AN INVESTIGATION OF PRE-SERVICE MATHEMATICS TEACHERS' SEMIOTIC REPRESENTATIONS AND MODELING ROUTES IN A MATHEMATICAL MODELING ACTIVITY

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

MERVE ÇETİNBAŞ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMATICS EDUCATION IN MATHEMATICS AND SCIENCE EDUCATION

SEPTEMBER 2022

Approval of the thesis:

AN INVESTIGATION OF PRE-SERVICE MATHEMATICS TEACHERS' SEMIOTIC REPRESENTATIONS AND MODELING ROUTES IN A MATHEMATICAL MODELING ACTIVITY

submitted by **MERVE ÇETİNBAŞ** in partial fulfillment of the requirements for the degree of **Master of Science** i**n Mathematics Education in Mathematics and Science Education, Middle East Technical University** by,

Prof. Dr. Halil Kalıpçılar
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. Erdinç Çakıroğlu
Head of the Department, **Math. and Sci. Edu.** ________________

Assoc. Prof. Dr. Bülent Çetinkaya
Supervisor, **Math. and Sci. Edu., METU** ________________

Examining Committee Members:

Prof. Dr. Ayhan Kürşat Erbaş
Math. and Sci Edu., METU ________________

Assoc. Prof. Dr. Bülent Çetinkaya
Math. and Sci. Edu., METU ________________

Assoc. Prof. Dr. Yasemin Sağlam Kaya
Math. and Sci. Edu., Hacettepe Uni. ________________

Date: 01.09.2022

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Merve Çetinbaş

Signature :

ABSTRACT

AN INVESTIGATION OF PRE-SERVICE MATHEMATICS TEACHERS' SEMIOTIC REPRESENTATIONS AND MODELING ROUTES IN A MATHEMATICAL MODELING ACTIVITY

Çetinbaş, Merve
Master of Science, Mathematics Education in Mathematics and Science Education
Supervisor: Assoc. Prof. Dr. Bülent Çetinkaya

September 2022, 149 pages

This study aimed to examine the characteristics of pre-service mathematics teachers' mathematical modeling processes in terms of semiotic representations and the modeling routes varying within the scope of these characteristics. Participants were 13 pre-service teachers studying at a state university in Ankara and enrolled in an elective course entitled "Mathematical Modeling for Teachers." Data were collected through a technology-integrated mathematical modeling activity. The pre-service mathematics teachers' modeling processes were audio and video recorded. Also, the groups' written work, including their solutions, was collected at the end of the class, and the presentations of their final models were video-recorded. The findings revealed that prospective teachers were included in all semiotic registers discussed in this study, and the semiotic register in which the groups were included differed according to the purposes of the actions in the different parts of the modeling activity. While the registers in which the students were involved in the actions for determining the shape varied as algebraic and geometric, the registers included in the actions related to area measurement changed as algebraic and numeric. In determining the shape, it was seen that the group, which was accepted as the algebraic model, performed more conversion transformation than others, which was seen as a more complex semiotic action. The findings also revealed that pre-service teachers could not be assigned to particular semiotic characteristics when their whole modeling processes were examined due to the context of the modeling activity. In addition, although the semiotic characteristics could be determined in different parts of the modeling activity, no pattern was found in the groups' Modeling Transitions Diagrams (MTDs). Regardless of the semiotic characteristics of the groups, it was observed that there were completed cycles in almost all MTDs and both backward and forward movements between the modeling transitions. Moreover, the findings revealed that the preferences and purpose of the groups to use technology and group dynamics changed the characteristics of the modeling routes represented in MTD.

Keywords: Mathematical Modeling, Semiotic Representations, Modeling Routes, Modeling Transition Diagram, Technology Integration

ÖZ

MATEMATİK ÖĞRETMEN ADAYLARININ SEMİYOTİK GÖSTERİMLERİ VE MODELLEME ROTALARININ BİR MATEMATİKSEL MODELLEME ETKİNLİĞİ ARACILIĞIYLA İNCELENMESİ

Çetinbaş, Merve
Yüksek Lisans, Matematik Eğitimi, Matematik ve Fen Bilimleri Eğitimi
Tez Yöneticisi: Doç. Dr. Bülent Çetinkaya

Eylül 2022, 149 sayfa

Bu çalışma, matematik öğretmen adaylarının matematiksel modelleme süreçlerinin göstergebilimsel temsiller açısından özelliklerini ve bu özellikler kapsamında değişen modelleme rotalarını incelemeyi amaçlamaktadır. Katılımcılar, Ankara'da bir devlet üniversitesinde öğrenim gören ve "Öğretmenler için Matematiksel Modelleme" başlıklı seçmeli bir derse kayıtlı 13 öğretmen adayıdır. Veriler, teknoloji entegrasyonlu matematiksel modelleme etkinliği aracılığıyla toplanmıştır. Matematik öğretmen adaylarının modelleme süreçleri ses ve video kaydına alınmıştır. Ayrıca grupların çözümleri de dahil olmak üzere yazılı çalışmaları dersin sonunda toplandı ve nihai modellerinin sunumları videoya kaydedildi. Bulgular, öğretmen adaylarının bu çalışmada ele alınan tüm göstergebilimsel kayıtlarında yer aldıklarını ve modelleme etkinliğinin farklı bölümlerinde grupların yer aldığı göstergebilimsel kayıtların, eylemlerin amaçlarına göre farklılık gösterdiğini ortaya koymuştur. Öğretmen adaylarının şekil belirleme işlemlerinde yer aldıkları kayıtlar cebirsel ve geometrik olarak çeşitlilik gösterirken, alan ölçümü ile ilgili işlemlerde yer alınan kayıtlar cebirsel ve sayısal olarak değişmiştir. Şekli belirleme sürecinde

cebirsel model olarak kabul edilen grubun, daha karmaşık bir göstergebilimsel eylem olarak görülen dönüşüm eylemini diğer gruplara göre daha fazla gerçekleştirdikleri görülmüştür. Bulgular, modelleme etkinliğinin bağlamı nedeniyle, tüm modelleme süreçleri incelendiğinde öğretmen adaylarının belirli göstergebilimsel karakteristiğe atanamadıklarını ortaya koymuştur. Ayrıca modelleme etkinliğinin farklı bölümlerinde göstergebilimsel karakteristik belirlenebilse de grupların Modelleme Geçiş Diyagramları'nda (MTD'ler) herhangi bir örüntüye rastlanmamıştır. Grupların semiyotik özelliklerinden bağımsız olarak, hemen hemen tüm MTD'lerde tamamlanmış döngüler ve modelleme geçişleri arasında hem geriye hem de ileriye doğru hareketlerin olduğu görülmüştür. Ayrıca bulgular, grupların teknoloji kullanma tercihleri ve amaçları ile grup dinamiklerinin MTD'de temsil edilen modelleme rotalarının özelliklerini değiştirdiğini ortaya koymuştur.

Anahtar Kelimeler: Matematiksel Modelleme, Göstergebilimsel Temsiller, Modelleme Rotası, Modelleme Geçişleri Diyagramı, Teknoloji Entegrasyonu

To all my loved ones

ACKNOWLEDGMENTS

I would like to express my deepest appreciation to my supervisor, Assoc. Prof. Dr. Bülent ÇETİNKAYA for his close support and guidance, invaluable patience and encouragement, and important critiques and suggestions throughout all stages of this thesis.

My special thanks go to my thesis committee members, Prof. Dr. Ayhan Kürşat ERBAŞ and Assoc. Prof. Dr. Yasemin SAĞLAM KAYA for their valuable time, important comments, and feedback to improve this thesis.

I would like to extend my sincere thanks to Res. Assist. Murat KOL for his invaluable time, contributions, and help in important parts of this study.

My heartfelt thanks also go to all my close friends. Special thanks to Ahu CANOĞULLARI, who spent her precious time sharing her valuable experiences with me, motivated me with her invaluable patience, and kept my belief that I will finish this work throughout the study. My special thanks also go to Tuba ARSLANDAŞ for her close support and numerous help whenever I needed her. Many thanks to my friend Yasin GÜNDOĞDU for his priceless encouragement he has given me to complete this work and always being there for me. Thanks also to my friend, Burak ÇELİK for his precious help whenever I needed.

I'm deeply indebted to my family for their priceless encouragement and profound belief in my study. I'm extremely grateful to my father, Hüseyin ÇETİNBAŞ, my mother, Selma ÇETİNBAŞ, my sisters, Habibe Meltem ÇETİNBAŞ and Melike İKİZ, my brother, Özgür ÇETİNBAŞ, and my brother-in-law, Ali İKİZ, for their endless support and unwavering belief to me in all my life. My heartfelt thanks also go to my little nephews/nieces, who always were a source of motivation for me with their sweetness. Endless thanks to all my loved ones who will always be by my side and give me strength with their belief and love for me.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVIATIONS

ABBREVIATIONS

DMS	Dynamic Mathematics Software
MAD	Modeling Activity Diagram
MTD	Modeling Transition Diagram
MEA	Model-Eliciting Activity
TRSR	Theory of Registers of Semiotic Representations

CHAPTER 1

INTRODUCTION

Communication is a fundamental process that people need in daily life to make sense of any situation and to take action towards this understanding. This process that we are all familiar with can sometimes occur via channels such as languages, gestures, or even symbols. That is exactly why mathematics is very similar to daily life. As much as we need communication daily, the same is true for mathematics. From a similar point of view, it can be said that representations like channels provide communication in mathematics. In this regard, representations have great importance in the mathematical world because they have a role in both understanding the meaning of the mathematical concepts, ideas, or processes and expressing one's knowledge of this meaning to another one. In this process, it is also crucial to learn the relations in mathematics in giving meaning to mathematics. In mathematics, an abstract science with many complex structures, multiple representations are used to reinforce these relationships and meanings. It can be said that multiple representations support individuals to produce different strategies, and they have a more meaningful process related to mathematics in this way.

Individuals can develop their learning processes more meaningfully by producing various strategies with different representations. At this point, making transformations between these representations can also both support the mathematical understanding and give information about this understanding of the individuals as well as their use of different representations. Actually, obtaining information about the mathematical understanding of individuals can inform educators about the difficulties of individuals while realizing this understanding. To understand these difficulties, Duval (2006) stated that we need to "determine the cognitive functioning underlying the diversity of mathematical processes" (p. 103). In the cognitive functioning of representations, in fact, the existence of semiotic representations is mentioned, and they are considered as tools to "produce new knowledge" and "communicate any particular mental representation" (Duval, 2006, p. 104). Semiotic representations serving such purposes are critical aspects of mathematical tasks, and they are powerful tools to make sense of individuals' mathematical processes and their difficulties in this process. Thus, semiotic representations are considered to have a critical role in comprehending mathematics meaningfully.

Within the scope of learning mathematics in a meaningful way, researchers emphasized that seeing mathematics in their daily life significantly impacts individuals' conceptual understanding (Altay et al., 2017; Moore et al., 2015). When real life and mathematics are considered together, the existence of mathematical modeling, serving as a bridge providing the transition between them, can be mentioned there. Mathematical modeling is a process requiring translations between mathematics and reality which means mathematics outside, including nature, society, daily life, and other disciplines (Blum & Borromeo Ferri, 2009). This nature of mathematical modeling, which integrates real-life situations into mathematics, improves mathematical understanding, seeing the applicability of mathematics to real life, and develops skills like communication, working collaboratively, creativity, and making meaningful choices (Stohlmann, 2017). The process of mathematical modeling, implemented via model-eliciting activities, including well-structured realistic problems in an interdisciplinary context, provides students with skills such as collaboration, metacognition, multiple processes, self-directed learning, self-assessment, fostering of ownership, and model development (Chamberlin & Moon, 2005). The model-eliciting activities are powerful instructional tools, including authentic real-life context, group work, open-ended questions, multiple entry points, and interdisciplinary relations (Lesh & Zawojewski, 2007).

The world continues to improve each day as conditions change fast, but developments in almost every field accelerated in the last decades. In this process, the focus of the researchers turned to educational innovations in the direction of these developments. As a necessity of the digital world, technology has started to be used in education to promote individuals' understanding of the subject matter. In the same direction, it is evident that students' mathematical reasoning can be enhanced, and they can be supported to make sense of challenging mathematical concepts with the help of technology integration into mathematics lessons (Suh et al., 2008). Students can be more active, motivated, and engaged in technology-integrated mathematics classes (Kim et al., 2003; Wolf et al., 2011). As the indispensable technological tool of this digital process, perhaps the most preferred one is Dynamic Mathematics Software (DMS). Straesser (2002) emphasized that DMS is a helpful tool providing great convenience in creating shapes and expanding the range of accessible geometrical solutions. It is also reported that this environment helps students to build complex mental models about shapes at an increasingly higher level and improve their understanding of analyzing the property of this shape (Battista, 2002). One of the most known DMS is undoubtedly GeoGebra Software, a mathematical environment enabling the work dynamically on graphs, geometry, algebra, spreadsheets, and much more (GeoGebra, n.d.). Having dynamic properties, such as dragging, provides a rich learning environment for students to experience many mathematical situations conveniently.

Mathematics is generally perceived as a difficult subject by students, and a negative attitude is developed towards mathematics due to this misperception (Ünlü, 2007). Students could not see how mathematics is related to their daily lives. Within the scope of the used mathematical content in the current study, this is also true for the area concept because students prefer to memorize area formulas of geometric shapes instead of understanding what it means conceptually and what their applications are in the real world (Clements & Stephan, 2004; Tan Şişman & Aksu, 2016; Van de Walle et al., 2012). In light of this information, it is important to include well-prepared tasks related to real life in mathematics courses and to provide students with experience in using mathematics to make sense of real-life situations. More specifically, instead of traditional teaching in classrooms, the inclusion of tools such as mathematical modeling or technology integration into mathematics lessons to enrich the course content has great importance for individuals to learn mathematical concepts in a meaningful way. However, the relevant literature shows that model-eliciting activities are not integrated adequately in mathematics classes (Borromeo Ferri & Blum, 2013). In Turkey, this is due to factors such as mathematics teachers' unfamiliarity with mathematical modeling and model-eliciting activities and insufficient time to implement these activities in the intense curriculum, according to the teachers (Urhan & Dost, 2016). On the other hand, it is also stated in the literature that prospective mathematics teachers have difficulties in their modeling processes (Korkmaz, 2010).

It is also critical to learn how students are involved in modeling tasks, which are important for the cognitive development of individuals. This is valuable to develop various tasks for individuals with different mathematical backgrounds and abilities and to raise awareness of their difficulties or strengths to improve their learning processes. To acquire a deep understanding of these learning processes, determining the modeling routes followed by the students between the modeling stages having different requirements plays a crucial role in modeling tasks. On the other hand, being aware of diverse learners who differ in terms of their mathematical representations and their progress between the modeling stages is necessary to ensure their cognitive development in the further process. In light of all this information, it is important to introduce pre-service teachers to mathematical modeling tasks because they may include such activities in their teaching in the future. In addition, supporting pre-service teachers' knowledge about mathematical modeling with such activities will improve them about when and how they should intervene with individuals in case of difficulties experienced by students while implementing these tasks (Shahbari & Tabach, 2020). On the other hand, the characteristics determined by the semiotic representations used by pre-service teachers, who may be more likely to use different representations due to their higher level of mathematical knowledge, will shed important light on how the modeling routes of individuals with different mathematical proficiencies change.

Therefore, this study used a technology-supported modeling activity that involved the concept of an area of irregular shapes and that can be solved in multiple ways. Additionally, we focused on the semiotic representations of pre-service teachers in their mathematical models and modeling processes and examined the modeling routes and the relation between the modeling routes and different semiotic representations.

1.1 Statement of the Purpose and Research Questions

The purpose of this study was to investigate the semiotic representations of pre-service mathematics teachers in their modeling processes in a technology-integrated modeling activity and their changing modeling routes according to these representations. The research questions that guided the current study were as follows.

1. What are the characteristics of pre-service mathematics teachers' mathematical models and modeling processes in terms of the semiotic representations used in a technology-integrated model-eliciting activity?

2. What are the features of pre-service mathematics teachers' modeling routes in a technology-integrated model-eliciting activity?

2.1 How do pre-service mathematics teachers' modeling routes differ according to the semiotic characteristics of their mathematical models and modeling processes in a technology-integrated model-eliciting activity?

1.2 Significance of the Study

The way individuals understand the mathematical concept and their learning processes for these concepts may differ from each other. This situation has increased the interest in different teaching methods and promoted researchers to find new ways to encourage learners. Integrating mathematical modeling tasks and using the modeling approach in teaching mathematics can reverse students' perception of mathematics as a subject where only numbers, formulas, and routine problems exist (Lesh & Doerr, 2003). The importance of mathematical modeling, a process requiring translations between mathematics and the real world, is exhibited with its contributions to individuals, such as providing a better understanding of the real world, promoting mathematical understanding, and developing mathematical competencies (Blum & Borromeo Ferri, 2009). Although different tools such as mathematical modeling that contribute to students' mathematical understanding are common in educational settings, there are also mathematical concepts in which students cannot achieve this mathematical understanding and have difficulties. More specifically, one of them is the area measurement concept within the scope of this study. Studies attributed the reason for the difficulties experienced by the students in the concept of area measurement to the lack of experience in area measurement, the learning of this concept according to traditional teaching methods, and reported that students memorized the area formulas instead of understanding the meaning of it (Clements & Stephan, 2004; Muir, 2007; Tan Şişman & Aksu, 2016; Van de Walle et al., 2012).

One of the subjects that gained importance with the mathematical modeling approach was the "individual modeling routes" that showed how individuals progressed through the modeling transitions. The modeling route, defined by Borromeo Ferri (2007, p. 267) as "the individual modeling process on an internal and external level," is a valuable research tool to acquire a better understanding of the individuals' thinking processes while involving in a modeling task (Borromeo Ferri, 2017; Shahbari & Tabach, 2020). This understanding has a critical role in improving the learning processes of students and teaching styles of teachers because each individual is unique in the world and each one has a different thinking process. Investigating their modeling processes deeply can help educators increase their awareness of the existence of different learners and attempt to diversify their teaching in the direction of these different thinking processes, individuals' difficulties and strengths in the modeling tasks. Thus, it is critical to investigate pre-service teachers' modeling processes and their modeling routes as they will be the ones who use modeling tasks in teaching mathematics.

Borromeo Ferri (2017, p. 136) stated that "modeling routes reveal how students follow different steps of the cycle" and "these routes also correspond to their mathematical thinking styles". Specific to these mathematical thinking styles, Borromeo Ferri mentioned that she determined three thinking styles having different characteristics in her empirical study (Borromeo Ferri, 2004). In the mentioned study, it was observed that students with different thinking styles proceeded differently in their modeling processes. The different thinking styles that Borromeo Ferri (2004) can be associated with systems containing different semiotic representations, which Duval (2020) sees as the power of mathematical thinking. Duval (1998) emphasized that the mathematical process requires different semiotic representations and there would be no mathematical task without semiotic representation (Duval, 2020). In this regard, he said that the development of the learning and thinking process can be realized by transitions between these semiotic representations (Duval, 1998). Revealing the semiotic representations is important to understand how students make a transformation between different semiotic representations in a cognitive point. This provides us seeing students' conceptual understanding or difficulties in the mathematical concepts that can be expressed with different representations. This is vital to diagnose the particular difficulty of student because any unresolved difficulty may prevent seeing the connections between mathematical concepts and transforming between those mathematical concepts in the further process.

In his study, Hıdıroğlu (2012) found that technology had a positive effect on different modeling competencies of pre-service teachers and emphasized that they could focus better on their cognitive processes by reducing the complexity of the operations. These prove that technology emerged as a helpful tool for preservice teachers in different modeling transitions/phases. In this regard, it has a great importance to understand how technology is involved in these modeling transitions/phases, which largely reflect the thinking processes of students. To evaluate it from another perspective, technological processes can provide rich course content for students especially in mathematics courses. Moreover, technology can play a role both as an assistant and a developer in their thinking processes. Therefore, it has a critical role to shed light on which modeling stages technology is needed and how it is used in these stages in order to understand how students' thinking processes are shaped and to monitor how these processes of students with different characteristics change.

To follow the modeling processes of students, the modeling route was visualized differently by different researchers (Ärlebäck, 2009; Borromeo Ferri, 2007; Czocher, 2016). In the literature, the modeling route is mainly represented by arrows on the modeling cycles. This may be a challenging way to show their complex modeling processes. On the other hand, the graphical representation of modeling routes may enable us to follow groups' modeling processes chronologically. Therefore, the graphical representation was preferred in this study examining the modeling routes of groups. According to the literature, in their graphical representation of modeling routes, while Ärlebäck (2009) used modeling activities parallel to the modeling transitions, Czocher (2016) represented the modeling routes of the individuals by using modeling transitions. However, the graphical representations in these studies did not include the technological world, included just the real and mathematical world in the modeling cycle. In this regard, using graphical representation in representing the modeling routes and including the technology stage can offer a distinct perspective to the literature.

Lastly, this study was also significant as it is a study testing the rubric that Czocher (2013) created by adhering to the literature to determine the modeling transitions and extending that rubric within the scope of a different modeling activity.

1.3 Definition of Important Terms

Mathematical Modeling

Mathematical modeling is the process including to create a model for open-ended, practical problems based on real-life situations by using mathematics to represent, analyze, make assumptions, and make sense of this model (Consortium for Mathematics and Its Applications [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2016; Stohlmann & Albarracin, 2016).

Model-Eliciting Activity (MEA)

Model-eliciting activity is an instructional tool including authentic real-life context, group work, open-ended questions, multiple entry points, and interdisciplinary relations (Lesh & Zawojewski, 2007).

Models

Models are described as “conceptual systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system” (Doerr & English, 2003, p. 112).

Modeling Route

Borromeo Ferri describe the modeling route as “the individual modeling process on an internal and external level” (2007, p. 265).

Representation

Representation is described as “the sign and its complex associations” by Duval (2006, p. 104).

Register (Semiotic System)

The notion of the register is described as “the semiotic systems which fulfill a specific cognitive function” (Duval, 2020, p. 724).

Semiotic Representation

The signs, including natural language, iconic/non-iconic figures, symbolic writings and diagrams/graphs in semiotic registers described as “the semiotic systems which fulfill a specific cognitive function” (Duval, 2006, 2020, p.724).

CHAPTER 2

LITERATURE REVIEW

This study aimed to investigate pre-service mathematics teachers' characteristics of their mathematical models and modeling processes in terms of semiotic representations and their changing modeling routes according to these characteristics in technology-integrated modeling activity. In the direction of this aim, models and modeling perspective was initially included in this section. Then, it is followed by the part of the theoretical framework for the semiotic registers and the related studies within the scope of the aims of this study.

2.1 Mathematical Modeling

Models and modeling perspective is a theoretical approach that includes conceptual systems as cognitive objectives of mathematics education, and through these systems, mathematics-related real-life situations are constructed, described, or explained (Lesh & Doerr, 2003). According to this perspective, models are described as a conceptual system of elements, operations, relationships, and rules by providing to describe, explain, construct, interpret, and predict situations for some specific purpose (Doerr & English, 2003; Richardson, 2004). Models are powerful tools open to sharing and reuse in daily life since they help individuals make sense the real-life situations by providing to interpret them for a specific purpose (English et al., 2005). On the other hand, the mathematical model is the construction of real objects, data, relations, and conditions translated into mathematics (Blum, 2002).

To realize the models and modeling perspective in mathematics, it can be considered the existence of the mathematical modeling process. Mathematical modeling includes creating a model for open-ended, practical problems based on real-life situations by using mathematics to represent, analyze, make assumptions, and make sense of this model (COMAP & SIAM, 2016; Stohlmann & Albarracin, 2016). This process requires translations between mathematics and reality which means mathematics outside, including nature, society, daily life, and other disciplines (Blum & Borromeo Ferri, 2009).

2.1.1 Model Eliciting Activities (MEAs)

Model-eliciting activities, allowing implementation of the mathematical modeling process, are instructional tools including authentic real-life context, group work, open-ended questions, multiple entry points, and interdisciplinary relations (Lesh & Zawojewski, 2007). In model-eliciting activities, including well-structured real-life problems instead of traditional word problems like in textbooks (Lesh & Doerr, 2003), the aim is to provide individuals to create a mathematical model for real-life problems and use this model to generate a solution to the existing problem. Model-eliciting activities are one of the tools to gain individuals various skills valuable in mathematics education, such as model development, metacognition, multiple processes, self-assessment, etc. (Chamberlin & Moon, 2005).

2.1.2 Modeling Cycles and Modeling Routes

The notion of mathematical modeling is too often confronted in the literature as applications and modeling. This is because this term expresses the interaction between reality and mathematics with products and processes (Blum, 2015). The modeling process that encompasses this interaction is visualized by Pollak (1979, p. 233) with a model that illustrates the transitions between the “rest of the world” and mathematics science, as in Figure 2.1 below. Along with Pollak’s modeling cycle, which is accepted as the origin, it can be seen that researchers have revealed many different modeling schemas to date. In the literature, one of the most common modeling cycles varying in their transitions and stages is the cyclic model given in Figure 2.2 (Blum & Leiß, 2007). In their modeling cycle, which is the research framework of the study, there are six transitions and six stages to express the modeling process. While the transitions are "understanding," "simplifying/structuring," "mathematizing," "working mathematically," "interpreting," and "validating," the stages are named "real situation," "situation model," "real model," "mathematical model," "mathematical results" and "real results" as seen in Figure 2.2. Over time, the rapid development in science and technology in the world has also shown itself in mathematical modeling. Along with this process, modeling cycles, which show the transitions and phases between the real world and the mathematical world, have evolved and included the technology world (Greefrath et al., 2011).

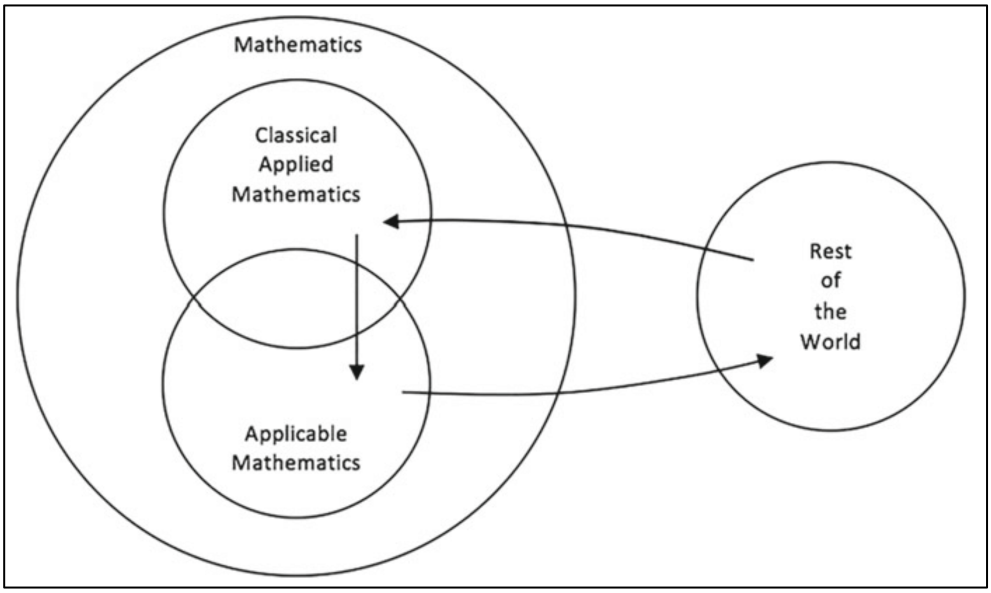


Figure 2.1 Modeling Cycle of Pollak (1979, p. 233)

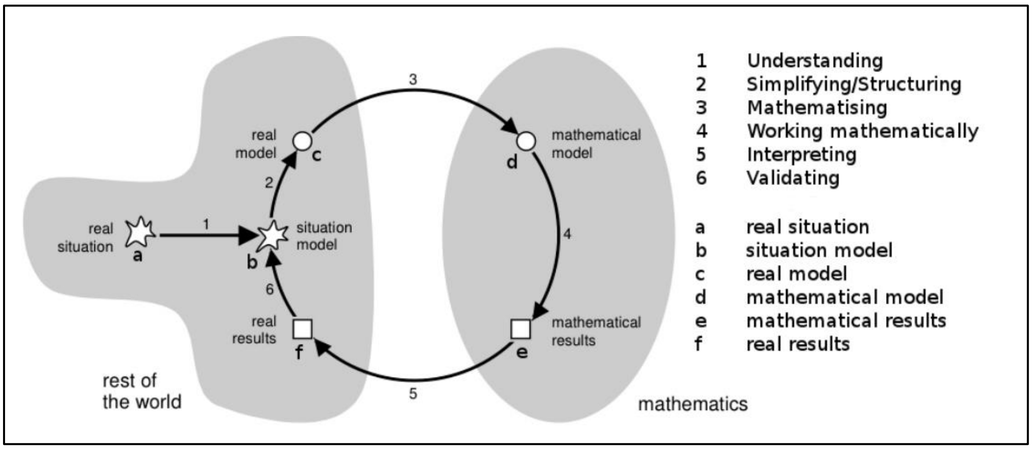


Figure 2.2 Modeling Cycle of Blum & Leiß (2007, p. 225)

In addition to the modeling process of individuals, how they progressed in the modeling stages during this process is another subject that gained importance. Specifically, this progression is named “modeling route” and defined as “the individual modeling process on an internal and external level” by Borromeo Ferri (2007, p. 265). The modeling route is formed by starting with a particular stage and going through other stages according to the preferences of the individuals. At this point, Borromeo Ferri (2007) stated that visible modeling routes could be determined by “verbal utterances and external representations.”

