x)\gamma_\mu S_q(x)\gamma_5 \\
& +2S_Q(x)\widetilde{S}_{Q'}(x)\gamma_\mu S_q(x)\gamma_5 \\
& +2S_q(x)\gamma_5 \,\mathrm{Tr}\left[\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\right] \\
& -S_{Q'}(x)\gamma_5 \,\mathrm{Tr}\left[S_Q(x)\gamma_\mu S_q(x)\right] \\
& -S_Q(x)\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& -S_q(x)\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& +S_{Q'}(x)\widetilde{S}_q(x)\gamma_\mu S_Q(x)\gamma_5 \\
& +S_Q(x)\gamma_5 \,\mathrm{Tr}\left[\widetilde{S}_{Q'}(x)\gamma_\mu S_q(x)\right] \\
& -S_q(x)\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\gamma_5 \\
& +2\beta S_{Q'}(x)\gamma_5\widetilde{S}_Q(x)\gamma_\mu S_q(x)
\end{aligned}
\tag{C.11}
$$

$$
\begin{aligned}
&-2\beta S_Q(x)\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_q(x)\\
&+2\beta S_q(x)\,\mathrm{Tr}\left[\gamma_\mu S_{Q'}(x)\gamma_5\widetilde{S}_Q(x)\right]\\
&-\beta S_{Q'}(x)\,\mathrm{Tr}\left[\gamma_5\widetilde{S}_q(x)\gamma_\mu S_Q(x)\right]\\
&-\beta S_Q(x)\gamma_5\widetilde{S}_q(x)\gamma_\mu S_{Q'}(x)\\
&-\beta S_q(x)\gamma_5\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\\
&+\beta S_{Q'}(x)\gamma_5\widetilde{S}_q(x)\gamma_\mu S_Q(x)\\
&+\beta S_Q(x)\,\mathrm{Tr}\left[\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_q(x)\right]\\
&-\beta S_q(x)\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\Big\}\left|0\right\rangle
\end{aligned}
$$

The correlation function gets contributions from perturbative and non-perturbative parts. To get the perturbative effects, one needs to replace the propagator of the quark that emits the photon by

$$
S^{ij}_{\alpha\beta} \longrightarrow \left\{ \int d^4y S^{free}(x-y)\not{A} S^{free}(y) \right\}^{ij}_{\alpha\beta} \tag{C.12}
$$

To get the non-perturbative contributions, the light quark propagator should be replaced as [39];

$$
\left\langle\gamma(q)\right| S_q \left|0\right\rangle \longrightarrow -\Gamma_j \frac{1}{4} \left\langle\gamma(q)\right| \overline{q}\Gamma_j q \left|0\right\rangle \tag{C.13}
$$

where $\Gamma_j = \left\{\mathbf{1}, \gamma_\alpha, \gamma_5, i\gamma_5\gamma_\alpha, \frac{1}{\sqrt{2}}\sigma_{\alpha\beta}\right\}$. Full set of $\left\langle\gamma(q)\right| \overline{q}(x)\Gamma_j q(0)\left|0\right\rangle$ is given as [36]:

$$
\begin{aligned}
&\left\langle\gamma(q)\right| \overline{q}(x)\sigma_{\mu\nu}q(0)\left|0\right\rangle\\
&= -ie_q\left\langle\overline{q}q\right\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 du e^{i\overline{u}qx}\left(\chi\varphi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right)\\
&- \frac{i}{2(qx)}\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du e^{i\overline{u}qx}h_\gamma(u)
\end{aligned}
$$

$$
\left\langle\gamma(q)\right| \overline{q}(x)\gamma_\mu q(0)\left|0\right\rangle = e_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 du e^{i\overline{u}qx}\psi^v(u)
$$

$$
\left\langle\gamma(q)\right| \overline{q}(x)\gamma_\mu\gamma_5 q(0)\left|0\right\rangle = -\frac{1}{4}e_q f_{3\gamma}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^\nu q^\alpha x^\beta\int_0^1 du e^{i\overline{u}qx}\psi^a(u)
$$

$$
\begin{aligned}
&\langle\gamma(q)|\,\overline{q}(x)g_sG_{\mu\nu}(vx)q(0)\,|0\rangle\\
&=-ie_q\,\langle\overline{q}q\rangle\,(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{S}\,(\alpha_i)
\end{aligned}
$$

$$
\begin{aligned}
&\langle\gamma(q)|\,\overline{q}(x)g_s\tilde{G}_{\mu\nu}(vx)i\gamma_5q(0)\,|0\rangle\\
&=-ie_q\,\langle\overline{q}q\rangle\,(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\tilde{\mathcal{S}}\,(\alpha_i)
\end{aligned}
\tag{C.14}
$$

$$
\begin{aligned}
&\langle\gamma(q)|\,\overline{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5q(0)\,|0\rangle\\
&=e_qf_{3\gamma}\,(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{A}\,(\alpha_i)
\end{aligned}
\tag{C.15}
$$

$$
\begin{aligned}
&\langle\gamma(q)|\,\overline{q}(x)g_sG_{\mu\nu}(vx)i\gamma_\alpha q(0)\,|0\rangle\\
&=e_qf_{3\gamma}\,(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{V}\,(\alpha_i)
\end{aligned}
\tag{C.16}
$$

$$
\begin{aligned}
&\langle\gamma(q)|\,\overline{q}(x)\sigma_{\alpha\beta}g_sG_{\mu\nu}(vx)q(0)\,|0\rangle=e_q\,\langle\overline{q}q\rangle\Bigg\{\\
&\Bigg[\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\nu}-\frac{1}{qx}(q_\alpha x_\nu+q_\nu x_\alpha)\right)q_\beta\\
&-\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\nu}-\frac{1}{qx}(q_\beta x_\nu+q_\nu x_\beta)\right)q_\alpha\\
&-\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\left(g_{\alpha\mu}-\frac{1}{qx}(q_\alpha x_\mu+q_\mu x_\alpha)\right)q_\beta\\
&+\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)\left(g_{\beta\mu}-\frac{1}{qx}(q_\beta x_\mu+q_\mu x_\beta)\right)q_\alpha\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1\,(\alpha_i)\\
&\Bigg[\left(\varepsilon_\alpha-q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\mu\beta}-\frac{1}{qx}(q_\mu x_\beta+q_\beta x_\mu)\right)q_\nu\\
&-\left(\varepsilon_\alpha-q_\alpha\frac{\varepsilon x}{qx}\right)\left(g_{\nu\beta}-\frac{1}{qx}(q_\nu x_\beta+q_\beta x_\nu)\right)q_\mu\\
&-\left(\varepsilon_\beta-q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\mu\alpha}-\frac{1}{qx}(q_\mu x_\alpha+q_\alpha x_\mu)\right)q_\nu\\
&+\left(\varepsilon_\beta-q_\beta\frac{\varepsilon x}{qx}\right)\left(g_{\nu\alpha}-\frac{1}{qx}(q_\nu x_\alpha+q_\alpha x_\nu)\right)q_\mu\Bigg]\int\mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_1\,(\alpha_i)
\end{aligned}
$$

$$\frac{1}{qx}\left(q_\mu x_\nu - q_\nu x_\mu\right)\left(\varepsilon \alpha q_\beta - \varepsilon_\beta q_\alpha\right)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_3\left(\alpha_i\right)$$
$$\left.\frac{1}{qx}\left(q_\alpha x_\beta - q_\beta x_\alpha\right)\left(\varepsilon \mu q_\nu - \varepsilon_\nu q_\mu\right)\int \mathcal{D}\alpha_i e^{i(\alpha_{\bar{q}}+v\alpha_g)qx}\mathcal{T}_4\left(\alpha_i\right)\right\} \tag{C.17}$$

In above expressions, χ is the magnetic susceptibility of the quarks, $\varphi_\gamma(u)$ is the leading twist-2 distribution amplitude, $\psi^v(u)$, $\psi^a(u)$, $\mathcal{A}$ and $\mathcal{V}$ are twist-3 distribution amplitudes and $h_\gamma(u)$, $\mathbb{A}$, $\mathcal{T}_i$ are twist-4 photon distribution amplitudes.

The non-perturbative contributions to the correlator is given as:

$$\begin{aligned}
\Pi^S_{np}(p) = i\int d^4x e^{ipx}\frac{-1}{2\sqrt{6}}\Big\{ \\
& S_{Q'}(x)\gamma_5 Tr\big[\gamma_\mu S_Q(x)\widetilde{\Gamma}_j\big] \\
& - S_Q(x)\widetilde{\Gamma}_j\gamma_\mu S_{Q'}(x)\gamma_5 \\
& - \Gamma_j\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& - S_{Q'}(x)\widetilde{\Gamma}_j\gamma_\mu S_Q(x)\gamma_5 \\
& + S_Q(x)\gamma_5 Tr\big[\widetilde{\Gamma}_j\gamma_\mu S_{Q'}(x)\big] \\
& + \Gamma_j\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\gamma_5 \\
& + \beta S_{Q'}(x) Tr\big[\gamma_\mu S_Q(x)\gamma_5\widetilde{\Gamma}_j\big] \\
& - \beta S_Q(x)\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x) \\
& - \beta\Gamma_j\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x) \\
& - \beta S_{Q'}(x)\gamma_5\widetilde{\Gamma}_j\gamma_\mu S_Q(x) \\
& + \beta S_Q(x) Tr\big[\widetilde{\Gamma}_j\gamma_\mu S_{Q'}(x)\gamma_5\big] \\
& + \beta\Gamma_j\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\Big\}
\end{aligned} \tag{C.18}$$

where $\widetilde{\Gamma}_j = C\Gamma_j^T C$

Same procedure holds for the η_A as well.

$$
\begin{aligned}
\Pi^A(p) = i \int d^4x e^{ipx} \frac{-1}{6\sqrt{2}} \Big\{ & \\
& 2S_{Q'}(x)\widetilde{S}_Q(x)\gamma_\mu\Gamma_j\gamma_5 \\
& +2S_Q(x)\widetilde{S}_{Q'}(x)\gamma_\mu\Gamma_j\gamma_5 \\
& +2\Gamma_j\gamma_5 \, \mathrm{Tr}\left[\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\right] \\
& -S_{Q'}(x)\gamma_5 \, \mathrm{Tr}\left[S_Q(x)\gamma_\mu\Gamma_j\right] \\
& -S_Q(x)\widetilde{\Gamma}_j\gamma_\mu S_{Q'}(x)\gamma_5 \\
& -\Gamma_j\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x)\gamma_5 \\
& +S_{Q'}(x)\widetilde{\Gamma}_j\gamma_\mu S_Q(x)\gamma_5 \\
& +S_Q(x)\gamma_5 \, \mathrm{Tr}\left[\widetilde{S}_{Q'}(x)\gamma_\mu\Gamma_j\right] \\
& -\Gamma_j\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\gamma_5 \\
& +2\beta S_{Q'}(x)\gamma_5\widetilde{S}_Q(x)\gamma_\mu\Gamma_j \\
& -2\beta S_Q(x)\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu\Gamma_j \\
& +2\beta\Gamma_j \, \mathrm{Tr}\left[\gamma_\mu S_{Q'}(x)\gamma_5\widetilde{S}_Q(x)\right] \\
& -\beta S_{Q'}(x) \, \mathrm{Tr}\left[\gamma_5\widetilde{\Gamma}_j\gamma_\mu S_Q(x)\right] \\
& -\beta S_Q(x)\gamma_5\widetilde{\Gamma}_j\gamma_\mu S_{Q'}(x) \\
& -\beta\Gamma_j\gamma_5\widetilde{S}_Q(x)\gamma_\mu S_{Q'}(x) \\
& +\beta S_{Q'}(x)\gamma_5\widetilde{\Gamma}_j\gamma_\mu S_Q(x) \\
& +\beta S_Q(x) \, \mathrm{Tr}\left[\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu\Gamma_j\right] \\
& -\beta\Gamma_j\gamma_5\widetilde{S}_{Q'}(x)\gamma_\mu S_Q(x)\Big\} \left\langle\gamma(q)\right| \overline{q}(x)\Gamma_j q(0) \left|0\right\rangle \left|0\right\rangle
\end{aligned}
\tag{C.19}
$$

Appendix D

SAMPLE CALCULATION

In the calculations, there are terms in the form of:

$$\Pi = i\int d^4x \frac{K_{n_1}(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^{n_1}} \frac{K_{n_2}(m_{Q'}\sqrt{-x^2})}{(\sqrt{-x^2})^{n_2}} \frac{1}{(x^2)^{n_3}} e^{ipx}\int_0^1 du e^{iuqx}\mathrm{DA}(u) \quad (D.1)$$

The integral representation of the modified Bessel functions can be used as:

$$\frac{K_n(m_Q\sqrt{-x^2})}{(\sqrt{-x^2})^n} = \frac{2^{n-1}}{m_Q^n}\int_0^\infty dt t^{n-1} e^{tx^2-\frac{m_Q^2}{4t}} \quad (D.2)$$

And for $\frac{1}{(x^2)^n}$:

$$\frac{1}{(x^2)^n} = \frac{(-1)^n}{\Gamma(n)}\int_0^\infty dt t^{n-1} e^{tx^2} \quad (D.3)$$

can be used.

