

PRE-SERVICE MATHEMATICS TEACHERS' TECHNOLOGY-ENHANCED  
COLLECTIVE ARGUMENTATION

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ENHANCED COLLECTIVE ARGUMENTATION**

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## ABSTRACT

### PRE-SERVICE MATHEMATICS TEACHERS' TECHNOLOGY- ENHANCED COLLECTIVE ARGUMENTATION

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Education

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This study aims to investigate the nature of pre-service mathematics teachers' collective mathematical argumentation processes in a technology-enriched learning environment.

This design-based study designed a course to improve pre-service mathematics teachers' technology competencies. In this context, pre-service teachers' mathematical argumentation processes were evaluated with data obtained during a semester. The data presented in this study were obtained from 10 pre-service teachers who enrolled in the course in the fall semester of 2018 and continued to the primary school mathematics teaching program at a state university in Ankara. The activities using argumentation in the lesson lasted for seven weeks. Multiple data collection methods were used in the study, and qualitative analysis methods were used in data analysis.

The data obtained from the study revealed that technology, instructor, and pre-service teachers, who are the actors of the collective argumentation processes in a technology-enriched learning environment, played different roles and functions

during the interactions. During the activities, it was observed that the teacher's instrumental orchestration functioned in argumentation to varying degrees. In addition, the technological behavior styles of teacher candidates in the collective argumentation process also varied according to the components of Toulmin's argumentation model. The analysis of the study's data also allowed the researcher to observe how the pre-service teachers use technology in inductive and deductive reasoning types that emerge integrated into the argumentation process.

Keywords: Collective mathematical argumentation, Mathematical reasoning, Mathematics education, Classroom teaching experiment, Mathematics education technologies.

## ÖZ

### **MATEMATİK ÖĞRETMEN ADAYLARININ TEKNOLOJİ İLE ZENGİNLEŞTİRİLMİŞ KOLEKTİF ARGÜMANTASYONLARI**

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Bu çalışmanın amacı, teknoloji ile zenginleştirilmiş bir öğrenme ortamında matematik öğretmen adaylarının kolektif matematiksel argümantasyon süreçlerinin doğasını araştırmaktır.

Tasarım-tabanlı bu çalışmada, matematik öğretmen adaylarının teknoloji yeterliklerinin geliştirmek için bir ders tasarlanmış ve bu bağlamda öğretmen adaylarının matematiksel argümantasyon süreçleri, bir dönem süresince elde edilen verilerle değerlendirilmiştir. Bu çalışmada sunulan veri 2018 güz döneminde derse kayıt yaptıran ve Ankara’da bulunan bir devlet üniversitesinde ilköğretim matematik öğretmenliği programına devam eden 10 öğretmen adayından elde edilmiştir. Dersteki kolektif argümantasyonun kullanıldığı etkinlikler yedi hafta sürmüştür. Çalışmada birden çok veri toplama yöntemi kullanılmış olup, veri analizinde nitel analiz yöntemleri kullanılmıştır.

Çalışmadan elde edilen veriler, teknoloji ile zenginleştirilmiş bir öğrenme ortamında sürdürülen kolektif argümantasyon süreçlerinin bileşenleri durumundaki öğretmen, öğretmen adayları ve teknolojinin etkileşimi sırasında farklı rollerde ve işlevlerde

bulduklarını ortaya koymuřtur. Etkinlikler boyunca retmenin enstrümantal orkestrasyonunun argümantasyondaki işlevinin de çeřitli derecelerde olduėu gözlemlenmiřtir. Ayrıca retmen adaylarının kolektif argümantasyon sürecindeki teknolojik hareket tarzları da Toulmin'in argümantasyon modelinin bileřenlerine göre çeřitlilik göstermiřtir. Çalışmanın verilerinin analizi ayrıca, retmen adaylarının argümantasyon sürecine entegre bir şekilde ortaya çıkan tümevarımsal ve tümdengelsel muhakeme çeřitlerinde teknolojiyi nasıl kullandıklarını da gözlemler fırsatı sunmuřtur.

Anahtar Kelimeler: Kolektif matematiksel argümantasyon, Matematiksel akıl yürütme, Matematik eğitimi, Sınıf retimi deneyi, Matematik eğitimi teknolojileri.



Dedicated to my family

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## **LIST OF ABBREVIATIONS**

### ABBREVIATIONS

CTE: Classroom Teaching Experiment

DBR: Design-Based Research

DGE: Dynamic Geometry Environment

FMNT: Focus on Mathematics to Notice Technology

FMUT: Focus on Mathematics with the Use of Technology

FTNM: Focus on Technology to Notice Mathematics

IK: Instrumental Knowledge

MK: Mathematical Knowledge

PST: Pre-service teacher

TECA: Technology-enhanced collective argumentation

TELE: Technology-enhanced learning environment

TPACK: Technological Pedagogical Content Knowledge



## CHAPTER 1

### INTRODUCTION

Mathematical thinking and reasoning skills, which include constructing a conjecture and developing solid arguments, are essential for mathematics education since these skills promote the development of new perspectives and further studies (NCTM, 2000). One of the five standards determined by NCTM is “reasoning and proof”. In addition, other mathematics education curricula list the skills that students should acquire, including mathematical reasoning, problem-solving, argumentation, constructing and testing conjectures, and proof (MoNE, 2013).

It is an accepted fact that the relationship between reasoning and argumentation is a complex one. According to Iordanou et al. (2016), argumentation is a social interaction process between at least two people in which people try to convince each other about the correctness of their own ideas. Regarding education, argumentation takes the form of social interaction in which teachers and students are involved in the classroom environment. However, not every social interaction is an argumentation. Argumentation in the classroom learning environment depends on a discussion and the presentation of data to convince the other about the correctness of the idea or information (Durand-Guerrier et al., 2011; Sriraman & Umland, 2014). Of course, for forming such a learning ecology, students must first create a classroom norm within the dual relationship of “learn to argue” and “argue to learn”. In fact, as Krummheuer (2007) stated, mathematics learning is argumentative learning by its nature. In addition to the critical mention of curricula, many studies (Alibert & Thomas, 2002; Francisco & Maher, 2011; Schwarz & Asterhan, 2010) revealed that argumentation improves students’ conceptual understanding, problem solutions, and their ability to defend their correctness.

Toulmin’s (2003) model, which achieved significant success with the argumentation, includes core elements of argumentation such as data, warrant, and claim, as well as

auxiliary elements such as qualifier, rebuttal, and backing. Data and warrant play a role in argumentation as evidence supporting the claim. In this functional argumentation model, a warrant can be a verbal statement and visual evidence. Knipping and Reid (2019) state that this model is frequently used in educational research because it can be used to reconstruct local argumentation in the classroom.

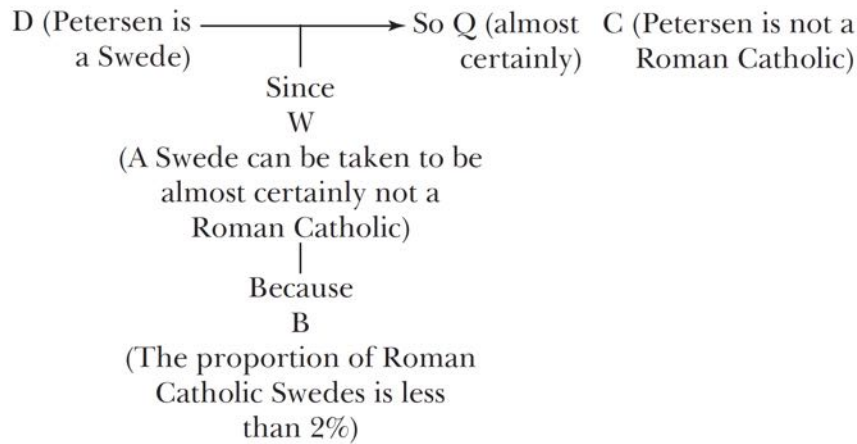


Figure 1.1: Toulmin's argumentation diagram (Toulmin, 2003, p. 103)

Krummheuer (1995) suggested that argumentation was previously considered a logical reasoning chain constructed by a person to persuade the other. However, it is not just a mental process that takes place in a person's mind, and the process progresses interactively in a social environment. Because of this interaction, he put forward the process in which argumentation is built in mutual interaction within a community, which he calls collective argumentation.

What are the dynamics of this collective argumentation process? Besides teacher and student interaction, what is the nature of the interaction between students? In such a learning environment, what will be the actors' roles who can be assumed to have a division of labor? These are subjects worth researching, and detailed studies are needed due to their complex nature (Knipping & Reid, 2019).

Ever since technology entered human life, everything in our lives has changed incredibly and rapidly. The duration of technology literacy has decreased to such a short time that a person who cannot always keep himself up-to-date on technology may be very competent at the beginning of his professional life but may be quite

behind in the middle of this period. With the advent of technology, many things that used to take much effort to do have now become very easy. In addition, perhaps even more impressive, things that were not possible before have become indispensable in our lives with technology.

Despite this strong effect of technology in our daily lives, unfortunately, the change due to its effects on education has not been that fast. While expressing the main reason for this, Kaput (1992) stated this lack of change is not because of technological limitations but because of limited human imagination and the constraints of old habits besides the social structures. Similar to the situation mentioned above, technology has gradually affected mathematics education. In comparison, some concepts that were difficult to explain before became more understandable with the help of technology, and subjects that were not in the curriculum began to enter thanks to technology. Moreover, as Mariotti (2002) points out, the issue's complexity is still not fully resolved, despite all the studies examining the effects of technological tools in learning and teaching.

Even in learning environments involving technology, the teacher is still the most effective agent of learning. For this reason, if technology is desired to be more effective in mathematics education, the shortest and most direct way to do this should be to train pre-service teachers with the necessary equipment, as Garofalo et al. (2000) stated. Because according to the accepted view (Clarke & Kinuthia, 2009; Dinçer, 2018) educating teachers, which is the main factor that will change mathematics education, is more important than providing the tools and technology they will use.

Suppose we accept the widespread view that the teacher is the most critical actor who can bring about this technological transformation. Nevertheless, from that moment on, another problem arises: How will this actor prepare for his role? As Gillow-wiles and Niess (2014) stated, most of the courses offered in many teacher training programs to improve pre-service teachers' technology knowledge are insufficient. As Hollebrands and Lee (2016) agree, authentic learning environments should be offered where teachers will learn the interaction of technology,

mathematics, and students that they need, rather than courses dedicated to learning technology itself.

Regarding technology in mathematics education, it is possible to mention various groupings for mathematics education software. Some studies have also made a taxonomy of these software (e.g., Kurz et al., 2005). However, with the emergence of free and open-source dynamic mathematics software, much of the research inevitably shifted in that direction. Although this software fills an essential gap with its multi-representation capabilities, it is acceptable that it would not be the right approach to persistently try to choose this software for every subject to be taught. The correct approach, on the other hand, should be to develop the ability of the teacher to correctly determine the technology and pedagogy triad suitable for the selected content, as can be deduced from the theoretical framework of TPACK (Mishra & Koehler, 2006). It is undisputed that there should be more than just one software in a technology integration course offered for pre-service mathematics teachers because each software has its features and usage areas. Considering the mathematics curriculum, having a single software that covers all subjects is almost impossible. In addition, considering the unique features of the software and the affordances they offer in teaching, it is imperative for teachers to benefit from different software as much as possible in terms of their technology competencies.

Let us assume that the teacher acquires the necessary components for mathematics education, which I have tried to summarize so far. So, how do students and teachers who are equipped with technology use these tools for their own purposes? In answer to the part of the question related to student portion, Vérillon and Rabardel's (1995) framework of instrumental genesis may provide a basis investigation. This theoretical framework, which we can summarize as the transformation of an artifact into an instrument, has been used quite frequently in the literature. If we consider a technology tool as an artifact, we can express the student's use of this tool for his/her particular purpose as instrumental genesis. However, this interaction is not unidirectional. The tool can also change how the user thinks, just as the user uses the artifact for its intended purpose and maybe changes it. In other words, it is possible



to talk about two-way interaction. In order to describe this interaction, Guin and Trouche (1998) introduced the concepts of instrumentalization and instrumentation. Instrumentalization is the student's transformation of the tool by his own way of thinking and purpose, while the instrumentation is the tool shaping the student's way of thinking.

In response to the teacher's question about the purpose of the second part of the above question, Assude et al. (2006) came up with a concept that could be an answer. In this definition, which they named instrumental integration, they stated that the teacher's intervention during the use of technology could be in two general modes. These interventions may vary depending on the student's familiarity with technology. While at the lowest level, it is only aimed at teaching the features of technology, at the highest level, the primary purpose is to increase mathematical knowledge by using technology effectively. Another study conducted to explain the interaction in a technology-enriched learning environment is the didactic tetrahedron metaphor put forward by Olive et al. (2010). It emerged with the addition of technology, the new member of this environment, to the previously known didactic triangle of Steinbring (2005): student, teacher, and mathematical knowledge. Hollebrands and Lee (2016) proceeded through this didactic tetrahedron metaphor and determined the focal points of the questions they asked while examining the interventions of pre-service teachers in a DGE-supported learning environment. They determined different foci by associating them with the movement on the edges of the regular tetrahedron.

A student proving should naturally enter the reasoning and argumentation process beforehand (K. F. Hollebrands et al., 2010). Probably, for this reason, most of the studies have examined argumentation as a part of the proving process instead of directly investigating argumentation in technology-supported learning environments since it is a process that leads to proof. Another remarkable issue is the preferred technology and topic selection in technology-supported argumentation studies. Although there are studies with promising results in teaching GeoGebra's Algebra and Calculus subjects in the systematic literature review conducted by Campbell and

Zelkowski (2020), it is noteworthy that the vast majority of the studies use the GeoGebra's dynamic geometry module. However, as NCTM (2000) emphasizes, proof, reasoning, and argumentation are issues that should be spread throughout the curriculum, not just in a unit of geometry or logic.

The importance given to mathematics in all of the courses given in mathematics teaching departments can be accepted as a fundamental principle that ensures the integration of different disciplines. For this reason, the focus in the courses offered in the mathematics education departments should be on teaching mathematics. For example, in an assessment and evaluation course, the context should be chosen as a math course, and all examples, prepared questions, and assessment tools should be related to mathematics. Similarly, a technology integration course should also focus on teaching mathematics. Even if generic technology software is to be taught, its utilization in mathematics classrooms should be included.

In line with the above reasons, including argumentation processes in teaching mathematics subjects to be covered in a technology integration course can help to improve PSTs' both technology and mathematics knowledge competencies. To this end, creating a learning ecology that simulates the collective argumentation processes that occur in natural classroom settings becomes crucial. At this point, the need for pre-service mathematics teachers to draw a detailed picture of collective argumentation processes in learning environments enriched with different types of mathematics education technologies emerges. Because of the interaction of various components in such a rich learning ecology with different teaching agents, their roles and functions in such an environment can be an essential step toward understanding the nature of learning in the environment. The fact that the course will continue for one semester becomes even more critical, given that most similar studies are carried out in less than a week (Campbell & Zelkowski, 2020). Under these circumstances, it becomes remarkable to draw a holistic picture of how the pre-service teachers' (PSTs') collective argumentation in the described learning environments are shaped under different concepts and technologies.

## 1.1 Purpose of the Study and Research Questions

Briefly summarized above, the necessity to fill the gap of examining the nature of the collective argumentation processes of pre-service teachers in learning environments enriched with different mathematics education technologies is the main reason for the researcher to choose this subject. Accordingly, this study aims to investigate the contribution of a TELE within a technology-supported math teaching course to pre-service mathematics teachers' collective argumentation.

Learning ecology in the course environment designated as context and specially designed for this purpose will have different dimensions as might be expected. The interactions, roles, and functions of the instructor, PSTs, and technology in the argumentation processes, which are the actors of the learning ecology, stand out as issues worth examining.

Given the lack of research regarding pre-service teachers' technology-enhanced collective argumentations (TECAs) within a semester-long technology integration course aiming to improve both technological competencies and mathematical knowledge, this study aims to investigate the pre-service teachers' collective argumentation within TELEs as part of a technology-supported math teaching course. The aim of this study is also to contribute to the body of knowledge base about how technology, instructor and pre-service teachers function during a TECA process working with a set of representative mathematics education technologies.

The main research question and its sub-questions shaping this study as follows:

How can a technology-enhanced learning environment within a technology-supported math teaching course effect pre-service teachers' collective argumentation?

1. How do technology, instructor, and pre-service teachers interplay in technology-enhanced collective argumentation (TECA)?

- a. What are the characteristics of the roles of technology, instructor, and peer interaction for developing mathematically correct argumentation?
  - b. What are the characteristics of the obstacles that inhibit pre-service teachers from developing mathematically correct argumentation?
  - c. What are the characteristics of the argumentative function of technology in pre-service teachers' TECA?
  - d. What are the characteristics of the technological actions for supporting the TECA?
2. How does technology operate pre-service teachers' mathematical reasoning emerged in TECA?

## **1.2 Significance of the Study**

Reasoning and argumentation, among the vital skills of mathematics curricula, are concepts whose importance is undeniably accepted (MoNE, 2013; NCTM, 2000, 2009). From this point of view, many studies emphasize the importance of argumentation and reasoning (Ball & Bass, 2003; Erna Yackel, 2001; Golanics & Nussbaum, 2008; Stein, 2001). It is imperative for teachers to have this equipment before starting their professional life since the most important agent that can take learning to the next level in learning environments where technology is used is the teacher. Examining and reporting the collective argumentation of teacher candidates in combinations of different technology and mathematics topics, with a more holistic perspective, without being tied to a single type of technology and concept, can be seen as an essential contribution to closing the gap in this field.

Determining the characteristics of the roles of technology, instructor, and prospective teachers during the collective argumentation processes in a TELE can contribute to the knowledge base of understanding the argumentation process. For this reason, determining the characteristics of these roles will contribute to understanding how teachers who want to teach mathematics and their students'

argumentation skills should behave and design the task accordingly, taking into account the role of the technology to be used.

Another expected contribution is to determine the nature of factors that may prevent pre-service teachers from developing a correct collective mathematical argumentation in TELEs. Thus, it will ensure that researchers who want to conduct research in an environment similar to the one in this study or teachers who aim to create a similar environment in their lessons will be aware of these obstacles. Accordingly, estimating the obstacles that can be observed regarding the task to be used in the lesson can help him/her prepare to remove them.

Another issue that the study will contribute to is the determination of technology functions according to Toulmin's model, which has been carried out quite rarely. While pre-service teachers are constructing TECA, determining which components of the Toulmin model's roles and how technology plays will contribute to the body of knowledge base about the structure of local argumentations.

Studies in which dragging models are determined in studies conducted to learn geometry concepts using DGE are essential for understanding the use of technology, especially in processes such as conjecture generation and verification. Determining the modalities of technological actions taken by students in environments where different mathematical concepts are taught with other technological tools can be seen as an essential contribution to this knowledge base.

Finally, identifying the argumentations in which different types of mathematical reasoning take place and determining the function of technology in types of reasoning will help to understand the relationship between technology and mathematical reasoning. Whether such task designs are for formal proof or to create a conjecture, the role of technology can be planned in advance and can guide researchers and teachers who want to shape the design.

### 1.3 Definition of the important terms

This section defines the several terms included in the research questions constitutively and operationally.

*Technology-enhanced learning environment (TELE):* TELEs are defined by Wang and Hannafin (2005) as technology-based learning and teaching systems in which students acquire skills or knowledge with the help of teachers, learning support tools, or technological resources. In this sense, the difference between TELEs from conventional learning environments can be depicted as the use of computers to enable technology to guide and enrich learning. In such environments, technology has the potential to shape all forms of communication between the learner, the teacher and the computer.

*Argumentation:*

Argumentation is defined as “the act or process of forming reasons and of drawing conclusions and applying them to a case in discussion” by Merriam-Webster dictionary. Walton (1990) states that argumentation refers to the global process of defending and criticizing a thesis or point of view that encompasses the entire context of discussion. Therefore, argumentation can be understood as the presentation of supporting or refuting ideas about a raised issue in a structure that supports as a logical chain.

*Collective argumentation:* In any collective argumentation the participants will at least try to find collectively valid statements on the basis of which one of the possible answers to a disputed question can be converted into a collectively valid statement. Collective argumentations constitute a fundamental interpersonal method for the formation of collective beliefs (Miller, 1987). Conner et al. (2014a) define collective argumentation as people working together to make claims and provide evidence to support them. In this respect, they state that the class discussion can be considered a collective argumentation, considering the potential of different students to contribute to the components of the argument during the lesson.

*Technology-enhanced collective argumentation (TECA):* Technology-enhanced collective argumentation can be defined as a collective argumentation that takes place in a TELE. Therefore, technology provides essential support with the affordances it provides in shaping the collective labor to support the claim in the collective argumentation process. Depending on the nature of the subject discussed, technology can scaffold by changing the course of collective argumentation beyond providing support.

*Mathematically correct argumentation:*

From a logical standpoint, an argument can be true or false. However, the logical and pragmatic values of the argumentation are independent. Successfully stating an argument does not mean it is true. If the framework of an argument is sufficient to support the claim, the argumentation is considered valid (Narița, 2020). If there is no problem when the sufficiency of the framework is evaluated mathematically, the argumentation can be considered mathematically correct.

*Technological actions:* They are the actions taken by the learners in a TELE while using the mathematics software to solve the problem during the activity. Different patterns can be observed depending on the software used, even when using different software tools. The patterns that emerge from the observations are called technological actions.

*Mathematical Reasoning:* Mathematical reasoning is a critical skill that allows people to analyze a statement without referring to a specific context or meaning. Although this process, in which the expression's truth value is determined in the analysis process, is an essential part of all disciplines, it has an exceptional role in mathematics (NCTM, 2009). It is drawing conclusions based on evidence or assumptions.





## CHAPTER 2

### LITERATURE REVIEW

The aim of this study to add to the knowledge base about how pre-service teachers' collective mathematical argumentation develops in TELEs using the classroom teaching experiment methodology (Gravemeijer & Cobb, 2006). Thus, in this chapter, review of the argumentation studies, technology integration in mathematics education which constitute the two main pillars of the study will be presented.

#### 2.1 Theoretical Background

In this study, there are basically two different environments to consider: the mathematical environment of technology-supported mathematics teaching course offered for pre-service mathematics teachers and the phenomenological environment of empirical attempts including the collective argumentative experiences enhanced by mathematical software programs.

##### 2.1.1 Argumentation

The purpose of argumentation, which can find its place in educational studies quite often, is to persuade (Antonini & Martignone, 2011; Schwarz & Asterhan, 2010). Van Eemeren, et al. (1996) defines argumentation as “*a verbal and social activity of reason aimed at increasing (or decreasing) the acceptability of a controversial standpoint for the listener or reader, by putting forward a constellation of propositions intended to justify (or refute) the standpoint before a rational judge*” (p.5).

While argumentation is defined as a process in research on argumentative reasoning, the product that emerges in this process is called argument (Iordanou et al., 2016). The term argument, in written or oral form, refers to the support by a warrant of a claim made by an individual with one or more supporting reasons or evidence (Toulmin, 2003). However, argumentation is a social interaction between at least two people, in which people understand each other and convince the other about the correctness of their ideas (Iordanou et al., 2016). In the definitions of argumentation in mathematics education, the interaction in the classroom and the arguments produced by the teacher and the students are emphasized (Sriraman & Umland, 2014). Durand-Guerrier et al. (2011) also emphasized the social debate environment in the classroom while defining it as “we regard argumentation as any written or oral discourse conducted according to shared rules, and aiming at a mutually acceptable conclusion about a statement, the content or the truth of which is under debate”. Sriraman and Umland (2014) takes the “mathematical argument” in the context of a math class as a line of reasoning that aims to show or explain why a mathematical result is correct. The aforementioned mathematical result can be a general statement about some mathematical objects or a solution to a posed mathematical problem. In this sense, a mathematical argument can be a formal or informal proof, an explication of how a student or teacher reaches a particular conjecture or elucidating how a student or teacher reasoned to solve a problem. It can also merely be a series of calculations that lead to a numerical result. This view of argumentation is very contextual, and mathematics can no longer be seen as a purely individual epistemic activity but as a social activity with certain principles. Therefore, there are studies on the place of argumentation in mathematics education to understand how to assist students in adopting this critical scientific practice (Conner et al., 2014a; Dogruer & Akyuz, 2020; Forman et al., 1998; Krummheuer, 2007; Mueller et al., 2014; Patty Anne Wagner et al., 2014; Yackel, 2002). Argumentation is not just about designing mathematics teaching for “learning to argue” but “argue to learn”. Considering the amount of interaction in mathematics learning environments in the classroom, one can assume that engagement in the argumentations is relatively high.

In this context, we can state that learning mathematics is a type of argumentative learning (Krummheuer, 2007). Regardless of how argumentation is presented, the combined conclusion of the studies is that engaging in argumentation improves conceptual understanding (Schwarz & Asterhan, 2010). In mathematics education, the importance of students creating arguments to support their solutions to problems and then defending them is undeniable (Alibert & Thomas, 2002; Balacheff, 1991; Ball & Bass, 2003; Francisco & Maher, 2005, 2011; Mueller et al., 2014; Yackel & Hanna, 2003). High school curricula (MoNE, 2013; NCTM, 2000) have placed a considerable emphasis on mathematical reasoning, argumentation, and proof, which are essential parts of the disciplinary practices of mathematics. NCTM (2009) even drew attention to the importance of the subject by publishing a book series.

#### **2.1.1.1 Uses of Toulmin’s Argumentation Model**

Toulmin (2003) proposed a triple model for the inference step in argumentation, consisting of a claim, data, and warrant, as well as auxiliaries such as qualifier, rebuttal, and backing (see Figure 2.1). In argumentation, warrants may consist of visual evidence or, for example, empirically constructed features. Although Toulmin recognizes that the distinction between data and warrant may not always be clear, he states that their functions are different, with the phrase “in one situation to convey a piece of information, in another to authorize a step in an argument” (p. 99). Depending on the context, the exact phrase may serve either data or warrant (p. 99). Toulmin’s functional model paves the way for reconstructing local mathematical argumentations in the classroom environment (Knipping & Reid, 2019).

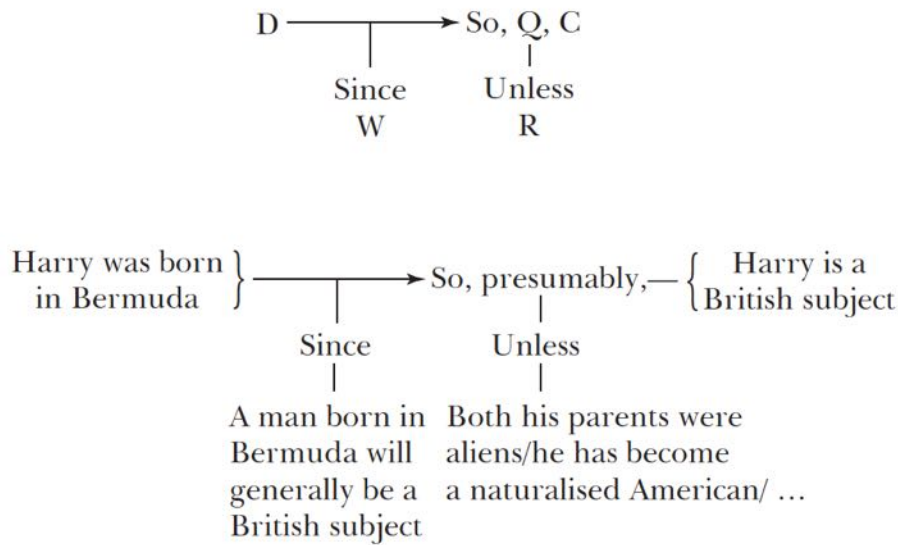


Figure 2.1: Toulmin's (2003, p.94) argumentation model

In his cornerstone work, Krummheuer (1995) states that argumentation was previously seen as a process carried out against an audience to be persuaded by a single person. However, he states that argumentation should be seen as a specific feature of social interaction since learning occurs due to a social process in the interaction between people, not a phenomenon that a person performs only by employing his mental processes. As a necessity of this social interaction, it should be considered an activity involving more than one participant. He defines these circumstances as collective argumentation.

Collective argumentation emerges as the effort of a group (mostly between students and a teacher) to show the truth of a claim (Conner et al., 2014b). Krummheuer (1995, p. 243) considers the part of the argument consisting of data, warrant, and claim as the core. The reason for this is that the backing component is implicit for the observer and mostly inaccessible. Of course, some researchers object to this and state that the components of qualifiers and rebuttal should be examined in collective argumentation (Inglis et al., 2007). However, it has also been argued that the argumentation in mathematics learning environments in schools differs from the argumentations produced by advanced mathematics students. Therefore, the components other than data, warrant, backing, and claim are unnecessary (Knipping

& Reid, 2019). In order to present a holistic picture of the argumentation process observed in the classroom environment, studies stating that the local argumentation structured with Toulmin's functional model can also be structured as a chain within an interconnected global argumentation (Erkek & Işıksal Bostan, 2019; Knipping & Reid, 2015; Krummheuer, 2015) is essential. In other words, when a claim is supported by data and warrant in a structure similar to the lemma used in mathematical proofs, it can now be used as data or warrant in the following argument. Thus, a nested body of argumentation emerges.

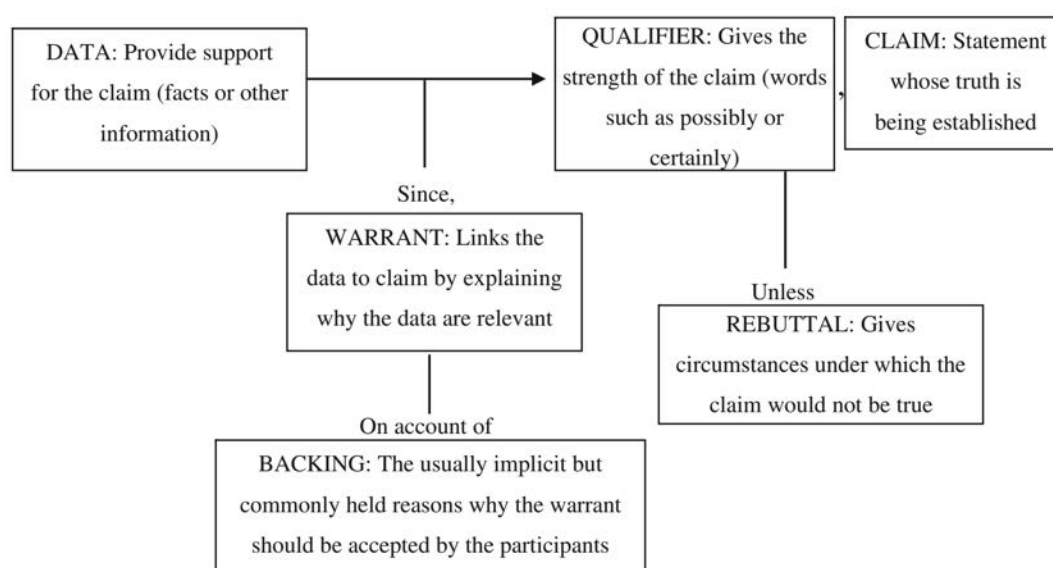


Figure 2.2: Adapted version of Toulmin's diagram (Moore-Russo et al., 2011, p. 6)

Knipping and Reid (2019) mention a component they call refutation to be used in the Toulmin model. Refutation completely rejects part of the argumentation, while rebuttal refers to exceptions to the conclusion in an argument.

Patsiomitou (2012) states that theoretical-dragging in dynamic geometry environment serves as a non-linguistic warrant in the argumentation structure. Relying on this inference, she reframed the Toulmin's model of argumentation based on the instances of dynamic geometry software utilization.

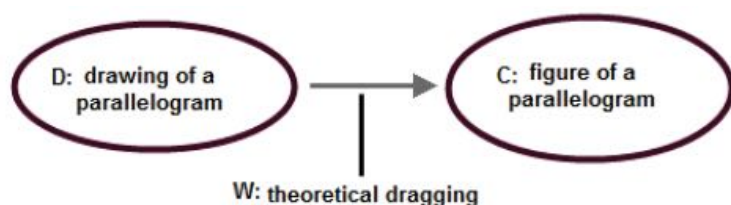


Figure 2.3: An example of a Patsiomitou's reduced pseudo-Toulmin's model

According to this model, while the data in the argumentation can be an element or an object in dynamic geometry environments, a warrant can be a tool or a command in the same environment that supports the claim. She defines theoretical dragging used for a warrant in this model as “in which the student aims to transform a drawing into a figure on screen, meaning s/he intentionally transforms a drawing to acquire additional properties” (Patsiomitou & Ubuz, 2011).

How the teacher will carry out the task in a learning environment where collective argumentation takes place is also a remarkable issue. Although the roles of the agents in a learning ecology vary according to the nature of the task, it can be considered that there is a division of labor. As Herbst (2002) states, students contribute to the argument, which is expected in active learning, but the teacher is primarily responsible for the structure and accuracy of the argument. Therefore, there is a division of labor throughout the argumentation process. The actors involved in such a collective argumentation process participate in the argumentation respectively and produce the general argumentation with what they say depending on each other (Knipping & Reid, 2019). Because no matter how vital a task's design is in providing learning, its implementation by the teacher is at least as important (K. F. Hollebrands & Lee, 2016).

### 2.1.2 Mathematical Reasoning

Although mathematical reasoning is emphasized in various curricula (MoNE, 2013; NCTM, 2000), the information about how these skills can be gained is rather vague (Jeannotte & Kieran, 2017). In fact, Yackel and Hanna (2003) state that many

mathematicians and mathematics educators use mathematical reasoning without any clarification and elaboration.

Some may view the mathematical reasoning in the curriculum as just an esoteric embellishment for their primary objectives to be achieved. It does not matter much to them, even if it is removed. However, mathematical reasoning is undoubtedly no less critical than what is written as the primary goal (Ball & Bass, 2003). Considering that it is impossible to achieve “meaningful” learning without mathematical reasoning, it is better understood that it does not receive as much attention as it deserves. It is noteworthy that Ball and Bass (2003) compared the importance of mathematical reasoning that “the more important it is to understand a text in reading, the more fundamental is mathematical reasoning in order to know and use mathematics properly”. Especially when there is a demand for previously learned but fading knowledge, mathematical reasoning is essential to revive it.

In addition to reviving old knowledge, mathematical reasoning is vital in more permanent and meaningful learning as it acts as an interrogator mediator in discovering new ideas. For this reason, NCTM (2000, p. 56, 2009, p. 1) argues that mathematical reasoning and proof cannot be learned only in a unit reserved for logic or by making proofs in geometry lessons. On the contrary, reasoning and sense-making should be a crucial part of all mathematics lessons because mathematical reasoning plays a central role in verifying or proving mathematical claims. Students learning mathematics usually go through mathematical reasoning and sense-making before creating the steps of a formal proof (K. F. Hollebrands et al., 2010).

Although the reasoning usually takes place within the argumentation framework, it does not necessarily occur in a discussion environment (Walton, 1990). Although these two concepts are mostly intertwined, Schwarz and Asterhan (2010) state that the distinction between these two concepts is overlooked in educational and psychological literature. Walton (1990) states that reasoning may not be purposeful. However, argumentation is conducted for a purpose. Thus, the reasoning in the argumentation becomes purposive.

### **2.1.2.1 The Model of Identifying Mathematical Reasoning**

Despite the studies summarized above on the deep and complicated relationship between mathematical reasoning and argumentation, there were few studies to bring this relationship to the surface. Especially in the collective argumentation process, it was imperative to examine what kind of reasoning was used to reveal the claims of the participants and to consider the relationship between these two mental processes.

Conner et al. (Conner et al., 2014a) examined a teacher's argumentative classroom episodes to reveal the relationship. They revealed the difference between inductive, deductive, and abductive reasoning in the collective argumentation process. Because they felt that Toulmin's model fell short of distinguishing between different kinds of mathematical reasoning, for this purpose, they combined Peirce's (Peirce, 1956) and Toulmin's (1958/2003) perspectives and proposed a new model. Accordingly, they blended rule, case, and conclusion, which Peirce refers to as different elements of reasoning, and data, warrant and claim, which Toulmin refers to as different components of an argument.

While they generally prefer to examine Toulmin's entire model, like Inglis et al. (Inglis et al., 2007), in examining collective argumentation processes, this new model, like Krummheuer's (1995) analysis, focused only on the core of the argument. They explained this as their focus on determining the type of mathematical reasoning rather than the accuracy of mathematical reasoning.



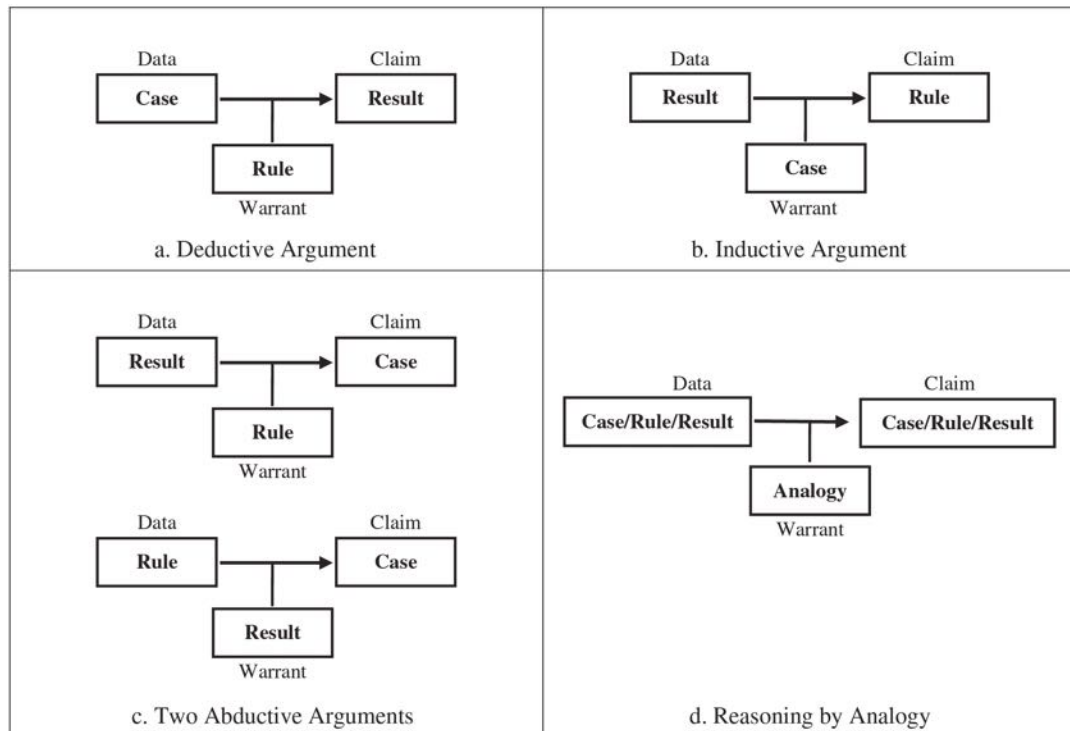


Figure 2.4: Toulmin-style diagrams of different reasonings (Conner et al., 2014a, p. 186)

Many mathematics education researchers were at risk of putting a deductive structure in their Toulmin diagrams, perhaps because of the prejudices they created by considering epistemological structure of mathematics. However, Conner et al. (2014) stated that the reasoning in the argumentation does not have to be only in a deductive structure. Therefore, they argued that their model could be essential in identifying different kinds of argumentation. They suggested that by using the model they created, they could not only determine the existence of argumentation but also investigate how mathematical reasoning types take place in argumentation processes.

An induction is an inference that allows a claim to be made by generalizing some particular cases. In inductive argumentation, generalization plays an important role. The inductive reasoning person reaches abstractions and generalizations by using the information about the cases obtained from individual observations. Reid and Knipping (2010) defined this process as a transition from the specific to the general.

In mathematics lessons, students form a mathematical hypothesis using inductive reasoning and the patterns they notice due to their observations. However, the assumptions made by generalizing are not mathematically precise.

In deductive reasoning, the inference process is from general propositions to particular propositions (Morris, 2002). Rigorous logical proof, a unique primary feature of studies in mathematics, is created using deductive reasoning (Ayalon & Even, 2008) and is considered the only reasoning type that leads to definite conclusions (Conner et al., 2014a).

## **2.2 Technology in Mathematics Education**

Today's students were born in a world where technology is actively involved in every aspect of life. It is pretty natural for this generation, who is familiar with technology in large part of their daily lives, to use it in their mathematics lessons, and it is an issue they probably think it should be. For this reason, isolating technology from classroom environments is against the flow of life (Powers & Blubaugh, 2005). For this reason, the courses in the faculties of education aim to prepare future teachers, to ensure that the technologies that will be considered normal in mathematics education are learned, and to ensure that their students adapt to the world of mathematics. However, as Mariotti (2002) states, although there have been many studies on using technological tools in teaching and learning, the subject is still quite complex. Therefore, as mathematics education researchers, it is still an important issue to investigate why and how new technologies affect mathematics education. According to Mariotti (2002), the computer has the potential to transform the task itself and even the user's relationship with essential knowledge beyond just being a powerful resource that the user will use to accomplish the task. Because due to the ability to allow multiple representations, technology students have the opportunity to approach the problem presented to them from different perspectives, support their solutions with different technologies, and test the applicability of the

answers (Erbaş et al., 2005). When listing some of the potential benefits of using the computer in education, Kurz (Kurz et al., 2005) counts four:

*“First, technological tools help to support cognitive processes by reducing the memory load of a student and by encouraging awareness of the problem-solving process. Second, tools can share the cognitive load by reducing the time that students spend on computation. Third, the tools allow students to engage in mathematics that would otherwise be out of reach, thereby stretching students’ opportunities. Fourth, tools support logical reasoning and hypothesis testing by allowing students to test conjectures easily” (p.124)*

While listing the different ways of using technology, Garofalo et al. (2000) states that the most direct and effective way for students to learn mathematics, which is the main goal, is to prepare pre-service teachers who will facilitate students’ mathematical thinking by supporting them with technological tools. In addition, there is a dominant opinion that the factor that will improve learning in mathematics education is the teachers responsible for correctly using that technology rather than the technology itself (Clarke & Kinuthia, 2009; Dinçer, 2018). In line with this prevailing view, courses and courses are reserved for technology integration in almost every teacher training program. However, Ntuli (2018) also reported in her study that although most teachers took technology courses during their university years, they lacked the necessary skills.

The fact that the focus of most educational technology courses in current teacher training programs is technology manipulation (Gillow-wiles & Niess, 2014) is a remarkable result of this issue. Although generic technology courses are widely offered, they are considered a limited contribution to teacher development. Because it is crucial to equip the teacher with knowledge about how to use it in his/her field, as well as just learning technology. It may be sufficient for the person working in any business line where technology is required to learn how to use the relevant

technology. Because using that technology may be enough for him/her to know what affordances it provides that will make his job more manageable. However, when it comes to teaching, things change. Because a teacher is supposed to use technology for the sake of teaching rather than solving their own problems. Dick (2011, p. 2) states, “the value of technology to the teacher lies not so much to the answers it provides, but rather to the questions it affords”. In other words, when the teacher uses the relevant technology in his/her lessons, unlike a student, it is necessary to learn what kind of questions that technology allows him/her to ask in order to make his/her students understand the concepts. Hollebrands and Lee (2016) mention that technology changes one’s way of thinking and, therefore, one’s view of mathematics. Teachers should consider the interaction between mathematics, technology, and student thinking in detail. The ideal ground that comes to mind and is suitable for developing these thinking skills stands as educational technology courses.

Putnam and Borko (2000) stated that the training containing simulations of real teaching experiences successfully supported PSTs’ technological developments. There are many studies on the issue of the content and approach of the technology integration courses offered to teacher candidates (Bush et al., 2015; Hew & Brush, 2007; Hughes, 2005; Ntuli, 2018; Trgalova et al., 2017). These studies agree on the opinion that can be accomplished most accurately by not developing their technology skills solely but blends the technology with mathematics. Therefore, it was considered necessary for pre-service teachers to experience lesson learning environments that simulate how the technological pedagogy knowledge required in classroom education is blended with mathematics.

### **2.2.1 Software Used in Mathematics Education**

Technological tools used in mathematics education have a broad spectrum. Dynamic geometry environments, computer algebra systems, graphics software, spreadsheet software, data visualization tools, microworlds, and data collection and analysis tools can be listed as the first software groups that come to mind. Although it is impossible

to teach all the software in each group in detail in teacher training programs, teacher candidates must be aware of these groups, even at the general level of culture. Thus, when it comes to technology in mathematics education, it can be prevented from developing a mindset based on only a few software. Niess (2005) stated that teachers should redesign their teaching and curricula according to the technology they will use, following the nature of the subjects in the curriculum. In this context, it can be ensured that they gain a perspective in determining the appropriate technology for specific subjects and all subjects in their curriculum.

In their study, Kurz et al. (2005) proposed a taxonomy of the mathematics education technologies currently used at that time. Numerous technologies many people would not even conceive in the taxonomy are covered. In this context, the taxonomy includes the following grouping: (a) review and practice: technologies used to reinforce previously learned topics; (b) general: technologies designed for use in various fields (c) specific: technologies designed for use in teaching only a particular field of mathematics; (d) environment: technologies used to teach mathematical concepts contextually; and finally (e) communication: technologies aimed at ensuring effective communication between the stakeholders of learning. They mention that it is possible for the lecturers using this taxonomy to reach all of the pre-service teachers with the examples they will present according to different learning philosophies. Thus, they state that it will create an environment where pre-service teachers can compare technologies and discuss which ones might be more useful than others.

### **2.2.1.1 Dynamic Mathematics Software**

Dynamic mathematics software, which is used in many fields with its very different features and can be used in a wide area with its affordances, is perhaps the most preferred software type in the mathematics education literature. However, most of these studies are related to dynamic geometry environments. In their systematic literature review and meta-analysis study, Chan and Leung (2014) reported that

teaching based on dynamic mathematics software had a statistically significant effect on students' test scores and that this software should effectively transform traditional teaching.

This software provides environments where multiple representations of mathematical objects can be observed in the same environment, dynamically observing how a change in one affects the other. Since tools such as dragging and slider provide a basis for observing variant and invariant properties of defined objects, they are frequently used in studies (e.g. see Antonini & Baccaglioni-Frank, 2016; Arzarello et al., 1998; Baccaglioni-Frank, 2019; Baccaglioni-Frank & Mariotti, 2010; Guven, Cekmez, & Karatas, 2010; Leung, Baccaglioni-Frank, & Mariotti, 2013; Mariotti, 2012, 2014; Mariotti & Pedemonte, 2019) based on constructing and testing conjectures, and their benefits have been reported.

Arzarello et al. (1998), who examined students' drag in the dynamic geometry environment, determined that these uses changed according to three different goals. The first type is "wandering dragging", which is used to look for regularities and properties that appear when objects are randomly dragged. The second type is "lieu muet", the dragging the student makes while maintaining a little of the regularity introduced in the previous process. The third type is dragging to determine the correctness of an assumption they call a "dragging test". Mariotti (2014) mentions that in a dynamic geometry environment like Cabri, the dragging function transforms students' perceptual data into an assumption. In this process, the student uses the drag feature to observe the existence of a meaningful relationship between objects. In addition, as Trocki and Hollebrands (2018) stated, dynamic geometry software also allows easy construction of geometric objects. However, its usefulness goes beyond constructing these objects, allowing the observation of variants and invariants while dynamically manipulating them.

Martinovic and Karadag (2012) emphasized that dynamic mathematics software is also beneficial in teaching calculus concepts such as limit, which is inherently dynamic and much more advantageous than traditional teaching. They express that

choosing the traditional way is a loss and an unnecessary challenge for the student when software that provides such a great advantage can be used. Takači et al. (2015), in their study of GeoGebra's features and graphics of functions, state that dynamic mathematics software offers students a rich learning environment for exploration and construction.

### **2.2.1.2 Spreadsheets**

The origin of spreadsheets is not related to the education world. They were created for accounting purposes. However, the software's mathematical and statistical abilities required for calculation paved the way for its use in mathematics education. Therefore, an increase is observed in the number of countries that add it as a skill to their curricula (Haspekian, 2014). Considering that they can be added to mathematics education with the proper planning, their integration into mathematics education lags far behind, especially when compared to dynamic geometry software, whose popularity is undeniable.

Spreadsheets are considered a handy tool that can provide students with a smooth transition from arithmetic to algebra (Beigie, 2017; Dettori et al., 2002; Friedlander, 1998). However, it is necessary to be aware of the limitations of the software during this transition. Because spreadsheets only allow manipulations with numbers, cells containing numbers or functions, direct manipulation with variables or unknowns important for algebraic representation is impossible (Dettori et al., 2002). Niess (2005b) states that spreadsheets can also increase their ability to capture patterns and generalize with their functions of writing formulas and plotting data in cells. Where Dettori et al. (2002) enumerate the benefits of using the spreadsheet, he mentions, as the last point, perhaps his most important contribution as the introduction of generalization, abstraction, and synthesis, the core cognitive abilities in mathematics.

In spreadsheets, a cell plays the role of the variable or independent variable in mathematics, and the cell containing the formula written in that cell plays the role of

the dependent variable. Thus, it may be possible to create a cognitive infrastructure that will form the basis of algebraic expressions and the concept of function. In addition, data analysis, recursive sequences, and functions in the secondary and high school curriculum offer opportunities such as solving optimization problems typically taught in higher grades so that younger students can understand them (Beigie, 2017).

When a student working in a spreadsheet environment tells the computer what to do, he/she learns to think and express this cognitive process both algebraically and arithmetically. Ploger et al. (1997) also state that since a student can instantly observe the response to this command, s/he will have the opportunity to test the accuracy of her/his reasoning by receiving immediate feedback. In addition, using technology in conjunction with the simulation of random events and data analysis can also increase students' interest. Yet another significant contribution is, according to Ploger et al. (1997), when working with a set of cells defined as dependent on a starting cell, they can observe the answer to a question like "what happens if we change the first number". For this reason, they state that students can explore mathematical possibilities.

### **2.2.1.3 Statistics, Data Collection and Visualization Tools**

Because statistics inherently contain arithmetic calculations and many formulas, these calculations by hand were seen as a burden by most students and considered drudgery (Bratton, 1999). However, this burden has decreased with the development of technological tools such as computer software, graphic calculators, and web-based applets. Thus, instead of wasting too much time with tedious calculations in statistics classes, it allowed focus more on the conceptual foundations of data analysis and probability.

Statistical literacy has come to the fore for the meaningful analysis and interpretation of data we encounter in every field in today's life. As a reflection, studies on the



inclusion of statistics in curricula have increased (e.g. see Burrill & Biehler, 2011; Gould, 2017; Rumsey, 2002; Sharma, 2017; Tishkovskaya & Lancaster, 2010; Watson, 2003). Teaching statistical reasoning supported by these new technological tools has been a breakthrough in advancing the relatively young field of statistical education (Ben-Zvi & Garfield, 2008). According to (Olive et al., 2010), making statistics visual and dynamic and focusing on concepts rather than calculations makes statistics attractive for students. For example, they state that instead of directly teaching how to calculate the mean, they can first examine and compare the height distributions of two groups using technology's visualization capabilities.

Biehler et al. (Biehler et al., 2012, p. 650) divide the technological tools used in statistics education into seven groups:

- (a) statistical software packages, (b) spreadsheets, (c) applets/stand-alone applications, (d) graphing calculators, (e) multimedia materials, (f) data repositories, and (g) educational software.*

TinkerPlots is software that falls into the last category above and is specifically designed to develop the statistical reasoning abilities of younger students. Its dynamic structure and simulation options allow data analysis without using symbolic inputs (Biehler et al., 2012). Since it is built on the same platform as Fathom, designed for teaching statistics in older age groups, the transition can be smooth. Kazak et al. (2014) state that students can create their own chance models using different options, especially with the sampler tool added in the second version of the software, allowing students to conduct virtual experiments and quickly collect and analyze the outputs of these experiments. In this way, they stated that they provided the students with a platform to make more generalizations and abstractions by having the computer handle the operation, which can be considered complex, and necessary for analysis.

Another software that has started to find its place in the literature is the statistics and probability module of GeoGebra, which is a complete dynamic mathematics

software (e.g. see Hewson, 2009; Phan-Yamada & Man, 2018; Prodromou, 2014). The fact that GeoGebra is open source and free to use is, of course, an essential factor in this increase. In their study using GeoGebra, Nabbout et al. (2017) stated that interactively exploring the dynamic link between different graphic representations allows them to focus more on conceptual knowledge.

### **2.2.2 Technology as a Cognitive Tool**

The idea of the “microworld”, which Papert (1980) first introduced to the education community, seemed to herald how vital technology would have in cognitive education. This idea initiated the ground for analyzing learning environments where students’ mathematical knowledge is aimed to be structured with the help of technology. According to Mariotti (2002), microworlds differ from traditional teaching as they are essential in constructing students’ conceptual knowledge. This role acts as a mediator between the teacher and the learners and is used in the evolution of the cognitive thoughts of the students in the class. In this sense, technology functions as a semiotic tool (Maria Alessandra Mariotti, 2002).

According to Kurz et al. (2005), technology should be used not only as a cognitive tool and to help them learn mathematics better or faster, even without technology, but also to learn fundamental mathematics in a different process with the affordances it provides. Mariotti (Mariotti, 2014) states that there is an ambiguity in the use of this mediation term. What he means by this ambiguity is that the tool includes using two different and interrelated potentials, the successful completion of a task or the stimulation of learning processes related to mathematical ideas. Along with analyzing the role and functions of technological tools in learning environments, the analysis of how these tools will be used in learning and teaching processes also emerges as a topic that has often taken place in research.

### 2.2.3 Technological Actions

Mathematical software programs provide an environment where users obtain immediate feedback with the visual perception and the movement of sliders and mouse by dragging. These environments may serve as a medium to mediate the gap between experimental and theoretical knowledge by an argumentation process (Arzarello et al., 1998). Lopez-Real and Leung (2006) classified the dragging in two broad categories, namely, confirmatory and exploratory modalities. The former was defined as an inductive confirmation via dragging to check the robustness of the construction. They described the latter modality as checking whether one can determine the invariant properties related to the figure while dragging the independent elements.

The classification of drag types mentioned above is essential for educational research. According to Zbiek et al. (2007), examining these dragging forms provides opportunities to examine the qualitative meanings of actions possible with a technological tool towards a goal. In addition, associating students' goals with their actions while using technological tools can help explain students' behavior in such learning environments.

Hollebrands (2007) examined students' actions using GSP during his research on geometric transformation, which lasted for seven weeks in a geometry class with 16 tenth-grade students. As a result of this study, he named two different types of strategies students used: reactive and proactive. She stated that what she considers vital in these strategies is that they are related to the student's ability to predict the outcome of the action and interpret the result by blending their understanding of mathematics with the technology tool they use. After students observed the screen, they continued with another action. However, while doing this, they may not guess the following action and may not have any idea prior to taking action. She calls this type of action as reactive strategy because it depends on the screen produced by GSP. However, they can take their actions according to certain expectations by predicting what kind of results their actions will yield. Therefore, in line with the plans before

the start of the movement, the use of technological tools is revealed in a way that is shaped according to their geometric understanding. Hollebrands points out that these two strategy types differ from the drag types mentioned above. She explains that the reason for this is that these strategies do not only focus on how the tools are used; rather, the way students work with the tool is directly related to their mathematical understanding.

#### **2.2.4 Technology Use and Its Orchestration**

One of the most common frameworks used for how students in technology-enriched learning environments use technology is the instrumental genesis developed by Vérillon and Rabardel (1995). This framework examines the transformation of an artifact into an instrument while the student uses technology. Based on the task to be completed, the student must learn how to use technology. While an artifact is associated with a physical or symbolic object, feature, or algorithm, the instrument is associated with transforming that instrument in a way that incorporates it into the student's way of thinking. In other words, in a sense, an instrument is related to the mental equivalent of the physically existing artifact adapted for a specific purpose. As Mariotti (2002) stated, there is no real difference between these two terms; it is only related to how students use them. In this sense, she comments on that the instrument does not have a permanent structure, and the transformation can continue dynamically concerning how the student uses it.