In the literature, it is seen that the researchers used the arrows between the stages in the modeling cycles to represent the modeling routes (Borromeo Ferri, 2007; Shahbari & Tabach, 2020). On the other hand, there is also another representation style to represent the modeling route shown in a graph (Albarracin et al., 2019; Ärlebäck, 2009; Czocher, 2016). Ärlebäck (2009) introduced a two-dimensional diagram called the Modeling Activity Diagram (MAD), an example of which is shown in Figure 2.3. Considering the other studies in the literature on the representation of the modeling route and the studies of Borromeo Ferri, MAD differed from the other studies with some points. Ärlebäck (2009) divided the modeling activities into six parts: reading, making model, estimating, validating, calculating, and writing, respectively, as seen in the vertical axis of MAD given in

Figure 2.3. As a different aspect, unlike the literature, Ärlebäck (2009) gave place to new activities for the MAD. While estimating and writing were added as new for the modeling routes, there are also matching activities with Borromeo Ferri's (2007) modeling transitions. Specifically, Ärlebäck's (2009) reading and calculating are similar to Borromeo Ferri's (2007) understanding of the task and working mathematically, respectively; making model is the incorporation of simplification/structuring and mathematizing, and validating is the incorporation of interpreting and validating transitions. Another important and different point about MAD is that Ärlebäck (2009) marked the horizontal axis of MAD as time. Thus he made it possible to see when individuals are in a specific activity and how much time they spend on it.

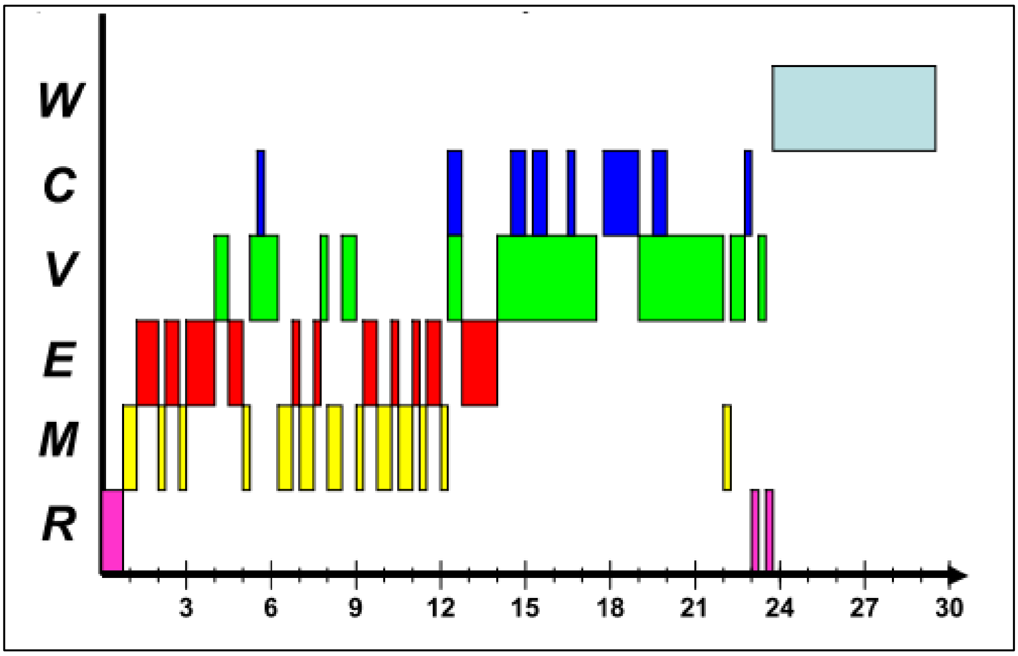


Figure 2.3 A sample Modeling Activity Diagram (MAD) in Ärlebäck (2009)

In her study, Czocher (2016) also used the two-dimensional diagram created by Ärlebäck. Unlike Ärlebäck (2009), Czocher (2016) used Borromeo Ferri's (2007) six transitions in the Modeling Transitions Diagram (MTD) in her study. An example of MTD used by Czocher (2016) is displayed in Figure 2.4 below. Czocher (2016) emphasized that it is difficult to determine how much time individuals spend in any specific transition, and therefore she preferred to mark the time with a dot when a specific transition was first observed in MTD, as seen in Figure 2.4.

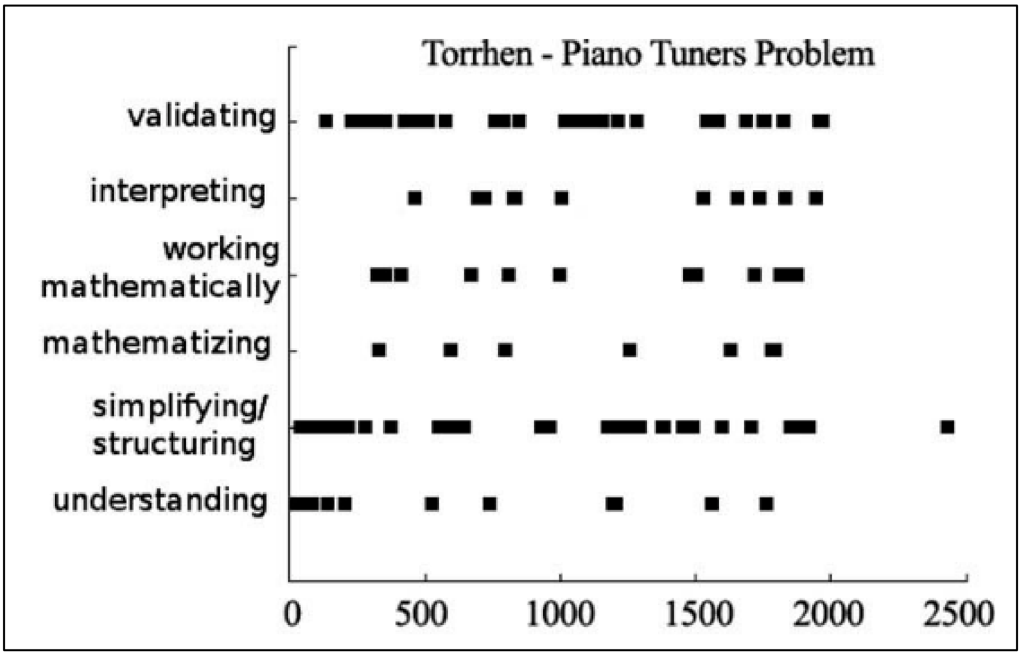


Figure 2.4 A sample Modeling Transition Diagram (MTD) in Czocher (2016, p. 97)

2.2 Theory of Registers of Semiotic Representations (TRSR)

In mathematics, representation can be considered as a way of expressing an individual's mathematical knowledge in different forms, such as using graphs, symbols, shapes, or language. Specifically, representations can also be "the sign and its complex associations," according to Duval (2006). At this point, the existence of the notion of semiotic representations can be mentioned. Duval (2020, p. 724) describes "the semiotic systems which fulfill a specific cognitive function" as a register. To classify the semiotic representations, Duval uses the transformations between these registers. According to TRSR, conversion is a transformation between two representations belonging to different semiotic systems, and treatment is a transformation between two representations belonging to the same semiotic system.

To clear up the semiotic representations and the registers to which they belong, Duval (2006) created the table showing their categorization. To begin with, according to TRSR, there are two kinds of the register to classify semiotic representations. These are the multifunctional and monofunctional semiotic systems (see the first and second rows in Figure 2.5). While the multifunctional register includes the natural language and geometrical shapes as a representation type, the monofunctional register includes symbolic writings (numerical systems, algebraic expressions), graphs, and diagrams. On the other hand, for the columns, semiotic representations are classified whether they are discursive or non-discursive representations. While discursive representations include numerical or algebraic expressions, definitions, and descriptions, non-discursive representations include geometrical figures, graphs, and diagrams. This classification can be seen in the following figure.

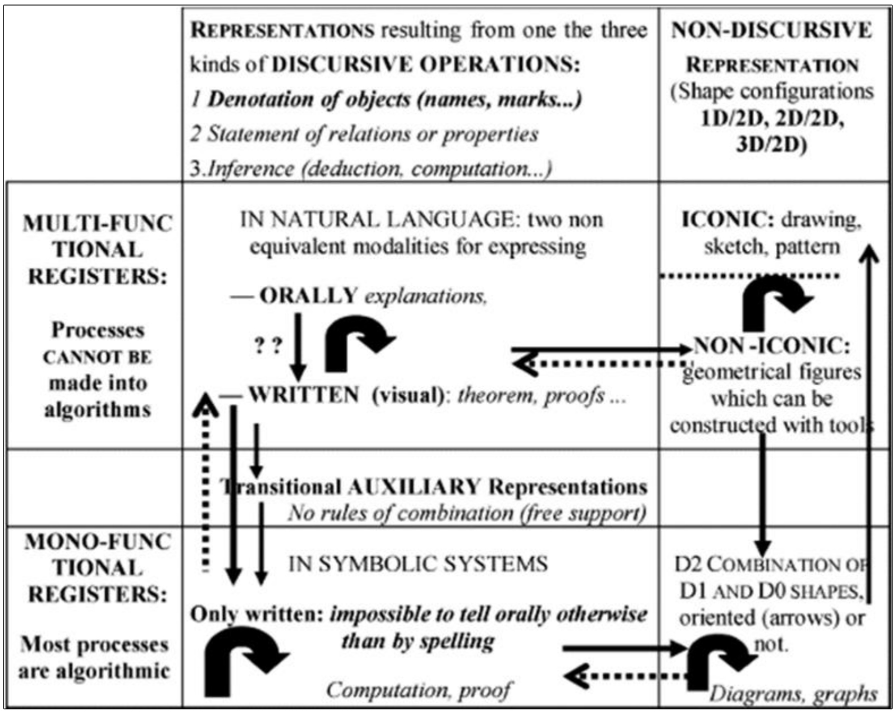


Figure 2.5 Classification of the registers (Duval, 2006, p.110)

As seen in the figure above, while rows show the multifunctional and monofunctional register, columns show the discursive and non-discursive representations, respectively. On the table, Duval (2006) used the arrows to stand for the semiotic actions named treatment and conversion. The curved arrows in the figure are treatment, meaning the transformation of the semiotic representations within the same register. On the other hand, the straight arrows (also dotted arrows) in the figure are conversion, meaning the transformation of the semiotic representations between two different semiotic registers. In other words, if any semiotic representation changes the cell to be expressed in another form, the action is named conversion. But if it remains in the same cell, the action is called a treatment.

2.3 Related Studies

2.3.1 Studies on Area of Irregular Shapes

The concept of the area has an essential place in the measurement strand of the curriculum. In the literature, there are various definitions of the area that is one of the measurement concepts. While Van de Walle and his colleagues (2012, p. 384) define the area as “two-dimensional space inside a region,” Dickson’s (1989, p. 79) definition of area is “the amount of surface of a region.” On the other hand, area measurement can be expressed as covering a region conceptually (Muir, 2007).

In measuring the area of a shape, students can develop or use different strategies. While measuring the area of a closed shape, children use strategies such as using area formulas for regular shapes, estimating by using standard and non-standard units, dividing the shape into sub-shapes, and reshaping to obtain shapes whose area formula is known (Civil & Khan, 2001; Kordaki & Potari, 1998; Muir, 2007; Rejeki, 2015). Although students can develop different strategies in measuring the area, they also have some difficulties in this topic. As Dickson (1989) stated, children generally see the area concept as multiplying the length by width. At this point, it is important to help students understand that the area means covering the surface rather than teaching it as a formula to be calculated (Muir, 2007). Indeed, difficulties with this concept result from the traditional teaching of area measurement, including rote memorization (Clements & Stephan, 2004; Tan Şişman & Aksu, 2016; Van de Walle et al., 2012). One of the most common difficulties is confusing the area with a perimeter (Huang & Witz, 2013; Machaba, 2016; Rejeki, 2015; Ryan & Williams, 2007). Similarly, on the concept of a circle, Dikkartin Ovez (2012) found that students have difficulty in making a distinction between the area and circumference of the circle. She also stated other students' challenges in mixing up the radius and diameter while computing the area of the circle. Moreover, children may not be aware of what is being measured while determining the area, and this may cause another misconception. Specifically, researchers reported that children use the length units in area measurement instead of area units (e.g., cm instead of cm^2) (Baturo & Nason, 1996: Tan Şişman & Aksu, 2016). Indeed, this misconception can be considered as the result of the difficulty in comprehending the relationship between dimensions and the interdimensional transition in length, area, and volume measurement; in other words, children think of an area in one dimension.

To overcome students' difficulties in area measurement, activities providing various experiences related to the area become an important matter. To illustrate, working with irregular shapes may be a great source of experience for students in area measurement. Several studies aim to investigate the issues of finding the area of irregular shapes (Civil & Khan, 2001; Kordaki & Potari, 1998; Muir, 2007; Papadopoulos & Dagdilelis, 2008; Rejeki, 2015; Stehr et al., 2018). When they encountered the irregular shape to determine its area, 3-4th grades students made an estimation through grid paper, used nonstandard units of measurement like the length of their finger, span, or multiplied the width and length of the shape (Civil & Khan, 2001) while 6th-grade students divided the irregular shape into familiar shapes which they know the area formula (Kordaki & Potari, 1998; Papadopoulos & Dagdilelis, 2008) and reshaped the irregular shape by using the cut-and-paste technique (Papadopoulos & Dagdilelis, 2008; Rejeki, 2015).

In a study with gifted middle school students, Şengil Akar and Yetkin Özdemir (2020) revealed that students could develop their creative thinking and generate products in high quality, originality and variability via the MEA in their study, including finding the area of irregular shape. According to this study, these students could display creative approaches to find the area of the shape given in “Guilt Problem.” On the other hand, Moore et al. (2015) conducted a study using an MEA named “The Pelican Colonies,” in which students tried to find the number of pelican nests placed within a given irregular-shaped area. They mentioned in their study that while elementary school students focused on the covering aspect, middle school students focused on decomposing irregular shapes into familiar shapes whose formulas were known. They reported students' solution strategies in “The Pelican Colonies” task as covering the entire irregular shape with centimeter cubes with gaps and square units in a regular and not regular array.

Within the scope of this study, the literature provides several studies, including technology-supported activities aiming to find the area of the irregular region (Palmas et al., 2020; Papadopoulos, 2004; Papadopoulos & Dagdilelis, 2008, 2009; Stehr et al., 2018; Yunianto, 2015). In technology-supported activities, while technology is used primarily to verify the solution strategies, students also use it in dividing the shapes into sub-shapes for the area measurement with the trial-and-error method and using distance and area tools (Papadopoulos, 2004). Indeed, this is evidence that the use of the technological tool in such activities facilities the learning process of students (Papadopoulos & Dagdilelis, 2009), and thus they can be focused on their conceptual understanding of the specific subject.

Another study, including the irregular shape activity supported by technology, was conducted by Stehr et al. (2018). In their study, 3-5 grades students tried to find the area of an irregular-shaped puddle via an applet. In this study, students initially had difficulty covering the area of the puddle since they were given only rectangles having a vertical and horizontal orientation to cover the irregular region in the applet. At this point, their difficulties pushed students to devise new strategies to fill the irregular region; thus, their thoughts about filling an area and covering meaning of the area measurement were supported through the technological tool. Also, Yunianto (2015) conducted a case study to teach area measurement by using an applet. In his study, allowing students to display different approaches to finding an area of the given shape, their understanding of the conservation of area was examined. According to the study, students could reshape the given figure into a rectangle through the cut and paste activities via the technological tool, but they used the trial-and-error method to obtain a rectangle when they encountered the irregular-shaped figure. At this point, the study reported that technological tools enabled students to create various strategies and to try these strategies in an easier way. Also, students' understanding of the area measurement was supported by the technological tool providing different strategies on area, and they were more creative and enthusiastic about finding new solutions thanks to the technological tool.

2.3.2 Studies on Semiotic Representations

Moyer-Packenham and her colleagues (2022) investigated the relationship between the semiotic actions of students aged 9-12 and their mathematics performance outcomes in digital mathematics games by conducting a study with a mixed methods design. To display this relationship, pretest and posttest were implemented for students in this study. In these tests, students' verbal responses and external representations were coded according to the four types of semiotic representation language, images, symbols, and gestures. The frequency of semiotic actions (treatment or conversion) between these semiotic representations was determined. According to the result of this study, while images and symbols were the most used semiotic representations by the students respectively, the most common semiotic action made was conversion. Furthermore, they revealed that students' familiarity with the representational transformations is related to their mathematics outcomes, and their performances from pretest to posttest are more likely to be affected positively if they are familiar with these transformations.

In the literature, there is another study based on semiotic representations and actions implemented in Turkey (Özcan et al., 2022). In the mentioned study, the conceptual understanding of middle school students in the circle concept was examined according to the semiotic representations in the TRSR framework of Duval (2006) by conducting a teaching experiment method. Within the scope of the flipped classroom approaches with the 5E inquiry model, Özcan et al. (2022) implemented tasks to the participants to teach the relationship between "the central angle and the length of the arc," "radius and perimeter," and the"π value" within the scope of the circle context by using GeoGebra Software. Participants' representations were examined in five semiotic systems, drawing, verbal, numeric, visual, and algebraic. According to their findings, it is seen that participants made a transformation between two representations belonging to different semiotic systems. Their study also stated that the GeoGebra tasks supported the conceptual understanding of the participants who made this transformation named conversion.

Shahbari & Tabach (2020) carried out a study to examine the modeling routes of preservice teachers having different characteristics in terms of their semiotic representations and the relation between preservice teachers' modeling sub-competencies and semiotic characteristics. In their study, two different types of learners having different semiotic characteristics were encountered when the mathematical models of pre-service teachers working as a group were examined. Also, it was observed that there were differences in the modeling routes of these groups, which were determined as numeric and algebraic models. According to the results of this study, while the groups determined as an algebraic model had a more complex modeling route compared to the groups using the numeric model, it was also reported that the modeling routes of the numeric models proceeded more sequentially.

2.3.3 Studies on Modeling Routes

Considering the part of this study about the modeling route, it can be mentioned that there are numerous studies on this subject in the literature (Albarracin et al., 2019; Ärlebäck, 2009; Borromeo Ferri, 2010; Czocher, 2016; Çakmak Gürel & Işık, 2021; Hankeln, 2020; Shahbari & Tabach, 2020). Within the theoretical framework of this study, there are studies examining the modeling routes and using MAD structure in the representation of that routes in this part.

Ärlebäck (2009) investigated the mathematical problem-solving behaviors of students at the upper secondary level in his study and introduced mathematical modeling with Fermi problems in different contexts. He developed the MAD framework, a graphical representation of modeling routes that reflects students' modeling processes (see Figure 2.3). There were six modeling activities in MAD, reading, making model, estimating, calculating, validating, and writing as alternatives to the six modeling transitions. This diagram enabled Ärlebäck (2009) to determine how much time was spent on the different modeling activities and follow which modeling activities were done by groups simultaneously. According to the result of this study, Ärlebäck (2009) revealed that group dynamics, such as group discussions, and sharing of group opinions and preferences, are crucial in shaping the modeling processes and problem-solving behaviors of groups.

In another study examining the modeling routes based on the MAD framework, Albarracin and his colleagues (2019) aimed to extend the MAD framework by implementing Fermi problems with secondary school students. They also expected to examine the potential and possibilities of the extended MAD. According to the result of this study, Albarracin and his colleagues demonstrated the complex problem-solving processes of students in modeling cycles on the MAD, and thus they could present a more detailed analysis tool for the students' problem-solving processes having their different choices and actions.

Lastly, Czocher (2016) constructed the Modeling Transition Diagram (MTD) (see Figure 2.4), developed based on the MAD, to examine the modeling routes of four undergraduate students studying engineering. In MTD, Czocher used the six modeling transitions, understanding, simplification/structuring, mathematizing, working mathematically, interpreting, and validating instead of six modeling activities, and therefore changed the name of MAD to MTD. Czocher (2016) developed a methodological tool including various indicators specific to particular modeling transition according to the actions of participants in different Fermi problems. According to her study, there were presented findings such as identifying no pattern in MTDs and mostly no sequential progression, and that mathematical/nonmathematical knowledge was essential for students' progression in mathematical modeling. Her study also supported that modeling is a complex process.

CHAPTER 3

METHOD

This study aimed to examine the characteristics of the pre-service mathematics teachers' mathematical models and modeling processes in terms of semiotic representations and how their modeling routes differed based on these characteristics in a technology-integrated model-eliciting activity. This chapter includes sections on the design of the study, the context of the study and participants, model-eliciting activity (MEA), data collection procedures and data sources, data analysis procedures, researchers' role and trustworthiness, respectively.

3.1 Research Design

A qualitative study is a research type that researchers aim to search and understand specific phenomena by collecting and analyzing data where the researcher is the primary instrument and presenting a rich description for that (Merriam & Grenier, 2019). According to the nature of the study aims and the research questions, qualitative research techniques were adopted in this study. Specifically, the researcher conducted a case study research method to answer the research questions. A case study is a research method aiming to explore the bounded system profoundly during the large-scale data collection process where a case represents individuals, organizations, processes, programs, institutions, or events (Creswell, 2011; Fraenkel et al., 2012; Yin, 2017).

Yin (2003) divided the case study design into four types, single-case (holistic), single-case (embedded), multiple-case (holistic), and multiple-case (embedded) according to the number of cases and unit of the analysis. In other words, while Yin (2003) categorized the case studies as a single or multiple according to the number of the case, he also divided the case studies into two as holistic and embedded according to the number of the unit of analysis. More specifically, embedded single case studies include one case and more than one unit of analysis (Yin, 2003).

In the current study, there was one case, the modeling processes of five groups consisting of pre-service mathematics teachers. In this regard, to investigate these processes, I determined two units of analysis: the shortest uninterrupted dialogue, and general solutions and actions of groups. Therefore, an embedded single case study approach Yin (2003) was adopted in the current study as a research design to answer the research questions.

3.2 Context of the Study and Participants

The participants of this study were 16 junior and senior prospective mathematics teachers studying in the elementary mathematics education program at a public research university in Ankara, Turkey. This department provides a certificate for them to be a mathematics teacher in middle school covering grades 5 to 8. They were enrolled in an elective course entitled "Mathematical Modeling for Teachers." In the program of elementary mathematics education, there were 43 courses including 37 must and six elective courses (see Appendix A). More specifically, in addition to general education courses and elementary mathematics education courses, must courses also included higher level mathematics courses such as Calculus, Differential Equations, Linear Algebra, and Elementary Geometry. On the other hand, courses such as Physics, Turkish and English language, Statistics and History were among must courses. Before participating in this study, participants reported that they had not taken any mathematical modeling courses.

In the course entitled "Mathematical Modeling for Teachers", which was taken for the first time by the participants in the context of "Model and Modeling Perspective", different objectives were aimed for the development of pre-service teachers. The main objectives of this course were explaining the characteristics of modeling activities and their difference from the other mathematics problems, improving pre-service teachers' modeling competencies, using their mathematical knowledge in modeling activities, and implementing modeling activities and using technology in their teaching. In line with these objectives, the pre-service teachers both acquired knowledge about mathematical modeling and worked on various modeling tasks such as "Water Tanks" and "The Summer Job" during a 14-week course period. The modeling task that was used to collect data was implemented after the prospective teachers had familiar with model eliciting activities and the process of mathematical modeling. Pre-service teachers took the course in the "Mathematics Laboratory" classroom where there was access to technological devices and mathematical tools.

Pre-service teachers working as a group of 2-3 actively participated in the lesson during the three-hour weekly lesson period by the course requirements. In this sense, each of them was expected to share their knowledge and ideas within the group and class discussion, and take on a task such as using technology, making calculations, writing a report, and presenting their modeling process in a modeling activity. Additionally, the responsibilities of the pre-service teachers after the lesson were to read the articles of the course, write a reflection paper related to mathematical modeling and implement a modeling activity as a final project created by them.

At the beginning of the semester, the participants were asked to form the groups that they wanted to be in. During the implementation of Model-Eliciting Activity (MEA), participants participated in modeling activity in six groups including 2 or 3 members. Although all groups wanted to be involved in the study, one of the groups having three participants was eliminated because of data loss resulting from the inaudible voices of group members. Therefore, the data of five groups were used in this study and the number of the remaining participants was 13 (all of them female).

Each group was named alphabetically in order of analysis and the names of the participants were replaced with pseudonyms with the first letter starting with the group name. Group A included three participants whose pseudonyms are Ahsen, Asya, and Aylin. The mathematical modeling process of Group A was audiotaped and videotaped during the MEA. Within the group, participants had various duties.

While Ahsen got involved in the process by writing a report predominantly, Aylin used the technology to represent their mathematical model. Apart from that, all group members contributed to the development of various solutions and models.

Group B also included three participants, Bahar, Beliz, and Berna. In this group, while Berna got involved in the process by writing a report and using the technology in the process, Bahar and Beliz mainly worked on calculations and measuring. In the development of the model, all members of the group contributed to the process.

Group C included two participants, Canan and Ceren. Both of the participants contributed to their modeling process by producing various solutions and creating a model. Predominantly, while Canan worked on calculations and measuring, Ceren used technology for the representation of the model.

Group D included two participants, Deniz and Doğa. Since the use of technology was optional in this study, this group did not use it due to the personal preferences. Except for this, participants of this group worked together on calculations and measuring for the development of the model.

Group E included three participants, Ece, Elçin, and Eylül. While Elçin worked on the writing of the report and Eylül used the technology mainly, all participants produced new ideas to improve their mathematical model.

Within the context of the current study, it was also important characteristic whether pre-service teachers were familiar with technology in advance or not. The prospective teachers' background information shows that most of them had taken an elective course entitled "Exploring Geometry with Dynamic Geometry Applications" before. In this lab-based elective course, some of the main goals were to introduce pre-service mathematics teachers with the GeoGebra Software and teach them how they use this program. In each group, there was at least one pre-service teacher taking this elective course.

3.3 MEA: Tumor Surgery

As an MEA, the “Tumor Surgery” task (Hall and Lingefjärd, 2017, p.136) was adapted and used in this study. In 14-week course period, “Tumor Surgery“ task was implemented in 8^{th} week. The context of the task is related to tissue with a tumor in the lung that could not be treated with medicine. In this task, groups are expected to determine the size and location of this irregular-shaped tumor to operate it provided that minimum healthy tissue is taken with the whole tumor. One of the important requirements expected from the groups is to prepare a practical guide for similar surgeries. The Tumor Surgery task is given in Figure 3.1.

“Tumor Surgery” task served three general course aims. One of these aims was related to development of pre-service teachers’ modeling competencies. Within the scope of this aim, the implemented task may enhance pre-service teachers’ modeling competencies such as understanding the real problem situation, constructing a mathematical model, solving mathematical questions by using the created model, interpreting the reached mathematical results in context of the problem and validating the conditions and assumptions by evaluating the results. Moreover, this modeling task can enable the pre-service teachers to apply their mathematical knowledge to solve nonroutine real-world problems. Lastly, “Tumor Surgery” task may also develop pre-service mathematics teachers’ reasoning and communication skills by providing an environement to use mathematical language and representations.

TUMOR SURGERY

A disc-shaped tumor has been diagnosed in one of the patients' left lung as shown by the lesion in the figure. Since it is not small enough to be treated with conventional medication, the doctors decided the patient needed surgery to remove the tumor.

Figure: Chest x-ray showing the lesion due to the tumor in the left lung

To be successful, the surgery needs to be as much precise as possible by removing minimum amount of tissue without leaving any piece of tumor. It is known that the right lung is 2.5 cm shorter than the left one (source: http://www.theodora.com/anatomy/the_lungs.html). As a member of the surgery team, you were asked to locate the exact position and the size of the tissue disseminated with tumor that is going to be operated on. As part of this task, you were also asked to prepare a (practical) guide for determining the size of any such tumor tissue for future surgeries.

Figure 3.1. MEA: Tumor Surgery Task [Adapted from Hall and Lingefjärd (2017, p.136)]

3.4 Data Collection Procedures and Data Sources

Data was collected in a mathematics classroom that provided an environment suitable for group work and provided easy access to needed technological and concrete materials. The seating arrangement of the classroom during the MEA was U-shaped, and Figure 3.2 shows where the groups sit. The groups freely selected the desk that they would use throughout the semester.