After replacements, Π becomes:

$$\Pi = i\frac{(-1)^{n_3}}{\Gamma(n)}\int d^4x dt_1 dt_2 dt_3 e^{ipx} \frac{2^{n_1-1}}{m_Q^{n_1}} \frac{2^{n_2-1}}{m_{Q'}^{n_2}} t_1^{n_1-1} t_2^{n_2-1} t_3^{n_3-1} \quad (D.4)$$
$$\times e^{(t_1+t_2+t_3)x^2} e^{-m_Q^2/4t_1 - \frac{m_{Q'}^2}{4t_2}} \int du e^{iuqx}\mathrm{DA}(u)$$

Let us define $P \equiv p + uq$ and $C = \frac{(-1)^{n_3}}{\Gamma(n)} \frac{2^{n_1-1}}{m_Q^{n_1}} \frac{2^{n_2-1}}{m_{Q'}^{n_2}}$. With the new definitions, Π becomes:

$$\Pi = iC\int d^4x du dt_1 dt_2 dt_3 e^{iPx} e^{(t_1+t_2+t_3)x^2} e^{-m_Q^2/4t_1 - \frac{m_{Q'}^2}{4t_2}} \mathrm{DA}(u) \quad (D.5)$$

With a Wick's rotation, $x^0 \to -ix^0$ and $P^0 \to iP^0$, the term becomes:

$$\Pi = C \int d^4x du dt_1 dt_2 dt_3 e^{-\tau\left(x^2 - \frac{iPx}{\tau}\right)} e^{-m_Q^2/4t_1 - m_{Q'}^2/4t_2} \mathrm{DA}(u) \tag{D.6}$$

where $\tau = t_1 + t_2 + t_3$. Now the x integral can be calculated using the expression below:

$$\int d^n x e^{-\alpha x^2} = \left(\sqrt{\frac{\pi}{\alpha}}\right)^n \tag{D.7}$$

To do this, first the exponential depends on x as should be rewritten as a square.

$$x^2 - \frac{iPx}{\tau} = \left(x - \frac{iP}{2\tau}\right)^2 + \frac{P^2}{4\tau^2} \tag{D.8}$$

Defining $y = x - \dfrac{iP}{2\tau}$, $dx = dy$, hence

$$\int d^4x e^{-\tau\left(x^2 - \frac{iPx}{\tau}\right)} = \int d^4y e^{-\tau y^2} e^{-P^2/4\tau} = \frac{\pi^2}{\tau^2} e^{-P^2/4\tau} \tag{D.9}$$

After taking the x integral, Π becomes:

$$\Pi = C\pi^2 \int du dt_1 dt_2 dt_3 e^{-1/4\left(\frac{P^2}{\tau} + \frac{m_Q}{t_1} + \frac{m_{Q'}}{t_2}\right)} \frac{t_1^{n_1-1} t_2^{n_2-1} t_3^{n_3-1}}{\tau^2} \mathrm{DA}(u) \tag{D.10}$$

Let us make the transformations as; $t_1 = tx$, $t_2 = ty$ and $t_3 = t(1 - x - y)$. Here τ becomes $\tau = t_1 + t_2 + t_3 = t$. With the transformation $dt_1 dt_2 dt_3$ becomes $dt_1 dt_2 dt_3 \to J dt dx dy$. The Jacobian of this transform can be calculated as;

$$J = \begin{vmatrix} \frac{\partial t_1}{\partial t} & \frac{\partial t_1}{\partial x} & \frac{\partial t_1}{\partial y} \\ \frac{\partial t_2}{\partial t} & \frac{\partial t_2}{\partial x} & \frac{\partial t_2}{\partial y} \\ \frac{\partial t_3}{\partial t} & \frac{\partial t_3}{\partial x} & \frac{\partial t_3}{\partial y} \end{vmatrix} = \begin{vmatrix} x & t & 0 \\ y & 0 & t \\ 1 - x - y & -t & -t \end{vmatrix} = t^2 \tag{D.11}$$

$$\Pi = C\pi^2 \int_0^1 du \int_0^\infty dt \int_0^1 dx \int_0^{1-x} dy t^{n_1+n_2+n_3-3} x^{n_1-1} y^{n_2-2} (1 - x - y)^{n_3-1} \times e^{-1/4t\left(P^2 + \frac{m_Q}{x} + \frac{m_{Q'}}{y}\right)} \mathrm{DA}(u) \tag{D.12}$$

Further simplification can be made by denoting $\alpha = P^2 + \frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y}$ and $t' = \dfrac{\alpha}{4t}$. With this replacement, $dt = -\dfrac{dt'}{4t'^2}$ and $t^{\sum n_i - 3} = \dfrac{\alpha^{\sum n_i - 3}}{4^{\sum n_i - 3} t'^{\sum n_i - 3}}$

$$\Pi = -\frac{C}{4^{\sum n_i - 2}} \pi^2 \int dudxdydt' x^{n_1} y^{n_2} (1-x-y)^{n_3} \frac{\alpha^{\sum n_i - 2}}{t'^{\sum n_i - 1}} e^{-t'} \mathrm{DA}(u) \quad \text{(D.13)}$$

Now the integral representation of the Gamma function can be used

$$\int dt' t'^{1-\sum n_i} e^{-t'} = \Gamma\left(2 - \sum n_i\right) \quad \text{(D.14)}$$

$$\Pi = -\frac{C}{4^{\sum n_i - 2}} \pi^2 \int dudxdy \frac{x^{n_1} y^{n_2} (1-x-y)^{n_3}}{\alpha^{2-\sum n_i}} \Gamma\left(2 - \sum n_i\right) \mathrm{DA}(u) \quad \text{(D.15)}$$

Here one more replacement for the α can be used as:

$$\frac{1}{\alpha^{2-\sum n_i}} = \frac{1}{\Gamma\left(2 - \sum n_i\right)} \int dt t^{1-\sum n_i} e^{-\alpha t} \quad \text{(D.16)}$$

After the replacement Π becomes;

$$\Pi = -\frac{C}{4^{\sum n_i - 2}} \pi^2 \int dudxdydt x^{n_1} y^{n_2} (1-x-y)^{n_3} t^{1-\sum n_i} e^{-\alpha t} \mathrm{DA}(u) \quad \text{(D.17)}$$

Since $q^2 = 0$, $P^2 = (p+uq)^2 = p^2 \overline{u} + (p+q)^2 u$. Hence $\alpha = p^2 \overline{u} + (p+q)^2 u + \dfrac{m_Q^2}{x} + \dfrac{m_{Q'}^2}{y}$

Now, the Borel transform can be performed as:

$$\mathcal{B}\left(e^{-p^2 ut}\right) = \delta\left(\sigma - ut\right) \quad \text{(D.18)}$$

where $\sigma \equiv \dfrac{1}{M^2}$ is the Borel parameter.

Detailed formula for a Borel transformation is given in Appendix-B. After the Borel transformation on p^2 and $(p+q)^2$, Π becomes:

$$\Pi = -\frac{C}{4^{\sum n_i - 2}} \pi^2 \int dudxdydt x^{n_1} y^{n_2} (1-x-y)^{n_3} t^{1-\sum n_i} e^{-t\left(\frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y}\right)} \quad \text{(D.19)}$$
$$\times \delta\left(\sigma_1 - \overline{u} t\right) \delta\left(\sigma_2 - ut\right) \mathrm{DA}(u)$$

The product of delta functions can be rewritten as

$$\delta\left(\sigma_1 - \overline{u}t\right)\delta\left(\sigma_2 - ut\right) = \delta\left(\sigma_1 + \sigma_2 - t\right)\frac{1}{\sigma_1 + \sigma_2}\delta\left(\frac{\sigma_2}{\sigma_1 + \sigma_2} - u\right) \tag{D.20}$$

Now using the definitions $\sigma_2 = u_0\sigma$ and $\sigma_1 = (1 - u_0)\sigma$, $\sigma_1 + \sigma_2 = \sigma$. This makes, $\delta\left(\frac{\sigma_2}{\sigma_1 + \sigma_2} - u\right) = \delta(u_0 - u)$.

t and u integrals can be calculated with the help of delta functions:

$$\Pi = -\frac{C\pi^2 \text{DA}(u_0)}{4^{\sum n_i - 2}} \int dxdydt x^{n_1} y^{n_2} (1 - x - y)^{n_3} \sigma^{-\sum n_i} e^{-\sigma\left(\frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y}\right)} \tag{D.21}$$

Denoting $\sigma = \frac{1}{M^2}$ Π becomes

$$\Pi = -\frac{C\pi^2 \text{DA}(u_0)}{4^{\sum n_i - 2}} \int dxdy x^{n_1} y^{n_2} (1 - x - y)^{n_3} M^{2\sum n_i} e^{-1/M^2\left(\frac{m_Q^2}{x} + \frac{m_{Q'}^2}{y}\right)} \tag{D.22}$$

Now, s can be introduced to have an expression for the spectral density:

$$\Pi = -\frac{C\pi^2 \text{DA}(u_0) M^{2\sum n_i}}{4^{\sum n_i - 2}} \int_0^\infty ds e^{-s/M^2} \tag{D.23}$$
$$\times \int dxdy x^{n_1} y^{n_2} (1 - x - y)^{n_3} \delta\left(s - \frac{m_Q^2}{x} - \frac{m_{Q'}^2}{y}\right)$$

Writing the expression for C and denoting the spectral density:

$$\rho(s) = \int dxdy x^{n_1} y^{n_2} (1 - x - y)^{n_3} \delta\left(s - \frac{m_Q^2}{x} - \frac{m_{Q'}^2}{y}\right) \tag{D.24}$$

and making continuum subtraction as having s_0 as the upper bound of the s integral, Π becomes

$$\Pi(M^2) = -\frac{(-1)^{n_3}}{\Gamma(n)} \frac{2^{n_1 + n_2 - 2}\pi^2 \text{DA}(u_0)}{m_Q^{n_1} m_{Q'}^{n_2} 2^{2\sum n_i - 4}} \int_0^{s_0} ds e^{-s/M^2} \rho(s) \tag{D.25}$$

In the calculations, one may have expressions with M^2 outside of the exponential. To get the spectral representation, the spectral representation $\rho(s)$ should be independent of the M^2. For the positive M^2 powers one may use the derivative of the spectral density $\rho(s)$, $\tilde{\rho(s)}$ and use the integration by parts.

$$I_n = \int_0^\infty dse^{-s/M^2} \left(M^2\right)^n \tilde{\rho}(s) = \int_0^\infty dse^{-s/M^2} \left(M^2\right)^n \frac{d^n}{ds^n}\rho(s) \tag{D.26}$$

Applying integration by parts

$$I_n = e^{-s/M^2} \left(M^2\right)^n \frac{d^{n-1}}{ds^{n-1}}\rho(s)\Big|_{s=0}^{\infty} + \int dse^{-s/M^2} \left(M^2\right)^{n-1} \frac{d^{n-1}}{ds^{n-1}}\rho(s) \tag{D.27}$$

For $s = \infty$, $e^{-s/M^2} = 0$ and for $s = 0$, $\dfrac{d^m}{ds^m}\rho(s) = 0$ since $\rho(s) = 0$ for $s < s^{th}$. Therefore we have

$$I_n = \int_0^\infty dse^{-s/M^2} \left(M^2\right)^n \frac{d^n}{ds^n}\rho(s) = \int e^{-s/M^2} \left(M^2\right)^{n-1} \frac{d^{n-1}}{ds^{n-1}}\rho(s) \tag{D.28}$$

After applying the integration by parts n times, I_n becomes

$$\begin{aligned} I_n &= \int_0^\infty dse^{-s/M^2} \left(M^2\right)^n \frac{d^n}{ds^n}\rho(s) \\ &= \frac{1}{\Gamma(n)} \int_0^\infty dse^{-s/M^2} \int_0^s d\alpha (s-\alpha)^{n-1}\widetilde{\rho}(s) \end{aligned}$$

where $\widetilde{\rho}(s) = \left(\dfrac{d}{ds}\right)^n \rho(s)$ For the negative powers of M^2, one can express the $(M^2)^{-n}$ as $\left(-\dfrac{d}{ds}\right)^n e^{-s/M^2}$

$$I_n = \int_0^\infty dse^{-s/M^2} \left(M^2\right)^{-n} \rho(s) = \int_0^\infty ds \left(-\frac{d}{ds}\right)^n e^{-s/M^2}\rho(s) \tag{D.29}$$

Doing integration by parts, I_n becomes

$$I_n = -\frac{d^{n-1}}{ds^{n-1}}e^{-s/M^2}\rho(s)\Big|_0^\infty + \int_0^\infty ds\frac{d^{n-1}}{ds^{n-1}}e^{-s/M^2}\frac{d}{ds}\rho(s) \tag{D.30}$$

Where, again the boundary terms vanish as in the positive M^2 case. Applying the integration by parts n times we get

$$I_n = \int_0^\infty dse^{-s/M^2} \frac{d^n}{ds^n}\rho(s) \tag{D.31}$$

Appendix E

RESIDUES OF OCTET AND DECUPLET BARYONS

In the calculations, residues of the octet and decuplet baryons are needed.