Mariotti (2002) noted that in the conscious use of existing technology for semiotic mediation, this two-fold use of artifacts in the environment should be considered in the activities and planned accordingly and carefully. In this respect, it expresses the necessity of deep knowledge about the instrumental genesis process. The conversion process of an artifact to an instrument is not by itself but by an instrumented action, according to Zbiek et al. (2007). Then they ask the question: "How, specifically, do artifacts such as calculators, or computer software become mathematical instrument tools that the user can employ for their own mathematical purposes?" In order to

understand this subject, Guin and Trouche (1998) put forward the concepts of instrumentalization and instrumentation, which they expressed in addition to the framework of instrumental genesis. Accordingly, instrumentalization is the user's shaping of the tool for his own purposes; instrumentation involves shaping the user's understanding of the instrument. In other words, it is necessary to be aware that the user and the tool interact reciprocally. According to Zbiek et al. (2007), the instrumental genesis construct helps researchers understand technology's role in technology-enriched learning environments. Because it explains that technology does not have the same power for every user, it also explains how both technical and conceptual knowledge is necessary for the intelligent use of technology.

Assude et al. (2006) introduced the concept of instrumental integration. Accordingly, the teacher should make strategic decisions about how artifacts should be used based on students' perceptions, knowledge levels, and familiarity with technology. In this process, they used some indicators to identify different modes in the decisions made by the teacher. First of all, is the focus on the tool itself or the math in the task given to the students? Secondly, regarding the strategy used to solve the activities, is it more essential to use math, technology, or both? In other words, is instrumental abilities (IA) critical in this process, mathematical knowledge (MK), or both (IA/MK) equally important?

The teacher's decision-making mechanism is shaped by the student's relationship with technology (Assude, 2007). If students are new to technology, the first mode is instrumental initiation. In this mode, students either do not know the technology or have limited knowledge required to solve the task. The teacher aims to enable his students to learn technology in general, and the relationship between IK and MK is minimal in this mode. In the next mode, instrumental exploration, although the student still does not know the vehicle, the teacher ensures that the students gain this knowledge through mathematical exploration. In this mode, the teacher aims to develop both IK and MK. The relationship between IK and MK can be in varying degrees, from the lowest to the highest, depending on the nature of the task. If students are not in the category of beginners in technology, the goal begins to evolve

towards using technology, not learning it directly. In instrumental reinforcement, the first mode of this category, students experience instrumental difficulties while trying to solve the mathematical task, despite their knowledge of technology. At this point, the teacher intervenes and gives information to overcome this difficulty, but the primary purpose is to develop mathematical knowledge. In this mode, the relationship between IK and MK is at the highest level because IK is vital to success in the task. In the instrumental symbiosis mode, the highest level of instrumental integration, students have already used technology, and both IK and IK develop while solving a task in which IK and MT are connected. In this mode, the relationship between IK and MK is at the highest level. Hollebrands and Okumuş (2018) name the first two modes “lower modes” since they are used in the first usage stages of the artifact. Similarly, he qualifies the last two modes as “upper modes” due to his experience in the artifact.

Olive et al. (2010, p. 136) introduced the didactic tetrahedron (see Figure 2.1) by adding technology to the Steinbring’s (2005) metaphor of the didactic triangle. Thus, they stated that this tetrahedron’s four different triangular faces could emphasize all possible relationships between teacher, student, mathematical knowledge, and technology.

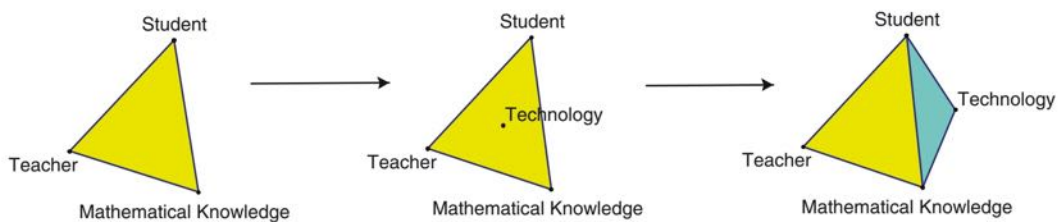


Figure 2.5: Transformation of didactic triangle to didactic tetrahedron

Hollebrands and Lee (2016) used the metaphor of the didactic tetrahedron to examine pre-service teachers’ implementation of technology-based activities. In their study, they identified four different categories about which dimensions of the didactic tetrahedron the teachers and students focus on while working on the task: (1) focus on technology, (2) focus on technology to notice mathematics, (3) focus on mathematics with the use of technology, and (4) focus on mathematics. Focus on

technology to notice mathematics (FTNM) can be elaborated in episodes where the main aim is to help PSTs explore a feature of the related technology to learn and teach a mathematical concept. However, when the focus on mathematics with the use of technology (FMUT) is utilized, the main aim is to help PST to understand the mathematics concept in focus by using the related mathematical software. In FTNM, the order of the components follows the way teacher-technology-mathematics-student, whereas, in FMUT, the order alters to teacher-mathematics-technology-student. During the FMUT episodes, technology is used to explore mathematical concepts. Similarly, in FTNM episodes, mathematical ideas are used as a tool to explore the related technology.

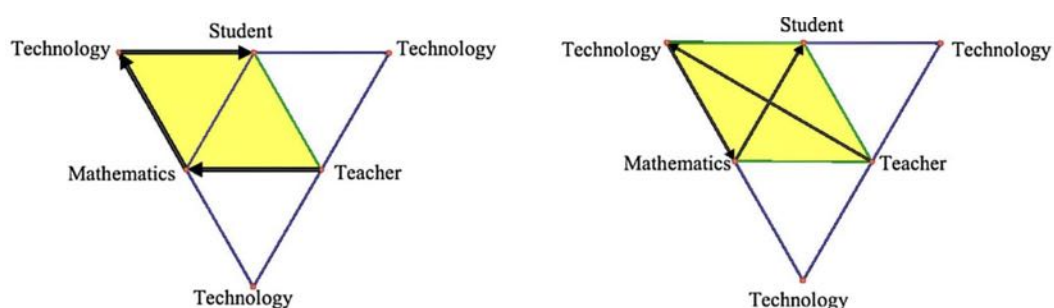


Figure 2.6: FMUT and FTNM on unfolded didactic tetrahedron

The first figure above, showing the unfolded form of the didactic tetrahedron, depicts the “focus on mathematics with the use of technology”. FMUT follows the teacher-mathematics-technology-student path. The second illustrates the “focus on technology to notice mathematics” intervention. FTNM follows the teacher-technology-mathematics-student path.

### 2.2.5 Technology, Argumentation and Mathematical Reasoning

In the context of dynamic mathematical software programs, users can explore the variant and invariant features related to the mathematical object under consideration. Hence the collective argumentation under the light of these mathematical software programs is an ideal environment for observing and conjecturing mathematical statements (Baccaglioni-Frank, 2010).

In the mathematics education literature, argumentation has been examined many times together with proof. Although formal proof and logic are the two most important characteristics that distinguish mathematics from other fields, students usually engage in reasoning and sense-making activities before entering the proof process (K. F. Hollebrands et al., 2010). There are many studies stating that students need to engage in argumentation activities to create a hypothesis in order for the proof, which is difficult for students to cope with cognitively, to be more manageable for them (Antonini & Baccaglini-Frank, 2016; Arzarello et al., 1998; Boero, 2017; Furinghetti & Paola, 2003; Maria Alessandra Mariotti, 2002). For this reason, in most studies, argumentation in a technology-enriched learning environment has not been directly examined but has been studied as a process that leads to proof.

Hollebrands et al. (2010) indicate that students do not use technology when stating explicit warrants in their argumentation processes and use technology more when they do not present explicit warrants. They stated that the reason for this might be the students' lack of technology familiarity, which is probably because they mostly use technology to make calculations or discoveries in other mathematics lessons. Another issue they drew attention to was that the students rarely stated the warrant while using the technology. The underlying reason for this situation was probably because students accepted technology as a warrant.

In his systematic literature review, Campbell and Zelkowski (2020) stated that technology supports argumentation in areas such as Calculus and Algebra, although few studies consider topics other than geometry. He expressed the contribution of GeoGebra's dynamic geometry module to processes such as creating conjectures in the field of geometry and testing its accuracy. In addition, he states that GeoGebra offers many opportunities for Algebra and Calculus as it supports visualization and conceptual understanding.

As mentioned in the previous section, the drag feature of geometry modules of dynamic mathematics software allows students to create a conjecture and test its accuracy with the visuality it provides (Prusak et al., 2012). Thus, students complete



an argumentation process in which they will be sure of the correctness before proceeding to the formal proof of a claim they put forward. Dogruer and Akyuz (2020) reported that the dynamic geometry environment increased students' learning of geometric concepts and allowed them to explain their reasoning along with their thoughts. Many studies show that dynamic geometry environments improve students' understanding of geometric concepts and reasoning skills (e.g. Connor & Moss, 2007; Forsythe, 2013; K. F. Hollebrands, 2007; Laborde, Kynigos, Hollebrands, & Strässer, 2006; Ng & Sinclair, 2015). Zembat (2008), in his study with university students, showed that students using GSP, TI-83, and spreadsheets could associate with more representations and exhibit more different types of reasoning than those using paper-pencil.

Campbell and Zelkowski (2020) states that most studies are completed in less than a week and that studies need to examine the effect of students' interactions with technology on their argumentations in a long-term study. Another critical issue he draws attention to is NCTM (2000), and mentions the importance of realizing the necessity of studies involving technology and argumentation in other fields as reasoning and proof are essential not only for geometry but also for all areas of mathematics. While doing this, he advises them to work using tools commonly used in mathematics classes, such as GeoGebra and Desmos, which will be more popular and widely used in the future.



## CHAPTER 3

### METHODOLOGY

This chapter will describe the study's design, research settings and context, collection and analysis of the qualitative data, and the trustworthiness and ethical issues.

#### 3.1 Design of the Study

The researcher employed the classroom teaching experiment methodology in this study, which was conducted as part of the technology-supported math teaching course offered in the Department of Mathematics and Science Education at a public university in Turkey. The aim of designing and developing this elective course was to improve the technology competencies of PSTs through a set of teaching episodes. After Steffe (1983) stated that a teaching experiment includes a series of teaching events, Steffe and Thompson (2000) later noted that these instructional episodes should comprise a teaching agent, one or more students, a witness of teaching episodes, and a method of recording what happens during the episode. These recordings can be used, if any, in preparing subsequent chapters and conducting a retrospective conceptual analysis of the teaching experiment. These elements apply to all teaching experiments. Researchers had to take responsibility for classroom instructional design for a long time to create more productive classroom environments. In doing so, the one-to-one teaching experiment methodology used at the beginning of the methodology's emergence was extended to classroom teaching experiments (Gravemeijer & Cobb, 2006).

Since mathematics educators were the first to use the teaching experiment methodology and make it widespread, the definitions are generally associated with mathematics education. For example, Steffe and Thompson (2000) define the

primary purpose of a researcher using this method as “to construct models of students’ mathematics.” The aim of the classroom teaching experiment is not merely to interpret what students understand through their interpretations but also to interpret the influence of their explanations on other students. Therefore, the social aspect of classroom discourse encompasses instructor-student and peer-to-peer interaction (Baker, 2016). In this environment, what matters is not just the comprehension of a single student but whether and how other students in the class accept and grasp each other’s insights. The researcher examined the interaction of the PSTs with each other and the technology in such an environment where technology use is essential, as well as the interplay between the instructor-student and peers mentioned in the literature.

In a classroom teaching experiment as a design research study, Gravemeijer and Cobb (2006) state that the main goal is to explain what happens in that classroom. They indicate that, unlike many other studies, it is aimed at understanding the ecologies of learning rather than making a statistical generalization based on samples. They define learning ecology as an environment with interacting factors rather than individual factors that affect learning. This study reveals PSTs within technology-enhanced learning ecology improve mathematics content knowledge and technology through TECA.

### **3.2 Research Settings and Context**

This study includes a pilot study and a subsequent main study conducted through classroom teaching experiment methodology. This section will first briefly describe the pilot study. The following sections will present more detailed information about the main study. Since the researcher conducted the study in the context of a previously mentioned designed educational technologies course, information about this course will be presented first.

### **3.2.1 Technology-Supported Math Teaching Course**

The researcher supposed that PSTs already learned mathematics topics essential for the course during their high school and university years. However, in his teaching experience of more than fifteen years, the researcher observed that high school students seek practical solutions to the possible questions to be asked in university exams without learning about the underlying reasons. They are content to memorize only enough to meet their short-term needs without processing the information with mathematical reasoning. It would not be wrong to expect that the PSTs included in the study will have similar shortcomings as they finish high school with the same process and prepare for the university entrance exams. The lack of content knowledge of some is especially evident when explaining the mathematical reasons for the solutions or statements they make in the lessons. The fact that many of them were pretty inadequate in this regard was a reality they admitted.

The researcher developed the tasks to teach the technologies that PSTs will use when they become in-service teachers with the lesson simulations based on mathematical reasoning and partially eliminate the deficiencies in the content knowledge. The motto about teaching technologies used throughout the course was “do not waste time for learning a learning technology.” The researcher did not plan comprehensive training for the technologies to be used, except for introducing the interface and demonstrating some basic features for those unfamiliar with them. Instead, it was aimed to learn by doing the technologies required in the activities every week, thus provide a more contextual learning environment. Hence, the course was designed to effectively use the limited time by providing simultaneous learning of mathematical content and technology knowledge, rather than being a generic technology learning course. In this way, it was aimed to shorten the time spent learning only the software features, and to focus on the use of these software in teaching mathematical concepts. This course’s primary aim was to develop teacher candidates’ technology competencies. However, the researcher aimed to approach this goal not from the perspective of generic educational technologies courses but from the TPACK

development standpoint. As seen from the weekly syllabus below (see Table 3.1), the researcher first introduced the theoretical framework of TPACK (Mishra & Koehler, 2006). He explained to the PSTs that it was aimed to develop their competencies in this framework. The researcher utilized formative and summative evaluations during the course to the extent of the achievement of this targeted development. For this purpose, he used observation notes, reflections, lesson plans prepared by PSTs individually and in groups, peer evaluation, e-portfolio, and interviews. The development of these competencies was measured using surveys used in the literature at regular intervals. All lectures were recorded with a video camera and audio recorders. The researcher used video recordings as a data source because this study focused on the collective mathematical argumentation processes during the tasks in the lessons from this large-scale data.

While determining the technologies to be taught in the syllabus, the researcher took dimensions in the theoretical framework of TPACK into account. According to the researcher's experiences and the literature, which technologies had significant advantages in teaching which mathematical concepts in the high school curriculum were influential in the decision-making mechanism. In other words, among numerous alternatives, the researcher determined the mathematical concepts to teach as the cornerstones of the curriculum and the most relevant technologies as much as possible. The contributions and uses of the selected technologies in mathematics education were summarized in the literature section.

While choosing the appropriate technologies for teaching and learning that they will need in a course offered for PSTs, the researcher did not only consider the technical characteristics of technology or even the educational beliefs he brings as instructor. He also examined the unique features of different media, providing various possibilities for learning and teaching. A much better understanding of the strengths and limitations of various media for teaching purposes is required if the necessary technologies for mathematics education are to be successfully selected. Considering the different contextual factors affecting learning, this choice can become much more complex. The researcher's experiences and the literature review influenced decision-

making in selecting technologies providing significant advantages in teaching math concepts in the high school curriculum. In other words, the researcher identified the mathematical concepts to be introduced as the cornerstones of the curriculum and, as far as possible, the most appropriate technologies among numerous alternatives. Contributions and uses of selected technologies to mathematics education are summarized in the literature section.

Table 3.1: Technology-supported math teaching course schedule

Week	Topics
1	First meeting & Course introduction
2	Implementation of a sample lesson plan using technology & Discussions about “using technology vs integrating technology”
3	TPACK framework and TPACK Levels
4	Teaching “transformations of function graphs” with <u>Graphing Software</u>
5	Teaching “equations and inequalities from graphing approach” with <u>Graphing Software</u>
6	Teaching “triangle inequality, sides lengths of a triangle, and inscribed angles of circles” with <u>Dynamic Geometry Environments</u> .
7	Teaching “sequences, infinite radicals and introduction to limits” with <u>Spreadsheets</u> .
8	Teaching “Descriptive statistics and types of probability” with <u>data visualization and modeling tools</u> .
9	TPACK Game
10	Teaching “quadratics” with a <u>data-collection and analysis software</u> .
11	Teaching “Average rate of change, definition of derivative, 1 <sup>st</sup> and 2 <sup>nd</sup> derivative tests” with <u>Graphing Software</u>
12	Group project implementation and presentation
13	Group project implementation and presentation
14	Wrap-up & ignite presentations

A total of three lesson periods were allocated for the lesson in the classroom environment each week. The researcher generally combined this period, and he carried out teaching episodes with only one break. With the presentation of the

prepared tasks on the smart board, the instructor ensured that the PSTs discussed the same parts simultaneously. In order to start the discussion on each sub-title in the task, first, the relevant part of the presentation was reflected on the board and the process was started. During the time determined for this stage, the instructor observed the argumentation processes of the groups in the class and intervened where necessary. After PSTs completed each activity in the task as a group discussion, the researcher presented the subsequent activities one at a time using the prepared presentation file. Thus, he made sure that the argumentation processes developed simultaneously, ensuring that all groups were discussing the same activity of the task at the same time. He noted the situations that stood out during these observations and that he considered important for the next teaching episodes. A conclusion period was determined as a class to arrive at the conclusion of each of the sub-argumentation processes in the tasks. During this time, the instructor ensured that each group explained their results and reached a common conclusion.

**TRANSFORMATION OF GRAPHS** (10 mins)

Open a graphing software to investigate the activities in this lesson.

1. Graph the following functions on the same set of axes.

a.  $y = 2x + 1$ ,  $y = 2x + 3$ , and  $y = 2x - 1$ .

b.  $y = x^2$ ,  $y = \frac{1}{2}x^2$ , and  $y = -2x^2$ .

c.  $y = (x - 1)^3$ ,  $y = (x + 2)^3$ , and  $y = (x + 4)^3 + 2$ .

d.  $y = \frac{1}{x}$ ,  $y = \frac{3}{x - 2}$ , and  $y = \frac{-2}{x + 1} - 1$ .

Discuss your discoveries and write a brief report on it.

Figure 3.1: A sample discussion activity for ensuring simultaneous group discussion

If the creation process of the software files to be used in the tasks would contribute to the development of the targeted learning outcomes, the researcher freed the pre-service teachers to create the software according to their knowledge and approach to the solution. The methods used in such cases and the argumentation process



developed differently. However, if the relevant teaching episode’s learning objective was not directly related to constructing the learning environment itself, the researcher provided the ready-made file on the course’s Moodle page. He expected them to develop an argumentation by playing with the variants and invariants on the file.

### 3.2.2 Pilot Study

The pilot study includes teaching episodes in the first semester in which the course was offered. A total of fifteen PSTs, twelve females and three males, participated in this study. All PSTs were third-year and fourth-year students. In other words, all of them had taken the required mathematics and education courses before. This enabled the instructor to continue teaching episodes, assuming they already knew basic math concepts, lesson plan preparation, and educational pedagogies, considered prerequisite knowledge for the course. In addition, as a requirement within the scope of the course, it was said at the beginning of the course that they should have an environment for the prepared lesson plans to be applied and reported in a natural classroom environment. For this reason, the researcher ensured pre-service teachers attending schools within the scope of school experience or internship take the course.

Table 3.2: Data sources and data collection timeline for the pilot study

Data Source	Time	Purpose
Interview	2 <sup>nd</sup> week	
TPACK-practical survey	2 <sup>nd</sup> week	To assess the PSTs self-reported TPACK-practical level
Assessment of a lesson plan based on TPACK Levels Rubric	3 <sup>rd</sup> week	To help participants to become familiar with TPACK Levels rubric.
TPACK levels activity video record	3 <sup>rd</sup> week	To investigate PSTs’ TPACK levels within a lesson plan evaluation activity

Table 3.2 (Continued)

Reflection 1	3 <sup>rd</sup> week	To investigate PSTs' TPACK levels within a lesson plan evaluation activity
Individual Lesson Plan 1	6 <sup>th</sup> week	To investigate the individual TPACK level
Peer evaluation of lesson plans	7 <sup>th</sup> week	To investigate PSTs' TPACK approaches
Reflection to individual lesson plan 1	8 <sup>th</sup> week	To make participants to critique a lesson plan according to TPACK Levels rubric. To make participants face the complexity
TPACK Game	9 <sup>th</sup> week	of determining the appropriate combination of the components
Group Lesson Plan	10 <sup>th</sup> week	Let participants practice developing detailed lesson plans.
Implementation report	11 <sup>th</sup> week	Let participants evaluate a lesson plan according to TPACK level rubric
Individual Lesson Plan 2	12 <sup>th</sup> week	To identify the individual TPACK level
TPACK-practical Survey	13 <sup>th</sup> week	To assess the PSTs' self-reported TPACK level
Online discussion logs	Each week	To make PSTs get familiar being a member of online community of practice.
e-Portfolios	Last week	Let PSTs get familiar using e-portfolios
Final Interviews	Last week	To get information about how the course affected PSTs' sense of technology integration.

The researcher collected the data in the table above (see Table 3.2) according to the planned schedule. At this stage, the aim was to test whether the data collection

instruments were working and determine to what extent the teaching episodes based on the designed tasks served the targeted learning outcomes. In this context, the researcher revised the tasks and selected the ones to be implemented within the limited weekly lesson period. In the pilot study, the time allotted for each collective argumentation was more limited. The process was accelerated by the instructor, especially in concluding. In other words, if it would take time for pre-service teachers to complete their group discussions, the instructor ensured that the whole class reached a collective conclusion more quickly. Because the aim was to test as many teaching episodes as possible in the tasks, this goal has also been achieved. The researcher took notes about the teaching episodes conducted every week, and he revised the parts that the pre-service teachers had difficulty with or in which they reached a conclusion without encountering any challenging situation. For example, the researcher simplified the file prepared during the spreadsheet week by removing three sheets and revising the remaining ones for the main study. (see Figure 3.2).

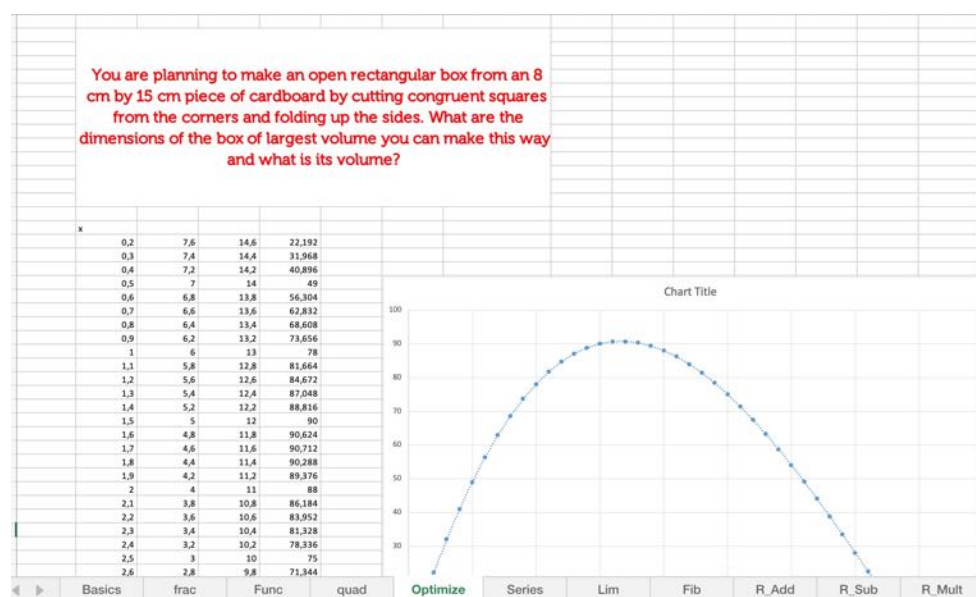


Figure 3.2: The spreadsheet used in a teaching episode

Besides the data not used in this study, especially concerning the technology competencies and developments of teacher candidates, the interviews also shaped the main study. Because these interviews were conducted at the beginning and the end of the term, the teacher candidates were also asked questions about the course.

To the questions about the tasks in the lessons and the development of technology content knowledge, the pre-service teachers said that they realized they did not know the mathematical concepts they thought they knew before. This data also supported the researcher's initial assumption that pre-service teachers memorize the subjects mostly without mathematical sense-making and reasoning. The answers to which tasks contributed more to them in this context were also crucial in determining the tasks to be used in the next stage.

Plomp (2013, p. 16) defines *development studies* within the scope of design-based research:

*“In the case of development studies, the purpose of educational design research is to develop research-based solutions for complex problems in educational practice. This type of design research is defined as the systematic analysis, design and evaluation of educational interventions with the dual aim of generating research-based solutions for complex problems in educational practice, and advancing our knowledge about the characteristics of these interventions and the processes of designing and developing them.”*

In this sense, the pilot phase and following main phase of the study was carried out as a development study, as it included the design and evaluation of the teaching episodes and, in this context, the understanding of the characteristics of the episodes to an advanced level. The data collected in the pilot study, which were not used in revising the tasks and structure of the main study, were analyzed and stored for use since they are the subject of other future studies.

### **3.2.3 Main Study**

After analyzing the data obtained in the pilot study, the researcher started the main study. The Technology-Supported Math Teaching course has been offered again, with minor changes in its weekly content. The pre-service teachers first discussed

the designated topics for each week online in a group created on Edmodo. These discussions pertained to the technology and pedagogy components of the TPACK framework identified for each week. Using Diigo, a social bookmarking platform, they shared the news or websites they found on the internet about related topics they deemed essential, together with their comments. In addition, articles that can provide information about the relevant TPACK components of each week, appropriate for their level and basic ideas, have also been added to the course Moodle page. If possible, the instructor expected prospective teachers to read these articles before class and even before discussions on Edmodo. Thus, it was aimed for them to have preliminary knowledge before the discussions. In addition, the pre-service teachers who would lead the conversation each week had the questions they prepared beforehand approved by the instructor, shared them on Edmodo, and became the discussion leaders. Until the lesson was held in the classroom environment, pre-service teachers were provided mental preparation for the technology and pedagogies to be discussed.

Table 3.3: Math education technologies used in teaching episodes

		<b>Math Education Technologies</b>							
		GeoGebra	Desmos	GeoGebra	MS Excel	VuStat	GeoGebra	TinkerPlots	Logger Pro
<b>Math concepts</b>	Transformation of function graphs	✓	✓						
	Periods of trigonometric functions	✓							
	Solving equations and inequalities graphically	✓							
	Absolute value equations	✓							
	Extraneous solutions	✓							
	Triangle inequality			✓					
	Geometric construction & Golden Mean			✓					
	Inscribed angles in circles			✓					

Table 3.3: (Continued)

<b>Math concepts</b>	Optimization problems	✓		
	Sequences and series	✓		
	Limits of functions	✓		
	Fibonacci Sequence & Lucas Numbers	✓		
	Infinitely nested radicals	✓		
	Distributions & Sampling distribution		✓	
	Descriptive statistics			✓
	Probability types			✓
	Data Analysis and Modelling	✓		✓
	Derivatives		✓	

At the beginning of each lesson in the classroom, the discussion leaders were required to summarize the discussions with a presentation. Thus, the instructor completed the target of warming up for the class beforehand. If the PSTs were utterly unfamiliar with the technology to be used that week, there was a session aiming to introduce the details of the technologies by explaining the interfaces and basic features. After all these introductory parts, the instructor followed the presentations prepared for each class and then conducted the teaching episodes.

### 3.2.3.1 Participants of the Main Study

One of the participants in the main study was an in-service mathematics teacher pursuing a master's degree. The remaining ten participants were pre-service mathematics teachers. All pre-service mathematics teachers are third and fourth-year students. Besides the female in-service mathematics teacher, eight of the teacher candidates were female and two were male. As in the pilot study, the researcher selected teacher candidates who will take this course from those close to graduation.

Thus, it was assumed that pre-service teachers with competence in mathematics content knowledge and educational pedagogies would participate in the study. Because they had to implement the lesson plans to be prepared during a lesson in an actual classroom environment. In other words, they had to be equipped to appear in front of students in the classroom and give lectures. They tested the extent to which the lesson plans worked and accordingly revised them in the next step. After the implementation of the lesson plan in an actual classroom setting, they prepared a report about this process, presented it to their classmates, and created a brief discussion environment.

As Creswell and Poth (2013) stated, purposeful sampling is used in qualitative research. Because this is to understand the phenomenon at the center of the research question, the researcher should determine the sample suitable for his purpose. For this reason, he wanted to assure the pre-service teachers had the expected background to participate in this study by taking the offered course. In addition, as a result of the pilot study, the researcher limited the course quota to eleven students to ensure that the discussions in the classroom environment were not influencing each other. According to the observations and notes were taken during the teaching episodes conducted throughout the term, intensity sampling was used for determining the sample to be used in the data analysis. Creswell and Poth (2013) defined this sample type as “information-rich cases that manifest the phenomenon intensely but not extremely”. Using this sample type, the researcher selected the groups in which the mathematical argumentation phenomenon was observed intensely in the learning environments whose characteristics were mentioned above.

#### **3.2.4 Researcher’s Background and Role**

In the classroom teaching experiment, the instructor’s role is twofold: First, he/she plays a crucial role as a development agent responsible for promoting reflection on solution effectiveness. Second, he/she acts as an observer accountable for taking notes, interpreting and analyzing students’ comments, and engaging in the classroom

discourse (Baker, 2016). In this study, the researcher took on both roles mentioned above. He has designed the tasks to require the TECA process to be used in each lesson. Secondly, he participated in the discussions held in the context of these tasks when needed. This participation has been as creating a discussion environment in the classroom as well as following the discussions in the group by constantly going to them. In addition, he made observations during the lesson and noted the issues that the researcher should consider for the following teaching episodes.

In a qualitative study, the researcher must be conscious of the prejudices, experiences, and values. Creswell and Poth (2013), who define this as reflexivity, state that this concept consists of two parts: First, the researcher should talk about his experiences with the study. Any work or school experience could be associated with the phenomenon in this research. Second, it refers to how these experiences shape the researcher's interpretation of the phenomenon studied.

Before starting his academic life, the researcher had various experiences such as teaching for twelve years, teacher training, and writing a textbook. He then served as a teaching assistant in almost every mathematics education course offered in the department during her nine years of master and doctoral studies. In addition to this formal professional experience, he had the opportunity to observe the mathematics learning processes of high school students by tutoring them for many years. These experiences have been decisive in shaping the teaching episodes designed by the researcher. Because he has experienced how students have difficulties in certain mathematics subjects, he has prepared lesson simulations to overcome those difficulties. In addition, the challenges that the pre-service teachers might experience while teaching these subjects, which he observed while working as a teaching assistant, also became an essential part of his design.

While talking about the role of the instructor in design research studies, Plomp (2013) states that the instructor should take a proactive role during teaching episodes. The researcher, who designed the teaching episodes with these assumptions, made observations throughout the teaching episodes and played a proactive role during the formative evaluation. He did this not simply by supporting students' learning, as a



teacher would typically, but by investigating trainees' understanding and reasoning and finding out why they used specific approaches. Throughout this study, the researcher primarily designed teaching episodes based on his experiences and literature. Based on the researcher's experiences as a teaching and teaching assistant, he identified the mathematical concepts where there is more meaningful learning in the discussion environment. However, since he knew that these concepts were concepts that students did not learn meaningfully without mathematical reasoning but learned by rote, he turned to them. Because they probably memorized the PSTs by going through a similar process. Thus, these concepts could be learned for them in an environment of argumentation and discussion. He received support from the literature in scanning the sources used to teach these concepts. In this context, he created the concepts' main structure by using different textbooks and internet resources. In determining the technologies to be used in teaching these concepts, the technologies he used during his teaching years were also decisive in the selection. Another factor in determining technologies was the researcher's experience in technology courses during his teaching assistant years. In addition, by supporting these experiences in literature, which technologies were used in teaching mathematics subjects and what kind of benefits they provided were one of the critical pillars of the design.

Afterward, this course made the application of simulations within the time allocated for each course. The aim was to enable pre-service teachers to observe a simulation of how the mathematics topic determined for the relevant course could be taught with the specified technology, with both student and teacher glasses. During the teaching episodes, he constantly wandered around the classroom and observed the collective argumentation processes of the pre-service teachers. Thus, the researcher gave them an idea of the teacher's role in a mathematics lesson designed with this approach. In addition, he observed the argumentation processes of the pre-service teachers and took field notes.

### 3.3 Data Collection Sources and Procedures

The data collection tools and timeline used in the pilot study were also valid in the main study. Data were collected following this schedule (see Table 3.2). Creswell and Creswell (2017) describes the qualitative audiovisual and digital materials as a category of qualitative data. The audiovisual data to be analyzed in this study were teaching episodes recorded with video cameras and voice recorders. During these teaching episodes, pre-service teachers worked in groups with three people in three groups and two in one group. Four video cameras are positioned near the desks where each group works (see Figure 3.3). The cameras were set to center the computer screen during group work. Thus, the interactions of the pre-service teachers with each other and the computer were fully recorded.

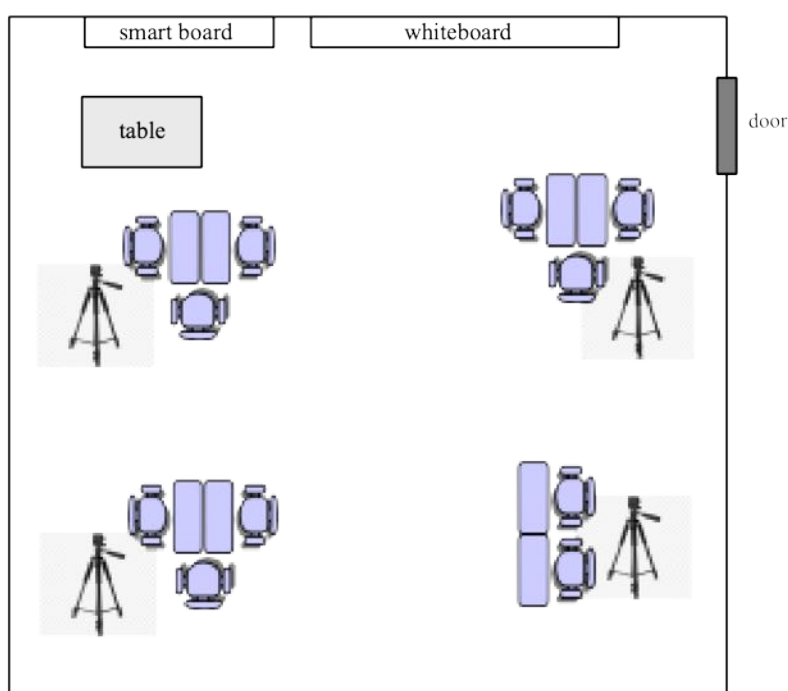


Figure 3.3: Classroom layout throughout the data collection

The researcher also took field notes regularly on PSTs' behavior and activities during the teaching episodes as a formative evaluation. In these field notes, the researcher recorded the activities in the learning ecology in an unstructured way.

### 3.4 Data Analysis

There is a recommended process for data analysis in qualitative research. Creswell and Poth (2013) suggest that this process consists of preparing and organizing the data for analysis, then reducing the data to themes by coding and condensing the codes, and finally representing them. They suggested different strategies for this process. In this study, the analytical strategy was preferred among these strategies. The analytic strategy includes taking notes, summarizing field notes, working with words, identifying codes, reducing codes to themes, counting the frequency of codes, and relating categories (Creswell & Poth, 2013). In this study, the researcher first examined the field notes he took during the teaching episodes and created preliminary codes using the literature. Afterward, he made the first analysis of the recorded videos and provided these codes. He removed the codes that did not appear in the videos, although they were included in the field notes. However, the new codes which are not noticed during the teaching episodes and detected in the initial video analysis were added to the code list. In the next stage, he determined themes by associating the codes. He determined the repetition frequency and interrelationships of the codes that he classified according to the themes. He made the transcripts of the sections with the determined codes in the videos and converted them into written text. The results obtained from this whole process are reported in the Findings chapter.

Toulmin's (2003) model was utilized as a lens to observe the mathematical argumentation within learning ecology. Toulmin (2003) determined that an argument consists of four parts: claim, data, warrant, and backing. A claim is what participants determine to be true in an argumentation process. The data are used to explain why the participant's claim is valid. Data are usually procedures or facts that lead a participant to a conclusion. A warrant is used to verify how the data gives rise to and often supports the claim. In other a warrant acts as a link between the data and the claim. In cases where a warrant fails to support the claim, a backing comes into play that explains why the claim is justified.

Table 3.4: Components of Toulmin's argumentation model

Component	Definition	Example
Data	The facts and evidence supporting the claim.	The measure of the interior angles of the $\Delta ABC$ are $53^\circ$ , $68^\circ$ , and $59^\circ$ .
Warrant	Associates the data with the claim and legitimize it by showing the relevant justifications.	A triangle with three acute angles is called an acute triangle.
Claim	the assertion that the author wants to ascertain to their audience.	$\Delta ABC$ is an acute triangle

In this study's data analysis, only the main components in the Toulmin model were preferred. The reason for this is that the main focus of the study was to examine the functions and roles of agents in learning ecology in the collective argumentation process. The main components were deemed sufficient for this purpose.

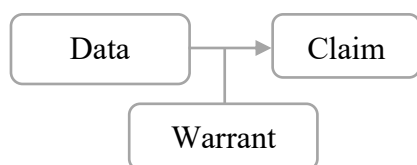


Figure 3.4: Core components of Toulmin's diagram

As suggested by Knipping & Reid (2019), the researcher focused on the conclusions or claims stated during the collective argumentation process to identify the argumentation streams.

### 3.5 Trustworthiness

Fraenkel et al. (2011) define the concept of validity in quantitative research as “the appropriateness, meaningfulness, and usefulness of the inferences made based specifically on the data they collect”. They also define the concept of reliability as

“the consistency of these inferences over time, location, and circumstances”. Instead of these concepts in quantitative research, trustworthiness is used in naturalistic studies (Creswell & Poth, 2013; Fraenkel et al., 2011; Lincoln & Guba, 1985; Merriam & Tisdell, 2015). With this concept of trustworthiness, the researcher acknowledges that a set of reasonable analyzes can be made from a given dataset for different purposes.

Table 3.5: Criterion to establish trustworthiness by Lincoln and Guba

Criterion area	Technique
Credibility (Internal validity)	(1) activities in the field that increase the probability of high credibility <ul style="list-style-type: none"> <li>a. prolonged engagement</li> <li>b. persistent observation</li> <li>c. triangulation (sources, methods, and investigators)</li> </ul>
	(2) peer debriefing
	(3) negative case analysis
	(4) referential adequacy
	(5) member checks (in process and terminal)
Transferability (External validity)	(6) thick description
Dependability (Reliability)	(7a) the dependability audit, including the audit trail
Confirmability (Objectivity)	(7b) the confirmability audit, including the audit trail
All of the above	(8) the reflexive journal

Much of the analysis in a qualitative study depends on the researcher’s perspective, and all researchers have certain biases. Accordingly, because different researchers can see some things more clearly than others, qualitative researchers use several techniques to control their perceptions and ensure they are not misinformed

(Fraenkel et al., 2011). Different techniques have been used to promote trustworthiness (validity and reliability). For example, Merriam and Tisdell (2015, p. 259) suggested “triangulation, member checks, adequate engagement in data collection, reflexivity, peer review, audit trail, rich or thick descriptions, maximum variation” as the strategies for promoting validity and reliability. However, the criterion stated by Lincoln and Guba (1985, p. 328) will be used in this study. They used the terms credibility, transferability, dependability, and confirmability criteria to establish trustworthiness (see Table 3.5).

### **3.5.1 Credibility**

Internal validity deals with the match between reality findings and reality. Searching for answers to questions such as how the research results are compatible with reality, whether it captures what is in the research field, and whether the researcher observes or measures what he thinks he is measuring are questions to be considered for internal validity (Merriam & Tisdell, 2015). The term corresponding to internal validity in qualitative research is credibility. Creswell and Poth (2013) defines credibility as an examination of whether the results correctly interpret what the participants meant.

As the first criterion for credibility, the researcher invested enough time learning their culture and gaining their trust by conducting the lessons with the teacher candidates for one semester (sixteen weeks). For the second criterion, persistent observation, he constantly kept field notes in the classes. These notes have been essential criteria for the researcher to determine the general characteristics of the prospective teachers and the learning ecology in which they are concerning the research question. He used triangulation as the third criterion. Although different methods are mentioned as triangulation, in this study, it was preferred to compare the data obtained at different times, relying on the definition of triangulation as “comparing and cross-checking data collected through observations at different times or in different places” (Merriam & Tisdell, 2015). Thus, the study also

provided this criterion for the credibility of the findings. Another criterion is what Lincoln and Guba (1985, p. 308) defined peer debriefing as the “process of exposing oneself to a disinterested peer in a manner paralleling an analytical session and for the purpose of exploring aspects of the inquiry that might otherwise remain only implicit within the inquirer’s mind”. The primary purpose of this criterion is to ensure that the researcher is honest about himself with another experienced eye. The second purpose is to provide an opportunity to evaluate the opportunities that may arise in the design methodologically. Another purpose is to clear emotions and feelings in the researcher’s mind that may hinder making sound decisions in the following process. Another mathematics educator holding a doctorate degree from another university analyzed the study’s findings for this purpose. He evaluated all the episodes in the findings based on the thick descriptions of the codes. As a result of the mutual evaluation of the feedback given, it was concluded that there was no difference in the reporting of the findings.

### **3.5.2 Transferability**

Another concept used to establish trustworthiness in qualitative research is transferability, which corresponds to external validity. However, these two terms are not precisely the same. Similar to external validity in quantitative research, which is the ability to generalize from the sample to the population, Lincoln and Guba (1985, p. 297) state that transferability “depends on the degree of similarity between sending and receiving contexts”. In order to talk about the transferability of the study in design-based research, the researchers should state domain-specific instructional theories and conjectures used (Cobb & Gravemeijer, 2014). Thus, it allows another researcher who may be interested in transferring that work to decide whether such a transfer is possible. The researcher used thick description defined by describing the context in which questions asked and situations are observed (Fraenkel et al., 2011) and purposive sampling among the suggested transferability strategies in this study. He described the implementation of teaching episodes, the learning ecology during

the teaching episodes, the data analysis, and the participants' characteristics and background in detail. Thus, by making the necessary definitions of the context of the study, the ecology of learning, and the application of the tasks, he sheds light on the extent to which researchers can adapt this study so that it can be replicated or transferred to their context.

### **3.5.3 Dependability and Confirmability**

The third criterion for establishing trustworthiness in qualitative studies is dependability, which corresponds to reliability in quantitative studies. Fraenkel et al. (2011) use the phrase "consistency of the scores (interpretations)" when describing reliability. However, to ensure reliability in qualitative studies, they recommend "to compare one informant's description of something with another informant's description of the same thing." This strategy can also be related to the previously mentioned peer debriefing strategy. Merriam and Tisdell (2015) list triangulation, peer examination, investigator's position, and audit trail while listing techniques that a researcher can use for dependability. As mentioned in the previous section, the researcher utilized peer debriefing and triangulation to establish trustworthiness.

Confirmability is the last criterion used for trustworthiness, which is the counterpart of objectivity in quantitative research. A researcher is expected to report his inferences objectively by diminishing the influence of his biases. Besides examining only variables in quantitative research, qualitative research with naturalistic inquiry also considers factors such as the participants themselves, the context, and the role of the researcher, along with the variables. The researcher used triangulation and thick description, which are suggested strategies so that biases do not affect the interpretations (Lincoln & Guba, 1985; Shenton, 2004). Regarding the triangulation and thick description strategies exploited by the researcher, the relevant sections characterized beforehand can be referred.



### **3.6 Ethical Issues**

Ethical issues in research involve more than just the researcher seeking and obtaining permission from institutional review committees or boards. In addition, and even more importantly, it means that the researcher is aware of and has addressed all ethical issues in the study (Creswell & Poth, 2013). Creswell states that ethical issues are not just some procedural actions that must be done at the beginning of the research; instead, there are steps to be considered at every stage of the research. Before starting this study, an application was made from the institute for ethical permissions, and permissions were obtained (see appendix). At the beginning of the research, he mentions the necessity of giving all participants general information about the study's purpose. A pre-interview was held with the students who wanted to register for the course during the add/drop week, and the work to be done in the content of this course was explained in general. However, it was stated that participating in the study was not a prerequisite for taking the course, and they did not feel pressure to sign the form. During the data collection phase, it was avoided to deceive the pre-service teachers by explaining how and for what purpose the already collected data would be used. While recording the teaching episodes with the camera, the researcher placed all cameras behind the participants according to their sitting positions. Since the cameras focused on the computer screens, the faces of the participants were prevented from being recorded. During the data analysis, they were told that pseudonyms would be used to respect the participants' privacy, and it was done as they were told. In the reporting phase of the data, the evidence, data, and findings were reported honestly without falsification. Creswell and Poth (2013) states all these stages as requirements for ethical issues.



## CHAPTER 4

### FINDINGS

This chapter focuses on the findings that emerged from the data analysis in two main sections, which address the research questions provided in the first chapter. The findings related to the first main research question, that is “How do technology, instructor, and pre-service teachers interplay in TECA?” will be summarized in the first section.

The results presented in this section depend on the tasks used in the study and the instructor’s behavior and role in these activities. As described in detail in chapter 3, the researcher developed the activities used in the study to exemplify the teaching of the topics in the high school mathematics curriculum by a mathematics teacher in a school using technology. Therefore, during the activities, the aim was to eliminate the shortcomings of the PSTs in the subjects they learned with insufficient mathematical reasoning and to design a lesson environment where they could see how they could teach these subjects by enriching them with technology. Therefore, tasks would require PSTs to create a conjecture with collective mathematical argumentation and mathematical reasoning during the lesson. The parts that require technology support to create a conjecture were specially injected into the tasks. In this way, it aimed to make teacher candidates aware of the importance of technology in creating conjectures and gaining experience in this field.

Since the instructor aimed to model the expected behaviors of a teacher in the classroom during these activities, he sometimes participated in the discussion where the PSTs got stuck in the collective argumentation processes. The instructor supported the discussion with the interventions he deemed necessary to make the argumentation process that started to deviate from the goal go as expected. However, since the researcher targeted specific goals within one lesson hour, he made some

interventions to speed up the process of forming the conjecture, considering the time constraint.

The next section presents the findings related to the second main research question “in what ways does technology transform PSTs’ mathematical reasoning during TECA”.

#### **4.1 Technology-Enhanced Collective Argumentation (TECA)**

One of the main purposes of this study was to examine the nature of PSTs’ TECA. For this purpose, the video records of all the lessons were examined to reveal the PSTs’ mathematical argumentation episodes during all the tasks they were supposed to complete. Toulmin’s (2003) model served as a lens for examining these episodes. Findings emerging during the developed mathematical argumentation were discussed in three sections, as seen in the Figure 4.1.

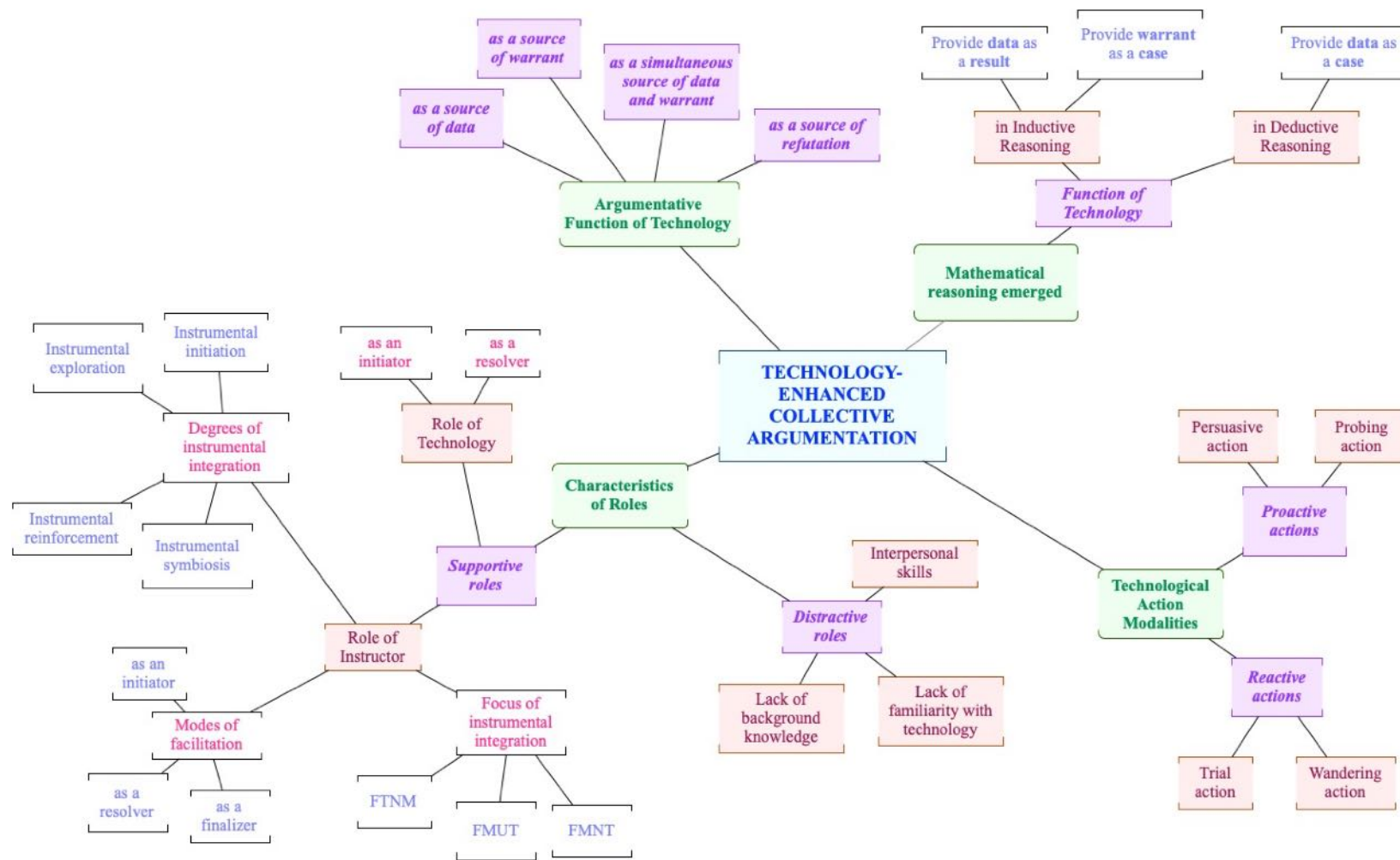


Figure 4.1: Structure of the findings

#### 4.1.1 The Characteristics of the Roles in TECA

The findings of the sub-questions of the first main question will be presented in this section. The findings of the first of these sub-questions, “How do technology, instructor, and PSTs as students interplay in TECA?” will be discussed in this section. Data analysis revealed the supportive and distractive factors during the technology-enhanced mathematical argumentation process.

##### 4.1.1.1 Supportive Roles in Mathematically Correct Collective Argumentation

Investigating the argumentation episodes revealed that both the technology and the instructor played a supportive role for PSTs in the process of developing a mathematically correct argumentation. The emerging codes after the data analysis will shape the following sections (see Figure 6).

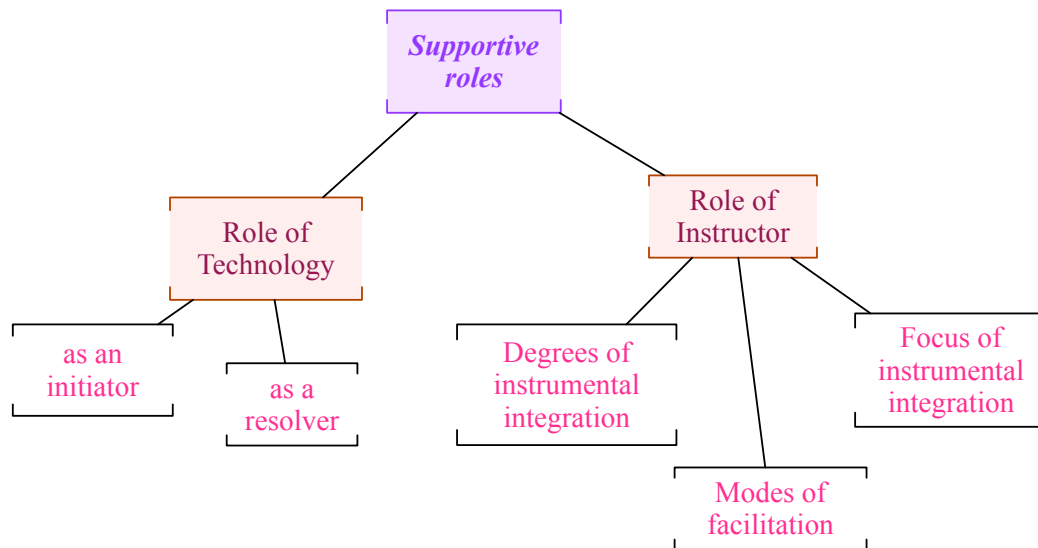


Figure 4.2: Supportive roles

#### 4.1.1.1.1 Role of Technology

In the episodes of technology-enhanced mathematical argumentation, two different roles of technology affecting the progression and formation of argumentation were identified. Technology served either as an initiator or as a resolver for the related argumentation unit.

##### 4.1.1.1.1.1 Technology as an initiator

Technology has the role of triggering the argumentation by providing the essential components. When PSTs do not have enough ideas to start mathematical argumentation or are unsure where to start, technology provides data and ideas to get started.

Episode 1

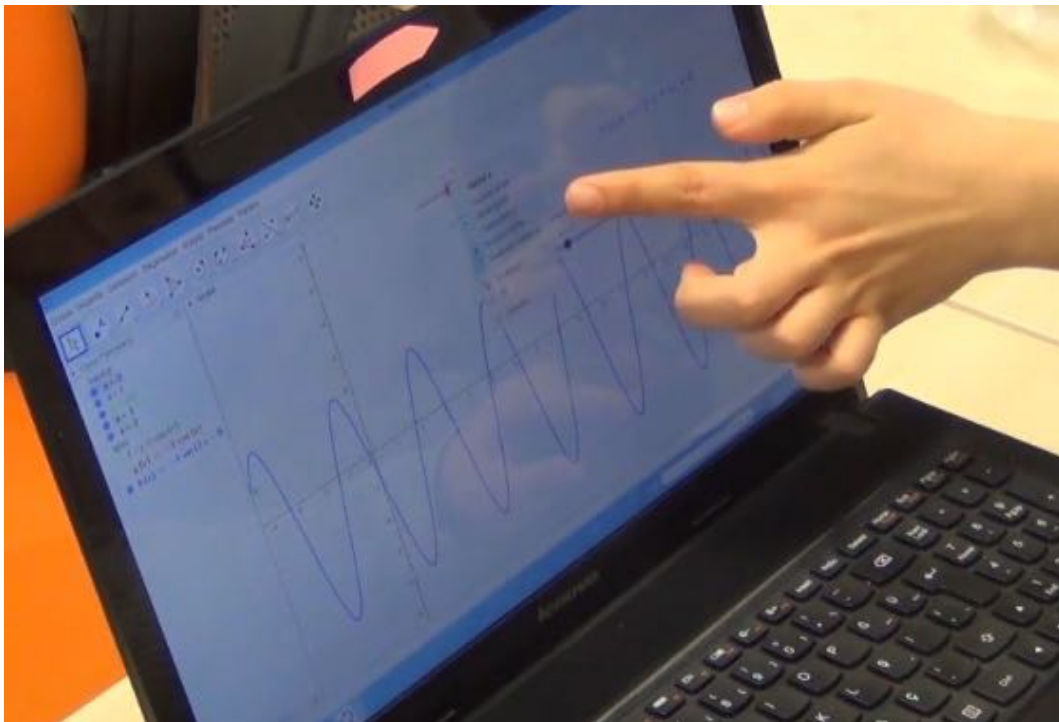


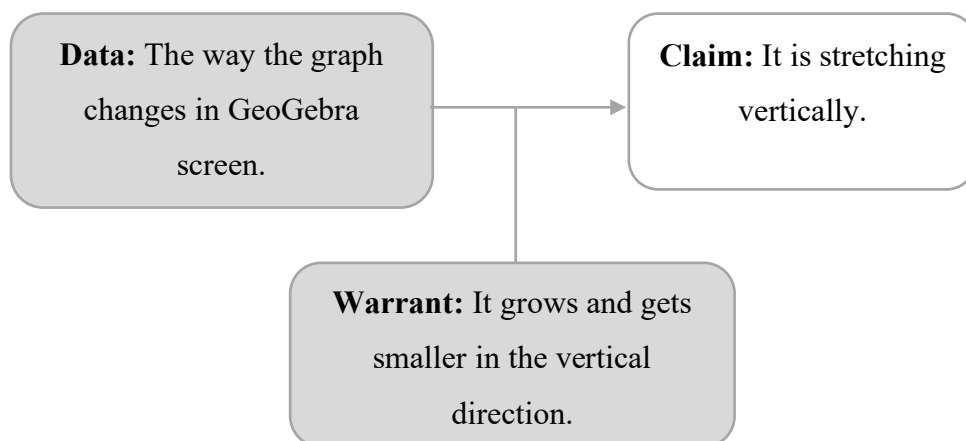
Figure 4.3: Technology as an initiator in transformation of graphs

In this episode, Burcu, Eda and Nisa want to observe the effects of the sliders they determined in the task of the fourth week where the effect of coefficients on transformations of functions is examined. For this purpose, they activate the sliders animation option and observe the change. As a result, they realize that the change of the graph of the function has a stretching or a shrinking effect in the vertical direction. In this argumentation process, GeoGebra takes on the role of initiating argumentation by providing data as well as warrants to PSTs.