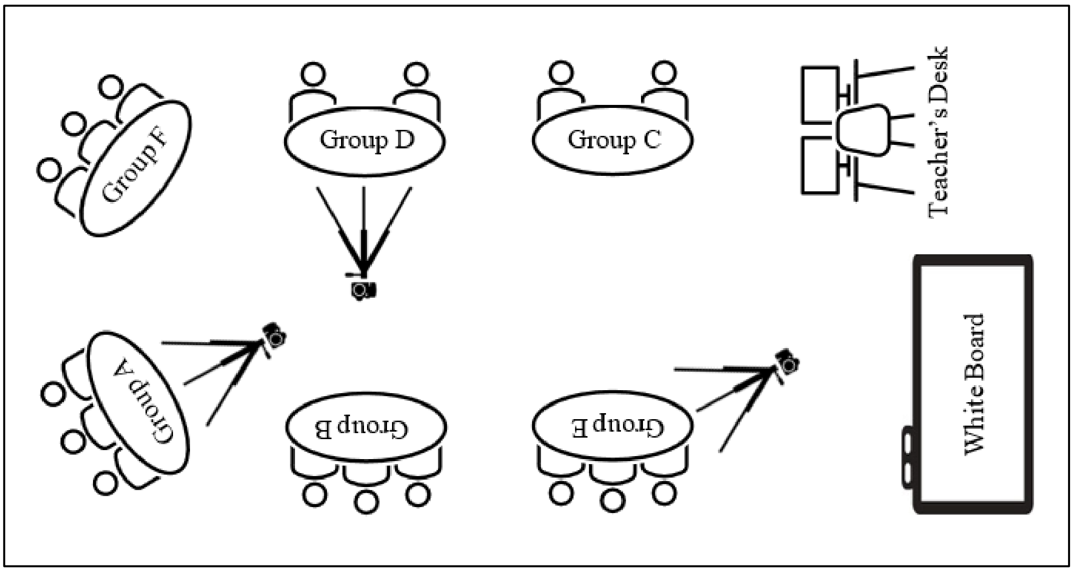


Figure 3.2 Seating Arrangement

Pre-service mathematics teachers were provided a (digital and printed) copy of an x-ray image of a lung having tumor tissue, a grid paper, a ruler, a compass, and a notebook. During the data collection process, they were informed that they can use any technology or application whenever they felt they needed. All groups using technology preferred GeoGebra Software and each group had an access to it during the activity. The data were collected in a three-hour course period. In the implementation of the MEA, there were two instructors and one assistant involved. Instructors conducted all processes, guided pre-service teachers' modeling processes, and facilitated small group and whole class discussions throughout the lesson. In addition to these, one of the instructors and the course assistant dealt with technical issues such as managing audio recorders and cameras to collect data, and helped pre-service teachers with their questions regarding the use of GeoGebra Software. The course assistant also was responsible for providing necessary instructional materials to the groups during the implementation of the activity.

Three groups (Group A, Group D, and Group E) were both audiotaped and videotaped, the remaining (Group B and Group C) were just audiotaped during the activity, and their GeoGebra files including their solutions (apart from Group D because the technology was not used) were recorded. Furthermore, the groups recorded their process in their report including their mathematical models, findings, and guide for future operations. After the implementation of the modeling task, all groups shared their mathematical models and findings with the whole class. Thus the main data used to analyze pre-service mathematics teachers' modeling processes were audiotaped and videotaped classroom observations. The groups' solutions and reports on the modeling task were used to better understand their modeling processes.

3.5 Data Analysis

As it was mentioned before, the main data for the study was gathered via groups' audiotaped and videotaped observations while they worked on the modeling task. Thus in analyzing the data, I initially watched all the videos and listened to the audio recordings carefully. Then, all of these recordings were transcribed word by word to prepare the data for the analysis. To answer research questions, I aimed to determine the modeling routes of the groups and the characteristics of their modeling processes in terms of semiotic register. In this section, I first explained the development of the data analysis approach, then briefly describe the rubric developed to analyze modeling routes and the details of data analysis procedures.

3.5.1 The Process of Deciding on Data Analysis Procedures

A data analysis framework was developed to identify modeling routes and semiotic representations of the groups. In this section, I explain how the data analysis framework was developed by focusing on the ways of determining the representation of the modeling route and modeling transitions, the rubric for determining the modeling route, and methods of determining the semiotic representations.

3.5.1.1 Determination of the representation of the modeling route and modeling transitions

One of the main goals of this study was to determine the modeling routes of groups. Because of this, as a first step, I tried to divide the whole transcript into dialogues. This is needed to assign groups' actions to the correct modeling transitions and determine their modeling routes more easily since transcripts contained long modeling processes that took almost 2 hours to complete the modeling task by the pre-service teachers in the current study. Ärlebäck (2009) also used the same method and stated that the categorization of the transcript was done according to the modeling sub-activities in the dialogues formed with the sequence of utterances made by the groups. Then, the data of the two groups were organized by adopting the data formatting structure developed by Shahbari and Tabach (2020). According to their research, groups' general solutions and actions are placed in a table so that modeling phases and transitions are ordered from understanding to validating. This system did not work in this study because the modeling processes of groups took a long time, and the categorization of the modeling transitions of the whole transcript was too difficult. Moreover, some problems were observed in the representation of modeling transitions when two groups were analyzed. One of the problems was that most of the data were ignored and not included in the modeling routes of the groups. Another problem was that the representation of these modeling routes in the extended modeling cycle, including the real, mathematical, and technological world

(Greefrath et al., 2011), was too complex in this study because participants' actions were placed many times in different modeling transitions. It was almost impossible to show them with arrows clearly. Lastly, the structure of modeling transitions continuing step by step in an orderly way from understanding to validating did not fit this study's data. Therefore, the representation of modeling routes with arrows as in the Shahbari and Tabach's (2020) study was not preferred in this study. This study was not used in the parts related to the representation of the modeling routes, but their study sheds light on the current study in the determination of the semiotic representations according to the particular modeling stages/transitions mentioned in the following parts of the data analysis (see Table 3.5).

As a result of all these reasons, I needed a new modeling cycle structure in which all recorded modeling transitions can be shown, and new stages such as technology can be included. At this point, I decided that Ärlebäck's (2009) Modeling Activity Diagram (MAD) and Czocher's (2016) Modeling Transitions Diagram (MTD) would be more useful in representing and analyzing our data. In his study, Ärlebäck (2009) created a graphical representation called MAD whose horizontal and vertical axis represent time and modeling activities, respectively. As mentioned in the literature review chapter, in the MAD, Ärlebäck (2009) categorized the modeling activities into six: reading, making model, estimating, validating, calculating, and writing. Constructing a similar structure as in MAD, Czocher (2016) uses Borromeo Ferri's six transitions in the MTD: understanding, simplification-structuring, mathematizing, working mathematically, and interpreting-validating. Borromeo Ferri (2007) used the mentioned transitions in the modeling cycle of Blum & Leiß (2007) (see Figure 2.2) in order to represent individual modeling routes of students with arrows on that modeling cycle. In the MAD, Ärlebäck used the phrase "modeling activities" instead of "modeling transitions". In his framework, Ärlebäck included activities similar to Borromeo Ferri's modeling transitions, but also he added new activities to the MAD, like estimating and writing. Ärlebäck's reading and calculating activities are similar to Borromeo Ferri's understanding of the task and working mathematically transitions; making model activity is similar to simplification/structuring and mathematizing, and validating activity is similar to interpreting and validating transitions in Borromeo Ferri's model (see Figure 2.3 for Ärlebäck's MAD and "modeling activities").

For this study, using Ärlebäck's (2009) MAD and Czocher's (2016) MTD was meaningful as they allowed displaying the modeling transitions exactly when they were observed. Czocher's (2016) MTD was also helpful in developing a rubric to identify and interpret the modeling routes of the groups. In this study, in the preliminary analysis, Ärlebäck's framework was used with three revisions (see Table 3.1 below). One of them was removing the estimating activity from the framework since it was not used in the Tumor Surgery problem. Thinking about the nature of the modeling task, I also found that "working mathematically" would explain the modeling process better than "calculating". In addition to these, the technology stage was added to the framework. As a result of these changes, in this study, the following transitions were initially used: reading, making model, working mathematically, technology, validating, and writing.

After the preliminary data analysis using this framework, I noticed that the actions of all groups were intensively assigned to the "making model" transition. This may be caused by the scope of the making model transition which encompasses simplification/structuring and mathematizing transitions, and thus it had too many indicators in the developed rubric to assign the transitions. Furthermore, activity terminology was found to be not handy in analyzing and explaining the transformation between semiotic registers, which was the other issue explored in this study. Ultimately, I decided to use Borromeo Ferri's transitions like in Czocher's study with some additions and changes. While six transitions were used verbatim, using technology was added to MTD. Also, the writing stage was changed to reporting because groups reported their processes by writing a report, conveying it to the instructor, and presenting it at the end of the class. In Table 3.1 below, modeling transitions used in this study were given together with Ärlebäck's MAD and Czocher's MTD frameworks.

Table 3.1 The modeling activity/transitions frameworks of Ärlebäck (2009), Czocher (2016), and the current study

Ärlebäck's MAD	*Czocher's MTD*	*The Current Study*
Reading	Understanding	Understanding
Estimating + Making Model	Simplification/Structuring + Mathematizing	Simplification/Structuring + Mathematizing
Calculating	Working Mathematically	Working Mathematically
Validating	Interpreting + Validating	Interpreting + Validating
-	-	Using Technology
Writing	-	Reporting

3.5.1.2 Development of a Rubric for Modeling Routes

In this study, the methods used in Ärlebäck's and Czocher's studies were a guide in analyzing and representing modeling routes. One of these methods was related to identifying the modeling transitions. Since the data included long and intense dialogues, assigning modeling transitions using sub-competencies mentioned in the study of Maaß (2006) was inadequate as these competencies were too general for this study. On the other hand, the rubric provided in Czocher's (2016) study was handy as it included an indicator table for each particular observed event in order to assign modeling transitions of participants. This rubric was used in this study with various additions and changes. While some of the indicators were used as they were, the wordings in some of the indicators were revised to better explain the data collected in this study. For example, Czocher's "mentioning variables, parameters, constants" indicator was changed as "mentioning components of the model (e.g., Radius, center, focus, origin…) in the extended rubric because groups' models were related to shapes instead of equations and they spoke intensely on the components of that shapes. Secondly, since the modeling task, Tumor Surgery, involved requirements such as technology use and report writing, new indicators were added to meet these requirements. To do this, the studies including technology-supported MEAs and mathematical modeling competencies were examined. Moreover, the works of Maaß (2006), Galbraith and Stillman (2006), Ärlebäck (2009), and Hıdıroğlu (2012) were used in adding and refining the indicators of the rubric. After a pre-analysis of the data, a few new indicators were added (see the explanations below). The extended rubric is given in Table 3.2 below.

Table 3.2 Extended Rubric Including New Indicators

Modeling Transitions	*Code*	*Indicators*	*Source*
Understanding	U1	Reading the task	Czocher (2016)
	U2	Returning to elements in the statement of the task	Czocher (2016)
	U3	Clarifying what needs to be accomplished	Czocher (2016)
	U4	Explaining/Expressing the problem/task in simple terms	Hıdıroğlu (2012)
	U5	Questioning the given information	Researcher
Simplification/ Structuring	S1	Making assumptions to "simplify" the problem	Czocher (2016)
	S2	Referring to assumptions	Czocher (2016)
	S3	Listing possible solutions/models/strategies	Czocher (2016)
	S4	Mentioning variables, parameters, constants	Czocher (2016)
	S5	Mentioning components of the model (e.g., Radius, center, focus, origin…)	Czocher (2016)
	S6	Specifying conditions	Czocher (2016)
	S7	Introducing outside knowledge	Czocher (2016)
	S8	"Running out" of conditions, assumptions, variables, parameters	Czocher (2016)
	S9	Drawing or labelling sketches that correspond to stated or implied conditions/assumptions/variables/parameter	Czocher (2016)
	S10	Identifying strategic entit(ies)	Galbraith & Stillman (2006)
	S11	Specifying the correct elements of strategic entit(ies)	Galbraith & Stillman (2006)
	S12	Counting/Measuring by using material to solve the task	Researcher
Mathematizing	M1	Writing/Drawing mathematical representations of ideas (e.g., symbols, equations, graphs, tables, shapes etc.)	Czocher (2016)
	M2	Speaking in terms of symbols, operations, or relationships	Czocher (2016)
Working Mathematically	WM1	Explicit mathematical operations that may not be arithmetic/algebraic (e.g., comparing, rounding, partitioning)	Czocher (2016)
	WM2	Making inferences and deductions without reference to nonmathematical knowledge	Czocher (2016)

Table 3.2 (continued)

	WM3	Carrying out symbolic or verbal operations (e.g., solving an algebraic equation, taking a derivative)	Czocher (2016)
	WM4	Changing mathematical representation	Czocher (2016)
Interpreting	I1	Speaking about the result in context of the problem	Czocher (2016)
	I2	Referring to conditions/ variables/parameters from "simplifying/ structuring"	Czocher (2016)
Validating	V1	Implicit or explicit statements about the reasonableness of the answer/model	Czocher (2016)
	V2	Checking extreme cases by using the developed model	Czocher (2016)
	V3	Comparing an answer to a known (theoretical or practical) result	Czocher (2016)
	V4	Comparing merits of different models	Czocher (2016)
	V5	Generalizing solutions (strategies) that were developed for a special situation	Maaß (2006)
Using Technology	UT1	Choosing the appropriate technology to represent the model *This also belongs to the simplification/structuring transition.*	Hıdıroğlu (2012), Galbraith & Stillman (2006)
	UT2	Switching between technological and mathematical representation	Hıdıroğlu (2012)
	UT3	Utilizing the (visual) opportunities of technology (enlargement of the image, making background image, using GeoGebra grids as base, encolouring, thickening)	Hıdıroğlu (2012)
	UT4	Obtaining necessary data/information for the model by using technology	Hıdıroğlu (2012)
	UT5	Using technology to produce geometric representations *This also belongs to mathematizing transition.*	Galbraith & Stillman (2006)
Reporting	R1	Writing a report	Ärlebäck (2009)
	R2	Reporting/Conveying what has been done to instructor	Researcher
	R3	Presenting of the modeling process to the class	Researcher

As seen in Table 3.2, the extended rubric includes modeling transitions, codes, indicators, and references columns. The codes are created according to the first letter of the related transition and the number of indicators belonging to that transition. In the extended rubric, there are four indicators created by me. Their codes are U5, S12, R2, and R3. U5 includes the action of making sense of and questioning the given information, which is the difference between lungs' sizes. S12 corresponds to the actions regarding groups' measure of the size of an x-ray for establishing a proportion between the size of it in reality and in the x-ray. R2 includes the conversation between group members and instructors, and R3 includes the presentations of the groups.

In the current study, two indicators of the Using Technology transition/stage were used in another transition. UT1 was included in the Simplification/Structuring transitions and UT5 was included in the Mathematizing transition. These indicators were taken from the study of Galbraith and Stillman (2006) for the technology stage of the extended rubric. I initially included them in the transitions of Mathematizing and Working Mathematically as in the study of Galbraith and Stillman (2006). Then, the transitions that they are involved in were changed, due to the content of MEA. Since the key elements in Tumor Surgery task were the area and location of the tumor, geometrical shapes used to remove this tumor aimed to both create a mathematical model and generated a mathematical representation of the ideas. Then, groups continued their actions by finding an area and a location through this model. Therefore, it was more plausible to include UT5 in the Mathematizing transition instead of Working Mathematically. On the other hand, UT1 was involved in the Simplification/Structuring transition since the choice of technology is related to the structuring of the given situation in the activity.

3.5.1.3 Determination of Semiotic Representations

In the current study, the work of Shahbari and Tabach (2020) was used as a base for analyzing semiotic representations of groups. In their study, groups' modeling actions were included in six modeling transitions/phases. Then, within the scope of Duval's (2006) framework related to semiotic representations, these actions were analyzed using three types of semiotic registers, natural, numeric, and algebraic registers; and two types of semiotic actions, treatment and conversion. In this part of the data analysis, I initially watched videos and listened to audio recordings carefully to determine the semiotic registers that the groups used. Here, I identified another semiotic register in addition to the registers in Shahbari and Tabach's (2020) study. As the context of the implemented modeling task required, the groups were also used geometric register in addition to the natural, numeric, and algebraic semiotic registers. Therefore, in the current study, four types of semiotic representations were used to determine the semiotic characteristics of the groups.

3.5.2 Data Analysis

The data were analyzed in two steps in the current study. The first one was to determine the modeling routes of the groups, while the second one is to determine what characteristics the groups have in terms of semiotic registers. Hence, this section explains how data was analyzed in depth regarding modeling routes and semiotic registers.

3.5.2.1 The Analysis of Modeling Routes

As mentioned before, to prepare data for analysis, each word spoken by the group members was transcribed word by word. The completeness of the transcript for this part of the analysis was important to determine the units of analysis. The unit of analysis can be each expression (utterance) as in Czocher's (2016) study where she analyzed each individual's modeling routes or it can be the shortest uninterrupted dialogue as in Albarracin and his colleagues' (2019) study where they analyzed student's modeling routes as groups. In the current study, since we examined the modeling processes of the groups, like in Albarracin et al. (2019), it was more appropriate to determine the unit of analysis as the shortest uninterrupted dialogue. Thus, to analyze data, the dialogues of the groups whose modeling processes were transcribed were broken into the shortest uninterrupted dialogues. These dialogues were then transferred into a spreadsheet to organize the data for analysis. In the spreadsheet, the following headings were used to systematically analyze the data:

time in the format of seconds and hh:mm:ss, dialogues, indicators, indicators' codes, and modeling transitions, respectively.

After splitting the transcript into dialogues and getting the spreadsheet ready to organize the data, the first step was to analyze each of the units in itself. The analysis of dialogues proceeded as follows. To begin with, the dialogue was examined in detail, and the video recording was carefully monitored simultaneously. Then, using the rubric (see Table 3.2) the expressions in the dialogue were assigned to the indicators of the relevant modeling transitions. Although some expressions were sufficient to assign the dialogue to any transition, it was important to look at the dialogue holistically. Because participants worked as a group in this study, dialogues might include some expressions independent from the process of the group. At this point, it was more reasonable to analyze dialogue with a holistic approach instead of micro-level analysis.

In any unit of analysis, there may be one or more indicators involved in the same or different modeling transition. Thus, a dialogue may include different modeling transitions. For instance, while understanding the situation, a group may also make assumptions as a next step in the same dialogue. Thus, in analyzing the data, the group's actions were sometimes assigned to two different transitions. Also, any expression or action in each unit of analysis might be assigned to two different indicators or modeling transitions. Specifically, when indicators of UT1 and UT5 (see Table 3.2) were assigned for any action, it was also placed in the simplification/structuring and mathematizing transitions. This situation was also possible for the other indicators and illustrated in the sixth row of the data analysis table given below (see Table 3.3). Lastly, some of the expressions and utterances that were not related to the focus of the study were not included in the analysis.

Considering all of these, another important point here was time. In this study, as Czocher (2016) did in her study, the first moment when each new transition observed in the dialogue was recorded. In the time (hh:mm:ss) column, the exact time of the observed action was recorded according to the time of the video recording. Then, it was converted to seconds in order to represent the recorded time in the MTD framework, and it was placed in the first column of the analysis table (see Table 3.3). In addition to this, the start time of the dialogues was also recorded to easily follow the dialogues. Another thing that was ignored in the analysis was break times. Because it may cause misinterpretation while reading MAD, the break time of the group was not involved in MAD.

The graphical representation of MTD was created by using GeoGebra. The sample MTD created in this study is displayed in Figure 3.3 below. While recorded times were placed in the horizontal axis of the MTD, the modeling transitions were in the vertical axis. After each of the dialogues was assigned to modeling transitions, they were transferred to MTD with recorded time for the representation of modeling routes of groups. In MTD, each modeling transition is enumerated from 1 to 8 respectively. While transferring identified transitions and recorded times, the command (in the format of recorded time in seconds, the number of related modeling transition) was entered into the input in GeoGebra. This operation helped me to mark the transitions with recorded time in MTD.

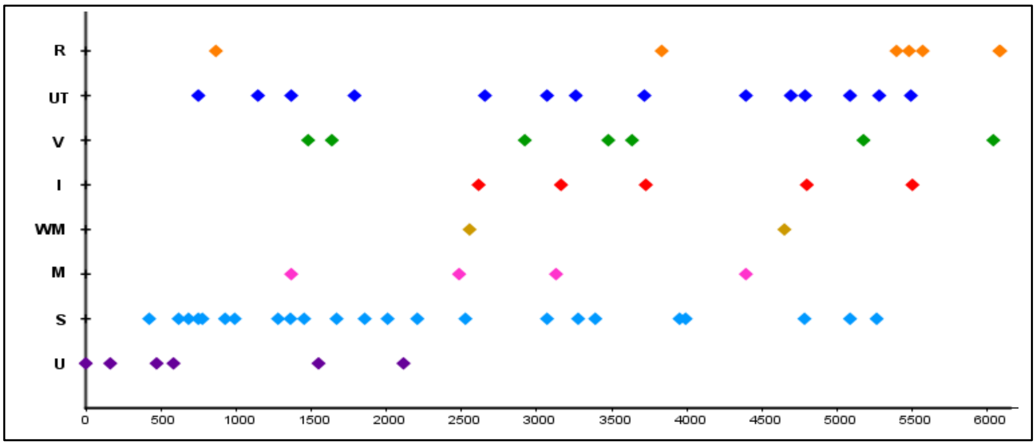


Figure 3.3 A Sample Modeling Transitions Diagram (MTD) created in the current study.

The abbreviated "modeling transitions", understanding, simplification/ structuring, mathematizing, working mathematically, interpreting, validating, using technology, and reporting take place in the vertical axis of MTD.

Table 3.3 A Sample Data Analysis Table for Modeling Routes

Time (Seconds)	Time (hh:mm:ss)	Sample Dialogues	Indicator	Code	Modeling Transition
237	00:03:57	Ahsen: We'll actually draw a circle.	Making assumptions to "simplify" the problem	S1	Simplification/ Structuring
243	00:04:03	Ahsen: So, what does the fact that the right is 2.5 cm shorter than the left show us? The tumor is already on the left. Aylin: Yes, I did not understand it. Maybe, the tumor enlarges 2.5 cm on the left! Asya: Sir, I think we don’ t understand.	Questioning the given information	U5	Understanding
255 261	00:04:15 00:04:21	Deniz: It's going to be something like reporting again, like why did I think of that. T: Yes. You will prepare a practical guideline that can also be used for further operations. Ahsen: Hmm. Are we going to do something like what it would be like if it was on the right? Asya: Are we going to decide where to remove that tissue from?	Clarifying what needs to be accomplished	U3	Understanding
283 289 295	00:04:43 00:04:49 00:04:55	T: Let's not think of the liver as a volume right now; you can just think of it as a surface area. Ahsen: For example, when we think of it as a circle… T: So, think it in two dimensions. Imagine you remove it in the form of a two-dimensional disk. The area of the disk… Ahsen: Okay, it’ s a circle.	Making assumptions to "simplify" the problem Implicit or explicit statements about the reasonableness of the answer/model	S1 V1	Simplification/ Structuring Validating

Table 3.3 (continued)

3863 3875	01:04:23 01:04:35	Aylin: Now, where will the focus points of that ellipse be? Ahsen: The corner points… Asya: Those two. Aylin: Shall we accept these two as the focus points? Ahsen: Let' s accept the farthest. Asya: She says let's draw two ellipses, try it, how will it be? Aylin: This one is too big, This one doesn't cover it, so let me draw another one. Asya: For example, we can draw an ellipse to cover it, but what will be its plus and how will we determine the focus points? Aylin: Yes. The intersections of those two or something!	Mentioning components of the model (e.g., radius, center, focus, origin…) Making assumptions to "simplify" the problem Using technology to produce geometrical representations Speaking about the result in context of the problem	S5 S1 UT5 M I1	Simplification/ Structuring Using Technology Mathematizing Interpreting
4369 4388	01:12:49 01:13:08	Ahsen: How can we generalize it now? Aylin: By choosing the farthest thing and… Asya: Draw the most absurd thing in the world… Aylin: Determine the two farthest points… Asya: For example, if the tumor is like this, will we make an ellipse again or a circle in here? Aylin: I' m looking for tumor shapes on the Internet Ahsen: Isn't it already going to approach the circle at one point? Here, for example, you drew something round. What will you do? I assumed the farthest points are these. I took the midpoint of the midpoint of them, and I'll draw an ellipse through there. Ellipse already becomes a circle somewhere if that length is equal to that length. So, it will fit all tumors when I draw an ellipse.	Checking extreme cases by using the developed model Making inferences and deductions without reference to nonmathematical knowledge	V2 WM2	Validating Working Mathematically

3.5.2.2 The Analysis of Semiotic Registers

The second step of the data analysis was to determine the characteristics of the groups in terms of semiotic registers. In the Theory of Registers of Semiotic Representations (TRSR) framework, Duval (2006) includes various semiotic representations such as symbolic writings and natural language. In this study, considering the content of the modeling task, I categorized representation types into four groups: natural, geometric, numeric, and algebraic. At this point, the descriptions of three types of semiotic representations (natural, numeric, and algebraic) were taken from the Shahbari and Tabach's (2020) study because their analysis method was used in a similar way in the current study. As a different semiotic representation, the description of geometric representation was incorporated from the Duval's (2006) TRSR framework. To be more precise how I coded these four types of representations, the explanations of semiotic representations with their examples are given in Table 3.4 below.

Table 3.4 Explanations and Examples of Semiotic Representations

Semiotic Representations	*Explanations*	*Examples*
Natural	Verbal statements not including numeric or algebraic notation (Shahbari & Tabach, 2020)	- Explaining and simplifying the situation (e.g., Tumor can be considered as circle.) - Interpreting the created model or mathematical results (e.g., The created shape does not cover the entire tumor.) - Validating actions verbally (e.g., Ellipse model is more appropriate than the circle for the different tumor shapes.)
Numeric	Representations including just specific numbers (Shahbari & Tabach, 2020)	- The size of the tumor is 53.5 cm^2 - Establishing a ratio between image and real distance between two lungs as 14:2.5

Table 3.4 (continued)

Geometric	Geometrical shapes created by tools, drawings, and sketches (Duval, 2006)	- Drawing a circle around the tumor on x-ray - Created geometric shapes such as ellipse, circle, and pentagon in the GeoGebra environment - GeoGebra CAS (Computer Algebra System) inputs not including algebraic notations such as FitImplicit({F, G, C, J, E, I, D, H}, 2)
Algebraic	Representations including variables (algebraic notations) (Shahbari & Tabach, 2020)	- The area formula of a circle is πr^2 - The location of the tumor is c: $(x - 8.8)^2 + (y - 18.7)^2 = 17.64$ - The input command including algebraic notations such as $(x\text{-}a)^2 + (y\text{-}b)^2 = r^2$

In the study, the groups' general solutions and actions were examined for the semiotic registers according to Duval's TRSR. In analyzing data, I used the suggestions of Shahbari and Tabach (2020) who also used Duval's TRSR. To begin with, three of the semiotic representations (natural, numeric, and algebraic) were coded as described by Shahbari and Tabach as seen in Table 3.4. Depending on the context of this study, the three registers discussed in their study were used in the same way, only the geometric register was used in accordance with Duval's (2006) description. Similar to Shahbari and Tabach's study, a unit of analysis for this part was determined as general solutions and actions of groups. In this regard, their presentation style in the categorization of the modeling competencies according to modeling transitions (understanding, simplification/structuring, mathematizing, working mathematically, interpreting, and validating), modeling phases (situational model, real model, mathematical model, mathematical results, and real results), semiotic representations (natural, numeric, geometric, and algebraic) and semiotic actions (treatment and conversion) was also used in the current study exactly the same like in the study of Shahbari and Tabach (2020). Initially, groups' general solutions and actions were determined in accordance with the modeling transitions and phases. Then, modeling transitions and phases were placed in a regular progression in a table as seen in Table 3.5 below. Here, regular progression means that modeling transitions/phases are put in order from understanding to validating without moving backward in the modeling transitions. Each new part of the solution was determined as a new cycle. As a next step, I classified each action according to four types of semiotic representation. Then, semiotic action was classified as a treatment if the representation types of the two adjacent actions are the same. If the representation types changed (for instance, changing from natural to numeric), the semiotic action was classified as a conversion. Table 3.5 illustrates the second part of the analysis of this study.

Table 3.5 The modeling transitions and semiotic actions of Group C in the first cycle

1st Cycle			
Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/ Structuring		Natural	Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by using the "Circle through three points" tool
	Conversion		For the location of the tumor: The chest x-ray can be put on a coordinate system.
Mathematizing		Geometric	Identifying several points around the tumor on GeoGebra.
	Treatment		Drawing a circle via "Circle through three points" tool on GeoGebra by using identified points

Table 3.5 (continued)

Mathematical Model		Geometric	Circle passing through the identified points around the tumor (K, L, M points)
	Conversion		
Interpreting		Natural	Interpreting to reality: Created circle could cover the whole tumor.
	Conversion		
Real Results		Numeric	The size of the tumor is 53.5 cm^2.
Validating	Conversion	Natural	Generalization: To identifying the location and area of the tumor, x-ray is needed to be switched in coordinate system in actual size, then the circle is drawn around the tumor by looking the farthest points of it.

3.6 The Role of the Researchers

"Tumor Surgery" task was implemented in advance within the scope of the elective course entitled "Mathematical Modeling for Teachers." The course was followed by two instructors and one assistant during the data collection process (see section 3.4 for the roles of instructors and an assistant). As a researcher of the current study, I was not involved in the modeling task implementation. In the research, I played a role in watching and listening to the modeling processes of groups, the transcription of data, data analysis, and reporting all of the processes from beginning to end.