The residues for the decuplet baryons are given in Table E.1 [37].

Table E.1: Residues of Decuplet Baryons

Baryon	Residue (GeV^3)
Ξ^*_{bc}	0.15
Ξ^*_{bb}	0.22
Ξ^*_{cc}	0.12

The values for the parameters M^2 and s_0 that appear in the sum rules and are used in [38] are shown in Table E.2. Woth the values shown in table E.2, the residues calculated in [38] can be written as:

$$
\begin{aligned}
&\lambda_{\mathcal{O}}\left(\Xi_{bc}\right) = 16.8 \times 10^{-3} GeV^3 \\
&\sqrt{18.7\left(\beta^2-1\right) + 6.88(\beta-1)^2 + 10.6(\beta(5\beta+2)+5)} \\
&\lambda_{\mathcal{O}}\left(\Xi'_{bc}\right) = 17.9 \times 10^{-3} GeV^3 \\
&\sqrt{2.1(\beta-1)(5\beta+1) + 2.3(\beta-1)(11\beta+13) + 10.7(\beta(5\beta+2)+5)} \\
&\lambda_{\mathcal{O}}\left(\Xi_{bb}\right) = 90.9 \times 10^{-3} GeV^3 \\
&\sqrt{7.2\left(\beta^2-1\right) + 4.1(\beta-1)^2 + 5.3(\beta(5\beta+2)+5)} \\
&\lambda_{\mathcal{O}}\left(\Xi_{cc}\right) = 4 \times 10^{-2} GeV^3 \\
&\sqrt{2.4\left(\beta^2-1\right) + 0.8(\beta-1)^2 + 1.5(\beta(5\beta+2)+5)}
\end{aligned} \tag{E.1}
$$

Selected M^2 and $\sqrt{s_0}$ values in the [38] are shown in the table E.2

Table E.2: Parameters for the residue calculation

Baryon	M^2	$\sqrt{s_0}$
Ξ_{bc}	8	7.5
Ξ'_{bc}	8	7.5
Ξ_{bb}	11	10.9
Ξ_{cc}	5	4.6

Appendix F

EXPRESSIONS

The expressions for the correlation functions before the continuum subtractions in this study are given as:

$$
\begin{aligned}
\Pi_1^S(Q,Q',q) =& \int_0^\infty ds e^{-s/M^2}\rho_1^{S1}(s) \\
&- f_1^{S1}(\mathcal{A}(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_2^{S1}(s) \\
&+ f_2^{S1}(\mathcal{A}(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_2^{S1}(s) \\
&+ f_3^{S1}(\widetilde{\mathcal{S}}(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_3^{S1}(s) \\
&+ f_3^{S1}(\mathcal{T}_2(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_4^{S1}(s) \\
&- f_3^{S1}(\mathcal{T}_4(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_5^{S1}(s) \\
&+ f_3^{S1}(\mathcal{T}_3(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_5^{S1}(s) \\
&- f_4^{S1}(\mathcal{T}_3(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_6^{S1}(s) \\
&+ f_4^{S1}(\widetilde{\mathcal{S}}(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_7^{S1}(s) \\
&- f_4^{S1}(\mathcal{T}_4(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_8^{S1}(s) \\
&+ f_4^{S1}(\mathcal{S}(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_9^{S1}(s) \\
&+ f_4^{S1}(\mathcal{T}_1(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_{10}^{S1}(s) \\
&+ f_4^{S1}(\mathcal{T}_2(\alpha_i))\int_0^\infty ds e^{-s/M^2}\rho_{11}^{S1}(s) \\
&+ \int_0^\infty ds e^{-s/M^2}\rho_{12}^{S1}(s) \\
&+ \int_0^\infty ds e^{-s/M^2}\rho_{13}^{S1}(s)
\end{aligned}
\tag{F.1}
$$

where

$$
\begin{aligned}
f_1^{S1}(DA(\alpha_i)) &= \int d\alpha_i \frac{DA(\alpha_i)}{\alpha_g^3}\Bigg\{ \\
&\quad \alpha_g \delta\Big(\frac{u_0-\alpha_{\overline{q}}}{\alpha_g}\Big)\theta(\alpha_g+\alpha_{\overline{q}}-u_0) \\
&\quad + \theta(u_0-\alpha_{\overline{q}})\Big[(\alpha_g+\alpha_{\overline{q}}-u_0)\delta\Big(\frac{\alpha_g+\alpha_{\overline{q}}-u_0}{\alpha_g}\Big) \\
&\quad + \alpha_g\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\Big]\Bigg\} \\
f_2^{S1}(DA(\alpha_i)) &= \int d\alpha_i \frac{DA(\alpha_i)}{\alpha_g^3}\Bigg\{ \qquad\qquad \text{(F.2)} \\
&\quad (\alpha_g+\alpha_{\overline{q}}-u_0)\delta\Big(\frac{u_0-\alpha_{\overline{q}}}{\alpha_g}\Big)\theta(\alpha_g+\alpha_{\overline{q}}-u_0) \\
&\quad + \alpha_g\delta\Big(\frac{\alpha_g+\alpha_{\overline{q}}-u_0}{\alpha_g}\Big)\theta(u_0-\alpha_{\overline{q}})\Bigg\} \\
f_3^{S1}(DA(\alpha_i)) &= \int d\alpha_i \frac{(\alpha_g+\alpha_{\overline{q}}-u_0)DA(\alpha_i)\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\theta(u_0-\alpha_{\overline{q}})}{\alpha_g^2} \\
f_4^{S1}(DA(\alpha_i)) &= \int d\alpha_i \frac{DA(\alpha_i)\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\theta(u_0-\alpha_{\overline{q}})}{\alpha_g}
\end{aligned}
$$

$$
\begin{aligned}
\rho_1^{S1}(s) &= \int_0^1 du\overline{u_0}h_\gamma(u)\theta(u-u_0) \\
&\quad \int_0^1 dx\frac{(1+\beta)e_u M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{8\sqrt{6}\pi^2}(m_{Q'}\overline{x}+m_Q x) \\
\rho_2^{S1}(s) &= \frac{(\beta-1)e_q f_{3\gamma}M^4\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{24\sqrt{6}\pi^2} \\
\rho_3^{S1}(s) &= \int_0^1 dx\frac{e_q M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}\beta(m_Q+m_{Q'}) \\
\rho_4^{S1}(s) &= \int_0^1 dx\frac{e_q M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}\frac{(\beta+1)(m_Q+m_{Q'})}{2} \\
\rho_5^{S1}(s) &= \int_0^1 dx\frac{e_q M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}\frac{(3\beta+1)(m_Q+m_{Q'})}{2} \\
\rho_6^{S1}(s) &= \int_0^1 dx\frac{e_q M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}\frac{3\beta+1}{4}\left(\frac{m_Q}{\overline{x}}+\frac{m_{Q'}}{\overline{x}}\right)
\end{aligned}
$$

$$\rho_7^{S1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q} q \rangle \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}$$
$$\left(\frac{m_Q(\beta(x-2)-x)}{4\overline{x}} + \frac{m_{Q'}(\beta(\overline{x}-2)-\overline{x})}{4x}\right)$$
$$\rho_8^{S1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q} q \rangle \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}$$
$$\left(\frac{m_Q(\beta(3\overline{x}+1)+(\overline{x}+1))}{4\overline{x}} + \frac{m_{Q'}(\beta(3x+1)+(x+1))}{4x}\right)$$
$$\rho_9^{S1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q} q \rangle \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}$$
$$\left(\frac{m_Q(\beta(3x-2)+x)}{4\overline{x}} + \frac{m_{Q'}(\beta(3\overline{x}-2)+\overline{x})}{4x}\right)$$
$$\rho_{10}^{S1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q} q \rangle \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2}$$
$$\left(\frac{m_Q(\beta(4x-3)+(4x+1))}{4\overline{x}} + \frac{m_{Q'}(\beta(4\overline{x}-3)+(4\overline{x}+1))}{4x}\right)$$
$$\rho_{11}^{S1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q} q \rangle \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{12\sqrt{6}\pi^2} \tag{F.3}$$
$$\left(\frac{m_Q(\beta(5x-4)+(3x-2))}{4\overline{x}} + \frac{m_{Q'}(\beta(5\overline{x}-4)+(3\overline{x}-2))}{4x}\right)$$
$$\rho_{12}^{S1}(s) = \int_0^1 dx \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right) \Bigg\{ \frac{-3(\beta-1)e_q \overline{x} x M^6}{16\sqrt{6}\pi^4}$$
$$+ \frac{e_q M^4}{16\sqrt{6}\pi^2}$$
$$\Big(2(3\beta+1)\langle \overline{q} q \rangle (m_{Q'}\overline{x} + m_Q x)\chi\varphi_\gamma(u_0)$$
$$+ (\beta-1) f_{3\gamma}\overline{x} x\Big(6\psi^a(u_0) - (u_0-1)\Big(4\psi^v(u_0) - \psi^{a\prime}(u_0)\Big)\Big)\Big)$$
$$- \frac{M^2}{768\sqrt{6} m_Q m_{Q'}\pi^4 \overline{x} x}$$
$$\Bigg\{ \left\langle g_s^2 G^2 \right\rangle \Bigg[e_Q m_Q x\left(2\beta m_Q x + (\beta-1) m_{Q'}\overline{x}(x + \overline{x} u_0)\right)$$
$$+ e_{Q'} m_{Q'}\overline{x}\left(2\beta m_{Q'}\overline{x} + (\beta-1) m_Q x(\overline{x} + x u_0)\right)$$

$$- 2e_q x\overline{x}\left(\beta\left(2m_Q^2(2x-3)+2m_{Q'}^2(2\overline{x}-3)\right)+(\beta-1)m_Q m_{Q'}(1+u_0)\right)\Bigg]$$

$$+ 48\pi^2 e_q m_Q m_{Q'} x\overline{x}\Bigg[$$

$$- 2f_{3\gamma}\psi^a(u_0)\left(x\overline{x}(\beta-1)\left(\frac{m_{Q'}^2}{\overline{x}}+\frac{m_Q^2}{x}\right)+2\beta m_Q m_{Q'}\right)$$

$$+ (3\beta+1)\left\langle\overline{q}q\right\rangle\mathbb{A}(u_0)(m_{Q'}\overline{x}+m_Q x)\Bigg]\Bigg\}$$

$$+ \frac{e_q\left(\frac{m_Q^2}{\overline{x}}+\frac{m_{Q'}^2}{x}\right)}{384\sqrt{6}m_Q m_{Q'}\pi^4}$$

$$\Bigg(\left\langle g_s^2G^2\right\rangle\left(2\beta x m_Q^2+(\beta-1)m_{Q'}m_Q+2\beta m_{Q'}^2\overline{x}\right)$$

$$- 12(3\beta+1)m_Q m_{Q'}\pi^2\left\langle\overline{q}q\right\rangle(\overline{x}m_{Q'}+m_Q x)\mathbb{A}(u_0)\Bigg)\Bigg\}$$

$$\rho_{13}^{S1}(s) = \int_0^1 dx\int_0^{1-x}dy\frac{(\beta-1)\delta\left(s-\frac{m_Q^2}{x}-\frac{m_{Q'}^2}{y}\right)u_0\overline{u_0}}{384\sqrt{6}\pi^4}\Bigg\{$$

$$\left\langle g_s^2G^2\right\rangle(e_Q+e_{Q'})+72M^4\Bigg[2e_q xy-(1-x-y)(e_{Q'}x+e_Q y)\Bigg]\Bigg\}$$

$$
\begin{aligned}
\Pi_1^A(Q,Q',q) &= \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_1^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_4(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_2^{A1}(s) \\
&- f_4^{A1}(\widetilde{\mathcal{S}}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_3^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_2(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_4^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_3(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_2^{A1}(s) \\
&- f_2^{A1}(\mathcal{V}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_5^{A1}(s) \\
&+ f_1^{A1}(\mathcal{V}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_5^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_3(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_6^{A1}(s) \\
&- f_4^{A1}(\mathcal{T}_1(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_7^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_2(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_8^{A1}(s) \\
&+ f_4^{A1}(\mathcal{T}_4(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_9^{A1}(s) \\
&+ f_4^{A1}(\mathcal{S}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_{10}^{A1}(s) \\
&- f_4^{A1}(\widetilde{\mathcal{S}}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_{11}^{A1}(s) \\
&+ \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_{12}^{A1}(s) \\
&+ \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_{13}^{A1}(s)
\end{aligned} \tag{F.4}
$$