Nisa: What is it doing? (After turning on the animation feature and observing the change for a while) It actually stretches upwards!

Burcu: It grows and gets smaller in the vertical.

Nisa: So, it is stretching vertically.



## Episode 2

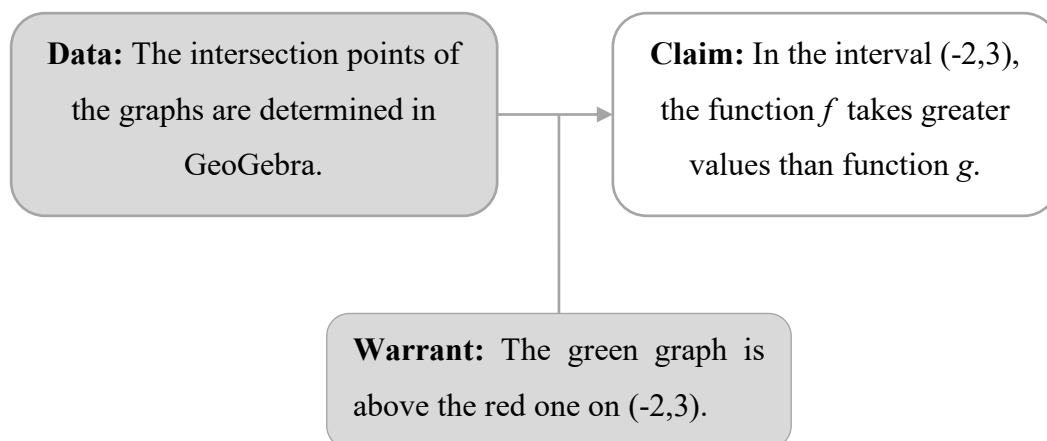
In the part of the task in the fifth week, in finding the solution of the “ $f(x) > g(x)$ ” inequality, the PSTs first found the intersection points by drawing the graphs of the functions in GeoGebra. Then they decided which function takes larger values in which interval by taking these intersections into account.

Some PSTs: The intersection points of the graphs of the functions are marked in GeoGebra. Then the positions of the functions relative to each other are examined on the screen.

Sude: In the interval  $(-2,3)$ , the function  $f$  takes greater values than function  $g$ .

Nuriye: The interval where the green graph is above red becomes the solution.





### Episode 3

In this episode, during the activity of the 7th week, PSTs are asked to cut a square-shaped piece of paper from the corners of a piece of paper and make an open-top prism-shaped container after it is folded. To find the size of the paper to be cut to obtain the prism with the largest volume, they need to find it in the activity. For this, PSTs want to find their product by writing down each prism dimension in a column. Here, they write the formula that will find the products by writing the values in the spreadsheet application to be able to observe the size that will give the greatest volume. Then they copy the formula by dragging and applying it to other cells.

Berk and Arda do not have any predictions about the maximum value. However, they write the relevant values into the cells, thinking that multiplying the values they will produce using the spreadsheet software can reach this value. Technology, therefore, plays a triggering role in reaching a conjecture.

Berk: Now look! The more we shorten 15, won't we have to shorten this that much, too? Now, 15 by 8. OK? If you entered 4 cm from here, you have to enter the same from both sides. In other words, when you cut the part you cut from here and make 1 cm from inside, two will decrease by one cm. The more I increase here, the less here. *(He talks about values inserted in different columns.)*

Arda: Shouldn't it be 2 cm less? When you cut by 1 cm, it will be 1 cm less from the other corner, too. So, decrease by 2 cm.

Berk: Right! You are right! It decreases by 2 cm. We will be able to enter 3 cm at most so that 3 comes from there and 3 from there. If we enter 4 cm, it won't happen anyway. Is it correct? What are we going to write there?

Arda: Product of three.

Berk: Exactly. This is the product of three. B2, C2, and D2. Teacher, the greatest volume is 88.

#### Episode 4

In the following episode, Berk and Arda need to observe whether the ratio of the successive terms of the Fibonacci sequence approaches a value. For this purpose, they write the necessary formulas into the cells to obtain the terms of the Fibonacci sequence. Then they write a new formula next to the relevant column that gives the ratio of consecutive terms to each other and drag it down to examine how this value approaches a particular value. Although they do not have extensive knowledge of how to do this, the spreadsheet software initiates the argumentation process by providing necessary data and warrant to reach a claim.

Arda: Take it down a little bit.

Berk: Look, it didn't work that way.

Arda: Now drag them down again ... Exactly! There's nothing under it, no numbers, that's because of it.

Berk: Huh! You are right!

Arda: It will gradually approach zero (*after observing the screen for a while*). No, it's 1.6.

Berk: It looks like 1.6, but let's see.

#### Episode 5

In the following episode, Sude, Nuriye, and Tuba are seeking an answer to the question, "If the height of all students increases by 0.8 dm, what would change if they recalculate the values they found before?" They open a column next to the values they enter in GeoGebra's spreadsheet screen and add 0.8 to all of them. Then they calculate the descriptive statistics values of the values of this column. Accordingly, they are expected to reach a conclusion. In this activity, the PSTs had

ideas that they did not utter aloud. However, here again, we observe the role of technology in triggering the argumentation process, as they reach a conclusion by using technology to calculate these values quickly.

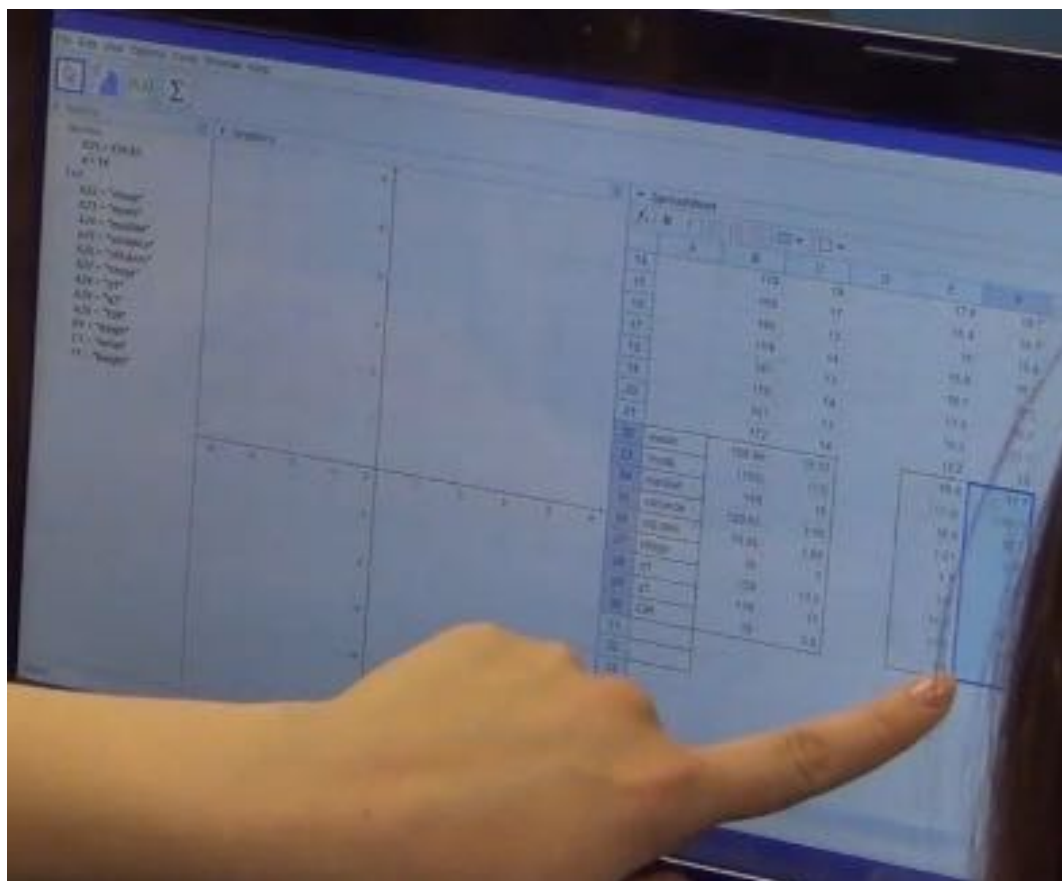


Figure 4.4: Technology as an initiator in a statistics related task

Nuriye: Now we will make +8 there. E2 +0.8

Tuba: No!

Sude: We added dm. (By comparing the values on the screen after the copying process is finished)

Nuriye: Variance changed; range remained unchanged.

Tuba: Standard deviation has not changed.

Nuriye: (After looking at the numbers on the screen for a while) What has not changed now? Variance did not change.

Tuba: Standard deviation has not changed.

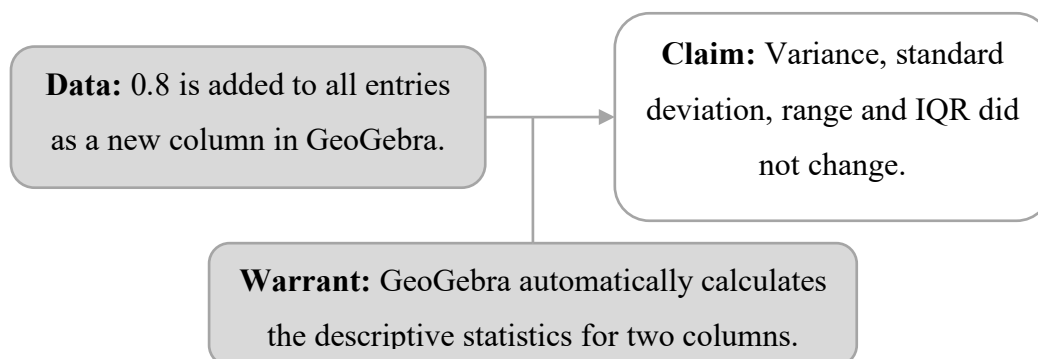
Sude: You will not be looking at that anyway. This is just divided into 10.

Nuriye: I am comparing the following two.

Tuba: Okay. Variance, standard deviation, range, IQR are unchanged.

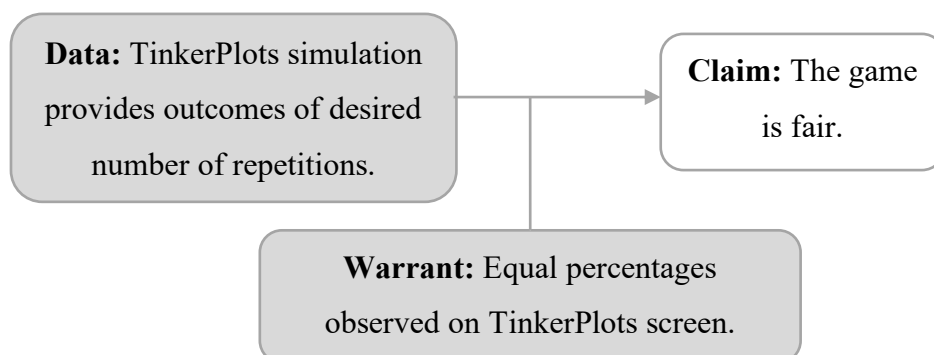
Sude: Median, mean, and mode are all increased by 0.8.

Sude: Of course, Q1 and Q3 increased by 0.8.



## Episode 6

In this episode from the probability activity that takes place in the eighth week, the PSTs make their predictions about a game. The first box of this game contains 4 red chips, and the second box contains 2 red and 2 blue chips. The game is won when chips of the same color are drawn from the boxes. Their first guess about this game was that the game was fair. They then have to verify the accuracy of these predictions using TinkerPlots. For this purpose, they create two mixers in TinkerPlots and start the simulation. In the brief argumentation process in this quote, they were not sure how to confirm the accuracy of their predictions that they first stated. TinkerPlots provides PSTs with data and warrant to support their predictions where they are unsure.



Sude: Now, these are results. Let us see the points for this. We will look at its joined view.

Nuriye: Won't you carry "Joined" down?

Sude: I'm trying to get that down. *(By grouping the data in the graph view of the TinkerPlots)* I threw them aside. I will see how many there are.

Tuba: How did we make it?

Sude: I will throw the same ones aside. I can combine both of these. *(After combining)* There were different ones, the same ones. We win when the same color comes.

Nuriye: Show me the percentages!

Sude: Where is it?

Nuriye: You will click on the percent sign there. *(After the percentages appear, pointing to the number of trials)* Let us increase it right here. Make 1000.

Sude: Let us speed it up. Let us not get the fastest so that we can have some fun. *(After the simulation is over)* Fair. Fair is out, sir!

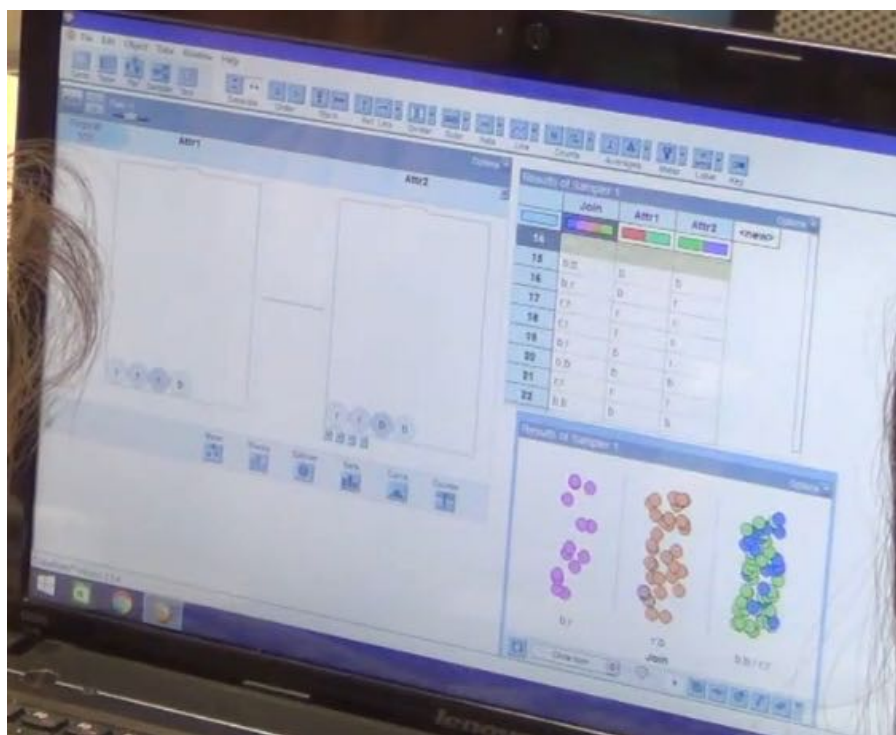


Figure 4.5: Technology as an initiator in probability related task.

## Episode 7

In this episode, PSTs are asked to estimate the slopes of some given curves at desired points. They are asked to make these estimates by using the zoom feature in the ready-made GeoGebra file provided to them, using the feature that all curves look like straight when zoomed enough. Although initially, they have no idea about the slopes of the given curves at desired points, technology contributes to the process by initiating argumentation in this activity. In the following part, PSTs use the slider feature in the GeoGebra file and zoom in enough to find the slope of the  $y = \sin(x)$  curve at the  $x = 3$ . They try to estimate the value of the slope of the part that looks like a line on the screen.

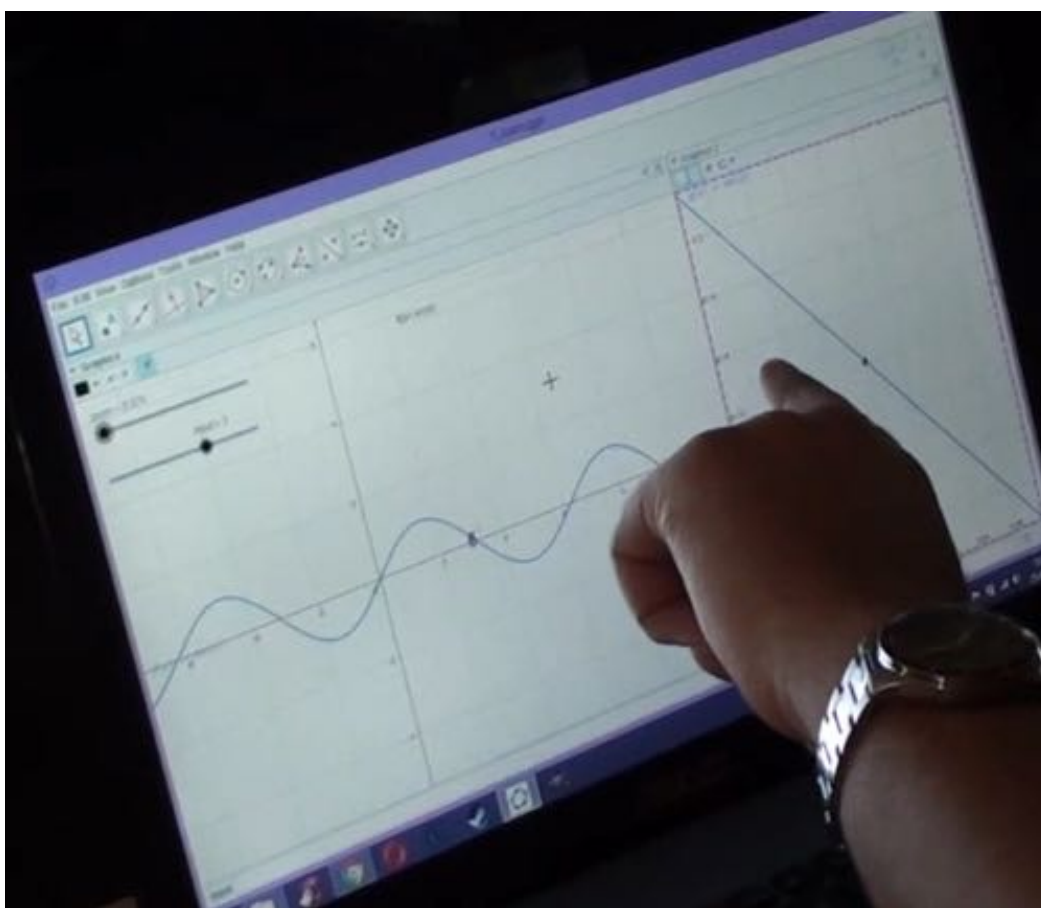
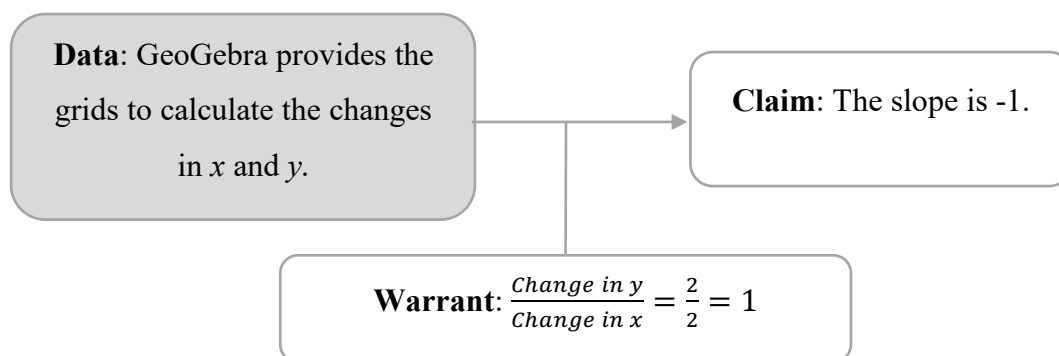


Figure 4.6: A snapshot of the PSTs' approximating the slope by "rise over run".

Berk: Look, we can find this in the same way. Here, one, two... one, two... (*Counting changes to use the formula Change in y over change in x*) The slope of this is 1. But just a second, is it -1?

Arda: Yes, - 1.



#### 4.1.1.1.2 Technology as a resolver

In this section, there are episodes to illustrate the second role of technology that support the argumentation process. This role of technology is observed in situations where the argumentation process cannot proceed as expected. In these cases, technology plays a crucial role in removing the obstacles to the blocked process by helping the emergence of argumentation units to support PSTs' predictions.

##### Episode 8

In this episode, PSTs were given some tasks to gain the habit of defining absolute value equations using the concept of distance. First, they learned to define absolute value equations by using the definition of a circle. Next, they examined how shifting the circle's center on the  $x$ -axis would affect the equation. They sought the solution to this problem primarily by algebraic means. Having trouble interpreting the effect of changing the circle's center algebraically, they opted to analyze it using GeoGebra's slider tool. Without observing the effect of the slider, they thought algebraically and concluded that they would find the roots by adding "a" to the distance from zero. However, after observing the effect of the slider, they realized that the distance did not change. They changed their interpretations by adding the value "a" to the points where the circle intersects the  $x$ -axis in the previous case.

With this move, GeoGebra provided them with data and warrants, removing the barriers blocking the collective argumentation process.

Sude: It's like we added  $a$  to the distances from zero.

Nuriye:  $x$  minus  $a$  squared is equal to  $r$  squared. So,  $x$  is equal to minus  $a$  ... Is  $x$  minus  $a$  squared in the square root of  $r$ .

Sude: Yeah.

Nuriye: So,  $x$  minus  $a$  became either  $r$  or minus  $r$ . Then  $x$  became  $r$  plus. (*After silence for a while*) Did we do something wrong? There is  $a$  minus  $r$  over there, right? (*Thinking a little*) Minus  $r$  plus  $a$ .

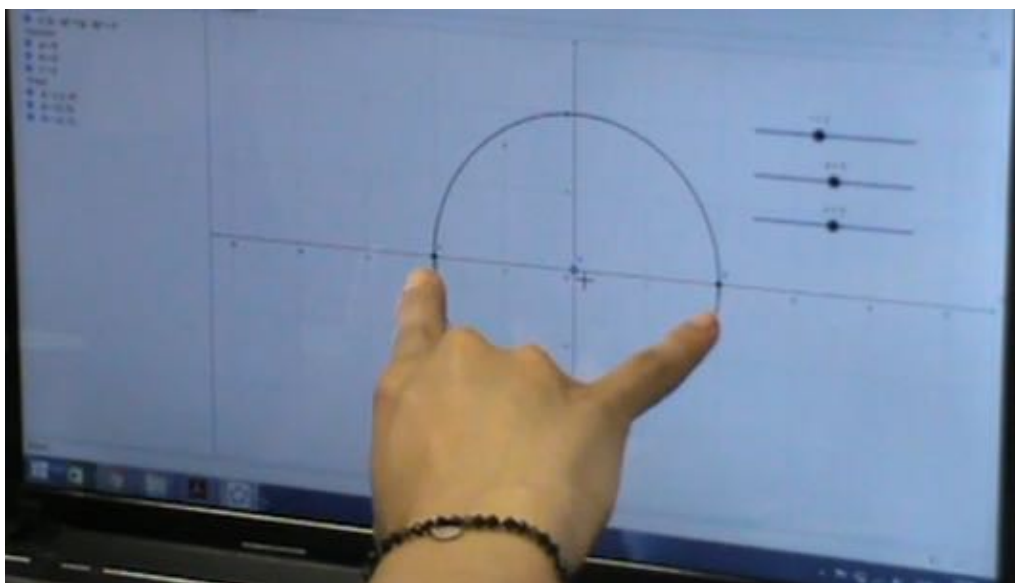


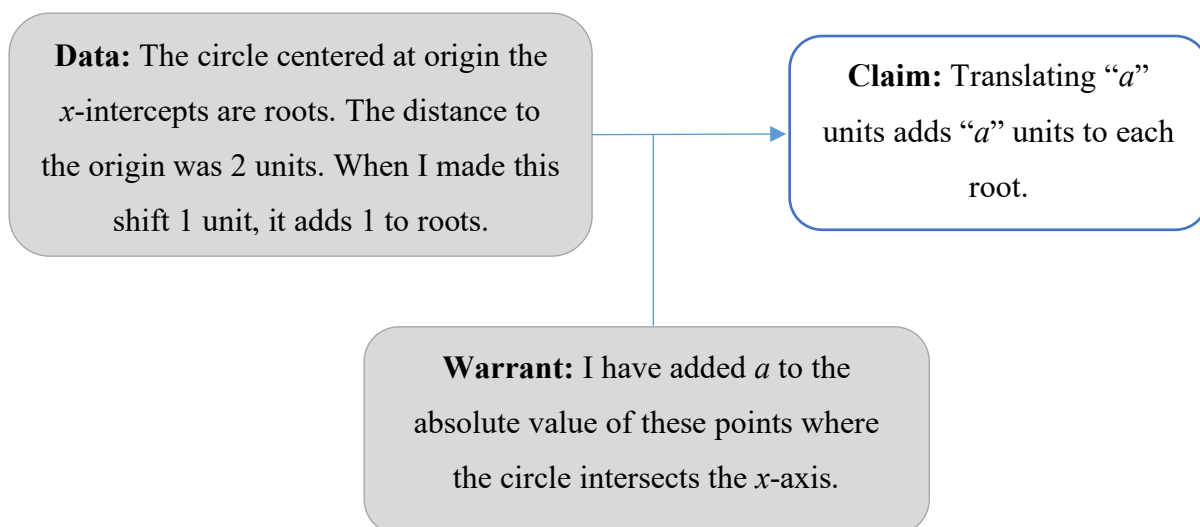
Figure 4.7: Technology as a resolver for absolute value task

Sude: Actually, if we think of  $x$  as the absolute value, it seems like we did not add  $a$  to its distance from zero. So, while you are in this thing, look! When we are at zero, that is our roots. I set it as points  $a$  and  $b$ . These points  $a$  and  $b$  were also ... points whose distance to the origin, I mean its absolute, were 2. In other words, But I made this  $a$  1. I have added  $a$  to the absolute value of these points where the circle intersects the  $x$ -axis.

Tuba: Translation occurs.

Sude: Exactly! We have translated as " $a$ " unit. It affects up to " $a$ " unit to its absolute value.





### Episode 9

In the following episode, PSTs are trying to learn how to make use of technology in the creation of a sign chart that examines the values that a quadratic function can take. In fact, they had memorized the creation of these tables as they were included in the textbooks of their high school years. Using the visual representation of GeoGebra in creating the table and making inferences, Berk tries to convince those who do not understand by using this dynamic visuality. Thus, Berk and his other friend, who understands what he is talking about, use technology as a tool to overcome the obstacles to making the desired conjecture.

Berk: We will look at the sign of “ $a$ ” again. Why is that? Because...

Burcu: Let me play with this “ $a$ ”. For example, move it to be negative.

Berk: This time, something will happen. After the roots... The smaller root and the greater root... This time, what was the name of this? Coefficient of “ $a$ ” due to transformation. This is like the slope here. The instructor always emphasizes not to say that, but the arms are down and the arms are up. Do you understand? Therefore,  $a$  determines whether the arms are pointing upward or downward here. For example, while  $a$  is positive, those smaller than the radical are positive and those that are greater than the major radical are positive. While  $a$  is negative, those that are less than the minor root are negative, those that are greater than the major root are negative.

Burcu: You are speaking as if you are reading from somewhere right now.

Nisa: Exactly!

Berk: No, I'm not.

Burcu: I can't see it. Could you show me what you said?

Berk: Now look, Burcu! This parabola has two roots, right? Here and here.  $a$  is positive, right? Are you aware that values less than the small root are always positive when  $a$  is positive? Greater than zero. Therefore, we can say that the function takes positive values when  $x$  takes values less than the small root. Of course, this only applies when  $a$  is positive.

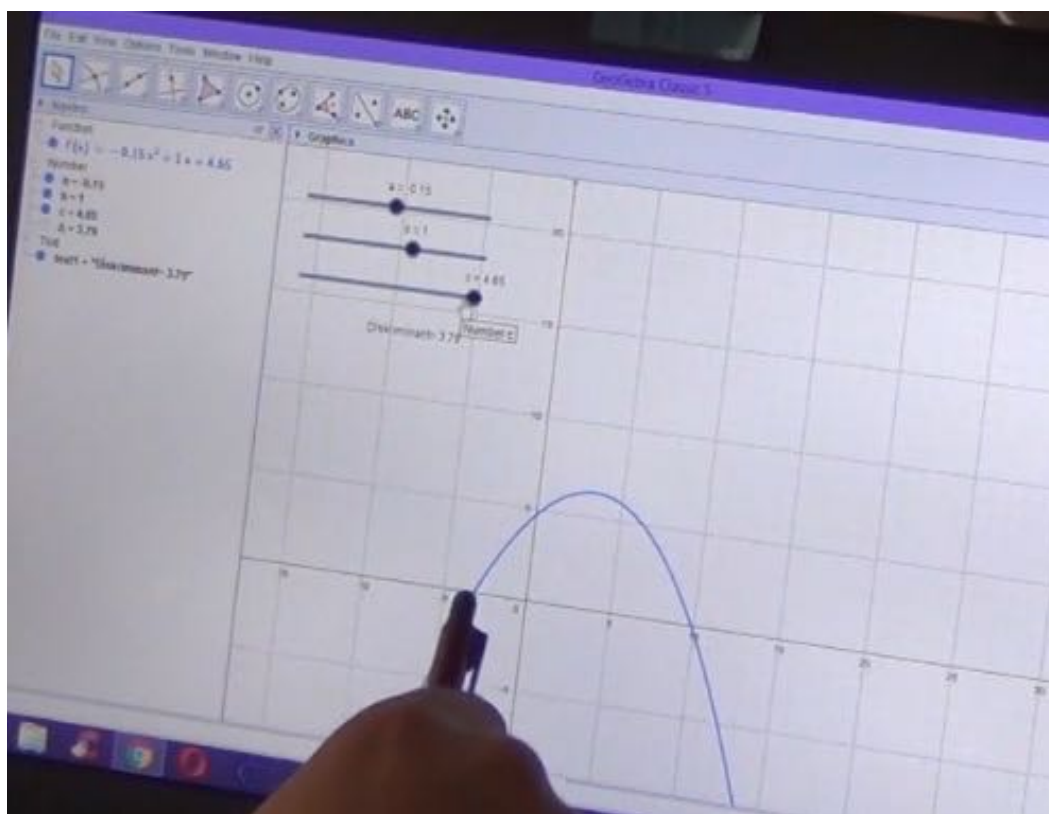


Figure 4.8: Technology as a resolver in constructing sign-diagram task

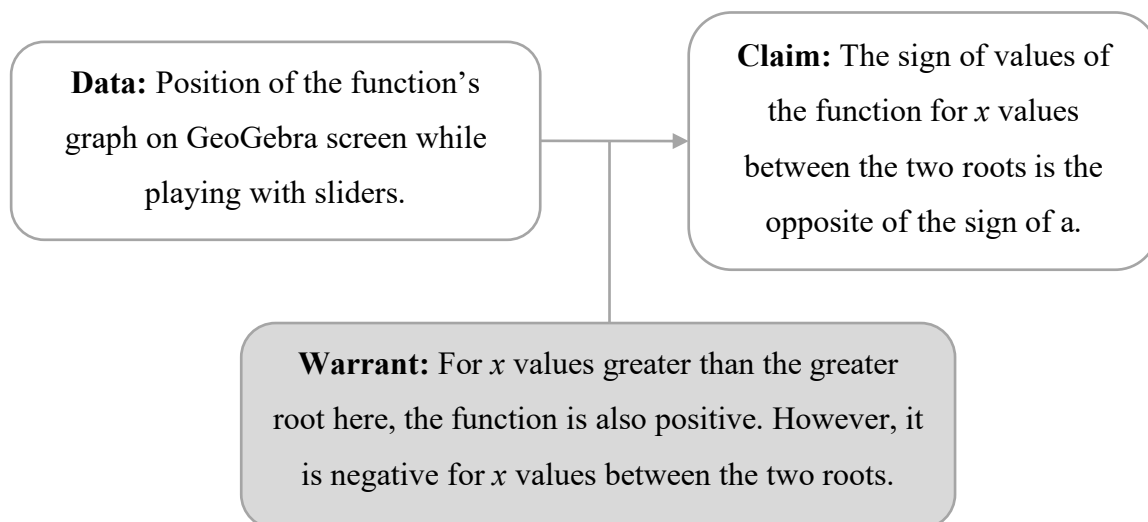
Nisa: Think about that side, Burcu! Look! The graph is above the  $x$ -axis.

Berk: While the  $x$  values are getting bigger than the greater root, the function has positive values again. But it has negative output values for the remaining input values between the roots.

Nisa: Let's say it this way: the function takes opposite values between two roots.

Berk: But look, when we make a negative... What happened this time? The function is negative for  $x$  values less than the smaller root, so here. Again, it is negative for input values greater than the major root. Again, the reverse is true for values between roots.

Nisa: Let's just say then. The sign of values of the function for  $x$  values between the two roots is the opposite of the sign of  $a$ .



### Episode 10

In the following episode, PSTs examine the effect of parameters on the transformation of functions. They were confused by remembering the information they had memorized from their high school years while inferring about the effect of parameters on the parabola in previous examples. While examining the same change in trigonometric functions, they were able to express the effects of the parameters on the graph more accurately by observing the GeoGebra screen.

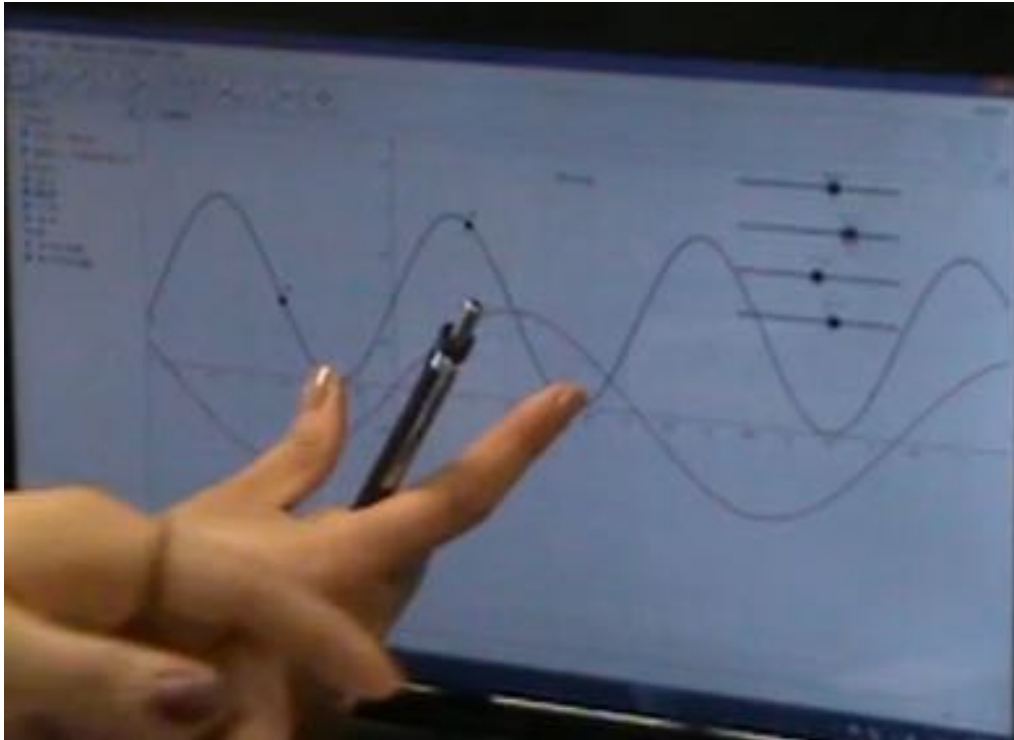


Figure 4.9: Technology as a resolver in periods of trigonometric functions

Sude: a is two. Only thing got bigger, b is two, things in between have changed. *(At that time, she is trying to make sense by looking at the graph, but she is not sure.)*

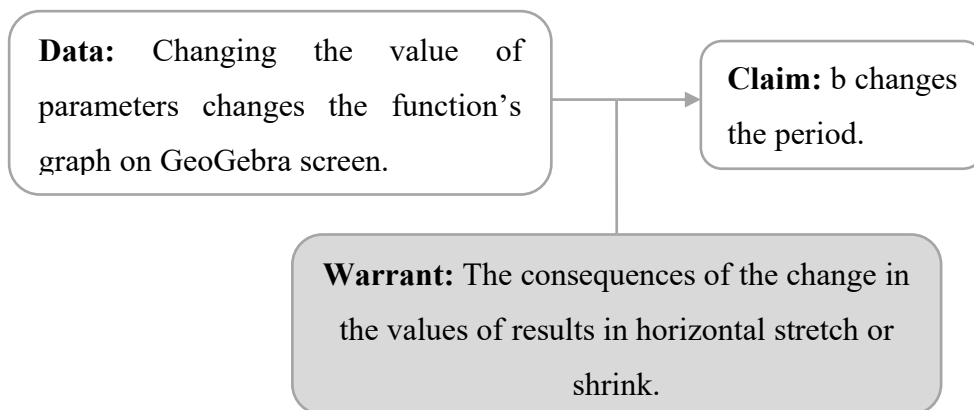
What number has changed? The number of waves in between.

Nuriye: That part is getting narrower because... Look! We just said that the change of a will change the arms! Let's change it like this. Makes it narrower on the x-axis! This largest one takes up smaller space. We said previously that it is greater in this and again greater in this. *(Showing the graphs  $y = x^2$ ,  $y = \frac{1}{2}x^2$ , and  $y = -2x^2$ )* But in the sine function it is just the opposite. These places do not change, they change when we change a.

Sude: For example, when we change b, there is a half wave between these two points; When I increase it by 1 there is a whole wave. Next time I increase it by 1, there one and half waves obtained by 3 times a half wave.

Tuba: The period is changing.

Sude: b changes the period.



### Episode 11

In this episode, PSTs want to find the maximum value of the box's volume to be created by cutting a square from each corner and folding it using spreadsheet software. They write the dimensions of the box formed after the cut piece in a column and calculate the value of their product by writing a formula. But they fall into the misconception that the size of the cut square should be just an integer. In order for them to realize this mistake, the instructor intervenes by using the affordance of technology.

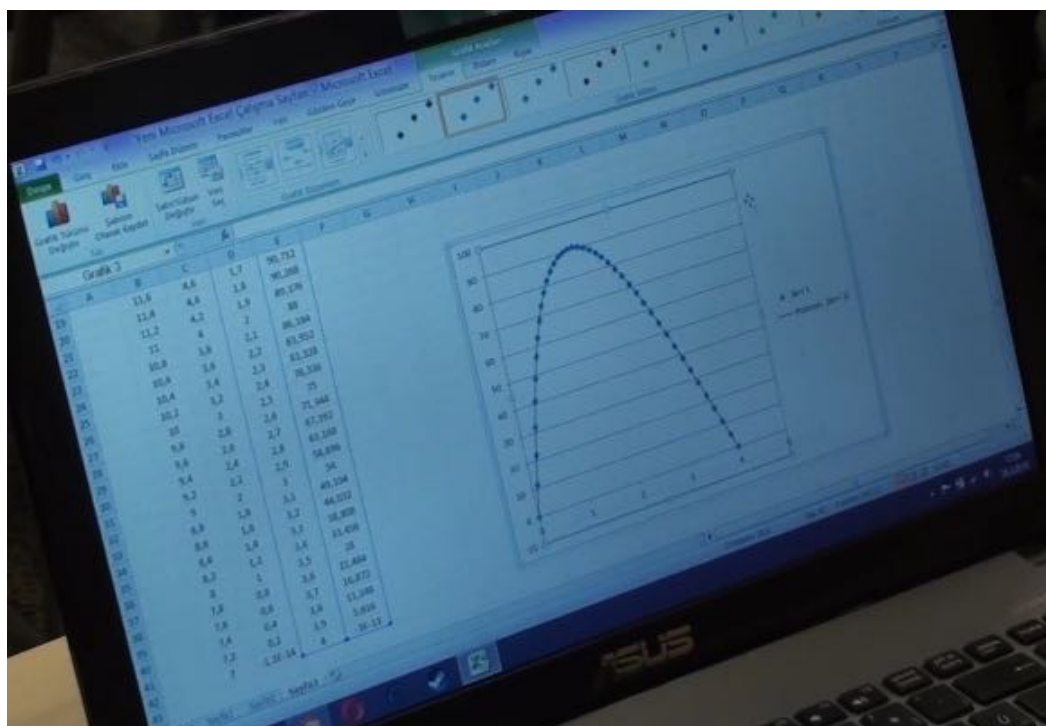


Figure 4.10: Technology as a resolver in an optimization task

Berk: Teacher, the maximum value of the volume is 88.

Instructor: Have you plotted its graph?

Berk: Graph?

Instructor: Yes. For example, draw a graph of a function so that the length of the square you cut is an argument, and the volume value is the dependent variable.

Berk: You said that the piece we cut is  $x$ .

Instructor: Plot a graph of the function with its volume as  $y$ . OK. Now, insert a graph with these data.

Berk: The graph looks weird because of the negative values. It won't be negative.

Instructor: You're right! What about the input values? Do  $x$  values have to be integers?

Berk: No. But the maximum value of this will be somewhere here in this column.

Instructor: What if it is not there?

Arda: He says if not 88. For example, if you plug a decimal value between these, you will get more. (*Looking at the graph on the computer here, he notices that the value will produce a larger volume once in a while.*)

Berk: Well, then we will have to look at the infinite value of this. Umm ... can't we see it from the chart anyway? We cannot see it.

Arda: We'll see.

Berk: No, I cannot see it because we will draw the graph according to these data.

Arda: Give it a try. Enter that data too.

Berk: Yes, this is the maximum value of the volume

Arda: Not precisely.

Berk: Exactly. There is a bigger one.

Arda: We need to do it in more detail. Are we going to enter decimal values like this?

Berk: Let it be 0.5. Then make it 1. (*Later, realizing that this will not be enough either, he adjusts the increment value to 0.1*)

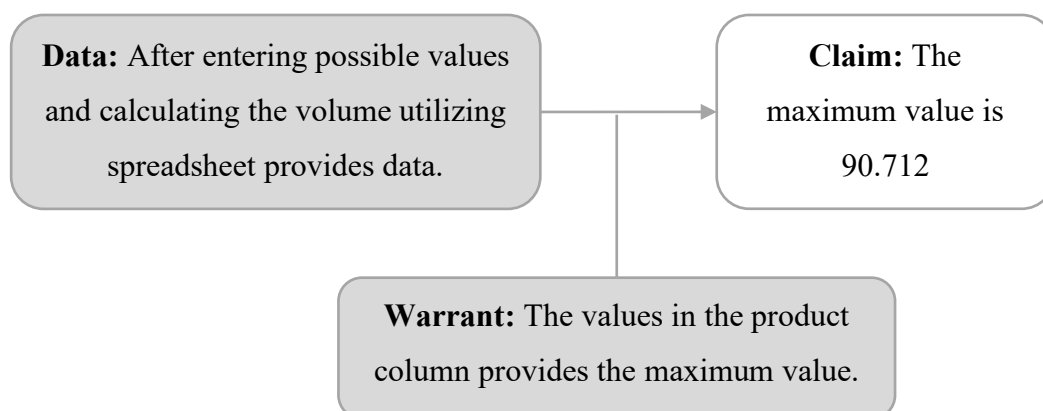
Berk: If it is 0.1, it will decrease by 0.2. Is it correct? It will be 14.6. Is it right when it decreases by 0.2? (*Then it fills the cells by selecting two cells and dragging them down*)

Arda: Will it be?

Berk: Yes

Arda: Draw the graph again because it is not updated. (*Then, he updates the related range for the graph.*)

Berk: Sir, we did it. Its maximum value is not 88. In fact, it is about 90.712.



## Episode 12

In this episode, PSTs are trying to find the limit of the function  $f(x) = \frac{3x+1}{x-1007}$  as  $x$  values approach infinity with the help of a spreadsheet. They stated the desired limit value as 3 when starting the activity using their prior knowledge from calculus lessons. However, when they start entering input values, it turns out that they are confused. When they continue to increase their input values without bound, the value they need to reach emerges, and the question marks in their minds are disappeared.

Berk: Three times C2 plus 1. (They enter the formula into a cell)

Arda: Divided

Berk: Divided

Arda: C2 minus 1007

Berk: (Copying the formula by dragging downwards) Why did this happen?

Instructor: What happened? Is there a problem?

Berk: No problem, sir ... It may be from the numbers. Did we reverse the numbers?

Instructor: No! Go on!

Berk: Shouldn't it be close to 3?

Arda: It is getting closer.

Instructor: Go ahead.

Berk: It came from eight and nines. Anyway, it will approach 3 as it goes towards infinity, but you are right. *(In the meantime, he continues to drag and copy.)*

Berk: Look! It is approaching 3. Do you see it? The maximum value is 9, but look, as it goes towards infinity ...

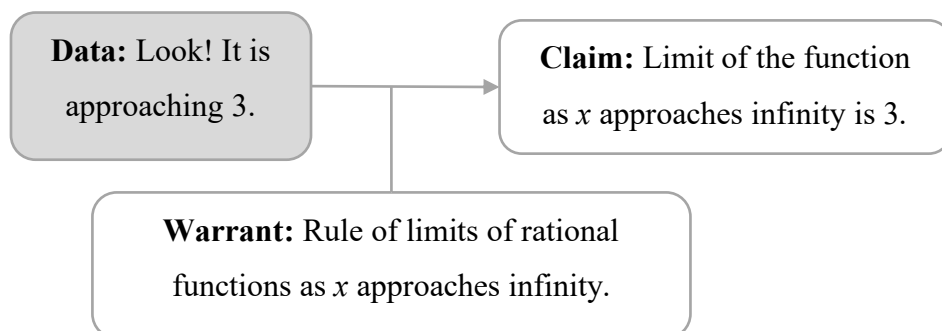
Arda: Let's move forward by increasing by 60000.

Berk: Right! *(He continues to copy by making the increment as 60000).* Sir, we came to 8,230,000. But here is the thing. Its highest value is different. Sounds like 9.

Arda: The highest value does not concern us!

Instructor: What happens to  $y$  values as  $x$  values increase?

Berk:  $y$  values are approaching 3, sir.



### Episode 13

In the following episode, PSTs examine the limit value of the ratio of consecutive terms of the Fibonacci Sequence as  $n$  increases indefinitely. Although they had no idea that this limit would converge to any value initially, they later realize that the



written formula converges to a value for increasing values by copying it into consecutive terms.

Nuriye: We will find it by dividing the second term into the first term. Divide the two by one and then pull it down to copy it. You have to go up a little bit. This is our first term and this is our second term. We will write something there.

Sude: Let us start with the thing. Let us start with the initials. Let us drag it down.

Nuriye: We will write  $A_2 / A_1$ . Equals,  $A_2 / A_1$ .

Sude: We will drag it now and the result will come.

Nuriye: Yes!

Sude: Now let me go to the hundred. (*She keeps dragging and stops.*)

Nuriye: 61 ... Is it decreasing? I think to drag a little more. It is very close to the hundred.

Sude: Yeah. Stop, I will increase their thing. (Increases the number of decimal points after the comma)

Nuriye: Let us go down.

Sude: I increased it a little more so that I could see the change.

Nuriye: I wonder if you stop once in a while? (Meanwhile, Sude continues to drag and copy downwards)

Sude: I stopped.

Nuriye: But let me say something, it stays around 1.60. We came to 600<sup>th</sup> terms, we are still at 1.6s.

Sude: I just took this up to 1000.

Instructor: OK

Sude: We can continue up to 1000 if you want.

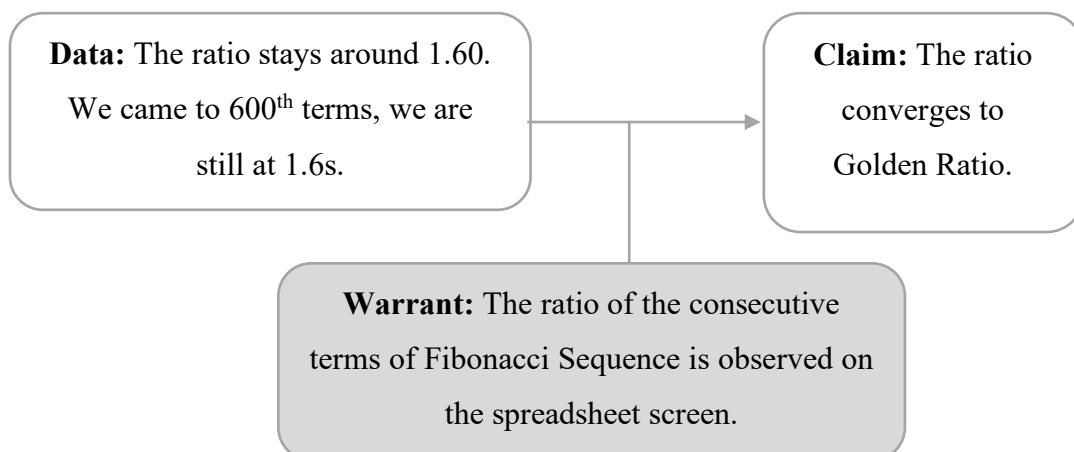
Nuriye: Okay, let us continue to drag up to 1000 again.

Sude: It does not change anymore. You can increase the decimal to see, but how much do we have to increase?

Instructor: What number do you say the number approaches?

Sude: I am at the 603<sup>rd</sup> term, but...

Nuriye: 1.61. We already knew that this was the golden ratio, sir.



#### Episode 14

In this episode, the PSTs are discussing a game to be won when the same color chip is drawn in the event of drawing one chip from a bag with 3 red and 1 blue chips, respectively, from a bag with 3 blue and 1 red chips. Before running the simulation, they discuss whether this game is fair or not and then check the accuracy of their conclusion with the help of the simulation. When they run the simulation, they realized that the result they obtained in the argumentation process was wrong.

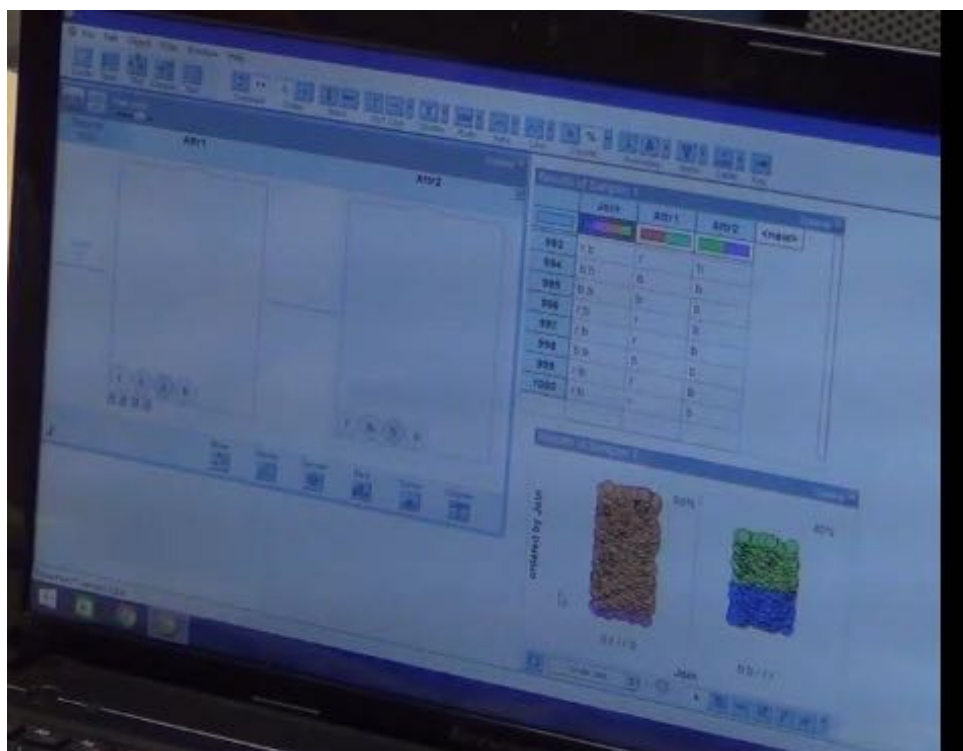


Figure 4.11: Technology as a resolver in probability game task

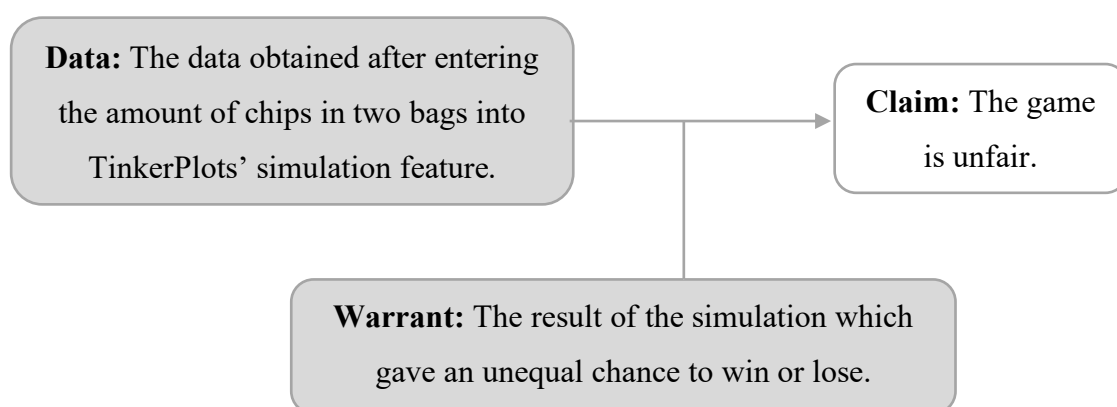
Sude: The second one is fair in an exciting way. *(She says this after calculating the theoretical probability using paper and pencil)* Let us look at the third game now. I am looking. Did the first have three reds and one blue?

Instructor: Is the game fair according to your decision?

Sude: We thought it was fair, but now we are a little skeptical, sir.

Instructor: Well, now decide by making your simulation.

Sude: *(After running the simulation)* Unfair! Sir, I already said we were in doubt. Please do not take it too seriously what we call fair.



## Episode 15

In the next episode, PSTs examine the derivative of a function at a certain point. They have the opportunity to observe the transition from a secant line to a tangent line using a file prepared with GeoGebra. The instructor asks them if there is a difference between  $h$  approaching zero from the left and right. They think it won't matter because the function they're observing is a polynomial function. However, the instructor asks them to enter another function and review their decisions accordingly. Then, when they plot this function in GeoGebra and re-examine the approach, they arrive at the proper conclusion.

Instructor: Should it always give the same result?

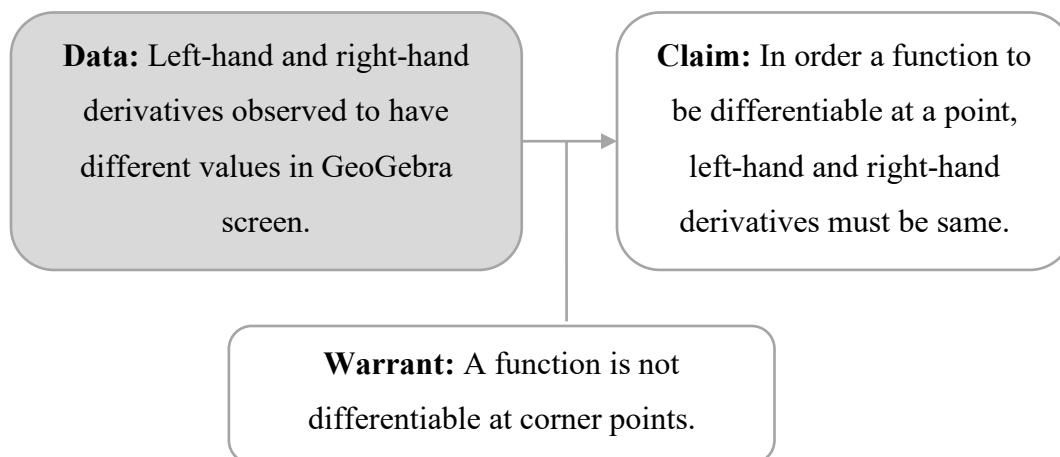
Berk: I think it should.