3.7 Trustworthiness

The fact that any research presents accurate and reliable findings for the reader is an important criterion for that study to be quality. In this regard, there are several techniques to make the research trusted and ensure validity and reliability issues. In qualitative studies, the validity and reliability terms change as "trustworthiness" concerning the persuasion of the reader that the study is worth reading (Lincoln & Guba, 1985). The terms such as internal validity, external validity, reliability, and objectivity in quantitative studies were discussed under different names in a qualitative study by Lincoln & Guba (1985) and called credibility, transferability, dependability, and confirmability in their book. Specifically, the trustworthiness of the current study was ensured with the credibility and dependability concepts.

The concept that appears as "internal validity" in quantitative studies is called "credibility" in qualitative studies; credibility is related to the correct reporting and interpretation of research findings (Lincoln & Guba, 1985). To ensure credibility, there are several techniques, such as member checking and triangulation. For this study, triangulation techniques were used to present correct findings, and two different triangulation methods were adopted, data triangulation and investigator triangulation. For the data triangulation, multiple data sources were used in the current study. Specifically, groups whose whole processes were audiotaped and videotaped also wrote a report and made a presentation reflecting their processes at the end of the modeling activity.

Moreover, GeoGebra files containing the mathematical models of the groups that preferred to use technology were also used as multiple data sources and contributed to the accurate presentation of the data. Secondly, investigator triangulation, recommended as another mode of triangulation to ensure trustworthiness by Denzin (1978), was employed in the current study. This mode was related to asking for coding the data from a second researcher. Therefore, a definite percentage of the data was sent to the second coder for individual analysis. The second coder was a master's student in a mathematics education program, and she had sufficient knowledge of analysis methods in educational research and was familiar with the context of the study because she had taken the elective course mentioned in this study in advance. In the literature, it is stated that the size of the data that the second coder analyzes should be between 10% and 25% of the total data (O'Connor & Joffe, 2020). In this regard, data of two random groups was sent to the second coder as two different analyzes of this study, semiotic representations and a data analysis table for the modeling routes. 23.5% of the data related to the analysis of the modeling route was sent, while 44.70% of the data covering part of the analysis of semiotic representation was sent to the second coder. At this point, the reason that more than 25% of semiotic representations data sent to the second coder was because one of the analyzed groups had too many modeling cycles. After the analysis process, we reached an agreement on the percentage of %84.21 over the total data. In the next step, we discussed the incompatible parts, and we finally reached a total agreement.

The concept that appears as “reliability” in quantitative studies is called “dependability” in qualitative studies; dependability is related to the consistency of the findings (Lincoln & Guba, 1985). In the current study, expert opinion was also taken on whether the analyzes made in the parts for analyzing the modeling route were correct, and the analyzes were checked. At this point, the trustworthiness of the study was ensured with expert opinion taken from my thesis advisor with expertise in mathematical modeling.

CHAPTER 4

RESULTS

This chapter presents the findings of the characteristics of the groups' mathematical models and modeling processes in terms of semiotic representations and their detailed modeling routes. Based on the research questions, the first part includes the findings of groups' semiotic representations according to their actions in each specific modeling transition. In the second part of the results section, groups' modeling routes are focused on finding an answer to the second research question. Here, the research questions of this study are given:

1. What are the characteristics of pre-service mathematics teachers' mathematical models and modeling processes in terms of the semiotic representations used in a technology-integrated model-eliciting activity?

2. What are the features of pre-service mathematics teachers' modeling routes in a technology-integrated model-eliciting activity?

    2.1 How do pre-service mathematics teachers' modeling routes differ according to the semiotic characteristics of their mathematical models and modeling processes in a technology-integrated model-eliciting activity?

4.1 Semiotic Representations Used by Pre-service Teachers to Develop Their Models

The Tumor Surgery task includes two parts to create a model. Specifically, these are (i) resizing the x-ray to its actual size by establishing a ratio and (ii) determining the area and exact location of the irregular-shaped tumor. Considering each requirement in itself, the modeling processes of the groups were reported first while reporting semiotic representations to acquire a general impression about their actions.

Preservice teachers' mathematical modeling process, including their answers for the task and the actions they presented in this process, were categorized according to modeling transitions progressing from understanding to validating. Within this scope, each new solution was represented in the new cycle. The analysis of data indicated that the groups demonstrated their actions and solutions in four different ways of representation. These were categorized as natural, numeric, algebraic, and geometric registers for each action.

4.1.1 The actual size of the X-ray

One of the main goals of the Tumor Surgery task is to resize the reduced chest x-ray. For this aim, the value of 2.5 cm, the difference between the lengths of the lungs, was given in the task. The groups were expected to use this value to establish a ratio between the image and the actual size of the x-ray at first. Then, they would determine the area and location of the irregular-shaped tumor. In line with these aims, the modeling processes of the groups were shown first here, and then their actions were classified according to the semiotic registers and modeling transitions.

4.1.1.1 Mathematical Modeling Processes of the Groups

After obtaining a general impression of the Tumor Surgery task, the classroom discussion was conducted to make sense of the real situation. During the discussion, the conditions for finding the size and exact location of the tumor were generally mentioned. As a next step, while some of the groups initially made assumptions about the shape of the tumor, some intensely questioned the given information about the difference between the lungs.

At the beginning of their modeling process, Group A was one of the groups questioning why the difference between lungs was given. Since they could not find any solutions to use this information afterward, they continued their process by making assumptions about the shape of the irregular-shaped tumor. In carrying the model to the technological world, this group began to create regular shapes that they assumed around the tumor without taking any action on proportioning for the actual size. Along with the instructor's guiding questions, Group A realized that the tumor size changed when the x-ray was enlarged on GeoGebra. This prompted the group to find a constant value to fix the size of the x-ray. Group members measured the length of the difference between lungs and the size of the x-ray by using grid paper. According to measured values for the difference between lungs, it was determined that 10 unit squares (1 cm) on the x-ray are equal to 2.5 cm in real. Using the measured values and the difference of 2.5 cm, a ratio between the image and actual size was established as 10:2.5. In the next step, they measured the height and width of the x-ray as 203×203, then it was rounded to 200×200. As a result of the calculations, the size of the x-ray was found as 50×50 cm^2 in real. The x-ray was transferred to the GeoGebra environment in its actual size.

After the class discussion, Group B expressed that the given information was necessary to find the actual size at the very beginning of their modeling process. To find the constant value for the actual size, they used grid paper and measured the difference between lungs at first. According to the measured value, Group B determined that 14 unit squares (1.4 cm) on the x-ray are equal to 2.5 cm in real. Using the required values, a ratio between the image and actual size was established as 14:2.5. Then, the image size of the x-ray was measured as 193×193, and the actual size was calculated as 34.66×34.66 cm^2 (but 34.46 was used mistakenly). The x-ray was transferred to the GeoGebra environment in its actual size.

The modeling processes of Group C also dwelled upon how they use that value in their calculations. Similar to Group A, the members of this group transferred the x-ray to the GeoGebra environment in its image size, then created regular shapes around the tumor. After realizing they were expected to find the actual size, they made the necessary measurements using the ruler. It was determined that the length between the lungs, which is 2.5 cm in real, corresponds to 1.3 cm on the image. Then, they found the image size of the x-ray as 20.2 cm, but they mistakenly used the value of 20.02 in their calculations. As a result, the proportionality constant was found as 1.3:2.5, and the actual size of the x-ray was calculated as 38.5 cm^2.

Group D also went through the same processes as Group A and C, except for transferring the x-ray to the technological world and questioning the given information intensely. While speaking about the tumor's location, the Group D members, who made all their measurements according to the given x-ray and determined the model they would create by using these measurements, realized these measurements were not the actual lengths. Using the ruler, Group D measured just the length of the difference between lungs, and its length was found as 1 cm on the image. A ratio between the image and actual size was established with the measured values and the difference of 2.5 cm. However, Group D did not use this ratio for the size of the x-ray. They used this ratio only for the operations they would do on the tumor. Since this group did not take any action regarding the actual size of the x-ray, they determined the tumor's location incorrectly according to the measurements in the image size.

Lastly, Group E was the other group questioning why the difference between lungs was given. Although this difference put a question mark on their minds, the members of Group E continued their operations related to the tumor's shape. With the supportive questions of the instructor, they realized why they needed to use that difference and how they would determine the actual size of the x-ray. As a different solution approach, Group E preferred to perform this process on GeoGebra instead of determining any ratio. They created a line segment whose length is 2.5 cm on the GeoGebra and enlarged the x-ray until the difference between the lungs was equal to the length of that line segment. Ultimately, the actual size of the x-ray was determined as 34.5×34.5 cm^2 on GeoGebra.

4.1.1.2 Actions of the Groups and their Classification

Groups' actions related to resizing the x-ray were classified according to the modeling transitions, and then semiotic registers of that actions were determined in the data analysis. At this point, it was observed that most of the groups progressed almost in the same way while deciding the actual size of the x-ray. Although the values they measured differed, their actions showed similarities in the specific modeling transitions. Therefore, the differences in actions were noted in this part. These actions for Group A, B, and C are shown in the following tables (see Appendix B for Group D and Group E).

Table 4.1 The modeling transitions/phases and semiotic actions of Group A

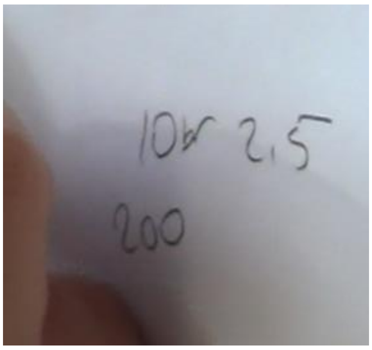
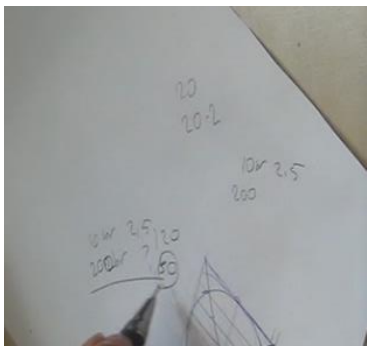
Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding	Treatment	Natural	Understanding the situation
Situational Model		Natural	The difference of 2.5 cm between lungs is needed to reach the actual size of the x-ray.
Simplification/Structuring		Natural	Simplifying and constructing relations
	Conversion		
Real Model	Treatment	Numeric	For the actual size of the x-ray: 10 square units (10 mm) on the grid paper is equal to the difference of 2.5 cm between lungs and the image size of the x-ray is 200 square units (200 mm).
Mathematizing		Numeric	Establishing a proportion between the image and the actual size of the x-ray 
Working Mathematically		Numeric	Finding the actual size of the x-ray by solving the proportion 
Mathematical Results		Numeric	The result is 50. The size of the x-ray in real: 50x50

Table 4.2 The modeling transitions/phases and semiotic actions of Group B

Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	The difference of 2.5 cm between lungs is needed to reach the actual size of the x-ray.
Simplification/Structuring		Natural	Simplifying and constructing relations
	Conversion		
Real Model		Numeric	For the actual size of the x-ray: 14 square units (14 mm) on the grid paper is equal to the difference of 2.5 cm between lungs and the image size of the x-ray is 193 square units (193 mm).
Mathematizing		Numeric	Establish a ratio between image and real as 14:2.5
	Treatment		Establish a proportion for the size of the x-ray whose length is 193x193 unit squares.
Working Mathematically		Numeric	Finding the actual size of the x-ray by solving the proportion ▪ If 14 unit squares equal 2.5 cm, what is the actual length of 193 unit squares?
Mathematical Results		Numeric	The result is 34.46. The size of the x-ray in real: 34.66x34.66.
	Conversion		
Interpreting		Natural	Interpreting to reality: It is not reasonable that the shape of our lungs is square.

Table 4.3 The modeling transitions/phases and semiotic actions of Group C

Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	The difference of 2.5 cm between lungs is needed to reach the actual size of the x-ray.
Simplification/Structuring		Natural	Simplifying and constructing relations
	Conversion		
Real Model		Numeric	For the actual size of the x-ray: 1.3 cm is equal to the difference of 2.5 cm between lungs and the image size of the x-ray is 20.2 cm.
Mathematizing		Numeric	Establish a ratio between image and real as 1.3:2.5
	Treatment		Establish a proportion for the size of the x-ray whose length is 20.2x20.2 unit squares.
Working Mathematically		Numeric	Finding the actual size of the x-ray by solving the proportion
Mathematical Results		Numeric	The result is 38.5.
	Conversion		
Interpreting		Natural	Interpreting to reality: The x-ray may belong to a man, so it is big.
	Conversion		
Validating		Numeric	Validate the result: There is a difference of 2.5 cm between lungs transferring to GeoGebra environment in actual size. (From 7 units to 9.5 units)

As seen in the tables above, all groups understood that they needed to use the given information to reach the actual size of the x-ray. After that, they started to simplify the situation by constructing relations between the values in the problem situation and the reality. In these modeling transitions and phases, groups used the natural register in their representations, and their semiotic actions were treatment. Unlike other groups, Group E did not take any action to construct relations mathematically during the simplification process. Instead of establishing a ratio and doing any mathematical operation on paper, they tried to reach the actual size via technology. The solution of Group E is given in Figure 4.1 below.

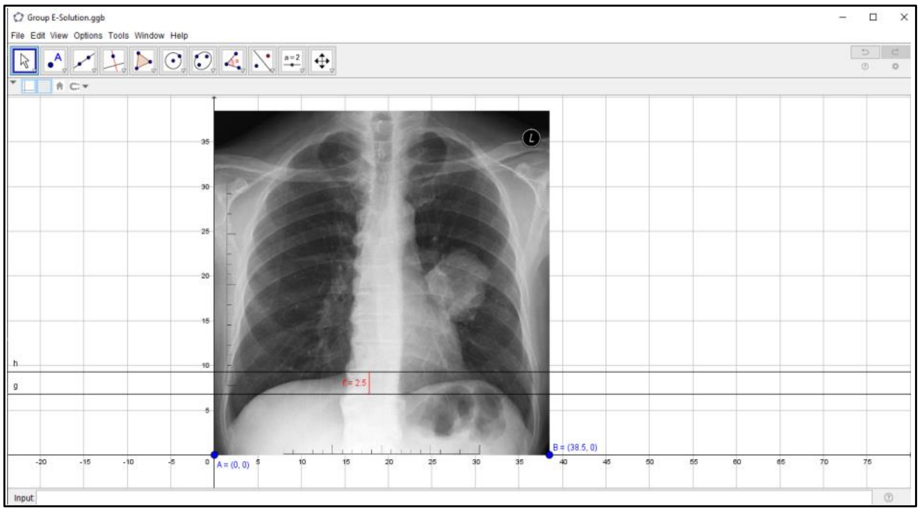


Figure 4.1 Solution way of Group E for the actual size of the x-ray

Group E constructed two lines and a line segment of 2.5 cm length in their solution. Then, they enlarged the x-ray until they provided the real conditions. For this reason, Group E could not go beyond the simplification transition in creating this model related to resizing the x-ray. Unlike other groups, Group E continued their process with the technological world instead of the mathematical one, obtaining computer results. After reaching the actual size of the x-ray, Group E did not move to other transitions to interpret and validate their result.

In the phases from real model to mathematical results, Groups A, B, and C reached the actual size of the x-ray by using the relationship they established. At this point, it was observed that their actions took place in the numeric register. While their semiotic action is conversion in the representation of the real model, semiotic action was treatment in the numeric representations they used one after the other.

Group D was another group also establishing a ratio for the actual sizes. While Group A, B, and C used the proportionality constant for the actual size of the x-ray, Group

D determined just the proportionality constant and used it repeatedly in operations on the tumor size. Here, Group D's process up to the mathematizing transition was similar to the other groups in terms of both semiotic action and registers.

Group A, D, and E ended their cycle related to resizing after finding a mathematical result. As seen in Tables 4.2 and 4.3, Group B and Group C continued their processes. Group B interpreted their result in terms of the shape of the lung. It did not make sense that the shape of the lung was square in reality. Then, they accepted the size of the liver according to the result they found. Group B's cycle ended with the interpreting transition. The representation in this transition was recorded as a natural register. This process is illustrated in the conversation below.

Berna: *How many centimeters does that equal now?*

Bahar: *Hmm, I don't know.*

Berna: *You just proportioned it.*

Bahar: *Hmm, as a thing. That ratio? Hmm, we need to calculate that, too*

Berna: *There and the upper part. Aren't they the same?*

Bahar: *Square?*

Berna: *But there is a bulge. I think this is the square. How long is that?*

Beliz: *Check it out. Hmm, we cannot check it out.*

Bahar: *Okay. That was 193. What came?*

Beliz: *34.66.*

Bahar: *Exactly. 3, 4, 5, 6, 7, 8, ..., 18.18. Now…*

Beliz: *I think it is 19. Let's draw a line here. Where do we draw the line? Will you measure the height of it up to here?*

Bahar: *Yes, at least up to here.*

Beliz: *I think it's 19x19; it's a square.*

Bahar: *Ok then, 19.*

Berna: *Let's see if that's it.*

Beliz: *Although it doesn't make sense for our lungs to be square…*

Group C was the only group that completed the cycle related to resizing. After finding the actual size, they interpreted their results in terms of lung size. When they interpreted this result in context, they realized that they found a higher result than normal when they compared it. Later, they concluded that a lung of this size could belong to a man. In the validating transition, Group C confirmed their result on the x-ray that they transferred to GeoGebra. They controlled whether the difference between the two lungs was 2.5 cm. During the interpreting transition, the action of Group C was recorded as a natural register. On the other hand, they used a numeric register in the validating transition because they verified their result using the subtraction operation. This process is given in the conversation below.

Ceren: *For example, let's do something like this. 2.5 corresponds to 1.3 on the ruler, for example. If we just measure its actual size with a ruler and then calculate its actual size with math calculation, is it possible? Proportional… For example, we said that 2.5 corresponds to 1.3 on the ruler. Let me measure this actual size with a ruler. I'll write. Then I'll find this proportionally, and find its actual size. I'm going to measure that length with a ruler.*

Canan: *Will we measure the full lung here?*

Ceren: *I guess so. Because we're going to transfer the full size of it, we'll place the coordinates accordingly.*

Canan: *It's 15. Up to the bottom.*

Ceren: *20.2?*

Canan: *Yes, 20.2. I guess that's a square. Exactly, it came out to be the same again.*

Ceren: *Good. Okay, it's a square.*

Canan: *22, 23. Let's say 23.*

Ceren: *23 or 20.2?*

Canan: *Don't say 23. Let's say 21.*

Ceren: *I say 21? But we took this as one point above. Eventually, a side of it… Okay, let's calculate these things after that. Multiply…1.3…*

Canan: *Oh, how long is that?*

Ceren: *2.5. I divided 20.2 by 1.3. [Group used 20.02 in the calculation instead of 20.2.]*

Canan: *30.*

Ceren: *It's equivalent here; one side of it was 38.5. So, do you understand? For example, we measured 20 here with a ruler. Its actual size is 38.5.*

Canan: *Hmm, that was it!*

Ceren: *Yes. Maybe, this belongs to a man.*

Canan: *Maybe, but 15-15*

Ceren: *So, it could be.*

The groups' semiotic registers are given in Table 4.4 below. All the groups used just the natural and numeric register to reach the actual size of the x-ray. The distribution of the registers according to the modeling transitions and phases is presented in Table 4.4. Capital letters from A to E in Table 4.4 represent the names of the groups.

Table 4.4 Groups' Semiotic Registers in each Modeling Transitions/Phases for Resizing the X-ray

Transitions/Phases	*Natural Register*	*Numeric Register*
Understanding	A, B, C, D, E	-
Situational Model	A, B, C, D, E	-
Simplification/Structuring	A, B, C, D, E	-
Real Model	-	A, B, C, D
Mathematizing	-	A, B, C, D
Mathematical Model	-	-
Working Mathematically	-	A, B, C
Mathematical Results	-	A, B, C, E
Interpreting	B, C	-
Real Results	-	-
Validating	-	C

4.1.2 Shape of the Tumor

Another aim of the "Tumor Surgery" task is to determine the size and location of the irregular-shaped tumor. To achieve this aim, groups were expected to identify the tumor's shape and decide which regular shape it resembles. Then, they would calculate the size of the irregular-shaped tumor with the area formula of regular shape and determine the location with the equation of that shape. Ultimately, they were expected to meet the conditions of removing the entire tumor and taking the least healthy tissue with a determined shape. To generalize their solution, they would also prepare a practical guide for future tumor surgeries. In the direction of these aims, the modeling processes of the groups were shown firstly in this part; then, their actions were classified according to the semiotic registers and modeling transitions.

4.1.2.1 Mathematical Modeling Processes of the Groups

After reading the problem and the class discussion, all groups understood that the irregularly shaped tumor should be taken as a two-dimensional disc. In this way, their process started by making assumptions about the tumor's shape to determine its area and location. Groups focused mainly on the three regular shapes, circle, ellipse, and pentagon.

At the beginning of the model development, one of the members of Group A concentrated on the circle for the irregularly shaped tumor. In light of the first assumption they made, Group A detailed their solution. While discussing possible solutions, they initially considered placing the chest into a coordinate system to determine the tumor's location and area. During their modeling process, Group A mostly dwelled on how they would determine the components of the circle accurately, whether the circle they will determine in the paper-pencil environment will really determine a circle, and how they can reduce the size of the tissue to be taken for the minimum healthy tissue. For these aims, they initially considered creating a circle inside a square, rectangle, or any quadrilateral. In the process, they put forward different ideas for determining this circle and its components. They transferred the x-ray to the technological environment and placed it into the coordinate system of GeoGebra. Then, they tried each of these ideas on GeoGebra. In each case, they could not meet the condition that covering the tumor with minimum healthy tissue, they tried to create the circle with a different method. Besides trying new ways to draw a circle, Group A also made assumptions about different shapes to improve their solution. For example, initially, they considered creating a rectangle to remove the tumor, but they decided that the rectangle was an inappropriate shape for different tumors. They also considered the possibility of using a polygon shape to remove the tumor because of its pentagon-like shape. They eliminated this idea as it would not be a practical way to develop a generalizable solution if there was a polygon with many sides. Lastly, they assumed that the tumor could be removed by creating an ellipse. Group A preferred to draw an ellipse instead of a circle around the tumor as a final model. According to them, an ellipse was more suitable than a circle for different tumor shapes. The final solution of Group A is given in Figure 4.2.

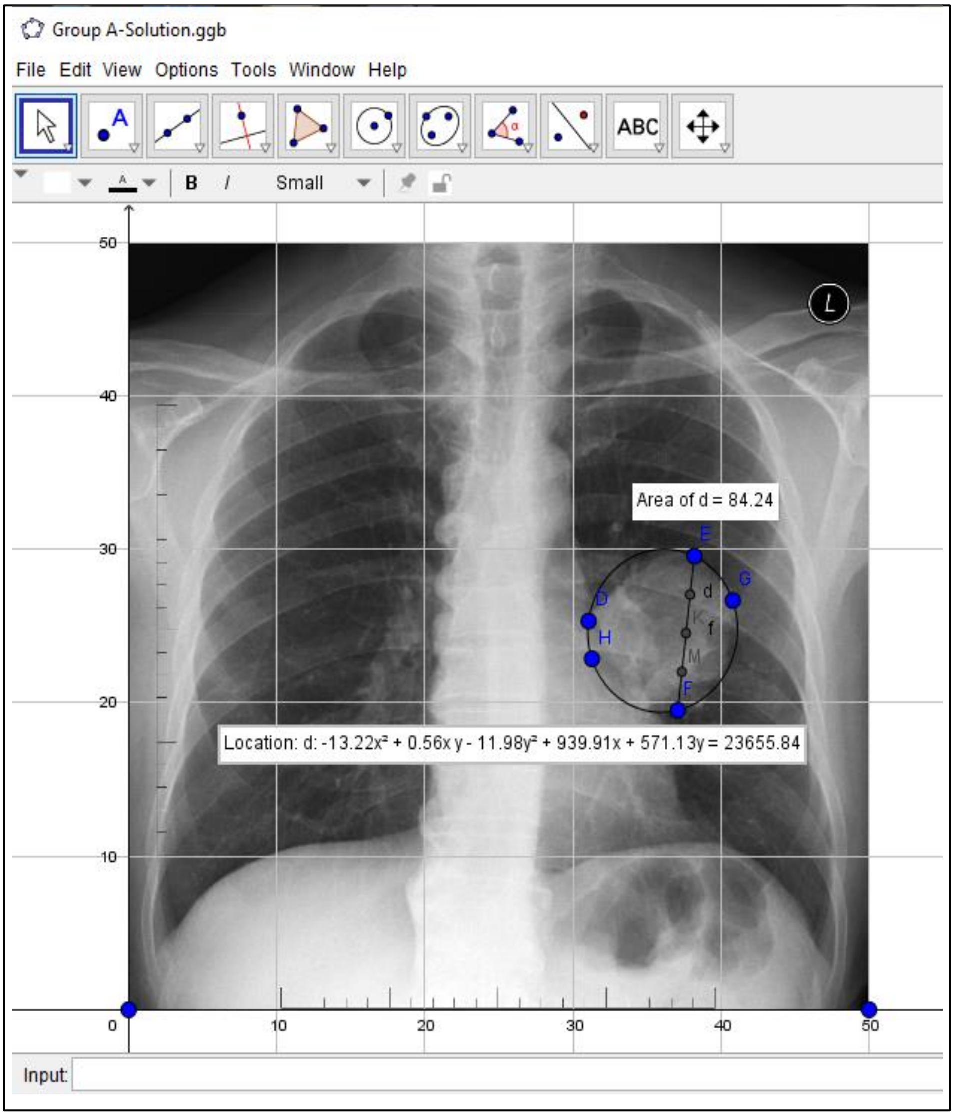


Figure 4.2 Final Model of Group A

Group B was another group that tried various shapes for the tumor. This group initially found a proportionality constant to be able to work on actual sizes. The determined constant was used again to find the actual lengths of the values in the area formula of the different shapes to be drawn around the tumor. Their assumption for the shape of the tumor was a circle at first. To determine the smallest circle around the tumor, they measured the distance between the farthest points of the tumor horizontally and vertically. At this point, one of the group's members (Berna) transferred the x-ray to the GeoGebra and placed it into the coordinate system of GeoGebra by putting the left bottom vertex of the x-ray in the origin. Then, she enlarged the x-ray in the determined proportion. In the next step, using the determined actual radius length, they created a static circle on GeoGebra. The purpose of Group B in using the technology at this point was to visually see and test whether the circle they had identified on paper covered the tumor. After verifying that they could obtain a circle covering the tumor, the group members used the center coordinates of the created circle to identify the equation of it. The area of this circle was calculated using the area formula of the circle in a paper-pencil environment, GeoGebra's area tool was not used for area measurement.

In the modeling process, Group B focused on the different shapes that could be drawn around the tumor. They considered square, regular pentagon, and ellipse, respectively. They eliminated the options of a square and a regular pentagon because they found that when they used these shapes, the area that needed to be cleared of tumor became larger than the area for a circle. Group B used the area formula of the mentioned shapes and reached the areas of these shapes in a paper-pencil environment. Finally, Group B created an ellipse to cover the whole tumor by using the input command in GeoGebra. They calculated the elliptical area in the paper-pencil environment instead of using the area tool of GeoGebra. Although they found the area of the ellipse was smaller than the circle, they chose the circle as the latest model. One of the reasons for this preference was that they could not calculate the area of the ellipse exactly on GeoGebra and could not determine its location because the created shape indicated a curve on GeoGebra. More specifically, the ellipse created with the input command did not indicate a region on GeoGebra, that was just a curve. Therefore, the area tool of the GeoGebra did not calculate the area of the ellipse. The final model of Group B is given in Figure 4.3.

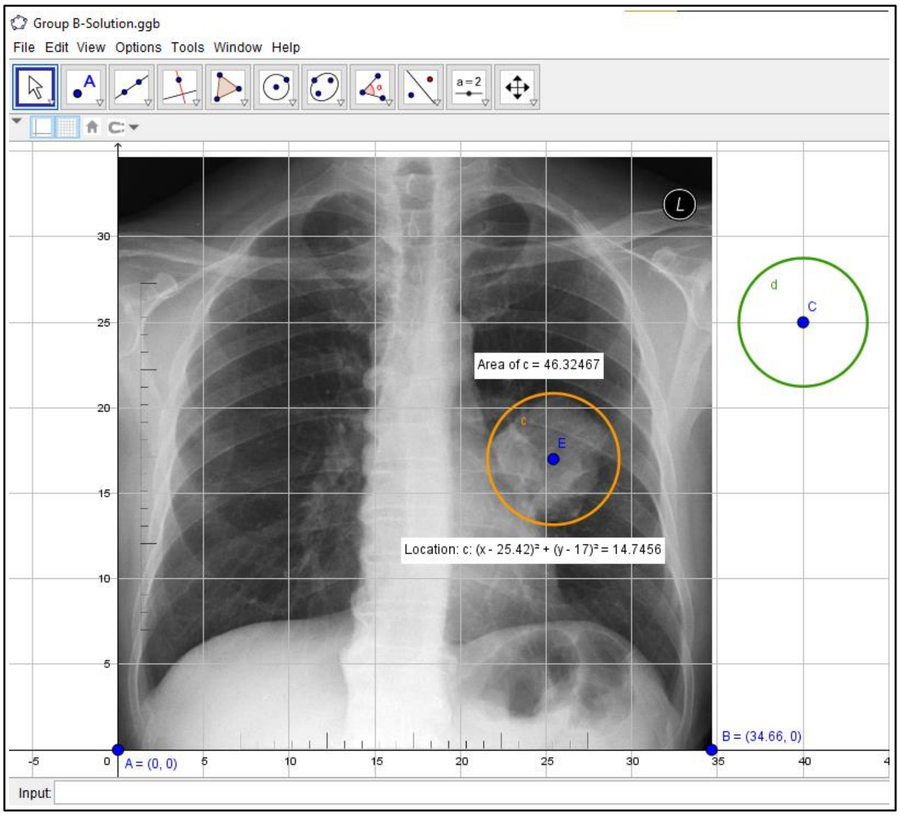


Figure 4.3 Final Model of Group B

Group C was a group that made only one assumption for the shape of the tumor and developed their solutions based on that shape. Like the other groups, Group C also considered drawing a circle around the tumor. After transferring the x-ray to GeoGebra in its actual size, they initially used one of the GeoGebra tools to draw a circle. Although they created a circle covering the whole tumor, they felt the need to draw this circle with more precision. To do this, they entered a command to the input on GeoGebra. Using the equation of the circle and sliders, they constructed a circle whose size and position can be dynamically changed to find a more precise area. The final model of Group C is given in Figure 4.4.