where

$$
\begin{aligned}
f_1^{A1}(DA(\alpha_i)) = \int d\alpha_i \frac{DA(\alpha_i)}{\alpha_g^3}\Big\{ \\
& \alpha_g \delta\Big(\frac{u_0-\alpha_{\overline{q}}}{\alpha_g}\Big)\theta(\alpha_g+\alpha_{\overline{q}}-u_0) \\
& +\theta(u_0-\alpha_{\overline{q}})\Big[(\alpha_g+\alpha_{\overline{q}}-u_0)\delta\Big(\frac{\alpha_g+\alpha_{\overline{q}}-u_0}{\alpha_g}\Big) \\
& +\alpha_g\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\Big]\Big\} \\
f_2^{A1}(DA(\alpha_i)) = \int d\alpha_i \frac{DA(\alpha_i)}{\alpha_g^3}\Big\{ \\
& (\alpha_g+\alpha_{\overline{q}}-u_0)\delta\Big(\frac{u_0-\alpha_{\overline{q}}}{\alpha_g}\Big)\theta(\alpha_g+\alpha_{\overline{q}}-u_0) \\
& +\alpha_g\delta\Big(\frac{\alpha_g+\alpha_{\overline{q}}-u_0}{\alpha_g}\Big)\theta(u_0-\alpha_{\overline{q}})\Big\} \\
f_3^{A1}(DA(\alpha_i)) = \int d\alpha_i \frac{(\alpha_g+\alpha_{\overline{q}}-u_0)DA(\alpha_i)\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\theta(u_0-\alpha_{\overline{q}})}{\alpha_g^2} \\
f_4^{A1}(DA(\alpha_i)) = \int d\alpha_i \frac{DA(\alpha_i)\theta(\alpha_g+\alpha_{\overline{q}}-u_0)\theta(u_0-\alpha_{\overline{q}})}{\alpha_g}
\end{aligned}
\tag{F.5}
$$

$$
\begin{aligned}
\rho_1^{A1}(s) &= \int_0^1 du\overline{u_0}h_\gamma(u)\theta(u-u_0) \\
&\int_0^1 dx \frac{(\beta+5)e_q M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_Q^2}{\overline{x}}-\frac{m_{Q'}^2}{x}\right)}{24\sqrt{2}\pi^2}(m_Q x - m_{Q'}\overline{x}) \\
\rho_2^{A1}(s) &= \int_0^1 dx \frac{(\beta-1)e_q(m_Q-m_{Q'})M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{72\sqrt{2}\pi^2} \\
\rho_3^{A1}(s) &= \int_0^1 dx \frac{(\beta+2)e_q(m_Q-m_{Q'})M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{36\sqrt{2}\pi^2} \\
\rho_4^{A1}(s) &= \int_0^1 dx \frac{(\beta+5)e_q(m_Q-m_{Q'})M^2\langle\overline{q}q\rangle\,\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{72\sqrt{2}\pi^2} \\
\rho_5^{A1}(s) &= \int_0^1 dx \frac{(\beta-1)e_q f_{3\gamma}M^4(x-\overline{x})\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{72\sqrt{2}\pi^2} \\
\rho_6^{A1}(s) &= \int_0^1 dx \frac{(\beta-1)e_q M^2\langle\overline{q}q\rangle\,(m_Q x-m_{Q'}\overline{x})\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}}
\end{aligned}
$$

$$\rho_7^{A1}(s) = \int_0^1 dx \frac{(\beta-1)e_q M^2 \langle \overline{q}q \rangle (m_Q x - m_{Q'}\overline{x})\delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}}$$

$$\rho_8^{A1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q}q \rangle \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}}$$
$$(m_Q x((\beta+11)x-6) - m_{Q'}\overline{x}((\beta+11)\overline{x}-6))$$

$$\rho_9^{A1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q}q \rangle \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}}$$
$$(m_Q x(\beta(x+4)+2-x) - m_{Q'}\overline{x}(\beta(\overline{x}+4)+2-\overline{x}))$$

$$\rho_{10}^{A1}(s) = \int_0^1 dx - \frac{e_q M^2 \langle \overline{q}q \rangle \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}} \tag{F.6}$$
$$(m_Q x(\beta(x-6)-x) - m_{Q'}\overline{x}(\beta(\overline{x}-6)-\overline{x}))$$

$$\rho_{11}^{A1}(s) = \int_0^1 dx \frac{e_q M^2 \langle \overline{q}q \rangle \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{144\sqrt{2}\pi^2 x\overline{x}}$$
$$(m_Q(\beta(3x-2)+9x-4)x - m_{Q'}\overline{x}(\beta(3\overline{x}-2)+9\overline{x}-4))$$

$$\rho_{12}^{A1}(s) = \int_1^0 dx\delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)\Bigg\{$$
$$\frac{(\beta-1)e_q \langle \overline{q}q \rangle \mathbb{A}(u_0)(m_Q x - m_{Q'}\overline{x})\left(\frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{96\sqrt{2}\pi^2}$$
$$-\frac{M^2}{2304\sqrt{2}\pi^4 m_Q m_{Q'} x\overline{x}}\Bigg[$$
$$-48\pi^2(\beta-1)e_q m_Q m_{Q'} \langle \overline{q}q \rangle x\overline{x}\mathbb{A}(u_0)(m_Q x - m_{Q'}\overline{x})$$
$$+\left\langle g_s^2 G^2 \right\rangle \Big[e_Q m_Q x(2(2\beta+1)m_Q x + 3(\beta-1)m_{Q'}\overline{x}(u_0\overline{x}+x))$$
$$-e_{Q'} m_{Q'}\overline{x}(2(2\beta+1)m_{Q'}\overline{x} + 3(\beta-1)m_Q x(u_0 x+\overline{x}))\Big]\Bigg]$$
$$-\frac{(\beta-1)e_q M^4 \langle \overline{q}q \rangle \chi\varphi_\gamma(u_0)(m_Q x - m_{Q'}\overline{x})}{24\sqrt{2}\pi^2}\Bigg\}$$

$$\rho_{13}^{A1}(s) = \int_0^1 dx \int_0^{1-x} dy\delta\left(s - \frac{m_{Q'}^2}{y} - \frac{m_Q^2}{x}\right)\Bigg\{$$

$$\frac{(2\beta+1)\langle\overline{q}q\rangle M^2\left(e_Q m_Q^2 y - e_{Q'} m_{Q'}^2 x\right)}{384\sqrt{2}\pi^4 m_Q m_{Q'} xy}$$
$$+\frac{(\beta-1)\langle\overline{q}q\rangle\overline{u_0}\left(e_{Q'} m_{Q'}^2 x - e_Q m_Q^2 y\right)}{768\sqrt{2}\pi^4 xy}$$
$$-\frac{M^4(x+y-1)}{32\sqrt{2}\pi^4 xy}$$
$$\Big[e_Q m_Q y(3m_Q y(\beta-1)\overline{u_0} - 2m_{Q'}(2\beta+1))$$
$$- e_{Q'} m_{Q'} x(3m_{Q'} x(\beta-1)\overline{u_0} - 2m_Q(2\beta+1))\Big]$$
$$-\frac{3(\beta-1)M^6(e_Q y(x\overline{u_0}-1) - e_{Q'} x(y\overline{u_0}-1))}{32\sqrt{2}\pi^4}\Bigg\}$$

$$\begin{aligned}
\Pi_2^S(Q,Q',q) = & \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_1^{S2}(s) \\
& - f_1^{S2}(\mathcal{A}(\alpha_i)+\mathcal{V}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_2^{S2}(s) \\
& + f_2^{S2}(\mathcal{A}(\alpha_i)+\mathcal{V}(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_2^{S2}(s) \\
& - f_3^{S2}(\mathcal{T}_2(\alpha_i),\mathcal{T}_4(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_3^{S2}(s) \\
& + f_3^{S2}(\mathcal{T}_4(\alpha_i),\mathcal{T}_2(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_3^{S2}(s) \\
& + f_4^{S2}(\mathcal{T}_1(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_4^{S2}(s) \\
& - f_4^{S2}(\mathcal{T}_3(\alpha_i)) \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_4^{S2}(s) \\
& + \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_5^{S2}(s) \\
& + \int_0^\infty ds e^{-\frac{s}{M^2}} \rho_6^{S2}(s)
\end{aligned} \tag{F.7}$$

where

$$
\begin{aligned}
f_1^{S2}(DA_1(\alpha_i), DA_2(\alpha_i)) &= \int d\alpha_i \frac{\theta(\alpha_g + \alpha_{\overline{q}} - u_0)\theta(u_0 - \alpha_{\overline{q}})}{\alpha_g} \\
&\quad (DA_1(\alpha_i) + DA_2(\alpha_i)) \\
f_2^{S2}(DA_1(\alpha_i), DA_2(\alpha_i)) &= \int d\alpha_i \frac{\theta(\alpha_g + \alpha_{\overline{q}} - u_0)\theta(u_0 - \alpha_{\overline{q}})}{\alpha_g^2} \\
&\quad (\alpha_g + \alpha_{\overline{q}} - u_0)(DA_1(\alpha_i) + DA_2(\alpha_i)) \\
f_3^{S2}(DA_1(\alpha_i), DA_2(\alpha_i)) &= \int d\alpha_i \int_0^1 dv \theta(-v\alpha_g + \alpha_g + \alpha_{\overline{q}} - u_0) \\
&\quad (DA_1(\alpha_i) + vDA_2(\alpha_i)) \\
f_4^{S2}(DA(\alpha_i)) &= \int d\alpha_i \int_0^1 dv DA(\alpha_i)\theta(-v\alpha_g + \alpha_g + \alpha_{\overline{q}} - u_0)
\end{aligned}
\tag{F.8}
$$

$$
\begin{aligned}
\rho_1^{S2}(s) &= \int_0^1 du \overline{u}(u - u_0) h_\gamma(u) \theta(u - u_0) \\
&\quad \int_0^1 dx \frac{e_q \delta\left(s - \frac{m_Q^2}{\overline{x}} - \frac{m_{Q'}^2}{x}\right)}{4\sqrt{6}\pi^2} \\
&\quad \left\{(1+\beta)\langle \overline{q}q\rangle (m_{Q'}\overline{x} + m_Q x) + 2(1-\beta)\psi^v(u) f_{3\gamma} M^2 x\overline{x}\right\} \\
\rho_2^{S2}(s) &= \int_0^1 dx \frac{(\beta-1)e_q f_{3\gamma} M^2 \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{24\sqrt{6}\pi^2} \\
\rho_3^{S2}(s) &= \int_0^1 dx - \frac{(\beta+1)e_q (m_Q + m_{Q'})\langle \overline{u}u\rangle \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{12\sqrt{6}\pi^2} \\
\rho_4^{S2}(s) &= \int_0^1 dx \frac{(\beta-1)e_q \langle \overline{u}u\rangle \left(m_{Q'}\overline{x}^2 + m_Q x^2\right)\delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{12\sqrt{6}\pi^2 x\overline{x}} \\
\rho_5^{S2}(s) &= \int_0^1 dx \frac{(\beta-1)e_q u_0 \overline{u_0} \delta\left(s - \frac{m_{Q'}^2}{x} - \frac{m_Q^2}{\overline{x}}\right)}{192\sqrt{6}\pi^4} \\
&\quad \left\{\left\langle g_s^2 G^2\right\rangle + 24\pi^2 x\overline{x} f_{3\gamma} M^2 \psi^a(u_0)\right\} \\
\rho_6^{S2}(s) &= \int_0^1 dx \int_0^{1-x} dy \frac{(\beta-1)u_0\overline{u_0}\delta\left(s - \frac{m_{Q'}^2}{y} - \frac{m_Q^2}{x}\right)}{384\sqrt{6}\pi^4} \\
&\quad \left\{\left\langle g_s^2 G^2\right\rangle (e_Q + e_{Q'}) - 72M^4\Big[(1-x-y)(e_{Q'}x + e_Q y) - 2xye_q\Big]\right\}
\end{aligned}
\tag{F.9}
$$

$$
\begin{aligned}
\Pi_2^A(Q,Q',q) = \rho_1^{A2}(s) \\
&- f_1^{A2}(\mathcal{T}_4(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_2^{A2}(s) \\
&+ f_1^{A2}(\mathcal{T}_2(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_2^{A2}(s) \\
&- f_2^{A2}(V(\alpha_i))\Big(\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_3^{A2}(s) \\
&+ f_3^{A2}(\mathcal{A}(\alpha_i),\mathcal{V}(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_3^{A2}(s) \\
&+ f_4^{A2}(\mathcal{T}_1(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_4^{A2}(s) \\
&- f_4^{A2}(\mathcal{T}_3(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_4^{A2}(s) \\
&+ f_4^{A2}(\mathcal{T}_4(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_5^{A2}(s) \\
&- f_4^{A2}(\mathcal{T}_2(\alpha_i))\int_0^\infty ds e^{-\frac{s}{M^2}}\rho_5^{A2}(s) \\
&+ \int_0^\infty ds e^{-\frac{s}{M^2}}\rho_6^{A2}(s)
\end{aligned}
\tag{F.10}
$$

where

$$
\begin{aligned}
f_1^{A2}(DA(\alpha_i)) &= \int d\alpha_i \int_0^1 dv\theta(-v\alpha_g+\alpha_g+\alpha_{\bar{q}}-u_0) \\
&\quad vDA(\alpha_i) \\
f_2^{A2}(DA(\alpha_i)) &= \int d\alpha_i \frac{\theta(\alpha_g+\alpha_{\bar{q}}-u_0)\theta(u_0-\alpha_{\bar{q}})}{\alpha_g^2} \\
&\quad (\alpha_g+\alpha_{\bar{q}}-u_0)DA(\alpha_i) \\
f_3^{A2}(DA_1(\alpha_i),DA_2(\alpha_i)) &= \int d\alpha_i \frac{\theta(\alpha_g+\alpha_{\bar{q}}-u_0)\theta(u_0-\alpha_{\bar{q}})}{\alpha_g^2} \\
&\quad \Big[(2\alpha_g+\alpha_{\bar{q}}-u_0)DA_1(\alpha_i)+\alpha_g DA_2(\alpha_i)\Big] \\
f_4^{A2}(DA(\alpha_i)) &= \int d\alpha_i \int_0^1 dv\theta(-v\alpha_g+\alpha_g+\alpha_{\bar{q}}-u_0) \\
&\quad DA(\alpha_i)
\end{aligned}
\tag{F.11}
$$

$$
\begin{aligned}
\rho_1^{A2}(s) &= \int_0^1 du\overline{u_0}(u-u_0)\theta(u-u_0)h_\gamma(u) \\
&\quad \int_0^1 dx \frac{(\beta+5)\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{12\sqrt{2}}(m_Q x - m_{Q'}\overline{x}) \\
\rho_2^{A2}(s) &= \int_0^1 dx \frac{(\beta+5)e_q(m_Q-m_{Q'})\left\langle \overline{q}q\right\rangle \delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{36\sqrt{2}\pi^2} \\
\rho_3^{A2}(s) &= \int_0^1 dx \frac{(\beta-1)e_q f_{3\gamma}M^2(x-\overline{x})\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{24\sqrt{2}\pi^2} \\
\rho_4^{A2}(s) &= \int_0^1 dx \frac{(\beta-1)e_q \left\langle \overline{q}q\right\rangle \left(m_{Q'}\overline{x}^2-m_Q x^2\right)\delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{36\sqrt{2}\pi^2 x\overline{x}} \qquad \text{(F.12)} \\
\rho_5^{A2}(s) &= \int_0^1 dx \frac{e_q \left\langle \overline{q}q\right\rangle \delta\left(s-\frac{m_{Q'}^2}{x}-\frac{m_Q^2}{\overline{x}}\right)}{36\sqrt{2}\pi^2 x\overline{x}} \\
&\quad \left(m_{Q'}\overline{x}(5\overline{x}-3+\beta(\overline{x}-3)) - m_Q x(5x-3+\beta(x-3))\right) \\
\rho_6^{A2}(s) &= \int_0^1 dx \int_0^{1-x} dy \frac{(\beta-1)u_0\overline{u_0}\delta\left(s-\frac{m_{Q'}^2}{y}-\frac{m_Q^2}{x}\right)}{384\sqrt{2}} \\
&\quad \left[\left\langle g_s^2G^2\right\rangle (e_Q-e_{Q'}) + 72M^4(1-x-y)(e_{Q'}x-e_Q y)\right]
\end{aligned}
$$

where $S(A)$ represents the symmetric(anti-symmetric) correlation functions.

Appendix G

FIGURES

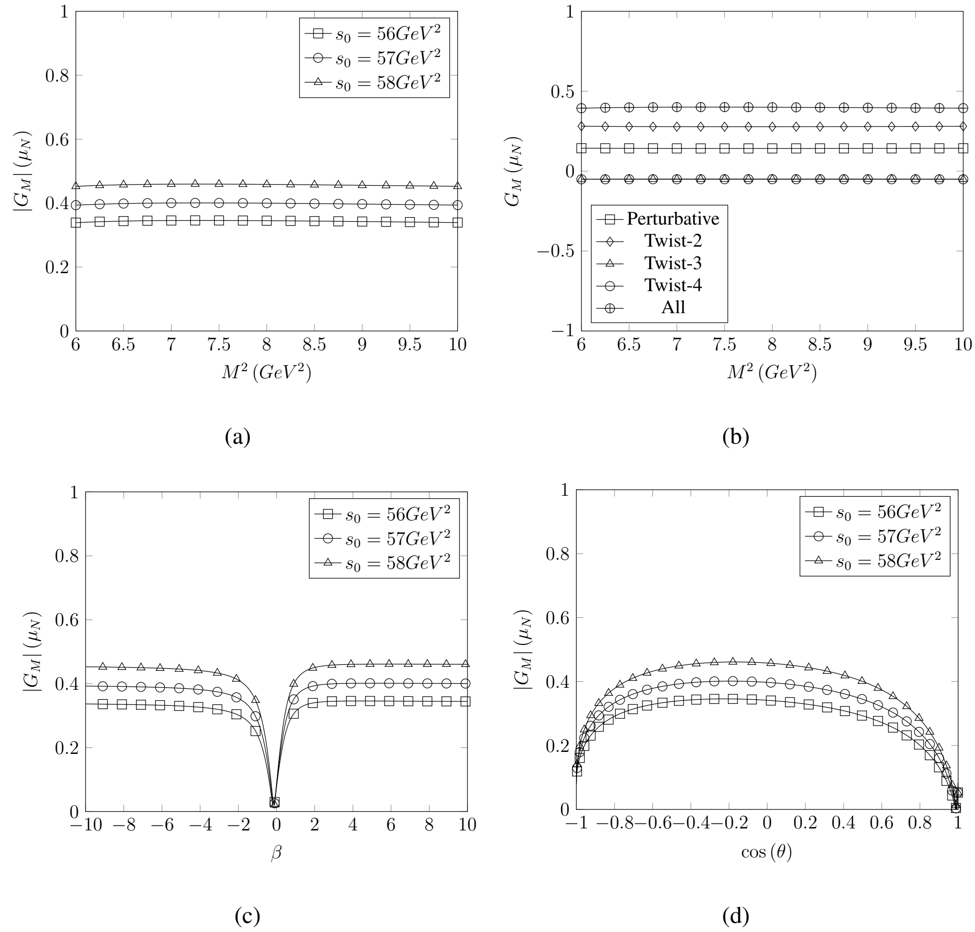


Figure G.1: **a)** M^2 dependence of the G_M for $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}\gamma$ transition with different s_0 values where $\beta = 3$, **b)** M^2 dependence of different twists and perturbative contributions to G_M for $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}\gamma$ transition where $\beta = 3$ and s_0 is the middle value of selected s_0 values($57GeV^2$), **c)** β dependence of G_M for $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}\gamma$ with different s_0 values where $M^2 = 8GeV^2$, **d)** $\tan(\theta)$ dependence of G_M for $\Xi_{bc}^{+} \to \Xi_{bc}^{*+}\gamma$ with different s_0 values where $\beta = \cos(\theta)$ and $M^2 = 8GeV^2$

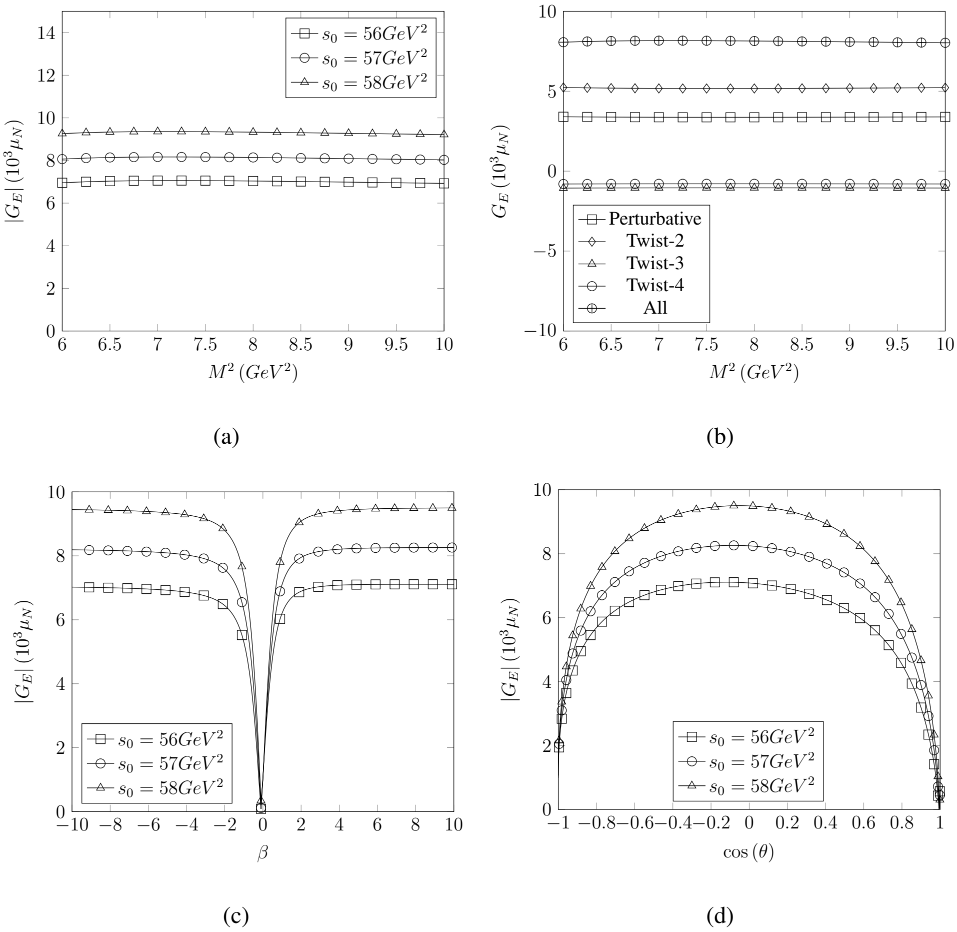
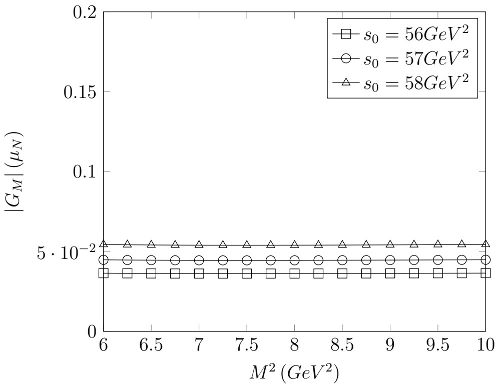
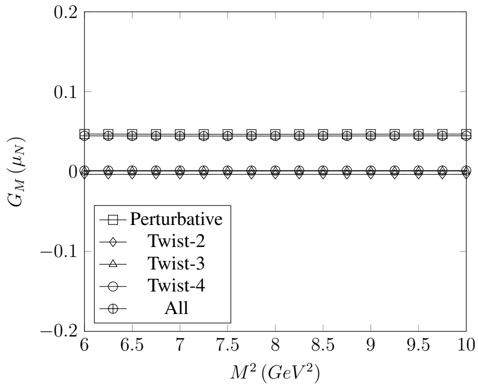


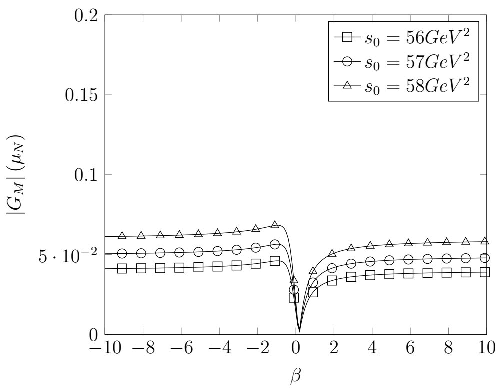
Figure G.2: Same as G.1 but for G_E



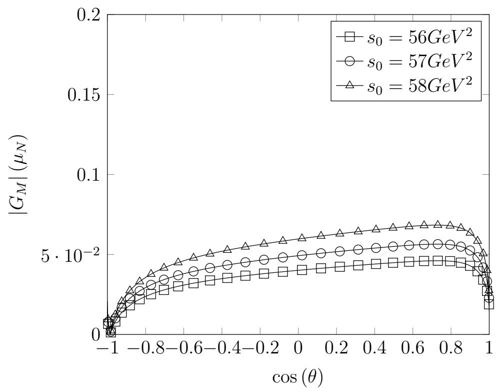
(a)



(b)



(c)



(d)

Figure G.3: Same as G.1 but for $\Xi_{bc}^{\prime +} \to \Xi_{bc}^{*+}\gamma$

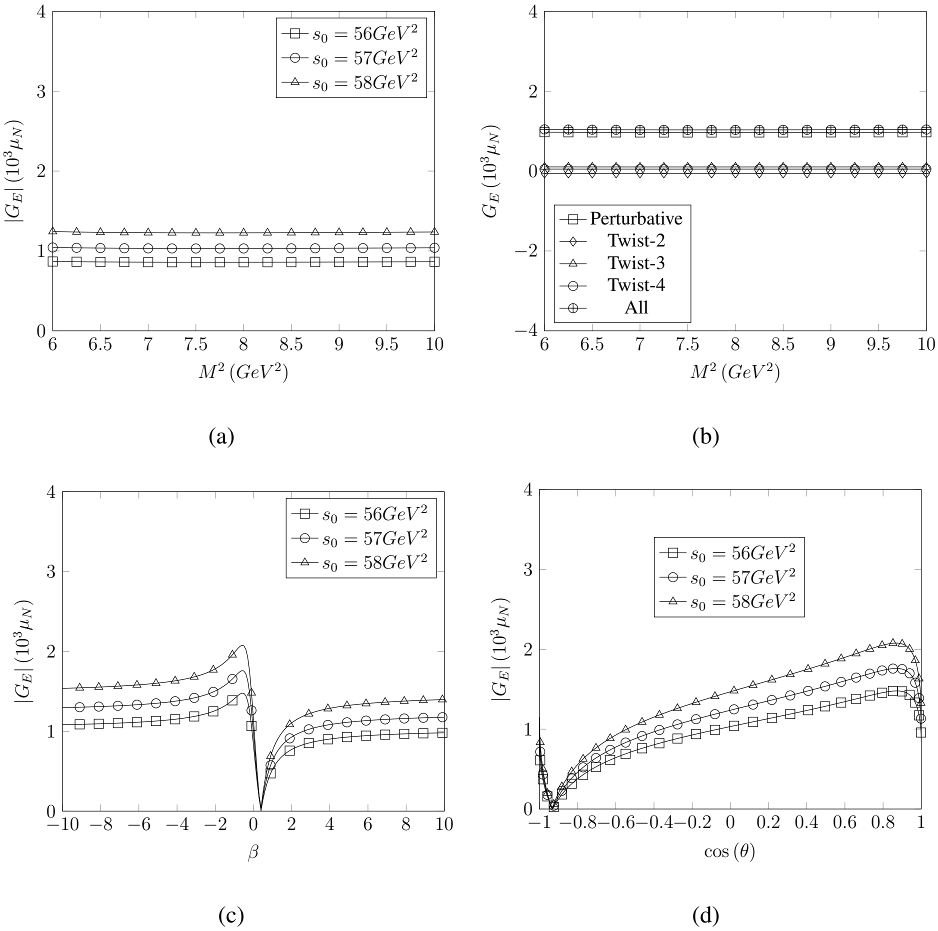


Figure G.4: Same as G.2 but for $\Xi_{bc}^{\prime +} \to \Xi_{bc}^{*+}\gamma$

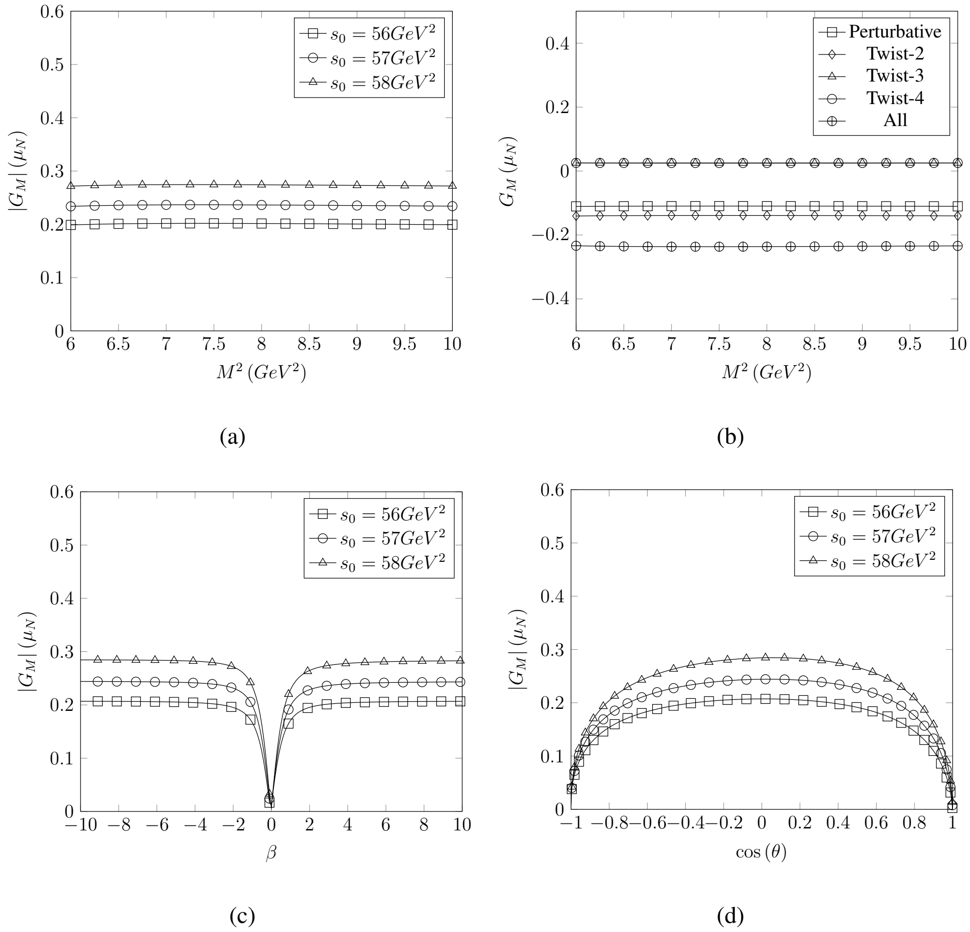


Figure G.5: Same as G.1 but for $\Xi_{bc}^{0} \rightarrow \Xi_{bc}^{*0}\gamma$

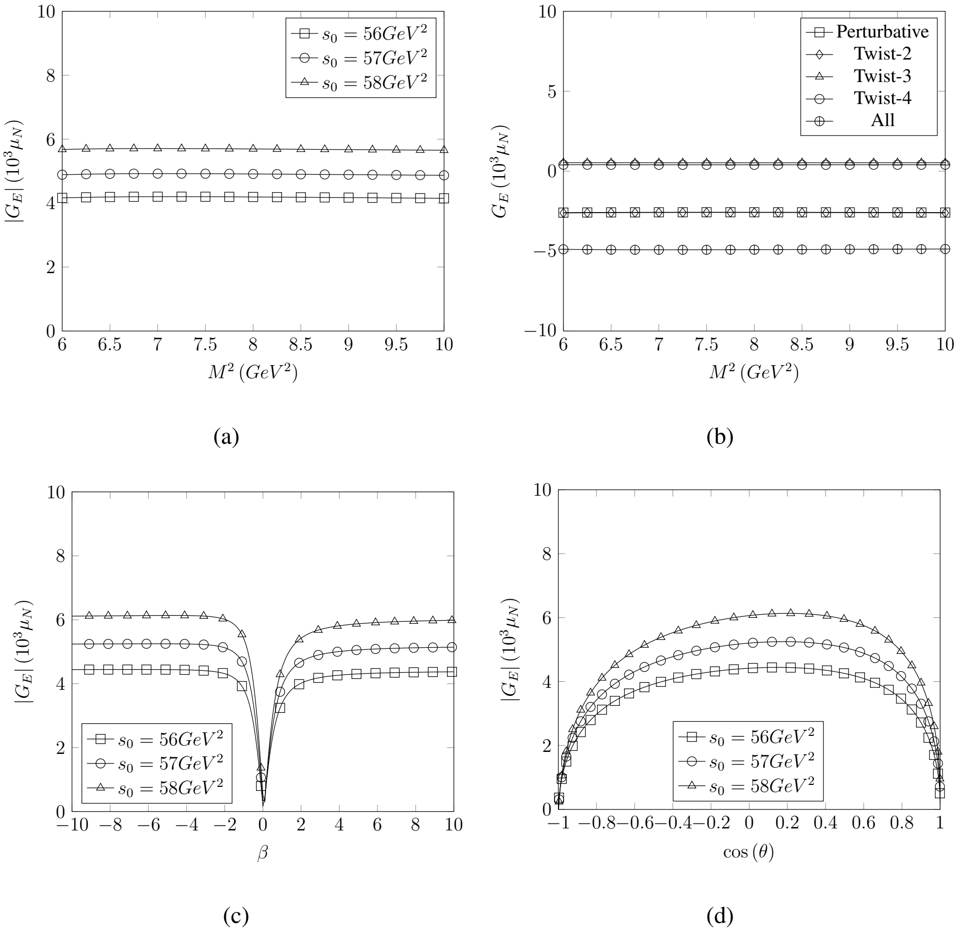


Figure G.6: Same as G.2 but for $\Xi^0_{bc} \to \Xi^{*0}_{bc}\gamma$

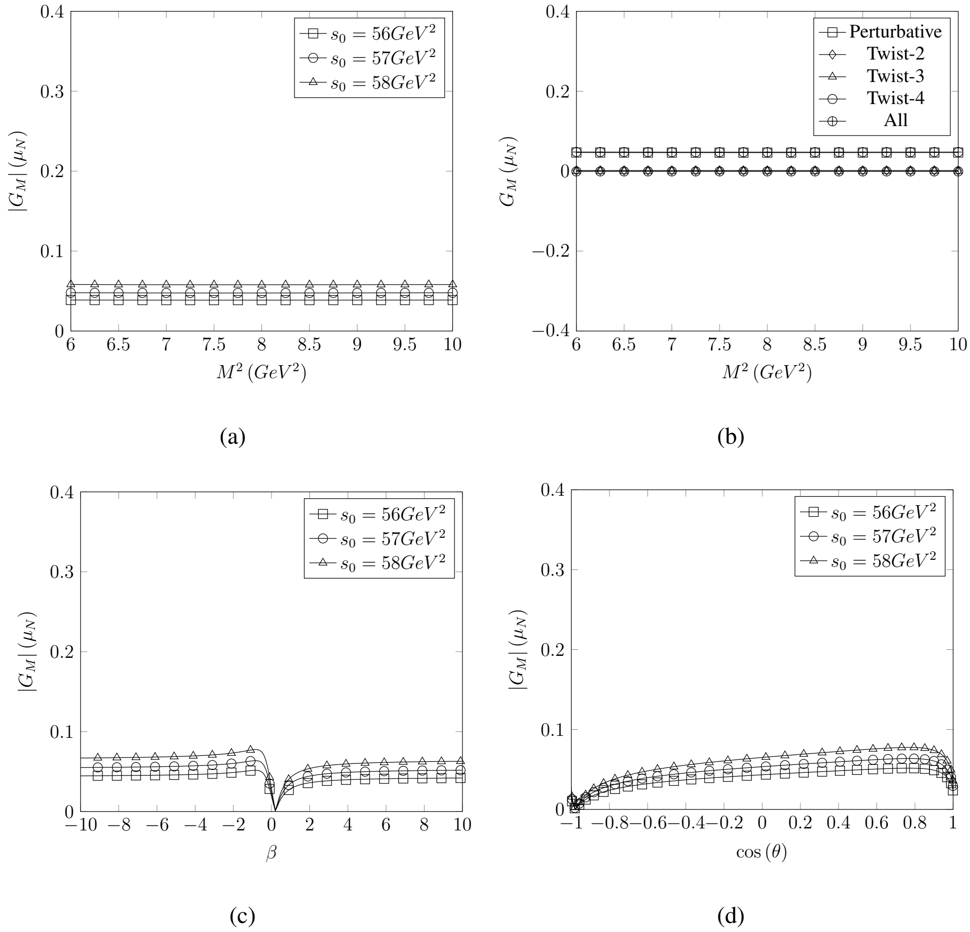


Figure G.7: Same as G.1 but for $\Xi'^0_{bc} \to \Xi^{*0}_{bc}\gamma$

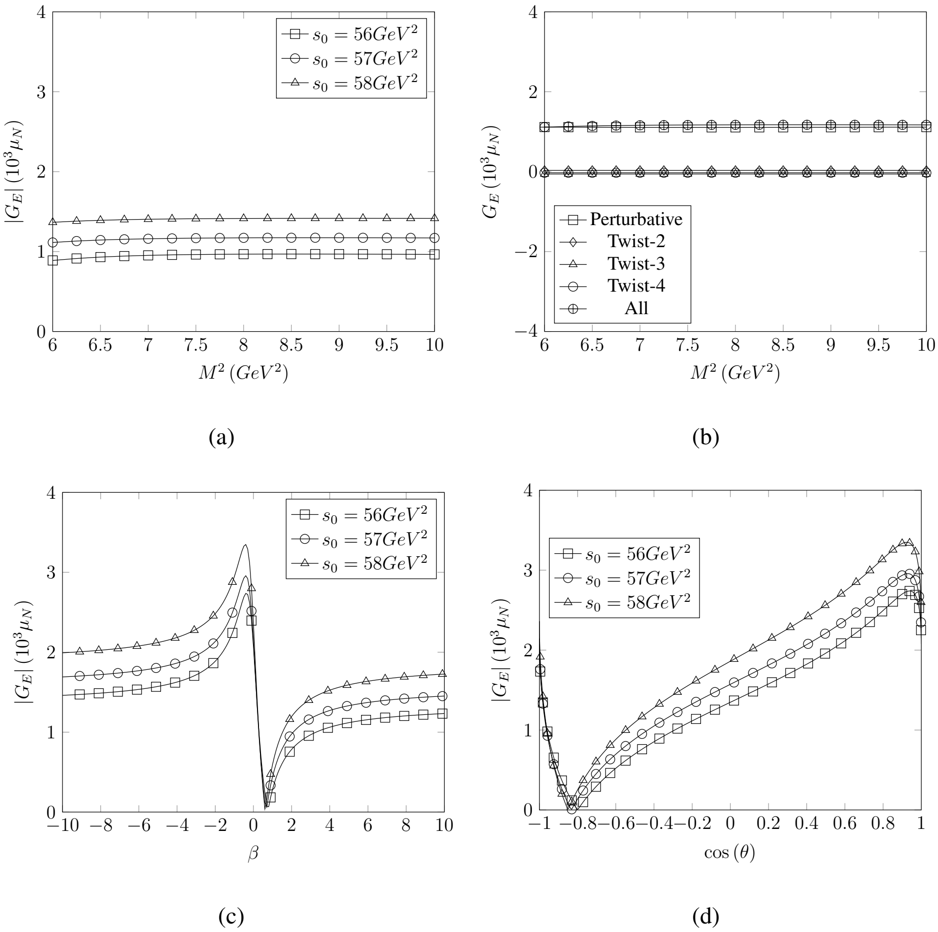


Figure G.8: Same as G.2 but for $\Xi'^0_{bc} \to \Xi^{*0}_{bc}\gamma$

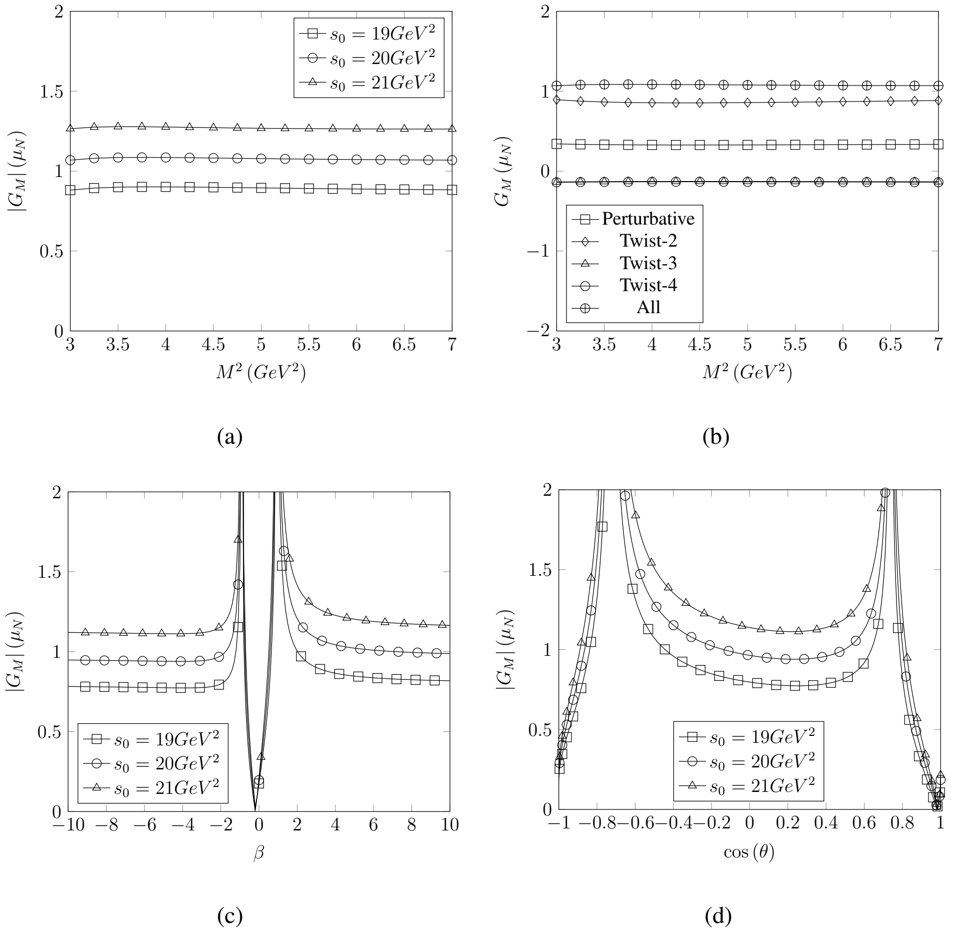


Figure G.9: **a)** M^2 dependence of the G_M for $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{*++}\gamma$ transition with different s_0 values where $\beta = 3$, **b)** M^2 dependence of different twists and perturbative contributions to G_M for $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{*++}\gamma$ transition where $\beta = 3$ and s_0 is the middle value of selected s_0 values($20GeV^2$), **c)** β dependence of G_M for $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{*++}\gamma$ with different s_0 values where $M^2 = 4GeV^2$, **d)** $\tan(\theta)$ dependence of G_M for $\Xi_{cc}^{++} \rightarrow \Xi_{cc}^{*++}\gamma$ with different s_0 values where $\beta = \cos(\theta)$ and $M^2 = 4GeV^2$

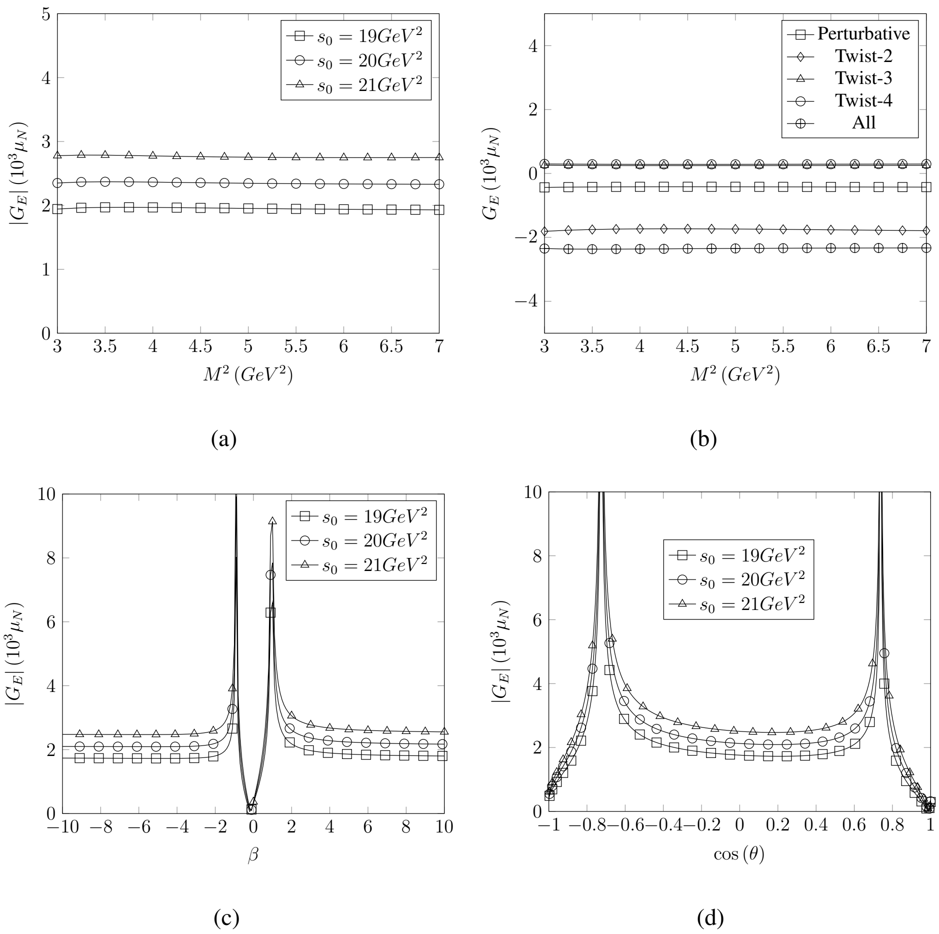


Figure G.10: Same as G.9 but for G_E

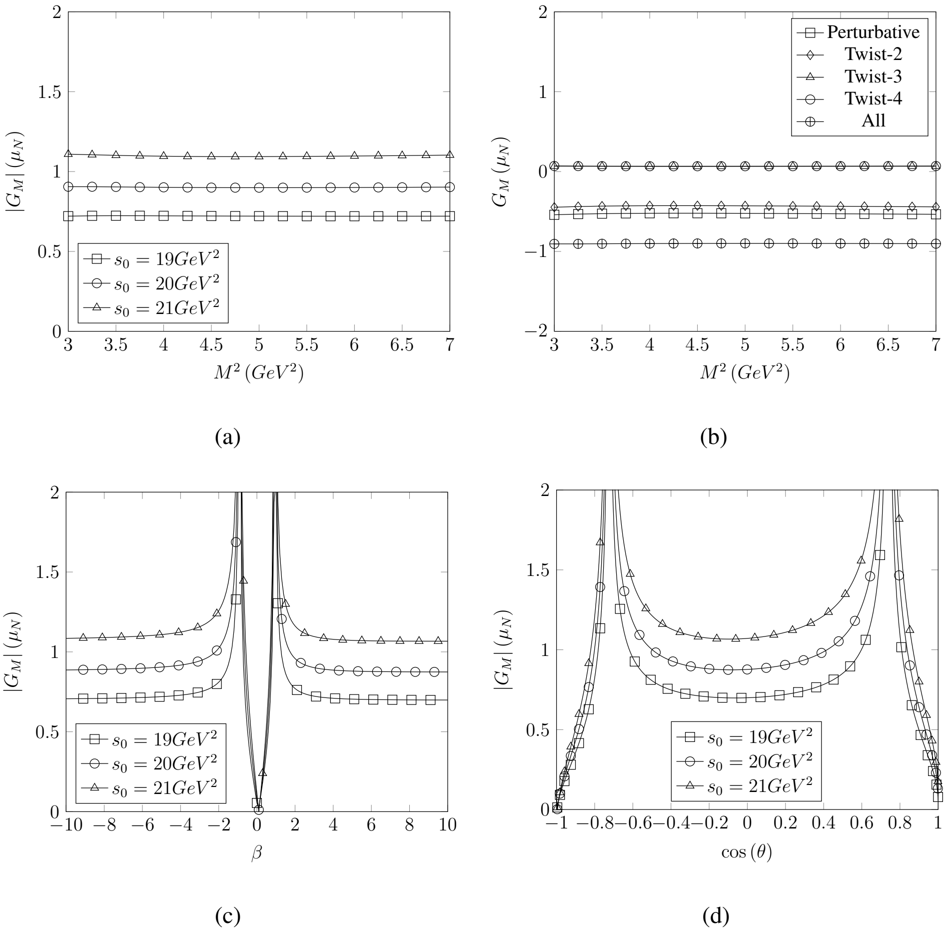


Figure G.11: Same as G.9 but for $\Xi_{cc}^{+} \rightarrow \Xi_{cc}^{*+}\gamma$

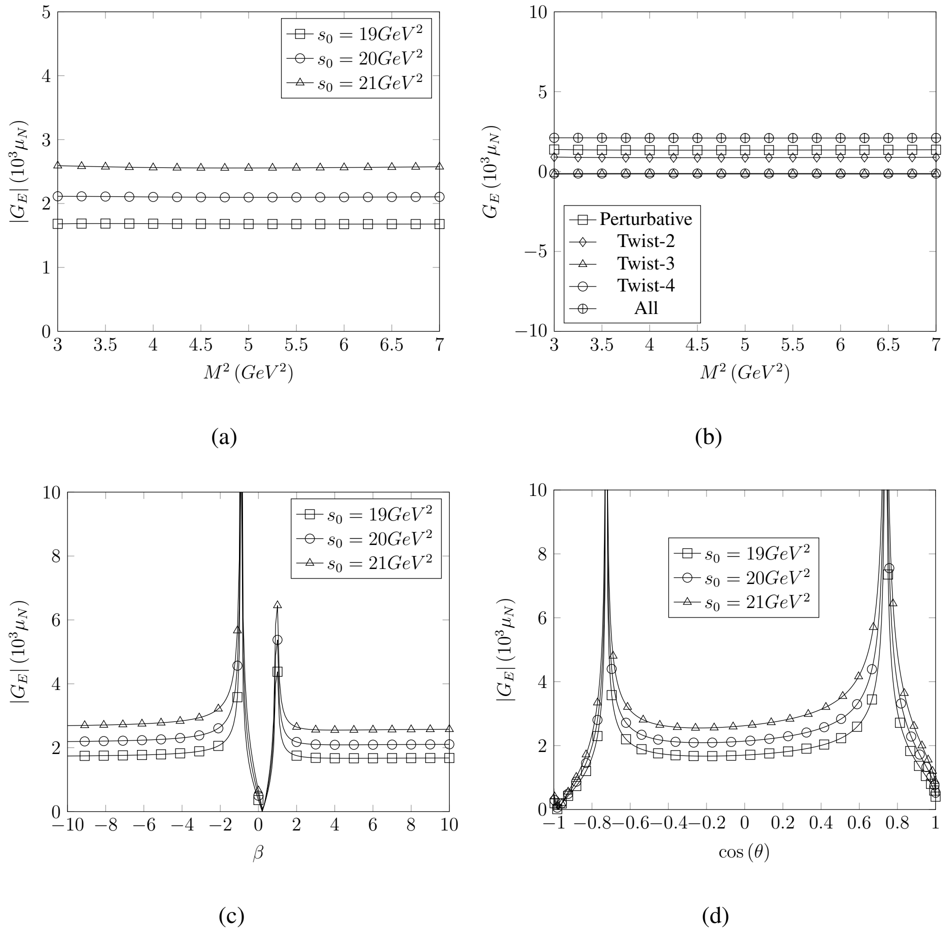


Figure G.12: Same as G.10 but for $\Xi_{cc}^{+} \to \Xi_{cc}^{*+}\gamma$

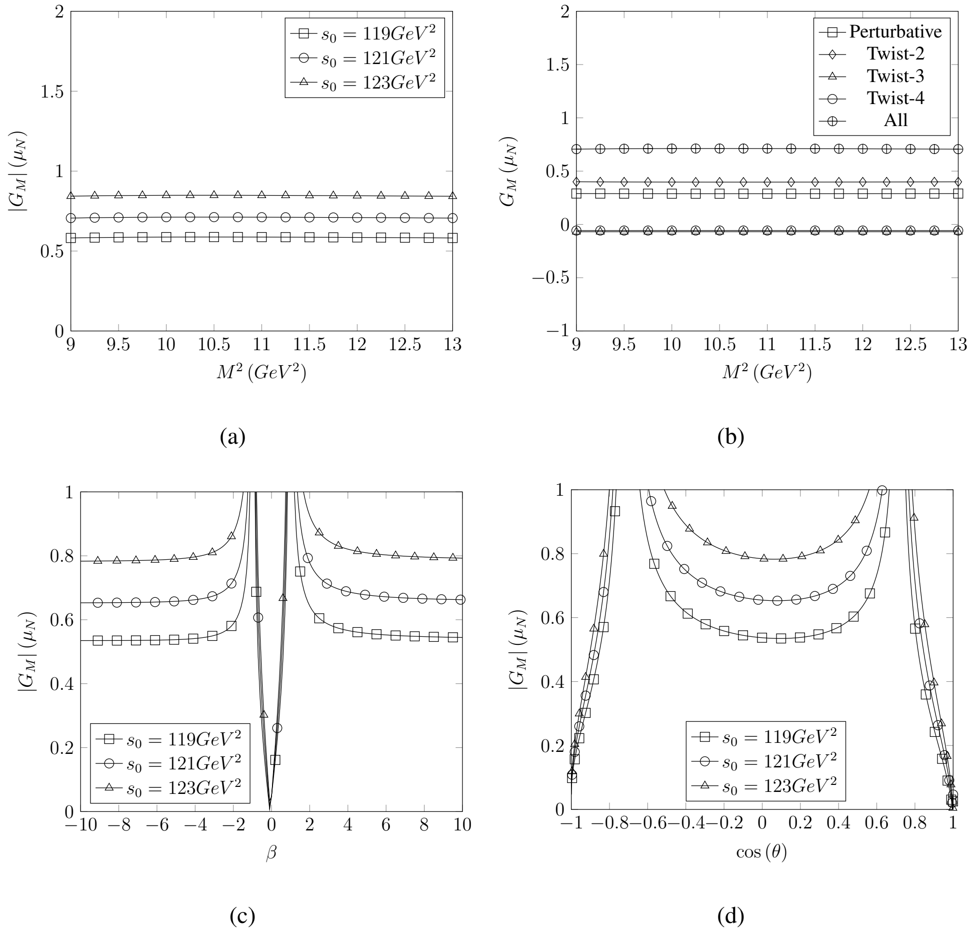
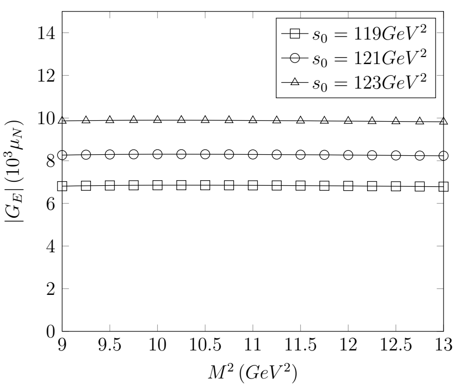
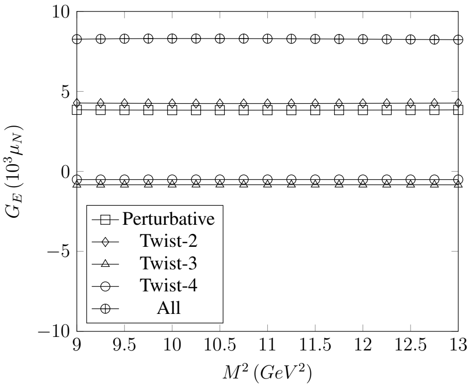


Figure G.13: **a)** M^2 dependence of the G_M for $\Xi_{bb}^0 \to \Xi_{bb}^{*0}\gamma$ transition with different s_0 values where $\beta = 3$, **b)** M^2 dependence of different twists and perturbative contributions to G_M for $\Xi_{bb}^0 \to \Xi_{bb}^{*0}\gamma$ transition where $\beta = 3$ and s_0 is the middle value of selected s_0 values($121GeV^2$), **c)** β dependence of G_M for $\Xi_{bb}^0 \to \Xi_{bb}^{*0}\gamma$ with different s_0 values where $M^2 = 12GeV^2$, **d)** $\tan(\theta)$ dependence of G_M for $\Xi_{bb}^0 \to \Xi_{bb}^{*0}\gamma$ with different s_0 values where $\beta = \cos(\theta)$ and $M^2 = 12GeV^2$



(a)



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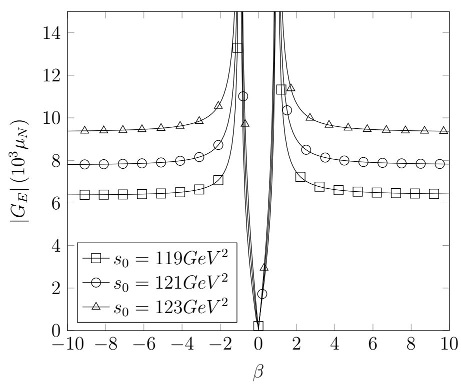

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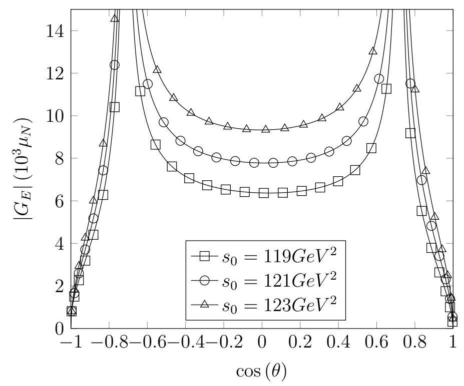

(d)

Figure G.14: Same as G.13 but for G_E

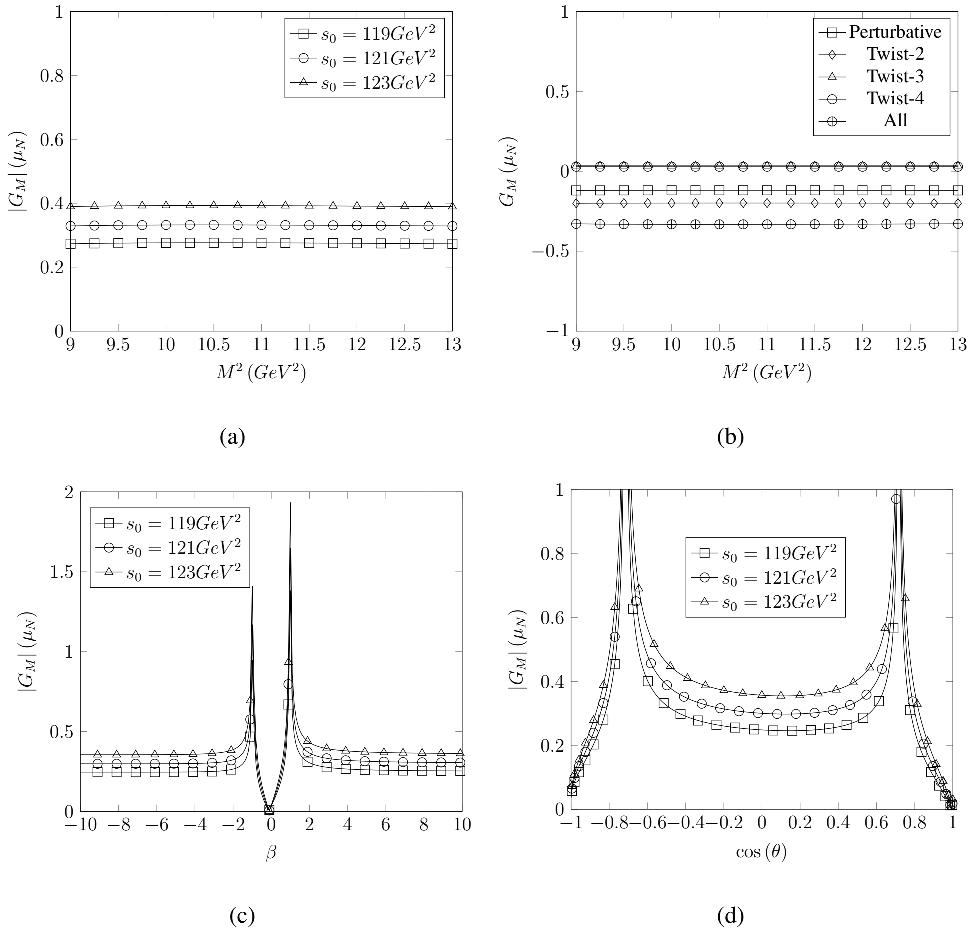

Figure G.15: Same as G.13 but for $\Xi_{bb}^{-} \rightarrow \Xi_{bb}^{*0}\gamma$

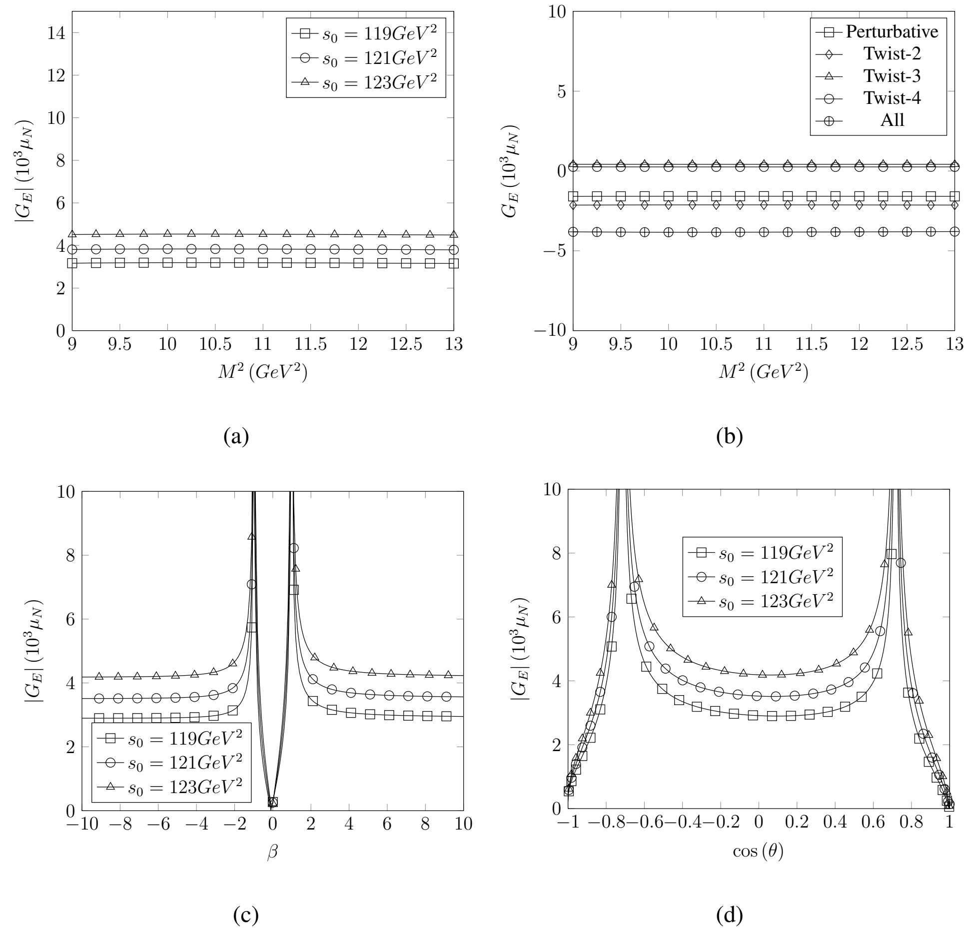

Figure G.16: Same as G.14 but for $\Xi_{bb}^{-} \rightarrow \Xi_{bb}^{*0}\gamma$