Arda: Maybe the graph is bad. What if the function is something angular like this. Or, what if it is not defined at that point?

Instructor: Here is what I want from you. Would you write  $x - 2$  in absolute value instead of our function? You can enter it as  $\text{abs}(x - 2)$ . Let us make the value of  $x_0$  two here first. Now move  $h$  closer to 0 from the left and right. What do you notice?

Arda: At that point, the function is not differentiable.

Berk: The slope of the tangent lines is approaching -1 and +1. The absolute value, of course, will be different. The limits are not equal. So, it does not exist.



The episodes in this section in which the supporting role of technology was emerged a total of twenty-six times, of which fourteen were “initiator” and twelve were “resolver”. Episodes that are considered adequately representative are reported. The use of technology sometimes assisted PSTs who had no idea about the problem in initiating the argumentation process by providing a starting point. Sometimes, it provided support by offering solutions in cases where the process went into a blockage and could not progress.

#### 4.1.1.1.2 Role of Instructor

##### 4.1.1.1.2.1 Instructor’s role with degrees of instrumental integration during the argumentation process

In this section, I analyzed student-centered mathematical tasks to obtain schemes of the intertwining nature of technology and mathematics in the given tasks utilizing the instrumental integration framework.

#### 4.1.1.1.2.1.1 *Instrumental initiation*

The findings in this section include the instructor's integrating technology into the task to eliminate the problems that PSTs may experience due to the lack of knowledge related to the core features of the technological tool that they will use in the argumentation process. The main aim is generally to inform PSTs to gain instrumental abilities (IA) regarding the software utilized in the process. There is little relation between instrumental ability and mathematical knowledge (MK) to maintain the argumentation.

##### Episode 16

In this episode, PSTs are expected to make inferences by drawing functions derived from a parent function on the same GeoGebra screen to examine the transformations of functions. When the teacher candidates had three function graphs drawn on the same screen, the graphs were intertwined, and in this case, it was difficult for them to make the expected inference. It was a piece of information that could make the process easier by making only the desired ones visible and comparing them by hiding the others. However, when the instructor realizes that the PSTs are having difficulties because they do not know this feature, he intervenes and shows it to them. Thus, it contributes to the initiation of the argumentation process. This intervention is only for teaching a GeoGebra feature and does not directly contribute to mathematical knowledge.

Arda:  $y$  is equal to 1 over  $x$ .  $y$  is equal to 3 over  $x$  minus 2.  $y$  is equal to minus 2 over  $x$  plus one, minus 1.

Berk: Okay. But it got all messed up.

Instructor: To hide or show any of these graphs, just click the circle near each function.

##### Episode 17

In the next episode, PSTs need to find the average rate of change of the function  $P(t) = -\frac{t^3}{6} + 15\frac{t^2}{2}$  between some time intervals starting from 10<sup>th</sup> minute. However, even though they knew that what they had to do on the function's graph was to create a secant line, they have no idea about how to do it. Here, the instructor

helps them advance the argumentation process by explaining the unknown feature of mathematical software.

Instructor: Create a new file. Minus  $x$  cubed divided by 6 plus 15 times  $x$  squared divided by 2. OK. Now let us see the zoom settings.

Berk: How do I set it up.

Instructor: Zoom out. Now, click on the  $y$ -axis and grab it while holding down the shift to make it visible. You will find the average value between 10 and 20, right? Now place a point. Let  $x$  be precisely 10. (*He does not know how to do it.*) Capital A is equal to... If you make a capital A it will indicate a point. 10 comma 0 since it will be on the  $x$ -axis. OK. The same for B.

Berk: 35?

Instructor: Yes, 35 comma 0. Now we need to define the corresponding points on the graph. Do you know how to do that?

Berk: No.

Instructor: Open brackets and write  $x(A)$  and comma  $f(x(A))$ . (*He is explaining how to write the needed expression step by step.*) So, we defined a point on the graph of the function. Do you understand? Now, do the same for the point B.

Arda: Now, we should draw the line joining these points on the graph.

Instructor: What are you looking for?

Berk: The slope of the line.

Episode 18

In the activity where the following episode takes place, the PSTs examine together the first and the second derivative tests in calculus. To accomplish this goal, they examined the graphs of a function, its first and second derivatives. They were expected to make inferences about the relevant tests by overlapping the graphs of the function itself and its first derivatives, respectively by responding to the activity's questions. However, since the critical component in this task is to compare the function and its derivatives, the instructor does not want to waste time finding the extreme points of the function. He makes such an intervention both to speed up the process and teach the relevant GeoGebra command.



Figure 4.12: Instrumental initiation in derivative tests task

Instructor: Did everyone open this file? Find where the function has local extrema by writing Extremum(g). When you write this, it shows the extremum points of the function directly (*This is a command in GeoGebra*). Do this first. Extremum, then type g in brackets.

Berk: There are two types of it here. One needs a polynomial, the other needs a function. Start  $x$ -value, end  $x$ -value.

Arda: I think the following thing. The instantaneous slope of the function.

#### **4.1.1.2.1.2 Instrumental exploration**

In this section, actions involving the instructor's contribution to the PSTs' argumentation processes will be reported, either by being personally involved in the discussion process or by the directions or questions involved in the task. Unlike the previous section, these actions aim to improve instrumental ability and contribute to the development of mathematical knowledge simultaneously.

## Episode 19

In the following episode, the instructor wants PSTs to learn the ability to draw the graphs of functions simultaneously on the same coordinate plane first by writing the directions in the activity and explaining the questions posed by the PSTs during the activity. Later, by interpreting this property, he wants to reach inferences about transformations of functions by discussing the concept of a family of graphs. Thus, the educator aims to contribute to the development of PSTs both in instrumental ability and mathematical knowledge.

Instructor: Friends, we expect you to do the following: Plot graphs of functions in groups a, b, c, and d separately on the same coordinate plane. Then discuss with your friends to make inferences on these graphs.

Berk: We draw the graphs of three functions in the group a (*He is waiting*) ...

Instructor: Yes, you are drawing all graphs in a group on the same coordinate plane.

Berk: After that, we comment on them.

Instructor: Yes.

## Episode 20

In this episode, PSTs are trying to solve an optimization question using a spreadsheet. They are asked to cut a square from the corners of cardboard to form an open-top box with the remaining piece. The aim is to find the maximum value of the volume of this box. They determine the length of the piece to be cut as an independent variable and write a formula for the box's volume. Then they copy this formula in other cells. However, since they determine the increase of the value one by one, they only examine the integer values. On the other hand, the instructor contributes to the development of the PSTs by emphasizing how inserting graphs in a spreadsheet software can be used in such situations and that the increment values in the formulas do not have to be only integers.

Instructor: What is the maximum value?

As a whole group: 88!



Instructor: Did everyone find the same value? Did you insert the graph of this data?

Berk: (After silence for a while.) Not 88 guys!

Nisa: What is it then?

Instructor: Do the values you enter have to be integers, friends?

Burcu: Of course not! Okay, we entered only integer values. We can enter non-integer values as well.

Episode 21

In this activity, the PSTs are trying to calculate the terms of the Fibonacci Sequence by utilizing the spreadsheet software. After the terms of the sequence were written, they were expected to respond to the task's questions. However, the PSTs in the group had difficulty in making the spreadsheet software to calculate the terms of the sequence. The instructor steps in here, intervening to improve PSTs' instrumental abilities and related mathematical knowledge.

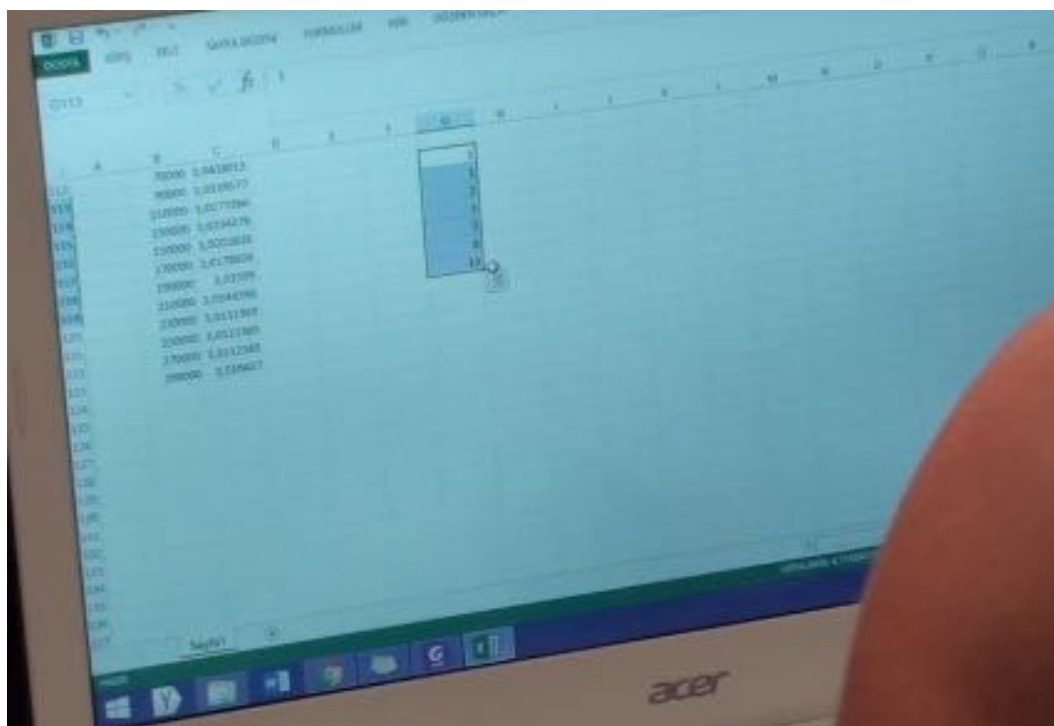


Figure 4.13: Instrumental exploration in sequences task

Burcu: We are going to call the first one something... We will write 1 to the first and the second. We will write 2 to the third. Then we will drag them down.

Eda: Exactly, okay.

Nisa: 1, 1, 2. Now it will come over here. Let me enter a few, for example. I will enter until 13. Let us see if it will detect the pattern.

Busra: Doesn't it perceive?

Nisa: I hope it will. I need to get it from here but (Selects all the cells with the value. After copying the formula by dragging it below). It did not!

Instructor: Did anyone find it? Have you written the Fibonacci Sequence to Excel?

Nisa: We could not write, teacher.

Instructor: One term will be the sum of the two previous ones, right? So, you will enter two initial values, and then write a formula to find the sum of the previous ones as the next one.

Nisa: I see. Okay! Let us say this is equal, for example, that one plus this one.

Busra: Will we drag to copy later?

Nisa: Yeah. When I drag this ... (She tries to obtain the terms of the Fibonacci Sequence by entering the terms in the cells, selecting them, and then dragging them down.)

Eda: But you have to complete the remaining terms as well.

Nisa: I will fill in (*showing cells that are blank*). Oh wait! It did not work.

Busra: Did not?

Nisa: That is because we have to take the terms themselves. Wait! Then I will write it here. (Now entering the formula in the right place, just below the first two cells by writing the sum of two preceding cells)

Instructor: Isn't it enough to just enter the first two?

Nisa: Yes teacher. I am going to drag this down now. (She is copying the formula he wrote down by dragging it down.)

Episode 22

In the next episode, the instructor's involvement in the process is to eliminate the flaws in the PSTs' instrumental abilities in the spreadsheet software. In doing so, he aims to develop both instrumental abilities by associating them with mathematical knowledge.

Instructor: Now it is time to answer the question Berk just asked. “What happens to the ratio of the successive terms if you change the initial terms as 2 and 1 instead of 0 and 1. So the initial terms should not be 0 and 1. Let us start with 2 and 1, for example? What will be the next term?”

Eda: Three.

Instructor: Then what you have to do is not change the formula. Isn't it enough if you just change these initial values?

Eda: Write as he said.

Instructor: Copy the cells that you calculated previously and paste them into the next column. Then change the initial values in the column. Check it out later and see what happens.

Burcu: Copy and paste here. Then let us change things. Let us observe them all.

#### ***4.1.1.1.2.1.3 Instrumental reinforcement***

In this section, the episodes where the instructor posed questions either verbally or in the prepared task so that PSTs were expected to utilize and develop a specific mathematical knowledge by investigating features of the mathematical software will be presented. During these instances, PSTs were faced with difficulties in instrumental knowledge while responding to the questions required in the posed task. The instructor aimed mainly to improve mathematical knowledge.

#### **Episode 23**

In the episode below, the PSTs examine the effects of the parameters they set on the transformation of a function's graph. When asked to share their conclusions, there are questions that the instructor asks the PSTs in the group in order for them to reach the correct conjecture. The questions address the difficulties that PSTs have encountered using the slider tool in GeoGebra to improve their relevant mathematical knowledge.

Instructor: What did you guys do?

Nisa: We did it, sir.

Instructor: Okay. Now please stop the animations first. Now first of all what effect does  $a$  have on the graph? In order to see that, please set the value of  $a$  to 1. The value of  $a$  is 1 right now. What happens if you make it 2?

Nisa: Stretch.

Instructor: So?

Burcu: All values are doubled.

Instructor: Okay. Would you make the value of  $a$  2? Have the values doubled? (*By examining the change on the screen*) It looks OK. What kind of transformation do you foresee for  $b$ ? Again, set the value of  $b$  to 1. Now if you change the value of  $b$  to 3 or 2, what will happen to the graph?

Eda: It is getting narrower.

Nisa: Its frequency is changing.

Instructor: Good. For example, make it 3. Okay, we saw the shrink effect. Now, what is your prediction for  $c$ ? For example, what do you expect when you increase it by 1 unit?

Nisa: One unit of horizontal shift.

Instructor: Okay. Let us increase it by 1 unit. How much was it translated? Now it has to shift from  $(0,0)$  to  $(1,0)$ , right this point? (*Showing the point passing through the origin on the screen*) Where did it move? (*They observed on the GeoGebra screen what they predicted was false.*)

Nisa: Could it be because our frequency is 3. Could that be the reason?

Instructor: Maybe, but I want you to show the effects of the parameters on the graph independently. So, you should be able to say something like that: “when we change this parameter this amount, the graph changes by this amount independently from the other parameters. Then, what about  $d$ ?

Eda: Up and down.

Instructor: Okay. Just think about how you can solve the issue related to horizontal transformation.

Episode 24

Nuriye: Teacher, can I ask you something?

Instructor: Of course.

Nuriye: We just changed it from  $x^2$  to  $-x^2$  and we called it reflection. Should we consider this as horizontal reflection?

Tuba: I think it should be vertical.

Nuriye: But I think it is horizontal. When we usually reflect any object to a horizontal line, we call it horizontal reflection.

Tuba: But I think it as follows: What are the initial and final positions of the reflected object? For example, from here to here (*it shows the movement from the top of the x-axis to the bottom*)

Sude: Then it becomes vertical.

Nuriye: I also say that it will be vertical reflection if we do it according to a vertical line.

Instructor: In general, the direction of movement of the object being reflected is considered as a basis in the definition. In other words, when the graph is reflected along the x-axis, it moves vertically. Therefore, it can be called a vertical reflection.

Nuriye: Sir, when the value of  $a$  increases, there is a shrinking effect on the graph.

Instructor: What if the function is different? Why are you stuck with the quadratic function?

Nuriye: Okay, it's the same with the x cube. Or the tangent function, for example. Where do we push it? Aren't we push from the side? Where do we shrink? It is from the x-axis.

Instructor: Well, consider the sine function.

Nuriye: OK, write  $\sin(x)$ .

Instructor: How do you know if a stretch is horizontal or vertical?

Tuba: I think it logically depends on where we push it. Where do I shrink? It tapers along the x-axis.

Instructor: What changes in the graph as it shrinks along the x-axis?

Nuriye: Values are getting closer to each other.

Tuba: Range is changing. Range or domain?

Nuriye: But those y values are shrinking.

Instructor: There are two essential concepts for a function. We call them variables. One is independent, and the other one is dependent. In other words, input and output. How do these are affected by your changes? For example, what changes in horizontal changes?

Nuriye: x-values.

Instructor: Input values are changing. Then you take a reference point for it and then think again. Is this horizontal or vertical? For example, what happens when you stretch the graph upwards here? Is it horizontal or vertical? Where is your reference?

Tuba: It's horizontal.

Nuriye: I think we are changing something in x. y takes values up to infinity.

Tugce: Exactly! We're changing independent values, after all.

Nuriye: Dependent values also change depending on it.

#### Episode 25

In this episode, the teacher asks PSTs to learn to interpret graphics with the questions in the activity. However, they do not use the previous comments in solving equations in the last step; instead, they prefer the solution they were accustomed to from high school. Therefore, he asks them to use their previous comments by asking a question to realize what they have learned.

Instructor: Did you answer this part? "Discuss the intervals where  $f(x)$  has greater values than  $g(x)$ ?" Let each group decide on this, please.

Berk: What do we want? What do we want to be bigger than what?

Burcu:  $f(x)$  than  $g(x)$ .

Berk: We are looking for places where  $f(x)$  is greater than  $g(x)$ .

Nisa: Pull it down, Berk. (To make the chart appear fully on the screen by clicking and dragging)

Instructor: Well! What if we ask: where does  $g(x)$  take larger values? How do you respond to that?

Berk: Don't we need to subtract the result just we had before from the set of real numbers?

Instructor: You said between -2 and 3. Does everyone agree? Let's see if it is? In the range of -2 to 3, does function f take larger values than function g?

Some PSTs: Yes!

Instructor: How did you figure it out?

Berk: From the chart!

Burcu: It stays above! (Can't be heard in class as she whispers)

Berk: From the y values it got.

Some PSTs: We looked at the values of the intersection points.

Berk: It depends on the intersection points, but this is not something that will only related to the intersection points. Don't we need to look at the y values?

Nuriye: When we look at the green graph, it is above.

Instructor: Yes, this is the answer I was waiting for. The green graph is above the red. What about g? In what interval does g take larger values?

Berk: What if we subtract this interval from real numbers?

Instructor: Now! What would you do if I ask "Solve the following equations for x."

(*the equation was  $x + 4 = x^2 - 2x$* )

Some PSTs: We'll collect all of them on one side of the equation. It would be  $x^2 - 3x - 4 = 0$ .

Berk: - 4 and 1 are the roots.

Instructor: So why do you think I might have asked this after the previous observations?

Nuriye: We look at it according to their graphs.

Berk: Two different graphs. Hmm... Intersection points?

Instructor: Draw it.

Berk: That's right! The points where the two graphs intersect will give the roots. Have you seen that already? I said -1 and 4. (*misremembers the previous response*)

Episode 26

The following episode examines a situation that PSTs used to do before while solving questions but did not question why. For this, associations are made with the

graphs, and they are asked to be questioned. The instructor aims to construct meaningful mathematical knowledge by using the features of the software.

Instructor: What is the root of function?

Berk: Any function, right?

Instructor: Yes.

Sude: The points where it intersects the x-axis.

Brooke: No! Values that make the function zero.

Instructor: Values that make the function zero?

Berk: OK, there are points where it intersects the x-axis.

Instructor: But why? (*after a while of silence*) How else can we express this? Roots of a function or? What do we call it?

Tugce: Solutions?

Instructor: Zeros of a function. Is it meaningful to have a solution to a function? What do we call something that has a solution?

Berk: Equation.

Instructor: There has to be an equation for it to be a solution. Now, what is the root of a function? Isn't that we're looking for  $x$  values that make this zero? (*he writes  $f(x)=0$  on the board and put ? on  $x$* ) Well, think about the last interpretations we made. (*Writing  $f(x)=g(x)$  on the board*) So what if we were to interpret the root of function utilizing these interpretations? Discuss this in your group. What to do graphically to find the so-called root? And why? We're talking about any function  $f$ . For example, when we wrote  $f(x)=g(x)$ , the solution set of the equation was the abscissa of the intersection points of the two graphs. What do you do to find the root of an equation, then?

Berk: The points where it crosses the x-axis.

Burcu: I make any of them equal to zero.

Sude:  $y = 0$ , I would take one of them as 0.

Instructor: The intersections with  $y = 0$ , so? The points where it intersects the x-axis. Therefore, we consider it correct to find the points where the graph crosses the x-axis to find the root of any equation. Although there is no such definition, when we



combine what we just talked about, we can think of it as the points where it crosses the  $x$ -axis.

Episode 27

In the following episode, PSTs are expected to observe how GeoGebra can be utilized when creating a sign chart for the linear inequality to be solved. In this direction, the teacher asks them to examine the graph why the sign changes before and after the root while the sign chart is created.

Instructor: OK. What is the solution set of the inequality  $2x + 4 > 0$ ?

Berk: Sir, from -2 to infinity.

Instructor: -2 to infinity. Let everyone see by drawing the graph, and please wait a bit. Remember your knowledge from high school but think of it as a teacher now. While solving the inequality, we created a sign chart considering the leading coefficient, the sign before the root, the sign after root, etc. Do you remember?

Some PSTs: Yes.

Instructor: Considering the algebraic expression represented by a line, how do you interpret its sign? For what values it takes positive and negative values?

Berk: It's derivative!

Nuriye: After finding the root, we determine the sign by giving values less than and greater than the root.

Instructor: You're talking about the sign chart. Do you say that we can make the table by plugging numbers?

Nuriye: For example, suppose that the root is -2. What happens when we plug in -3? What happens when we plug in -1? This is the way how we decide the sign by checking them.

Instructor: Good. We have a name for that. What's that? Does anyone know? We call it test value.

Berk: Sir, isn't the derivative possible?

Instructor: We're talking about the sign of the function. What does the derivative give us an idea about?

Berk: OK, when it's greater or less than zero. Umm... OK. Increasing and decreasing. True! And turning points.

Instructor: We'll talk about derivatives later. But think about it here. If I were to ask you: What would be the answer if we thought of this inequality as an equation?

Some PSTs: -2.

Instructor: Well, now you mean that values that are greater than -2. Don't you? So that's it (*showing it on the GeoGebra screen*). On what basis are you claiming this?

Nuriye: The graph is above zero.

Instructor: The graph is above the  $x$ -axis. So, can we say that it takes negative values here and positive here?

Episode 28

In the following episode, PSTs observed that the two ratios of the successive terms of the Fibonacci Sequence are approaching the golden ratio. The instructor aims to make them realize a mathematical fact using the feature of the spreadsheet software by asking the planned question.

Instructor: Do you see that the value approaches that value as you copy the formula downwards? We even named this one  $\phi$  and the other as  $\varphi$ . Do you remember? (*In the previous week of geometry*) Now let me ask you that Berk just asked me: What happens to the ratio of the successive terms if you change the initial terms? 2 and 1 instead of 0 and 1? So, we had started with 0 and 1 for Fibonacci. Now, let's start with 2 and 1 instead of 0 and 1. What happens with the next term?

Berk: 3.

Instructor: Then, you will change the initial values instead of changing the entire formula. Don't you? Copy and paste there with the necessary changes.

Arda: OK. Try here! There are two 1's.

Berk: What should I do, then? Let me write 2 and 1. OK, everything has changed. Teacher! The golden ratio again!

Instructor: Again? Didn't change?

Berk: As I said before. So Fibonacci isn't so important. The point here is to add the previous two terms.

### Episode 29

In this episode, PSTs need to find the length of the piece to be cut from the edges of the cardboard. By doing so, they need a box with maximum volume. For this aim, they enter values in Excel and write a formula to increase the values one by one. However, they think they have to write only integers in the formula for the increase. As a result, they make an estimation that is far from the actual value. The teacher makes them realize this by asking whether the increase they will determine has to be one at a time.

Nuriye: Teacher, what will we do after we find the maximum value? Shall we draw the graph?

Instructor: First of all, may I know what did you find as the maximum value?

Some PSTs: 88

Nuriye: 88.

Instructor: Everyone got the same result?

Some PSTs: Yes!

Instructor: Then, draw the graph, please.

Sude: Which cells should we select to draw the graph?

Instructor: I think you have increased the values for cells representing the length of the piece you cut 1 by 1. However, does it have to grow in such a way?

Nuriye: Ha! We didn't think about it.

Sude: Then let's not consider it one by one.

Nuriye: Take it 1.1, for example.

Sude: Let's not cut 1 unit as the initial value, then.

Nuriye: Yes. It could be 0.1, for example. OK. Let's drag it down now.

Sude: 0.2. I drag quite a bit. How much should I do? It is 2.2 now.

Nuriye: You can drag up to 3.

Berk: *(he interferes from the other group)* You can drag it up to 4.

Nuriye: Not 4.

Berk: It shouldn't be 4, exactly!

Nuriye: The others are the same. *(Sude is only copying by dragging the first column)*  
You can even drag the multiplication columns.

Instructor: What did you find?

Sudes: 90,712

### Episode 30

The following episode is from an activity designed to help PSTs arrive at a definition of the derivative they were initially unable to answer. First, they find the average rate of change of the given function at different intervals using GeoGebra. However, when they do not realize that this will lead them to the definition of the derivative, although the range for the average rate of change is getting narrower, the instructor's question helps them realize what to do.

Nuriye: What should come out when we calculate it? Shall we calculate?

Sude: Then do it on the calculator. Subtract the two and divide by 25.

Nuriye: It is the same, yes.

Sude: Good. We will proceed with this method.

Instructor: Which method is that? Let me see.

Sude: We drew it and looked at its slope.

Instructor: How did you find its slope?

Nuriye: There is a slope tool in the menu, Sir.

Instructor: OK, fine. Let me show you another way. Right click on the line in the algebra view. First, choose the way the equation displays. You can see it directly from there.

Sude: Yes! It is effortless to check the slope.

Nuriye: We can find the others in the same way.

Sude: Yes. Same way.

Nuriye: But is it the same? Shall we check them?

Sude: It is not the same thing. For example, it went here and came back, for example. It has come to the same place, but there is time passed. So, between 10 and 11, it comes out very different. For example, let us do another pair that is close to each other. Like 10 to 15.

Nuriye: I think we can drag it.

Sude: Look, for example, I have chosen a pair of points that are very close to each other. There has been a considerable increase in the slope right now.

Instructor: Now on to the next question. What is the speed shown on the speed dial of the car at the 10th minute?

Nuriye: We will draw a tangent from the point at the 10th minute.

Sude: Is it OK if I delete this?

Nuriye: Yes, do it! If not, we can take it back. Now there is a tangent creation tool. Let us select it. Where was it? (*looking at the menu*) Tangent! Will it be created when I click on the curve and the point?

Sude: It does. OK.

Tuba: We got the point in the 10th minute, right?

Nuriye: Yes.

Sude: Its slope is 1.02.

#### **4.1.1.1.2.1.4 Instrumental symbiosis**

This section reports the episodes where the instructor aims to improve PSTs' mathematical and instrumental knowledge with minor instrumental difficulties. Unlike the levels of instrumental genesis, the relation between instrumental and mathematical knowledge is maximal. The instructor intended to improve PSTs' mathematical knowledge by building new information on existing mathematical knowledge and instrumental abilities within the utilized mathematical software.

Episode 31

In this episode, PSTs are examining the effects of the parameters on the transformations of function graphs. For this purpose, they were asked to explore the changes in another function where they will write using parameters. They used the Slider tool in GeoGebra for the parameters. The PSTs in the group were sure about the correctness of the function they wrote. However, they realize that one of their assumptions is not valid. Here the instructor enables PSTs to recognize that their assumptions are false using the features of GeoGebra.

Instructor: What is your expectation when you double  $b$ ?

Berk: Horizontal stretch.

Instructor: Horizontal stretch? Let us see.

Berk: Sorry, shrink.

Instructor: So, you have shrunk the graph horizontally twice. What are you expecting when you change  $c$ ? What happens when you change the slider's value one unit?

Berk: We expect sliding one unit to the left.

Instructor: Well, it was so? Check it out. Bring to zero first. Look at the points where the graph crosses the  $x$ -axis. For example, it is currently passing through the origin. Right? Now increase it by one unit. It did not produce a translation of one unit. But, why?

Berk: Because  $a$  and  $b$  are two. When  $a$  and  $b$  are both 1, it will slide  $a$  units.  
(*Laughter*)

Instructor: OK. You understand the problem. Let us talk about  $d$ . Here, you should see the effects of parameters independently. They should not depend on others. When you scroll one unit, it produces a translation of half unit. However, you said it is true when  $b$  is one. But when you say it is true when  $b$  is 1, it becomes no longer a parameter.

Berk: It is not enough to have  $b$  equals one. Both  $a$  and  $b$  need to be one.

Instructor: Are you sure? Think about it.

Episode 32

In the following episode, the instructor asks PSTs, who had observed the effects of parameters on function graphs, to think about the change of the fundamental period of the function  $f(x) = a \cdot \cos^n(bx + c) + d$ . They can express the effects of changing the parameters on the period of the function based on the previous observations. However, although they observe the change on the GeoGebra screen, they cannot describe the impact of the change in  $n$  on the period. In this case, the teacher steps in and tries to trigger them to think a little more deeply with questions.

Instructor: The period of this function is  $\frac{2\pi}{b}$  while the other one is  $\frac{\pi}{b}$ . Why?

Berk: It is because... I know that, but how can I explain it right now?

Instructor: Consider it as follows. Do the roots change as you change the value of  $n$ ?

Berk: The roots are changing ... Wait! No! They do not!

Instructor: The roots are not changing. What is changing?

Berk: The shape of the part between the roots is changing.

Instructor: What change are you observing when you change  $n$  from 1 to 2? (*After a silence*) Focus on the negative parts.

Berk: It transforms the negative parts into the positives.

Instructor: And we know that the parts of the graph under and above the  $x$ -axis are symmetric.

Berk: Right!

Instructor: So, what happens when it becomes positive?

Berk: It is carried above the  $x$ -axis.

Instructor: So what happens in this case?

Berk: The new period becomes half the previous one.

Episode 33

In this episode, the instructor, together with the PSTs, first makes sense of the absolute value of a number based on the circle's definition. Then, he asks PSTs to discuss how to make sense of different absolute value equations using transformations of functions. First, the instructor wants PSTs to create parameters to write an equation of a circle and bring its center to the origin by changing the parameters' values. Then, he asks the PSTs what transformation is needed to keep the center of the circle on the  $x$ -axis. Then he expects them to think about how to write an absolute value equation describing the points where the circle crosses the  $x$ -axis. Thus, the instructor aims to improve their mathematical knowledge of absolute value equations by associating the circle's points intersecting the  $x$ -axis on the GeoGebra screen and the parameters.

Instructor: Now, I want you to move the circle on the  $x$ -axis by changing the slider values. As you may remember from the transformation of functions, the parameters  $a$  and  $b$  will move the circle horizontally and vertically. What should we change here?

Some PSTs: The value of  $a$ !

Instructor: So there will be a horizontal translation. Well, let's play with some values. Now, when you slide the circle on the  $x$ -axis, how can you represent the  $x$ -intercepts by an absolute value equation? Think about it.

Sude: It's like we added  $a$  to the distances to zero.



Nuriye: Then  $x$  minus  $a$  became  $r$  and negative  $r$ . Then  $x$  became  $r + a$ .

Tuba: There is a shift.

Nuriye: Teacher! We think that it is translating along a vector from 0 to  $a$ .

Instructor: Along vector  $(a,0)$ ! So, it was initially  $|x| = r$ . Now?

Sude:  $x$  becomes negative  $a$ .

Instructor: *(By showing the circle's location on the GeoGebra screen and the points where it crosses the  $x$ -axis)* Could you then tell me an absolute value equation with these numbers as the solutions?

Berk: The absolute value of  $x$  minus 2 is equal to 3.

Instructor: OK! So what does that mean? How would you interpret this?

Berk: A distance of 3 units from 2 on the number line.

Instructor: Well, let's put it more mathematically by editing it a bit more.

Berk: Points with a distance of  $r$  units from point  $a$  on the number line.

#### Episode 34

Before the following episode, PSTs using the spreadsheet first observed that the limit of the ratio of consecutive terms of the Fibonacci Sequence approached the golden ratio. Then they repeated the same procedure using the Lucas Sequence instead of Fibonacci. The teacher asks the PSTs teachers who made the aforementioned observations whether the values entered at the beginning are essential. They ask them to reach a hypothesis by using technology and think about the mathematical reason for this.

Instructor: Well, now let's look at this: Why the ratio of the successive terms approaches the same number regardless of the initial terms? We have just tried Lucas. Now enter two random initial values. Do we always get the same result?

Berk: I think so. I will try 3 and 5. *(He enters the values and checks the result)*. Sir, we are getting the same ratio again.

Instructor: So, if you define a recursive relation this way, regardless of the initial values, we see that this ratio is the golden ratio of the limit. OK, but why?

In this section, where the roles of the instructor are reported according to various degrees of instrumental integration, the most striking episodes were selected among forty-one episodes in total. In these episodes, mostly consisting of advanced instrumental integration (instrumental reinforcement - 22 and instrumental symbiosis - 11), the role of the instructor in the collective argumentation process was reported, taking into account the interventions of the instructor.

#### **4.1.1.1.2.2 Instructor's role with modes of facilitating the argumentation**

##### ***4.1.1.1.2.2.1 The instructor as an initiator***

The findings to be reported in this title include the episodes when PSTs had difficulty initiating the argumentation processes. Even though the activities included what they should do, there were moments when they could not decide how to start the argumentation process. At these moments, the instructor pushes them into the argumentation process by taking the first step.

Episode 35

In this episode, the teacher asks the PSTs to think about the possible transformations on graphs of functions. When PSTS faced this question, they stuck with the previous knowledge about the transformation of objects and accordingly responded as symmetry, reflection, rotation, and translation. They cannot start mathematically correct argumentation because they think that the transformations of the objects, they learned in other courses are also valid for functions. Realizing this, the instructor steps into the discussion and initiates argumentation by asking the question.

Instructor: Let us assume that a function's graph is given. What kind of transformations can there be on the graph of this function? For example, let me tell you one. You just said shifting left or right. What else could it be like this? Get together and discuss briefly.

Berk: Isn't it symmetry, reflection, rotation, and translation? It was too short for us, Sir.

Instructor: Have you made your decision?

Berk: Yes. Reflection, translation, rotation. There was something else.

Instructor: How about functions? Think about it.

Berk: Wouldn't it be wrong, Sir, on functions too?

Instructor: We're not talking about types of transformations in geometry, guys. How can we change the graph of the function by changing the parameters? What transformations can we make?

#### Episode 36

In the following episode, PSTs write a parametric function in GeoGebra to observe the effects of the parameters on the function graph. PSTs activated the animation feature to observe the results of changes in the parameters. However, they cannot reach a detailed and accurate conclusion about the transformations. The change they could notice from the animation was limited to a general result like "the graph expands vertically as  $a$  increases". At this stage, the teacher steps in and asks the following question to initiate the argumentation process necessary for them to reach a more specific conclusion.

Instructor: What did you do? Did you write?

Berk: Sir, we are all right. We made the model and revived it.

Instructor: Well, let us see then. The initial value of  $a$  is 1. Now, make it 2, for example. What do you expect to happen on the graph when you double it? Based on your decision, what kind of change do you expect if you set the value to 2?

After this question, PSTs focused on the numeric values in the argumentation process and discussed how a one-unit change in each parameter affected the graph. Thus, they went through a mathematically correct argumentation process by responding to the question that guided them through the process.

#### Episode 37

In this episode, PSTs discussed why the formula they used to find the periods of trigonometric functions during their high school was written in that way. Since they

had studied the transformations of functions before, they understood why  $2\pi$  and  $\pi$  should be divided by  $b$  in the formula. But they failed to realize why it was different for even and odd powers of the function. To initiate this discussion, the teacher starts the argumentation process by asking the following question.

Instructor: OK! But what about the power in the formula? Why it depends on the even and odd values of  $n$ ? What happens to the graph when you change the values of  $n$  by adjusting the slider?

#### Episode 38

In this section, PSTs interpreted different function families by drawing them on the same coordinate plane to introduce function transformations. Using this experience, they will be expected to make a generalization about the effects of coefficients of a quadratic function on its graph. To this end, the instructor asks the following question to initiate the argumentation process:

Instructor: How is the graph of the function affected when  $a$ ,  $r$ , and  $k$  change? Please discuss this without plotting the graph in GeoGebra.

On this question, PSTs start the argumentation process using the information from their high school years and the static graphs they have just observed.

#### Episode 39

In this episode, PSTs are expected to interpret the graph of functions to provide solutions to inequalities. The goal of the task was to examine the positions of the two graphs relative to one another, making them realize that they could find solutions to inequality. Therefore, it was to observe which graph was positioned above the other. However, when they fail to start, the teacher wants to be reminded of the intersection point of the lines first, as they did previously in equations. But they are conditioned to plot the inequality through the software and respond accordingly.

Instructor: We have two lines, right? Do they intersect?

Berk: Yes, they do.

Instructor: Sure?

Berk: Yes, because they are both linear.

Instructor: Do each pair of lines intersect?

Berk: Yes, they do. No, it doesn't! They are not parallel, so they intersect.

Nisa: They intersect because they are not parallel.

Berk: Their slopes are different, so they are not parallel. And they are not coincident. So, they intersect.

#### Episode 40

In the following episode, PSTs were aimed to make sense of absolute valuable equations based on the distance concept by referencing the definition of the circle. They were expected to write an absolute value equation with solutions as the x-intercepts of the circle whose center is on the x-axis. However, while they knew the circle definition as a set of points equidistant from a fixed point, they could not recognize the situation. In the meantime, the teacher opens the discussion with a question.

Instructor: OK. Take an integer value for  $r$  now. What can you say about the circle and its intersecting points?

Berk: The x-intercepts are negative 7 and...

Nisa: The points which are five units away from negative 2.

Burcu: What are the x-intercepts? Negative seven and?

Nisa: Three

Instructor: We have concluded that the x-intercepts of the circle centered at origin are the solution to the equation  $|x| = r$ , right? What happens to the equation when you translate the circle along the x-axis?

This question triggered the discussion and argumentation process, and they were engaged in a fruitful discussion to obtain a correct result.

#### Episode 41

In the following episode, PSTs used a spreadsheet to observe the limit of successive terms of the Fibonacci and Lucas sequences approaching the same value. However, they did not realize to test whether this outcome only applies to these specific sequences. After writing the formulas, they could change the initial values and

observe the result. Since they cannot take this step, the instructor starts this discussion with the question he asks here.

Instructor: Well, now let us look at this: Why the ratio of the successive terms approaches the same number regardless of the initial terms? We just tried Lucas. Now enter two random initial values. Do we always get the same result?

Berk: I think so. I will try 3 and 5. (*He enters and checks these values*). Sir, it is the same value again.

Instructor: So, if you define a recursive relation in this way, regardless of the initial values, this limit value approaches the golden ratio. Okay, but why?

After this question, PST try to find an answer by following an algebraic way about why this might happen.

Episode 42

In the following episode, PST observed the change in descriptive statistics values when a number was added to or subtracted from each value given in the dm unit in a data set. Even though they generalize the results calculated in cm units by assigning sliders, they claim that nothing will change when they are asked to compare with the situation in dm. Then, the teacher asks his question and allows them to experience an argumentation process within the group.

Instructor: Now the next question. What happens if you do these calculations in cm? We calculated it as dm, right? Do descriptive statistics values change when we convert these data to cm?

Sude: Nothing will change!

Instructor: What will you do when you convert these values to cm?

Sude: We will multiply all the data by ten.

Instructor: Well, don't you think multiplication changes anything? Can you validate your claim by opening a new column for the required operation?

Berk: (After calculating the required values for a new column) It's changed. We multiply them all by ten.

Instructor: When every value is multiplied by ten, is the variance multiplied by ten?

Sude: Yes, it is also multiplied by ten.

Berk: No! The variance is multiplied by one hundred (*based on the evidence obtained from a new column*).

Sude: Yes, variance is multiplied by one hundred, sir.

Episode 43

In this section, PST examines a function's rate of change on an activity prepared in GeoGebra. For varying values of the parameter  $h$ , they are expected to notice the left-hand and right-hand derivatives while examining the slope of the secant line in the case of  $x_0 + h$  approaching the  $x_0$ . Seeing that they could not realize these concepts they were supposed to know, the teacher initiates the discussion by asking the following question.

Instructor: What I want to ask is actually this. Specify a point and let us name it as  $x_0$ . Is there any difference between the tangent line that emerges when we  $h$  approaches 0 from right, I mean with positive values, and the tangent line that occurs when approaching from the left?

Berk: No, sir! Doesn't it already give the same value? After all, won't it give the same value when  $h$  is close to 0?

Instructor: Should it always give?

Berk: I think it should.

Instructor: Isn't there any case where it doesn't? Is there?

Arda: What if the graph of the function is bad. What if the graph has a corner something like this. Or, what if it's not defined at that point?

Episode 44

In the next episode, PSTs examine a ready-made GeoGebra file. They are trying to visually ensure that the line segments on the graph are tangent by vertically replacing the orange dots (see Figure 4.14). After completing all of them, they need to make the derivative of the primary function visible and see that these points are located on the derivative function. Then they are expected to search for answers to why these points are located on the derivative function. However, when the instructor realizes

that they cannot enter this discussion after examining the file, he asks the following question.

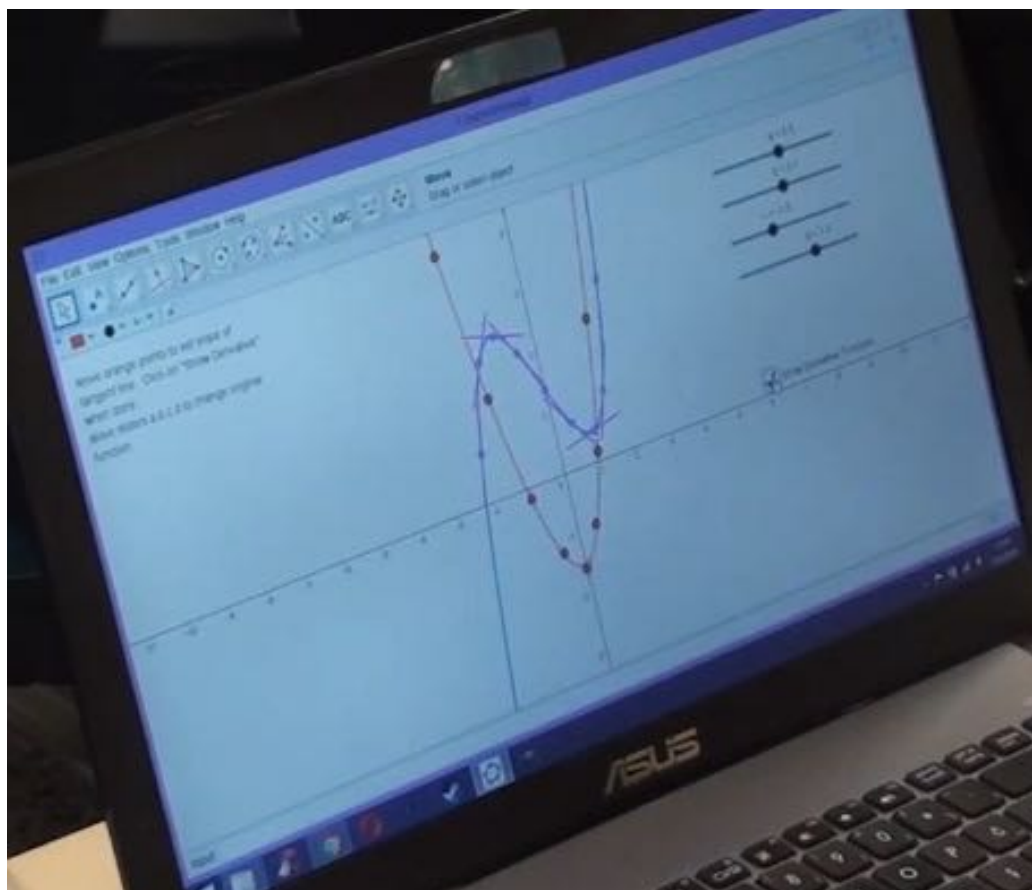


Figure 4.14: Instructor as an initiator in derivative activity

Instructor: It seems that all groups completed. Did the points lie on the derivative function? Everyone has found the answer to the question. Yes! My next question is: Why do the orange points always lie on the derivative function? So, if you had tried different functions, those orange dots would always be on the derivative function. Why is that? What provides this?

Berk: Tangents, right?

Instructor: How should those orange dots be defined so that they are always on the derivative function?

Arda: I have no idea. You?



Berk: I have no idea either. (*They start examining the properties of objects by activating the algebra window from the view in the GeoGebra file.*)

#### Episode 45

In this episode, PST examine a file that can superimpose the graphics of a function and its derivatives in a previously prepared GeoGebra file. They are expected to investigate the graphs and identify critical points. However, although they have learned derivative tests in their previous courses, they cannot notice the relationships. The teacher leads them with the following question to start the discussion.

Instructor: Guys, here's what I want you to focus your attention on considering the abscissas of the extremum points of the  $g$  function on the graph. What can you say about  $g'$ ?

Some PSTs: The points where the function crosses the  $x$ -axis.

Instructor: So, what's that mean for the initial function? Which function's  $x$ -intercepts are you talking about, and what do these points mean?

Some PSTs: The function has extreme values at the points where the derivative crosses the axis.

#### **4.1.1.1.2.2.2**     *The instructor as a resolver*

In this section, the instructor's role will be reported to solve the minor problems that arise so that the PST can progress in the argumentation process. However, these problems are interventions that have gone to a certain point in the process but can be considered a slight push to move forward. In other words, they are not significant enough to change the course of the argumentation process.

#### Episode 46

In this episode, PSTs examine the transformations of functions and the effect of the changes in the parameters they determined on a function's graph. Although they looked at the impact of other parameters in the GeoGebra environment and came to a correct conclusion, they have a slight problem with the parameter that creates the horizontal translation. However, they overlook this issue. For them to realize this, the instructor pushes them by asking questions.

Instructor: All right. What do you expect to happen when you double " $b$ "?

Berk: Horizontal stretch.

Instructor: OK! Let's see!

Berk: Sorry, horizontal shrink.

Instructor: Shrink! So, when you make it 2, you have shrunk the graph twice. What do you expect when  $c$  changes? For example, what will happen when you increase it by one unit?

Berk: We expect a one-unit shift to the left.

Instructor: So, it happened. Check it out. Set it to zero first. For example, look at the points where the graph intersects the x-axis. For example, it is passing through the origin right now. Now let's increase it by 1 unit. Where will it cross the x-axis? Not a one-unit shift. Why is that?

Berk: Because  $a$  and  $b$  are two right now. When  $a$  and  $b$  are both one, it will shift one unit. (*Laughter*)

Episode 47

In this episode, PST investigate creating the sign chart by examining the multiplicity of the roots of a polynomial function. They assigned a parameter for each zero, the degree of the factors, and the leading coefficient of the polynomial function. However, there was a shortcoming in the conjecture they created because they ignored the leading coefficient of the polynomial while discussing it. The instructor makes them realize this with the following intervention.

Nuriye: Shall I say something? It's always positive when they're both even.

Sude: Yes. Because they're all squares anyway. It cannot come out negative. So, it will be positive.

Instructor: What's positive?

Sude: If  $m$  and  $n$  are even numbers.

Nuriye: So, if their sum is even

Sude: No, if both are even numbers.

Instructor: Well, did you change  $k$ ?

Nuriye: We haven't changed it yet, sir.

Instructor: Can you change it? (*She increases it with positive values*) Make it negative, for example. Has anything changed in your previous comments?

On this question, they try to reach a generalization by taking into account the negative values of  $k$ .

Episode 48

In the next episode, PST try to calculate the maximum volume of a box that can be made with cardboard utilizing a spreadsheet. Although they enter the values in the cells and write the formulas correctly, they have difficulty reaching the desired result. First, they get inappropriate values because they only take integer or even negative values for the square's side length to be cut from the corners of the cardboard. When they say that they have found the desired value, the instructor helps them by asking a question to correct their deficiencies.

Berk: Teacher, the maximum value of the volume is 88.

Instructor: Have you plotted its graph?

Berk: Graph?

Instructor: Yes. For example, draw a graph of a function so that the length of the square you cut is an argument, and the volume value is the dependent variable.

Berk: You said that the piece we cut is  $x$ .

Instructor: Plot a graph of the function with its volume as  $y$ .

Instructor: Yes. Insert a graph with these data.

Berk: The graph looks weird because of the negative values. It won't be negative.

Instructor: You're right! What about the input values? Do  $x$  values have to be integers?

Berk: No. But the maximum value of this will be somewhere here in this column.

Instructor: What if it is not there?

Arda: He says if not 88. For example, if you plug a decimal value between these, you will get more. (*Looking at the graph on the computer here, he notices that the value will produce a larger volume once in a while.*)

#### Episode 49

In the next activity, PST investigate finding the limit of a function as  $x$  approaches infinity with a spreadsheet. Although they know the formula and immediately tell the answer, they have difficulty interpreting the values obtained when copying the formula inserted into the cells. After a while, they think they have made a mistake and look for another method. After the instructor recognizes this issue, he helps them solve the problem.

Berk: (*Copies the formula by pulling it down*) Why did it just happen like this?

Instructor: What happened? Is there a problem?

Berk: There is no problem, sir... It may be from the numbers. Did we reverse the numbers? Shouldn't it approach 3?

Arda: It's getting close already.

Instructor: Why did you stop copying? Is it important what happens at some specific values? You are looking for the limit of the function as  $x$  approaches infinity.

Berk: The values were 8 or 9. As  $x$  approaches infinity, it will approach 3, but you are right. (*In the meantime, he continues to copy by pulling down*) It's getting close to 3, see? The maximum value was nine initially, but as  $x$  takes higher and higher values, it approaches 3.

#### Episode 50

In this activity, PSTs experience finding the sum of nested infinite radical expressions using the spreadsheet. Normally, they could follow the expected procedure with a single formula based on two cells to represent the degree of the root and the radicand. However, they get far from the expected process while writing the formula. The main reason for this is that they cannot reach the desired value when they copy the formula without the use of absolute reference. Instead, they tried to increase the number of cells included in the formula. Realizing this, the instructor

first observes what they are doing and, after some guidance, allows them to enter the route they should be.

Instructor: You took its power too.

Nisa: Yes, we did it as  $1/n$ , but an unexpected result arose when we dragged it down. We also need to fill these because the  $x$  and  $n$  values are required.

Eda: We gave  $x$  the value of four.

Instructor: Have you tried pinning? (Reminds of the absolute reference feature of the spreadsheet)

Nisa: Ah! Okay, thank you very much, teacher.

Burcu: How are we going to do it?

Nisa: Dollar sign.

Burcu: Ah, yes.

Nisa: Let me delete these. (After updating and copying the formula, it deletes unnecessary data in other columns.)

Burcu: You will also put a dollar sign here. That's it!

Nisa: OK, sir. We fixed it, and it worked.

Episode 51

In this episode, PSTs are working with a GeoGebra file about the average rate of change of a function over an interval. They were required to observe the evolution of the secant line when the  $h$  value, which determines the width of the related interval, approaches zero. However, they cannot notice this evolution because they focus on the situation where the  $h$  value is zero rather than the change in the line. Realizing this, the instructor turns the discussion in the expected direction with the following intervention.

Instructor: What did you do?

Nuriye: Sir, as the  $h$  value gets smaller till zero, its value becomes  $f(x_0)$ .

Instructor: But we are examining the values it will get as it gets closer to zero. Remember that when we find the limit of a function at a point, we approach that value, but it will never be that value.

Sude: When we make a zero, there is no such line.

Instructor: Okay. If it's zero, it's not a line. It's not our problem because we don't care if it's zero. So, what happens when you get close?

Nuriye: The secant line is approaching the tangent line.

Sude: Yes. Look, the secant line is approaching the tangent line. The secant line disappears.

Nuriye: When it comes to that point, the secant line becomes the tangent line.

Sude: That's why the secant line disappears.

#### **4.1.1.1.2.2.3     *The instructor as a finalizer***

Due to the design of the course, since the primary purpose was to develop PSTs' mathematical content knowledge and technological competencies, it was sometimes left to the instructor to conclude where the argumentation got stuck and could not be terminated. In cases where PSTs cannot progress in finalizing the argumentation, the instructor leads the process and helps to bring the argumentation to a conclusion. These episodes will be presented in this section.

#### **Episode 52**

In the next episode, PSTs are expected to rediscover a formula used to find the periods of trigonometric functions. In fact, although they had used this formula before, they had difficulty understanding it because they did not question the way the formula was stated. Despite arguing among themselves, they could not conclude why the period formula changes according to the power of the function. At this point, the instructor takes the initiative and intervenes below to finalize this issue.

Tuba: Well, I don't understand what it has to do with being odd or even.

Sude: Sorry, the period is halved. (*She sets  $n$  to 3*) Hey! What is this?

Nuriye: It was very different.

Sude: It was very stylish, but why did it happen like this? (*Laughter*)

Nuriye: Can I say something? Here comes the difference between odd and even.

Sude: It's very different when it comes to negatives. Do you want to see it? But the general logic does not change. At least the wave endpoints don't change; only the number of waves changes.

Instructor: Well! What changed if I set the value to two? Let me put the value of  $b$  one and focus on how changing  $n$  affects the graph. Now when  $n$  is two, did the roots change?

Some PSTs: No.

Instructor: Roots did not change. Let's make the value three. Have the roots changed? Four, five, six... It doesn't matter. So, the roots don't change.

Sude: But the odd-even thing is changing.

Instructor: What is changing? When  $n$  is two, we are squaring the outputs. Don't we have positive values when squaring these lower parts? (*Pointing to the parts of the graph below the x-axis*)

Berk: Both are symmetrical.

Instructor: Since the root hasn't changed, isn't the bottom green and top green symmetrical? Therefore, the period is reduced from two  $\pi$  to  $\pi$ . Thus, when the power is even, the period becomes  $\pi$ , not  $2\pi$ . This is why the period changes according to the parity of  $n$ .

Episode 53

In the next episode, PSTs examine the multiplicity of the roots and create the sign chart of polynomial functions. They made some inferences by observing various polynomial functions with the help of the slider tool. However, most of them could not express how the sign changes according to the multiplicity of the roots. Realizing this, the instructor addresses the class in general and states the expected conclusion from these discussion processes.

Instructor: So, what's your final comment?

Berk: If  $n$  is even, the sign does not change.

Instructor: Why doesn't it change? What happens when  $n$  is even?

Berk: Because it's tangent.

Instructor: When there is an even multiplicity, the graph is tangent to the x-axis. What about an odd multiplicity?

Berk: The sign changes.

Instructor: When the multiplicity of a polynomial's root is even, the graph is tangent to the x-axis, so the sign does not change. When it is odd, the sign changes because the graph crosses the x-axis.

#### Episode 54

In the next episode, PSTs are working on making sense of absolute value equations. They can solve such absolute value questions with methods they have memorized before. But they didn't quite know how to make sense of absolute value. For this reason, it is aimed to enable them to learn this concept in a meaningful way by starting from the idea of points that are equidistant from a point, that is, the circle. They observe the points where the circle centered on the x-axis intersects the x-axis. However, they cannot make the desired definition when they are asked how they can write it as an equation containing absolute value. At this moment, the instructor expresses the desired result by taking the initiative.

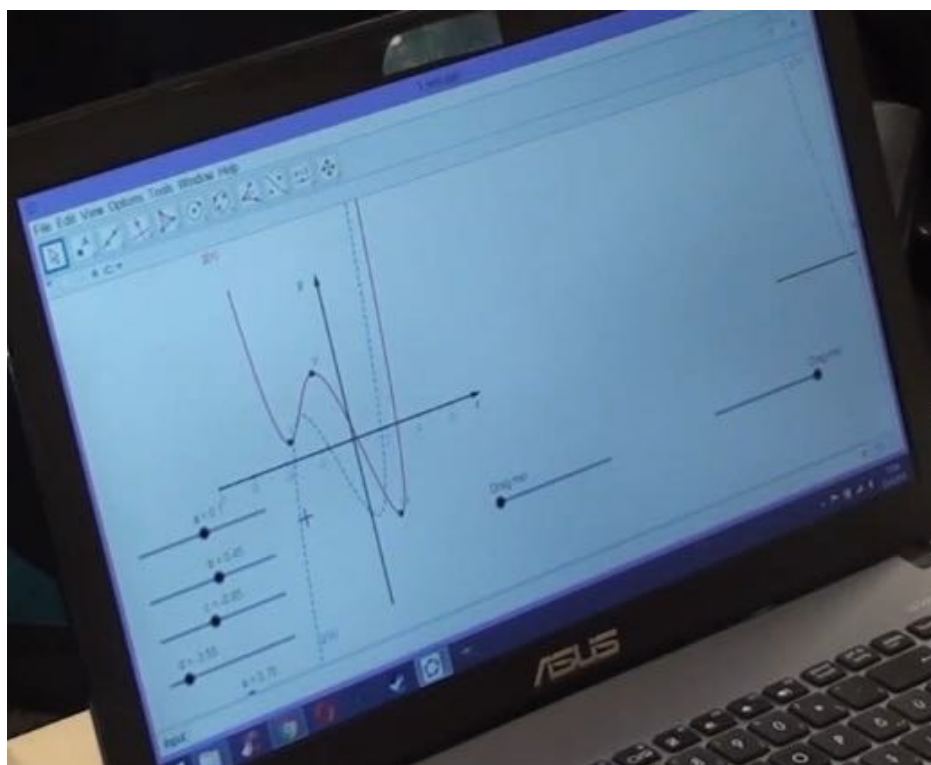


Figure 4.15: The instructor as a finalizer in derivative tests activity

Instructor: Let's check. When we substitute -1 and 5, it becomes true. OK! So what does that mean? How would you interpret this?



Berk: The distance of a point on the number line to -2 will be three units.

Instructor: What you said is true, but let's state it more appropriately.

Berk: Points that are 3 units away from -2 on the number line.

Instructor: Is it -2?

Berk: 2, sorry! Points that are three units away from 2.

Instructor: Well, would you state that? (*Shows the equation  $|x - a| = r$* )

Berk: Points with a distance of  $r$  units from point  $a$  on the number line.

Instructor: That's it! Then we can state that the distance between two points on the number line is the absolute value of their difference; that is, the distance from  $x$  to  $a$  is  $|x - a|$ .

Nuriye: Actually, it's like the definition of a circle, sir.

Episode 55

PSTs are expected to rediscover the first derivative test in the next episode. A GeoGebra that allows the graphs of a function and its derivative could be superimposed is provided to them (see Figure 4.15). Although they had taken Calculus before, they could not remember this feature as a whole. After observing PSTs' fragmentary comments on the graphs, the instructor has the final word by making a point to an appropriate summary.

Instructor: How was this result called?

Berk: It was derivative.

Instructor: Ok, it's about derivatives, of course, but there were derivative tests, right?

What test is this?

Some PSTs: First derivative test!

Instructor: Yes, first derivative test. Now let's summarize what you talked about; First, the  $x$ -coordinates of the extreme points of the function are the points where the derivative is equal to zero. Second, the function increases in the interval where the function's derivative is positive and decreases if it is negative.

The data analysis revealed too many coded segments in this title (74 initiator, 25 resolver, and 20 finalizer). However, those sufficient to explain the findings coded by this name were reported.

#### 4.1.1.1.2.3 The Focus of the Instrumental Integration

This section presents the findings related to the instructor’s instrumental integration during the activities designed for the collective mathematical argumentation. In this context, whether the focus of the task is on technological tools or mathematics and their levels will be presented in categories. In addition to the categories stated by Hollebrand and Okumuş (2018), one more category will be added, and the findings will be reported. The interventions, including questions or verbal directions during the implementation of the tasks, shaped the collective argumentation process. While these interventions are sometimes embedded in the specific task, sometimes the instructor himself is involved in the process spontaneously.

Table 4.1: Categories of The focus of the instrumental integration

	Purpose	Definition
Focus on Technology to Notice Mathematics (FTNM)	Focus on specific objects, actions, measures or meeting conditions	Focus on the technological objects, actions or measure to notice or consider mathematical knowledge without specifying the particular action to perform.
Focus on Mathematics with the Use of Technology (FMUT)	Focus on connecting mathematics with technological representations	Focus on using mathematics to make sense of technological representations and to describe variances/invariances in the representation.
Focus on Mathematics to Notice Technology (FMNT)	Notice a mathematically useful feature of the technological representation	Focus on mathematics to make learners feel the need for a specific affordance of technological representation.

#### 4.1.1.1.2.3.1 Focus on Technology to Notice Mathematics (FTNM)

The interventions in this section are technological, and it is aimed that PSTs notice the target mathematical knowledge by using technology. This section reports the findings involving such interventions.

Episode 56

Before this question, PSTs examined solving inequalities containing linear and quadratic functions. After that, they are asked to obtain a conjecture about the inequality solutions of the general polynomial functions. In his question, Instructor asks them to focus on GeoGebra's slider tools, write a function, and generalize accordingly about the sign diagram for the resulting graph.

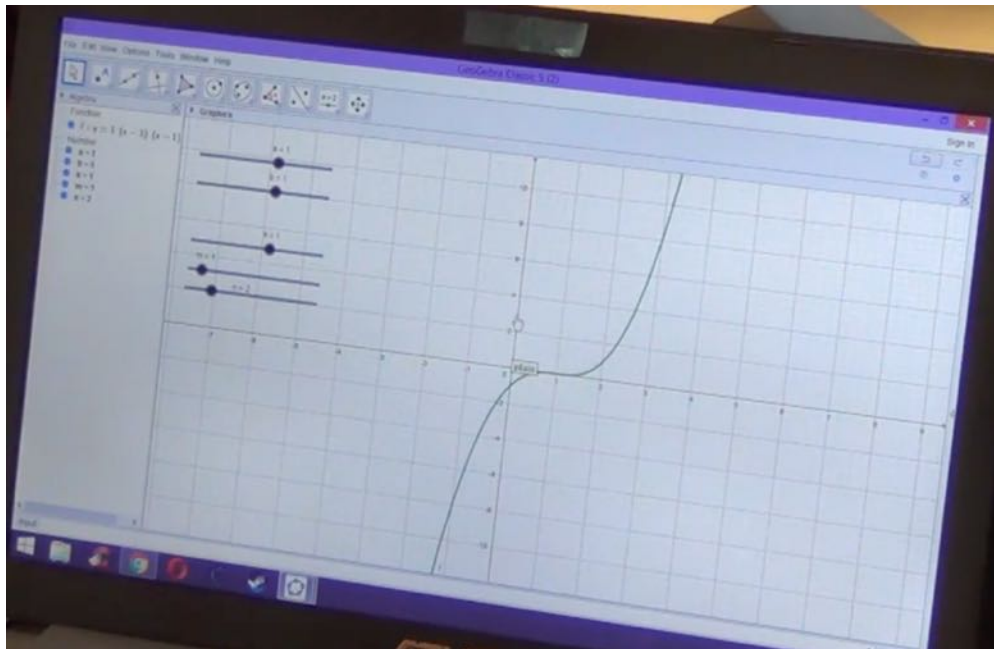


Figure 4.16: FTNM for creating a sign diagram for solving an inequality

Instructor: What happens to the function's graph at a root with odd multiplicity? And a similar one for an even multiplicity. "Create sliders  $k$ ,  $a$ ,  $b$ ,  $m$  and  $n$  to investigate the family of the graphs" First of all, there are five parameters here. Of course, you need to pay attention to that  $m$  and  $n$  must be natural numbers.  $k$ ,  $a$ , and  $b$  can be any real numbers. Therefore, please pay attention to these limitations when creating sliders. Accordingly, when a graph is drawn, examine the effect of these parameters on the graph and thus on the sign chart.

## Episode 57

With the question in this episode, the instructor focuses on defining the points in the pre-prepared GeoGebra file used by the PSTs and expects them to realize the underlying mathematics.

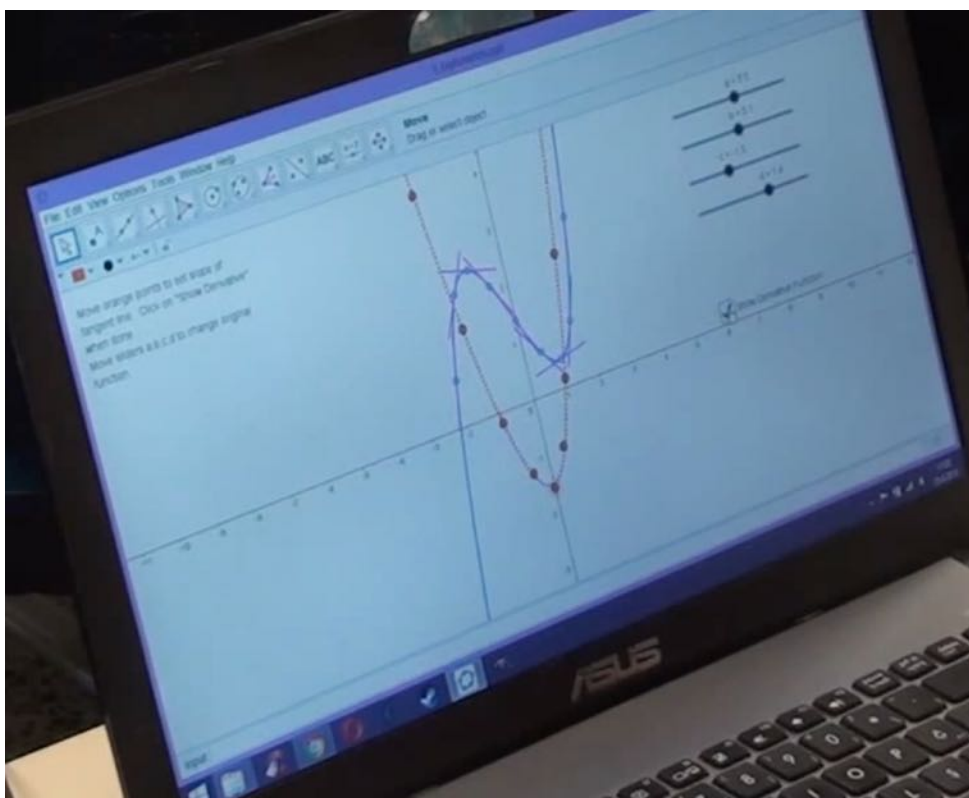


Figure 4.17: FTNM for derivative function

Instructor: OK. You observed that those orange dots are always on the derivative function. My new question is: Why always do the orange points lie on the derivative function? So if you had tried different functions, those orange dots would always be on the derivative function. Why? What provides this feature? How should those orange dots be defined to meet this requirement consistently?

### 4.1.1.1.2.3.2 *Focus on Mathematics with the Use of Technology (FMUT)*

The instructor's intervention contains a mathematical question or statement in this section. It is supposed that PSTs need to use technology to find an answer to them. Thanks to the unique opportunities provided by technology, they are expected to reach mathematical knowledge.

## Episode 58

Before the question in this episode, PSTs constructed a triangle with specific side lengths using Greek Construction Rules. The Instructor expects them to notice the triangle inequality with the following question.

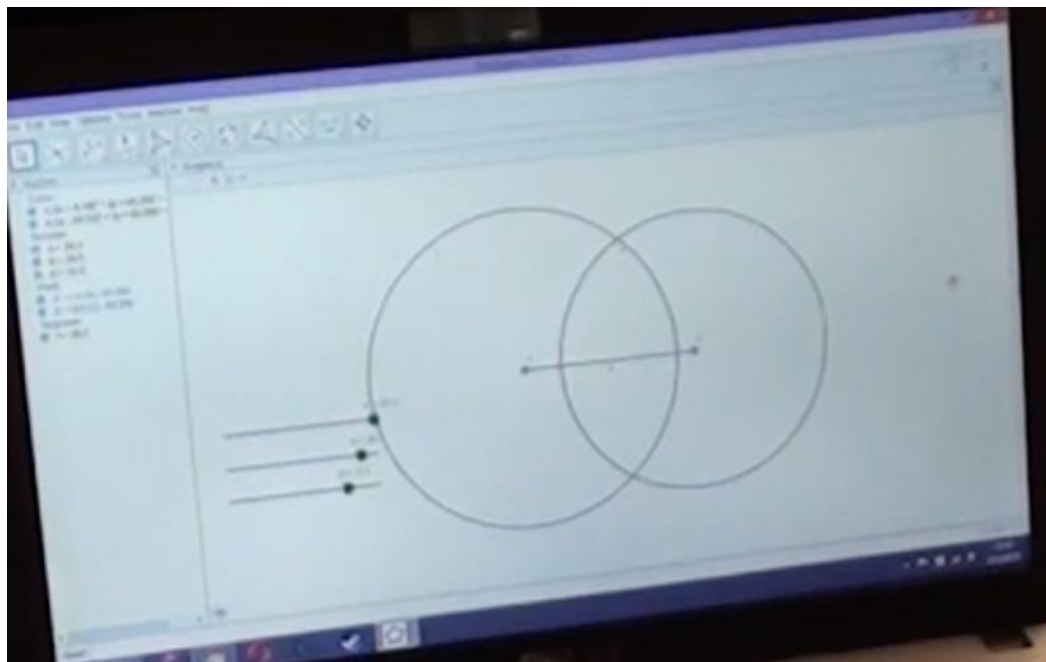


Figure 4.18: FMUT for noticing triangle inequality

Instructor: Now, create three sliders for  $a$ ,  $b$ , and  $c$ . Set the length of the line segment to be  $a$ . Draw two circles with radii of  $b$  and  $c$ , with the endpoints of the line segment as centers. Then observe their intersection.

## Episode 59

In the questions and interventions of this episode, PSTs wrote a function parametrically using a given function. The Instructor asks the following questions to enable them to observe the effect of these parameters. Here, he prepares an environment for them to observe the variant and invariant properties of the graph and wants them to notice the function transformations with the help of technology.

Instructor: Change the value of “ $a$ ” to 2. What kind of change do you expect in the graph when it is doubled?

Instructor: All right. It looks like it did. What do you expect to happen when you double the value of “ $b$ ”?

Instructor: Contraction. So, when you do 2, you have compressed the graph twice. Likewise, what do you expect when you change the “ $c$ ” value? For example, what should happen when you increase one unit?

Instructor: So, it happened? Check it out. Please set it to zero first. Now, for example, look at the points where it intersects the x-axis. For example, it is passing through the origin right now, isn't it? Now let's increase it by one unit. Where will it pass? Not slid by one unit. Why?

### Episode 60

In this episode, teacher candidates observe which parameters affect the period of trigonometric functions and how. They could express the effect of parameters other than the parameter that changes the strength of the function. When the Instructor realizes this, he helps them solve this problem using GeoGebra with the following questions:

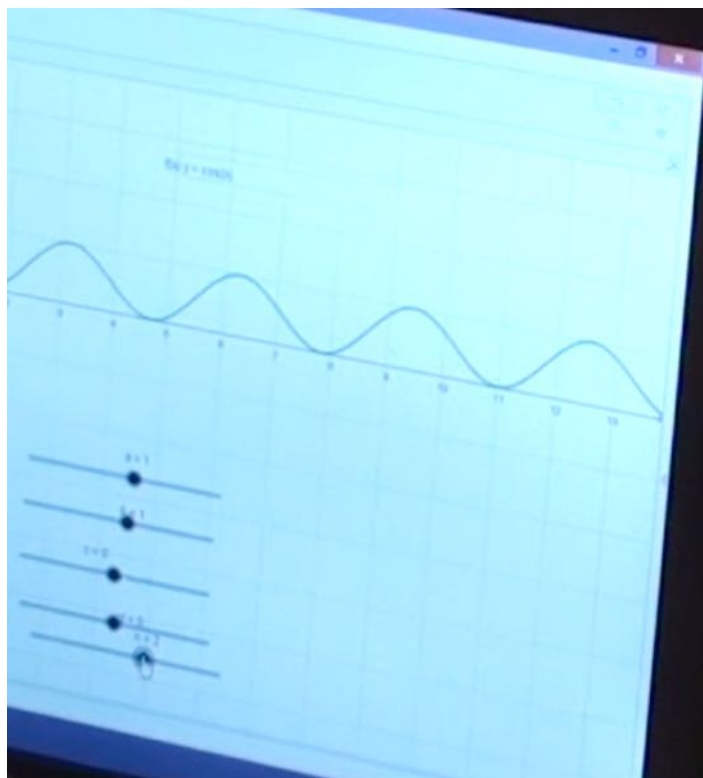


Figure 4.19: FMUT for periods of trigonometric functions

Instructor: Now, do you understand why the formula changes for odd and even powers of this function? The period of the function is  $\frac{2\pi}{b}$  in one case and  $\frac{\pi}{b}$  in the other. Why?

Berk: I know this, but how can I explain it now?

Instructor: Do the roots change as you change the value of  $n$ ?

Berk: Roots are changing... No, they don't!

Instructor: Roots don't change. So, what's changed?

Berk: The shape of the part between the roots is changing.

Instructor: So, what kind of change do you observe when you change the value from one to two? (*When you wait for a while and don't get an answer*) Focus on the negative parts, for example.

Berk: Negative parts turn into positive.

Instructor: So, what happens because the same goes from negative to positive?

Berk: It's moved up.

Instructor: So, how is the period affected in this case?

Berk: The period is halved.

#### Episode 61

In this episode, PSTs first obtained the circle equation starting from the distance definition on the coordinate plane. Then, examining special cases made the absolute value definition using distance. Using this definition, they are asked to interpret the absolute value equations according to the parameters assigned with the slider tool in GeoGebra. The Instructor expects them to reach this mathematical information by analyzing the dynamic figure obtained with GeoGebra.

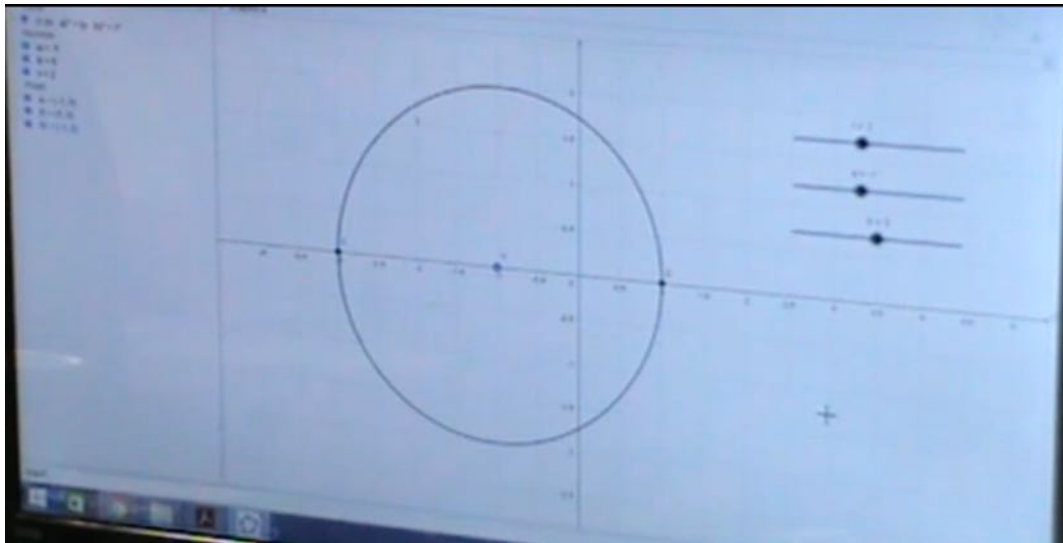


Figure 4.20: FMUT for absolute value equations

Instructor: So, the circle will be shifted to the right and left. Now how do you interpret the situation that occurs when you change the values of  $a$ ? I expect you to stop somewhere and talk about it. Interpret this according to the absolute value definition we just made.

#### Episode 62

In this episode, PSTs use spreadsheet software to find the maximum value of a box's volume they can make from cardboard. Since they take the length of the piece to be cut from the corners of the cardboard as an integer, they cannot reach the actual maximum volume. To realize these mistakes, the instructor advises them to add a chart using the data they obtained by writing a formula. Thus, according to the resulting graph, the instructor prepares the base for them to realize their mistakes and focus on a more precise maximum value.



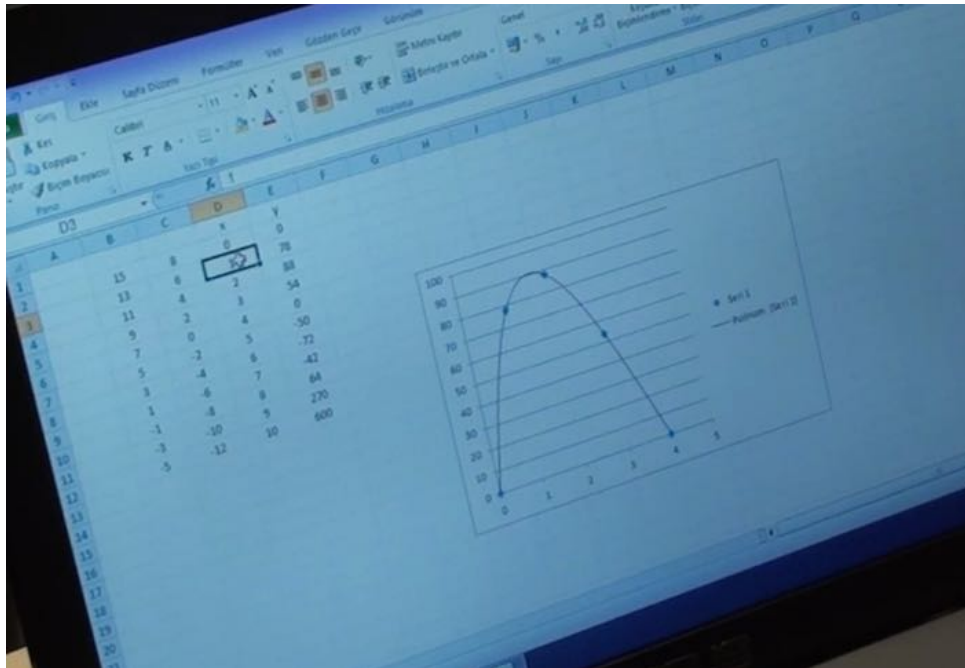


Figure 4.21: FMUT for the optimization problem

Berk: Teacher, the maximum value of the volume is 88.

Instructor: Have you plotted its graph?

Berk: Graph?

Instructor: Yes. For example, draw a graph of a function so that the length of the square you cut is an argument, and the volume value is the dependent variable.

Episode 63

With the question below, the instructor asks PSTs to use GeoGebra's statistics module to observe how multiplying the values in the data set by the same number will affect the descriptive statistics values.

Instructor: Now the next question. What happens if you do these calculations in cm? We calculated it as dm, right? Do descriptive statistics values change when we convert these data to cm?

Then, PSTs write a formula for multiplying each value by ten in the next column and copy the formula for all values. They can easily observe how the values are affected when the new descriptive statistics are obtained.

## Episode 64

In this episode, PSTs examine the approximate slope of any function at a point. However, although they have previously learned about this linear approximation, they cannot establish a relationship.

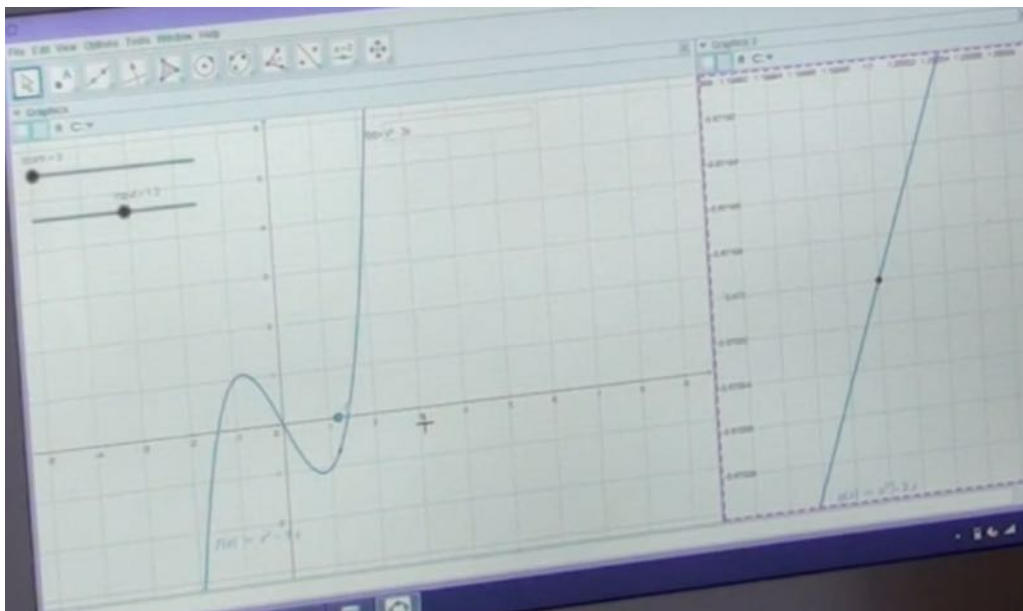


Figure 4.22: FMUT for linear approximation

Instructor: Now, let's look at it together. What does the image in 1.2 turn into when zoomed in? Is there anything you notice in all the charts?

Berk: It becomes linear.

Instructor: The function seems to be linear once you zoom in enough. It's about a mathematical concept, but I'm not sure you've learned about it.

Some PSTs: Tell me, sir.

Instructor: Have you seen linear approximation?

Berk: We saw it, sir.

Eda: Yes, we saw it in Calculus.

Instructor: Great! So, what was it?

Berk: If I remember correctly, we were calculating the length of a piece.

Eda: Didn't we see it in the limits of functions?

Some PSTs: We had a formula.

Instructor: So, what do you know besides the formula? (*Laughter in class*) Okay, no problem. Apparently, you don't remember. You said any function seems linear when you zoom in enough, right? Then the value of a line passing through that point and the value of the function will be very close, correct?

Some PSTs: Yes.

Instructor: So, which line is it?

Some PSTs: The tangent line.

Instructor: Can you find the equation for that tangent line?

Some PSTs: Yes. We find it with the help of derivatives.

Instructor: That's the logic of what we call linear approximation. It means approaching any curve with a straight line.

#### 4.1.1.1.2.3.3 *Focus on Mathematics to Notice Technology (FMNT)*

In the interventions in this section, the instructor focuses on mathematics and makes PSTs feel the need for specific features and affordances of technology. Thus, it is aimed to learn the affordances of technology in a meaningful and contextual way. The main motive for this intervention is not solely to teach technology features but to teach a feature that would be beneficial to use in other similar contexts. In other words, these interventions aim to increase teacher candidates' technological content knowledge, albeit indirectly.

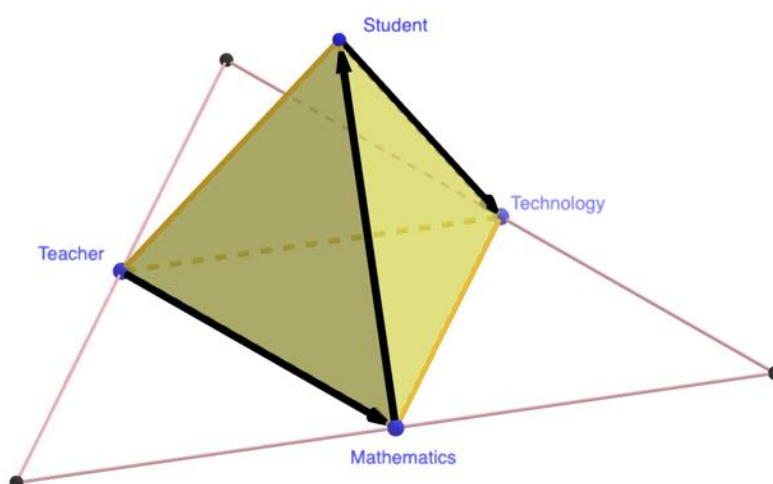


Figure 4.23: Didactic tetrahedron depicting FMNT

Like Hollebrands and Okumuş's (2018) representations of the didactic tetrahedron focus on mathematics to notice that technology intervention follows teacher-mathematics-student-technology edges of the tetrahedron. In other words, by using a statement, question, or demonstration about a mathematical concept, the teacher intervenes to make the students notice an important technology feature by making them feel the need to use it. This type of intervention emerged in the analysis as a contribution to the literature.

#### Episode 65

In the section just before this intervention, PSTs draw the graph of the function to solve an equation with the help of GeoGebra. Because the degree of the polynomial function is a large number, they can only observe a limited region of the graph. Zooming doesn't help them either. The Instructor asks if they can do anything else besides zoom but gets no response.

Instructor: Guys, I'd appreciate it if you could take a look here. This is what I was talking about earlier. In GeoGebra, you could press shift and narrow the axes as follows. So, you just said there are only two roots in the previous graph. And you don't see the rest of the graph. Why didn't you change the ratio of the axes to the visible area?

Berk: I didn't know.

Nisa: Sir, we zoomed out, but it still hasn't appeared. That's why we couldn't see it.

Nuriye: Sir, we haven't learned this before.

Instructor: When you graph any function, it will likely overflow the screen. Ok, no problem. Now you have learned. But why is this important? The outputs of some functions, such as the exponential function, increase very quickly. For this reason, the 1:1 aspect ratio is useless for the graph to be in the observable range. Therefore, narrowing the axis you want is a handy feature.

Berk: Sir, can we achieve this by zooming in on the axes instead of narrowing them?

Instructor: As long as you keep the ratio 1:1, you may not be able to capture the desired view of the function by zooming. For example, let's consider the function  $f(x) = \sin(100x)$ . Zoom as much as you want on this screen, but you will not be

able to capture a meaningful image. But if you expand the x-axis like this without making any changes in the y-axis... Because there is a horizontal compression here. Likewise, under normal conditions, you can see only a small portion of an exponential function which increases or decreases rapidly. But if you compress the y-axis, you expand the visible area.

Episode 66

In the part of this intervention, PSTs are working on drawing an equilateral triangle using Greek Construction Rules. However, since they use a tool for drawing a line segment of a certain length, they cannot obtain the desired drawing.

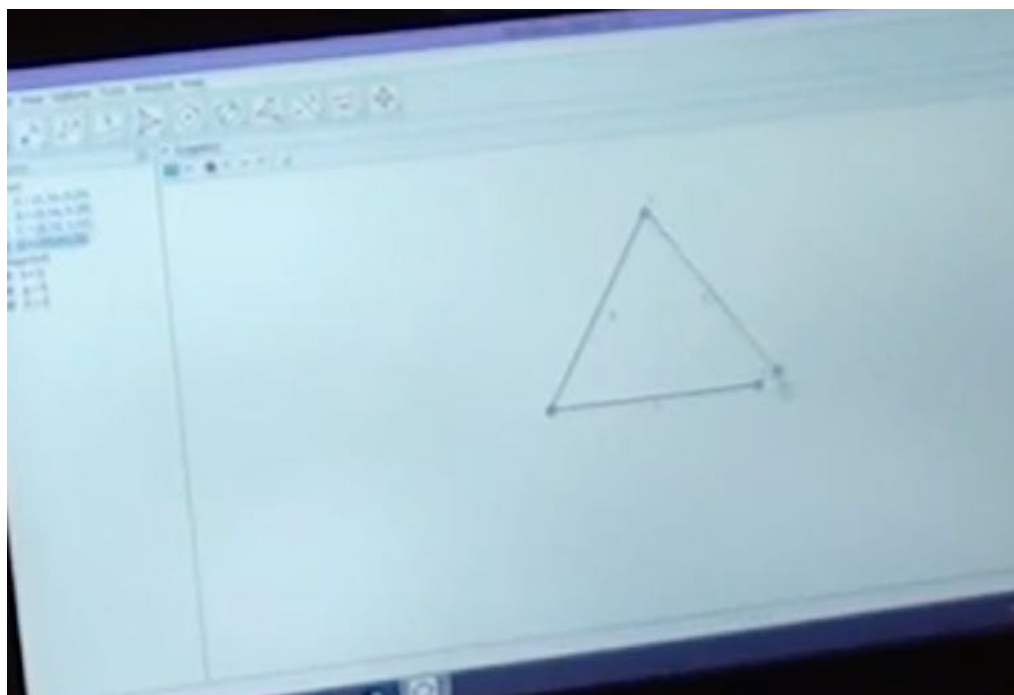


Figure 4.24: FMNT for constructing an equilateral triangle

Instructor: Think of it this way. With which tool can you create points at a certain distance within the rules of Greek Construction?

Berk: Compass.

Instructor: What tool can you use instead of the compass in GeoGebra?

Arda: You will draw a circle.

Brooke: Yes, okay.

Instructor: So, when making construction drawings, you can use the circle tool in GeoGebra when distance is required.

Episode 67

In this episode, PSTs are aware of an important GeoGebra but they cannot think of using it in an unfamiliar context. They are used to utilizing GeoGebra's slider tool in dynamic geometry environments and manipulation of parametrically drawn function graphs. However, they do not think they can use the same feature in GeoGebra's statistics module.

Instructor: So, how can we generalize this result? For example, if I ask you to examine the change in the descriptive statistics when you add  $k$  dm instead of 0.8 dm. How would you do this with GeoGebra? (*After a long silence*) You opened a new column here and added 0.8 dm to all of the data here, right? Let's make it dynamic by adding a parameter, like  $k$ , not 0.8.

Some PSTs: Slider!

Instructor: Yes! When you assign a slider and add the value of that slider to the new column, you can dynamically observe the change in the new column, right? Therefore, when you do this, an environment is created where we can dynamically observe the answer to the question that we usually ask students, what would change if the same wand was added to or removed from all values.

#### **4.1.1.2 Distractive Roles in Mathematically Incorrect Argumentation**

This section will present the factors that prevent PSTs from developing a mathematically correct argumentation process.

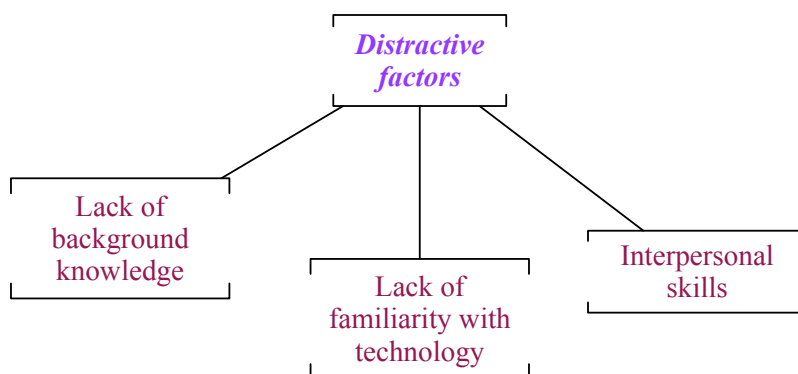


Figure 4.25: Distractive factors in technology-enhanced collective mathematical argumentation

#### 4.1.1.2.1 Lack of Background Knowledge

The first factor to be presented will be the PSTs' lack of mathematical knowledge observed during the argumentation process. In fact, the topics covered in the activities are thought to be known by PSTs. Still, there are episodes where significant deficiencies in their conceptual knowledge due to various possible reasons are noticed.

Episode 68

In the next episode, teacher candidates examine the multiplicity of the roots for a polynomial function. However, they cannot interpret the function's graph since they do not know how the multiplicity of a root will affect the graph. Another reason for developing a mathematically incorrect argumentation process is the mistake that every polynomial function must have real roots.

Berk: Minus 0.5. The other is 2. Yes  $-1/2$  and 2.

Instructor: How many roots should it have?

Some PSTs: There should be six.

Instructor: So why are there only them then?

Some PSTs: Multiplicity of the root is two.

Berk: It's not a two; it's a three.

Nisa: Can't four of them be  $-1/2$  and the other 2?

Brooke: Maybe. One of them can be  $-1/2$ , and five of them can be 2. Couldn't there be all sorts of different combinations?

Sude: How can we understand it from the graph?

Berk: You can't understand from the graph.

Episode 69

In this episode, PSTs are expected to analyze the solution set of an equation utilizing the graphing method. However, since they enter the given equation written on the board in GeoGebra, they first realize that there is no meaningful graph on the screen and begin to interpret it. But when they realize that there is no visible graph, they make a mistake trying to explain this situation, as if every equation must have a root. Therefore, this false conception steers the discussion in the wrong direction.

Berk: Sir, It cannot graph!

Instructor: What doesn't it draw? I do not know what you are trying to graph, but you do not have to. You can use any method you want.

Burcu:  $\pm\sqrt{3}i$ .

Berk: She found a complex root.

Eda: Doesn't the sine take a value between -1 and 1? When multiplied by two, it will be between -2 and 2. The other side is  $x^2 + 3$ . So, it will always take a value greater than three. How do you get 2 equal to 3?

Nuriye: It has a complex root.

Berk: Exactly! It must have complex roots; she is right.

After this comment, the discussion continues for a while to understand the nature of the non-existing roots.

Episode 70

In the next episode, PSTs are trying to find the box's maximum volume that can be made with the remaining part when squares of the same size are cut from the corners of cardboard using spreadsheet software.

Berk: Now look bro. The more we shorten 15, the less we have to trim it? Now think about it... If  $c$  is 4, you will cut 4 cm from here, right? You have to cut the same from both sides. In other words, when you cut 1 cm off from this part, you cut from here



and decrease by one cm. The more I increase here, the less it becomes here. (*He's talking about values in different columns.*)

Arda: Shouldn't it decrease by 2 cm? When you cut 1 cm, you will cut 1 cm from the other corner, too. That makes a 2 cm decrease

Berk: That's right. You are right. It decreases by 2 cm.

Arda: You do not have to do it that way. Don't we want these two numbers to be as close to each other as possible?

While they are aware that they want to find the maximum value of the product of three numbers, they think this value occurs when the numbers are close to each other. That is probably because they memorized in their high school years that the value of the product of two numbers gets its maximum when they are closest to each other. This thinking deficiency prevents them from progressing the argumentation process properly.

#### Episode 71

In this episode, another group cannot continue a mathematically correct argumentation process due to a similar mistake in the same part of the activity mentioned above.

Sude: Let's drag it down to 1, 2 now.

Nuriye: How much can we go?

Sude: We cannot cut 4 units.

Nuriye: Yes. We can cut at most 3.

Sude: Now let's find the multiplication. The volume will be the product of all three.

Nuriye: Equals, product... Select directly. Close now and pull down.

Sude: Of course, we had to guess that the middle one would be the maximum when there were three.

Nuriye: Teacher, what will we do after we find the maximum?

They accept this result as accurate without questioning it and try to move on to the next stage.

#### Episode 72

In this episode, PSTs examine the limit of a function using spreadsheet software. As the  $x$  values approach infinity, they need to explore the change in the function's output. However, they thought the function did not converge to any value after copying the formula through a limited number of cells below. Consequently, they checked an error in the formula and could not conclude the argumentation process.

Nuriye: Let me tell you something. We need to find out how close it is.

Sude: But our values are not approaching a specific value.

Nuriye: Let's make these numbers bigger.

Sude: Do you mean these numbers? Let me pull it down until 1500. It did not work!

Why did this happen? (When the output values did not converge, they thought they had written the formula wrong and tried to find an error.)

Nuriye: The numbers have changed.

Sude: But the formula is the same. It looks correct. Why did that happen?

### Episode 73

In the next episode, PSTs examine whether a probability game is fair. They were expected to observe the experimental probability by performing a simulation using TinkerPlots and comment on the theoretical probability. However, they comment on theoretical probability based on very few outputs. About the assumption made by one, others accept without objection.

Nisa: We looked at the probability of coming blue. Now we have to look at the probability of getting a red.

Burcu: Red will come from the first...

Instructor: Now I want you to simulate this with TinkerPlots.

Burcu: Can you count the same ones?

Eda: 9 and 10.

Burcu: What did you count?

Eda: The ones of the same color.

Nisa: Yes, 10 of them. (They are counting by checking the outputs one by one)

Eda: So unfair.

### Episode 74

In the next episode, they speculate about the “slope” of the function at that point, based on the fact that when a function is zoomed in enough, its graph will look like a straight line. However, all of the group members find the  $y/x$  ratio directly instead of using the rate of change when calculating the slope of the line. Since the first example is linear, they cannot notice their errors. However, in the second one, they get different values when the function they are examining is a third-degree polynomial. Since they also know that the result, they need to find must be equal to the function’s derivative at that point, they realize that this different result is a mistake. However, they cannot find the cause of this error.

Bus: 0.672. minus? Ah yes. With minus 0.672... minus 0.672 over 2.3.

Nuriye: Actually, we need to calculate the derivative. Three  $x$  squared minus 2

Sude: 1.2 times 1.2 times three minus 2 (*entering the values into the calculator*). It does not matter! Why is this happening? What could we be doing wrong? Here is the graph, and here is the slope. (*pointing to two different screens*) What could be wrong with that?

Instructor: What happened? What are you discussing?

Sude: There was no derivative slope. Why did that happen? When you put it in its derivative, it does not give the slope of the  $x$  value.

Nuriye: We do not do anything wrong.

Sude: Okay, but why doesn’t it come out the same? Shouldn’t it be the same? Slope. We find the slope at a point by derivative. It is what we know.

#### Episode 75

In another group that performs the same activity as in the above episode, a proper argumentation process cannot proceed due to the lack of background knowledge. As can be seen from the screenshot below (see Figure 4.26), although it is evident that the slope of the linear function on the right of the screen is negative, they cannot notice the inaccuracy of the value they find. The PSTs in this group also find the  $y/x$  ratio when calculating the slope of the line. Even though they know that the value they find must be equal to  $\cos(3)$ , they do not realize that the 1.1213 they get is not possible for  $\cos(x)$ . While calculating the value of  $\cos(3)$ , they make a calculation

error in degrees and add a new one to their mistakes, considering it should be close to zero.



Figure 4.26: Screenshot from the activity for finding the derivative

Eda: Let's look at the sine function. We will also look at this at  $x=3$ .

Burcu: I'm writing  $\cos(3)$  to it.

Nisa: 1.1213 comes out here.

Burcu: Can you calculate the  $\cos(3)$  value over the phone

Nisa:  $\cos(3)$  shouldn't come out like this.

#### 4.1.1.2.2 PSTs' Lack of Familiarity with Technology

In some parts of the collective argumentation process that required technology, basic technology knowledge was necessary for PSTs to develop a mathematically correct argumentation. However, in cases where this information was insufficient, PSTs could not proceed correctly. This section will present some examples of this inadequacy.

## Episode 76

In this episode, PSTs are asked to find the roots of a sixth-degree polynomial utilizing any method that they want. They preferred to draw the function graph as they did before. However, instead of drawing a function graph, they are faced with an unfamiliar graph as they enter it as an equation into GeoGebra instead of entering it as a function. While trying to interpret this graphic view that they cannot make sense of, the argumentation process cannot proceed on the right path when their deficiencies in content knowledge are combined.

Berk: Minus 0.5. The other is 2. Yes,  $-1/2$  and 2.

Instructor: How many roots should it have?

Some PSTs: There should be six.

Instructor: So why are there only them then?

Some PSTs: Multiplicity of two.

Berk: It's not a multiplicity of two; it's three.

Nisa: What is the problem?

Berk: Why didn't it draw this?

Nisa: Can't four of them be  $-1/2$  and the other 2?

Berk: Maybe. One of them can be  $-1/2$ , and five of them can be 2. Couldn't there be all sorts of different combinations?

## Episode 77

In this section, PSTs are first asked to enter the terms of the Fibonacci Sequence into the spreadsheet. Later, it is expected that some features related to the sequence will be moaned and they will form an assumption. But they want to get the other elements by typing the first eight elements of the array and then selecting these cells and dragging them down. Therefore, they cannot proceed to the next stage.

Burcu: We're going to write 1 to the first two cells and two to the third one. Then we'll pull them down.

Eda: Right, okay.

Nisa: 1, 1, 2. Let me add a few up to 13. Let's see if it'll detect it.

Burcu: Doesn't it detect?

Nisa: I hope it detects the relationship. I need to get it from but (*Selects all cells and drags them down*). It didn't work!

Instructor: Did anyone find it? Have we entered the Fibonacci sequence to Excel?

Nisa: We couldn't enter, sir.

Episode 78

In the next episode, teacher candidates are trying to prepare a file for the general solution of a problem involving nested square roots. However, they cannot obtain the expected values because they do not use the absolute reference property while writing the formula (see Figure 4.27). While dragging the formula they wrote down, they cannot realize the reason for the erroneous results. Meanwhile, observing the situation, the instructor makes them realize these shortcomings with a question, and the process starts to progress as it should.

Burcu: Now we have written the first one. Write power to the second one, and  $1/n$  of this plus this. Isn't it?

Eda: Exactly.

Nisa: What were we writing?

Burcu:  $x$  to the power of... Okay, drag it down now. We did not enter numbers. Do we need to add another one? Has anything changed?

Nisa: Teacher, we tied it like this. Here we did  $x$  to the  $n$ th power. Then we made the second one as  $C4$  plus the value we calculated here to the power of  $1/n$ .

Instructor: You took its power too.

Nisa: Yes, we did  $1/n$ . It happened when we pulled it down, but we also need to fill in the following because it requires the  $x$  and  $n$  values.

Eda: We assigned four as the  $x$  value.

Instructor: Have you tried the absolute reference feature?

Nisa: Ah... Okay, thank you very much, teacher.

Burcu: How are we going to do it?

Nisa: Dollar sign.

Burcu: Ah, yes.

Nisa: Let me delete these. (After updating and copying the formula, it deletes unnecessary data in other columns.)

Burcu: You will also put a dollar sign in front of that. That's it!

Nisa: Okay, sir. We fixed it, and it worked.

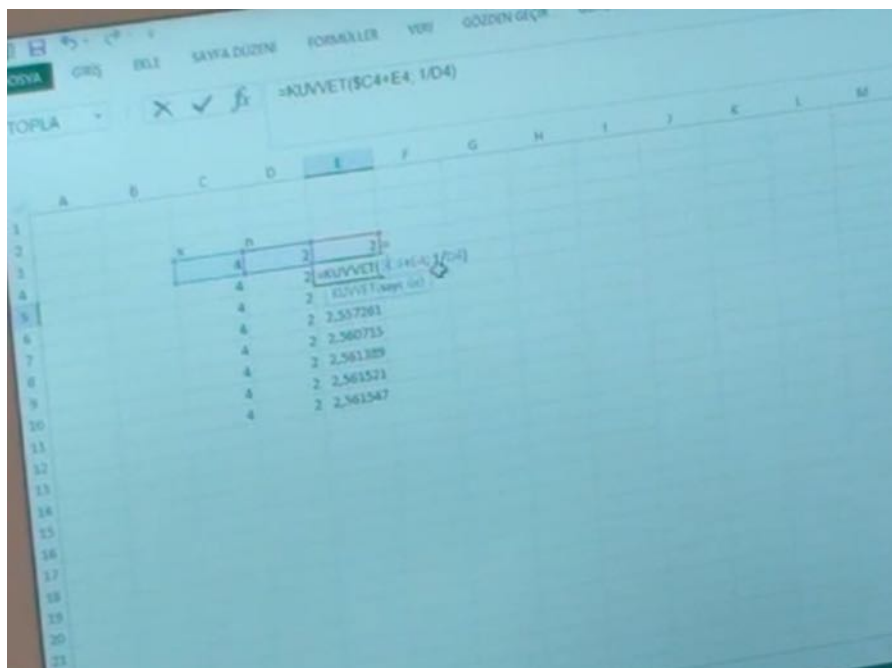


Figure 4.27: Lack of familiarity with a spreadsheet software

#### 4.1.1.2.3 Interpersonal skills

In argumentation processes, PSTs take actions unconsciously that prevent them from developing mathematically correct argumentation and disrupt the progression of the argumentation. In this section, situations will be presented due to the fact that while the argumentation process continues, some PSTs insist on their ideas even though they are not valid. Some cannot defend their ideas sufficiently even though their views are correct. This deficiency, which is necessary when working as a team, stands out as one of the obstacles to developing a mathematically correct argumentation process. Communication skills appear as a remarkable obstacle, primarily due to the inappropriate use of body language and tone of voice. But more importantly, in group work, the problems arising from not listening properly to the

other person and focusing on their ideas without fully understanding what they are saying come to the fore.

#### Episode 79

In this episode, PSTs are working on an activity related to the properties of the inscribed angle of the circle. They tried to achieve the desired result by drawing different shapes on GeoGebra for a while, but they were unsuccessful. Meanwhile, a PST says an idea that comes to mind. Although this idea is an idea that will lead them to the right solution, they give up on this initiative due to the dominant attitude of the other PST controlling the computer.

Arda: Did we just make a circle over there? (*He reminds Berk of their activity using paper and pencil.*) I think we should use the same here.

Berk: Okay, imagine that the ship is anywhere. For example, it could be here. It could be here too.

Arda: Are we going to get the most significant distance that the two made with an acute angle?

Berk: Then this angle intercepting this arc and the one here must be equal. Isn't it? For example, this angle must be equal to this angle.

Arda: Can you draw arcs in GeoGebra?

Berk: Yes, I can.

Arda: Draw an arc passing through these two. (*Proposes to draw an arc that will pass through the two lighthouse locations*)

Berk: (*After several unsuccessful attempts*) No, it's not that...



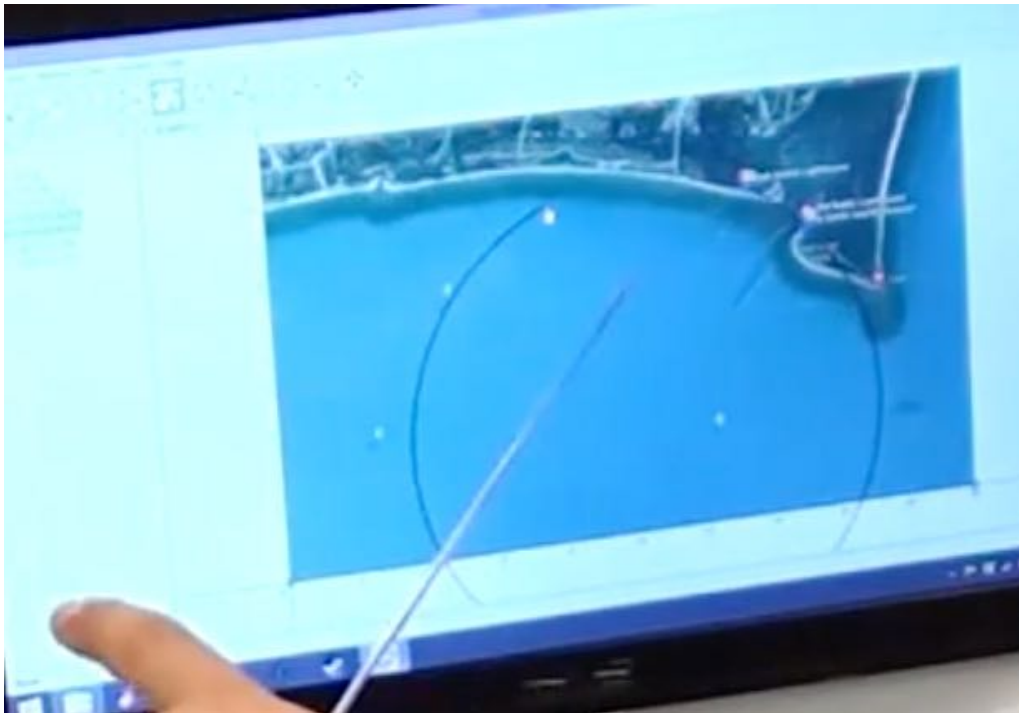


Figure 4.28: Lack of interpersonal skills in Angle of Danger task

#### Episode 80

In the next episode, PSTs add two more to a data set as part of a statistics-related activity without changing the median. Although there is an inclusive answer at the beginning, it is possible that the answer is not listened to in the group and focused on what they will say. Then they overlooked that idea and continued with a minimal solution.

Instructor: Add two values that do not change the median if possible.

Nuriye: Okay. Let's enter two Q2 values or one below and one above Q2.

Tuba: Half of their total should give Q2. So, we need to add values whose mean is the median.

Sude: Values whose sum is half of Q2.

Nuriye: Exactly. In other words, there should be values that give the mean median.

Sude: What weren't we changing?

Nuriye: The median will not change.

Sude: We need to add the median so that the median does not change.

Nuriye: We can add 169, but we can also add 168 and 170.

Tuba: Half of the sum of the two is 169.

Sude: Hmm... We need to add two. Understood.

#### Episode 81

This episode includes part of an activity where PSTs examined the average rate of change. Although Sude correctly stated what they needed to do, the argumentation process could not proceed correctly because the other two group members ignored this idea and continued with another idea.

Sude: First, it advanced, then it came back. Considering the location, it went forward and then came back. So, it went in the negative direction. After stopping, it moved in the positive direction, stopped there again, and moved in the negative direction again.

Nuriye: Doesn't that change anything when calculating the average speed?

Tuba: I don't think it will change it. Does it change? Shall we draw a tangent line to that now and find its slope?

Sude: Isn't it the change between the two points we're considering?

Nuriye: Okay. That's the slope right there.

Tuba: Yes, we must draw a line passing through it and find its slope.

Sude: Let me draw a segment then. Okay?

Tuba: Wasn't it in the Intersection tab?

Nuriye: What are you talking about, Tuba? Are you asking about the tangent?

Tuba: We need to draw a tangent line to that line.

Nuriye: Tangent? It has to be over there.

Sude: It's not tangent. Shouldn't it pass through these two points?

Tuba: Two points, but this is the slope of the tangent. Then there are two points. Don't we look at the change between the two?

Nuriye: So, at which point do we take the tangent? The tangent at  $x$  equals 25?

#### Episode 82

In the next episode, PSTs analyze the average velocity of a moving object whose position-time equation is given by drawing a graph. Eda tries to go that way by suggesting finding the average rate of change. However, since Burcu knows that the

derivative of the position-time function gives the velocity-time function, she offers to find the values in the velocity function and take the average. When insisting on this issue, the right path at the beginning is ignored, and a wrong process is entered.

Burcu: What do we need to find? Let's look at 10 to 35 now.

Eda: Are we going to take the time as 35? Because we're going to divide the position difference by time.

Burcu: We should take it as 35-10. We need to look at what's between the two.

Eda: Now, look! This is the value when it's 35. That's the distance when it's 10.

Burcu: There's no need for that. When we put these values in the first derivative, shouldn't it give us the velocity anyway? Aren't we going to find the average velocity right now? It says average velocity between the time intervals. We don't need to divide it by anything again.

Eda: So, are we going to add these values and divide them by two?

Burcu: Here it is between those two. The sum of the two will not give me the speed anyway. Is not it?

Eda: Okay, let's do that.

#### **4.1.2 The Characteristics of the Argumentative Function of Technology**

This section will report the functions of technology in a collective mathematical argumentation process. Episodes that will help answer the questions of what role technology plays as components of argumentation in the argumentation process in Toulmin's (2003) model and how it contributes to this process will be presented here.

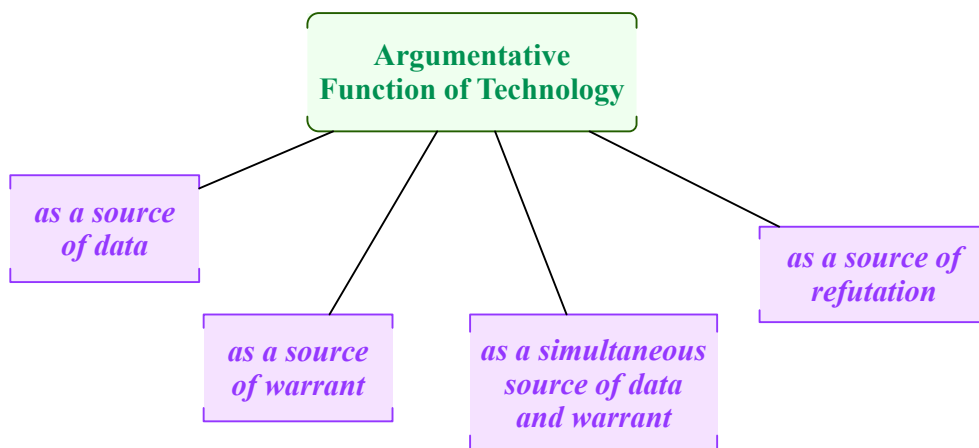


Figure 4.29: Argumentative function of technology

#### 4.1.2.1 Technology as Source of Data

This section presents the episodes where the function of technology as the source of data is the starting component of the argumentation in the Toulmin model. Mathematics education technologies used in the activities undertake a vital function in providing data that will prepare the ground for starting the argumentation process, thanks to the rich environments they contain. Some episodes selected from a large number of examples will help to clarify this situation.

Episode 83

In this episode, PSTs examine the transformations of function graphs. In the activity, they try to conclude by comparing the graphs of the functions given to them to draw and interpret their graphs (see Figure 4.30).

Berk: Things got even more complicated here. But the same logic again. For example, he multiplied by three and subtracted two from  $x$ . That's why it's widened a bit and shifted on the axis. What's the difference between these? It both expands a little more and slides on the axis. In this one, it doesn't slide but expands. Why?

Arda: It's sliding.

Beril: Is it sliding or not?

Arda: Exactly, you're right!

Berk: The input is changing. It also rotates upside down and slides both horizontally and vertically.

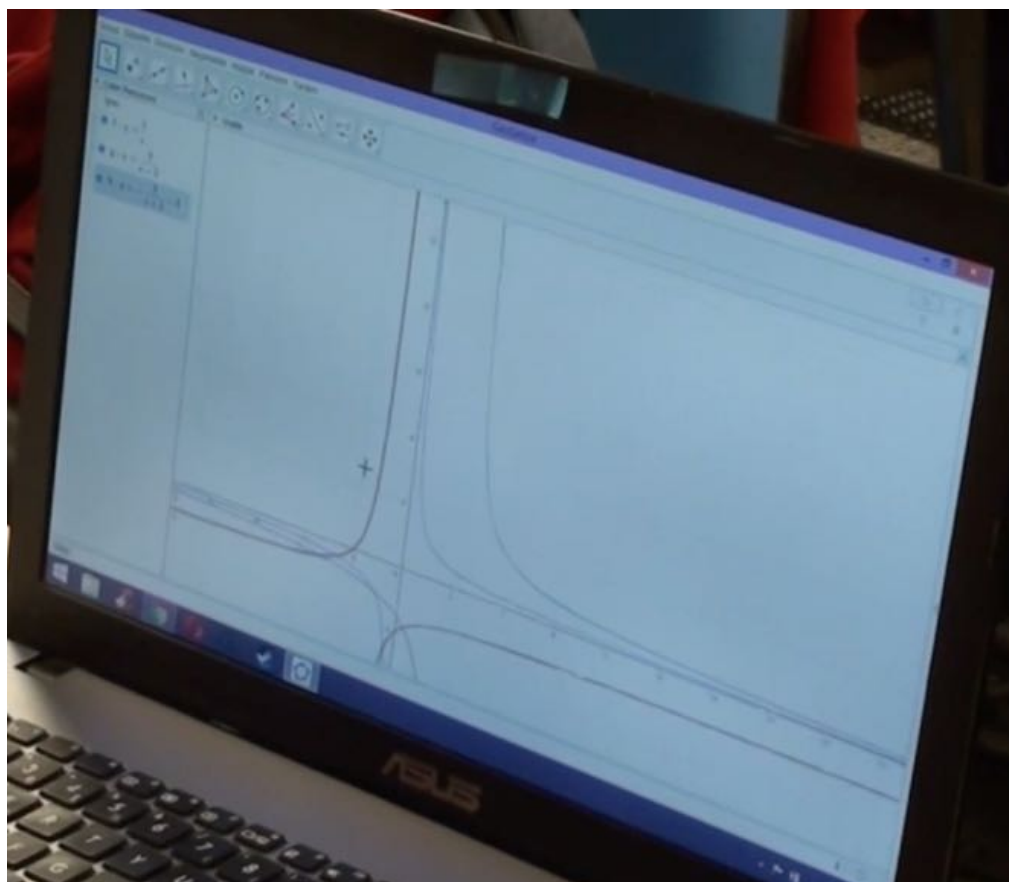


Figure 4.30: Source of data for transformation of graphs

#### Episode 84

In the next episode, PSTs examine the effect of coefficients on the transformations of trigonometric functions. They explore the impact of changing the parameters in the graphics they draw using GeoGebra's slider tool (see Figure 4.31). They build their assumptions on the data they get from these observations.

Burcu: Oh, let's see.

Nisa: Let's animate.

Eda: How do we open it?

Nisa: Right-click on the slider. The third is the animation. Now... How does this affect the graph? It's stretching upwards. So, it stretches vertically.

Burcu: Where is it precisely horizontal?

Nisa: It should be at zero.

Burcu: It has to be at zero, right?

Nisa: Two, one, zero. Yes, that's true!

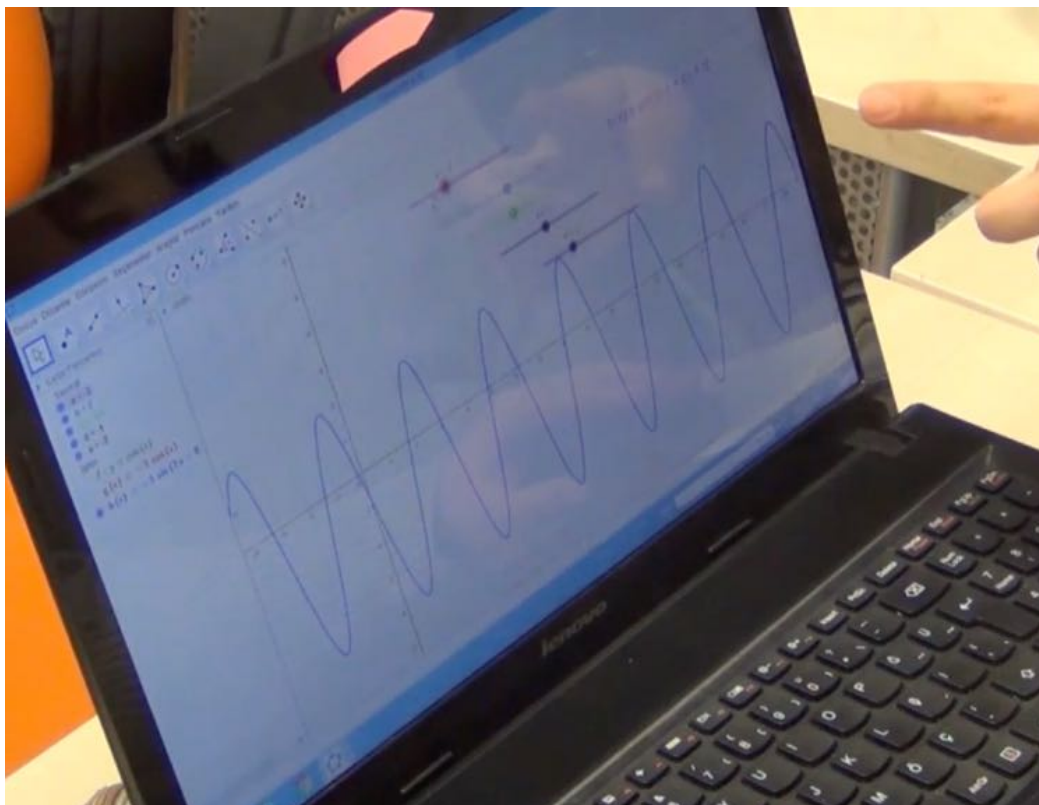


Figure 4.31: Source of data for transformation of circular functions

### Episode 85

In the next episode, PSTs assign the sliders they created in GeoGebra as parameters and examine the changes of these parameters on the graph of a quadratic polynomial function. They interpret the data obtained by observing the dynamically changing parameters' effect on the graph on the GeoGebra screen (see Figure 4.32).

Nuriye: Now, let's change these sliders. Let's move forward with what we're sure of first. What was the effect of  $d$ ? There was a shift effect in the vertical direction.

Sude: (They change the slider assigned to the  $d$  parameter and observe its effect) OK.

Nuriye:  $c$  also had a horizontal shift effect.

Sude: It is at zero right now. Negative one... Shifted one unit to the right. As it increases negatively, it shifts in the positive direction.

Nuriye: What did we say about the effect of  $a$ ? We said stretch and shrink, right? Shall we take a look at it too?

Sude: Then let me reset the others. Now let's change the values of  $a$ . OK, what we said is out. Vertical shrink.

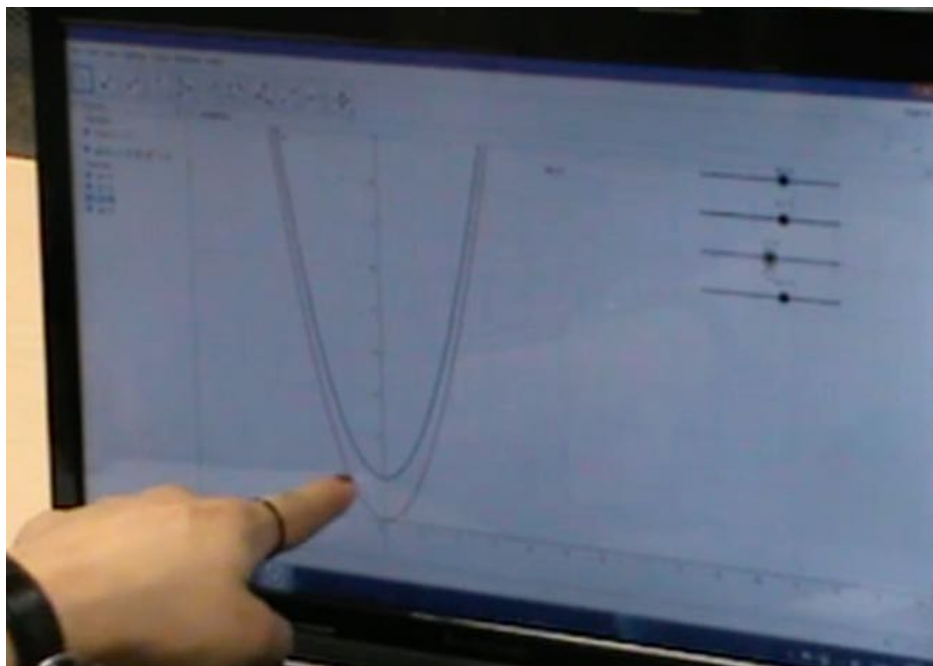


Figure 4.32: Source of data for transformation of the quadratic functions

#### Episode 86

In the next episode, PSTs are doing the sign analysis of polynomial functions. For this purpose, they examine the graphs of the functions for the table to be created with GeoGebra. They discuss the change of the assigned parameters with the help of the slider in the graph and use the data they obtained in the argumentation process.

Burcu: I can't see it. Could you show me what you said?

Berk: Now look, Burcu! This parabola has two roots, right? Here and here.  $a$  is positive, right? Are you aware that values less than the small root are always positive when  $a$  is positive? Greater than zero. Therefore, we can say that the function takes positive values when  $x$  takes values less than the small root. Of course, this only applies when  $a$  is positive.

Nisa: Think about that side, Burcu! Look! The graph is above the  $x$ -axis.

Berk: While the  $x$  values are getting bigger than the greater root, the function has positive values again. But it has negative output values for the remaining input values between the roots.

Nisa: Let's say it this way: the function takes opposite values between two roots.

Berk: But look, when we make a negative... What happened this time? The function is negative for  $x$  values less than the smaller root, so here. Again, it is negative for input values greater than the major root. Again, the reverse is true for values between roots.

Burcu: Okay, just the opposite.

Episode 87

In the next episode, PSTs will write the desired function using transformations based on a given mother function in an activity prepared using Desmos. They conclude by using the data they have obtained by counting the changes of the function on the Desmos screen as unit squares (see Figure 4.33).

Nuriye: Now it just changed horizontally.

Sude: So only the  $c$  value has changed.

Nuriye: So how many units have changed? Nothing else has changed, by the way, has it? Let's count those squares.

Tugce: One, two... five, six.

Nuriye: It's ok.

Sude: No, this is  $f(x)$  anyway. It will match green. We're trying to hover over the blue.

Nuriye: Write minus 6.

Tugce:  $f(x-6)$

Sude: Ok. No, it didn't.

Tuba: We write this function in terms of this. Plus six, then.

Nuriye: Isn't six too many?

Sude: How many units are there between them?

Tuba: 3 units.

Sude: Then we need to write  $f(x+3)$ .



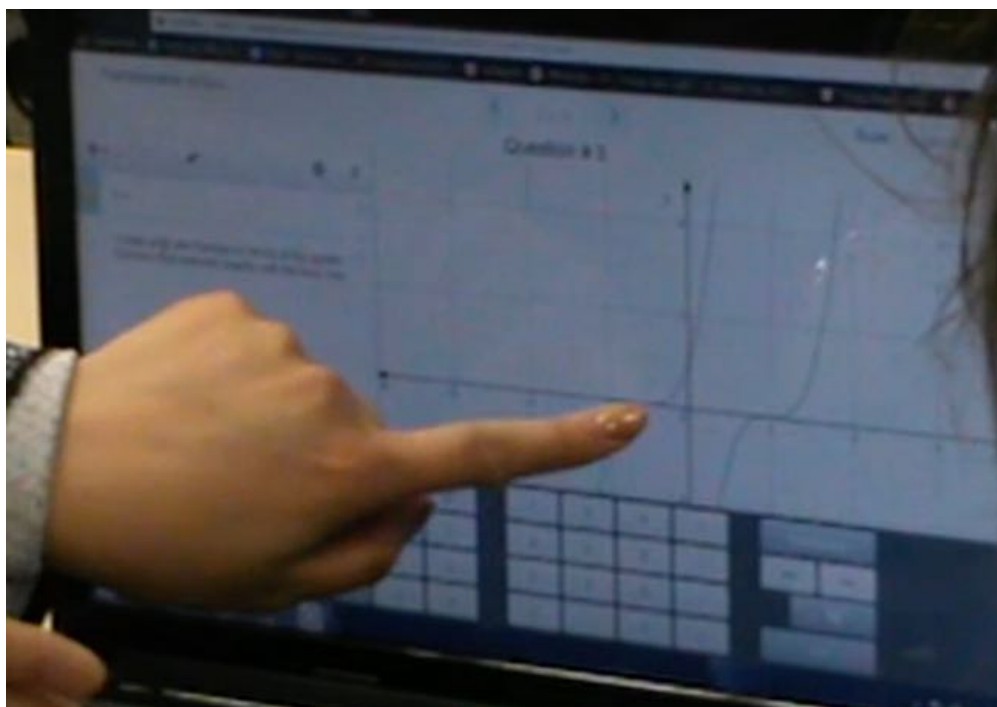


Figure 4.33: Source of data for transformation of functions using Desmos

#### Episode 88

In this episode, PSTs examine the points where a circle intersects the x-axis, starting from the definition of the circle, and try to establish their relationship with the absolute value equation. When the center of the circle is on the x-axis, they observe the coordinates of the points where it intersects the x-axis and expresses this situation as an absolute value equation (see Figure 4.34).

Nuriye: I think negative three and one.

Sude: It was like a distance from -1. When I brought the value of  $a$  to -1, its center also changed to -1.

Nuriye: Isn't our radius two right now?

Sude: Yes, two. Accordingly, it will be at a distance of two units in both directions.

Nuriye: Then the points where it crosses the x-axis are always  $a - r$  and  $a + r$ .

Sude: Exactly! We subtract  $a$  from its distance from zero.

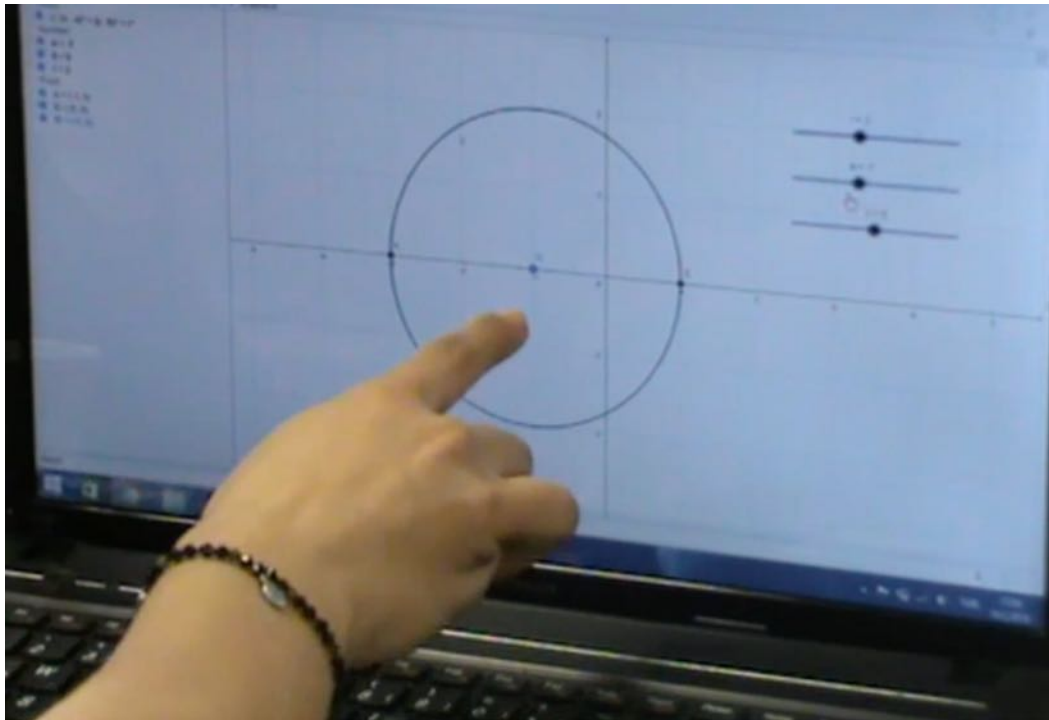


Figure 4.34: Source of data for absolute value equations

#### Episode 89

In the next episode, PSTs examine the angle properties of a circle on a given picture using GeoGebra. Using various tools of GeoGebra, they try to answer the questions posed to them by discussing them. Meanwhile, GeoGebra has provided enough data to make assumptions and claims (see Figure 4.35).

Sude: Sir! It should not pass over the stone. Stay a little outside.

Instructor: You decide that.

Sude: Don't let them notice when they're about to hit a stone. I think it's a little out of the way.

Nuriye: Can you zoom out the E point a little bit? Okay, this is good. Now let's create the angle and measure it. Let's make the segments and measure the angle.

Sude: Yes! This angle will be the answer.

Nuriye: If we bring it closer... Does the angle change?

Sude: Yes, the angle changes.

Nuriye: Okay. Make it closest.

Sude: This is the closest version.

Nuriye: Let it stay at 27.54 degrees.

Sude: I think we did.



Figure 4.35: Source of data for angle of danger activity

### Episode 90

In the next episode, PSTs observe the infinity limit of a function using a Spreadsheet. They examine the value of the function by dragging and copying the formula they wrote using the spreadsheet. They can comment on the value that the output values approach in response to the increase in the input values of the function (see Figure 4.36).

Arda: C2 minus 1007

Berk: *(Copies the formula by pulling it down)* Why did it just happen like this?

Instructor: What happened? Is there a problem?

Berk: There is no problem, sir... It may be because of the numbers. Did we plug the numbers in the wrong way?

Instructor: No! Go on!

Berk: Shouldn't it be closer to 3?

Arda: It's getting close already.

Berk: Sir, it came from eight and nine. Anyway, as the input values approach infinity, the output values will approach 3, but you are right. (*In the meantime, he continues to copy by pulling down*)

Berk: It's getting close to 3, see? Something with a maximum value of 9 at the beginning, but look as it goes to infinity...

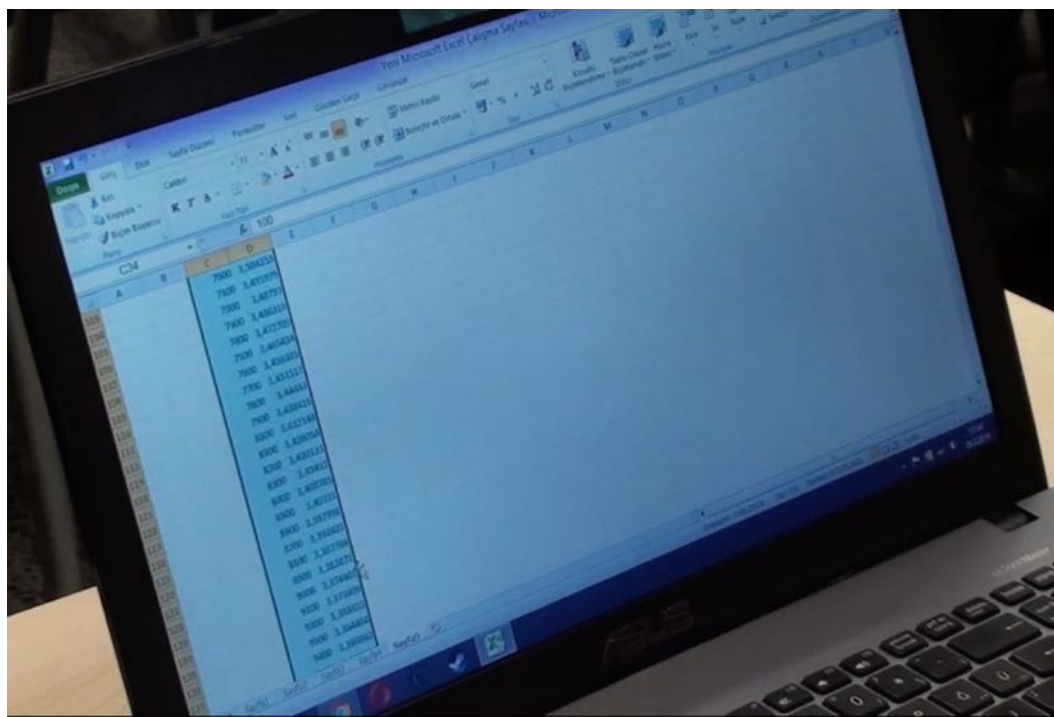


Figure 4.36: Source of data for limits using a Spreadsheet

### Episode 91

In this episode, PSTs examine the limit of the ratio of the consecutive terms of the Fibonacci sequence using a Spreadsheet. Then, they look for an answer to the question of how the limit would be affected if there were another series by changing the first two terms of the series. In response to this question, in the cells containing the formula prepared with Spreadsheet, they easily observe the effect of changing the value of only the initial two cells on the result and use this data to make assumptions.

Instructor: So, you're going to change the initial values instead of changing the formula, right? If you want, copy and paste it next to it and make the necessary changes.

Arda: OK, OK, it is here. Two became one.

Berk: What should I do there then? Let me do 2 and 1. OK, everything has changed. Sir, golden ratio again.

Instructor: Again? Didn't it change?

Berk: I told you so. So, Fibonacci wasn't that important then. The point is adding the previous two numbers.

Episode 92

In this episode, PSTs decide whether a probability game is fair using TinkerPlots. Using the simulation feature of TinkerPlots, they prepare a simulation as if they have played this game many times and examine the win rates. According to the data they obtained, they reveal their claims about the game (see Figure 4.37).

Sude: Now, these are the results. Let's see the notes for it. We'll look at the joint.

Nuriye: Aren't you going to carry the Joint status to the bottom of the table?

Sude: I'm trying to get this down. (*By grouping the data in graphical view in TinkerPlots*) I threw them aside. I'll see how many there are.

Tuba: Where did we make it?

Sude: I will throw the same ones aside. I can combine both of these. (*After combining*) The different ones became the same ones. We win when the same color comes.

Nuriye: Show me the percentages!

Sude: Where is it?

Nuriye: There's that percent sign; you'll click it. (*After the percentages appear, point to the number of attempts*) Let's increase it right here. Make 1000.

Sude: Let's speed it up. Let's not take the fastest one, so we can have some fun. (*After the simulation is over*) It's fair, sir.

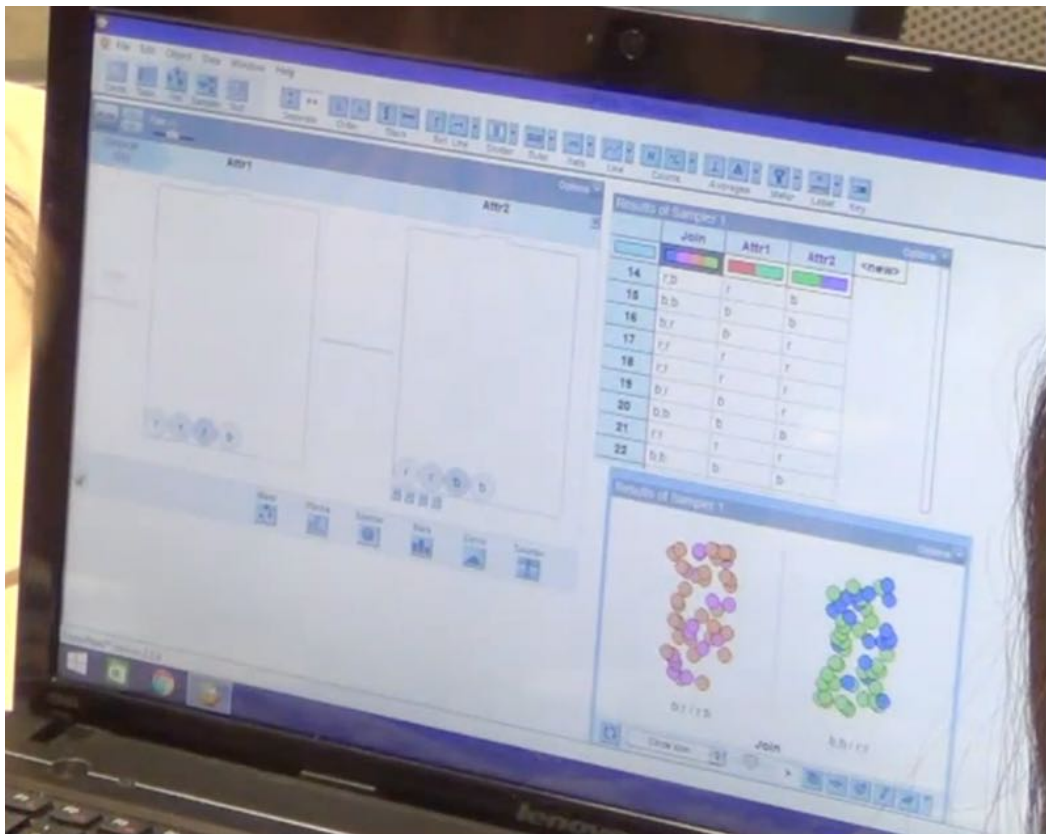


Figure 4.37: Source of data for probability game in TinkerPlots

#### 4.1.2.2 Technology as Source of Warrant

In TECA environments, technology plays an important role here as PSTs search for the warrant to support the data contained in the Toulmin model while constructing a conjecture. As Patsiomitou (2011) stated, technology provides a non-linguistic warrant and supports the continuation of the process in its ordinary course. This section will report the findings that the technology has been used as a non-linguistic warrant.

##### Episode 93

In this episode, PSTs are trying to understand why a formula about the periods of trigonometric functions is expressed that way. They examine the graphs for different values before and make an assumption. When they want to explain these assumptions to the instructor, they use GeoGebra to support their claims (see Figure 4.38).

Instructor: Now, do you understand why the formula changes for odd and even powers of this function?

Berk: Sir, we are talking about cosine right now. Because when the power is even, it cannot take a negative value. For example, now I see the following: (*Makes the slider value assigned for the power, an even number on the GeoGebra screen*) When  $n$  is even, the function does not take negative values. When  $n$  is odd, the graphs are very similar but only seem to change slightly at the points where they intersect. But it is always positive when  $n$  is even.

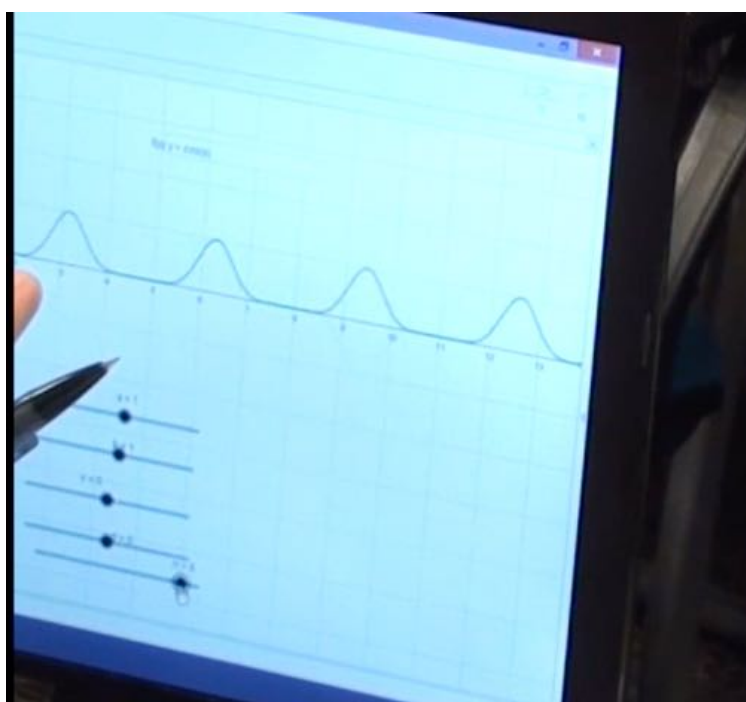


Figure 4.38: Source of warrant for periods of trigonometric functions

#### Episode 94

In the next episode, PSTs examine the transformations of trigonometric functions. For this purpose, they observe the graph of the function written based on the sliders they have created. She tries to back up his claim by using the GeoGebra as a warrant to persuade a group member of her hypothesis.

Sude: The period is not narrowing, it is expanding.

Nuriye: Can you increase a value a little more, Sude? Look, it is both narrowing and increasing from above!

Sude: I don't think it expands horizontally.

Nuriye: It is not expanding; it is already narrowing. It decreases as  $a$  increases. Doesn't it shrink?

Sude: It doesn't shrink. It stays the same. We can understand it like this. Let me put one point here. I put the x-axis where it cuts. (*As she changes the value of  $a$ , she wants to observe whether the coordinate of the point changes.*) Look at the x-intercepts! They stay in the same position.

Episode 95

In this episode, PSTs examine creating tables to generalize inequality solutions. First, when reviewing the signs of a quadratic polynomial function, they observe the effect of its coefficients on the graph (see Figure 4.39). Accordingly, they form assumptions about how they should make the sign chart.

Burcu: I can't see it. Could you show me what you said?

Berk: Now look, Burcu! This parabola has two roots, right? Here and here.  $a$  is positive, right? Are you aware that values less than the small root are always positive when  $a$  is positive? Greater than zero. Therefore, we can say that the function takes positive values when  $x$  takes values less than the small root. Of course, this only applies when  $a$  is positive.

Nisa: Think about that side, Burcu! Look! The graph is above the x-axis.

Berk: While the  $x$  values are getting bigger than the greater root, the function has positive values again. But it has negative output values for the remaining input values between the roots.

Nisa: Let's say it this way: the function takes opposite values between two roots.

Berk: But look, when we make  $a$  negative... What happened this time? The function is negative for  $x$  values less than the smaller root, so here. Again, it is negative for input values greater than the major root. Again, the reverse is true for values between roots.

Burcu: Okay, just the opposite.





Figure 4.39: Source of warrant for inequalities

#### Episode 96

In the next episode, PSTs are looking for answers to the questions of what is the extraneous solution and when it will be revealed. While answering the questions in the activity, they observed the effects of the algebraic operations they performed graphically using GeoGebra. Thus, while answering why an extraneous solution emerges, they can present a convincing justification with the help of GeoGebra.

Instructor: How many solutions does it have?

Berk: Two. Look now; these graphs intersect at  $x=5$ . They usually only cross at  $x=5$ , right? But these two graphs (*squared versions of the original functions*) intersect at  $x = 5$  and  $x = -3$ .

Nisa: Here, it appeared as a  $-3$  root.

Berk: For example, the value  $x = -3$  does not satisfy the original equation.

Burcu: When you write  $x = -3$ ... Of course not.

#### Episode 97

In the next episode, PSTs examine the properties of angles in the circle. They observe by dynamically changing the measure of the angle they created on the figure to support their assumption about the angle of danger in the activity.

Instructor: Now think about it this way. For example, there is a ship approaching this coast. What kind of change happens in the angle measure?

Sude: The angle gets bigger and bigger.

Nuriye: However, our limit is 27.54. After this value, the ship enters the danger zone.

Sude: For example, it can always travel safely around here since the angle is smaller than the angle of danger here. As you see, the angle measure increases as it gets closer to that area.

#### Episode 98

In the next episode, PSTs examine the limit of the ratio of consecutive terms of the Fibonacci Sequence using a Spreadsheet. After assuming this limit, they test it using the Spreadsheet. Thus, they obtain a warrant to support their assumptions.

Nuriye: But can I say something? It stays at 1.60. We came to the 600th term, and it is still at 1.6.

Sude: I am going to copy it till 1000.

Nuriye: Okay, let's do it until 1000.

Sude: It doesn't change any more. You see, I am increasing the number of decimal digits, but there is no significant change.

Instructor: So, what number do you think it approaches?

Nuriye: 1.61. We already knew that this was the golden ratio, sir.

#### Episode 99

In the next episode, PSTs are doing an activity about statistical concepts. They first discuss the question they asked within the group and make an assumption. Then, when they enter two more data as desired, they observe the change in the data they added in the GeoGebra statistics module. They use this action as a warrant to support their claim, as the software updates the formula according to the new values added and returns the result.

Berk: Look now. What is the median of this data set? 174, right? I'm adding 173 here. But the median is something different, isn't it? Shall I add 175 as well?

Arda: Okay.

Berk: Then the average is 174.

Arda: It is one and a half away and a half away.

Berk: No, but if the median is the number just in the middle, it should be. If not, we will do as you say because it must be the median value. Are we going to make this 23? *(He asks about the new number of data entries to update the formula)* Okay, done. Sir, we found it!

Episode 100

In this episode, PSTs first discuss whether the various probability games included in the questions in the activity are fair or not. They then simulate these games with TinkerPlots and experimentally test the accuracy of their assumptions (see Figure 4.40). They can confidently assert their claims at this stage, as the simulation provides a warrant.

Berk: It's not fair.

Arda: Fair! Look, the sum of the same colors is 38 plus 12.50. Different colors are 12 plus 38. So, they're both equal.

Berk: Okay, right! Sir, this game is fair too.



Figure 4.40: Source of warrant for fairness of a probability game

### Episode 101

In the next episode, PSTs realize that a function's derivative becomes another function based on the tangents drawn on the graph. They support their claims by using the feature in the prepared GeoGebra file (see Figure 4.41).

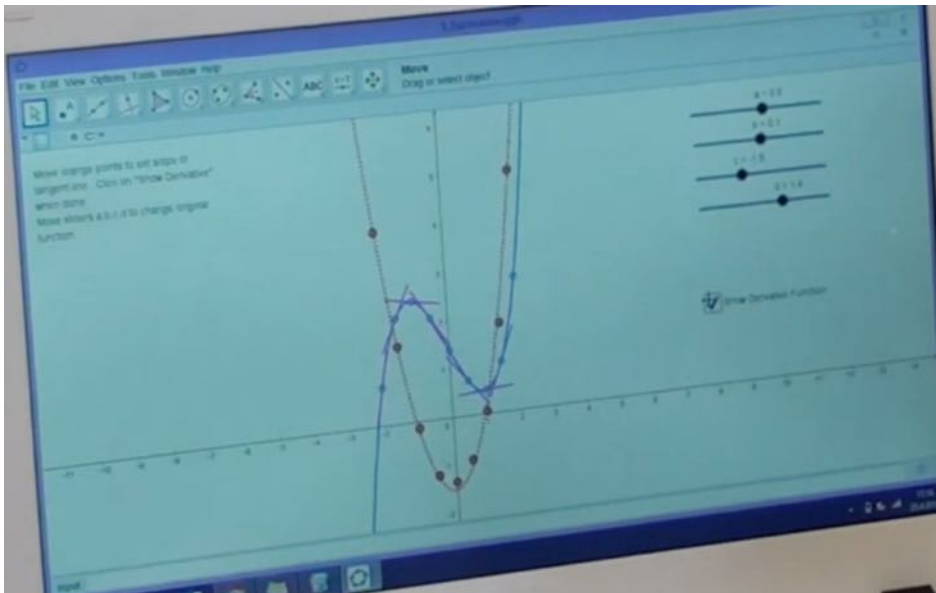


Figure 4.41: Source of warrant for the first derivative function

Nisa: It's like that, isn't it? Is that good?

Burcu: For example, we can make a tangent like this. Isn't it?

Nisa: Sure? It isn't happening.

Burcu: No. It should be on the hill.

Instructor: Are you done?

Nisa: We do, sir. Is that good?

Eda: Yes.

Burcu: All will follow each other according to a path.

Eda: This thing will come out.

Burcu: Up. The other is up. Click here. Shouldn't it go over it? Look at it! First derivative function.

#### 4.1.2.3 Technology as a Simultaneous Source of Data and Warrant

While seeking for a piece of evidence for the claim of the argumentation unit, technology sometimes may serve as both data and warrant. For instance, while PSTs work on the effect of the parameters on transformations of functions, they manipulate the sliders assigned for each parameter and see whether the predicted outcome has occurred or not. Since this value change in parameter effects its related visual (e.g., graph of a function, a geometric shape, etc.), it provides a warrant for their claim to support while it also provides data for the argumentation.

Episode 102

In this episode, PSTs examine the parameters' effects related to the functions' transformations on the graph. Using the tools in GeoGebra, they observe whether the impact of the parameters is as they predicted by dynamically changing the values. The screen (see Figure 4.42) provides the data they obtained during this observation and undertakes a task to support their assumptions.

Instructor: Super. Well, let's see now. Now let's change the values. (*At that time, typing  $\sin(x)$  in the input box changes the function*) You write  $\sin(x)$ , okay, good. Zoom in a bit so we can see more clearly. Okay, that's enough. Change "a", for example, make it 2. What do you expect to happen on the chart when it doubles?

Initially, the value is 1. Based on your claim, how will it affect the graph when you make it 2?

Berk: We expect a horizontal stretch.

Instructor: Horizontal or vertical?

Berk: Sorry. Vertical.

Instructor: Stretch vertically! Let's see if it will? Did it happen?

Berk: Yes.

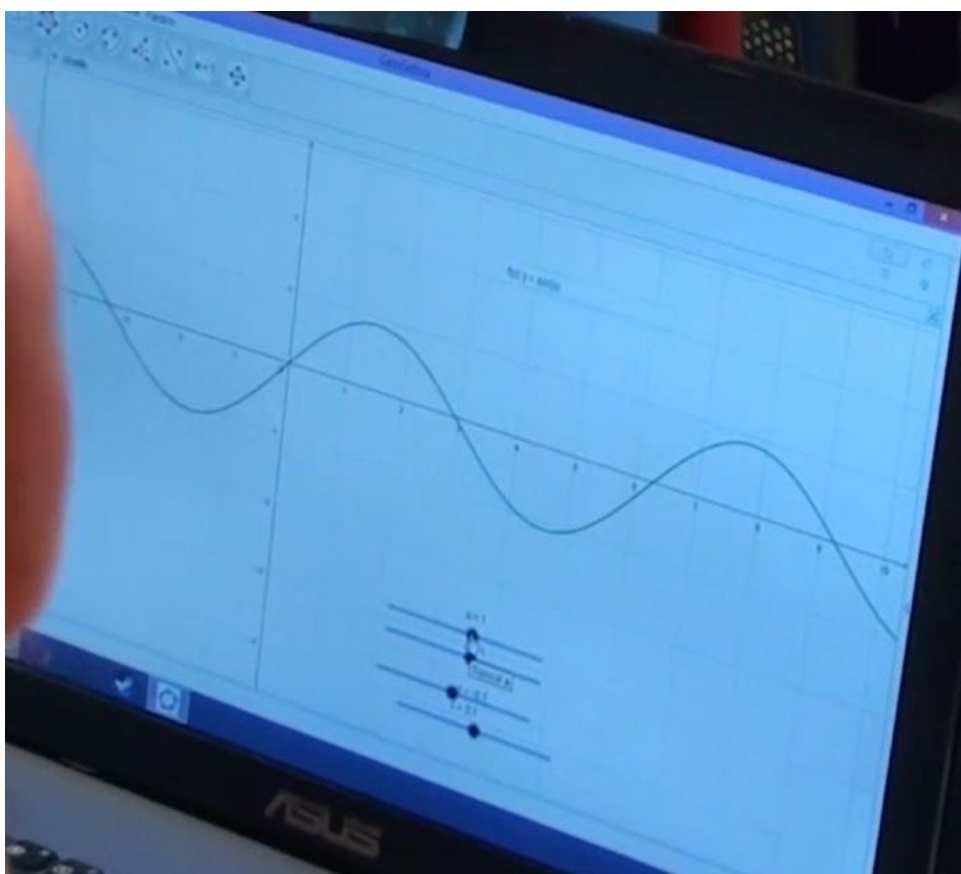


Figure 4.42: Simultaneous source of data and warrant for transformations of functions

### Episode 103

In the next episode, teacher candidates examine the relationship between the multiplicity of the roots of polynomial functions and the shape of the graph. Accordingly, they are expected to comment on how the graph can be drawn. Meanwhile, while examining the dynamic change effect of GeoGebra's tools on the

graph, the screen gives them data (see Figure 4.43). However, this screen also serves as a warrant in the argumentation process that progresses iteratively.

Berk: That's the root. Any other root? Is there a root here?

Nisa: Okay, but isn't it very unified over there?

Brooke: Okay. There's a root there.

Nisa: For example, isn't one a root right now?

Berk: Yes, there is one root right now. But for example, the factor where one is the root is also third order. Then we need to change the thing (*changing the values of a and b, which affects the root values*)

Burcu: Right now, the value of a is 5. One of the roots must be 5.

Berk: Yes, one of the roots has to be five, and it is.

Burcu: The other root is one because the value of b is 1.

Brooke: Okay. Let's say the root from a is 5; this is 1 (*they write the function they graphed in GeoGebra on paper*)

Burcu: Let them stay fixed, but let's change things.

Berk: But look, for example, 5 came from a, and since we took the value of m, which is the power of a, as 1, it became an odd multiplicity. Therefore, the sign of the function with a value of 5 will change. Look, it's changed. What happened here? It was negative and negative again because this root has an even multiplicity.

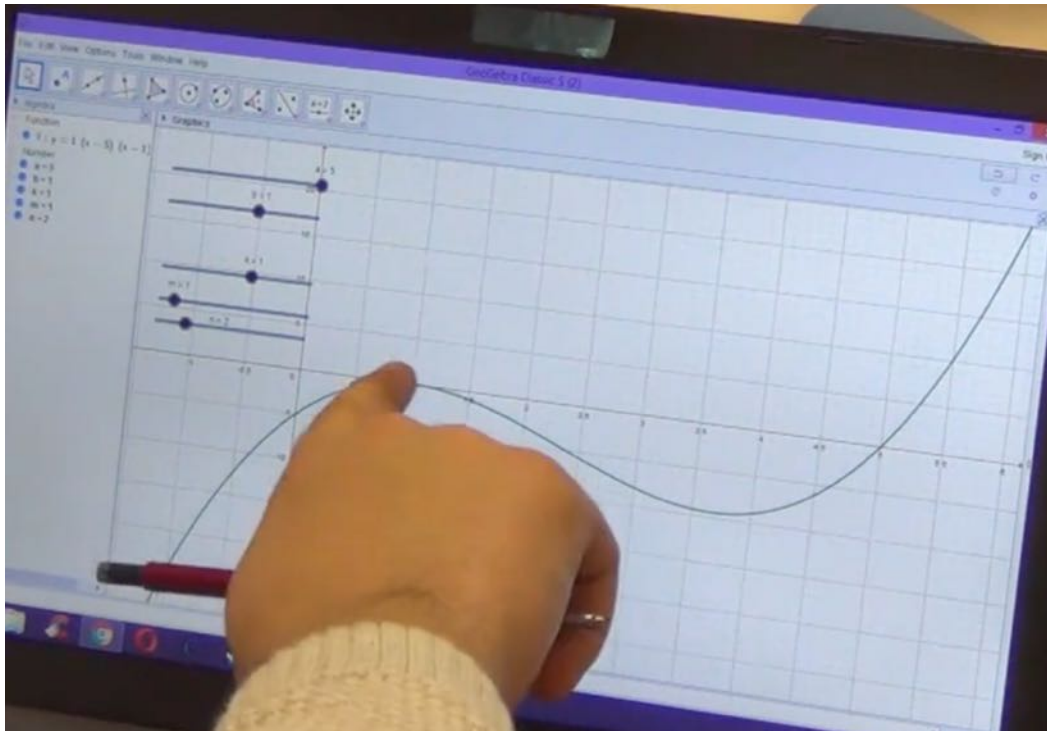


Figure 4.43: Simultaneous source of data and warrant for the multiplicity of polynomial functions

#### Episode 104

In this episode, PSTs try to make sense of absolute values by associating equations with the distance definition of absolute value. Using GeoGebra's slider tool, he observes the translation of the circle (see Figure 4.44). They make a claim using the data from this observation, but the same screen also allows them to test this claim.

Sude: It's like we added  $a$  to the distances to zero.

Nuriye: It's going to be  $x$  minus  $a$  squared equals  $r$  squared. So,  $x$  is equal to negative  $a$ ... Is  $x$  minus  $a$  squared in the square root of  $r$ ?

Sude: Yes.

Nuriye: Then  $x$  minus  $a$  became an  $r$  and  $a$  negative  $r$ . Then  $x$  became an  $r$  plus  $a$ . (After a moment of silence) Did we do something wrong? There's  $a$  negative  $r$  over there, right? It became negative  $r$  plus  $a$ .

Sude: Actually, if we think of it as absolute value  $x$ , it's as if you didn't add an  $a$  to its distance from zero. At zero, these are our roots. I set it as points  $a$  and  $b$ .  $a$  and  $b$  were also... They were points with an absolute value of 2, the distance from the



origin. But I made this  $a$  1. I have added  $a$  to the absolute value of the points where it intersects the x-axis.

Tuba: There is a shift.

Sude: Exactly! We're translating it for  $a$  units. It affects the absolute value up to one.

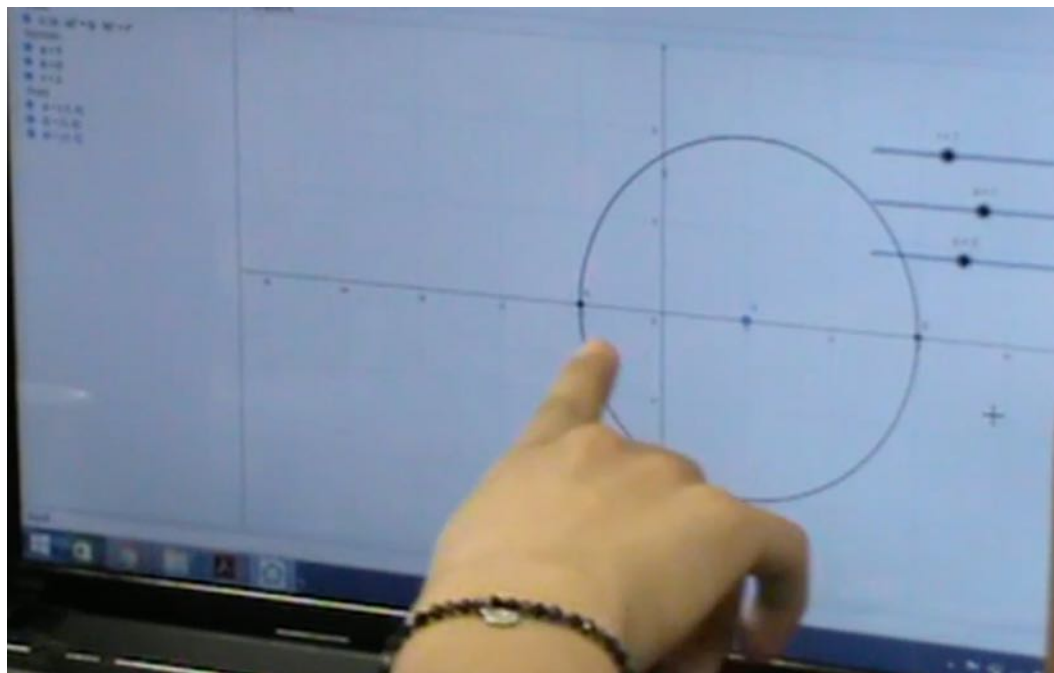


Figure 4.44: Simultaneous source of data and warrant for absolute value equations

### Episode 105

In this episode, PSTs try to find the box's volume with the maximum volume made from cardboard. They use Spreadsheets for this purpose. They see the volume by entering each of the dimensions of the box in a cell and finding their product (see Figure 4.45). They then comment on the maximum volume by entering different values. When only the cell view of the Spreadsheet misleads them, the Spreadsheet screen helps them make a more accurate claim by drawing graphs with the help of the instructor.

Arda: Instructor says if it's not 88. For example, if you plug a decimal value between these, you will get more. (*Looking at the graph on the computer here, he notices that the value will produce a larger volume once in a while.*)

Berk: Well, then we will have to look at the infinite value of this. Umm ... can't we see it from the chart anyway? We cannot see it.

Arda: We'll see.

Berk: No, I cannot see it because we will draw the graph according to these data.

Arda: Give it a try. Enter that data too.

Berk: Yes, this is the maximum value of the volume

Arda: Not precisely.

Berk: Exactly. There is a bigger one.

Arda: We need to do it in more detail. Are we going to enter decimal values like this?

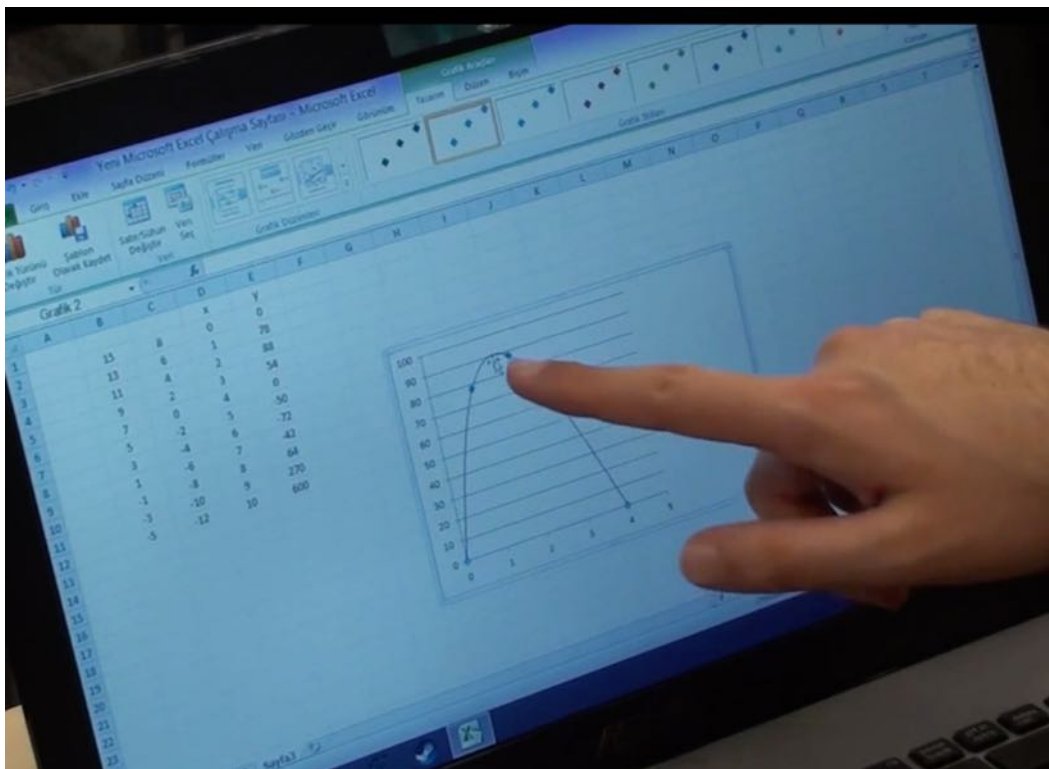


Figure 4.45: Simultaneous source of data and warrant for an optimization problem

### Episode 106

In this episode, PSTs use GeoGebra's statistics module to observe the effect of multiplying all the values in the data set by the same number. When you write the multiplied values in a new column and copy the previous formulas, the values are

updated automatically. Thus, the GeoGebra display simultaneously provides data and warrants to establish and support its claims.

Instructor: What happens when you convert to cm?

Berk: Does the standard deviation increase tenfold with variance? Yes, it does.

Instructor: You are multiplying each data by ten.

Berk: It's all changing. We're going to multiply all descriptive statistics by ten.

Sude: All of them will be multiplied by ten.

Instructor: So, if the variance is multiplied by ten, is the standard deviation also multiplied by ten?

Berk: Not! Standard deviation... Wait a minute. The variance is multiplied by one hundred. Here it is.

Sude: Yes, variance is multiplied by one hundred, sir.

#### **4.1.2.4 Technology as a Source of Refutation**

According to Toulmin's (2003) argumentation model, no matter how accurately a process progresses, there may still be cases where the assertion made is refuted in some cases. This component is called rebuttal in the model. A rebuttal is an acceptable presentation of another valid opinion contrary to a claim. Knipping and Reid (Knipping & Reid, 2019) added a "refutation" component to the argumentation model. Although it can be understood as the same as "rebuttal" in the Toulmin model, they explain the differences. A rebuttal is local to any step in argumentation and specifies exceptions to the result. However, refutation completely rejects an argument. Despite focusing on the core components of Toulmin's model in this study, situations emerged that ultimately rejected and concluded the ongoing collective argumentation. Therefore, the conditions that emerged in the analysis were coded as a refutation, as described above.

This section will report the situations where technology plays a role in putting forward a valid refutation after a claim is presented in the collective argumentation process.

## Episode 107

In the next episode, PSTs try to draw a triangle with given side lengths using GeoGebra. When they make the drawing that they think will get the result, they can see that their thoughts are wrong by receiving direct feedback from the GeoGebra screen (see Figure 4.46). Thus, they realize the lack of their claims with GeoGebra and continue revising the argumentation process.

Tuba: Can't we use the circle tool with a certain radius?

Nuriye: He will use it anyway. Their radii will be 9, 6, and 5 units.

Sude: This is nine.

Nuriye: Press OK.

Sude: We will also make a circle with a radius of 5 units over it. Now, what happened here? No, it didn't.

Nuriye: No, I think it did. If we draw it from here, the side length of nine units will be satisfied. If we draw it from here, it satisfies six units.

Tuba: Can't you connect it up here?

Nuriye: Let's put it together here. We had nine units from here. We also had six units from here. So how do we get the 5?

Sude: This should have been five units automatically, but it didn't. This method should have worked. What is the problem?

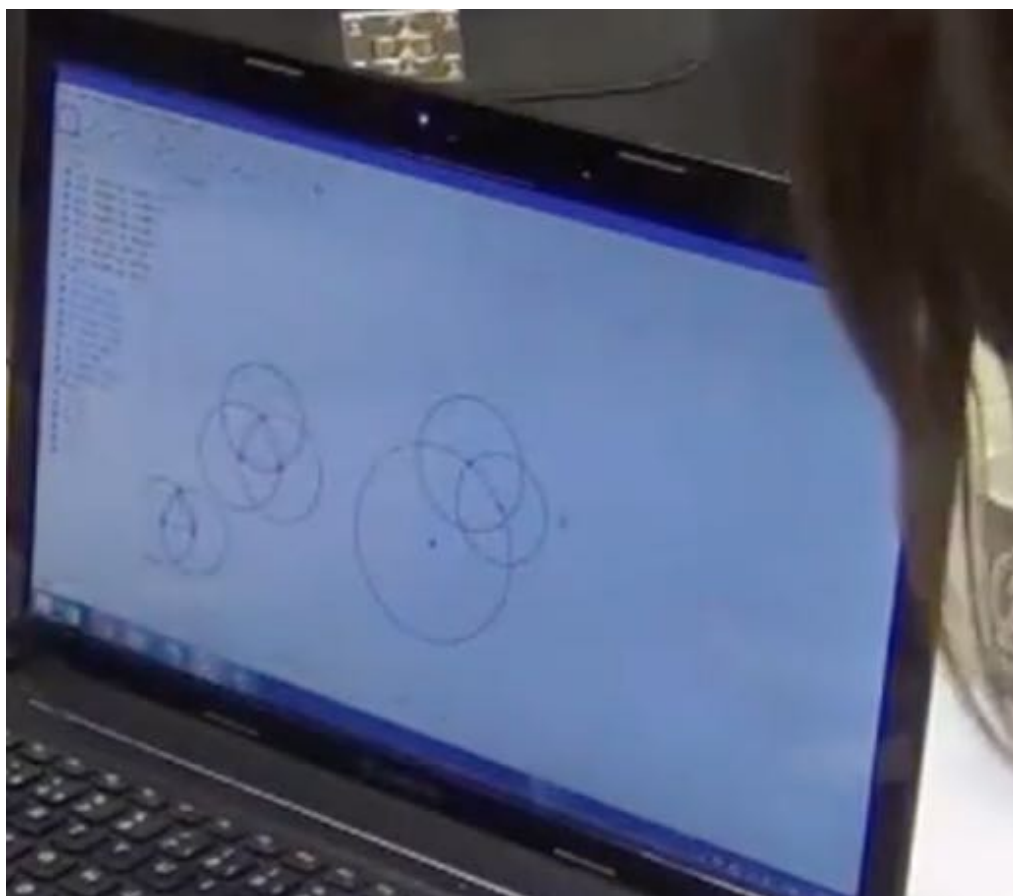


Figure 4.46: Source of refutation for triangle construction

#### Episode 108

In the next episode, teacher candidates examine the value to which the ratio of consecutive terms of the Fibonacci sequence approaches. They assume by looking at the ratio of the first terms. However, the value they see on the Spreadsheet screen does not support their assumptions (see Figure 4.47). First, they think they made a mistake and copy the formula again. When they notice that the same value comes out, they realize their argument is wrong. They rearrange their claims according to the values on the screen.