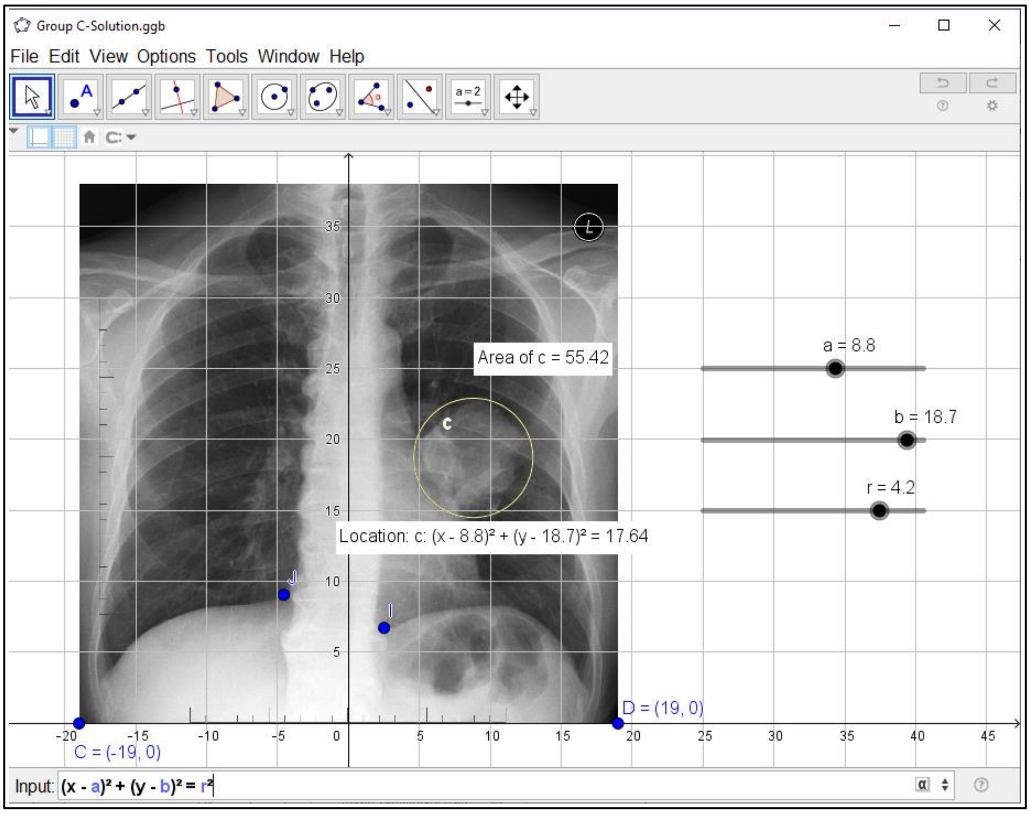


Figure 4.4 Final Model of Group C

Group D's assumption for the tumor's shape was a circle. They thought of creating a circle inside a square that would be drawn around the tumor. Unlike other groups, this group continued their solutions on paper rather than using technology. Initially, they found the midpoint of the x-ray for the reference point and accepted it as an origin. As a next step, they measured the distance between the farthest points of the tumor horizontally and vertically. Among these distances, they chose the larger one and formed a square according to that length. Thus, Group D created an incircle tangent to the sides of the square. They used the circle model in their solution, but they also gave place to the ellipse in the generalization part of the problem for the different tumor shapes. The final model of Group D is shown in Figure 4.5.

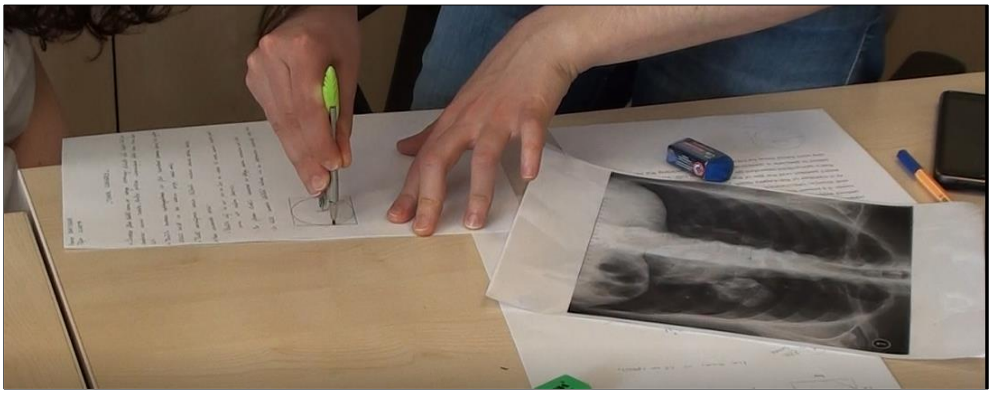


Figure 4.5 Drawings of Group D

Group E was the only group to use technology from beginning to end of their modeling processes. After determining the actual size of the x-ray on GeoGebra, Group E members made assumptions about the shape of the tumor. As a first assumption, they considered drawing a circle around the tumor. To draw this circle, they identified the estimated farthest points of the tumor. They determined different pairs of points right across each other and created different circles by using GeoGebra's circle drawing tool. Then, Group E compared those circles according to their area and whether they covered the entire tumor. After choosing the most suitable circle, that covers the entire tumor and has the smallest area than the other ones, they continued their process with a different model. Considering drawing a pentagon as a second model, Group E identified five different points around the tumor and constructed a polygon. Although they found that the area of the pentagon was smaller than the circle, they preferred the circle as a final model because they thought it was more practical and appropriate for different tumor shapes. Models of Group E are given in Figure 4.6.

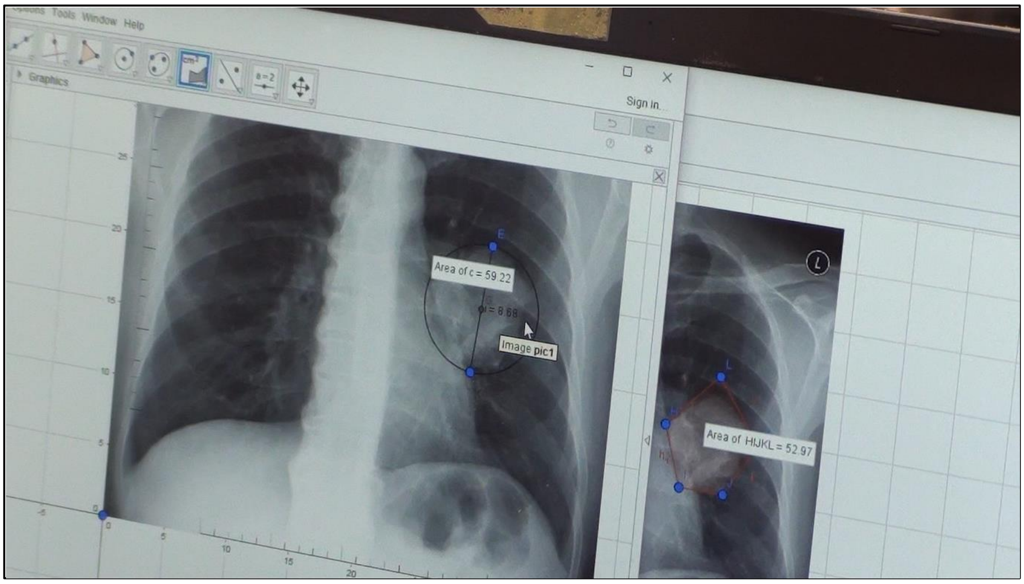


Figure 4.6 Models of Group E

4.1.2.2 Actions of the Groups and their Classification

Groups' actions related to finding the area and location of the irregular-shaped tumor were classified according to the modeling transitions. Then semiotic registers of these actions were determined in the data analysis. The actions of the groups regarding a specific model and different solutions for that model were presented as a new cycle each time in the findings section. As mentioned in the previous part, the modeling processes of the groups were quite different in terms of assumed shapes for the tumor, preference for using technology, and the results at the end of their process. Therefore, the actions of the groups and their categorization were given in detail in the following parts.

Group A

The actions of Group A took form around three different assumptions about the tumor's shape. These were circle, rectangle, and ellipse models. Group A's data on the tumor's shape were categorized with a total of eight cycles. These cycles included three different models and different solution methods to create these models. Because there were many cycles to categorize Group A's actions, not all of these cycles were given in this section as a table in detail. The following table shows the registers that Group A used in each transition/phase and modeling cycle (see Appendix C for the detailed table including modeling transitions and semiotic actions of Group A).

Table 4.5 Group A's Semiotic Registers Specific to Transitions/Phases and Modeling Cycles

Transitions/Phases	Modeling Cycles							
	1st	2nd	3rd	4th	5th	6th	7th	8th
Understanding	NaR							
Situational Model	NaR							
Simplification/Structuring	NaR	NaR	NaR	NaR	NaR	NaR	NaR	NaR
Real Model	NaR	NaR	NaR	NaR	NaR	NaR	NaR	NaR
Mathematizing	GR	GR	GR	GR	GR	GR	GR	GR
Mathematical Model	GR	GR	GR	GR	GR	GR	GR	GR
Working Mathematically								
Mathematical Results			NuR			NuR		NuR & AR
Interpreting	NaR	NaR	NaR	NaR	NaR	NaR	NaR	NaR
Real Results								
Validating		NaR				NaR		

Note. NaR: Natural Register; NuR: Numeric Register; AR: Algebraic Register; GR: Geometric Register

After understanding the task, they simplified the problem by making assumptions. In the first cycle, a circle model was created inside a quadrilateral. To represent this model, Group A created a mathematical model using the necessary tools on GeoGebra. Then, they interpreted the model in the problem context. Within the scope of this model, the members of Group A observed that created circle could not cover the whole tumor, and it covered much more healthy tissue. Therefore, they looked for a different solution and made new assumptions in the next cycle. During the first cycle, Group A's actions from understanding to the real model were determined as a natural register. In addition to these transitions/stages, the group also used a natural register in the interpreting transition. Another representation used in the first cycle was the geometric register. Actions in the mathematizing transition and mathematical model phase were recorded as a geometric register. Similarly, the registers in the first cycle were also observed in the other two cycles (2^{nd} and 5^{th} cycle) of the group that included the circle model.

As a different method to create a circle, Group A drew various circles passing through different points by using the GeoGebra tool called "Circle through three points" in the second and fifth cycles. The phases and transitions of the second cycle, in which the natural register was used, were simplification, real model, interpreting, and validating. On the other hand, the group used the geometric register for their representation in mathematizing transition and mathematical model stage of this cycle. The registers used in the fifth cycle were exactly the same as in the first cycle in terms of their type and the transition/stage in which they were observed.

Another assumption the group made was removing the tumor as a rectangle. In the third cycle, the actions of the group about the rectangle were included. The natural register was used in the simplification process and in determining the real model, mostly including group's discussion on the assumptions regarding the tumor's shape. As a next step, the rectangle model was created on GeoGebra, and its area was calculated with the area tool. While Group A used the geometric register in the representation of the rectangle, their semiotic register was numeric when they determined its area in the mathematical results phase. The last action of the third cycle was observed in the interpreting transition. Group A considered that the rectangle was not appropriate for different tumor shapes, and it was not practical because drawing a rectangle and expressing the location of that rectangle with coordinates were more difficult than drawing a circle and expressing the location of that circle with the equation of it. This interpretation was also recorded as the natural register.

In the remaining cycles, actions on the ellipse model took place with different solutions to create it. After simplifying the problem by making an assumption about the ellipse, Group A tried to create it on GeoGebra. First, they used the “ellipse” tool on GeoGebra and drew an ellipse covering the whole tumor. However, they observed that the enclosed area with an ellipse was greater than a circle, and it covered much more healthy tissue because they used the farthest points of the tumor as focus points. As seen in Table 4.5, in the fourth cycle, the group used natural registers while assuming and interpreting their model in context. On the other hand, their actions related to creating their ellipse model were included in the geometric register.

After returning to the circle model in the fifth cycle, Group A decided to draw the ellipse again in the sixth cycle. To create it, they identified the focus points of the ellipse by finding the midpoints of the farthest ends of the tumor. With this method, they were able to create an ellipse covering the entire tumor. When they compared the area of the ellipse with the circle they constructed in the previous cycle; they saw that the area of the circle was larger. In light of this comparison, Group A decided that the ellipse was a more reasonable model for different tumors at their validation transition because they thought that if the major axis and minor axis of the ellipse were equal, it would indicate a circle. Table 4.5 shows which registers were used in the sixth cycle. The group used the numeric register in the mathematical results phase and the natural register for the validation. Table 4.6 shows the fifth and sixth cycle of Group A in detail and illustrates the actions mentioned above.

Table 4.6 The modeling transitions and semiotic actions of Group A in the fifth and sixth cycle

Transitions	*Semiotic Actions*	*Registers*	*Explanations*
			5th Cycle
Simplification/ Structuring	Treatment	Natural	Return to situation for the circle model Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by using "Circle through three points" tool
Mathematizing	Treatment	Geometric	Drawing a circle via "Circle through three points" tool on GeoGebra by using identified points
Mathematical Model	Conversion	Geometric	Different circles (E, F, D points, E, F, H points and E, F, C points) passing through the identified points around the tumor
Interpreting		Natural	Interpreting to Reality: Created circles (E, F, D points and E, F, H points) could not cover the whole tumor. Created circle (E, F, C points) could cover the whole tumor
			6th Cycle
Simplification/ Structuring	Treatment	Natural	Return to situation for the ellipse model Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created according to the farthest points around the tumor Determining focus points of ellipse according to the midpoints of the midpoint of the farthest points around the tumor

Table 4.6 (continued)

Mathematizing		Geometric	Identifying focus points by using the farthest two point Drawing an ellipse via “Ellipse” tool on GeoGebra
	Treatment		
Mathematical Model		Geometric	Ellipse created by using the different focus points
	Conversion		
Mathematical Results		Numeric	Area of the circle passing through E, F, C points: 103.33 Area of the ellipse: 103.76
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created ellipse could cover the whole tumor, but it has a bit more tissue than the circle. .
	Treatment		
Validating		Natural	Generalization: Ellipse model is more appropriate than the circle for the different tumor shapes. Also, ellipse model is more reasonable because ellipse approaches to the circle when its diameters are equal. Validating the models by checking them on the tumor having different shape.

In the seventh cycle, Group A tried to create an ellipse by using the input command of GeoGebra. The input command was FitImplicit({F, G, C, J, E, I, D, H}, 2). This command was accepted as a geometric register in the mathematizing transition as different from the other cycles. After entering the command, they obtained an ellipse that they could change its endpoints. Because the constructed ellipse indicated a curve, Group A could not reach its area via the area tool. Therefore, they tried another way to create an ellipse in the last cycle. Using the GeoGebra tool named “Conic through five points,” the group could draw an ellipse covering the whole tumor and having a smaller area. In the mathematical results phase, while the area of the ellipse was represented in a numeric register, the equation for expressing the tumor's location was represented in an algebraic register.

Group B

Four different assumptions were made for the shape of the tumor by Group B. These were circle, square, regular pentagon, and ellipse models. Similar to Group A, the actions of this group also were presented in seven cycles. Because there were many cycles to categorize Group B's actions, not all of these cycles were given in this section as a table in detail. Table 4.7 shows the registers that Group B used in each transition/phase and modeling cycle (see Appendix D for the detailed table including modeling transitions and semiotic actions of Group B).

Table 4.7 Group B's Semiotic Registers Specific to Transitions/Phases and Modeling Cycles

Transitions/Phases	Modeling Cycles						
	1st	2nd	3rd	4th	5th	6th	7th
Understanding	NaR						
Situational Model	NaR						
Simplification/Structuring	NaR			NaR	NaR	NaR	NaR
Real Model	NaR			NaR	NaR	NaR	NaR
Mathematizing	NuR	GR					GR
Mathematical Model		GR	AR	AR	AR	AR	GR
Working Mathematically	NuR	NuR	NuR	NuR	NuR	NuR	
Mathematical Results	NuR	AR	NuR	NuR	NuR	NuR	AR
Interpreting				NaR	NaR	NaR	NaR
Real Results							NuR & AR
Validating			NaR			NaR	NaR

To begin with, the group understood the problem situation and made assumptions about the tumor's shape. As mentioned in the previous part, Group B used the determined proportionality constant in most cycles. The necessary lengths to determine the area of the assumed shapes were found by calculating their actual lengths at every turn. Table 4.8 shows the first cycle of Group B in detail and illustrates the actions mentioned above. From understanding to the real model phase, the representations of the actions were included in the natural register. In the modeling process, the length of the diameter of the circle was determined, and its actual length was calculated. The register of these actions was recorded as numeric.

In their second cycle, Group B created a static circle whose diameter was determined in the previous cycle on GeoGebra. These actions were recorded as geometric registers and included in the mathematizing and mathematical model. As a next step, the group used the coordinates of the center point and the length of the radius to determine the equation of a created circle in the working mathematically transition. Here, they submitted the determined values in the circle equation for the location of the tumor. This substituting action was included in the numeric register. At the end of this action, the mathematical result in which they found the location of the tumor was represented as the algebraic register.

Table 4.8 The modeling transitions and semiotic actions of Group B in the first cycle

1st Cycle			
Transitions	*Semiotic Actions*	*Registers*	*Explanations*
Understanding	Treatment	Natural	Understanding the situation
Situational Model	Treatment	Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/ Structuring	Treatment	Natural	Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created according to the farthest two points around the tumor horizontally and vertically. For the location of the tumor: The chest x-ray can be put on a coordinate system and the equation of the circle can be used.

Table 4.8 (continued)

Mathematizing	Treatment	Numeric	Establish a proportion for the diameters whose lengths are 42 (vertical) and 43 (horizontal) unit squares.
Working Mathematically		Numeric	Finding the actual length of the diameters by solving the proportion: ▪ If 14 unit squares equal to 2.5 cm, what is the actual length of 42 unit squares? Then, half of it is radius. ▪ If 14 unit squares equal to 2.5 cm, what is the actual length of 43 unit squares? Then, half of it is radius.
Mathematical Results		Numeric	The length of the vertical diameter is 7.5 cm, and the radius is 3.75 cm. The length of the horizontal diameter is 7.68 cm, and the radius is 3.84 cm.

The third cycle includes the process of the calculations on the circle area. While determining the area of the circle, Group B preferred to find it with the area formula of the circle instead of the area tool on GeoGebra. To do this, they used the formula of "πr^2" as a mathematical model and substituted relevant values into this formula to find the area. They obtained the area of the circle as 46.30. As seen in Table 4.8, the register of the mathematical model is algebraic. On the other hand, their actions of substituting the values and finding a result are represented in numeric registers. To validate their result, Group B also considered checking other shapes that would be constructed around the tumor.

Group B worked on the square, regular pentagon, and ellipse models in the subsequent three cycles. They first made their assumptions for these models to simplify the problem situation. Then, they used the area formula of the determined shape as a mathematical model. In the mathematical working transition, they both calculated the actual lengths of the values they will use for the formula and determined the area of the shapes. They then obtained a mathematical result for each shape and compared it with the areas of other shapes in context. In these three cycles (4th, 5th, and 6th), the natural register was used in the simplification process, real model phase, and interpreting transition. On the other hand, while the mathematical model was represented through an algebraic register, the register in the working mathematically, and the mathematical result was numeric.

In the seventh cycle, the group considered creating the ellipse whose area was smaller than the circle on GeoGebra. Using the same method as Group A, this group also used the input command to draw an ellipse passing through determined points around the tumor. While the actions in the simplification process were represented in a natural register in this cycle, the group's mathematizing process and mathematical model were in a geometric register. In the next step, Group B obtained the equation of the ellipse but could not find the area of it because the created shape did not indicate a region on GeoGebra. The created shape/curve was just a curve that the area tool of GeoGebra did not work when group members clicked on that shape/curve. Therefore, their mathematical result was included in the algebraic register. Group B preferred to use the circle as a final model because they thought that the shape of the tumor was too close to a circle. Their representations in interpreting and validating transitions were included in the natural register. In the real model phase, the size and location of the circle were represented in a numeric register and an algebraic register, respectively.

Group C

Group C was a group that made an assumption on a single model and developed their solutions to build that model. The actions of the group choosing the circle model for the shape of the tumor were presented in two cycles. These cycles are shown in Table 4.9 below.

In their first cycle, the group initially understood the problem situation, then simplified it by assuming the shape of the tumor as a circle. To create that circle, they transferred the x-ray to GeoGebra in actual size and identified three points around the tumor. Then, the "circle through three points" tool in GeoGebra was used to draw a circle. Within the scope of the context, the group observed and interpreted whether the constructed circle covered the whole tumor to identify the appropriate circle. In the next process, the size of the tumor was determined, and the generalization was made within the mentioned solution. From understanding to real model transitions/stages, the group used the natural register in this cycle. In addition to these transitions/stages, a natural register was also used in the interpreting and validating processes. While their actions on drawing a circle on GeoGebra were presented in a geometric register, the real result was represented in a numeric register.

In the next cycle, aiming to draw a circle more precisely, Group C used the sliders and input command. Initially, three sliders were created at the estimated intervals and named a, b, and r to represent the x-y coordinates of the center and radius of the circle, respectively. The equation of the circle was entered into the input in the next step, and the circle was created as a mathematical model. Dragging the sliders, they determined the almost perfect circle around the tumor. Their actions in the process of simplification, interpretation, and validation were involved in the natural register. While their real results were represented in algebraic and numeric representations, the group used the algebraic and geometric representations in the mathematizing and mathematical model phases, respectively.

Table 4.9 The modeling transitions/phases and semiotic actions of Group C

1st Cycle			
Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/ Structuring		Natural	Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by using the "Circle through three points" tool For the location of the tumor: The chest x-ray can be put on a coordinate system.
	Conversion		
Mathematizing		Geometric	Identifying several points around the tumor on GeoGebra. Drawing a circle via "Circle through three points" tool on GeoGebra by using identified points
	Treatment		
Mathematical Model		Geometric	Circle passing through the identified points around the tumor (K, L, M points)
	Conversion		
Interpreting		Natural	Interpreting to reality: Created circle could cover the whole tumor.
	Conversion		
Real Results		Numeric	The size of the tumor is 53.5 cm^2.
Validating	Conversion	Natural	Generalization: To identifying the location and area of the tumor, x-ray is needed to be switched in coordinate system in actual size, then the circle is drawn around the tumor by looking the farthest points of it.

Table 4.9 (continued)

2nd Cycle			
Transitions	*Semiotic Actions*	*Registers*	*Explanations*
Simplification/ Structuring		Natural	Return to the situation to look for a more precise circle Simplifying and making assumptions
	Treatment		
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by input command on GeoGebra. For the location of the tumor: The chest x-ray can be put on a coordinate system, and the equation of the circle can be used.
	Conversion		
Mathematizing		Algebraic	Creating three sliders (a, b, and r) representing center coordinates and the length of the radius of the circle Entering a circle equation depending on sliders to input on GeoGebra to draw a circle around the tumor Input Command: $(x\text{-}a)^2 + (y\text{-}b)^2 = r^2$
	Conversion		
Mathematical Model		Geometric	Circle with the center of (8.8, 18.7) and the radius of 4.2
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created circle could cover the whole tumor.
	Conversion		
Real Results		Numeric	The size of the tumor: 55.42 cm2
	Conversion		
		Algebraic	The location of the tumor: c: $(x - 8.8)^2 + (y - 18.7)^2 = 17.64$
	Conversion		
Validating		Natural	Generalization: Firstly, it is needed to find the actual size of the x-ray and it is switched to a coordinate system in actual size. Then, the smallest circle covering the whole tumor is found by entering the circle equation to input on GeoGebra. The location of the tumor is also determined by using the equation of the created circle.

Group D

Group D was another group that assumed just a circle model for the irregular-shaped tumor. As mentioned before, Group D used the proportionality constant for the operations related to the tumor's shape instead of for the actual size of the x-ray because they did not prefer to use technology and transfer the x-ray in actual size to GeoGebra. The detailed actions of Group D are shown in Table 4.10 below.

To begin with, Group D understood the problem situation and simplified it by assuming the tumor could be represented with a circle. Their actions showed similarities to Group A and B in some ways to determine that circle. While these actions were similar to Group A with the idea of drawing a circle inside a square, they were also similar to Group B in measuring the farthest points of the tumor horizontally and vertically to determine the diameter of the circle. These actions from understanding to the real model were determined as a natural register, as seen in Table 4.10. In the next process, the group visualized their circle model on paper using the necessary materials. Their visual representations in mathematizing transition and mathematical model phase were included in the geometric register.

On the other hand, Group D also determined the area formula of the circle as a mathematical model. In the working mathematically transition, they initially calculated the actual length of the radius. They then substituted the identified values in the area formula and reached the size of the tumor. While they used an algebraic register to represent a mathematical model of the area of the tumor, their actions in calculating the operations and reaching the mathematical results were represented in a numeric register. For the location of the tumor, Group D identified the coordinates of the center of the circle by measuring the x-ray in image size. Still, these coordinates were wrong because they had not calculated the actual size of the x-ray. Then, they used these coordinates and the length of the radius for the equation of the circle to express the location of the tumor. As the last action, the group made a generalization for future surgeries and completed their process. Specifically, the group used the algebraic register for representing the location and the natural register to generalize their results.

Table 4.10 The modeling transitions/phases and semiotic actions of Group D

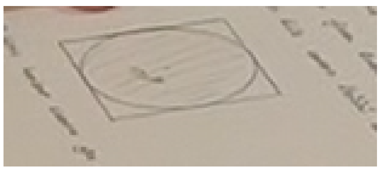
1st Cycle			
Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/ Structuring		Natural	Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created inside a square.
	Conversion		The diameter of the circle is also one of the side of the square that will be determined according to the long distance horizontally or vertically For the location of the tumor: The chest x-ray can be put on a coordinate system.
Mathematizing		Geometric	Drawing a square having the side of 4 cm (image length) around the tumor and creating a circle (with the diameter of 4 cm) in that square to cover the whole tumor in minimum size.
	Treatment		
Mathematical Model		Geometric	Circle with a radius of 2 cm (image length) inside the square created in paper-pencil environment. 
	Conversion		

Table 4.10 (continued)

	Conversion	Algebraic	The area formula of a circle for the size of the tumor is πr^2
Working Mathematically		Numeric	Image length of the radius of 2 cm is equal to the actual length of the radius is 5 cm where the ratio is 1:2.5.
	Treatment		Substituting the value of r=5 and π=3.14 in the formula of πr^2
Mathematical Results		Numeric	The area of the tumor is 78.5.
	Conversion		
		Algebraic	The location of the tumor is $(x-11.5)^2 + (y+0.5)^2=25$
	Conversion		
Validating		Natural	Generalization: Firstly, the tumor is resembled to geometrical shape. Then, the distance is measured both vertically and horizontally. If this distance is equal, then the tumor is circular area. If not, it is elliptic area. For the location, x-ray is placed in coordinate system and the ratio between the image and actual size is found. According to this ratio, the coordinate of the center is identified, then the equation of the shape is determined. Lastly, the size of the tumor is found by using the area formula of determined shape.

Group E

In developing their mathematical model for the task, Group E concentrated on two different shapes for the irregular-shaped tumor, a circle, and a pentagon. The following table shows the registers that Group E used in each transition/phase and modeling cycle.

In the first cycle, Group E initially understood the task and made an assumption about the shape of the tumor. As a first solution, they considered creating a circle on GeoGebra according to the farthest points on the tumor's boundaries. After identifying the estimated most distant points across each other and their midpoint, they created different circles via GeoGebra's “Circle with center through point” tool. Then, they chose the almost perfect circle covering the whole tumor. Group E used the natural register in their understanding, simplification, and interpretation process in the first cycle, as seen in Table 4.11 (see Appendix E for the detailed table including modeling transitions and semiotic actions of Group E). On the other hand, they used the geometric register to create their circle model. Lastly, the size and location of the tumor were determined as a real result by using the information obtained from the GeoGebra. Their registers were numeric and algebraic, respectively.

The group’s other assumption was to remove the tumor as a pentagon because the shape of the tumor was quite like a pentagonal. The actions related to the pentagon were presented in the second cycle. When the members of Group E made assumptions in the simplification process, they used the natural register like in the first cycle. As a next step, five points were identified at the edges of the tumor. Then, the polygon tool was used to draw a pentagon around the tumor. They found that the constructed pentagon covered the entire tumor and had less area than the circle. As the last action, they made a generalization on their findings and concluded that the circle model was more appropriate than the pentagon for different tumor shapes and more practical to determine it. In forming the pentagon, the group used the geometric register, and their interpretations of the pentagon in the context were recorded as a natural register. In addition to these registers, the mathematical result on the area of the pentagon was numeric, and the group’s generalization was a natural register in the last cycle.

Table 4.11 Group E's Semiotic Registers Specific to Transitions/Phases and Modeling Cycles

Transitions/Phases	Modeling Cycles	
	1st	2nd
Understanding	NaR	
Situational Model	NaR	
Simplification/Structuring	NaR	NaR
Real Model	NaR	NaR
Mathematizing	GR	GR
Mathematical Model	GR	GR
Working Mathematically		
Mathematical Results		
Interpreting	NaR	NaR
Real Results	NuR & AR	NuR
Validating		NaR

4.1.3 Summary of the Semiotic Representations

In this section, the semiotic registers that the groups used in their modeling process are presented to examine their characteristics holistically. Table 4.12 displays which register is used in which part of the "Tumor Surgery" task by the group.

As seen in Table 4.12, each group used both natural and numeric registers to determine the actual size of the x-ray. Similarly, semiotic representations that groups used also show similarity in terms of their types in the determination of the location of the tumor. Here, natural and algebraic registers were used in the process of each group.

Table 4.12 Characteristics of the Groups in terms of the Semiotic Registers

	Registers	*Actual Size of the X-Ray*	*Shape of the Tumor*	*Area of the Tumor*	*Location of the Tumor*
	Natural	+	+	+	+
Group A	Numeric	+		+	
	Geometric		+		
	Algebraic				+
	Natural	+	+	+	+
Group B	Numeric	+		+	
	Geometric		+		
	Algebraic			+	+
	Natural	+	+	+	+
Group C	Numeric	+		+	
	Geometric		+		
	Algebraic		+		+
	Natural	+	+	+	+
Group D	Numeric	+		+	
	Geometric		+		
	Algebraic			+	+
	Natural	+	+	+	+
Group E	Numeric	+		+	
	Geometric		+		
	Algebraic				+

Differences in the groups' semiotic registers were seen in the parts related to the shape and area of the tumor. Actions of Group A for the shape of the tumor were included in both natural and geometric registers. In addition to Group A, the semiotic registers of the other groups for the shape of the tumor were also involved in the natural and geometric registers. At this point, it is seen that Group C also used the algebraic register for the shape of the tumor, unlike the other groups. For the area of the tumor, the same semiotic registers were used in the representations of Group A, C, and E. Their actions were included in both natural and numeric register for this part of the problem. On the other hand, Group B and D also used the algebraic register in addition to the natural and numeric register for the area of the tumor.

As the last finding, the natural register is the only register that all groups used in each part of the problem.

4.2 Modeling Routes of Groups

This section includes the five parts presenting groups' modeling routes in the "Tumor Surgery" modeling task. Their modeling processes are shown in a graphical representation named Modeling Transitions Diagram (MTD).

4.2.1 Group A: Ahsen, Asya and Aylin

Preservice teachers in Group A worked on various models and explored different solution methods for these models during their process. Group A's MTD showing their detailed modeling routes is shown in Figure 4.7 below.

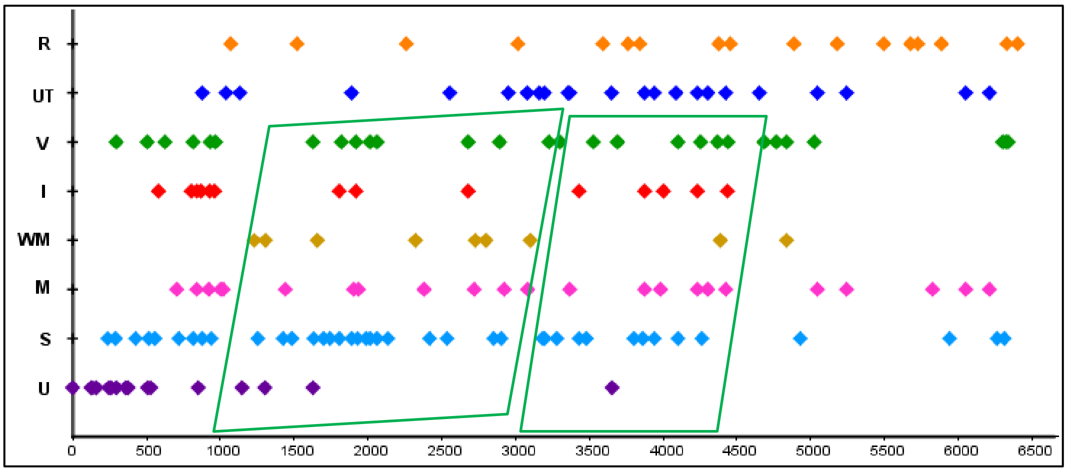


Figure 4.7 Modeling Transition Diagram of Group A and their Modeling Route

The MTD of Group A indicated that the modeling process of the group was too complex from the beginning to the end. Here, Group A had too many entries, especially in the simplification/structuring compared to the other transitions. In addition to a large number of entries, their simplification process continued almost to the end, just like in mathematizing, validating, using technology, and reporting stages. On the other hand, the frequencies of incidences in understanding, working mathematically, and interpreting transitions were visibly less, and they ended earlier compared to the other ones.

In Group A's MTD, it was difficult to follow the entries and determine the different cycles. In Figure 4.7, the green box on the left specifically included the entries related to the actions of Group A in resizing the x-ray and mainly their assumptions on tumor shapes. Here, Group A focused on the different parts of the modeling task. The green box on the left did not represent a cycle for a particular model or solution, but it included different cycles related to the mentioned parts of the modeling task. On the other hand, the green box on the right also had different cycles starting with simplification/structuring transition and ending with validating transition. More specifically, in the green box on the right, the mentioned cycles included the actions related to different models such as circle and ellipse, and the different solution methods for these models. Here, in the first of the boxes mentioned above, it seemed that the cycles did not proceed on a regular path. When the entries in this box were examined, it was observed that the group frequently moved forward and backward between transitions. On the other hand, when the cycles in the green box on the right were examined, it could be observed that these cycles had more sequential progression from the simplification to the validating transition.

Another important detail that needs to be mentioned is that the transitions in which the group was involved did not progress regularly. As seen in the MTD, the group was not always moving forward; they often had backward actions in terms of modeling transitions. To mention this regular progress in the modeling route from another perspective, it could be said that the group continued their process by jumping some transitions at many moments. Explaining this event with an example may make the situation more understandable. Specifically, the group initially acquired the general impression of the task and simplified it right after. At this point, the group jumped to the validating from the simplification/structuring transition at the very beginning of their process. One of Group A's members, Ahsen, had used the implicit statement about the reasonableness of her assumption on the shape of the tumor. This statement is given in the following dialogue.

> ***Teacher:*** *Let's not think of the liver as a volume right now; you can just think of it as a surface area.*
>
> ***Ahsen:*** *For example, when we think of it as a circle…*
>
> ***Teacher:*** *So, think of it in two dimensions. Imagine you remove it in the form of a two-dimensional disk. The area of the disk…*
>
> ***Ahsen:*** *Okay, it's a circle.*

In the above episode, it was seen that Ahsen likened the tumor to a circle. At that moment in the classroom conversation, the statements confirming the model produced by Ahsen on the teacher's explanations about the task moved the group from the simplification/structuring to the validating transition. It can be observed that such jumps and going backward in transitions were widespread in the modeling route of Group A.

According to the markings in the MTD, the group is repeatedly involved in the same transition in some moments. In other words, they show a horizontal progression. For example, while they took place in the understanding level for about 250 seconds at the beginning of their processes, it was observed that they displayed this horizontal progress at the reporting level towards the end of their modeling processes. When the progress of the group was examined second to second, it was noticed that the only transition where there was no horizontal progression was the interpreting transition.

Some markings of different transitions in the MTD coincided with the same second. Especially, it was seen that the dots in the using technology stage were marked simultaneously with both the simplification/structuring and mathematizing transitions. This situation was also observed when any statement in the unit of analysis was assigned to more than one indicator belonging to the different transitions. The episode below illustrates the mentioned situation.

Ahsen: *For example, how did we fix this right now?*

Aylin: *We placed the bottom on the x-axis; we accepted the middle of it as (0,0).*

Asya: *These 10 small squares are equal to 2.5 cm. We know that. We have this.*

Aylin: *We have now determined the coordinate system according to our way.*

Ahsen: *No. The teacher said, for example, that when we enlarged this shape, it changed. How do I make sure it doesn't change?*

Aylin: *Now, we have ensured that it does not change in this way. According to this measure, our 10 square units is 2.5…*

Asya: *Do you know how it happened! For example, you enlarged this. If this is equal to 2 units, then a hundred percent, that is, the whole, will be equal to 4 units.*

Ahsen: *But we will have defined it in the coordinate system.*

Asya: *After that, this will also enlarge. 2 units equal 2.5, 4 units become 5 again.*

Aylin: *Yes, exactly.*

Ahsen: *Okay. I got it.*

In the dialogue above, the group members discussed fixing the x-ray on GeoGebra by finding its actual size. At the beginning of the dialogue, Ahsen's question on understanding the solution and what they were doing was included in the understanding transition. In contrast, the other group members attempted to justify their solutions and convince Ahsen were included in the validating step.

4.2.2 Group B: Bahar, Beliz and Berna

Group B was also another group working on various models during their process. Their MTD is displayed in Figure 4.8 below.

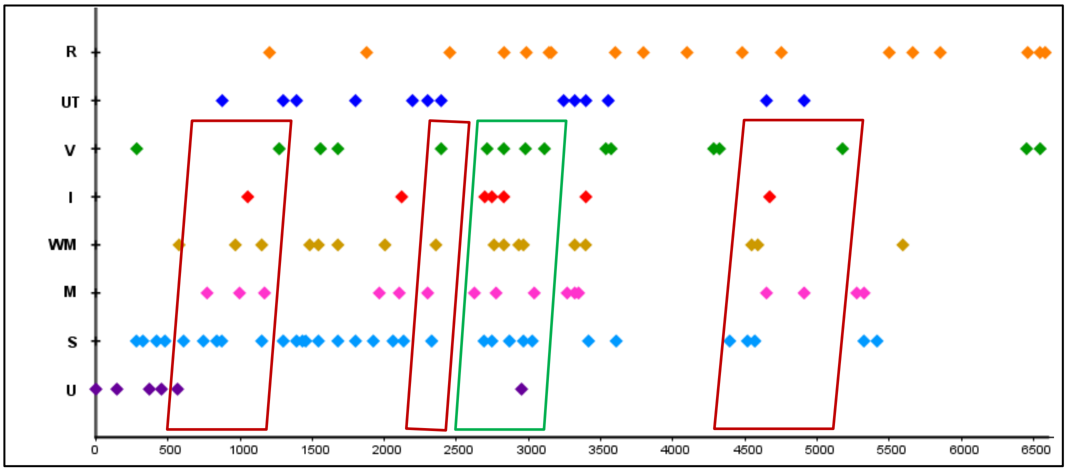


Figure 4.8 Modeling Transition Diagram of Group B and their Modeling Route

Similar to Group A, the modeling process of the group was also too complex from the beginning to the end according to their MTD. While the number of entries in the understanding and interpreting was less than the number of entries in the other transitions, the transition with the most entries was the simplification/structuring as in Group A. Furthermore, it was seen that the entries in the understanding and interpreting transition of the group ended earlier than the others. Another visible detail regarding the frequencies of the entries was that Group B, whose process was as complex as Group A, had much fewer entries at the using technology stage than Group A.

In Group B's MTD, it was also too difficult to follow the entries and determine the different cycles. In Figure 4.8, the red box on the left specifically included the entries related to the actions of Group B in resizing the x-ray. When this cycle was examined, it was observed that the group frequently moved forward and backward between transitions in their modeling process. In the next red box just after the 2000th second, there was a cycle starting with mathematizing transition and ending with validating transition. Here, it was also observed that the group did not have sequential progression within the cycle because they frequently moved forward and backward between transitions in their modeling process. On the other hand, the green box in Group B’s MTD included entries resulting by the actions of Group B in the various models, such as square and regular pentagon. Specifically, Group B made an assumption on mentioned models for the tumor’s shape and calculations on the area of the determined shapes in a paper-pencil environment at that moment. Lastly, it was also determined one more cycle in Group B’s MTD. In the red box on the right, the mentioned cycle included the actions related to the ellipse model. When the entries in this box were examined, it was observed that the group frequently moved forward and backward between transitions similar to other cycles.

Like other groups, Group B's process was also not progressing on a steady path. Although it was observed that the entries moved forward between the transitions, there were also backward entries at some moments. In addition to this irregular progress, it could be observed that the group progressed in their modeling process by jumping some transitions.

In each transition except interpreting, Group B's entries were repeated to be included in the same transition. To illustrate this, their first two entries were marked in the understanding, while the entries in the reporting stage towards the end of their process were marked in a row. On the other hand, the fact that the horizontal progression was not observed in the interpreting transition was similar to the MTD of Group A.

The MTD of Group B showed that there were also some markings belonging to different transitions coinciding at the same second. Some of them were seen simultaneously in the using technology stage and the simplification/structuring or mathematizing transitions. The episode below illustrates one of the simultaneous markings other than entries at the using technology stage.

Beliz: *There is something like that, isn't there? For example, let's say this is a circle; the longest chord has to be the diameter. But here, the longest chord is not the diameter.*

***Bahar**: How?*

Berna: *How not?*

Beliz: *For example, this one. This is 43, and this is also 43. But, this is not the diameter because it does not pass through the center.*

Bahar: *This is 42.*

Berna: *No, Think about it in this way.*

Beliz: *For example, something like this remains, do you understand? It does not pass from here. Here is 43…*

Berna: *No. We have a tumor like this, right? For example, we're thinking about how much this will be covered in minimum as a circle. If we think it covers this as a minimum circle, this will be our center, which you drew here. We don't look at it as a thing. We don't say that part of the fluctuations; for example, we don't say this is the widest. We perceive that tumor as a circle now. I mean, that's the least tissue we could remove, other than that because we cannot remove it like that.*

Bahar: *We'll remove it, too. There will be no tumor left. We'll create the smallest circle inside it.*

Berna: *Exactly.*

In the dialogue above, the group members discussed the diameters of the circle model in horizontal (43 unit squares) and vertical (42 unit squares). According to their solution, they would use the longer diameter to draw a circle covering the whole tumor. At the beginning of the conversation, Beliz argued that the longest chord, that was, the diameter of the circle to be drawn, would not pass through the center when she considered the intersection point of the diameters both in horizontal and vertical. Here, while the group was involved in the simplification/structuring transition by mentioning the components of the circle, they also moved to the mathematical working transition with Beliz's mathematical inference that the longest chord of the circle must be the diameter. In addition to these transitions, the group was also involved in the validating transition while Beliz was questioning whether the model was reasonable, and the other group members were attempting to justify their solutions and convince Beliz.

4.2.3 Group C: Canan and Ceren

Preservice teachers in Group C worked on just one model and improved their model with the different solution methods during their process. Group C's MTD is displayed in Figure 4.9 below.

Compared to Group A and B's MTD, Group C's MTD had a simpler appearance in terms of the number of entries. While their statements fell intensely into the simplification/structuring and using technology stages, there were just two entries in the working mathematically transition during their modeling process. It was evident that the number of entries in the remaining transitions was less than ten. On the other hand, the entries in the understanding transition ended at the beginning of their process. Their mathematizing and working mathematically processes were also ended earlier than other transitions although those processes continued towards the end of the modeling process.

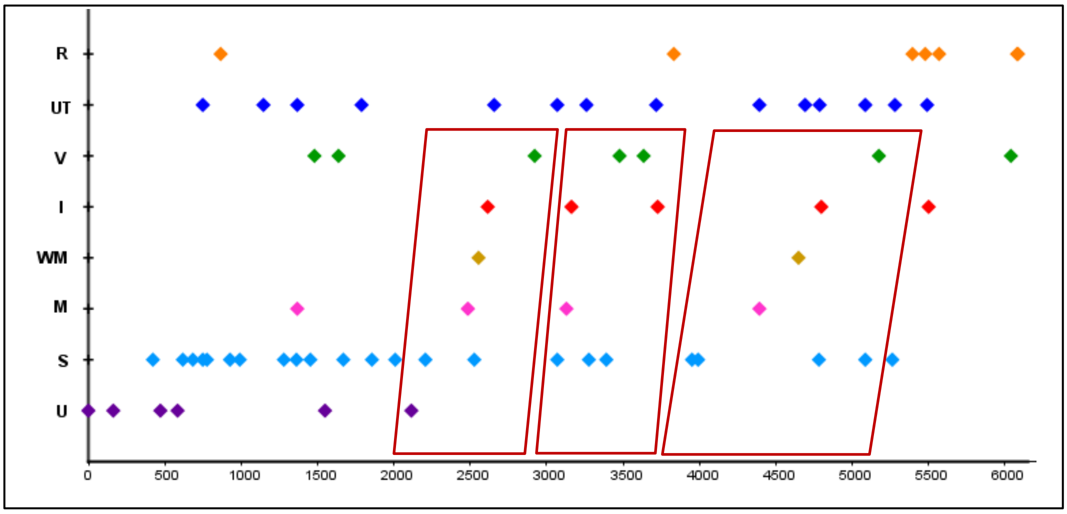


Figure 4.9 Modeling Transition Diagram of Group C and their Modeling Route

In the previous two MTDs, it was too difficult to follow the entries in regular paths. When we examined Group C's MTD, we could determine the different cycles. These cycles were represented on MTD in Figure 4.9. Based on mathematical modeling transitions, it could be said that the group had more actions on simplifying the problem or establishing the systemic structure until the 2000th second. As an anomaly in this section, the fact that the group did not attempt working mathematically and interpreting, but proceeded to the validating transition directly, could be given as an example. Just after the 2000th second, there was a cycle where the group completed all the modeling transitions. The group worked on the actual size of the x-ray at that moment. Also, two different cycles were not progressing in regular paths after the 3000th and 4000th seconds. In the first of the aforementioned cycles, the group returned to the simplification transition several times and completed the cycle by jumping to the mathematical working transition. In the next cycle, the group again returned to the simplification stage several times, as in the previous cycle, and then completed the cycle with no jumping any of the modeling transitions. As mentioned before, the group had two different solution methods for the circle model. These cycles represented specifically these methods.

According to MTD, there were four transitions in which the entries of Group C were involved in the same transition in a row. Unlike the previous two groups, these transitions observed the horizontal progress were understanding, simplification/structuring, validating and reporting. The MTD of Group C also showed that there were some markings belonging to different transitions that coincided at the same second. The dots in the same second were because the group is also included in either the simplification or mathematizing transition while using technology.

4.2.4 Group D: Deniz and Doğa

Preservice teachers in Group D worked on just one model during their process, like Group C. The MTD of Group D is given in the Figure 4.10 below.

While the statements of Group D fell intensely into the simplification/structuring like the other groups, there was no entry in the using technology stage. In the other transitions, the number of entries ranged from 4 to 13. From another perspective, the entries in the understanding transition continued almost until the end of their modeling process, unlike the aforementioned groups.

Similar to Group C, following the entries in Group D's MTD as a cycle was easier than the first two groups. After acquiring the general information on the task, Group D had continued their process by making assumptions on the shape of the tumor. At the same time, the reasonableness of their model was discussed, and the model was interpreted in the group. These processes could be seen as entries at the beginning of their MTD. Here, there was an anomaly that was jumping to the interpreting and validating transitions without passing through the mathematizing and working mathematically transitions. After the 1500th second, there was a cycle that included all modeling transitions but did not proceed in the regular path. Another completed cycle in which there was no jumping in any of the transitions was seen after 3500th seconds.

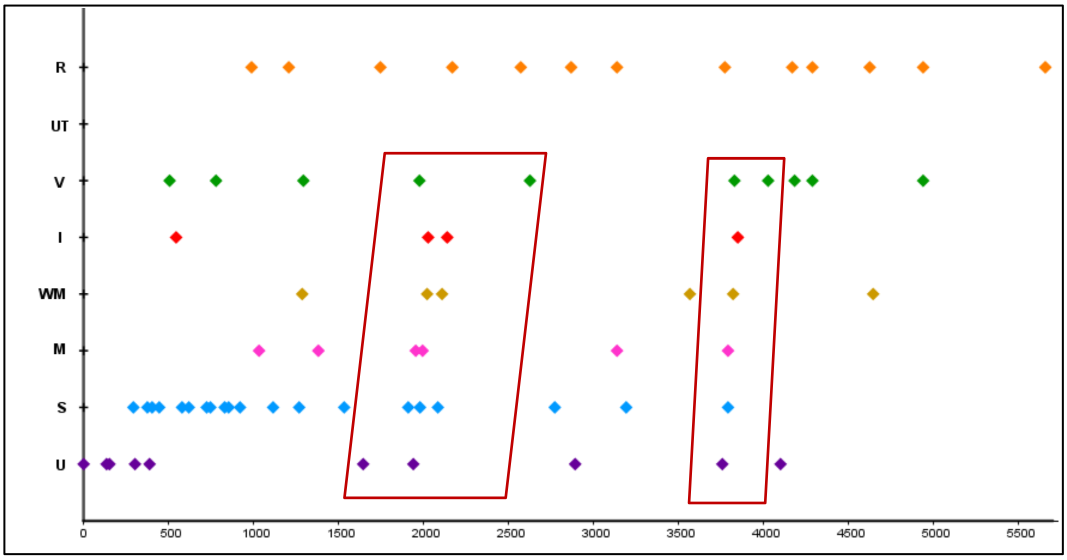


Figure 4.10 Modeling Transition Diagram of Group D and their Modeling Route

According to MTD, there were four transitions in which the entries of Group D were involved in the same transition in a row. Unlike the first two groups, horizontal progress was observed in the understanding, simplification/structuring, validating, and reporting transitions, similar to Group C.

The MTD of Group D showed that there were two markings belonging to different transitions that coincided at the same second. This coincidence was seen at about the 3100^{th} second in the transitions of mathematizing and reporting. Here, the group members wrote a report and drew mathematical representations related to their circle model. To illustrate, Group D initially determined the length of the diameter after the necessary measurements were made on the x-ray, and they wrote this process to the report. To visualize their model, they drew a square and a circle inside that square by using a ruler and compass as seen in Figure 4.11. Therefore, their actions were included in both reporting and mathematizing.

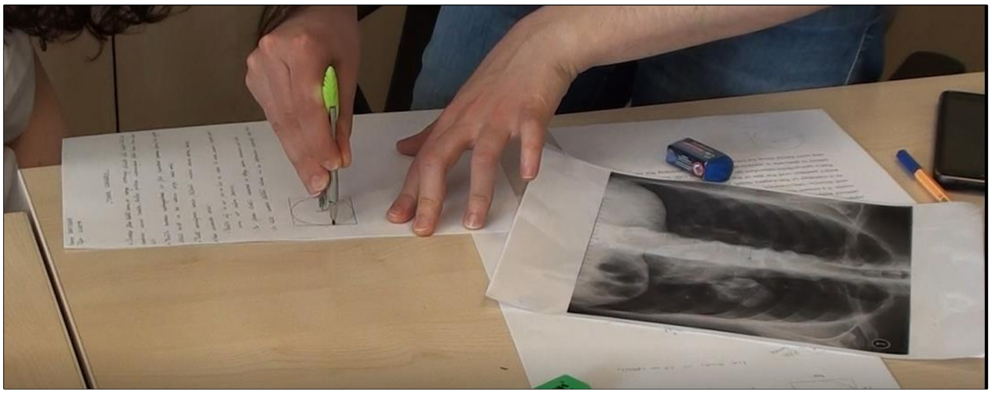


Figure 4.11 Drawings of Group D on their model and their reporting process

There was also a coincidence at about the 3700th second in the transitions of simplification/structuring and mathematizing. Specifically, the group discussed on the area of the tumor and mentioned the area formula of a circle and the values in that formula. The following dialogue illustrates the mentioned situation. In the middle of the dialogue, Doğa mentioned the formula of πr^2 and the value of "π" number. Here, the group's discussion of the πr^2 formula was included in the mathematizing transition, while their discussion of the values to use for π and r was included in the simplification/structuring.

Deniz: *Okay, Now let's calculate the area.*

Doğa: *Was that being asked?*

Deniz: *Size.*

Doğa: *What does size mean?*

Deniz: *That's the size.*

Doğa: *Is size related to the area? Shall we remove it as an area?*

Deniz: *The tumor we're going to remove…*

Doğa: *It's area. Is it called it's area?*

Deniz: *It's the amount.*

Doğa: *It's amount… Do we find the amount of the circle from the area of the circle?*

Deniz: *We'll find. That is…*

Doğa: *πr^2? We take π as 3.14.*

Deniz: *The square of 5 cm…*

Doğa: *I'm calculating now.*

Deniz: *Let's talk about an ellipse in generalization.*

Doğa: *So, if it was a square, I would remove it as a square. If it was a rectangle, I would remove it as a rectangle… Multiply 2.5…*

Deniz: *What is 2.5? Multiply 2.5 by 3.14.*

Doğa: *78.5. So, do we remove 78.5 gram?*

Deniz: *cm^2*

Doğa: *But, tumor is not expressed as cm^2. Isn't a tumor something like weight?*

Deniz: *We think it in two dimensions.*

4.2.5 Group E: Ece, Elçin and Eylül

Group E worked on two different models during their process, and they used technology at every stage of the modeling process. The MTD of Group E is given in the Figure 4.12 below.

As also seen in the aforementioned groups, the simplification/structuring was the only transition having the most entries, unlike the other transitions according to the MTD of Group E. On the other hand, surprisingly, there was no entry in the working mathematically transition of this group. As seen in Figure 4.12, while the left side of the MTD had many entries involved in the different transitions, the right side was formed intensely by the entries in the reporting stage. In other words, this might be considered as evidence that Group E spent more time in reporting their findings from about the 4000th second. From another perspective, the group's simplification and interpretation processes ended much earlier while the entries still continued to be seen towards the end of the process.

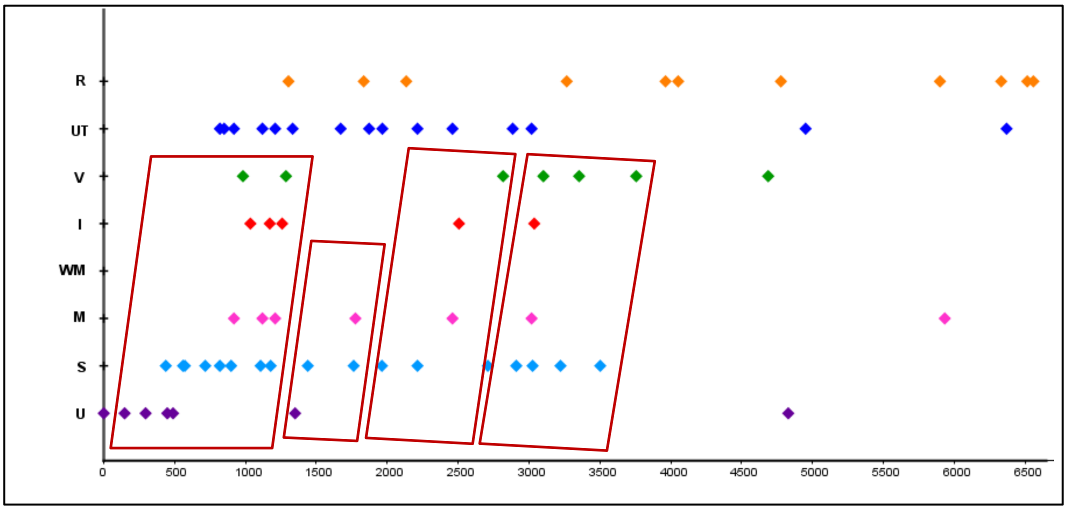


Figure 4.12 Modeling Transition Diagram of Group E and their Modeling Route

When the MTD of Group E was examined, it was clearly seen that there was a cycle at the beginning of their process. According to MTD, after the reading and understanding, Group E was placed in the simplification process for a while. Then, the group constructed a model based on their assumptions, as understood by the entries included in the using technology and mathematizing transitions marked at the same second. After that, they were involved in the interpreting and validating transitions while evaluating the model in the context and verifying it. Between 1000th and 1500th seconds, the second cycle occurred. Specifically, this realized when the group understood that they needed to find the actual size of the x-ray. Here, the group was intensely involved in the simplification/structuring and using technology stages. In the next process, there were also two cycles that the group worked on, the circle model and the pentagon model. These cycles did not progress in a regular path because there was no entry in the working mathematically process. Also, it was evident that the group returned backward sometimes instead of moving forward.

Based on the mentioned cycles and the modeling transitions, there was also an uncompleted cycle at about the 1500th second.

According to MTD, there were three transitions in which the entries of Group E were involved in the same transition in a row. These transitions observed in the horizontal progress were understanding, simplification/structuring, and reporting. Unlike Group C and D, there was no horizontal progression in the validating transition of this group.