Instructor: Now, I want you to look at this question. Does the ratio  $\frac{F_{n+1}}{F_n}$  get closer to a specific value as  $n$  increases?

Eda: 1/0, but this is not possible. The next one becomes 1/1. Next would be 2/1.

Nisa: If I write a formula. (She writes a formula to find the desired ratio and copies it by dragging it down)

Burcu: What are we trying to find?

Nisa: How does this value change as  $n$  increases?

Eda: Hmm... It's getting smaller.

Burcu: Is it getting smaller?

Eda: We started with 2, then it takes smaller values.

Burcu: It's always been the same around here, Nisa. Why?

Nisa: Yes, I may have done wrong while dragging it.

Burcu: Why is there a mistake?

Eda: Maybe the limit is 1.6. They are approaching the golden ratio.

Nisa: Yes. It seems you are right! It doesn't change.

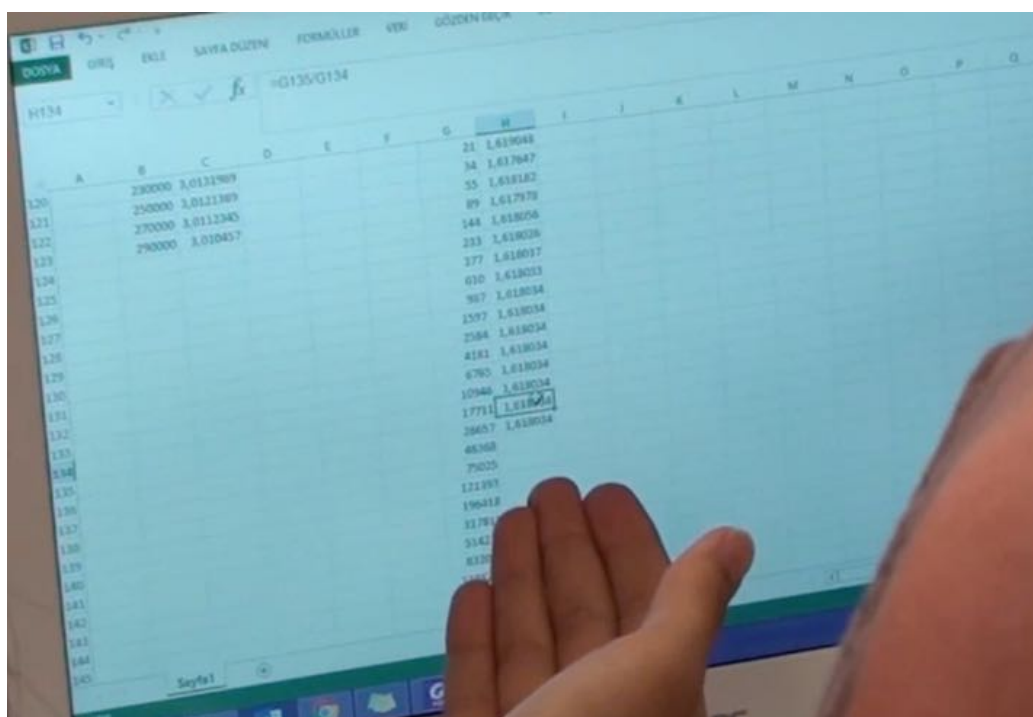


Figure 4.47: Source of refutation for limit of a ratio of two terms

### Episode 109

In this episode, PSTs examine the transformations of functions. They set their claim about the effect of parameters on the graph. The Instructor comes to the group and tests the accuracy of these claims together. Orienting themselves allows them to see the error in one of their claims. Of course, in seeing this error, GeoGebra plays the

role of refutation and helps them understand that they should make a change in their claims.

Instructor: Okay. Increase it by 1 unit. How many units have been shifted? Now it has to shift from (0,0) to (1.0), right? (*It shows the point passing through the origin on the screen*) Where did it come from? (*They noticed that it was not true.*)

Nisa: Sir, could it be because our frequency is three?

Instructor: But I was hoping you could show the effects of the parameters on the graph independently of each other. In other words, when I change this parameter to this, you should be able to say that this is the effect on the graph independently of the others. What will be the effect of  $d$ ?

Eda: Up-down.

Instructor: Okay. Just think about how you can solve that horizontal translation problem.

#### **4.1.3 The Characteristics of Technological Action Modalities During Argumentation**

As PSTs performed collective argumentation during the technology-enhanced tasks, they performed actions, afforded and mediated by the software programs utilized in the course, which helps to construct argumentation units. This section first presents these actions in two broad categories defined by Hollebrands (2007) as reactive and proactive actions. These two broad categories will be further elaborated on and presented in sub-categories.

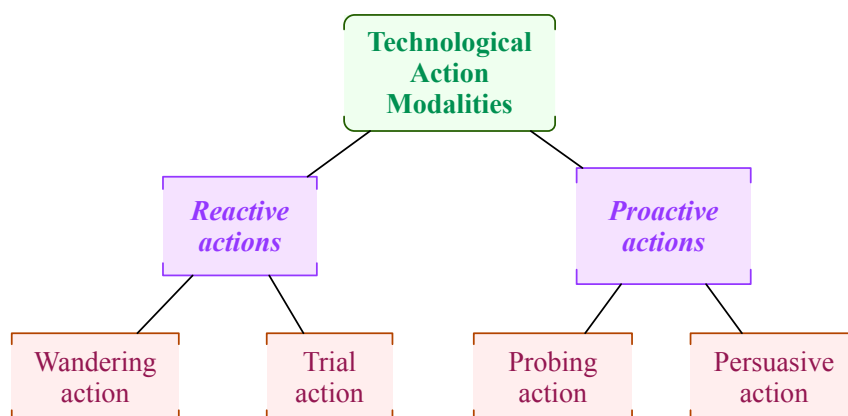


Figure 4.48: Technological action modalities

#### 4.1.3.1 Reactive Actions

By playing with variant features of the software like sliders, dragging, etc. PSTs explore invariants, regularities to move from empirical stage to theoretical stage. These are the situations in which teacher candidates' reasoning develops depending on the visual appearances provided by technology. In other words, they start these actions without knowing precisely what the technology will provide them visually before they started the action. They can be considered as the actions bringing PSTs to the next step according to the data they have obtained without any expectations. These actions can be grouped into two sub-categories, namely, wandering actions and trial actions.

##### 4.1.3.1.1 Wandering Action

This section reports the actions performed during the task to discover or find out some essential features related to the investigated object without a plan. PSTs demonstrate this action while using math software independently, without any suspicions or expectations. In other words, the aim is to use the software's features within the framework of the task given to them, like someone idly wandering around



in a shopping mall. If they notice something unusual, they use it as a transition to the following stages.

#### Episode 110

In this episode, PSTs are expected to comment by examining the graphs of the given functions. They also begin to analyze the functions by drawing their graphs using GeoGebra (see Figure 4.49). They observe the similarities and differences of the graphs as the first step that will underpin the process of creating a conjecture.

Berk: Okay. This is so confusing, though. Here things got even more complicated. But the same logic again. In the following function, for example,  $x$  is multiplied by three and subtracted by 2. That's why it's widened a bit and shifted on the axis. What's the difference between these two? It both expands a little more and slides on the axis. In that, it doesn't slide but expands. Why? Because there is nothing. But it can slide. Let me see.

Arda: It's sliding.

Beril: Is it sliding or not?

Arda: Exactly, right!

Berk: The input is changing. Here, too, it reflects upside down and slides on the axis. I mean, it shifts both horizontally and vertically.

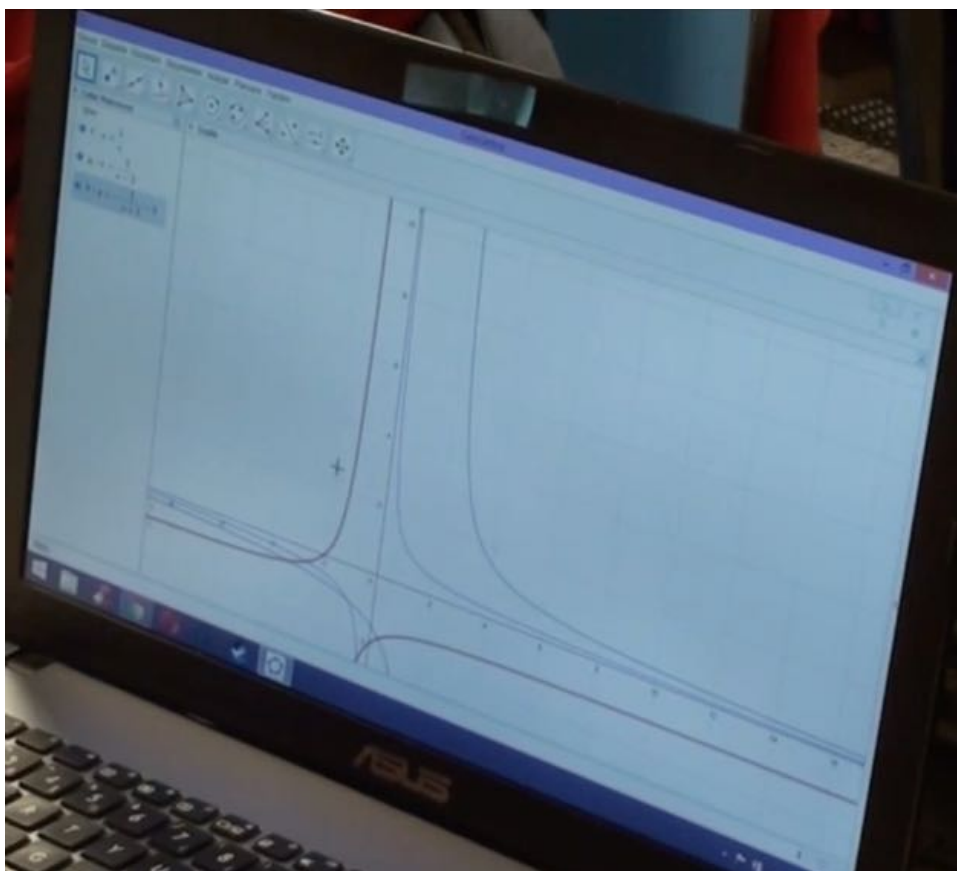


Figure 4.49: Wandering action for transformations of functions

#### Episode 111

In this episode, PSTs observe solving equations using a graphical approach. However, they are unaware of what might happen when they observe the graph using GeoGebra's features (see Figure 4.50). For this reason, they try to form an idea by examining how the equation they wrote creates an image.

Tuba: Sir, can we draw graphs in GeoGebra?

Instructor: You can do anything you want. Paper-pencil or technology; you can use whatever you want. Don't just solve the equation using a CAS like Wolfram.

Sude: What was the equation? Two times x to the sixth.

Nuriye: Minus three times x to the fifth, plus ten times x to the fourth, minus eighteen times x to the third, minus two x squared, minus fifteen x minus ten equals zero.

Sude: It's going. Something crazy happened? What's going on with it? (*They are trying to determine the point where it cuts the x-axis by zooming in on the screen*) It goes like this. It's approaching.

Tuba: I wonder if all the roots overlap.

Sude: The graph cut the axis precisely in half and two.

Nuriye:  $-1/2$  and  $2$ .

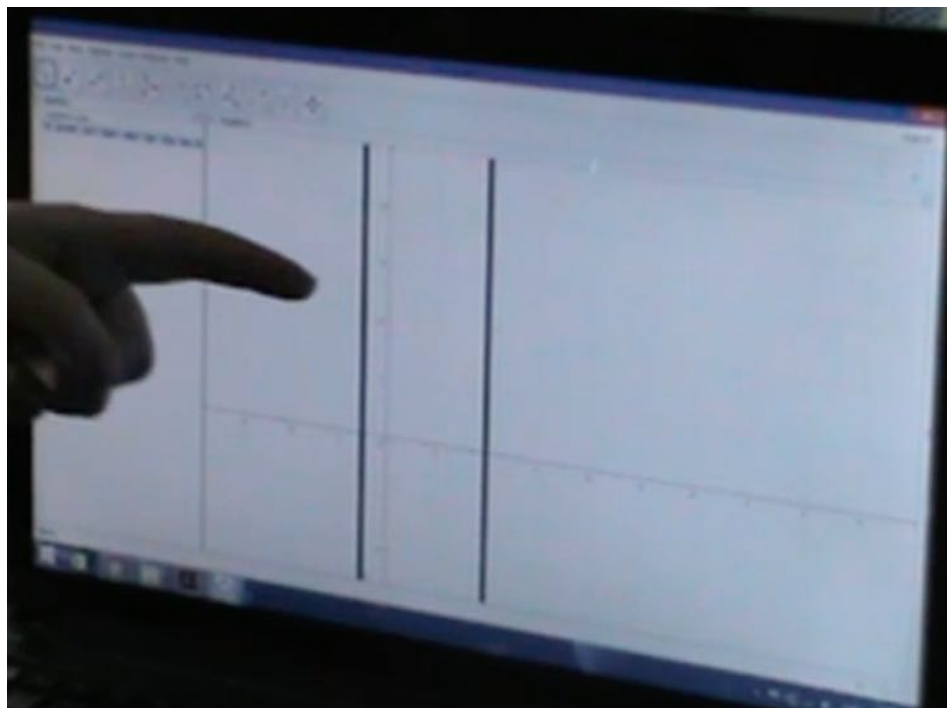


Figure 4.50: Wandering action for solving equations

#### Episode 112

In the next episode, PSTs learn to solve inequality solutions using the diagram method. They also observe how to create the diagram required for the solution by examining the signs of the functions and the multiplicity of the roots with a graphical approach in GeoGebra (see Figure 4.51).

Sude: Let's go up! I made the value  $2$ . The graph was like this because it was the third degree. For one, it was root with double multiplicity. At zero, there is an odd multiplicity. Yes, that's true! It was odd in one and even in two.

Nuriye: Isn't it like that because the rank is  $3$  in total?

Sude: Yes. But this will determine the multiplicity of the root. Look, for example, now both of them are even numbers.

Nuriye: Okay then we have  $m + n$  roots.

Sude: Exactly! We have  $m + n$  roots.

Nuriye: Shall I say something? It's always positive when they're both even?

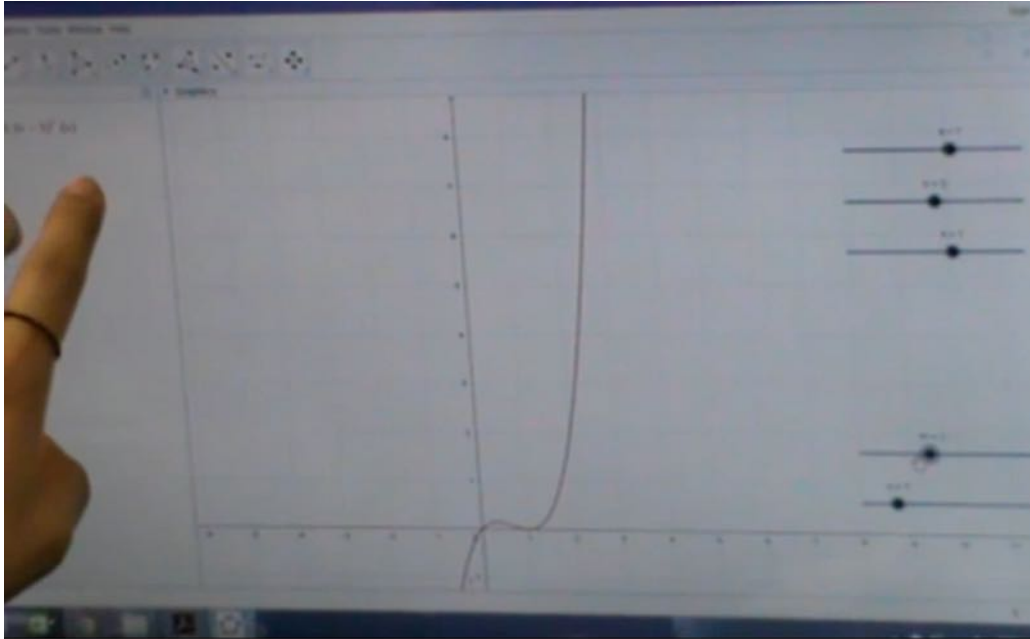


Figure 4.51: Wandering action for the multiplicity of the roots

### Episode 113

PSTs are asked to draw an equilateral triangle using Greek Construction Rules in this episode. However, they think they can create this triangle just by drawing, and they proceed by trial and error (see Figure 4.52).

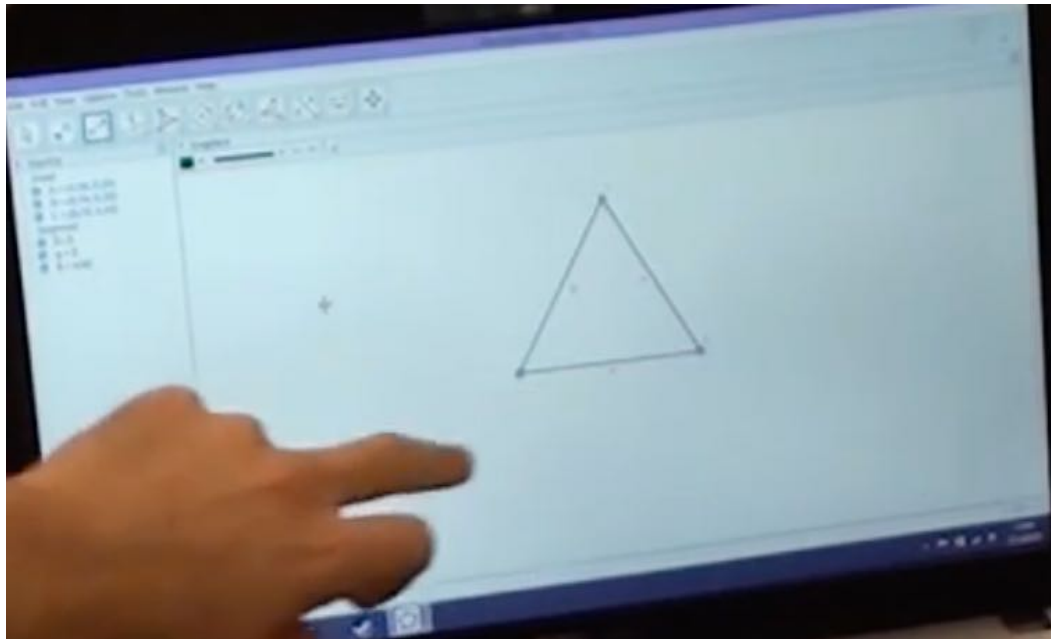


Figure 4.52: Wandering action for geometric construction

Berk: If I can use it anyway, it's ok. For example, if I choose from here, it's five units. It didn't happen.

Arda: It happened; it did. Could you do it again? Change the coordinates of that C. And do the same from B.

Instructor: Remember, guys! You can use only Greek Construction Rules. I don't want you to use GeoGebra's ready-made tools. So, imagine you have a compass and a ruler. So, the only things you can use are the straight-line drawing tool and the compass.

Berk: Ok, now it's the same thing. Sir, is it like this?

Instructor: How did you do it?

Arda: No, it didn't happen. Delete this and from B...

Berk: Is it because of the angle? We drew another one of these.

Arda: Connect points D and B.

Berk: They don't come together.

Arda: Let's move point C. It will be ok when they meet.

Berk: That's the angle you need to adjust.

Episode 114

In this excerpt, PSTs will construct the geometric figure formed by inscribed angles that pass through two points and have the same measure. However, since this is not given explicitly to them at the task, they should notice this for themselves. They're using GeoGebra's tools to realize that (see Figure 4.53). At this stage, since they have no idea about the shape, they try to make a drawing under the desired conditions by testing the tools.



Figure 4.53: Wandering action for angle of danger task

Nuriye: Let's put a point here first. Add another one here. Now there should be an arc tool over there. But this requires a center.

Sude: We need an arc drawn without the need for a center. There is this tool, semicircle by two points. But we don't want a semicircle.

Nuriye: I think we should put a dot at the end of the area where these rocks are. Let's draw an arc passing through three points.

Sude: We can do this if we put one more point (*meaning the arc drawing tool passes through three points*). Let's put a few here. We have to accept the largest of these.

Nuriye: Let me tell you something. There's nothing like a circle here.

Sude: I think so. It doesn't look like a circle at all right now.

Nuriye: Click that, click an E, and click here.

Tuba: But we have to take it in the broadest possible way.

Sude: Yes, it will be like that.

Nuriye: Let's draw another one from F. It should also contain that stone... But that stone remained outside. Then it's not what we want.

Sude: What if I move point E a little bit inwards?

Episode 115

In this episode, PSTs examine the limit of a rational function when  $x$  values approach infinity using spreadsheet software. However, since the vertical asymptote value of the function is 1007, they lead to a wrong result when they increase the values up to that value one by one. But they know from the calculus course's limit information that the function's limit must be 3. That's why they try to observe why the value they get is not the expected value.

Burcu: It will change depending on  $x$ .

Nisa: I will write equals here. It should be  $3x+1$ .

Burcu: You will put it in parentheses. Divide it by three. Then times, open parenthesis...

Nisa: If it's a problem for you to do it slowly, you can take it.

Burcu: No, no. We learn together.

Eda: I don't understand Excel at all.

Nisa: When we pull this one down, these will change.

Eda: What would we find? What does the input say when it gets the highest value?

Nisa: What happens to these values as  $x$  values increase? We will look at it. Let's copy a little further down.

Burcu: What happens to the number?

Nisa: Look, after -0.04, it becomes -0.09. Then it becomes -0.10. Values are getting smaller and smaller.

Burcu: Yes, the number is getting smaller. What happens when  $x$  becomes 1008?

Can we give value directly here?

Nisa: Let me do this 1008 right now. Hmm.. Where does that change?

Eda: Where is it positive?

Nisa: It was getting smaller in negative values.

Burcu: Then something must have happened.

Episode 116

In the next episode, PSTs examine the fairness of a probability game. For this purpose, they try to find the probability of winning the game by using the features of TinkerPlots (see Figure 4.54). Although it has more useful features, they try to decide by only counting the outputs TinkerPlots provides after simulation.

Nisa: We looked at the probability of coming blue. Blue - blue. Now we have to look at the likelihood of getting a red.

Burcu: Red will come from the first one... Say the same.

Eda: 8, 9, and 10.

Burcu: What did you count?

Eda: The ones of the same color.

Nisa: Yes, 10 of them. (They are counting by checking the outputs one by one)

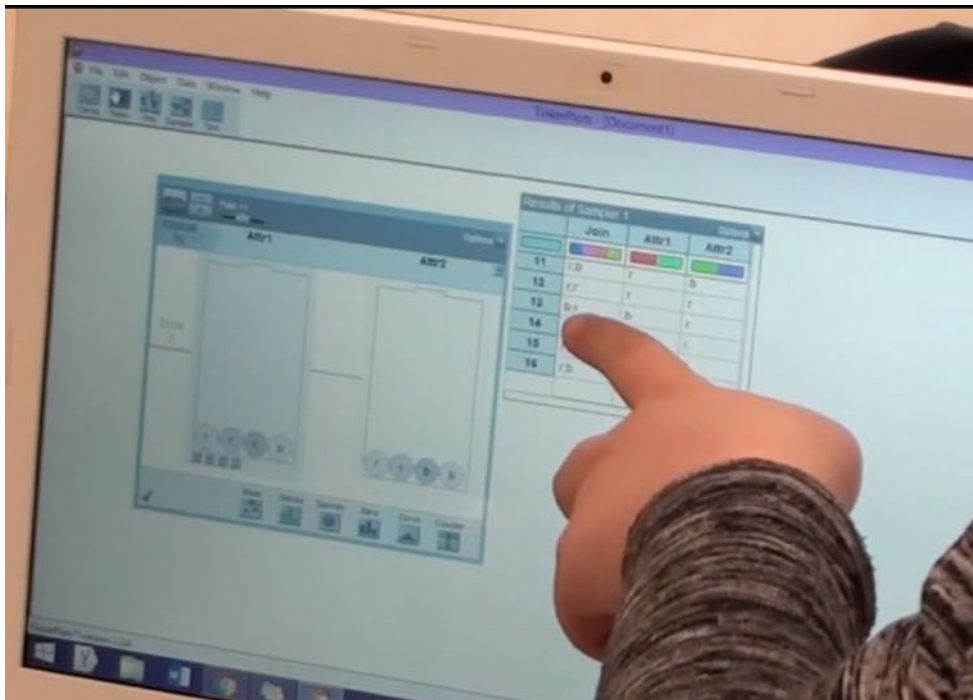


Figure 4.54: Wandering action for fairness of a probability game

Episode 117



In this episode, PSTs examine the graph of the derivative of a function. As stated in the question, they are trying to get an idea about the locus of point C according to possible situations by dragging point A.

Instructor: Now, guys, the question is: The yellow line is tangent to  $f(x)$  at B. Slope of the tangent is displayed. Move point A by dragging it or using the slider a. What can you say about the position of point C? You can also change the function if you want. But don't open the algebra view?

Berk: (*observing point C by dragging point A*) Sir, point C is always on a line.

#### 4.1.3.1.2 Trial Action

This type of action primarily builds on some features that stand out as a result of wandering action. It is tested by trial and error that salient features are random or provide a specific pattern or feature. This trial and error is the basis for claiming due to conscious actions in the next stage. The actions performed during the task kept the discovered features constant while altering the others to establish a conjecture. During this action modality, PSTs generally realize some features but cannot name them meaningfully. This section will report trial action findings.

Episode 118

In this excerpt, PSTs examine the relationship between the roots of a quadratic polynomial function and its coefficients. They test the accuracy of their previous observations about the function transformations as they are specifically working on the parabola (see Figure 4.55).

Nuriye: For example, what happens when a is 0?

Sude: It becomes linear. Because the square no longer exists. When c is zero, it does not affect the roots. (They only comment on the shape by playing with the parameters but don't have any thoughts about the points where it intersects the x-axis.) I set b to zero so it doesn't affect it. It changes on the y-axis. since a already affects the existence of  $x^2$ .

Nuriye: As a gets bigger, then its solution set gets smaller.

Sude: Because what happens as a gets bigger? Shrink!

Nuriye: Bring b to negative values.

Sude: It became -2.

Nuriye: Let's change c as well.

Sude: We already know that c makes vertical translation.

Nuriye: Now let's change a. I'm telling it to cut the x-axis.

Sude: Okay, I took it down a bit.

Nuriye: Okay. Pull down a little more.

Sude: I made c a negative one.

Nuriye: Let's change a now.

Sude: I increased a. It makes it shrink.

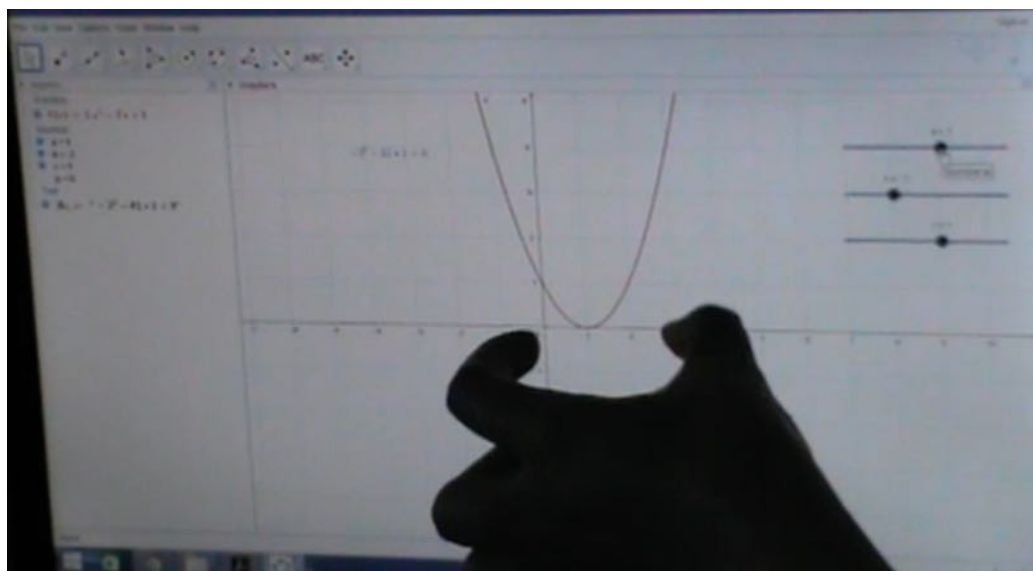


Figure 4.55: Trial action for the effects of the coefficients of a parabola

#### Episode 119

In the following excerpt, PSTs are trying to draw the triangle given the side lengths according to the construction rules. They realized that the distance from the points they obtained from the circles' intersection to the circles' centers could be thought of as the side length of the triangle (see Figure 4.56). They are experimenting with drawing the desired triangle with three different side lengths in this direction.

Arda: We get circles in these radii. Their intersection points become the vertices of the triangle.

Berk: Yes, you're right. So, let's draw a line segment again. No need for that, right?

Arda: Yes! Directly draw three circles.

Berk: Okay, but where will they intersect?

Arda: Are we going to take your tangents?

Berk: I think we will draw a 9-unit long line segment.

Berk: Look, I found it. The radius of that big circle is 9 units, that one is 5 units, and that one is 6 units. Did you understand?

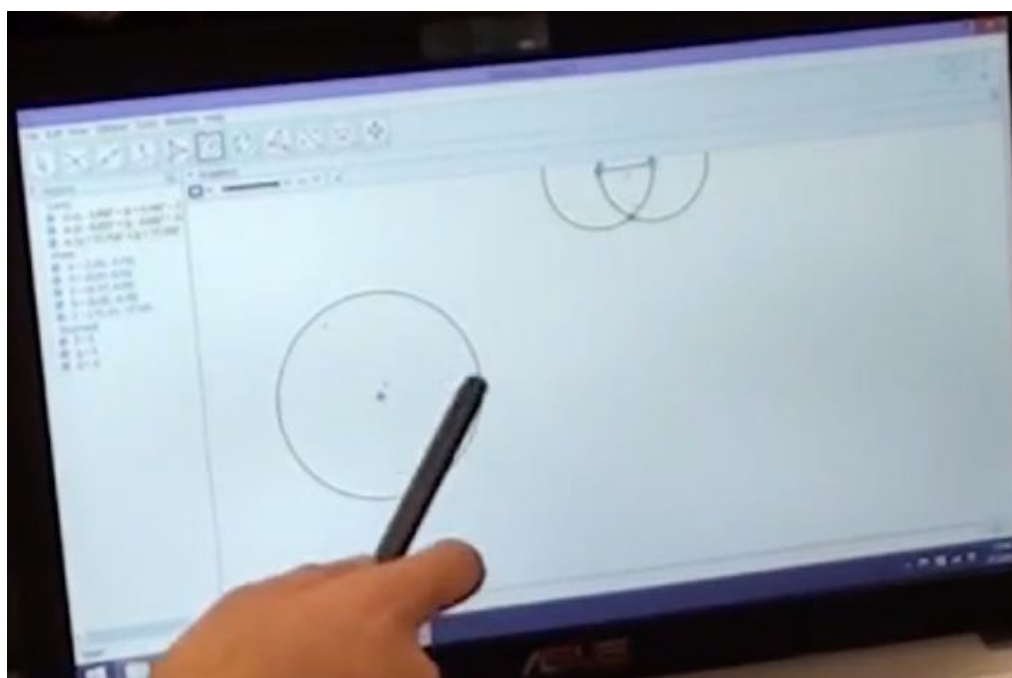


Figure 4.56: Trial action for constructing a triangle

#### Episode 120

In this episode, PSTs must draw a circular arc with two points to answer the task question correctly. This circle should be the smallest circle to cover all the rocks in the danger area. For this purpose, they try to find out what kind of drawing they should make by choosing the appropriate tool in GeoGebra (see Figure 4.57).

Nuriye: Click on it, click on point E, and click here.

Tuba: But we have to take it broadly.

Sude: Yes, it will be like that.

Nuriye: Let's draw from F if it also includes this stone. That stone was left outside. Then this is not the circle we want.

Sude: What if I move the E a little bit inwards?

Nuriye: Now, we will create the angle CEF.

Sude: Let me move point E a little. It would have been enough if we had made one like this initially. We would measure the angle accordingly.

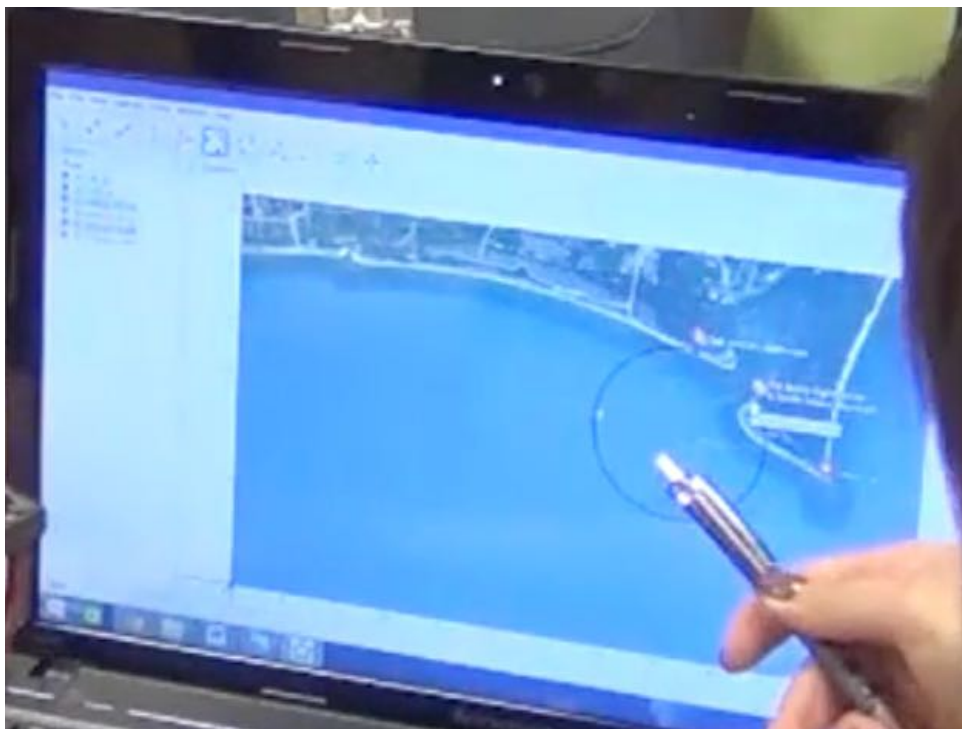


Figure 4.57: Trial action for angle of danger task

#### Episode 121

In the following excerpt, PSTs examine the ratio of consecutive terms of the Fibonacci Sequence. They observe whether this ratio approaches a particular value as the term index increases. They try to reach the desired number by trying different things in the Spreadsheet software (see Figure 4.58).

Arda: Drag it a little further down.

Berk: Look, it's not like that.

Arda: Now drag them again... Exactly. There is nothing under it, there is no number at the bottom, so it is zero.

Berk: Yes, that's exactly why! You are right.

Arda: It will gradually approach zero. No, it's 1.6.

Berk: It looks like 1.6 but let's see.

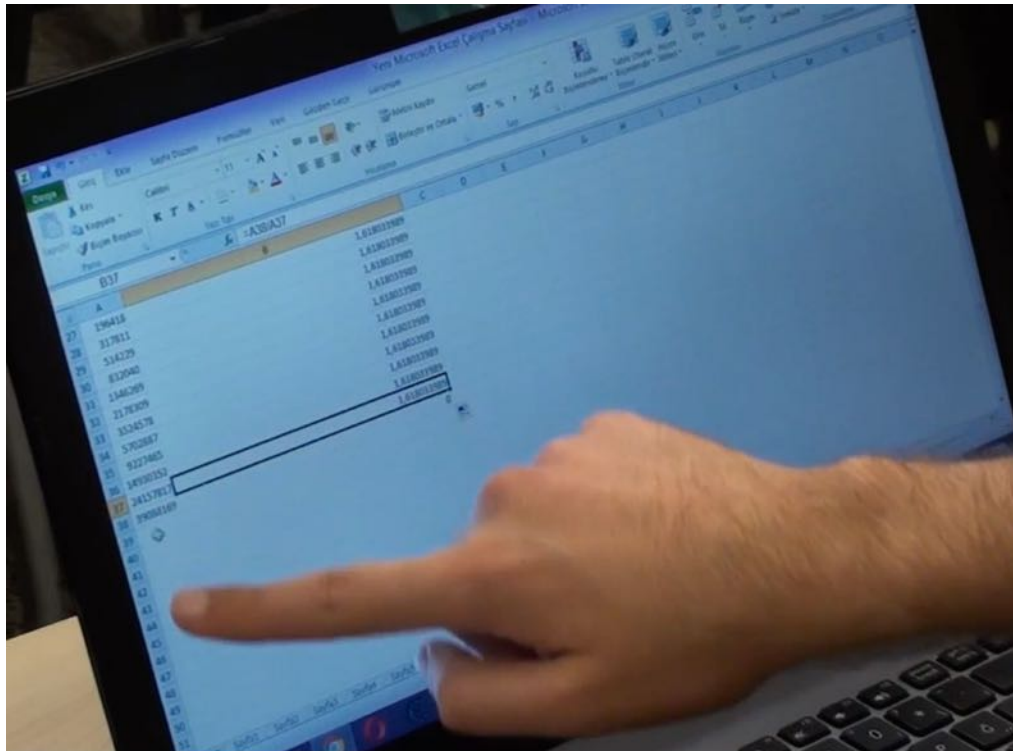


Figure 4.58: Trial action for ratio of consecutive terms of Fibonacci Sequence

### Episode 122

In this episode, PSTs decide on the fairness of a probability game. For this purpose, they use TinkerPlots software to simulate the results of playing the desired game multiple times. They are testing the accuracy of their observations from the previous stage by experimenting with TinkerPlots features.

Sude: Now, these are the results. Let's see the notes for it. We'll look at the joint.

Nuriye: Aren't you going to carry the Joint status to the bottom of the table?

Sude: I'm trying to get this down. *(By grouping the data in graphical view in TinkerPlots)* I threw them aside. I'll see how many there are.

Tuba: Where did we make it?

Sude: I will throw the same ones aside. I can combine both of these. *(After combining)* The different ones became the same ones. We win when the same color comes.

Nuriye: Show me the percentages!

Sude: Where is it?

Nuriye: There's that percent sign; you'll click it. (*After the percentages appear, point to the number of attempts*) Let's increase it right here. Make 1000.

Episode 123

In this episode, PSTs examine the approximation of a secant line to a tangent line using GeoGebra. They observe the effect of the change on the  $h$  value determined by the slider assigned, which determines the width of the interval.

Berk: Look now! As it approaches zero, it does not change! Approaching 9.5. Same with negative.

Arda: It increases and decreases, but when it is at zero, it becomes equal to its derivative at that point. (*He makes this conclusion based on the derivative information he remembers from the Calculus course*)

Berk: So, the left-hand limit and the right-hand limits are the same.

Arda: Yes.

Episode 124

In the next excerpt, PSTs are working on the average rate of change of a function over a particular range. They realize that this ratio is the slope of the secant line drawn in the desired interval. Then they try out how to calculate this slope using different GeoGebra tools.

Nuriye: Actually, the slope of this is like this. Actually, that's what we do. We found the difference in position divided by the difference in time.

Sude: So actually (*pretending there is a straight line going through points A and B*) is this slope. Well, let me ask you something. If I draw a straight line here, is there a tool in GeoGebra that gives the slope of that line? (*She draws the line passing through two points.*) How does it show the slope of this line?

Nuriye: Click on Advanced. Maybe there is.

Tuba: Wasn't it there?

Nuriye: Yes, it should be a slope tool.

Tugce: Exactly!

Nuriye: Yes, it is. Its slope is 58 points...

Sude: Can't I take it like this? It doesn't matter, though.

Nuriye: What should come out when we calculate it? Shall we calculate?

Sude: Then do it on the calculator. Subtract the two and divide by 25.

Nuriye: It's the same, yes.

Sude: Good. We proceed with this method.

PSTs' reactive action modalities were frequently emerged in the data analysis. Trial action spawned 54 times while wandering action spawned 35 times. PSTs performed these action modalities during data collection, usually needed in the initial stages of argumentation. Like treasure hunters outing to explore an unfamiliar area, PSTs tried to scout when they did not know where to start with the problem within the task. These action modalities have played a critical role as it is the data collection stage on which the argumentation will be built.

#### **4.1.3.2 Proactive Actions**

In the group that Hollebrands (2007) names proactive actions, it refers to actions that users expect about what might happen within a specific plan beforehand. These actions are grouped into two sub-categories, namely, probing actions and persuasive actions.

##### **4.1.3.2.1 Probing Action**

The actions performed during the task to confirm the experimented conjecture are named probing action modality. When a conjecture has arrived, PSTs do affirmative steps for the truth of the conjecture by repeating the procedures in a correct sequence. They have expectations about what might happen during the actions before they start playing with the technology. This section reports the actions PSTs try to observe and test whether these expectations are accurate or not.

Episode 125

In this episode, PSTs observe which of these parameters affect the period of trigonometric functions written with parametric coefficients. In their previous

observations, they had come to a conclusion about which parameter caused the period change. Building on this knowledge, they are now experimenting with being aware of what they need to change in order to come to a conclusion about the difference in odd and even values of the power of the function (see Figure 4.59).

Beril: Let  $n$  stay at 5. Let's play with  $b$ .

Berk: Very good. Compressing, isn't it?

Beril: Yes. Now change  $c$ .

Berk: It shifts horizontally. Look, when it is even... The function does not take a negative value when the exponent is even. It's like the negatives are reflected upwards. Do you see? Think of it as taking its absolute value. But when the value of  $n$  is odd... By the way, notice that the graph of  $n$  at 3 and graph at 5 are almost the same.



Figure 4.59: Probing action for periods of trigonometric functions

### Episode 126

In this episode, PSTs examine the transformations of functions. They observe the changes supporting their claim about the effect of the parameters determined on the



graph. For this purpose, by changing the values of the slider tool in GeoGebra, they check whether the changes they predicted occur.

Burcu: Let's start with the values of  $a$ .

Nisa: Let's animate it.

Eda: How do we start to animate?

Nisa: Right-click on the slider. The third is the animation. What effect does this have? It's stretching upwards. So, it's vertical stretching.

Burcu: When does it go straight?

Nisa: It should be at zero.

Burcu: It should be at zero, exactly.

Nisa: Two, one, zero. Yes!

Episode 127

In this episode, PSTs use the graph of a quadratic polynomial function to create a sign diagram to be used in inequality solutions. They examine different situations in GeoGebra, as the sign diagram will change depending on the presence of roots and the leading coefficient. They check the compliance of these states with their assumptions.

Berk: We will look at the sign of  $a$  again. Why? Because this time too...

Burcu: Play with that  $a$ . Take negative values, for example.

Berk: This time, we look at the roots. We consider the cases where it is greater or less than the root values and the values between the roots. What was the name of it? Transformation! The effect of the coefficient  $a$ ! The arms upwards or downwards thing! Do you understand? Since  $a$  determines whether the arms are pointing up or down here, we follow the roots... For example, when  $a$  is positive, the function gets positive values when  $x$  values are less than the smaller root; and positive when  $x$  values are greater than the bigger root. When  $a$  is negative, it's negative for  $x$  values less than the smaller root, and it's also negative for  $x$  values greater than the bigger root.

## Episode 128

In the next excerpt, PSTs created a conjecture about solving inequalities using the graphic method. Accordingly, they take actions to verify the inequality solution that includes a linear function, using GeoGebra (see Figure 4.60).

Sude: They intersect at (4,5).

Nuriye: Shall we treat the other's sign as negative? What do we equate with the other? They intersect only at the point  $x = 4$ .

Sude: They have to intersect at one point anyway. Since they are both linear functions, now let's substitute four here. So, we'll take the points where  $x$  is less than four.

Sude: After four,  $g(x)$  will take larger values. At points less than four,  $x + 1$  is greater. We want the parts where the first function is smaller. So, the interval from minus infinity to four will be the solution.

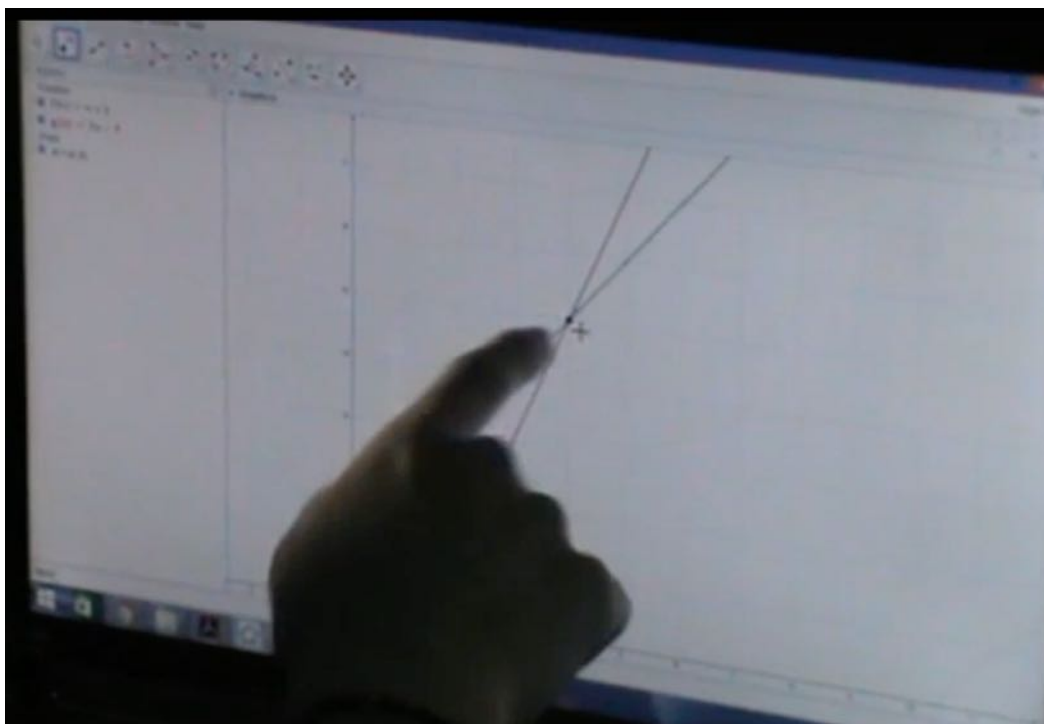


Figure 4.60: Probing action for solving inequalities by graphing

## Episode 129

In this episode, PSTs examine the angle in the Angle of Danger task. To support their claims made as a result of their previous experiments, they use GeoGebra tools to

create the desired angle and verify under which conditions this value is preserved (see Figure 4.61).

Arda: Click that first. Now click on a point here. And click on this spot.

Berk: Okay.

Arda: There has to be something like this for it to be the same everywhere, right? Because it is passing through these two points.

Berk: It's the same right now. For example, let's take this point on the circle. Then this angle should be the same as that angle. All right, done! But can't it be here too? Or can't it be over there? (*Shows points inside and outside the circle*)

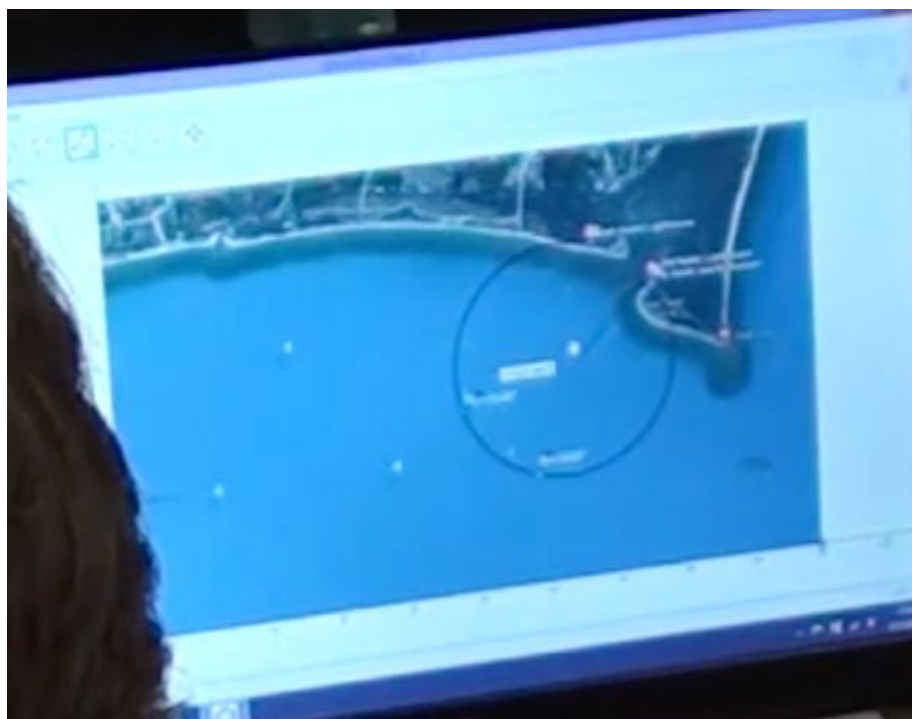


Figure 4.61: Probing action for Angle of Danger Task

Arda: Give it a try. Make a larger similar arc. Okay, now let's measure an angle on it. Will we get the same angle now?

Berk: Logically, it should be the same. Wait! Are you asking if there will be the same measure as these?

Arda: Yes.

Berk: It's not the same.

Episode 130

In this episode, PSTs examine the limit of a rational function at infinity. Based on their Calculus knowledge, they know that the function values should approach three. They also want to observe and verify this approach using a Spreadsheet.

Berk: (*Copies the formula by pulling it down*) Why did it just happen like this?

Instructor: What happened? Is there a problem?

Berk: There is no problem, sir... It may be from the numbers. Did we get the numbers wrong? Shouldn't it be closer to 3?

Arda: It's getting close already.

Instructor: Keep copying down!

Berk: Teacher, the values have come from eight or nine. Anyway, as input values approach infinity, output values will approach 3, but you are right. (*He continues to copy the formula by pulling the cells down*)

Berk: It's getting close to 3, see? Its initial value was something like 9, but look, it's really getting closer to three as it goes towards infinity.

Episode 131

In this excerpt, PSTs decided how to find the instantaneous speed of a vehicle on the speedometer in the derivative task. At this stage, they verify the question they asked by examining the location-time graph with GeoGebra.

Instructor: Now on to the next question, guys. What is the speed shown on the car's speed dial at the 10th minute?

Nuriye: We will draw a tangent from the point at the tenth minute.

Sude: Is it OK if I delete this?

Nuriye: No problem. At worst, we undo. Now there's tangent creation; let's select it. Where was he? (*looking at menus*) Tangent! Does it happen when I choose the curve and click on the point?

Sude: It does. OK. Now I will change the color so that it does not tire our eyes.

Tuba: We got the point in the tenth minute, right?

Nuriye: Yes.

Sude: (*Changes the equation format of the tangent line in GeoGebra to slope-intercept form*) The slope is here, 1.02.

#### 4.1.3.2.2 Persuasive Action

This action modality includes the actions that occur while convincing someone else that a claim obtained from the other phases is valid. At this stage, it is tried to obtain convincing data by playing with the variant and invariant properties of the object. This section will report the findings in which this action modality of the collective argumentation process has been observed.

Episode 132

In the next excerpt, PSTs examine the graphs of functions defined parametrically. With the instructor's guidance, they test the validity of their claims about the parameters' effect based on their previous observations.

Berk: Sir, we are all right. We made the model and examined the animation.

Instructor: Well, let's see. Now let's change the values. (*Berk is typing  $\sin(x)$  in the input box changes the function*) You wrote  $\sin(x)$ , okay, good. Zoom in a bit so we can see more clearly. Okay, that's enough. Let's examine the value of  $a$ . The value is initially 1. What kind of change do you expect in the graph when you double it? Change  $a$  to 2 as an example. Based on your claim, what do you expect to happen when you do this right now?

Berk: We expect a horizontal stretch.

Instructor: Horizontal or vertical?

Berk: Sorry. It will be stretched twice vertically.

Instructor: Let's see if it does? Did it happen?

Berk: Yes.

Episode 133

In this episode, PSTs examine the transformations of functions. They previously determined the different types of transformations and their conjectures about what parameters cause them. They evaluate the accuracy of these assumptions using Desmos (see Figure 4.62). They try to write a function of the desired graph using the graph given on the Desmos screen.

Nuriye: Now it just changed on the x-axis.

Sude: So, the value of  $c$  has changed.

Nuriye: So how many units have changed? Nothing else has changed, by the way, has it? Let's count those squares.

Tugce: One, two... five, six.

Nuriye: It's ok.

Sude: No, this is  $f(x)$  anyway. It will match green. We're trying to hover over the blue.

Nuriye: Write minus six.

Tuba: You should write  $f(x-6)$ , though.

Sude: Ok. No, it didn't work.

Tuba: We write the function of this graph using this graph. Six units to the left. Plus six, then.

Nuriye: But six is too much.

Sude: How many units between these?

Tuba: Three units.

Sude: Then it should be  $f(x+3)$ .

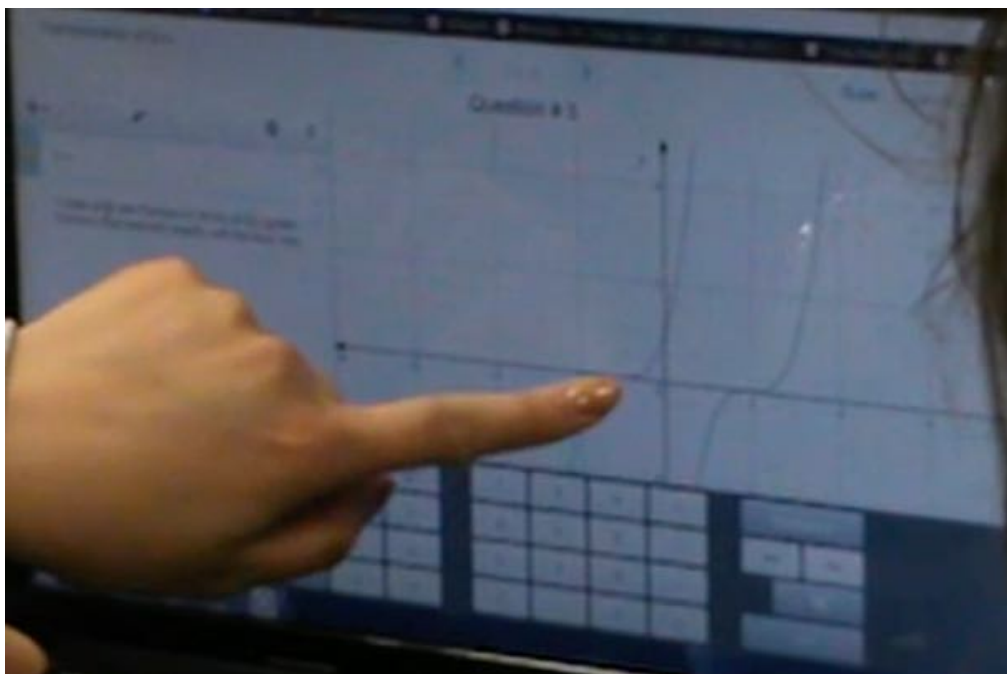


Figure 4.62: Persuasive action for transformations of functions

Episode 134

In this episode, one of the PSTs tries to convince his other friends about the truth of what he said before. They need to write the general form of sign diagrams of inequalities containing quadratic polynomial functions. For this purpose, he supports his claims by changing the graph he has drawn parameters using a slider in GeoGebra.

Burcu: I can't see it. Could you show me what you said?

Berk: Now look, Burcu! This parabola has two roots, right? Here and here.  $a$  is positive, right? Are you aware that values less than the small root are always positive when  $a$  is positive? Greater than zero. Therefore, we can say that the function takes positive values when  $x$  takes values less than the small root. Of course, this only applies when  $a$  is positive.

Nisa: Think about that side, Burcu! Look! The graph is above the  $x$ -axis.

Berk: While the  $x$  values are getting bigger than the greater root, the function has positive values again. But it has negative output values for the remaining input values between the roots.

Nisa: Let's say it this way: the function takes opposite values between two roots.

Berk: But look, when we make  $a$  negative... What happened this time? The function is negative for  $x$  values less than the smaller root, so here. Again, it is negative for input values greater than the major root. Again, the reverse is true for values between roots.

Burcu: Okay, just the opposite.

### Episode 135

In this excerpt, PSTs think they have found the answer to the danger angle in the task. By answering the Instructor's questions, they try to convince him of the certainty of their solution.

Instructor: What did you do?

Sude: That's how we drew it. It should not approach this area. There is an angle of around 27.55 from everywhere.

Nuriye: Sir, first, we determined these two points. We then picked and drew a point just outside the cliffs. We can already move this point E. We created it to cover all the rocks.

Sude: We created a circular arc passing through three points.

Nuriye: Sir, for example, the segment coming from here is the ray coming from the lighthouse, right?

Instructor: Yes. As long as the ship is around this area.

Nuriye: The angle where the two meet is the angle of danger, right?

Instructor: Yes. Take a point outside, for example, for any position of the ship.

Nuriye: Angle that will be smaller. Because this angle will get bigger as you get closer, isn't it?

Instructor: How many degrees did you find?

Sude: 20

Instructor: Now think about it this way. For example, that ship is approaching this coast. What kind of change is happening?

Sude: The angle gets bigger and bigger.

Nuriye: So, our limit is 27.54. After that value, it enters the danger zone.

Sude: For example, it can always travel safely around here.

Episode 136

In the next episode, PSTs examine adding data to the values in the dataset in a way that does not change some descriptive statistics values. For this purpose, they try to convince the instructor by showing the correctness of their conclusions.

Instructor: Okay. If so, look at the other one now. Add two more values that won't change the mean.

Berk: Okay, then.

Arda: We can directly add the average.

Berk: We can add the mean again or two numbers that are equidistant from the mean.

173.9, isn't it?

Arda: Exactly.



Berk: Look, this average will change if we include these now. Let's add 173.8 and 174. It's done, you see?

Episode 137

In this episode, PSTs discuss a probability game's fairness. Based on their theoretical probability information, they concluded whether the game was fair. At this stage, they try to provide a convincing basis for the accuracy of this result by making a simulation with TinkerPlots.

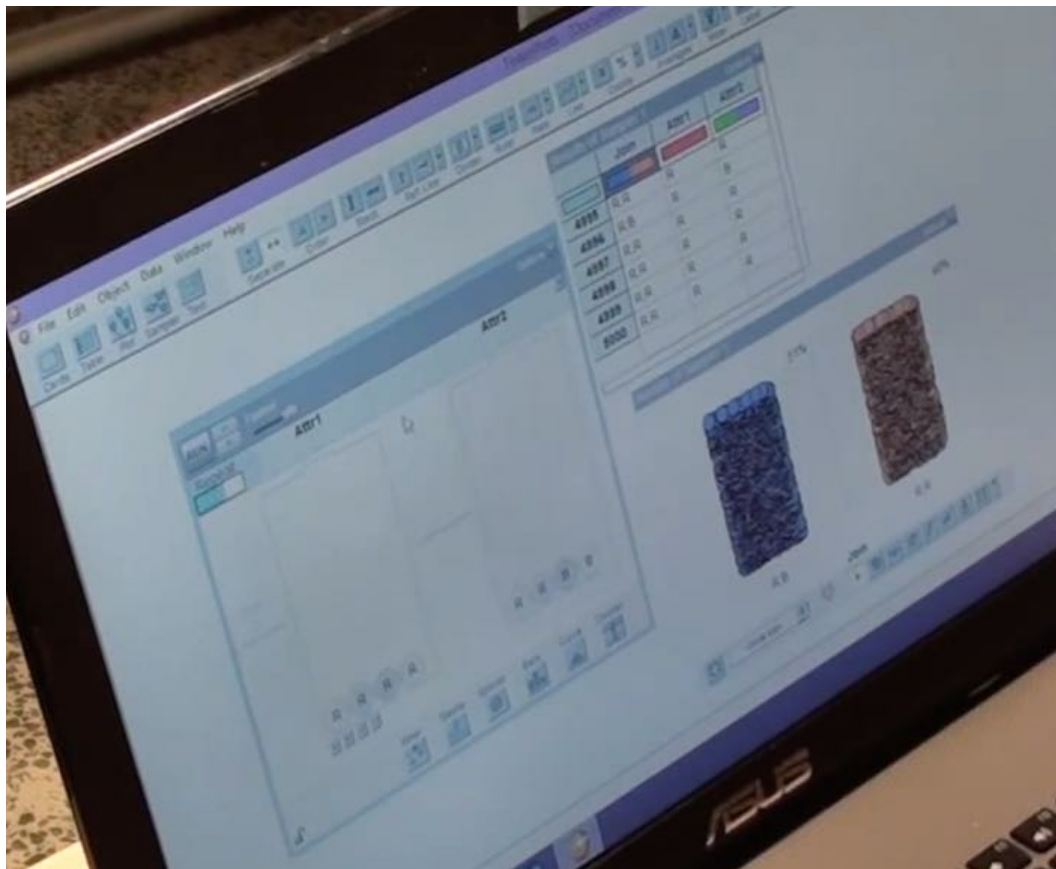


Figure 4.63: Persuasive action for fairness of a probability game

Berk: Ok, let's look at the percentages. They are 51% and 49%. What happens if I repeat this a million times?

Instructor: It doesn't need to be that big. It may be so slow, depending on your computer's processing speed.

Berk: Let it be 5000. Still same. Let me make it bigger. Aha!

Arda: It became 50-50.

Berk: Sir, ok. We got it after 100,000 tries. Ok, I'm changing that now. Sir, we found it in the hundred-thousandth attempt.

Instructor: Can you say fair as it increases?

Berk: Yes. We have obtained the same result as the theoretical probability value.

Actions of PSTs, called proactive action modalities, emerged quite frequently in data analysis, as in the previous one. Both probing and persuasive action modalities emerged 47 times. A sufficient number of episodes were reported to understand the modalities. The PSTs exhibited these action modalities after the argumentation phase of constructing a conjecture. It has been observed that these action modalities play a crucial role in argumentation, as they are the stages that enable an argumentation to be accepted as mathematically correct.

#### 4.1.4 The Characteristics of Mathematical Reasoning Emerged in TECA

Examining the mathematical reasoning in the collective mathematical argumentation process is very important in shedding light on how the claims or conjectures in the process are created and supported (Conner et al., 2014a). Considering technology's function in mathematical reasoning, this section reports the findings examining this process from various dimensions. The shaded parts show the components of the argumentation diagrams where technology plays a role.

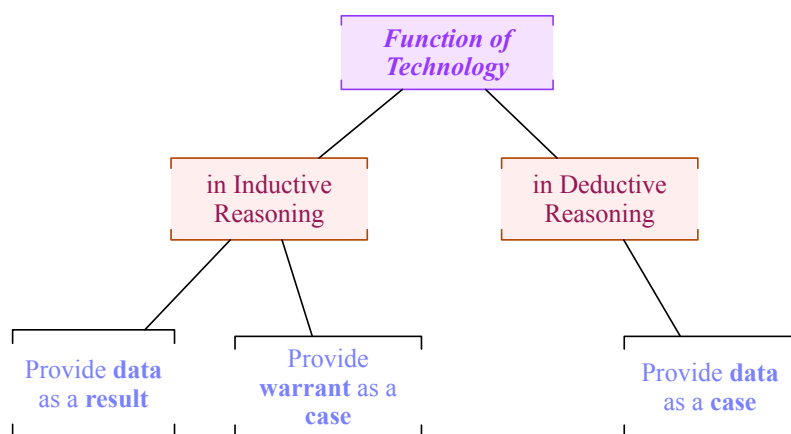


Figure 4.64: Function of technology in mathematical reasoning

#### 4.1.4.1 The Function of Technology in Inductive Reasoning

This section will report the findings regarding the process from specific to general in the collective argumentation that the PSTs made based on their observations. The following inductive reasoning diagram, which Conner et al. (2014) put forward by blending Peirce (1956) and Toulmin (2003) models, will be used.

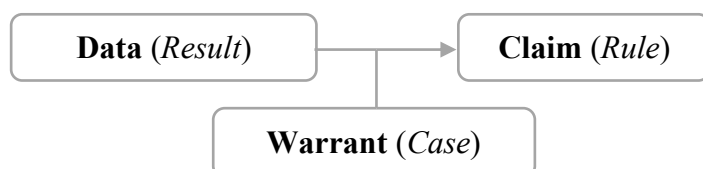


Figure 4.65: Inductive argument diagram

##### 4.1.4.1.1 Provide Data as a Result and Warrant as a Case

This section will report the situations where technology plays a role in the data (or result) component of the inductive argument in the PSTs' collective argumentation process.

Episode 138

In this episode, the PSTs use GeoGebra's slider tool to examine the transformations of a trigonometric function graph. They have the opportunity to observe the changes in the parameters assigned to the sliders by changing the relevant slider values. The different view of the function's graph at each value of the slider provides data for argumentation. In addition, the dynamically observed change in the function graph provides a warrant for argumentation.



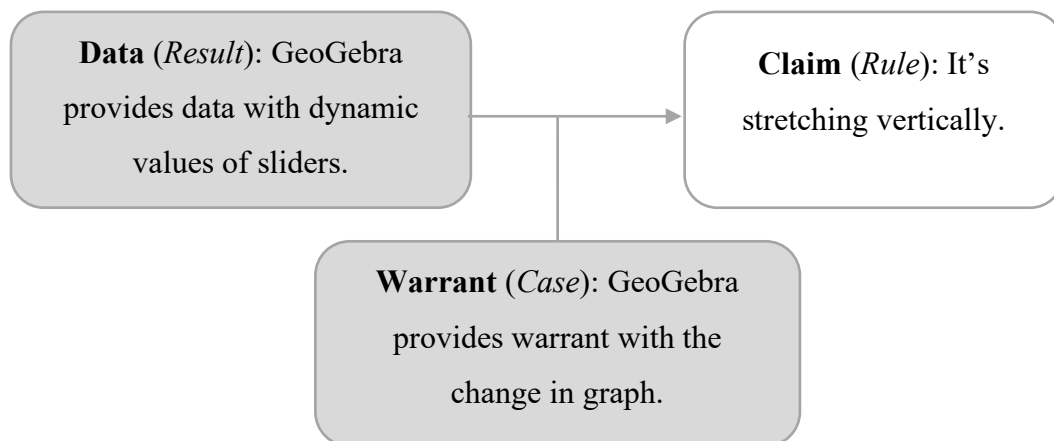
Figure 4.66: Technology provides result and case

Burcu: Let's examine the value of a.

Nisa: Let's turn on the animation.

Eda: How do we turn it on?

Nisa: Right-click on the slider. There is the animation in the third row; click on it.  
 OK! How does this affect the graph? It's actually stretching upwards. So, it's stretching vertically.



### Episode 139

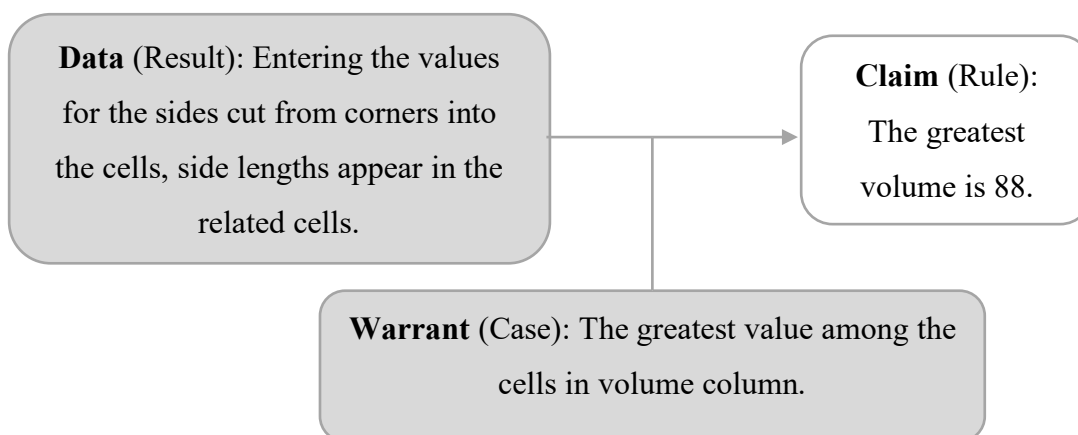
Berk: Now look! The more we shorten 15, won't we have to shorten this that much, too? Now, 15 by 8. OK? If you entered 4 cm from here, you have to enter the same from both sides. In other words, when you cut the part you cut from here and make 1 cm from inside, two will decrease by one cm. The more I increase here, the less here. *(He talks about values inserted in different columns.)*

Arda: Shouldn't it be 2 cm less? When you cut by 1 cm, it will be 1 cm less from the other corner, too. So, decrease by 2 cm.

Berk: Right! You are right! It decreases by 2 cm. We will be able to enter 3 cm at most so that 3 comes from there and 3 from there. If we enter 4 cm, it won't happen anyway. Is it correct? What are we going to write there?

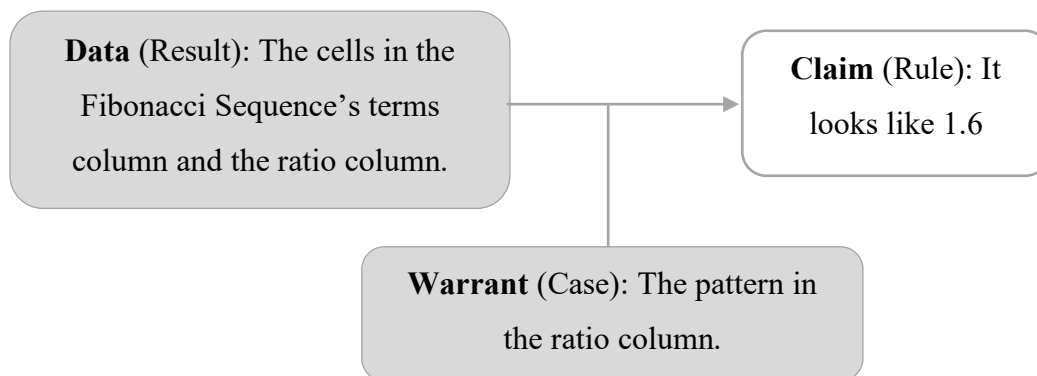
Arda: Product of three.

Berk: Exactly. This is the product of three. B2, C2, and D2. Teacher, the greatest volume is 88.



### Episode 140

In this excerpt, PSTs examine the variation of the ratio of consecutive terms of the Fibonacci sequence using the spreadsheet. If the ratio of consecutive terms approaches a value, they copy the formula they wrote into the cell for the ratio by dragging it. Thus, they have the opportunity to examine the existence of a limit value of this ratio. They are looking for an answer to how the result will be affected if a change is made in the starting values, which is considered one of spreadsheet software's most significant mathematical advantages.



Arda: Take it down a little bit.

Berk: Look, it didn't work that way.

Arda: Now drag them down again. Exactly! There's nothing under this cell, no entry! That's why we get zero there.

Berk: You are right!

Arda: It will gradually approach zero (*After checking the values in the ratio column*). No, it's 1.6.

Berk: It looks like 1.6.

#### 4.1.4.2 The Function of Technology in Deductive Reasoning

In the process of deductive reasoning, conjectures arise as a logical result of assumptions or conditions already known or stated during the argumentation process.

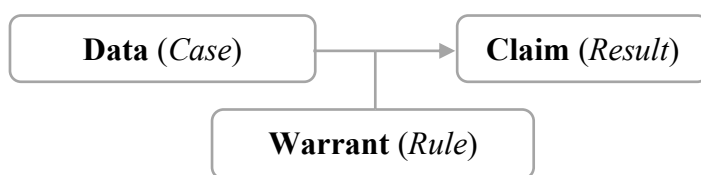


Figure 4.67: Deductive argument diagram

##### 4.1.4.2.1 Provide Data as a Case

This section will report the findings that the technology provides a case for the data component of the deductive argument diagram.

## Episode 141

In this episode, PSTs find the solution to inequality with a quadratic polynomial function on both sides. Using GeoGebra's graphical capabilities, they conclude by examining the relative positions of the functions in the intervals determined by the intersection points of the two functions.

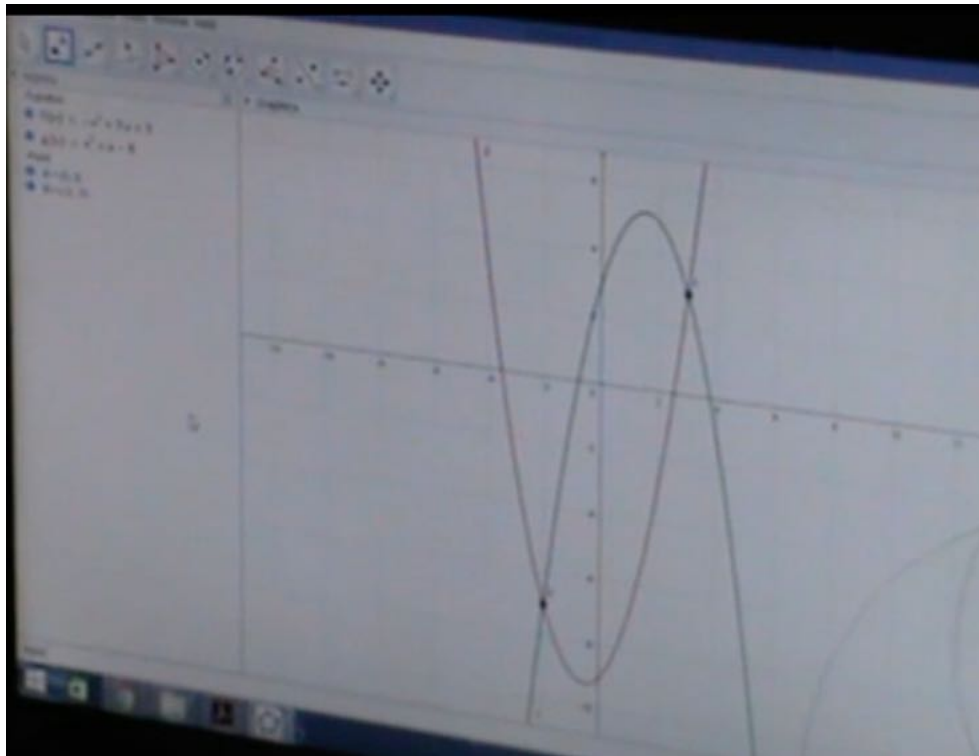


Figure 4.68: Technology provides data as a case in inequality task

Instructor: In what interval is  $g(x)$  greater?

Bus: Between -2 and 3.

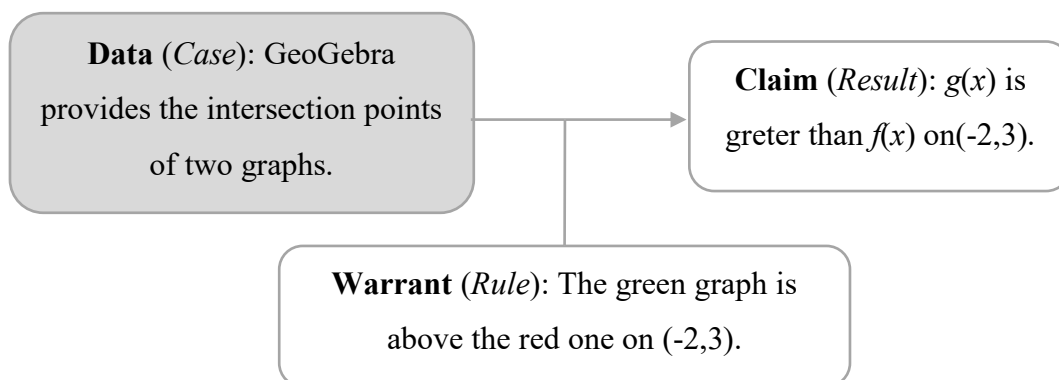
Instructor: How did you find this range?

Sude: We looked at the abscissas of the intersection points.

Berk: Does it depend only on the abscissa of the intersections?

Nuriye: We also looked at the y values. We looked at the interval where the green graph is above the red.

Instructor: Yes. When we examine it visually, the green graph is above the other in this interval.



### Episode 142

In this episode, PSTs try to find the approximate “slope” of any function at a point with the help of a file prepared with GeoGebra. They examine the slope of the line on the zoom screen, as each function looks linear when zoomed enough, similar to the logic of linear approximation.

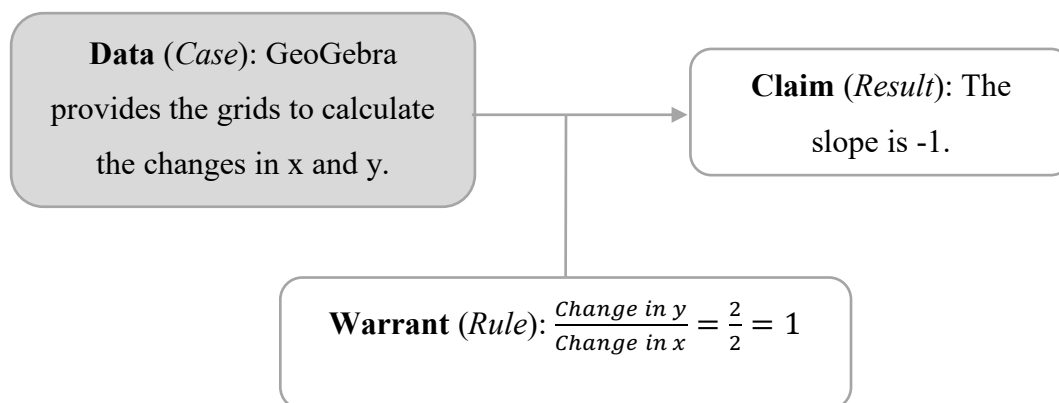


Figure 4.69: Technology provides data as a case in slope task

Berk: Look, we can find this in the same way. Here, one, two... one, two... (*Counting changes to use the formula Change in y over change in x*) The slope of this is 1. But just a second, is it -1?

Arda: Yes, - 1.





#### 4.1.4.2.2 Provide Data as a Case and Warrant as a Rule

In this section, there will be findings where the screen provided by the technology is used both as a case used as data in the argumentation process and as a warrant supporting the relevant rule.

Episode 143

In this episode, PSTs examine the fairness of a probability game. With TinkerPlots, they can see the percentages of each case in the game after the simulation ended. After increasing the number of draws, they can decide the game's fairness according to the percentages on the screen at the end of a simulation.

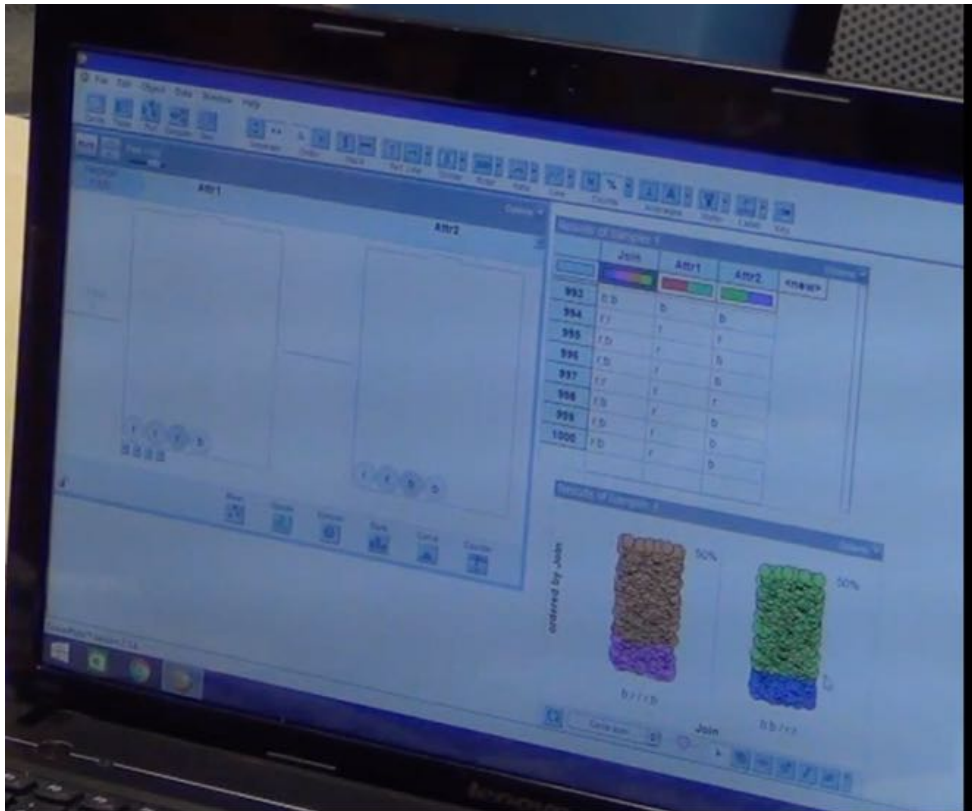


Figure 4.70: Technology provides data as a case and warrant as a rule

Sude: Now, these are the results. Let's see the notes for it. We'll look at the joint.

Nuriye: Aren't you going to move the joint under the graphic?

Sude: I'm trying to get this down. *(By grouping in the graphical view of the data in TinkerPlots)* I sent them to the side. I will look at the number of each.

Tuba: Where did we do that before?

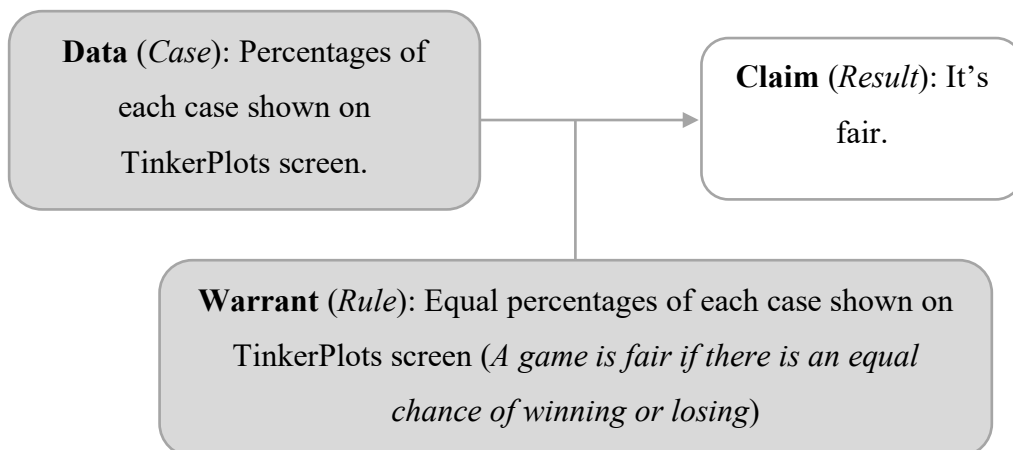
Sude: I will throw the same ones aside. I can combine both of these. *(After combining)* We have made two groups, the ones with different colors and the ones with the same color. We win the game when we choose the same color.

Nuriye: Show the percentages!

Sude: Where is he?

Nuriye: There's that percent sign. *(After the percentages appear, by pointing to the number of attempts)* Let's increase it right here. For example, make it 1000.

Sude: Let's speed it up. *(After the simulation is over)* It's fair, sir.



#### 4.1.5 Remarkable Intersection Frequencies of Coded Episodes

In this section, remarkable results of some common coded segments will be reported when examining the frequencies of the codes in the analysis made with MaxQDA. Although these frequency values are not meaningful on their own, they can provide meaningful information when evaluated in context.

Code System	SoD	SoW	SoR
ARGUMENTATION			
Argumantative Function of technolc			
SoD		230	9
SoW	230		8
SoR	9	8	
Technological Action Modalities			
Reactive actions			
WanAct	37	21	2
TriAct	77	51	4
Proactive actions			
PrAct	79	79	5
PeAct	74	79	4
Σ SUM	506	468	32

Figure 4.71: Argumentative function vs. Action modalities

The first intersecting codes that emerged in the analysis will be the “argumentative function of technology” and “technological action modalities”. The figure (see Figure 4.71) shows that the repetition frequencies of the codes specified in the matrix

form are given at the intersections. Proactive actions contributed more to the warrant component of argumentation.

Secondly, the codes that draw attention to the intersections to be reported are “modes of facilitation” and “degrees of instrumental integration”. Interestingly, the low-level modes in “degrees of instrumental integration” do not intersect with any code in “modes of facilitation” (see Figure 4.72). However, higher-level types of instrumental integration, such as instrumental reinforcement and instrumental symbiosis, mainly emerged in episodes where the instructor played the initiator role.

Code System	Mod_Fac_Ini	Mod_Fac_Res	Mod_Fac_Fin
ARGUMENTATION			
Characteristics of Roles			
Supportive roles			
Role of Instructor			
Modes of facilitation			
Mod_Fac_Ini		5	6
Mod_Fac_Res	5		2
Mod_Fac_Fin	6	2	
Degrees of Instrumenta			
D_Ins_Int_Ini			
D_Ins_Int_Exp			
D_Ins_Int_Rei	14	4	
D_Ins_Int_Sym	13	4	2
Σ SUM	38	15	10

Figure 4.72: Modes of facilitation vs. Degrees of instrumental integration

Finally, another exciting code intersection is “modes of facilitation” and “focus of the instrumental integration”. The mode of facilitation, which has the most intersection with the Focus of the instrumental integration codes, is the instructor as initiator code. It is also noteworthy that the role of the instructor as resolver only emerges at the intersection with FMUT. It is another engaging result that FMNT is observed only when the instructor plays the initiator role.

Code System	Mod_Fac_Ini	Mod_Fac_Res	Mod_Fac_Fin
ARGUMENTATION			
Characteristics of Roles			
Supportive roles			
Role of Instructor			
Modes of facilitation			
Mod_Fac_Ini		5	6
Mod_Fac_Res	5		2
Mod_Fac_Fin	6	2	
Focus of the Ins. Integration			
FTNM	8		2
FMUT	70	16	2
FMNT	4		
Σ SUM	93	23	12

Figure 4.73:



## CHAPTER 5

### DISCUSSION, CONCLUSION AND IMPLICATIONS

The aim of this study was to illustrate the rich collective argumentation processes of pre-service mathematics teachers in a technology-enriched learning environment. Different agents were thought to play a role in the collective argumentation process. A detailed examination of these agents' interactions affects learning and teaching in this learning ecology has been made. To be mentioned in more detail, the interaction of the instructor, technology, and pre-service teachers with each other, the roles they play, and the examination of their functions in the defined learning ecology were presented in detail.

This chapter discusses the findings and outlines the results of the study, its limitations, and recommendations for future studies. The chapter is organized according to the findings examined around the research questions; the roles played, the function of technology, the modality of technological actions, and the nature of mathematical reasoning will be discussed, respectively.

#### 5.1 The Nature of Roles in TECA

Data analysis revealed that the different components involved in an argumentation process could have supportive and distractive roles. First, if we talk about the supporting roles, two role players came to the fore. The first of these was the technology as a tool, and the other was the instructor, who mediated the meaningful use of this tool.

*Role of technology:* The first observed role of technology emerged in initiating the argumentation process. Especially the observations made by the pre-service teachers

using technology when they were not sure how to start the argumentation process about the questions in the tasks sparked the first spark of argumentation. For example, they could not predict the effect of the parameters they used on the graph while expressing the transformations of the graphs of the function in the first episode. Over time, they started to observe the change in the graph as they changed the slider values and wrote them as a new function by associating the parameters with the sliders in GeoGebra. Then, as Lopez-Real and Leung (2006) mentioned, the GeoGebra screen is triggered at the beginning of the mediation process by providing them with visual data to start the argumentation as a semiotic instrument. This issue is also related to the semiotic mediator role of technology stated by Mariotti (2002). Gestures, objects, images, or motion simulations provided by technology can play a vital role in situations where participants have an idea but cannot determine how to start a mathematical argument, and sometimes even they have no idea.

The first point to be considered for this role to emerge is that this role should be taken into account in the design of the task. While designing the task as a teaching agent, the argumentation process of students must start in a natural flow. The data part of the argumentation can be presented more implicitly if it is planned that students learn some necessary information in a more authentic learning environment by realizing it themselves. Thus, by using the affordances of technology, it can be ensured that students reach the first data that will initiate argumentation through experience by making an effort rather than by providing them. In this sense, motivation can be increased as an active learning environment can be created. However, the difficulty in the learning process should not be exaggerated here. In this regard, considering the Flow Theory (Csikszentmihalyi & Csikszentmihaly, 1990) in the task design according to the student's cognitive levels so that the students should not give up hope of completing the task without any ideas. However, if they still have difficulties, the instructor should step in, which will be a dimension to focus on in the next section.

In addition, when the global argumentation process cannot progress and gets stuck in local argumentation, technology can play a role in solving the problem when used



correctly. In such situations, it is essential to ensure students are open to trying new things using technology features. If the students are insufficient in this regard or their attempts are ineffective in solving this stuttering, the instructor must carefully observe the class so that he is aware of this situation and can make the necessary intervention.

*Role of instructor:* In addition to the passive role of technology in the argumentation process, the instructor's role in the argumentation process is entirely active. In the argumentation process, the instructor can take on roles in different positions, depending on the degree of instrumental genesis (Guin & Trouche, 2002). These roles can be performed by infusing the content during the design of the task, or it can be like the instructor's attitude when observing the argumentation and in a situation that requires intervention. Learning outcomes targeted with the collective argumentation process should play an essential role in determining these interventions' level of instrumental genesis in task design. Relatively low-level interventions such as instrumental initiation and instrumental exploration can be planned, especially in the sections aimed at establishing familiarity with technology at the beginning of the process. However, especially after mastering the features of technology, students should be guided with questions that are expected to contribute directly to argumentation. In particular, students should be supported to develop correct mathematical argumentation with interventions at instrumental reinforcement and symbiosis to develop the content knowledge they need to gain through argumentation. As can be noticed from the results obtained from the data, reaching the expected mathematical knowledge in instrumental genesis interventions at high levels has been more impressive. Its contribution to the process is remarkable, as interventions at higher levels are more in the form of providing warrants, which play a crucial role in the development of argumentation. However, these interventions do not always go as planned. Because factors such as the readiness of the pre-service teachers in the group, their familiarity with technology, and their content knowledge can affect their handling of the process. During active observation, the instructor should be able to foresee where the intervention he should

take in cases he notices can lead the process. The way to do this is to have a solid knowledge of content and technology and know the process of the task well.

Considering the interventions of the instructor at these different levels of instrumental genesis, his role in the argumentation process may also differ. As with the role of technology, the instructor can act as an initiator and resolver in the argumentation process. While the students should play a more active role and do the necessary trials themselves when technology plays a role, they can be more passive when the instructor plays the same role. The point to be considered here is that to avoid missing the touchpoint in collective argumentation; the instructor should take the initiative in his role and be careful not to put the students in a passive state. In addition to these two roles, if all of the students unexpectedly failed to conclude due to insufficient time or difficulty (in which the task should be reconsidered), the teacher should conclude the argument using the obtained data and warrant. The purpose of this role can be considered to finalize the local argumentation so that the global argumentation process is not interrupted.

As reported in the Findings Chapter, the low-level modes in the “degrees of instrumental integration” did not intersect any code in the “facilitation modes” (see Figure 4.72). In addition, it is not a coincidence that higher-levels of instrumental integration, such as instrumental reinforcement and instrumental symbiosis, occur mainly in episodes where the instructor plays the initiator role. It is because the instructor performs the necessary interventions to initiate the argumentation, not while aiming at developing instrumental knowledge but aiming to develop mathematical knowledge. Because the stage of recognizing the technological tool has been left behind, the argumentation in which mathematical knowledge will be included to find an answer to the problem must begin.

*Distractive roles:* Data analysis also revealed cases where the collective argumentation process was not mathematically correct. Although the mathematics topics for the tasks were chosen from the high school curriculum, some obstacles emerged in pre-service teachers’ development of a mathematically correct

argumentation process. These roles were named as lack of mathematical content knowledge, technological knowledge, and interpersonal skills. It was predicted that there might be incorrect or incomplete argumentation processes due to the lack of mathematical content knowledge, and cases emerged to support Hewitt (Hewitt, 2005). This result was expected and inevitable for some cases since eliminating such deficiencies had an important place in the course design. For this reason, it should be ensured that PSTs have the knowledge considered as prerequisite for the tasks designed to improve their argumentation skills and mathematics content knowledge. Otherwise, it may be necessary for the instructor to intervene more than usual and provide enough support to unblock local argumentation without distracting other groups. The effect of the lack of familiarity with technology on the development of a mathematically incorrect argumentation or the complete termination of the process was an expected result. Because if the use of technology is required in the task and this knowledge is lacking in the pre-service teachers, the expected development does not occur. This has sometimes led them to develop a false argumentation to support Hollebrands et al.'s (2010) study, perhaps due to their lack of technology knowledge because they rely too much on technology and accept it without question when they observe a screen that supports their claim. Sometimes, on the contrary, they did not notice the supporting screen they expected due to their lack of familiarity with technology and their lack of knowledge, causing them to abandon the argumentation process and try to move on to another argumentation. Because they could not use the relevant tool of technology in a way to support their claims, they concluded their claims were false. In addition, the fact that mathematics software offers perceptual reality and evidence and pre-service teachers do not question this may also cause a mathematically incorrect argumentation (Mariotti & Pedemonte, 2019). For this reason, it should be ensured that whichever technology will be used in the TECA process, pre-service teachers have at least a level of knowledge about those technologies that will not interfere with the process. In addition, information should be given about the mathematical affordance and limitations of the mathematics software used.

Another unexpected result was the lack of interpersonal skills of teacher candidates. Although they are adults, problems such as not listening to each other enough during the argumentation process and insisting on their own opinion to dominate prevented the development of mathematically correct argumentation. Since these situations did not stand out in his observations in the classroom and only appeared in video analyses, the researcher could not take any precautions in this regard. For the argumentation process to proceed correctly, discussing a general statement about interpersonal skills in the name of establishing a class norm could make a difference.

The emergence of distractive factors in the argumentation process may vary depending on the nature of the task. However, the most prominent role as a distractive factor in the tasks in this study was the lack of “background knowledge” of the PSTs, encountered in 37 different episodes. When a similar study is to be carried out, the student’s background knowledge should be determined well in advance, and it should be ensured that they do not have the least problems in the tasks to be prepared. However, due to the possibility of problems during the argumentation, the teacher should closely follow the processes of the groups in order to support them when they need it. Apart from that, “lack of familiarity with technology” and “interpersonal skills” emerged 8 and 6 times, respectively, as situations that should be considered in the planning of tasks. These problems are less encountered because there is a preparation phase before the tasks, during which basic information about the technology to be used is learned. From this point of view, before the tasks in which new technologies will be used, it should be ensured that the learners have the basic skills of the relevant technologies.

## **5.2 The Nature of the Argumentative Function of Technology**

In the teaching ecology targeted in the teaching episodes of the study, it was aimed that PSTs acquire mathematical content knowledge through a TECA process. The mathematical software can be a potential bridge to fill the gap between the empirical and the mathematical environments mentioned in the literature (Lopez-Real &

Leung, 2006). The data analysis also supports this emphasis because pre-service teachers use various technology functions during their argumentation processes. The findings are essential in terms of examining how technology can function in filling this gap. In the TECA processes, technology has functioned sometimes by providing data, sometimes by providing warrants, and sometimes by providing both simultaneously. In addition, technology has sometimes functioned as a source of refutation in argumentation to support the invalidity of the arguments made. Because testing the accuracy of a claim with technology can sometimes be very easy and powerful. Since this skill is related to knowing the affordances of technologies used, knowing how teacher candidates will do that verification should also be included in designing the tasks as a crucial learning outcome.

Analyses also revealed that PSTs appreciated using computers to support learning mathematical concepts through detailed mathematical reasoning. For example, this was noticed during the video analysis of the task of examining and concluding the first and second derivative tests with a graphical approach. A PST said, “Do you see how we forgot these?” when no one could comment on the question asked. Then another replied, “Actually, we never learned.” Then someone said, “We were taught these methods and we didn’t learn? I cannot accept this!”. It is remarkable that they are aware of and appreciate that they experience a more meaningful learning process thanks to the support provided by the technologies used in such an argumentation process by playing the data, warrant, and refutation functions. At first glance, refutation using technology may seem like a negative situation as it disrupts the argumentation process. However, it also had the effect of teaching pre-service teachers that they should continue with a broader perspective than an argumentation process. Thus, while considering warrants to support a data, it also played a supportive and instructive role for them to have a more comprehensive way of thinking by considering possible exceptional cases where their claims might be false.

Hollebrands et al. (2010) stated that students did not use technology while stating explicit warrants in their argumentation processes. In cases where they do not provide an explicit warrant, they indicate that they use technology more. According

to them, this may be due to the student's lack of technology familiarity. Because these students probably used technology more to make calculations or discoveries in other math lessons. The researcher observed similar situations in the first weeks of the lesson. However, since there is no short-term study and a semester continues, this deficiency has begun to fade in the following weeks. Because after a while, although no particular emphasis was given, they began to form a consensus on how students should express explicit warrants with technology (which the researcher named "a socio-techno-mathematical norm"). For students not in such an environment to gain this skill, participating in activities involving verification in other technology-based mathematics courses may be a complementary factor for their development in providing explicit warrants.

### **5.3 The Nature of Technological Action Modalities**

While describing the technological action modalities of students in technology-enriched learning environments, Hollebrands (2007) divided them into two different groups. He named these groups reactive and proactive actions. According to the results of the data analysis of this study, these categories were detailed and divided into subcategories. In the action modality defined as reactive actions, it was defined as the fact that the students did not have any prior knowledge or expectations about the results of the actions they would take. However, the researcher noticed variations of this action modality during the analysis. In the data analysis, the researcher realized that the pre-service teachers were in two different action modalities, which the researcher named them as wandering actions and trial actions. As PSTs often use these two action modalities for initial task investigation and exploration, they were mainly observed when pre-service teachers were trying to collect data for initiating the argumentation process.

Similarly, the subcategories of the proactive action modality were named probing and persuasive action modalities. These action modalities, on the other hand, have been observed as actions that primarily provide warrant (see Figure 4.71) to the

TECA process. Because these actions mainly were those that support the claim or the actions of the person who stated a claim to persuade others. Since the PSTs acted consciously in these types of actions, they were very effective in supporting their mathematics content knowledge. The researcher noticed that PSTs' sense of having the ability to use technological tools supporting their claims plays a role in increasing their confidence. While designing the tasks in another study that will include a similar learning ecology, it may be beneficial for developing mathematics content knowledge to prepare an environment where pre-service teachers can do the movements in this group more.

#### **5.4 The Nature of the Mathematical Reasoning Emerged in TECA**

Conner et al. (2014) presented a way to distinguish different types of mathematical reasoning by combining the Peirce and Toulmin models. Examining various mathematical reasonings in the collective argumentation process is vital in explaining how pre-service teachers engage in hypothesis formation and examination processes. Thus, it provides an opportunity to examine what kinds of reasoning and in which context they prefer to support their hypotheses. In addition, how does the game change when technology is included in the argumentation process? What is the function of technology in different kinds of mathematical reasoning? In the light of these questions, what stood out in the analysis made with the lens presented by Conner et al. was that technology assumed the function of providing data to reach the rule, which is perhaps the most critical part of the argumentation processes using inductive reasoning. However, in argumentation processes using deductive reasoning, the function of technology was observed as a data and warrant provider. Of course, the thing to note here is that the possible kinds of reasoning that might arise are related to the design of the tasks. It is possible to predict what kind of reasoning an argumentation will develop to how the questions in the tasks are asked. For this reason, knowing which types of reasoning, the preferred technologies are more prone to may gain importance as a skill that the

teacher educator should have in the task design process. For example, spreadsheet software is beneficial while providing the opportunity to generalize from more numerical data. For this reason, asking questions in tasks where spreadsheets are used in a way that leads to inductive reasoning may be a factor that can increase the efficiency of the task.

## **5.5 Implications**

The implications of this study are multifaceted. First, the methodological approach outlined in this study provides an example of how interventions (tasks, classroom interaction, the discursive and interventional role of the instructor, and the function and role of technology) will emerge based on instructional design and conceptual framework. It also gives a big picture of how the concept of TECA emerges in a classroom with the classroom teaching experiment methodology. In this sense, for those who want to create and work on a similar learning ecology, a rich picture has been depicted about the interactions of the components in the environment and the things to be considered during the design.

When determining the tasks to be used for a similar study, the subjects that they will teach when they become teachers should be chosen instead of observing the teaching of a subject by pretending to be a learner. Because such an environment is far from reality as it assigns pre-service teachers an artificial role. Therefore, the selected tasks should challenge PSTs' various components of pedagogical content knowledge so that they are enrolled in a real learning environment as true learners. The problems must be at a level that will challenge them to go through a natural argumentation process. Thus, by remembering the argumentation processes within the course, they may have an idea about how they should organize tasks in their professional life. When choosing the mathematics topics in the tasks, it would be appropriate to select the topics that PSTs presumably memorized in their student years, without going through mathematical reasoning. Because by using mathematical reasoning in the argumentation process in learning such concepts, PSTs will not only reinforce their



own mathematical content knowledge but also can have experience about how they should manage a similar approach when they become teachers.

This study revealed that using a relevant technology effectively to solve the task in a challenging learning environment by going through the argumentation process depicting a natural classroom improves PSTs' both mathematical and instrumental abilities. Because they may have an opportunity to experience examples of how that software can be used in teaching by going through a meaningful learning process. In the argumentation process, they have experienced at TECA, they are expected to know what kind of process their own students can go through, as they have experienced crucial mathematical skills such as conjecture generation, testing, and verifying.

If, as in this study, the researcher aims to improve the collective argumentation skills of teacher candidates along with their technology knowledge, determining the appropriate combinations of technology and content knowledge components should be preferred because it creates a diverse learning environment. For this purpose, first, the basic concepts of the relevant curriculum should be determined among the subjects to be taught by the target teacher candidates. Alternative technologies that can be used in teaching the determined concepts should be determined by making use of the mathematics education literature. Then, the titles of these concepts that can create a rich discussion environment and TECA from the selected technologies should be determined. While creating the tasks that require argumentation processes in which the determined mathematical concepts and technologies are used, the design should consider the roles and functions of the technology and the instructor that emerge in the study. In a design where the aim is improving TECA, a wide range of mathematics and technology components should be included. Because a learning ecology based on only a few technologies or a few subjects may not be enough to enable the development of different perspectives.

It has been reported that creating a classroom norm, mainly based on failed mathematical argumentation processes, may cause PSTs to be unable to complete

the argumentation correctly, even in the subjects they should know under normal circumstances. A similar situation can be experienced by PSTs so that they are aware of it, and awareness can be raised about what a future teacher should know. In this context, it should be aimed to establish a socio-mathematical and socio-techno-mathematical norm in the classroom. This should be created by following the TECA processes carefully. Both established norms are an important phenomenon for TECAs to achieve the targeted learning outcomes and to be more efficient. In order to establish class norms, what is observed and accepted in the groups should be presented to the class and the common norms should be decided together. For example, a socio-techno-mathematical norm can be defined as what technology must provide in order to be considered a warrant in an argumentation unit.

The number of learners is also an important factor for the instructor to follow the groups closely during TECA and to keep the progress of the argumentation processes under control according to the norms to be created. Having too many students and groups can prevent the instructor from making the necessary observations. In order to create rich argumentation environments, the number of groups consisting of three PSTs should be at most four, according to the results of this study. Since the presence of more groups will make the control difficult, it may not be sure to improve the argumentation skills of the PSTs. It should be taken into account that if the number of PSTs within the groups is less than three, it becomes difficult to come up with diverse ideas, and a dominant character can continue the whole process according to their own ideas. At the same time, in an environment where there are more than three PSTs, some of them may disappear and their development may be limited as they do not contribute to the process.

Another important issue should be the responsible use of technology in the TECA process. Within the tasks, there should be parts where the affordances of technology are used sufficiently, but after a point, alternative ways such as paper and pencil are also used. Thus, PSTs should be prevented from acting like technology addicts, unnecessary use of technology should be prevented and they should be aware of this

important issue. In other words, it should be emphasized that technology is not a goal, but should remain a tool in mathematics teaching.

## **5.6 Limitations and Suggestions for Future Research**

The primary learning outcome of the course, which was the context of this study, was to develop teacher candidates' technology competencies within the TPACK theoretical framework. However, there were teaching episodes in which collective argumentation processes were required as a tool used to achieve this goal. In other words, developing mathematical argumentation and reasoning skills with technology was not the primary goal. This can be accepted as the first limitation of the study. It can be recommended to design further studies where the main goal is to examine the development of PSTs' collective argumentation and mathematical reasoning skills in TELEs.

As a reported contribution of this study, new action modalities in the argumentation process were stated. More detailed future studies can be done about these new action modalities in argumentation processes and different types of mathematical reasoning.

Activities that provide sense-making can be designed by passing the concepts that teacher candidates have previously memorized but have not undergone a meaningful learning process through abductive reasoning processes. Due to the design of the tasks in this study, the argumentation processes progressed without the need for any use of abductive reasoning. However, future studies can be considered in which the subjects memorized can be learned more meaningfully through an argumentation process that requires abductive reasoning, which is briefly defined as “inference to the best explanation”.

The learning ecology in this study was planned to depict a classroom environment in its naturalistic setting. Therefore, the issue of how the targeted learning outcome will be reached through an argumentation process has been left flexible. By doing

so, the effect of different factors in the learning ecology could be observed by creating a more diverse environment. However, future studies can also be conducted to reveal a more structured HLT that will be passed through the relevant argumentation processes to achieve learning outcomes for the targeted mathematical concepts.

Since the technologies and mathematics topics chosen in this study are limited, other parts can be covered in other courses that are a continuation of such a course. Thus, by going through the technology and argumentation process, the diversity of the combinations of the subjects and technologies examined can be increased, and the integrity of the high school curriculum can be achieved.

This study lacks emphasis on creating socio-techno-mathematical norms, especially in cases where the persuasive action modality is observed, which may be a justified criticism. Because what could be an acceptable explanation or justification emerged during the analysis as a process that the students perform spontaneously in the natural flow of the lessons. While the pre-service teachers in the groups were working, they formed their norms after a while and shared norms were formed in the classroom discussions, but there was no emphasis on this issue at the beginning. As a future study, a study can be conducted to seek an answer to the question “How socio-techno-mathematical norms regulate the argumentation process and influence the learning ecology”. In addition, design-based studies can be carried out to determine the HLT for teaching the main subjects of the high school mathematics curriculum by blending them with the most appropriate technologies. These HLTs may include clear identification of the role of the instructor and the specifically chosen technology in learning ecologies involving a productive TECA.

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## APPENDICES

### A. Ethics Permission

#### METU Human Subjects Ethics Committee's Approval for the study.

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15 ARALIK 2017

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Prof.Dr. Ayhan Kürşat ERBAŞ ;

Danışmanlığınızı yaptığımız doktora öğrencisi Murat KOL'un "**Matematik Öğretmen Adaylarının Teknolojik Pedagojik Alan Bilgilerinin Gelişimini Desteklemeye Yönelik Ders Tasarımı**" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **2017-EGT-184** protokol numarası ile **03.01.2018-31.12.2018** tarihleri arasında geçerli olmak üzere verilmiştir.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Ş. Halil TURAN

Başkan V

Prof. Dr. Ayhan SOL

Üye

Prof. Dr. Ayhan Gürbüz DEMİR

Üye

Doç. Dr. Yaşar KONDAKCI

Üye

Doç. Dr. Zana ÇITAK

Üye

Yrd. Doç. Dr. Pınar KAYGAN

Üye

Yrd. Doç. Dr. Emre SELÇUK

Üye





## B. Consent Form

### Gönüllü Katılım Formu

Değerli katılımcı,

Bu ders, “Matematik Öğretmen Adayları İçin Bir Teknoloji Entegrasyonu Dersinin Geliştirilmesi: Tasarım Tabanlı Araştırma” konulu araştırma kapsamında içeriği oluşturulmuş olan matematik eğitiminde teknoloji entegrasyonu için hizmet öncesi öğretmen eğitimini amaçlamaktadır. Araştırmamızın amacı, öğretmen adaylarına, matematik eğitiminde teknoloji entegrasyonu için gerekli becerilerin kazandırılmasını sağlayacak bir dersin tasarlanması ve dersin içeriğinde tasarlanan etkinliklerin öğretmen adaylarının muhakeme ve anlamlandırmalarına olan etkilerinin incelenmesidir. Bu amaçlar için tasarlanan dersin planlanan 14 haftalık çalışma süresince (i) TPAB-Uygulama Testi, (ii) SQD-Uygulama Testi, (iii) grup etkinlik çalışma kağıtları, (iv) ses kayıt ve video kayıt cihazlarıyla desteklenmiş gözlemler, (v) görüşmeler, (vi) hazırlanan ders planları, (vii) hazırlanmış akran-ders planını değerlendirme raporları (viii) değerlendirme raporları ve (ix) çevrimiçi tartışma gruplarının kayıtları ve (x) öğretmen adaylarının sunumları (mikro-öğretim) temel veri kaynakları olacaktır. Bu kapsamda toplanacak veriler doktora öğrencisi Murat Kol’un doktora tez çalışmasında kullanılacaktır.

Çalışma süresince toplanacak veriler tamamıyla gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bulgular tez çalışmasında ve bilimsel yayımlarda akademik etik kurallarına dikkat edilerek kullanılacaktır. Çalışmaya katılım tamamıyla gönüllülük temelindedir. Çalışma süresince katılımcılar için potansiyel bir risk öngörülmemektedir. Ancak, katılım sırasında farklı amaçlarla toplanan veya alınan dersin gerekleri olarak toplanacak verilerin bilimsel çalışma ve tez çalışması amaçları çerçevesinde kullanılmamasını isteyebilirsiniz. Bu durum ders performansınızın değerlendirilmesinde kesinlikle negatif bir durum oluşturmayacaktır.

Çalışma hakkında daha fazla bilgi almak için dersi veren öğretim üyeleri ve Araştırma Görevlisi Murat Kol (e-posta: mkol@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz

Bu çalışmaya tamamen gönüllü olarak katılıyorum ve istediğim zaman yarıda kesip çıkabileceğimi biliyorum. Verdiğim bilgilerin bilimsel amaçlı yayımlarda kullanılmasını kabul ediyorum. (Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad

Tarih

İmza

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## CURRICULUM VITAE

### PERSONAL INFORMATION

Surname, Name: Kol, Murat  
Nationality: Turkish (TC)  
Date and Place of Birth: 14 February 1977, İstanbul  
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### EDUCATION

Degree	Institution	Year of Graduation
PhD	METU Mathematics Education	2022
MS	METU Mathematics Education	2014
BS	METU Mathematics Education	1999
High School	Prof. Dr. Faik Somer High School, İstanbul	1992

### WORK EXPERIENCE

Year	Place	Enrollment
2012-2021	METU Dept. of Math. And Sci. Ed.	Research Assistant
1999-2011	Ankara	Mathematics Teacher

### FOREIGN LANGUAGES

Advanced English

### PUBLICATIONS

- Şat, M., Kol, M., Kayaduman, H., & Baran, E. (2014). The impacts of TPACK workshop in professional experiences and attitudes of in-service math teachers. In *Association for Educational Communications and Technology Conference*. Jacksonville, Florida.
- Kol, M., Erbaş, A. K., & Çetinkaya, B. (2015). Ortaokul matematik öğretmen adaylarının dikey matematikselleştirme sürecinin bir matematiksel modelleme

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3. Cetinkaya, B., Kursat, A., Kubra Celikdemir, E., Koyuncu, F., Kol, M., & Alacaci, C. (2017). Professional competencies that mathematics teacher educators should have: reflections from a workshop. In G. E. & N. J. (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 491–494). Indianapolis, IN: Association of Mathematics Teacher Educators.
  4. Cetinkaya, B., Alacacı, C., & Kol, M. (2017). Matematik öđretmeni yetiřtirmek için gerekli matematik bilgisi: Ne bilmeli? Ne düzeyde bilmeli? Ne řekilde bilmeli? Matematik öđretmen eđitimcisi yeterliklerinin belirlenmesi ve matematik öđretmen eđitimcilerinin yeterlik algıları. In *Türk Bilgisayar ve Matematik Eđitimi Sempozyumu*. Afyon.
  5. Erbař, A. K., Kol, M., Çetinkaya, B., & Kurt, Ü. C. (2017). Matematik öđretmen eđitimcisi yeterlikleri: Öđretmenlerin, doktora öđrencilerinin ve öđretmen eđitimcilerinin öncelikleri. In *EJER Bildiri Özetleri Kitabı* (pp. 987–989).