It can be seen there were also some markings belonging to different transitions coinciding at the same seconds when the MTD of Group E was examined. While using the technology, the actions of the groups were involved in either the simplification or mathematizing phase in some cases. For this reason, entries marked at the exact second appeared in the MTD of Group E.

CHAPTER 5

DISCUSSION AND CONCLUSION

This study aimed to investigate the semiotic representations of groups' mathematical modeling processes and groups' changing modeling routes regarding the semiotic characteristics via technology-integrated modeling activity. In this chapter, the findings of this study within the scope of the mentioned aim were initially discussed. Then, recommendations for further research, educational implications and limitations of the study were explained in the following parts.

5.1 Characteristics of Groups in terms of Semiotic Representations

It is obvious that each individual in the world is unique, and therefore individuals' mathematical understandings differ from each other. Thus, the mathematical thinking processes of individuals and their tendency to use different representations may develop by their mathematical knowledge and understanding. It is possible to consider that individuals' semiotic representations in different forms and transformation between these representations forms can promote individuals' conceptual understanding (Özcan et al., 2022). In connection with this, Borromeo Ferri and Lesh (2013, p. 58) mentioned that model development can occur with different dimensions, specifically "concrete-abstract, particular-general, situated-decontextualized, simple-complex, intuitive-formal." This model development is handled as an algebraic and numeric model, which can be considered a more specific dimension, in the study of Shahbari and Tabach (2020). Shahbari and Tabach (2020), put forward another dimension (algebraic and numeric model) based on Duval's (2006) Theory of Registers of Semiotic Representations (TRSR) framework. In this study, using Duval's (2006) framework by extending the study of Shahbari and

Tabach (2020), we attempted to determine the semiotic characteristics of pre-service teachers working as a group.

The findings obtained in the current study revealed that the characteristics of the groups varied according to the purpose of their actions in the "Tumor Surgery" task. The task consisted of different parts that needed to be dealt with to reach the solution. These parts were related to the actual size of the x-ray, shape, area, and location of the tumor, respectively (see Table 4.12). During the analysis process of this study's data, it was seen that the actions of the groups were constantly moving between these parts.

In the first part of the problem, groups had been expected to resize the x-ray to its actual size by proportioning the relevant values. As presented detailed in the results section, all groups except Group E had determined proportionality constant to resize the x-ray to its actual size. Group E, on the other hand, preferred to do this resizing on GeoGebra without establishing a proportion. Here, the actions of each group were similar in the specific modeling phases and transitions, and their semiotic representations were involved in just the natural and numeric registers. Therefore, the findings showed that there is no difference in terms of the semiotic characteristics of the groups in the actions related to resizing the x-ray to its actual size.

In the second part of the problem, all groups compared the irregular-shaped tumor to a regular shape. Here, the characteristics of the developed mathematical models can be evaluated in two different dimensions. Specifically, the first one is related to the developed mathematical models varying from specific regular shape to general one, and the other one is the algebraic and geometric models of groups. To illustrate, while one of the groups had created an ellipse as a final model, others had decided to remove the tumor as a circle. Regardless of the semiotic representations used, Group A preferred to use the ellipse model as a more general shape than a circle for both the given tumor tissue and different shapes of tumors; that may be because they considered that circle is a special case of an ellipse. Among the other groups, Group D is another group that generalized the ellipse model for future surgeries, although they preferred to use the circle model for the specific situation. However, when semiotic representations come into play, it is seen that the most general model belongs to Group C and the characteristic of their model differs from other groups. Group C generated a model in the algebraic register, while the other groups generated models in the natural and geometric registers. This is consistent with the findings of the study conducted by Shahbari and Tabach (2020). In parallel with our study, they found that the groups with algebraic models used natural and numeric and algebraic registers, while the groups with numeric models used just numeric and natural registers. As a more general model, Group C created their final model by using the equation of the circle whose variables could be changed with sliders; then they chose the best possible circle covering perfectly the whole tumor with minimum healthy tissue. The final models of the other groups, using natural and geometric registers, differed according to the method they used to draw a regular shape. Group A created a conic passing through five points, and they tried to reach the perfect ellipse by arranging the locations of the determined points.

On the other hand, Group E attempted to create a circle according to the possible two farthest points around the tumor as a less general solution and tried to find the smallest circle covering the whole tumor. Group B and Group D determined a circle according to the longest horizontal or vertical distance of the tumor as a more specific model. In brief, considering all these situations, our study supports that the groups whose semiotic characteristic was determined as an algebraic model, as mentioned in the findings of Shahbari and Tabach's (2020) study, is a more comprehensive model than the other models having different semiotic characteristic because the groups determined as algebraic model used the other two registers (natural and geometric) in their modeling processes while groups determined as geometric model used just geometric and natural register in the current study. According to the findings of our study, groups were included in algebraic, geometric and natural register in their modeling processes related to the determination of tumor’s shape. On the other hand, in Shahbari & Tabach’s (2020) study, groups were included in algebraic, numeric and natural registers. This shows that the two studies differ in terms of the registers that groups used. Additionally, regardless of semiotic characteristics, final models of groups for the shape of the tumor can be given as an example of the particular-general dimension mentioned in the study of Borromeo Ferri & Lesh (2013).

In the third part of the task, the groups were expected to determine the size of the tumor. Here, the actions of the groups varied according to the registers that they were involved in to reach the area of the determined shape. While Group B and Group D additionally used the algebraic register for this part of the task, other groups generated only natural and numeric registers similar to the situation mentioned for the second part of the task. According to the presented results, the characteristics of the groups varied according to the preference of using technology and the aim of using it. Groups A, C, and E used the technology to create different and more precise models and interpret them in the context of the problem. The area of the determined model was found via the area tool of GeoGebra, and a numeric result was obtained by these groups. However, the aim of Group B in using technology was just to test whether their model covered the whole tumor. To determine the area of the circle, Group B used the area formula of the determined shape. This was quite surprising because Group B used the area formula of the circle to find the area of the tumor, although some of the members had sufficient knowledge of the use of GeoGebra and were aware of the area tool. The same action related to finding an area was also observed in Group D. Because they did not prefer to use the technology in any part of their process, they also used the area formula of the determined shape. This is why these groups used the algebraic register in actions related to the area of the tumor. The findings related to this part of our study were exactly similar to the findings of Shahbari and Tabach (2020), and the semiotic characteristics of the groups could be determined as algebraic and numeric.

In the fourth part of the problem, the groups were expected to determine the location of the tumor. Here, the acts of each group were the same, and their semiotic representations were involved in the natural and algebraic register. Therefore, the findings showed no difference in actions related to the tumor's location, like in acts related to resizing the x-ray.

To summarize, the characteristics of the mathematical models that the preservice teachers used in terms of semiotic registers varied in the parts related to the shape and area of the tumor. Regarding semiotic registers, the most general model belonged to Group C, and their semiotic characteristic was determined as the algebraic model. In contrast, the others were the geometric model in the parts related to the tumor's shape. For the area of the tumor, the semiotic characteristic of Group B and Group D using the area formula of the circle was determined as an algebraic model. In contrast, the semiotic characteristic of other groups using the area tool in GeoGebra was determined as numeric, like in the study of Shahbari and Tabach (2020). As an important detail regarding the algebraic and numeric model in the process of finding the area of the tumor, the semiotic characteristics of the groups showed an alteration according to their preference for using technology and their purpose to use technology.

Contrary to the study of Shahbari and Tabach (2020), in which students' final models were categorized under two registers, algebraic and numeric, in this study, no specific semiotic register was determined to categorize the characteristics of the groups' models within each modeling cycle or their whole modeling process. In this study, it was observed that four types of semiotic representations were used by each group, but they showed alterations in each part of the modeling task. Considering the semiotic representations in each modeling cycle or groups' whole modeling process, the fact that the groups were not included in a certain semiotic characteristic can be explained by the nature of the modeling task. By its nature, the solution to the Tumor Surgery task was provided by four types of semiotic registers determined in this study. For example, pre-service teachers were expected to determine the equation of the created shape in determining the location of the tumor in the lung. This situation pushed the pre-service teachers to make a representation with algebraic notations in this process. This and similar situations may have resulted in all groups using all types of the registers in their representations in different parts of the task.

The nature of the modeling task may have affected groups in using certain registers in their whole modeling process. In the current study, the groups did not have a particular semiotic characteristic in their entire modeling process. Still, Shahbari and Tabach (2020) were able to assign groups to a particular semiotic characteristic. In the modeling task used by Shahbari and Tabach (2020), no numerical values were given. Pre-service teachers were expected to make assumptions about both the three-dimensional shape and the length of its dimensions in order to find a volume that could vary according to the assumptions. Then they were expected to compare the volumes in the old and new situations. On the contrary, in our modeling task, pre-service teachers were given a length and expected to determine the area of a shape with a certain size and find its location based on this length. In this way, two different modeling tasks with different expectations (comparing volumes by determining ratios and determining an area/location of irregular shape with a certain size) may have affected the pre-service teachers' preferences for using particular semiotic registers or various semiotic registers in their whole modeling processes. To illustrate, in Shahbari and Tabach's (2020) study, since the ratio between two different numerical values can be established, numeric models may have been chosen to use random numerical values in comparing the volumes in their whole process. Similarly, since the necessary results can be obtained for comparison of volumes when a ratio was made between the formulas, including algebraic notations, the semiotic registers of the algebraic models may have mainly changed as algebraic registers. However, in the current study, pre-service teachers' assumptions were related to the tumor's shapes, and this enabled pre-service teachers to create different regular shapes with various methods open to using different semiotic registers. This may have resulted in the use of more registers in our study and the emergence of variation of these registers in different parts of the modeling task.

5.2 Modeling Routes of Groups

In the process of mathematical modeling, there is an intense interaction between mathematics and the real world. In this regard, Borromeo Ferri (2007) has revealed a structure between these interactions, which she called "the individual modeling route," in which individuals' cognitive procedures were examined in the modeling process. The representation of modeling routes was displayed in the modeling cycle of Blum and Leiß (2007) by Borromeo Ferri (2007). Shahbari and Tabach (2020) used the same representation format, which shed light on this study. Still, the complexity of the modeling processes of groups in the current study prompted us to look for a different structure for the representation of modeling routes. As mentioned in detail in the methodology chapter (see section 3.5.1.1), the graphical representation of modeling routes was preferred in this study like in the studies of Ärlebäck (2009), Czocher (2016), and Albarracin et al. (2019).

In the presentation of the findings of the study, modeling routes of the groups were handled in terms of the complexity of the MTD, the frequency of being in modeling transitions, sequential progression of the transitions, completed cycles, horizontal progression in a particular transition, and overlapping of transitions. To begin with, the MTD of Group A and Group B had a more complex appearance than others. This may be due to the fact that these groups produced much more solutions than the other groups and therefore had more modeling cycles. Specifically, Group A had so many answers because they wanted to create a more general model with the smallest area and was more appropriate for possible tumor shapes. In this way, Group A had more modeling cycles reflecting their modeling processes as they produced both different shapes and different solution strategies in creating these shapes to make the necessary revisions during the validation process. On the other hand, Group B had also many modeling cycles that they worked on various shapes, from circle to pentagon, to verify that a model they had created was the best one having the least area. It was quite challenging to follow the modeling routes of groups having complex MTDs and to determine a certain cycle. On the other hand, noticeable cycles could be detected in MTDs having a simpler appearance (Group C, D and E).

In her study, Czocher (2013) stated that students were more involved in the simplification/structuring transition in the MTD belonging to "The Cell Problem" task, and it was observed that there were more entries in the mentioned transition in almost all MTDs belonging to other tasks. Similarly, in the current study, it was also observed that this step was represented much more in all MTDs regardless of the group. Obviously, there are too many indicators in the simplification/structuring transition in the rubric (see Table 3.2) used to determine the modeling transitions, and this made us think of the reason why there were more input in the simplification process. The fact that Czocher (2013) observed this situation in the MTDs of Fermi problems, which she determined as a validating activity, was consistent with our findings. More specifically, even in validating activity that requires more modeling competencies such as justifying and verifying, the reason why students were more involved in the simplification transition may be that there are more indicators in the simplification transition in rubric.

On the contrary, Czocher (2016) also reported that there were anomalies in MTDs of students because there is no students' actions in some transitions throughout their modeling process. A similar case was also observed in the current study. To illustrate, Group D had no action in using the technology stage because they did not prefer to use technology. Moreover, Group E had no action in working mathematically transition because they used the technology in each action. Their actions could not involve working mathematically transition while assigning indicators in the rubric. Regarding the frequency of entries in different transitions, another striking detail was that the transitions with fewer inputs ended earlier than the other transitions in almost all MTDs in this study.

The regular progression of the transitions in MTD was determined in the current study, as that groups moved forward from understanding transition to validating transition in their modeling routes. This was also visualized in the idealized MTD of

Czocher (2016). In this regard, there were some differences in the MTDs of groups. To illustrate, it was observed that all groups moved both forward and backward in their actions within any cycle. At this point, Group A, B, C, and D were the groups that did not have regular progression in their MTDs. On the other hand, a completed cycle was observed with no jumping of the particular transition in MTDs of these groups. The completed cycles (Shahbari & Tabach, 2020) having no regular progression presented in this study were similar to the studies in the literature (Ärlebäck, 2009; Czocher, 2016). Another remarkable detail regarding MTDs is that some of the groups (Group A, B, and C) jumped to the validating transition from the simplification/structuring at the beginning of their modeling processes. Pre-service teachers tended to justify their assumptions reasonably in this study. Specifically, the explicit statement of one of Group A's members can be given as an example of this situation. Ahsen, assuming that a circle could be drawn around the tumor, thought that circle was a reasonable model based on the teacher’s statements during the classroom discussion at the beginning of their modeling process. Here, this thought was involved in the validating transition and it was assigned to the indicator of “implicit or explicit statements about the reasonableness of the answer/model.” This and similar situations may have caused the early occurrence of validation transition in the MTDs of the groups. Similarly, Czocher (2016) also reported that validating transition occurred although students had no mathematical results to validate. This case was also realized for the interpreting transition. Czocher (2013) revealed with the data she obtained that students interpreted their models within the context of the real problem situation without taking any action towards working mathematically. Unlike the theoretical framework, the current study supported the findings of Czocher regarding the early occurrence of interpreting and validating transitions in MTDs.

5.3 The Relation Between Semiotic Characteristics and Modeling Routes of the Groups

One of the aims of this study was to examine how the modeling routes changed according to the semiotic characteristics of the groups. In the context of the mathematical model produced to find the size of the tumor, the semiotic characteristics of Groups B and D were determined as algebraic, while other groups were determined as a numeric model. On the other hand, in the context of the mathematical model produced for the shape of the tumor, the semiotic characteristic of Group C was determined as algebraic, and others were determined as a geometric model. When the MTDs of groups with the same and different characteristics were examined in detail, no pattern related to this semiotic characteristic was found in MTDs. Similarly, Shahbari and Tabach (2020) divided the groups into two groups as algebraic and numeric models and examined the modeling routes of these groups. According to their findings, they reported that the groups with algebraic models did not complete their first cycle, while the groups with numeric models completed it without jumping any transition. However, this study could not reach such a distinction between the MTDs of groups having different characteristics.

In regard to regular paths in modeling routes, the current study revealed different findings from the study of Shahbari and Tabach (2020). According to the presented modeling routes in their study, groups always moved forward in any modeling cycle. On the contrary, several empirical studies (Albarracin et al., 2019; Ärlebäck, 2009; Borromeo Ferri, 2007; Czocher, 2016) reported that the modeling routes may not always move forward. The findings of this study are consisted with this literature and they justify Ärlebäck (2009, p.353), who stated that “the view presented on modelling as a cyclic process is highly idealised, artificial and simplified” by presenting produced Modeling Activity Diagram (MAD) as evidence.

Regardless of the semiotic characteristics of the groups, there are some cases worth mentioning. Although we could not reveal any pattern in MTDs in terms of semiotic characteristics, we observed that the preferences and purposes of the groups to use technology have changed their modeling processes. As mentioned before, Group D did not prefer using technology while solving a modeling activity. On the other hand, Group A, B, and C made their solutions in both paper-pencil and dynamic geometry environments. The findings of the current study revealed that MTDs of the groups had completed but not regular progressing cycles. On the contrary of these groups, Group E used the technology in each action during their modeling process. Due to this, the group did not use working mathematically transition because the group's mathematical working processes were realized by GeoGebra. As a result of this, neither a regular path nor a completed cycle was observed in the MTD of Group E. Moreover, while an early validity transition was observed in the MTDs of the groups (Group A, B, and C) that used technology and a paper-pencil environment for almost equal periods of time, such a situation was not observed in the MTDs of the other two groups. They either used technology for each action or never used it. These cases can be associated with the group dynamics shaping modeling processes, such as groups' discussions, opinions, or preferences in a modeling activity (Ärlebäck, 2009).

5.4 The Place of Technology in the Modeling Processes of Groups in terms of their Semiotic Representation and Modeling Routes

The findings of the study revealed that the modeling processes of groups were significantly influenced and shaped by technology. When the modeling processes of the groups were examined, it was seen that the technology had various functions, such as developing solution strategies, testing and evaluating the model, and comparing the created models easily. Some of these functions of technology observed in the current study were also evident in studies focusing on the roles of technology in the mathematical modeling process (e.g., Aydoğan Yenmez, 2017; Saka & Çelik, 2018). Moreover, the strategies of pre-service teachers who used GeoGebra Software as a technology in their modeling processes were also significantly affected by this software because it had various tools for creating models and provided a dynamic working environment.

GeoGebra has interactive mathematics tools used to construct various geometric shapes and measure length, area, and volume. In addition, it has GeoGebra CAS (Computer Algebra System), which allows multiple representations with its inputs containing algebraic notations. Considering these functions of GeoGebra, it was observed that the semiotic representations of the groups were affected by the use of technology within the scope of the purpose of this study. Regarding finding the tumor's area, some groups (Group A, C, and E) preferred to use GeoGebra's area measuring tool. Moreover, one of the groups (Group B) created their models on GeoGebra, although they used the area formula in finding their solution. While this situation included the groups using GeoGebra in the numeric register in the area measurement part, it pushed the groups that did not prefer GeoGebra to use the algebraic register in this part. Here, GeoGebra, providing numeric data automatically in area measurement, affected the algebraic register usage of the groups and resulted in different semiotic characteristics between the groups using and not using technology. On the other hand, when creating the assumed tumor shapes, GeoGebra's CAS allowed one of the groups to transform algebraic representation into geometric representation. Group C created a circle, entering the circle equation as input to the GeoGebra's CAS for their models. However, in creating regular shapes for the tumor, the semiotic characteristics of the groups using ready-made tools of GeoGebra instead of its CAS were determined as geometric. The property of GeoGebra that makes such transformations with its CAS has affected the usage of algebraic registers.

Using technology also significantly shaped the modeling routes of the groups. For instance, GeoGebra supported the groups in working mathematically transition. GeoGebra provided numeric data to the groups that used the relevant tool for area measurement and allowed them to obtain a mathematical result. The clearest example of this situation can be observed in Group E's modeling route. As mentioned earlier, Group E was the only group to use technology in all their actions from the beginning to the end of their modeling processes. When the MTD of Group E was examined, it was observed that this group was never included in the working mathematically stage. The most frequently observed actions in involving groups in the working mathematically transition were in parts related to measuring the area of the created shape and resizing the x-ray. The fact that Group E, which performed these actions on the GeoGebra Software, was not included in the working mathematically transition might be due to technology usage preferences.

The parts where the groups used technology the most in their modeling processes were related to creating a regular shape around the tumor, interpreting the created shape in the context of the modeling task, and comparing the created models except for measuring the area. When the MTDs of the groups were examined, it was observed that mathematizing, interpreting, and validating transitions, which generally included the mentioned parts, coincided with the using technology stage separately. When the parts that did not coincide with the using technology stage were examined, it was remarkable that the instances where these transitions were observed usually came just after using technology. Therefore, this was an important detail that shaped the modeling processes of the groups.

5.5 Suggestions for Future Studies and Educational Implications

Considering the results of this study regarding preservice mathematics teachers' semiotic representations and the changing features of their modeling routes according to their semiotic characteristics, the following recommendations can be made for future research.

To begin with, this study could not identify any semiotic characteristics for groups when their whole modeling processes were examined. This may be caused by the context of the modeling activity implemented in this study. The "Tumor Surgery" task had distinct parts (multipart task), such as resizing the image, determining the shape of the tumor, and the size and location of the tumor, and it pushed groups to use mostly geometric representations during their modeling processes. Future studies may use modeling tasks not having distinct parts (single-part tasks) to be able to identify particular semiotic characteristics of the groups and examine how their modeling routes differ according to these characteristics.

Although the findings of the study revealed that there was no pattern in MTDs of the groups having the same or distinct characteristics, it was seen that group dynamics made the MTDs of the groups have a particular feature. Ärlebäck (2009) stated that group dynamics comprises preferences, sharing opinions, and discussions within the group, and these may be effective in shaping the modeling processes of groups. Within the scope of this study, pre-service teachers worked as a group, and they were free to use technology in their modeling process. In this study, there were groups that used technology for different purposes and a group that never used technology. For example, one group used technology to validate their models in the context of the modeling activity, while others used the technology to build mathematical models based on the various assumptions they made. We could not identify any pattern in all MTDs in terms of semiotic characteristics, but we found a pattern in the MTDs of the pre-service teachers in line with the aforementioned preferences related to technology. Based on these, we thought that pre-service teachers' preferences for technology could be included in Ärlebäck's (2009) group dynamics. In this regard, future studies can focus on group dynamics and, more specifically, preferences to use technology and purposes of using technology while determining the features of the modeling routes or MTDs.

Finally, Czocher (2013) developed a rubric containing students' modeling competencies based on the problems she implemented in her study. This rubric includes indicators for different modeling transitions. In this study, this rubric was both extended by adding new indicators specific to the implemented modeling task and tested. Future studies can also try this extended rubric with modeling tasks in different contexts and improve the it by making necessary revisions.

There are also several implications based on the findings obtained in the current study. One of these implications is related to the teaching of area measurement concepts. As mentioned before, the literature shows that traditional teaching of this concept may cause misconceptions and difficulties. Therefore, in teaching this concept, several tools and educational methods can be integrated to enrich the course content. Within the scope of the current study, a modeling task for area measurement was developed, and the modeling process of the participants was supported by technology. The modeling task used in this study can be seen as an exemplary activity that teachers can use while teaching the area measurement concept. This modeling task may allow students to establish the relationship between mathematics and real life and to produce more creative solutions to real-life problems by acquiring problem-solving skills.

Another implication is related to pre-service teachers' involvement in mathematical modeling courses. According to the literature, mathematical modeling tasks are rarely included in mathematics classes (Borromeo Ferri & Blum, 2013). Integrating modeling tasks can contribute to students' learning of mathematical concepts in a meaningful way and help them see the issues related to mathematical modeling. Moreover, the variety in semiotic characteristics of pre-service teachers and their complex modeling routes found in this study suggest diversifying the intervention strategies in developing pre-service teachers' modeling competencies.

5.6 Limitations

There are various limitations in the current study. These limitations are related to data collection and class environment. Audio and video recordings are some of indispensable recording methods for qualitative research since it provides opportunity to observe and listen all the actions of the participants over and over again. These methods are too important for the current study as the transcribed dialogues affect data analysis procedures. As it mentioned in data analysis part, almost each word spoken by participants is quite critical because expressions in their dialogues are examined in depth in order to assign them to the relevant modeling transitions. Previously, it is mentioned that one of the groups (Group F) was removed from the data analysis because of data loss. Group F was not videotaped during MEA, there was just an audio recording of this group to analyze their process. Conversations within Group F were not either heard or understood by the researcher since two members of the group were away from the audio recorder. This situation negatively affected the data richness of this study. In addition to Group F, the modeling processes of Group B and Group C were recorded only with an audio recorder. Although the conversations of these groups were clearly heard, in some instances, it was difficult to make sense of what they were doing exactly. This may have caused to assign the modeling transitions incorrectly. On the other hand, Group A was one of the groups whose modeling process was both audiotaped and videotaped. However, the videotape showed the group members instead of showing the computer screen of this group that created many different models. This situation caused that could not monitor exactly how they create these models, and this may be cause us not to be able to report these models properly.

The second limitation is related to the class environment. When MEAs are implemented in a classroom environment, they may contribute to the classroom discourse that participants interpret the various models they created. However, it can also affect the actions of groups in some instances. Moreover, as illustrated in Figure 3.2, the groups were quite close to each other. Although the participants were involved in the conversation within the group, they also interacted with other groups from time to time. This may cause that interactions between groups affect groups' modeling routes. In order to prevent this type of interactions, data may be collected separately for each group at different times.

REFERENCES

Albarracin, L., Ärlebäck, J., Civil, E., & Gorgorio, N. (2019). Extending Modelling Activity Diagrams as a tool to characterise mathematical modelling processes. *The Mathematics Enthusiast*, *16*(1), 211-230. https://doi.org/10.54870/1551-3440.1455

Altay, M. K., Yalvaç, B., & Yeltekin, E. (2017). 8th grade student's skill of connecting mathematics to real life. *Journal of Education and Training Studies*, *5*(10), 158-166. https://doi.org/10.11114/jets.v5i10.2614

Aydoğan Yenmez, A. (2017). Teknolojinin matematiksel modelleme sürecine etkileri. *Adıyaman Üniversitesi Sosyal Bilimler Enstitüsü Dergisi*, *9*(26), 603-646. https://doi.org/10.14520/adyusbd.306665

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems in introducing mathematical modelling in upper secondary mathematics. *The Mathematics Enthusiast*, *6*(3), 331-364. https://doi.org/10.54870/1551-3440.1157

Battista, M. T. (2002). Learning geometry in a dynamic computer environment. *Teaching Children Mathematics*, *8*(6), 333-339. https://doi.org/10.5951/tcm.8.6.0333

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, *31*(3), 235-268. https://doi.org/10.1007/bf00376322

Blum, W. (2002). ICMI study 14: Application and modelling in mathematics education — Discussion document. *Journal für Mathematik-Didaktik*, *23*, 262-280. https://doi.org/10.1007/bf03338959

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In Je Cho, S. (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and Attitudinal Challenges* (pp. 73-96). Springer Nature. https://doi.org/10.1007/978-3-319-12688-3_9

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of mathematical modelling and application*, *1(1),* 45-58. https://doi.org/10.1007/978-94-007-0910-2_3

Blum, W., & Leiß, D. (2007). Deal with modelling problems. In Haines, C., Galbraith, P., Blum, W., & Khan, S. (Eds.), *Mathematical modelling: Education, engineering and economics-ICTMA* (pp. 222-231). Elsevier.

Borromeo Ferri, R. (2004). Mathematical thinking styles–An empirical study. *European Research in Mathematics Education III*. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.485.3819&rep=rep1&type=pdf

Borromeo Ferri, R. (2007). Modeling from a cognitive perspective. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling: Education, engineering and economics* (pp. 260–270). Horwood Publishing. https://doi.org/10.1533/9780857099419.5.260

Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners' modeling behavior. *Journal für Mathematik-Didaktik*, *31*(1), 99-118. https://doi.org/10.1007/s13138-010-0009-8

Borromeo-Ferri, R., & Blum, W. (2014). Barriers and motivations of primary teachers for implementing modelling in mathematics lessons. *Proceedings of the Eighth Congress of European Research in Mathematics Education*, (pp. 1000-1009). Middle East Technical University. http://cerme8.metu.edu.tr/wgpapers/wg6_papers.html

Borromeo Ferri, R., & Lesh, R. (2013). Should interpretation systems be considered as models if they only function implicitly? In G. Stillman, G. Kaiser, W. Blum, & J. Brown (Eds.), *International Perspectives on the Teaching and*

Learning of Mathematical Modelling. Teaching mathematical modelling: Connecting to research and practice (pp. 57-66). Springer. https://link.springer.com/content/pdf/10.1007/978-94-007-6540-5.pdf

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Cham, Switzerland: Springer.

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *Journal of Secondary Gifted Education*, *17*(1), 37-47. https://doi.org/10.4219/jsge-2005-393

Civil, M., & Khan, L. H. (2001). Mathematics instruction developed from a garden theme. *Teaching Children Mathematics*, *7*(7), 400-405. https://doi.org/10.5951/tcm.7.7.0400

Clements, D. H., & Stephan, M. (2004). Measurement in pre-K to grade 2 mathematics. In D. H. Clements, & J. Samara (Eds.), *Engaging young children in mathematics. Standards for early childhood mathematics education* (pp. 299–317). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Consortium for Mathematics and Its Applications (COMAP) & Society for Industrial and Applied Mathematics (SIAM). (2016). *GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education*.

Creswell, J. W. (2011). *Educational research: Planning conducting, and evaluating quantitative and qualitative research* (4th ed.). Kindle.

Czocher, J. A. (2013). *Toward a description of how engineering students think mathematically* [Doctoral dissertation, The Ohio State University]. https://etd.ohiolink.edu/apexprod/rws_etd/send_file/send?accession=osu1371873286&disposition=inline

Czocher, J. A. (2016). Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking. *Mathematical Thinking* *and Learning, 18(2),* 77–106. https://doi.org/10.1080/10986065.2016.1148530

Çakmak Gürel, Z. & Işık, A. (2021). An analysis of pre-service mathematics teachers' behavior on mathematical modeling cycle. *Bartın University Journal of Faculty of Education*, *10*(3), 571-585. https://doi.org/10.1016/buefad.871566

Denzin, N. K. (1978). *The research act: A theoretical introduction to sociological methods.* Routledge.

Dickson, L. (1989). Area of a rectangle. In Johnson, D. (Ed.), *Children's Mathematical Frameworks: A Study of Classroom Teaching* (pp. 76–88). Berkshire: NFER-Nelson.

Dikkartin Ovez, F. T. (2012). The effectiveness of 4mat teaching model in overcoming learning difficulties in the perimeter and area of circle and perpendicular cylinder among the seventh year students. *Procedia-Social and Behavioral Sciences*, *46*, 2009-2014. https://doi.org/10.1016/j.sbspro.2012.05.419

Doerr, H. M., & English, L. D. (2003). A modeling perspective on students' mathematical reasoning about data. *Journal for research in mathematics education, 34 (2),* 110-136. https://doi.org/10.2307/30034902

Duval, R. (1998). Geometry from a cognitive point of view. In *Perspectives on the Teaching of Geometry for the 21st Century* (pp. 37-52). Springer, Dordrecht.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Duval, R. (2020). Registers of semiotic representation. In S. J. Cho (Ed.), *Encyclopedia of Mathematics Education* (pp. 724-727). Springer Heidelberg New York Dordrecht London.

English, L. D., Fox, J. L., & Watters, J. J. (2005). Problem posing and solving with mathematical modeling. *Teaching Children Mathematics*, *12*(3), 156-163.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). *How to design and evaluate research in education* (Vol. 7, p. 429). New York: McGraw-Hill.

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM*, *38*(2), 143-162. https://doi.org/10.1007/BF02655886

GeoGebra. (n.d.). *What is GeoGebra?*. Retrieved August 27, 2022, from https://www.geogebra.org/about

Greefrath, G., Siller, H.-S. & Weitendorf, J. (2011). Modelling Considering the Influence of Technology. In: Kaiser, G. et al. (Eds.), *Trends in Teaching and Learning of Mathematical* Modelling (ICTMA 14). Dordrecht: Springer, 315-329. https://doi.org/10.1007/978-94-007-0910-2_32

Hall, J., & Lingefjärd, T. (2017). *Mathematical Modeling: applications with geogebra*. John Wiley & Sons.

Hankeln, C. (2020). Mathematical modeling in Germany and France: a comparison of students' modeling processes. *Educational Studies in Mathematics*, *103*(2), 209-229. https://doi.org/10.1007/s10649-019-09931-5

Hıdıroğlu, Ç. N. (2012). *Teknoloji destekli ortamda matematiksel modelleme problemlerinin çözüm süreçlerinin analiz edilmesi: Yaklaşım ve düşünme süreçleri üzerine bir açıklama* [Masters dissertation, DEÜ Eğitim Bilimleri Enstitüsü]. http://hdl.handle.net/20.500.12397/6974

Huang, H. M. E., & Witz, K. G. (2013). Children's Conceptions of Area Measurement and Their Strategies for Solving Area Measurement Problems. *Journal of Curriculum and Teaching*, *2*(1), 10-26. https://doi.org/10.5430/jct.v2n1p10

Kim, Y., Grabowski, B. L., & Song, H.-D. (2003). Science teachers’ perspectives of web-enhanced problem-based learning environment: A qualitative inquiry. *American Educational Research Association Annual Meeting*, 1–12. https://eric.ed.gov/?id=ED475713

Kordaki, M., & Potari, D. (1998). Children's approaches to area measurement through different contexts. *The Journal of Mathematical Behavior*, *17*(3), 303-316. https://doi.org/10.1016/S0364-0213(99)80065-2

Korkmaz, E. (2010*). İlköğretim matematik ve sınıf öğretmeni adaylarının matematiksel modellemeye yönelik görüşleri ve matematiksel modelleme yeterlikleri*. [Doctoral dissertation, Balıkesir Üniversitesi]. https://hdl.handle.net/20.500.12462/336

Lesh, R. E., & Doerr, H. M. (2003). Beyond constructivism: *Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Lawrence Erlbaum Associates Publishers.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *The handbook of research on mathematics teaching and learning* (2nd ed., pp. 763-804). Reston, VA: National Council of Teachers of Mathematics; Charlotte, NC: Information Age Publishing.Lingefjard,

Lincoln, Y. S., & Guba, E. G. (1985). Establishing trustworthiness. Naturalistic Inquiry

Maaß, K. (2006). What are modelling competencies?. *ZDM*, *38*(2), 113-142. https://doi.org/10.1007/BF02655885

Machaba, F. M. (2016). The concepts of area and perimeter: Insights and misconceptions of Grade 10 learners. *Pythagoras*, 37(1), a304. http://dx.doi. org/10.4102/pythagoras. v37i1.304

Merriam, S. B., & Grenier, R. S. (2019). *Qualitative research in practice: Examples for discussion and analysis*. John Wiley & Sons.

Moore, T. J., Doerr, H. M., Glancy, A. W., & Ntow, F. D. (2015). Preserving pelicans with models that make sense. *Mathematics Teaching in the Middle School*, *20*(6), 358-364. https://doi.org/10.5951/mathteacmiddscho.20.6.0358

Moyer-Packenham, P. S., Roxburgh, A. L., Litster, K., & Kozlowski, J. S. (2022). Relationships between semiotic representational transformations and performance outcomes in digital math games. *Technology, Knowledge and Learning*, *27*(1), 223-253. https://doi.org/10.1007/s10758-021-09506-5

Muir, T. (2007). Developing an Understanding of the Concept of Area. *Australian Primary Mathematics Classroom*, *12*(4), 4-9. https://eric.ed.gov/?id=EJ793986

O'Connor, C., & Joffe, H. (2020). Intercoder reliability in qualitative research: debates and practical guidelines. *International Qualitative Methods*, *19*. https://doi.org/10.1177/1609406919899220

Özcan, Ş., Demir, M., Aksu, N., Urhan, S., & Zengin, Y. (2022). Ortaokul öğrencilerinin çember konusundaki kavramsal anlamalarının incelenmesi: 5e öğrenme modeli ile ters yüz edilmiş sınıf yaklaşımı. *Muğla Sıtkı Koçman Üniversitesi Eğitim Fakültesi Dergisi*, *9*(1), 110-133. https://doi.org/10.21666/muefd.988366

Palmas, S., Rojano, T., & Sutherland, R. (2020). Digital technologies as a means of accessing powerful mathematical ideas. A study of adults with low schooling in Mexico. *Teaching Mathematics and its Applications: An International Journal of the IMA*. https://doi.org/10.1093/teamat/hraa004

Papadopoulos, I. (2004). Geometry problem solving in a computational environment: Advantages and reservations. *ICME10, Copenhagen, Denmark*.

Papadopoulos, I., & Dagdilelis, V. (2008). Students' use of technological tools for verification purposes in geometry problem solving. *The Journal of Mathematical Behavior*, *27*(4), 311-325. https://doi.org/10.1016/j.jmathb.2008.11.001

Papadopoulos, I., Dagdilelis, V., & Education, H. P. (2009, July). Estimating areas and verifying calculations in the traditional and computational environment. In *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 305-312). Aristotle University of Thessaloniki & University of Macedonia.

Pollak, H. O. (1979). The interaction between mathematics and other school subjects. *New trends in mathematics teaching IV*, 232-248.

Rejeki, S. (2015). Using reshaping strategy to support students' understanding of area measurement of circles.

Richardson, S. L. (2004). *A design of useful implementation principles for the development, diffusion, and appropriation of knowledge in mathematics classrooms* [Doctoral dissertation, Purdue University].

Ryan, J., & Williams, J. (2007). *Children's mathematics 4-15: learning from errors and misconceptions: learning from errors and misconceptions*. McGraw-Hill Education (UK).

Saka, E., & Çelik, D. (2018). The Role of Technology in Pre-Service Teachers' Mathematical Modeling Process. *Adıyaman University Journal of Educational Sciences*, Special Issue, 116-149. http://dx.doi.org/10.17984/adyuebd.463293

Şengil Akar, S., & Yetkin Özdemir, I. E. (2020). Investigation of mathematical collective creativity of gifted middle school students during model-eliciting activities: the case of the quilt problem. *International Journal of Mathematical Education in Science and Technology*, 1-27. https://doi.org/10.1080/0020739X.2020.1768311

Shahbari, J. A., & Tabach, M. (2020). Features of modeling processes that elicit mathematical models represented at different semiotic registers. *Educational Studies in Mathematics*, *105*(2), 115-135. https://doi.org/10.1007/s10649-020-09971-2

Stehr, E. M., He, J., & Nguyen, H. (2018). Selecting, Sequencing, and Connecting: Using Technology to Support Area Measurement through Tasks, Strategies, and Discussion. In *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference* (Vol. 2, No. 1, pp. 12-23). https://doi.org/10.20429/stem.2018.020104

Stohlmann, M. (2017). Elementary mathematical modeling: Get in the GAIMME. *Banneker Banner Journal*, *30(2),* 4-11. https://www.marylandmathematics.org/uploads/7/2/6/7/72676687/the_banneker_banner_spring_2017.pdf

Stohlmann, M. S., & Albarracín, L. (2016). What is known about elementary grades mathematical modelling. *Education Research International*, *2016*. https://doi.org/10.1155/2016/5240683

Straesser, R. (2002). Cabri-Geometre: Does dynamic geometry software (DGS) change geometry and its teaching and learning?. *International Journal of Computers for Mathematical Learning*, *6*(3), 319-333. https://doi.org/10.1023/A:1013361712895

Suh, J. M., Johnston, C. J., & Douds, J. (2008). Enhancing mathematical learning in a technology-rich environment. *Teaching Children Mathematics*, *15*(4), 235-241. https://doi.org/10.5951/TCM.15.4.0235

Tan Şişman, G., & Aksu, M. (2016). A study on sixth grade students' misconceptions and errors in spatial measurement: Length, area, and volume. *International Journal of Science and Mathematics Education*, *14*(7), 1293-1319. https://doi.org/10.1007/s10763-015-9642-5

Urhan, S., & Dost, Ş. (2016). The use of mathematical modelling activities in courses: teacher perspectives. *Electronic Journal of Social Sciences*, *15*(59), 1279-1295. https://doi.org/10.17755/esosder.263231

Ünlü, E. (2007). İlköğretim okullarındaki üçüncü, dördüncü ve beşinci sınıf öğrencilerinin matematik dersine yönelik tutum ve ilgilerinin belirlenmesi. *Dumlupınar Üniversitesi Sosyal Bilimler Dergisi*, *19*, 129-148.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and middle school mathematics teaching developmentally*. London: Pearson Education UK.

Wolf, D., Lindeman, P., Wolf, T., & Dunnerstick, R. (2011). Integrate technology with student success. *Mathematics Teaching in the Middle School*, *16*(9), 556-560. https://eric.ed.gov/?id=EJ926577

Yin, R. K. (2003). Designing case studies. *Qualitative research methods, 5*(14), 359-386.

Yin, R. K. (2017). *Case study research and applications: Design and methods*. Sage publications.

Yunianto, W. (2015). Supporting students' understanding of area measurement through Verknippen applet. *Southeast Asian Mathematics Education Journal*, *5*(1), 73-82.

APPENDICES

A. Elementary Mathematics Education Program

1st Year-Fall	MATH111	Fundamentals of Mathematics
	MATH115	Analytic Geometry
	MATH117	Calculus-I
	EDS200	Introduction to Education
	ENG101	English for Academic Purposes I
	IS100	Introduction to Information Technologies and Applications
1st Year-Spring	MATH112	Discrete Mathematics
	MATH116	Basic Algebraic Structures
	MATH118	Calculus-II
	CEIT100	Computer Applications in Education
	ENG102	English for Academic Purposes II
2nd Year-Fall	PHYS181	Basic Physics I
	MATH219	Introduction to Differential Equations
	STAT201	Introduction to Probability & Stat I
	ELE221	Instructional Principles and Methods
	EDS220	Educational Psychology
	HIST2201	Principles of Kemal Atatürk I
2nd Year-Spring	PHYS182	Basic Physics II
	MATH201	Elementary Geometry
	STAT202	Introduction to Probability & Stat II
	ELE 225	Measurement and Assessment
	ENG211	Academic Oral Presentation Skills
	HIST2206	Principles of Kemal Atatürk II
3rd Year-Fall	MATH260	Basic Linear Algebra
	ELE341	Methods of Teaching Mathematics I
	ELE310	Community Service
	TURK305	Oral Communication
		Elective Course
3rd Year-Spring	ELE329	Instructional Technology and Material Development
	ELE342	Methods of Teaching Mathematics II

	EDS304	Classroom Management
	TURK306	Written Expression
		Restricted Elective
		Elective
4th Year-Fall	ELE301	Research Methods
	ELE419	School Experience
	ELE465	Nature of Mathematical Knowledge for Teaching
		Restricted Elective
		Elective
4th Year-Spring	ELE420	Practice Teaching in Elementary Education
	EDS416	Turkish Educational System and School Management
	EDS424	Guidance
		Elective

B. Group D's and E's Modeling Cycles Related to Actual Size of X-Ray

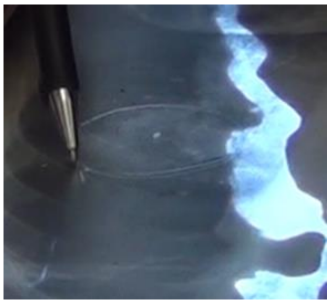
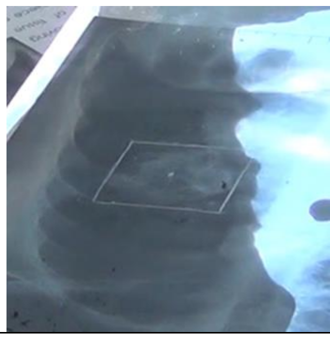
Group D

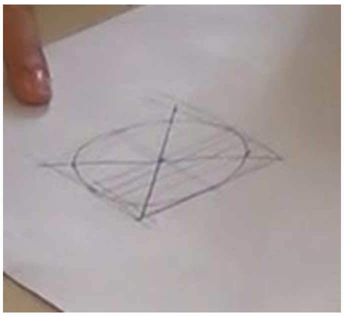
Transitions	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	The difference of 2.5 cm between lungs is needed to reach the actual size of the x-ray.
Simplification/Structuring		Natural	Simplifying and constructing relations
	Conversion		
Real Model		Numeric	For the actual size of the x-ray: 1 cm is equal to the difference of 2.5 cm between lungs
	Treatment		
Mathematizing		Numeric	Establish a ratio between image and real as 1:2.5

Group E

Transitions	*Semiotic Actions*	*Registers*	*Explanations*
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	The difference of 2.5 cm between lungs is needed to reach the actual size of the x-ray.
Simplification/Structuring		Natural	Structuring with GeoGebra Tools
	Conversion		
Mathematical Results		Numeric	The length of one side of the x-ray is 34.5 cm.

C. The modeling transitions/phases and semiotic actions of Group A

Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
		1st Cycle	
Understanding		Natural	Understanding the situation
Situational Model	Treatment	Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/Structuring		Natural	Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created inside a square/rectangle/ quadrilateral. For the location of the tumor: The chest x-ray can be put on a coordinate system.
Mathematizing	Treatment	Geometric	Drawing a square/rectangle/quadrilateral around the tumor and creating a circle in that square/rectangle/quadrilateral to cover the tumor in minimum size.  

		Geometric	Identifying the intersection point of diagonals of quadrilateral as center of the circle. 
		Geometric	Drawing a quadrilateral around the tumor by using segments and points, then identify the intersection point of diagonals of it on GeoGebra.
			Drawing a circle via “Circle with center through point” tool on GeoGebra
Mathematical Model	Conversion	Geometric	Circle inside the quadrilateral created on GeoGebra
Interpreting		Natural	Interpreting to Reality: Created circle could not cover the whole tumor, and it covers much more healthy tissue.
	2nd Cycle		
Simplification/Structuring	Treatment	Natural	Return to situation to look for the new model
			Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by using “Circle through three points” tool
Mathematizing	Treatment	Geometric	Identifying several points around the tumor on GeoGebra.
			Drawing a circle via “Circle through three points” tool on GeoGebra by using identified points

Mathematical Model	Conversion	Geometric	Circle passing through the identified points around the tumor
Interpreting	Treatment	Natural	Interpreting to Reality: Created circle could cover the whole tumor.
Validating		Natural	Generalization: To operate the tumor, it is needed to look extreme points of the tumor.
		3rd Cycle	
Simplification/Structuring	Treatment	Natural	Return to situation to look for the new model having smaller size. Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a rectangle.
Mathematizing	Treatment	Geometric	Drawing a rectangle around the tumor by using segments and points.
Mathematical Model	Conversion	Geometric	The rectangle passing through the identified points around the tumor.
Mathematical Results		Numeric	Area of the rectangle: 78.17
Interpreting	Conversion	Natural	Interpreting to Reality: The rectangle is not an appropriate for different tumor shapes and it is not practical.
		4th Cycle	
Simplification/Structuring	Treatment	Natural	Return to situation to look for the new model. Simplifying and making assumptions

Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created inside a rectangle.
	Conversion		Determining focus points of ellipse according to the farthest points around the tumor
Mathematizing		Geometric	Identifying focus points by using the farthest two point
	Treatment		Drawing an ellipse via “Ellipse” tool on GeoGebra
Mathematical Model		Geometric	Different ellipses created by using different focus points
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created ellipse have too much tissue, and it covers much more healthy tissue.
		5^{th} Cycle	
Simplification/Structuring		Natural	Return to situation for the circle model
	Treatment		Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created by using “Circle through three points” tool
	Conversion		
Mathematizing		Geometric	Drawing a circle via “Circle through three points” tool on GeoGebra by using identified points
	Treatment		
Mathematical Model		Geometric	Different circles (E, F, D points, E, F, H points and E, F, C points) passing through the identified points around the tumor
	Conversion		

Interpreting		Natural	Interpreting to Reality: Created circles (E, F, D points and E, F, H points) could not cover the whole tumor. Created circle (E, F, C points) could cover the whole tumor
		6^{th} Cycle	
Simplification/Structuring		Natural	Return to situation for the ellipse model
	Treatment		
			Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created according to the farthest points around the tumor
	Conversion		
			Determining focus points of ellipse according to the midpoints of the midpoint of the farthest points around the tumor
Mathematizing		Geometric	Identifying focus points by using the farthest two point
	Treatment		Drawing an ellipse via "Ellipse" tool on GeoGebra
Mathematical Model		Geometric	Ellipse created by using the different focus points
	Conversion		
Mathematical Results		Numeric	Area of the circle passing through E, F, C points: 103.33
	Conversion		Area of the ellipse: 103.76
Interpreting		Natural	Interpreting to Reality: Created ellipse could cover the whole tumor, but it has a bit more tissue than the circle. .
	Treatment		

Validating		Natural	Generalization: Ellipse model is more appropriate than the circle for the different tumor shapes. Also, ellipse model is more reasonable because ellipse approaches to the circle when its diameters are equal. Validating the models by checking them on the tumor having different shape.
		7^{th} Cycle	
Simplification/Structuring		Natural	Return to situation to look for the new model:
	Treatment		Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can
	Conversion		be created by input command including the identified points around the tumor on GeoGebra.
Mathematizing		Geometric	Drawing an ellipse via the conic command entering the input on GeoGebra.
	Treatment		FitImplicit({F, G, C, J, E, I, D, H}, 2)
Mathematical Model		Geometric	Ellipse covering the whole tumor
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created ellipse could cover the whole tumor, but it does not indicate an area.
		8^{th} Cycle	
Simplification/Structuring		Natural	Return to situation to look for the new model:
	Treatment		Simplifying and making assumptions

Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created by using “Conic through five points” tool
	Conversion		
Mathematizing		Geometric	Identifying several points around the tumor on GeoGebra.
	Treatment		Drawing an ellipse via “Conic through five points” tool on GeoGebra by using identified points
Mathematical Model		Geometric	Ellipse passing through the identified points around the tumor
	Conversion		
Mathematical Result		Numeric	The area of the ellipse: 84
	Conversion		
		Algebraic	The equation of the ellipse: d: $-13.22x^2 + 0.56xy - 11.98y^2 + 939.91x + 571.13y = 23655.84$
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created ellipse could cover the whole tumor, and it has the smallest size.

D. The modeling transitions/phases and semiotic actions of Group B

Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
		1st Cycle	
Understanding	Treatment	Natural	Understanding the situation
Situational Model		Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/Structuring		Natural	Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created according to the farthest two points around the tumor horizontally and vertically. For the location of the tumor: The chest x-ray can be put on a coordinate system and the equation of the circle can be used.
Mathematizing		Numeric	Establish a proportion for the diameters whose lengths are 42 (vertical) and 43 (horizontal) unit squares.
Working Mathematically	Treatment	Numeric	Finding the actual length of the diameters by solving the proportion: ▪ If 14 unit squares equal to 2.5 cm, what is the actual length of 42 unit squares? Then, half of it is radius. ▪ If 14 unit squares equal to 2.5 cm, what is the actual length of 43 unit squares? Then, half of it is radius.
Mathematical Results		Numeric	The length of the vertical diameter is 7.5 cm, and the radius is 3.75 cm. The length of the horizontal diameter is 7.68 cm, and the radius is 3.84 cm.

		2^{nd} Cycle	
Mathematizing		Geometric	Drawing a circle via “Circle: Center & Radius” tool on GeoGebra
	Treatment		
Mathematical Model		Geometric	Circle with a radius of 3.84 cm and a center of (25.32,17)
	Conversion		
Working Mathematically		Numeric	Substituting the values of r=3.84, a=25.32, and b=17 in the circle equation
	Conversion		
Mathematical Results		Algebraic	The equation of the circle: $(x-25,32)^2+(y-17)^2 = (3.84)^2$
		3^{rd} Cycle	
Mathematical Model		Algebraic	The area formula of a circle for the size of the tumor is πr^2
	Conversion		
Working Mathematically		Numeric	Substituting the value of r=3.84 and π=3.14 in the formula of πr^2
	Treatment		
Mathematical Result		Numeric	The size of the circle is 46.30.
	Conversion		
Validating		Natural	For validating the result by checking other geometrical shapes in order to take smaller area to cover minimum healthy tissue with the whole tumor.
		4^{th} Cycle	
Simplification/Structuring		Natural	Return to situation to look for the new model having smaller size.
	Treatment		Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a square, and the square can be created the farthest two points around the tumor horizontally and vertically
	Conversion		

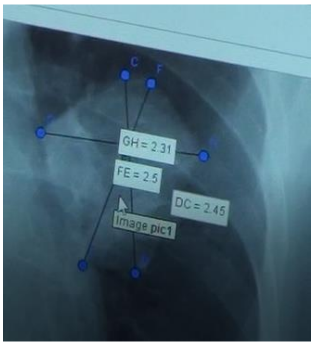
Mathematical Model	Conversion	Algebraic	The area formula of a square for the size of the tumor is a^2
Working Mathematically	Treatment	Numeric	Substituting the value of a=7.68 in the formula of a^2
Mathematical Results		Numeric	Area of the square: 58.98
Interpreting	Conversion	Natural	Interpreting to Reality: The area of the square is bigger than the circle.
		5th Cycle	
Simplification/Structuring		Natural	Return to situation to look for the new model having smaller size.
	Treatment		Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a regular pentagon, and the regular pentagon can be created according to the farthest two points around the tumor so that it is a side of the pentagon.
	Conversion		
Mathematical Model	Conversion	Algebraic	The area formula of a regular polygon for the size of the tumor is $\frac{5ar}{2}$
Working Mathematically	Treatment	Numeric	Substituting the value of a=5 and r=3.84 in the formula of $\frac{5ar}{2}$
Mathematical Results		Numeric	Area of the regular polygon: 48
	Conversion		
Interpreting		Natural	Interpreting to Reality: The area of the regular polygon is bigger than the circle.

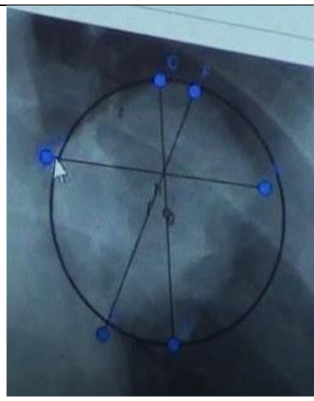
6^{th} Cycle			
Simplification/Structuring		Natural	Return to situation to look for the new model having smaller size.
	Treatment		Simplifying and making assumptions
Real Model		Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created according to the farthest two points around the tumor horizontally and vertically for the short and long diameter of an ellipse.
	Conversion		
Mathematical Model		Algebraic	The area formula of an ellipse for the size of the tumor is $\frac{\pi.a.b}{4}$
	Conversion		
Working Mathematically		Numeric	Substituting the value of a=3.75, b=3.84, π=3.14 in the formula of $\frac{\pi.a.b}{4}$
	Treatment		
Mathematical Results		Numeric	Area of the ellipse: 45.26
	Conversion		
Interpreting		Natural	Interpreting to Reality: The area of the ellipse is smaller than the circle.
	Treatment		
Validating		Natural	Using GeoGebra to validate the model by creating an ellipse covering the whole tumor.

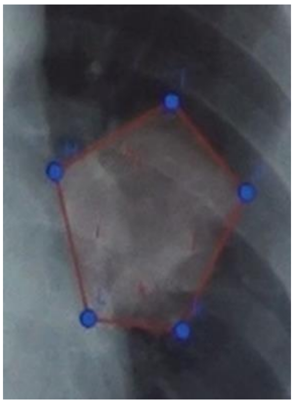
7^{th} Cycle			
Simplification/Structuring		Natural	Simplifying and making assumptions
Real Model	Treatment	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as an ellipse, and the ellipse can be created by input command including the identified points around the tumor on GeoGebra.
	Conversion		
Mathematizing		Geometric	Drawing an ellipse via the conic command entering the input on the GeoGebra.
	Treatment		FitImplicit({H, I, K, L, M, J}, 2)
Mathematical Model		Geometric	Ellipse covering the whole tumor
	Conversion		
Mathematical Results		Algebraic	The equation of the ellipse: b: $-0.00113x^2 + 0.0572x - 0.00098y^2 + 0.0335y = 0.9978$
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created ellipse could cover the whole tumor, but it does not indicate an area. Therefore, the circle model is selected finally.
	Conversion		
Real Results		Numeric	The size of the tumor: 46.30 cm^2.
	Conversion		
		Algebraic	The location of the tumor:
	Conversion		$(x-25,32)^2+(y17)^2=(3.84)^2$
Validating		Natural	If the tumor is disc-shape, a circle can be drawn around the tumor.

E. The modeling transitions/phases and semiotic actions of Group E

Transitions/Phases	*Semiotic Actions*	*Registers*	*Explanations*
		1st Cycle	
Understanding	Treatment	Natural	Understanding the situation
Situational Model		Natural	Location and area are strategic entities to determine where the tumor is and how much tumor will be operated.
Simplification/Structuring		Natural	Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a circle, and the circle can be created according to the farthest two points around the tumor.
			For the location of the tumor: The chest x-ray can be put on a coordinate system and the equation of the circle can be used.
Mathematizing	Treatment	Geometric	Identifying various points that will be the farthest points around the tumor and the midpoints of them.
			Drawing a circle via "Circle with center through point" tool on GeoGebra by using identified points and midpoints
Mathematical Model		Geometric	Circles created on GeoGebra according to the farthest points of the tumor.



	Conversion		 
Interpreting	Conversion	Natural	Interpreting to Reality: Created circle could cover the whole tumor.
Real Results	Conversion	Numeric	The area of the tumor: $59.22\ cm^2$
		Algebraic	The location of the tumor: c: $(x - 28.13)^2 + (y - 18.78)^2 = 18.85$
		2^{nd} Cycle	
Simplification/Structuring	Treatment	Natural	Return to situation to look for the new model: Simplifying and making assumptions
Real Model	Conversion	Natural	For the size (area) of the tumor: The shape of the tumor that will be operated is considered as a pentagon, and the pentagon can be created according to the boundaries of the tumor.

Mathematizing		Geometric	Identifying several points around the tumor on GeoGebra.
	Treatment		Drawing a pentagon via “Polygon” tool on GeoGebra by using identified points
Mathematical Model		Geometric	Pentagon passing through the identified points around the tumor
	Conversion		
Interpreting		Natural	Interpreting to Reality: Created pentagon
	Conversion		could cover the whole tumor.
Real Results		Numeric	The area of the tumor: 52.97 cm^2
Validating	Conversion	Natural	Generalization: Although the pentagon could cover the whole tumor and its area is smaller than the circle, circle model is more appropriate for different tumor shapes and more practical to determine it. To operate the tumor, it is needed to look extreme points of the tumor and identify the diameter of the circle. Then, the circle can be created so that it is cover the whole tumor.

F. Ethical Permission

Sayı: B.30.2.ODT.0.AH.00.00/126/08 – 868

15 Temmuz 2010

Gönderilen: Doç.Dr. Ayhan Kürşat Erbaş
Orta Öğretim Fen ve Matematik
Alanları Eğitimi

Gönderen: Prof. Dr. Canan Sümer
İAK Başkanı

İlgi: Etik Onayı

"Ortaöğretim Matematik Eğitiminde Matematiksel Modelleme Hizmet İçi ve Hizmet Öncesi Öğretmen Eğitimi" başlıklı ile yürüttüğünüz çalışmanız "İnsan Araştırmaları Etik Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

15/07/2010

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA