MIDDLE AND HIGH SCHOOL MATHEMATICS TEACHERS' PERSPECTIVES ON THE USE OF 3D DIAGRAMS IN TEACHING GEOMETRY

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ABSTRACT

MIDDLE AND HIGH SCHOOL MATHEMATICS TEACHERS' PERSPECTIVES ON THE USE OF 3D DIAGRAMS IN TEACHING GEOMETRY

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Master of Science, Mathematics Education in Mathematics and Science Education
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This study investigates the perspectives of middle and high school mathematics teachers on the application of three-dimensional diagrams in geometry teaching. A supporting aim was also to obtain their suggestions on supporting students' diagrammatic reasoning and addressing misconceptions and difficulties that may arise. The study's participants are 12 math teachers working in Turkey on face-to-face and online platforms. Data were collected through semi-structured, open-ended, and task-based interviews with every participant and examined using content analysis. Math teachers' perspectives on using 3D diagrams were categorized in terms of the role they assign to them in teaching geometry, their value as teaching resources, and their awareness of students' misconceptions and difficulties. While thought necessary to understand geometry, this study found that teachers prefer to use 2D and 3D diagrams in a two-dimensional way rather than digital ones when teaching. They also feel their students would benefit from materials integrating each question with a QR code so that questions can be better understood through multiple images, along with suggestions for digital courses for pre-service teachers and digital workshops for teachers.

Keywords: Geometry, 3D diagrams, math teachers, misconception, perspective

ÖZ

ORTAOKUL VE LİSE MATEMATİK ÖĞRETMENLERİNİN GEOMETRİ ÖĞRETİMİNDE 3D DİYAGRAMLARIN KULLANIMINA İLİŞKİN PERSPEKTİFLERİ

Yıldızel Saygılı, İlkay
Yüksek Lisans, Matematik Eğitimi, Matematik ve Fen Bilimleri Eğitimi
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Bu araştırmanın amacı ortaokul ve lise matematik öğretmenlerinin geometri öğretiminde üç boyutlu diyagramların uygulanmasına bakış açılarını incelemektedir. Ayrıca, bu çalışma ile öğretmenlerin, öğrencilerin 3 boyutlu geometri konusundaki kavram yanılgılarının farkında olup olmadıklarını anlamak ve matematik öğretmenlerinin diyagramlardan kaynaklanan kavram yanılgılarını azaltmaya yönelik önerilerini öğrenmek de amaçlanmaktadır. Çalışmada nitel araştırma yöntemleri ve fenomenolojik araştırma deseni benimsenmiştir. Araştırmanın katılımcılarını, Türkiye'de yüz yüze ve çevrimiçi platformlarda görev yapan 12 matematik öğretmeni oluşturmaktadır. Araştırmanın verilerini toplamak için her katılımcıyla yarı yapılandırılmış, açık uçlu ve göreve dayalı görüşmeler kullanılmıştır. Toplanan veriler içerik analizi kullanılarak incelenmiştir. Bulgular, matematik öğretmenlerinin 3 boyutlu diyagramları kullanma konusundaki bakış açıları, geometri öğretiminde kendilerine yükledikleri rol, öğretim kaynakları olarak değerleri ve öğrencilerin kavram yanılgıları ve zorluklarının farkında olmaları açısından kategorize edilmiştir Her kategori için en belirgin düşünce olarak diyagramların geometriyi anlamaya yardımcı olması ve öğretmenlerin derslerinde kullanmak üzere 2 boyutlu ve 3 boyutlu diyagramlara dijital diyagramlardan daha yakın oldukları ortaya çıkmıştır. Ayrıca, öğretmenlerin öğretmen adayları için dijital kurslar ve öğretmenler için dijital atölye çalışmaları için önerilerin yanı sıra soruların birden fazla görüntü aracılığıyla daha iyi anlaşılabilmesi için her soruyu bir QR koduyla entegre eden materyallerden öğrencilerinin yararlanacağını düşünmektedir.

Anahtar Kelimeler: Geometri, 3 boyutlu diyagramlar, matematik öğretmeni, kavram yanılgısı, perspektif

To My Beloved Family

&

All Teachers

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LIST OF ABBREVIATIONS

ABBREVIATIONS

NCTM	National Council of Teachers of Mathematics
MoNE	Turkish Ministry of National Education
SCK	Specialized Content Knowledge
2D	Two Dimensional
3D	Three Dimensional
IMAT	Integrating Multitype-Representations, Approximations, and Technology
PHI	Potentially Helpful Information
PMI	Potentially Misleading Information
CAD	Computer-Aided Design
EBA	Education Information Network for schools in Turkey
VITAMIN	An online educational support platform for K-12 by SEBIT
SEBIT	Education and Information Technologies Inc.
DGE	Dynamic Geometry Environment

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Geometry is the branch of mathematics that deals with the characteristics of space, the shapes of particular things, and the interactions between them in space. It is one of the oldest fields of study in mathematics and gets its name from Greek terms for measurement of the Earth having initially been devised to address real-world challenges found in land surveying. Geometry is a fundamental part of mathematics and is crucial to understanding our geometric world or cosmos. Additionally, it is an important subject that is being studied and learned about by students worldwide. In Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics reaffirmed the value of geometry by stating that "Geometry gives a means of describing, analyzing, and interpreting the world and perceiving beauty in its structures" (NCTM, 2000, p. 309).

The Principles and Standards for School Mathematics published by NCTM (2000) also highlights the significance of geometry and explains how it may be utilized as a tool for other mathematical and scientific disciplines alongside for understanding our world. Geometry is the natural subject of mathematics which also improves the reasoning abilities of the students (MoNE, 2005). Additionally, geometry plays a significant part in the Turkish National Education System. As claimed by the Turkish Ministry of National Education, teaching geometry and honing these skills are two of the main objectives of the modern mathematics curriculum.

Furthermore, geometry can be thought of as a starting point for mathematical visualization. To examine this further, looking at the papers and/or books published in the recent years dealing with visualization in mathematics education could be a good way. However, most of the available literature concentrates on teaching or learning calculus, followed by (pre-) algebra and number systems, then plane geometry, and only a small fraction on space geometry (Gutiérrez, 1996). Even though visualization is a critical component of teaching geometry, particularly space geometry, in the relevant field survey, effective methods for teaching these concepts have not been found currently.

When it comes to teaching geometry, it is possible to state that the effectiveness of the instructional techniques used by teachers remains a highly controversial matter. When choosing instructional materials, they can, for instance, use digital learning environments, virtual manipulatives, or more conventional methods such as textbooks depending on the grade levels of the students. Traditional approaches used in schools for teaching and learning geometry, however, are unsatisfactory since they do not meet all of the quality standards (Alakoç, 2003). The previous statement could be because the teachers did not know whether the resources they chose were appropriate for their learning goals or not. Students may struggle to visualize the location of the figures provided by the teacher on the chalkboard (Bako, 2003). These graphs on a blackboard which are a kind of diagram, are paired with concept representations in the literature. These examples demonstrate how students express concepts, theorems, and how issues in their reasoning can be affected by how they use diagrams.

It is also known that students have trouble physically imagining the analytic and geometric work in Euclidian 3D space. While working on spatial analytic geometry activities, chalkboard illustrations or homemade transparencies, primarily of the questionable quality of perception, are not a basis for developing an adequate spatial-geometric understanding. However, this does not preclude students from being able to solve those tasks algorithmically without spatial understanding (Schumann, 2003).

Teachers may use models, manipulatives, and diagrams to assist their students with the construction of solid conceptual pictures of three-dimensional geometric figures.

On the other hand, their perceptions and interpretations of the diagrams could lead to misunderstandings (Clements & Battista, 1992). Misconceptions about geometry might result from the knowledge of the teacher on the subject and particular content knowledge. For example, in a study of Euclidean geometry in Nigeria, participants included 37 mathematics instructors in senior classes at secondary school level. The instructors were asked to reply to four open-ended questions, and the items in the questionnaire required them to specify the knowledge gaps of the students and potential remedial measures. The study revealed that most teachers could not identify students who lacked proficiency in angles in parallel lines, and neither could they offer concrete solutions to the problems students had (Zuya & Kwalat, 2015).

Clearly, mathematics teachers need to know how to employ diagrams when teaching geometry and this knowledge should be reflected in their choice of teaching materials and use of 3D geometry in both middle and high school settings.

1.2 Purpose of the Study

The literature mentions several benefits that arise from using diagrams in teaching 3D geometry. Indeed, the processes by which mental pictures are produced and stored in a person's mind have been of interest to researchers for some time now. There are also many exams that have been created to gauge the proficiency of students in manipulating mental pictures, many of which forbid students from using pencil and paper or computers to complete the questions. Contrary to the viewpoint of cognitive psychologists, mathematics educators believe that mental pictures and exterior (i.e., non-mental) representations must work together for students to grasp concepts and resolve issues fully (Zimmermann & Cunningham, 1991).

The general use of diagrams is helpful to making concepts more concrete, but their inappropriate use can just as easily cause misconceptions and difficulties. Different types of diagrams are used in the in-depth analysis in both literature and the current study, each with a specific purpose. Although the consensus is that diagrams should be used to help students understand subjects more clearly, improper diagram usage can lead to misunderstandings and problems among students as well. For this reason, the present study investigates the perspectives of middle school and high school mathematics teachers working face-to-face and on online platforms using 3D diagrams in geometry education. The current study also aimed to understand whether teachers are aware of these difficulties and misconceptions of students on 3D geometry, and how teachers choose their teaching materials. However, not much research has been conducted on the specialized content knowledge and expertise of mathematics teachers regarding the role of diagrams in teaching and learning geometry.

The following research questions guided the study.

1. What specialized content knowledge do mathematics teachers have regarding 3D diagrams and their representations?
2. What is the mathematics teachers' awareness regarding difficulties and misconceptions that may occur in the students' minds while teaching 3D geometry in 2D environments?
3. How can these misconceptions and difficulties that the students have been reduced from the perspectives of mathematics teachers?

1.3 Significance of the Study

According to Tatsuoka et al. (2004), mathematics concepts and skills, including reasoning, knowledge, concepts, and qualities in judgmental ways, data management, and processing abilities are all intimately related to geometric knowledge. Studying geometry may help students become more mathematically proficient and develop their IQ level and other cognitive skills (Clements & Battista, 1992; Clements & Sarama, 2007). The authors also postulate that teaching geometry could serve as a springboard for teaching higher-order mathematical reasoning abilities. This study is valuable for seeing the perspectives of primary and high school mathematics teachers on the efficiency of diagrams in teaching three-dimensional geometry. The contribution of the ideas of teachers to the literature regarding the level of detecting difficulties and misconceptions arising from diagrams by students highlights another importance of this study.

On the other hand, the reasons behind the lack of knowledge of instructors regarding the suitability of the materials they select for teaching purposes are still not entirely discovered. Educators concur that instructor expertise is one of the key factors influencing student learning (Fennema & Franke, 1992). Investigating the particular subject knowledge of teachers is crucial for better mathematics, especially geometry education. By examining how mathematics teachers select their resources for teaching geometry and their opinions on the diagrams of 3D geometry, it is also possible to gain insight into designing future teacher education resources for geometry and professional development programs. Based on the findings of the current study, it will be possible to see how the teacher preparation program can be enhanced in terms of specialized mathematical subject knowledge and to get a sense of how the views of the teachers on diagrams alter depending on whether they work face-to-face or on digital platforms. Given the growing importance of technology, entrants to the profession can, in the opinion of in-service teachers, benefit from knowing how they apply it to enhance student learning (Koponen et al., 2016). In this way teachers will be able to base their instruction and methods for teaching on current best practice in their intended branch of mathematics.

In addition, the literature review revealed that few studies exist on using diagrams in teaching 3D geometry that involves the knowledge and perspectives of mathematics teachers. Especially in studies conducted abroad, the effects of using diagrams in evidence-related tasks were tried to be determined by students. The curriculum does not require that students make their own diagrams; rather, they are provided with all diagrams for classwork, homework, quizzes, and exams. The participants in this research believed that diagramming was necessary. Evidence also suggests that these students were unable to measure angles with a protractor. Students believed they could comprehend the issue after creating the diagrams. This discovery is significant (Mudaly & Reddy, 2016). At this point, it is essential to reveal the perspective of the teachers on issues and diagrams rather than proof.

Lastly, I would like to point out that I teach on the same digital platform as the mathematics teachers with whom I conducted this study. Indeed, the conceptual need for this research arose from a problem I encountered in the digital environment. This aspect of the study sets it apart from similar ones.

1.4 Definitions of Important Terms

The operational and conceptual definitions of the ideas and terms commonly used in this study are provided below.

Spatial Ability: According to Sjölinder (1998), "spatial ability" refers to the cognitive processes that allow people to successfully handle spatial relations, visual-spatial tasks, and object orientation in space. In the current study, the total spatial visualization and spatial orientation scores are referred to as the spatial ability score.

Geometry: It is the area of mathematics that deals with the characteristics of space, the shapes of particular things, and the interactions between them in space (Heilbron, 2022).

Diagrams: According to Accascina and Rogora (2006), diagrams are pictures that represent geometrical relationships and objects on a blank sheet. Diagrams can also be classified further. For example, sketch diagrams or outline drawings which, perhaps with the help of graphic conventions, illustrate particular two or three-dimensional geometric item features.

Specialized Content Knowledge: With the most common definition, "Specific sort of pure topic area knowledge" may be necessary for teaching, according to Ball et al. (2008, p.396).

Misconception: It refers to the mistaken ideas of the students on particular concepts, especially in the context of science. It was also defined as an incorrect or faulty concept understanding (Groff, 2020).

Perspective: Perspectives are collections of ideas, a way of thinking that each of us adopts that shapes how we view one another, our past, and the possibilities—or lack thereof—that lie ahead (Desautels, 2014).

Visualization: "The process of forming images (mentally, or with pencil and paper, or with the aid of technology) and using such images effectively for mathematical discovery and understanding" (Zimmerman & Cunningham, 1991, p. 3).

CHAPTER 2

LITERATURE REVIEW

The study on the perspectives of middle and high school mathematics teachers working face-to-face and on online platforms on the use of 3D diagrams in teaching geometry and relevant concepts are presented in this chapter. In this chapter, a presentation of geometry through spatial abilities and its teaching is presented first. Then a framework for teacher knowledge on teaching and learning geometry is discussed. Then, the role of diagrams in teaching geometry is presented. Lastly, a review and a detailed presentation of the research nexting to similar studies follow. The chapter concludes with a summary of the literature and its implication for the current study.

2.1 Geometry

“Every age, every grade, and every year should emphasize geometry. ‘What does this have to do with the actual world?’ is a common (student) question when teaching and learning mathematics. Geometry is the most applicable branch of mathematics. It is the foundation of all sciences, including physics, chemistry, biology, geology, geography, art, and architecture. The foundation of mathematics is geometry, though trendy abstraction disguised this fact for much of the 20th century. This is now changing because of computation and computer graphics, which enable the recovery of this core without sacrificing rigor. The students should be equipped with the skills they will need tomorrow through the middle school curriculum course material” (Senechal, 2005, as cited in Clements and Sarama, 2011).

Within a networked framework of concepts and systems of representation, geometry is the exploration of the properties, interconnections, and alterations of spatial objects. Geometry is supported by spatial reasoning, which enables pupils to mentally construct and manipulate those spatial objects (Clements & Battista, 1994). Consequently, geometry and spatial thinking should be studied concurrently (Battista, 2007).

According to Sherrard (as cited in Duatepe, 2004), geometry is essential for all students. After all, it facilitates communication because geometric definitions are used in speech. Thus, it is encountered in an ordinary routine. It helps students develop their spatial perception because learning geometry enables them for higher mathematics and science courses, many occupations requiring mathematical skills and because it helps promote general thinking and problem-solving skills. Unfortunately, presentations are frequently used while teaching mathematics, which only allows for superficial knowledge and requires learners to memorize mathematical information. One research finding suggests that students are not mastering geometry with related comprehension of the ideas, which calls into question the epistemological character of geometry (Ubuz & Üstün, 2004).

2.1.1 Spatial Abilities

Mathematical ability is influenced by spatial thinking, an essential human trait. It is a technique distinct from verbal thinking and uses several regions in the brain. (Shepard & Cooper, 1982). (Newcombe & Huttenlocher, 2000). Additionally, mathematical success and spatial ability are closely associated (e.g., Ansari et al., 2003). This argument is significant because some individuals suffer from poor spatial awareness and would benefit from learning more geometry and spatial skills (Casey & Erkut, 2005). Without the ability to communicate about object placement and relationships, provide and receive directions, and visualize changes in shape, size, or location, Smit (1999) emphasizes the significance of spatial abilities and contends that it would be challenging to function in society. In addition to being necessary for day-to-day tasks, spatial thinking is also needed for lower to higher-level cognitive functions (Basham, 2006).

Aptitudes involving spatial relations have long challenged the understanding of psychologists and mathematics professors alike. As noted by Wheatley (1998), there are virtually as many different ways to define spatial ability (and other related notions) as the number of academics who use this phrase. This is because there are so many different instruments available for gathering data. For example, the capacity to create, store, retrieve, and alter well-organized visual picture may describe spatial capacity (Lohman, 1996). Although there is debate about the exact meaning of the term, researchers agree that spatial ability is not a unitary phenomenon. Each spatial ability component that has been discovered emphasizes a different facet of image creation, archiving, retrieval, and alteration. Three of the most commonly addressed components of this ability are spatial vision, spatial orientation, and spatial interactions.

2.1.2 Teaching Geometry

For the past forty years, teaching geometry has been a topic of debate, and the place given to the empirical/theoretical duality has been subjected to significant changes. It was significantly impacted, particularly by a few nations supposedly reforming modern mathematics, which principally focused on the formal aspect of geometry without using pictures. It was argued that the usage of diagrams made geometry particularly challenging for students since diagrams confounded students with their empirical facts and the teachers insisted on employing deductive reasoning only (Laborde et al., 2006). On the other hand, new generations are influenced by a deficiency of geometry and geometrical teaching knowledge. For instance, a previous study discovered that kindergarten students were quite familiar with shapes and how to match them before even their training started. That showed that the teacher tended to elicit and confirm the previous knowledge of the students but did not appear to add or generate new information.

Studies have been undertaken in this area, such as the one by Ubuz and Üstün (2004), and their findings raise concerns about the epistemological character of geometry since they show that children are not learning geometry together with the conceptual understanding that is connected to it (Skemp, 2006). They, therefore, anticipate that students should be able to recognize, name, and analyze the properties of two-dimensional and three-dimensional geometric figures. This expectation is supported by the explicit recommendation of the National Council of Teachers of Mathematics that teaching programs of geometric figures should begin during the pre-school years (NCTM, 2000).

The ability to represent three-dimensional things in two dimensions and understanding the geometrical characteristics of three-dimensional forms in space are examples of three-dimensional geometry abilities. Fundamentally, 2D geometry is emphasized significantly more than 3D geometry in teaching geometry, according to my personal experience as a teacher. Obstacles and visual hurdles are put in front of learners due to the requirement to visualize 3D geometric figures using 2D drawings (Kali & Orion, 1996; Parzysz, 1988). On the other hand, according to Parzysz (1988), theoretical geometric objects are typically represented graphically by an infinite number of groups of forms that have similar properties (that is, instances of the idea). Thus, many students struggle to "penetrate" the two-dimensional drawings of three-dimensional forms and determine their correct three-dimensional representations (Kali & Orion, 1996).

2.2 Knowledge of Mathematics for Teaching

Knowledge of mathematics teachers is a rich topic of research in mathematics education. (e.g., see Ponte & Chapman, 2016). Despite the recent development of teacher knowledge research, various conceptualizations of mathematics teacher knowledge are already influencing the literature today (Petrou & Goulding, 2011; Rowland, 2014).

It is challenging to create an effective plan for a teacher education program if the role of the teacher's knowledge, the traits mentioned, and how they interact in teaching mathematics are not well understood. (Petrou & Goulding, 2011). Teacher expertise affects how well students grasp mathematics as well. The following is how Shulman and his colleagues rephrased the appraisal of the subject-matter expertise of teachers regarding the role of the material in education (Ball, 2008).

In 1987, in his seven-point proposal for teacher knowledge, Shulman argued that content was the framework that education research was missing. The parameters for teacher knowledge are introduced in Figure 2.1.

General Dimensions	**Content Dimensions**
General pedagogical knowledge	Subject Matter knowledge
Knowledge of learners	Pedagogical knowledge
Knowledge of educational context	Curricular knowledge
Knowledge of educational ends, purposes, and values	

Figure 2.1. The Components of Teacher Knowledge proposed by Shulman (1986, 1987).

Teachers must be knowledgeable about instructing students effectively, identifying their difficulties and misconceptions, and addressing them appropriately. In literature, comprehending incorrect responses is included in Common Content Knowledge (CCK), and identifying patterns in evaluating the nature of errors, particularly unexpected ones, requires Specialized Content Knowledge (SCK) (Ball, 2008).

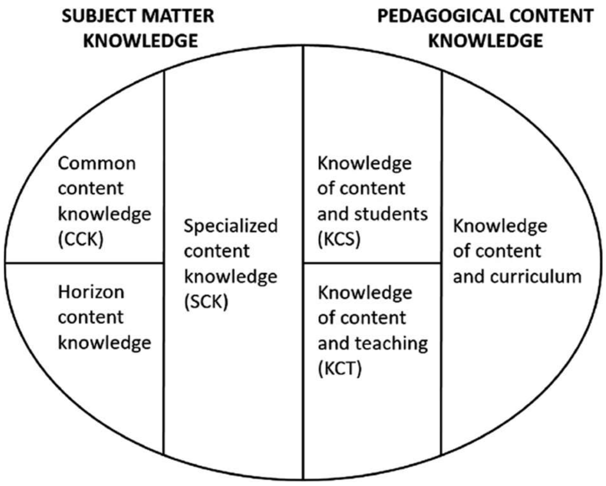


Figure 2.2. Zones of mathematical understanding for teachers (Ball et al., 2008, p. 403)

In other terms, according to Ball, as given in *Figure 2.2.* Zones of mathematical understanding for teachers (Ball et al., 2008, p. 403), and the Thames and Phelps (2008), "Specialized content knowledge (SCK) is the mathematical understanding and expertise particular to teaching." The abovementioned study also includes "how to exactly communicate numerical thoughts, supply numerical clarifications to fundamental principles and mechanisms, and inspect and appreciate peculiar arrangement approaches to problems" (Hill, Ball & Schilling, 2008, p. 377-378).

2.2.1 Specialized Content Knowledge of Mathematics Teachers

Mathematical understanding designed for a specific purpose in the teaching activity is defined as the understanding of specialized material. It is stated that instructors utilize it in their profession, but individuals with college degrees do not hold it and do not commonly use it for activities other than teaching. A "specific sort of pure topic area knowledge" may be necessary for teaching, according to Ball et al. (2008, p.396); it differs from the conceptual understanding discovered by Shulman and his collaborators because it is purified due to the reason that it is not integrated with a student or pedagogical knowledge. It is specialized because it is not essential for or used in settings other than mathematics instruction. (Ball et al., 2008).

As research on teacher knowledge has assumed a more fundamental role in mathematics education, the study of what separates the specialty in teacher knowledge has risen in relevance (Even & Ball, 2010).

The rigors of the mathematics teaching profession created a requirement for specialized mathematical knowledge structures (Ball, 2008). Cognitive processes in geometric activity include visualization, tool construction, and reasoning. Each has a distinct epistemic function, but due to their tight relationships and interdependence, "their combination is crucial for mastery in geometry" (Duval, 1998, p. 38). Teachers encounter misunderstandings if they fail to meet this need. These errors in geometry instruction might result from teachers' lack of understanding of diagrams, as described in the previous section on teaching geometry.

2.2.2 Relation between Specialized Content Knowledge of Mathematics Teachers and Diagrams

According to Scheiner et al. (2019), the first view emphasizes the complicated relationships between the application and purpose of expertise in teaching mathematics in an environment that demands specialization as a procedure instead of a condition of becoming. The position on epistemology is highlighted by the second point of view that mathematics teachers already possess and calls for awareness of the anthropological and cognitive foundations of mathematical findings. The last viewpoint emphasizes the intricate relationships between understanding components that produce active systems. Then, regarding the skill of mathematics teachers who specialize, they draw attention to overarching themes and points of convergence among these opposing viewpoints. Finally, rather than viewing specialization in mathematics teacher expertise as a type of knowledge, they suggest that it is viewed as a type of knowing (Scheiner et al., 2019).

Ball and her colleagues (2008) proposed a more complex diversification strategy in which the subject matter substance is carefully considered in a manner that is only intelligible to instructors. In other words, while the ideas of PCK and specialized content knowledge both imply the presence of a qualitatively different sort of knowledge, they differ in how much emphasis they place on each particular knowledge: The concept of pedagogical content knowledge of Shulman emphasizes a type of information specific to teachers (and not to disciplinary scholars), whereas the concept of specialized content knowledge of Ball and her colleagues (2008) emphasizes a type of knowledge specific to mathematics teachers (and not to teachers of other subjects). Following this, Scheiner and his colleagues (2017) summarized the solution of the three orientations to the question of what defines the specialization of mathematics teacher knowledge:

Requisite knowledge and skills in mathematics knowledge that is qualitatively different from that of other mathematicians, physicists, engineers, and other mathematicians, as well as the knowledge that is qualitatively different from that of professors of several other topics, including archaeology, geology, and chemistry. These knowledge requirements are outlined in the following paragraphs (teaching-oriented action).

Ball and her colleagues (2008) also identify and illustrate these subdomains in the next section. In the example above, the first domain corresponds to the initial stage: correctly calculating an answer or, more generally, solving a mathematical problem. They refer to this as common content knowledge (CCK) and define it as the mathematical expertise and information used in contexts other than formal education. The mathematical expertise and information specific to teaching constitute the second category, called specialized content knowledge (SCK). This is the area in which researchers have developed a great interest. According to a close examination, SCK is mathematical knowledge frequently not necessary for reasons other than teaching. Teachers must perform a particular type of mathematical work that others do not, such as searching for patterns in student errors or determining if a nonstandard strategy, like the one used for subtraction, will be generally effective. Herein, mathematics is unpacked in an odd way that is neither necessary nor desirable outside the classroom.

The mathematics teacher's specialized knowledge (MTSK) framework, built on and projects an intrinsic perspective whereby the idea of specialization is framed concerning the inseparability of knowledge and context, was previously illustrated (Carrillo et al., 2013).

In geometry, the ideas of concept image and concept definition introduced by Tall and Vinner (1981) have been included in the general cognitive theory of Figural Concepts developed by Fischbein. “In geometry, the ideal figural concept corresponds with the concept definition, while its mental reflection, with all its connotations and ambiguities, corresponds to a concept image” (Fischbein, 1993, p. 150).

As in earlier studies, the theoretical foundation of the present research is based on the Spatial Operational Capacity (SOC) hypothesis. The Spatial Operational Capacity (SOC) theory advises including activities that allow students to interact with actual three-dimensional objects, their two-dimensional representations, and other semiotic representations, as well as activities that allow information to be transformed between different representations to improve the 3D modelling abilities of students. The following is an explanation of the idea put out by Sack and van Niekerk (2009): “Scaled-down or full-scale replicas of essential things that the youngster can handle. The two models are semiotic, abstract symbolic representations that typically have no similarity to the authentic items and traditional models, which are two-dimensional (2D) graphic representations that resemble the authentic, three-dimensional (3D) objects. Diagrams of a vista and a floor layout are two examples” (p. 142).

2.3 Diagrams in Geometry

> The proverb *a picture is worth a thousand words*, or its equivalent *hearing a hundred times is not as nice as seeing once*, captures some of these reasons (Watson et al., 2013).

The teaching and learning of geometry is one of the areas of mathematics that makes extensive use of diagrams (Watson et al., 2013). According to the description given in this paper, a geometric illustration is a figure comprised of lines that are used to represent or specify something, or to assist in proving a claim. Such diagram usage in mathematics raises the possibility of similar use in mathematics education. In addition, "drawing diagrams is commonly cited as a heuristic for mathematical problem solving those students should engage in"; and as Samkoff et al. (2012) state on page 49, "diagrams are viewed by mathematicians and mathematics educators alike as an integral component of doing and understanding mathematics" (p. 50).

2.3.1 Definition of Diagrams

According to Accascina and Rogora (2006), diagrams are pictures that represent geometrical relationships and objects on a piece of paper. Diagrams can be classified further. For example, sketch diagrams or outline drawings that, perhaps with the help of graphic conventions, illustrate particular two- or three-dimensional geometric item features. Then, Euclidean 2D diagrams, or diagrams made with a ruler and compass, accurately represent a single instance of a specific geometric 2D configuration. Thirdly, Euclidean 3D diagrams or diagrams that are made by using a ruler and compass illustrate a particular 3D geometrical type arrangement using descriptive geometry techniques. Although some projection techniques can accurately maintain a few parallelisms and other features of the previous three-dimensional layout, these generally do not provide a faithful representation.

Furthermore, a subdivision can be made as digital diagrams using software, diagrams that display geometric objects and their relationships on a computer screen. Even though a more conceptual subdivision is feasible, they further separate according to the program or collection of applications used to create digital representations. They placed the diagrams made in CAD, Microsoft Paint, Cabri3D, and other programs.

On the other hand, Winn (1987), looking at diagrams generally, distinguished diagrams and picture formats (such as charts and graphs). According to this perspective, charts and graphs show how various factors relate. On the other hand, Winn claims that diagrams "describe whole processes and structures, frequently at levels of considerable complexity" (1987, p. 153).

2.3.2 Role of Diagrams in Teaching Geometry

Diagrams in teaching and learning mathematics are used every day for various reasons. Pittalis and Christou (2010) identified a variety of thinking styles and examined the relationships between various argumentation types and spatial awareness to define and evaluate the structures of 3D geometric thinking. They distinguished four distinct categories of reasoning: measurement, conceptualizing mathematical features, spatial structuring, and representation of three-dimensional (3D) objects. Regarding the 2D shapes that are used to represent 3D objects, Widder et al. (2014) suggested two different types of data. First, there is potentially helpful information (PHI), which is accurate information derived from two two-dimensional sketches of a three-dimensional picture. The second type of information is potentially misleading information (PMI), which is incorrect potential data derived from a two-dimensional shape representing a three-dimensional figure. There are two categories of PMI: (1) Modification of concealed information, such as secret vertices, edges, sides, and crossings of edges. (2) Addition of specific or erroneous information, such as edge crossings that do not exist, confluences of edges with straight lines that do not exist, and angles or sides that have been adjusted geometrically. The degree of visual difficulty can be perceived when the ratio of PMI to PHI is compared.

When teaching geometry, one of the earliest scholars to raise this problem was Freudenthal (1973). A growing chorus of voices clamoring followed him for the reintroduction of diagrams to geometry instruction, which eventually occurred towards the end of the 1970s and the beginning of the 1980s. And then Duval (1988, 1998, 2000) addressed this issue as well. Reasoning, tool-based building processes, and visual activities are the three types of cognitive processes that go into a geometrical exercise. Duval (2000) has identified numerous methods for drawing a geometric diagram and examined the function of visualization in the process of solving a geometry problem: a conversational approach anchored on the declaration outlining the problem's assumptions, an initial operational strategy used to identify sub-configurations helpful in resolving the issue; a wise technique that could be a barrier to the geometrical comprehension of the picture.

On the other hand, the figural and the conceptual components of geometrical notions are two parts that cannot be separated, according to Fischbein (1993), who developed a different psychological theory almost concurrently. The employment of two domains—the domain of diagrams and the domain of language—is the foundation of geometry instruction. In 2D geometry, diagrams have an unclear purpose, whereas language is a method for accurately specifying geometrical connections and issues. As multiple researchers have demonstrated, one effect is that students frequently believe that they can build simple visual clues to draw a geometrical figure or experiment to test a property (Chazan, 1993). The teacher expects students to use practical concepts while creating diagrams, yet they typically stay at the graphical stage and focus only on visual elements.

In contrast to this kind of instruction, researchers and teachers emphasized the significance of visualization in a geometry activity based on their research: geometry problems require more than just visualizing spatial relationships to be solved. It is generally accepted that teaching geometry should help students learn how to separate theoretical geometrical relationships from spatial graphical relationships, switch between conceptual elements and their visual portrayal, recognize geometrical relations in diagrams, and imagine every diagram that could be connected to a geometric object (Laborde et al., 2006). To develop a deductive argument that relates to the geometric properties of the specified idea rather than the specific attributes of the generic example, it is essential to use the diagram that underlies the geometric task as a generic example (Komatsu & Jones, 2020).

It is also known that the amount of data that can be found in a two-dimensional static graphic is far less than that is found in a body rotating in the 3-dimensional model, and the learner is not always able to complete the task in their mind. While moving from the actual thing to the drawn shape, learners may have no idea how much information is lost and continue to believe the structure accurately represents the original object. Many students face a visual barrier they may not even be aware of (Gutiérrez, 1996). Yet diagrams can help students understand numerous geometric concepts instantly and intuitively, especially while teaching flat Euclidean geometry. At the beginning of learning, however, diagrams are less helpful than manipulatives. Lack of manipulability may be one factor (Clements & Battista, 1992). "Diagrams can alter how students represent ideas, theorems, issues and misconceptions may develop out of how students interpret the diagrams themselves," according to research (Clements & Battista, 1992, p. 448). This is especially true for diagrams of 3D objects (Parzysz, 1988).

Parzysz lists three reasons why it can be challenging to comprehend diagrams representing three-dimensional objects correctly. The first reason is the loss of information due to projection. In a diagram, what one sees is a projection of what one has in space, and each point on the plane corresponds to an unlimited number of points in space. The second one is non-displayed parts of an object. For instance, planes are commonly modeled as quadrilaterals since one must only use a certain number of objects in a design. This makes the notion that two planes may meet at a segment or four points are required to determine a plane which seem reasonable to many students. And the final reason is lack of manipulation. As previously stated, this problem frequently arises in the illustration of geometrical objects using 2D diagrams.

According to Jones, diagrams are usually a good problem representation that allows complex geometric processes and structures to be represented holistically; therefore, their utility in geometry teaching and learning is not just due to the nature of geometrical objects. Diagrams can also deceive learners, so it is important to be careful while using them (Jones, 2013). Instead of seeing the picture as a representation of an abstract entity, students frequently remark on it as if it were the actual object (Gal & Linchevski, 2010; Laborde, 2005). Students who rely on a single drawing refer to it as the natural object and base their conclusions on the self-spatial-graphical characteristics of the drawing rather than the conceptual and theoretical geometrical characteristics of the real object (Fischbein, 1994; Laborde, 2005). The traditional method of teaching geometry, in which theoretical qualities are converted into graphical ones, completely ignores this confusing function of diagrams (Berthelot & Salin, 1998).

2.4 Studies on the Use of Diagrams in Teaching Geometry

2.4.1 Research Studies Abroad Related to Diagrams

In one of the earliest studies on 3D diagrams, Accassina and Rogora (2006) reviewed the literature on using diagrams in geometry instruction. Their research concentrated on how inadequate visual skills render 3D geometry instruction ineffective. They are interested in learning how Cabri 3D can help students enhance their visual thinking skills because standard methods are ineffective or require extensive training to be effective in doing so. Eight aspiring high school teachers used a multiple-choice test in 3D geometry as a pre-test in their studies before experimenting. They conducted tests to determine which questions students would have preferred using Cabri 3D over translucent and opaque solid models (Solid model vs. Cabri 3D). After that, students worked on cases independently at the first meeting. Two groups of students were set up when the two parties met again. They requested that they divide the cube into three pyramids equal to the one provided using Cabri 3D. They created a Cabri 3D diagram so the students could examine plane cube portions by dragging and altering the point of view, as seen in Figure 2.3.

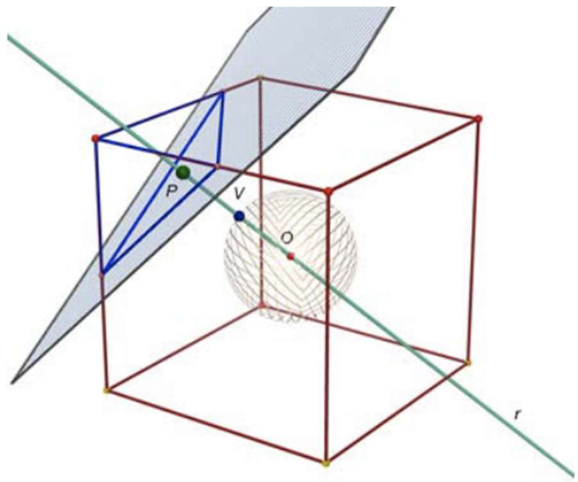


Figure 2.3. The points are used to regulate the plane section proposed (Accascina & Rogora, 2006, p. 6)

It is essential here to pay close attention to misunderstandings that could result from interpreting Cabri 3D diagrams. Their understanding of the perpendicularity of planes and lines was quite messy, their capacity to visualize 3D objects was limited, they had unexpected difficulties in understanding, and they had a limited understanding of the rationale underlying the definitions of 3D geometry. It was reported that Cabri 3D was highly user-friendly and that the students enjoyed using it because of the study. Since they had always had difficulty understanding 3D geometry using only static diagrams, some claimed to have finally discovered something helpful for examining the geometry of space. However, they have observed that Cabri 3D might be confusing because one can see things that are false while being unable to see obvious facts. They also observed that Cabri 3D helps students comprehend the combinatorial, topological, and affine features of a 3D construction.

In another study with a similar focus using IMAT (Integrating Multitype-Representations, Approximations, and Technology), Camou (2012) created a 3D geometry course for 140 high school students (integrating multi-type representations, approximations, and technology). The qualitative and quantitative data gathered from the experiment suggested that the method successfully enhanced the study of 3D geometry because students learned significant 2D and 3D geometry ideas over the 2-week experiment.

This outcome is in line with previous studies on technological tools that led the students to comprehend diagrams in which qualities are kept or modified (Komatsu & Jones, 2020). The findings support that using many diagrams could free students from adhering to the gestalt patterns of the archetypal examples and aid in their transition from uninformed conduct to more mathematical behavior (Duval, 1999).

Another study related to static diagrams can be mentioned at this point. The article aims to present findings from a recent investigation into the benefits and drawbacks of using diagrams in geometry instruction and focuses on recent studies on static diagrams in geometry education, even though part of this study has been on dynamic and interactive diagrams in geometry education (Jones, 2011). The remainder of this article concentrates on three uses of geometrical diagrams: those in textbooks for school mathematics, those in student geometrical problem-solving at school, and those used by teachers in the classroom to teach geometry. Jones, Fujita, and Kunimune (2012) used the geometry assignment using the diagram in Figure 2.4 on this paper with Grade 8 (Year 9) students in a recent classroom teaching experiment.

	Grade 8 (Year 9) textbook from Japan	Grade 8 (Year 9) textbook from England
Number of 'lessons' on geometry	34 'lessons' (out of 93 'lessons' in total for the year)	33 'lessons' (out of 121 'lessons' in total for the year)
Number of 'blocks' across the lessons	321 'blocks'	174 'blocks'
Proportion of 'Related graphic' blocks	14% of blocks	12.1% of blocks
Proportion of 'Worked examples with diagrams' blocks	8.4%	12.6%
Proportion of 'Exercise set with diagrams' blocks	16.2%	19.0%
Use of graphic or diagram	38.6% of blocks	43.7% of blocks

Figure 2.4. The frequency of geometrical diagrams in Grade 8 math textbooks (Jones et al., 2012, p. 39)

After more deliberation, a student who was sure that angle BGD would be 60 degrees proposed utilizing diagrams that show the cube in different orientations, as in Figure 2.5. The class was persuaded that triangle BGD is, in fact, equilateral, thanks to these alternate diagrams.

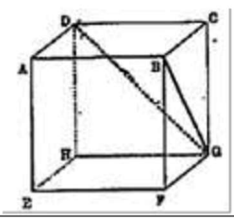
	What is the size of the angle BGD? State your reason why.

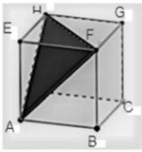
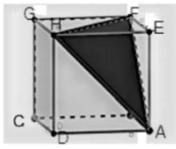
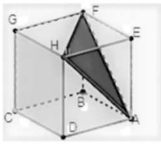
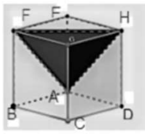
Figure 2.5. Cube Problem with Angle (Jones et al., 2013, p.40)

The appropriate diagram, together with appropriate language and symbols, helped to persuade the skeptical students—the interactions between diagrams, words, and symbols in the geometric problem-solving of the students need to be researched further, nonetheless. In conclusion, the three applications of these geometrical diagrams that have occupied the most of the paper's attention are as follows: diagrams in textbooks for secondary school mathematics, diagrams used by students to solve geometrical problems at school, and diagrams used by teachers to teach geometry. Each situation summarizes recent research highlighting some problems with geometrical diagrams. In each instance, many questions remain unanswered about the significance and application of geometrical diagrams. Using diagrams to aid teaching and learning is incredibly helpful; nonetheless, the procedures involved are relatively complicated (Samkoff et al., 2012). All of this suggests more study on diagrams in the teaching and learning of geometry.

There is also research on the usage of diagrams in proof cases. One is the study of Yerushalmy and Naftaliev (2011). They stated that whether the diagrams are static, like those found in books, or dynamic, like those found in digital technologies, is probably related to how secondary-school students engage with diagrams during a proving exercise. Elsewhere, however, it has been demonstrated how giving students a proof problem and more than one diagram, with a shape provided in various positions in each picture, influences their proof constructions and reduces their usage of PMI (Widder et al., 2014). The second study with a comparable aim is by Haj-Yahya (2021) on whether adding additional diagrams to a geometry problem could make it easier to prove specific three-dimensional geometry ideas. The research question "Does providing many diagrams in different orientations linked to a proof problem related to a 3D geometric idea lessen challenges in the proving process?" has been answered. Ninety students from an Arab high school in Israel made up the sample group. All students in three average-level classrooms where the participants studied mathematics with three different teachers participated in the study. A questionnaire that asked open-ended questions on two tasks was used. One of them is given below in Figure 2.6. Then, analyses, both qualitative and quantitative, were carried out. The survey looked at whether showing multiple diagrams with a proof task would reduce the use of the self-qualities of a single drawing rather than the essential concept attributes (in this case, related to the geometric space). The findings from the interviews supported the results of the surveys, and it was revealed in the interviews that adding more diagrams enables students to modify their proving procedures.

Figure 2.6. Questionnaire Questions (Haj-Yahya, 2021, p.7)

Table 1 The different positions of the diagrams presented in Version 2 of the questionnaire

	 Diagram 1	 Diagram 2	 Diagram 3	 Diagram 4
Non-existing added intersections of edges	FE and AH AF and CD	FA and HE	FA and HE DH and BC	FH and AE HA and CG FA and CG
Non-existing confluences of edges with a straight line				AE and CG
Geometrically altered sides	FA and AH are longer than HF	FA and AH are longer than HF	FA is longer than FH and HA	
Geometrically altered angles	The angles AHF and HFA are greater than the angle HAF	The angles AHF and HFA are greater than the angle HAF	Angle FHE = 90 degrees	

The results of this study suggest that displaying multiple data points associated with a proof task may impact proof construction. Specific designs had mathematically altered sides, while others only had geometrically altered angles. Nonetheless, other diagrams had both geometrically altered sides and angles. When dynamic software is not accessible, it is advised that teachers use the method of displaying several diagrams related to a geometric assignment to lessen the impact of visual obstructions. When a variety of diagrams are displayed, the students can adjust those diagrams to observe the variable sequence of examples that maintain the essential characteristics of the assignment while ignoring the optional characteristics. By developing such a mindset, mathematics teachers may be better able to identify and solve students' problems, perform better as instructors, and raise student performance levels.

2.5 Summary of Literature Review

The use of diagrams in teaching geometry has been addressed in the literature thus far in connection to various views and focuses on the subject. In one study, diagrams in textbooks for secondary school mathematics, diagrams used by students to solve geometrical problems at school, and diagrams used by teachers to teach geometry are compared. While some studies examined how usage of diagrams effect students' understanding in geometry, little is known about the perspectives of pre-service and in-service mathematics teachers. There are several proof studies that look into how students perceive geometry, however there are few studies about learning 3-dimensional Euclidean Geometry.

Additionally, most earlier studies examined the use of 3-dimensional diagrams in teaching quantitative research designs, particularly on proof tasks. The great majority of earlier studies used quantitative methods and quantitative study methodologies to explore the use of 3- dimensional graphics. Quantitative studies show that students learn geometry better when digital diagrams are used in the lessons, but the reasons behind these numerical results need to be examined separately with a deep understanding method.

Similarly, there are studies on digital diagrams, but the 2- dimensional reflections of the 3- dimensional objects is missing. It is also necessary to bear in mind that some teachers do not prefer to use digital diagrams, whether they are aware of how other visualization methods affect the knowledge level and understanding of their students, or not. When dynamic software is not accessible, it is advised that teachers use the method of displaying several diagrams related to a geometric assignment to lessen the impact of visual obstructions. We should learn what the teachers' views are on this point because they talk about this subject and technology is not used often with a deep understanding.

From the literature review, we appreciate there are numerous studies about diagrams related to geometry teaching; however, they are broad and consider the topic generally. Using diagrams to aid in teaching and learning is incredibly helpful. Still, the procedures involved are relatively complicated (Samkoff et al., 2012). All of this suggests more study of diagrams in the teaching and learning of geometry.

CHAPTER 3

METHODOLOGY

This study investigated the opinions of middle and high school mathematics instructors using in-person and online platforms to teach geometry and their ideas on the perspective of using 3D diagrams. This chapter first presents the study design, followed by descriptions of the participants and the study setting. Then, an explanation of the data gathering tool, the data collection process, and data analysis is presented. The credibility of the study will be the subject of discussion after that. Finally, the study's limitations will be the last discussion point of this chapter.

3.1 Overall Design of the Study

The phrase "an inquiry process of knowledge based on a specific methodological approach to inquiry that addresses a social or human problem" defines qualitative research (Creswell, 2013, p. 324). When it comes to methods of investigation, qualitative research is different from quantitative research. In contrast to quantitative research, qualitative research uses different assumptions, inquiry tactics, and techniques for gathering, analyzing, and interpreting data (Creswell, 2009).

In this research, qualitative research methods and phenomenological research design were adopted. The study was qualitatively designed because, according to Merriam and Tisdell (2016), "qualitative researchers are interested in learning how people perceive their experiences, how they create their reality, and what significance they give to their experiences″ (p.24).

There are various qualitative research methodologies in the literature. One of the qualitative research designs used in this study is the phenomenological design which aims to grasp the core of a collection of people's lived experiences surrounding a phenomenon. It focuses on the phenomena that we are conscious of but do not fully comprehend some concepts (Patton, 2002). To have a thorough grasp of the phenomena, would enable this study to investigate the perspectives of mathematics instructors on the phenomenon regarding misconceptions of 3D diagrams. Also, since generalizability is not a problem in qualitative investigations, this study is not concerned with producing conclusions that may be applied broadly (Creswell, 2013). Therefore, it would not be impossible to generalize the results of this study.

The aim is to examine the usage of 3D diagrams in geometry instruction from the views of middle-level and high school mathematics instructors who employ both traditional classroom settings and online learning environments. Three research questions were developed in line with the objectives of the study to find out the amount of specialized content knowledge of mathematics teachers in terms of 3D diagrams, the awareness of mathematics teachers about students' s difficulties and misconceptions in geometry, and the way of reducing these difficulties and misconceptions according to the ideas of the teachers.

Considering the purposes mentioned above, the following research questions will be studied to find related answers:

1. What specialized content knowledge do mathematics teachers have regarding 3D diagrams and their representations?
2. What are the mathematics teachers' awareness regarding difficulties and misconceptions that may occur in students while teaching 3D geometry in 2D environments, and how do they produce solutions?
3. How can the difficulties and misconceptions of students be reduced from the perspectives of mathematics teachers?

The interview questions were then created to gather information from mathematics teachers to address these study points.

3.2 Participants and Context of the Study

The present study was conducted with teachers who were working on an international online education platform. On this platform, an appropriate education plan for each child is determined and structured. Teachers teach their lessons via Zoom and using a graphic tablet. Apart from this, the teachers regularly attend seminars on education in digital environments. The platform works with teachers who are least equipped to provide education in the digital environment. Approximately 500 teachers work for this company in Turkey. Fifty of these teachers were determined as the target group, and an invitation was sent to them. These 50 teachers have been chosen with the idea that their answers to diagrams as a phenomenon will enable the researcher to answer the study questions with the help of their face-to-face and online experiences. Since participants are the source of information on the way to answering research questions; the data provider has a major impact on how well the research questions are answered (Plano Clark & Creswell, 2015).

The selected participants in this study consist of middle and high school mathematics (12 teachers) teachers working face-to-face and online. The participants selected for the present study consist of middle and high school mathematics (12 teachers) teachers working face-to-face and on online platforms whereas the participants involved signed up voluntarily to participate in the study. Some teachers work only on the online platform, and some in both face-to-face and online environments. Thus, the participant selection strategy of this study is in convenience way which is readily available to the researchers and is a goal of the participant selecting approach used by qualitative researchers. Participants from the target group who meet specific practical criteria, such as easy accessibility, geographic proximity, availability at the specified time, or the desire to participate are recruited for the study through convenience way(Gill, 2020). No age or gender restrictions were set to ensure maximum diversity. Table 3.1 below summarizes the demographic data of the participants, including their participant number, age, gender, and amount of experience. Since all teachers only had undergraduate degrees, this information is not shown in the table below.

Table 3.1 Participants' Descriptive Information

Participant	Age	Gender	School Types/ Teaching Environment	Years of Teaching Experience	Grade Level Teaching
T1	30	Female	Online	6-10	Middle
T2	27	Female	Both	0-5	Middle-High
T3	39	Male	Both	+16	High
T4	40	Female	Both	11-15	High
T5	25	Female	Both	0-5	Middle
T6	32	Male	Both	6-10	High
T7	45	Female	Online	11-15	Middle-High
T8	26	Female	Both	0-5	High
T9	26	Male	Both	0-5	High
T10	31	Female	Online	6-10	Middle-High
T11	33	Male	Both	6-10	Middle
T12	28	Male	Both	0-5	High

3.3 Data Collection Tool and Strategies

In-depth interviews with participants are the primary data gathering method in phenomenological studies since they allow participants to express their perspectives on the phenomenon and provide insights into their inner lives (Creswell, 2013). As stated above, 50 teachers were chosen with the thought that the answer on diagrams as a phenomenon will be able to answer the questions using their experiences in both face-to-face and online environments. As a result, as interviews are the principal approach for gathering data for phenomenological investigations (Yıldırım & Şimşek, 2011), interviews were used in the current study.

Figure 3.1 summarizes the procedures used to develop the data gathering tool for this study. Following a review of the literature, the first version of the open-ended, semi-structured, and task-based interview protocol was developed, consisting of two sections with nine major questions and several probes. The questions used in this study were previously studied with students in the literature research and were directed to teachers, and finally, code lists were prepared. Questions about demographics were included in the first section—the second part related questions about the teachers' perspectives on 3D diagrams.

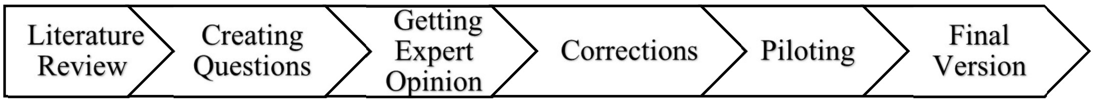


Figure 3.1. The Steps for Creating a Data Gathering Device

The examples of questions prompted in the interview will be provided in the following parts. All questions' Turkish and English versions and the main point of questions will be placed in Table 3.2. For the second part, with the first and second questions, the specialized content knowledge of mathematics teachers was measured by addressing the first research question. For example, ″1. What do you think about the role of diagrams in teaching geometry? ″ and ″2. When planning three-dimensional geometry topics/lessons, what kind of diagrams and how much do you use them, and for what purpose? You mentioned three-dimensional programs; how do you evaluate them? ″. The third, fourth, fifth, and sixth questions measure whether teachers recognize the misconceptions and difficulties that students may face by addressing the second research question.

Also, the seventh question included a digital version of diagrams in tasks, addressing the second research question. Students can better see the details they cannot see on paper in digital environments. For this reason, they were included in the subsequent tasks so that they did not affect the ideas on paper they saw in the digital environment. To illustrate, ″6. Suppose you ask your students about possible geometric shapes that can occur when a cube and a plane intersect. If the students received answers in the form of only three and four-sided polygons, as shown below, what would be your possible solutions, and how would you go about the difficulties they might encounter? ″. These tasks were previously asked students in different studies in the field survey, and the same questions were asked to teachers in this study. Finally, the eighth and ninth questions measure how teachers deal with the difficulties students have and what they propose for clarity of tasks and teacher education. For instance, ″8. As a teacher, how do you overcome your students' difficulties and misconceptions about diagrams of 2-D representations of 3-D geometric objects? ″ and ″9. The principle of openness in preparing teaching materials; is to carry out the features to be gained to the students and the activities to be organized for this purpose in a clear, easy, and understandable language and order for the student. How do you think this principle should be transferred to exams and course materials that include three-dimensional geometry? What are your suggestions for teacher training regarding this? ″

Table 3.2 Table of Specification for Data Collection Tool

Turkish Version	English Version	Main Points
1.Diyagramların geometri öğretimindeki rolü hakkında ne düşünüyorsunuz?	1.What do you think about the role of diagrams in teaching geometry?	Understanding role of diagrams in teachers' minds

Table 3.2 (continued)

2.Üç boyutlu geometri konularını/derslerini planlarken hangi tür diyagramlara ne kadar yer verirsiniz ve bunları ne amaçla kullanıyorsunuz?	2. When planning three-dimensional geometry topics/lessons, what kind of diagrams and how much do you use them, and for what purpose?	Understanding importance of diagrams in teachers' minds
3.Aşağıda MEB 12.sınıf ders kitabından alınan bir örnek verilmiştir. Sizce bu soruda öğrenciden ne yapması bekleniyor? Bu beklenti için bu tarz bir çizim uygun mu? Siz olsaydınız, böyle bir beceriyi öğrenciye kazandırmak için bu soruyu nasıl sorardınız?	3. Below is an example taken from the MEB 12th-grade textbook. What do you think the student is expected to do in this question? Is this kind of drawing suitable for this expectation? How would you ask this question to bring such a skill to the student?	Finding surface area of cube
4.Daha önce yapılan bir çalışmada, anket sorusu olarak öğrencilere A, M, F ve G noktalarının doğrusal olup olmadığı sorulmuş ve öğrencilerin Doğrusal olduğunu düşündükleri ortaya çıkmıştır. Öğrencilerin bu kanısını aşmak için ne önerebilirsiniz?	4. In a previous study, the question was given to the students in the questionnaire. It was asked whether points of A, M, F, and G were linear, and the students answered they were linear. To overcome this belief of students, what can you suggest?	Collinearity of A,M,F and G points
5. 12.sınıf geometri soru bankasından alınan bir soruya yer verilmiştir. Bu soruyu öğrencilerden nasıl çözmelerini beklersiniz? Sizce, öğrencilerin P noktasının yerini nasıl algıladıkları onların çözüm stratejilerini nasıl değiştirir? Bu tarz bir soruyu öğrenciler için daha anlaşılır kılmak için neler önerirsiniz?	5. A question taken from the 12th-grade geometry question bank is included. How do you expect students to solve this question? In your opinion, how do students perceive the location of point P? Does it change? What would you suggest making such a question more understandable for students?	Finding surface area of hemisphere and understanding place of P

Table 3.2 (continued)

6.Öğrencilerinize bir küp ile bir düzlem kesiştiğinde oluşan şekilleri sorduğunuzu varsayalım. Öğrencilerden, örnekleri gösterildiği gibi sadece 3 ve 4 kenarlı çokgenler şeklinde cevaplar gelseydi, olası çözüm yollarınız neler olur ve karşılaşabilecekleri zorluklar hakkında nasıl bir yol izlerdiniz?	6. Suppose you ask your students about possible geometric shapes that can occur when a cube and a plane intersect. If the students received answers in the form of only three and four-sided polygons, as shown below, what would be your possible solutions, and how would you go about the difficulties they might encounter?	Intersection of cube and plane
7.Verilen soruları 3 boyutlu dinamik ortamda yorumlamak isteseniz, nasıl cevaplar verirsiniz?	7.If you want to interpret the given questions in a 3D dynamic environment, how do you answer?	Understanding teachers' technological predisposition
8.Bir öğretmen olarak öğrencilerinizin 3 boyutlu geometrik objelerin 2 boyuttaki gösterimlerine ait diyagramlar ile ilgili kavram yanılgılarının üstesinden nasıl gelirsiniz?	8. As a teacher, how do you overcome your students' difficulties and misconceptions about diagrams of 2-D representations of 3-D geometric objects?	Understanding teachers' pedagogical predispositions
9.Öğretim materyalleri hazırlamadaki açıklık ilkesi; öğrencilere kazandırılacak özelliklerin ve bu amaçla düzenlenecek etkinliklerin, öğrenci açısından net, kolay ve anlaşılır bir dil ve düzende gerçekleştirilmesidir. Üç boyutlu geometri konularını içeren sınav ve ders materyallerine bu ilkenin nasıl	9. The principle of openness in preparing teaching materials; is to carry out the features to be gained to the students and the activities to be organized for this purpose in a clear, easy, and understandable language and order for the student. How do you think this principle should be transferred to exams and	To reveal teachers' advices about clarity of course materials on diagrams

Table 3.2 (continued)

aktarılması gerektiğini düşünüyorsunuz?	course materials that include three-dimensional geometry? What are your suggestions for teacher training regarding this?	

In terms of preparing the questionnaire, an expert with a Ph.D. and the title of Prof. who was working as a mathematics educator at a public university was given the initial draft of the semi-structured, open-ended, and task-based interview guide to validate the data collection tool. Some revisions were performed after receiving expert feedback.

The modified open-ended, semi-structured and task-based interview protocol is divided into two sections containing 4 and 9 core questions and sub-questions as probes. Four questions concerning personal information are included in the first section to reveal more about the participants. Since participants communicated in Turkish, the interview guide was also written in that language.

Pilot research was conducted with one middle school mathematics teacher and one high school mathematics teacher working in a digital education platform after the open-ended, semi-structured interview guide was revised following professional opinion. After it was piloted, several changes were made to the tasks. For some questions, prompt questions were added better to better understand the teachers' ideas and the phenomenon. For example, for the eighth question, ″ 8. As a teacher, how do you overcome your students' difficulties and misconceptions about diagrams of 2D representations of 3D geometric objects? ″ was the first version. After revision, questions of ″What features would you care to include in the diagrams? ″ and ″So, if you were to make the diagram drawings, what would you show and how? ″ were added as prompts. Also, for task-based questions, digital versions were decided to be added to understand how the ideas of mathematics teachers change based on the environment in which diagrams are shown. The interview protocol of the data collection technique (APPENDIX A) was finalized after the modifications and pilot with the help of the expert opinion.

3.4 Data Collection Procedures

The initial stage in the data gathering process was applying to the Human Subjects Ethics Committee (APPENDIX B) to get the approval necessary to conduct the study. After obtaining permission, a pilot study was conducted with two mathematics teachers, and through these interviews, an expert opinion was also taken by the researcher's advisor. Then, some revisions were made to the questions after these interviews. Then, a call for invitations (APPENDIX D) was made to invite volunteer teachers to participate and share a brief description of the study with them. Eligible teachers who fulfilled the requirements for selecting participants from the volunteer group were contacted by email and informed about the details of the study. Although it was stated to the teachers that they would be able to participate in the study at their own convenient time within two weeks, some teachers stated that they were out of town since it was during the summer vacation and that it would not be easy for them to access to the internet.

Additionally, participants were requested to use an interview schedule that the researcher had developed and shared with them on Google Drive, to indicate their availability. The calendar was shared via Google Drive so that the researcher and other participants could keep track of free and reserved time on the calendar at once to prevent any overlaps. After obtaining the approval of teachers to work on public, private, and online platforms, the interviews were conducted online. They were conducted using Zoom, a web-based communication platform. Participants completed the consent form (see APPENDIX C) and emailed it to the researcher before each interview. At the beginning of the interview, the participants were asked for their approval for the meeting recording. Meetings were recorded to capture the data and all essential information with the consent of all participants. Participants were also told that they could quit the interview whenever they wanted and that all recordings produced up until that point would be destroyed immediately. Each interview lasted between 30 to 45 minutes.

3.5 Data Analysis

The current study aimed to explore mathematics teachers' views in middle and high schools. Diagram use of teachers while teaching geometry and the study findings revealed verbal responses provided by participants to the interview questions. In this scope, the qualitative data for this study were gathered via 12 open-ended, semi-structured, and task-based interviews with participants. First, the transcripts of each interview recorded were completed, and then they were converted into a text format. The data is analyzed using MAXQDA 2022, as given below in *Figure 3.2*, to determine the critical issues in the transcribed interviews per group. Thus, using the software package MAXQDA 2022, which makes it easier for researchers to organize and sort material and construct codes and themes more quickly by permitting notetaking throughout the process, the data analysis was carried out rapidly and effectively.

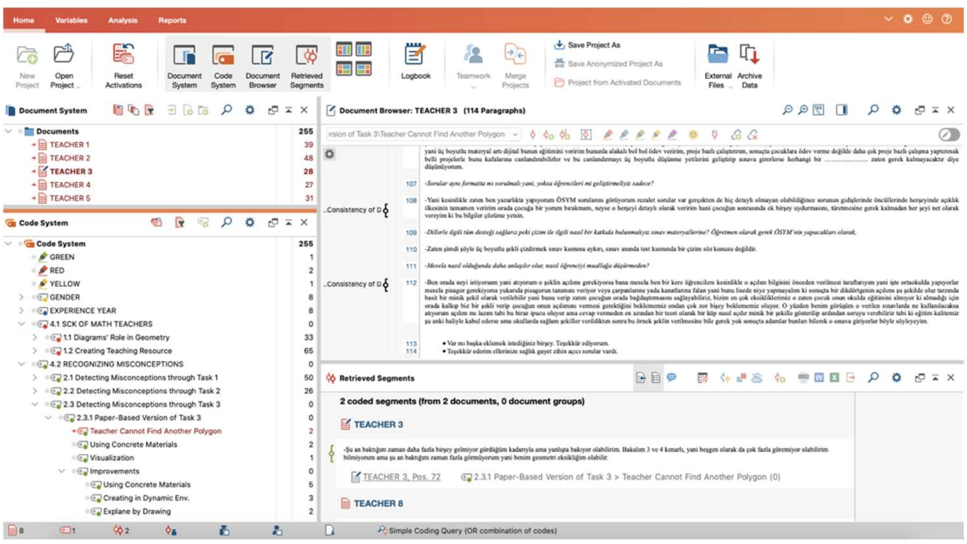


Figure 3.2. MAXQDA 2022 Coding Screen Used in This Study

Subsequently, examples of quotes and statements were chosen to provide textual descriptions to explain the significance of themes and codes formed by the connected responses of the participants (Creswell, 2013). The program also used research questions to code and group the data based on appropriateness. Finally, the data analysis was divided into three parts through research questions.

Chapter 4 contains all detailed codes and justifications in this chapter; all themes, sub-themes, codes, and teacher examples will be placed in Table 3.3, Table 3.4, Table 3.5, Table 3.6, Table 3.7, and Table 3.8.

Table 3.3 Coding for specialized content knowledge of mathematics teachers

Themes	Sub-themes	Codes	Sample Teacher Comments
Role of diagrams in teaching geometry	Diagrams as assistance in understanding geometry	Providing memorability	The teacher's drawing of colorful shapes increases the memorability of the lesson's subject. (T5)
		Attractive for students	Diagrams make geometry exciting and fun, like a puzzle. (T4)
		Connecting geometry with daily life	Children can establish a relationship between that real-life knowledge when they see diagrams. (T10)
	Diagrams as providing concretization	A way from abstract to concrete	Diagrams also help embody the subject. (T1)
	Diagrams as helping to the imagination	Visualization	I think it sure makes a kid imagine it in his head. (T7)
Diagrams' place in teaching resources	Through paper-based materials	Sketch	I give plenty of space to diagrams when drawing column and circle graphs. (T7)
		2-D	In Euclidean geometry, the Pythagorean theorem, etc. I use 2D drawings. (T1)
		3-D	I generally use it to show the open state of solid objects. (T1)
	Through digital materials	Digital software	GeoGebra / Thinkercad
		Mathematical packages	Vitamin/ Eba

Table 3.4 Coding of Task 1 for the mathematics teachers' awareness regarding difficulties and misconceptions

Themes	Sub-themes	Codes	Sample Teacher Comments
Recognizing Difficulties and Misconceptions and Improvements for Task 1	A paper-based version of Task 1	Incomprehensible	Even my eyes have difficulty perceiving, and I can't think of a solution. (T5)
		Unsuitable	That's why it will not be proper for the student. (T4)
		Diagonal is unclear	But does it go from A to G? (T12)
	Improvements for Task 1	Give extra info.	It would be clearer if it were stated first that G is the diagonal of the object. (T6)
		Changing the perspective view,	We could draw the view of the right isometric here. (T10)
		Coloring shape	One color from AM, another color from MF, and ED. (T5)
	Digital version of Task 1	Easy to explain	We can explain it to the child more quickly if we have this material. (T7)
		It makes the task enjoyable	It will get more enjoyable when the child turns two-dimensional after seeing this. (T10)
		Chance to see different views	Such programs contribute to learning and allow students to look at all aspects differently. (T11)

Table 3.5 Coding of Task 2 for the mathematics teachers' awareness regarding difficulties and misconceptions

Themes	Sub-themes	Codes	Sample Teacher Comments
Recognizing Difficulties and Misconceptions and Improvements for Task 2	A paper-based version of Task 2	Incomprehensible	When I first looked, it looked like it was in the air. I saw it as if it was going to the middle of the sphere but not reaching the top of the sphere. (T2)
		Misleading	I think P is on the surface. I cannot understand precisely what is shown in the figure. (T11)
	Improvements for Task 2	Giving extra info	With more info, the student would not be confused at all. (T6)
		Using concrete material	I would design a material for them to imagine. (T8)
		Coloring	This image is given in one color if it is drawn with different colors so that children can understand it better. (T8)
	Digital version of Task 2	Easy to imagine	Not every child can visualize it in their mind. Now we can explain this very well. (T7)
		Misconception remover	Students' misconceptions can be eliminated. (T1)
		Beneficial	It just helps the child to analyze that point better with questions. (T3)

Table 3.6 Coding of Task 3 for the mathematics teachers' awareness regarding difficulties and misconceptions

Themes	Sub-themes	Codes	Sample Teacher Comments
Recognizing Difficulties and Misconceptions and Improvements for Task 3	A paper-based version of Task 3	Not seeing extra polygons	I can see 3-4 sided polygons now; I don't see much so that it may be because of my lack of geometry. (T3)
		Explaining it by drawing	But I showed it by drawing the thing you know on the board with a pencil. (T4)
	Improvements for Task 3	Creating it in a dynamic environment	It is easier to benefit from programs that contain mathematical expressions. (T6)
		Using concrete materials	If there is an everyday cube-shaped object, I can pick it up and show it to the children so they can see it more realistically. (T10)
	Digital version of Task 3	Students may have difficulties	Could it be a difficulty, you know, it could be a problem in terms of use for children. (T10)
		Teachers may not handle technology.	Which is fine; I mean, it's an area where teachers should improve themselves. (T4)
		Improve the different thinking skills of students	I think it develops children's different thinking skills. (T12)

Table 3.7 Coding for the first part of reducing misconceptions from the perspectives of mathematics teachers

Themes	Sub-themes	Codes	Sample Teacher Comments
Principle of Clarity	Course materials	Bringing concrete materials to class	I put it somewhere in the classroom, so if the child cannot imagine it in the exams, I can show it. (T3)
		Giving detailed information	I recommend that the figures be given on detailed information and given correctly. (T2)
	Exam materials	Integrating each question with a QR code	We can go to a three-dimensional shape with a QR code reader. (T8)
		Consistency of drawings and information	Give everything clearly without the need for the child to make up. (T3)
		Exporting multiple images	Three shapes can be put and viewed from different angles. (T7)
		Coloring questions	Make it easier for children to see by adding more dimension by coloring. (T2)

Table 3.8 Coding for the second part of reducing misconceptions from the perspectives of mathematics teachers

Themes	Sub-themes	Codes	Sample Teacher Comments
Proposals for teacher education		Digital courses for preservice teachers	I think integrating technological lessons is missing in universities. (T11)
		The aesthetic aspect of teachers should be improved	I always think that a mathematician should have a very high aesthetic sense. (T1)
		Preparing digital workshops for teachers	Teachers need to be trained on 3D diagrams through digital courses. (T7)
		Integrating geometry and math as a discipline	Teachers already refer geometry lessons to each other. (T1)
		Creating math meetings for materials	

3.6 Trustworthiness of the Study

Regarding peer debriefing, a researcher who has been associated with another researcher in the research processes strengthens the study's credibility by being constantly informed about and expected to query the study's techniques, interpretations, and meaning (Creswell, 2013).

An analyst triangulation was employed in this study to support the validity of the data analysis. In analyst triangulation, the results are examined by different analysts. To reduce the potential bias of having a single individual participate in the analysis (Patton, 2002) and to provide the study with a single perspective, triangulating analysts in this study were obliged to have a fellow researcher assess the data simultaneously. To be able to provide triangulation, a second coder was involved in the coding process. A second coder who completed an MS thesis on geometry teaching coded the %10 part of the data since there were 12 participants, which is why the second coder completed the transcripts of two randomly selected teachers (among the total of 12). 85% agreement was attained once the analysis was concluded. The analysts met to discuss the points of disagreement with their associated second coder until they reached a full agreement.

CHAPTER 4

FINDINGS

This study aims to examine the views of mathematics teachers working in digital and face-to-face environments about diagrams in geometry lessons. With this aim, open-ended, semi-structured, and task-based online interviews were conducted with twelve mathematics teachers from Turkey working in both face-to-face and online settings. Three research questions were developed in line with the objectives of the study to find out the amount of specialized content knowledge of mathematics teachers in terms of 3D diagrams, the awareness of mathematics teachers about student's difficulties and misconceptions in geometry, and the way of reducing these difficulties and misconceptions according to ideas of the teachers. Content analysis was used to examine the qualitative information obtained from teacher interviews on mathematics. This research was conducted to ascertain how mathematics teachers see the use of diagrams while teaching geometry in three dimensions. The study intended to examine the perspectives of the participants on using diagrams and their knowledge of the subject matter. This chapter will provide results of the categories of conceptions of middle and high school mathematics instructors in three main sections.

4.1 Teachers' Specialized Content Knowledge on 3D Diagrams

The first research question of the current study seeks to evaluate the existence of SCK in mathematics teachers about 3D diagrams in geometry. For this instance, the first question was, "How much-specialized content knowledge do mathematics teachers have about 3D diagrams and their representations?" The opinions of the participant mathematics teachers on this topic were examined through content analysis.

Table 4.1 Specialized content knowledge of mathematics teachers' frequency table

Themes	**Sub-themes**	**Codes**	**Number of Participants**	**ID**
Role of diagrams in teaching geometry	Diagrams as assistance in understanding geometry	Providing memorability	7	T2, T4, T5, T1, T3, T8, T11
		Attractive for students	5	T2, T4, T6, T10, T11
		Connecting geometry with daily life	2	T10, T5
	Diagrams as providing concretization	A way from abstract to concrete	3	T12, T7, T11
	Diagrams as helping to the imagination	Visualization	3	T8,T7,T3,T6,T12
Diagrams' place in teaching resources	Through paper-based materials	Sketch	4	T3,T5,T7,T12
		2-D	6	T1,T2,T8,T9,T11 T12
		3-D	6	T1,T4,T5,T6,T8,T9
	Through digital materials	Digital software	3	T3, T6, T11
		Mathematical packages	4	T2,T4,T6,T10

The topics of the first study question, as shown in Table 4.1, are the roles of diagrams in geometry and the creation of teaching resources. The following sections will go into further detail on each of these topics.

4.1.1 Role of Diagrams in Geometry

The first aspect addresses mathematics teachers' specialized content knowledge on 3D diagrams in connection to themselves. Table 4.2 shows the sub-themes representing the perspectives of mathematics teachers who participated in the present study on the role of diagrams in geometry. The data revealed three sub-themes: assistant for understanding geometry, providing concretization, and helping with imagination.

Table 4.2 The sub-themes relating to the notions of mathematics teachers on the role of diagrams in geometry

Theme	Sub-themes
1. Role of diagrams in geometry	1.1. Diagrams as assistance in understanding geometry 1.2. Diagrams as providing concretization 1.3. Diagrams as helping with imagination

Three sub-themes of the topic titled ″diagrams' roles in geometry″ are shown in Table 4.2. The names of these sub-themes are assistants for understanding geometry, providing concretization, and helping with imagination.

4.1.1.1 Diagrams as Assistants for Understanding Geometry

Firstly, "assistant for understanding geometry" is the name of the first sub-theme of the diagrams' roles in geometry. The results of this section demonstrate that mathematics teachers see diagrams as helpful to students' understanding of geometry. It is a crucial step in the learning process for many reasons. Some teachers think that diagrams are helpful for teaching geometry thanks to their attractive properties, and they make memorability easier, as seen in some of the comments obtained from the participant mathematics teachers:

> Diagrams make geometry exciting and fun, like a puzzle. (T4)

> In other words, shapes are more common in children's minds. Because when I experienced it myself, I saw that what they saw and touched generally remained in their minds more. (T2)

> The teacher's drawing of colorful shapes increases the memorability of the lesson's subject. Even I remember the images my students draw on the board more quickly than they say. (T5)

Also, one teacher adds that including diagrams in the lesson is essential for students to perceive information better.

> I read something like this in a book when you read and see information, the percentage of being remembered is deficient compared to the percentage of being remembered only when you read it. That is why visuals help teach the subject. (T5)

Another teacher reported that connecting disciplines with daily life is essential to learning and helpful for the process.

> When students see a picture of a geometric shape, we have such an object in our home and school in daily life. In this way, children can establish a relationship between that real-life knowledge. (T10)

Moreover, a few teachers also stated their general thoughts on diagrams, which are given under this title.

> So, I think that if they do not, they cannot understand. For example, an ant is walking on the cylinder, so it is difficult to describe without drawing it. It is not possible, so these drawings are necessary. In this way, diagrams facilitate visual learning. (T4)

> Diagrams are indispensable in geometry, I think, even in mathematics. (T11)

4.1.1.2 Diagrams as Providing Concretization

The second sub-theme of the diagrams' roles in geometry is ″providing concretization″. The results indicate that while some see diagrams as a way of explaining themselves, some see them as a route from the abstract to the tangible.

Three mathematics teachers claim that using diagrams is an excellent approach to conveying oneself while teaching.

> When I draw a picture of a geometric object, I feel like I am describing it as if it were a recipe in a cookbook. (T12)

> Sometimes I can explain things much better by bringing the model of the things I cannot convey with the drawing to the class. (T7)

Others summarize the situation as:

> Because geometry is already a bit abstract, we need concrete material as teachers. Diagrams also help embody the subject. (T1)

4.1.1.3 Diagrams as Helping with Imagination

The last sub-theme of the geometry diagrams' functions is "helping imagination." A finding that indicates the mathematics teachers have opinions on the significance of imagination in teaching materials like diagrams and the progression of geometry learning.

Five mathematics teachers provided the following examples of their views on instructional materials:

> Students understand the visual better, a shape I drew on the board or a question on a book, for example, may not be precise enough. I think that when

> we show the figures to the children using digital applications from both the right and the left, in all directions, by turning them, children will understand better than seeing them in the book. (T8)
>
> For example, there is such a thing as the solid diagonal of a cube. The kid cannot picture it in his head when I say it. But when I explain it over the image, it becomes more apparent. I think it sure makes a kid imagine it in his head. (T7)

The following were some comments made by other mathematics teachers on the process of geometry learning:

> Diagrams are essential for the child to visualize the shape for permanent learning, especially when they are shown step by step; they are much more settled. Many students cannot visualize the shape. Maybe this is a problem caused by the education given in primary school, I do not know, but we have a big problem with it. (T3)

4.1.2 The Diagrams' Place in Teaching Resources

According to the study's findings, the placement of diagrams in teaching resources, which has two sub-themes, is the second aspect related to the specialized subject knowledge of mathematics instructors. Table 4.3 displays the sub-themes about the role of the diagrams in the instructional material. The data showed two sub-themes of the theme dealing with the diagrams' place in teaching resources through paper-based and digital materials.

Table 4.3 The sub-themes relating to the notions of mathematics teachers on the diagrams' place while creating teaching resources

Theme	**Sub-themes**
2. Diagrams' place in teaching resources	2.1. Through paper-based materials 2.2. Through digital materials

There are two sub-themes of diagram placement in teaching resources, according to Table 4.3. These sub-themes are presented as highlighting both paper-based and digital elements.

4.1.2.1 Through Paper-based Materials

The first sub-theme of the diagrams' place in teaching resources is the ideas of the teachers on the incorporation of diagrams ″through paper-based materials″, which comprise sketch diagrams, two-dimensional diagrams, and three-dimensional diagrams.

Primarily, some mathematics teachers mentioned the random sketches they drew on the board in their lessons. They stated that these were usually made up of concept maps; apart from that, graphics and probability were the mathematical subjects.

> I always try to draw a figure, if that's what we mean by the diagram, usually with the help of a pencil, etc. I used sketch diagrams on the topic of probability, generally. (T3)
>
> I do a lot of drawings on the blackboard in class. Before starting the topic, I draw concept maps to show children the topic's relationship with other topics; besides, I give plenty of space for diagrams when drawing column and circle graphs. (T7)

Sixth, teachers stated that they often included two-dimensional diagrams during their lessons and that these drawings were usually related to Euclidean geometry, such as drawing polygons.

> I'm using 2D diagrams for parabolas to show the situations of the arms. Again, in Euclidean geometry, the Pythagorean theorem, etc. I use 2D drawings. (T1)
>
> I want to teach that the interior angles of a triangle are 180 degrees in geometry lessons with geometric shapes. First, I explain this with a 2D drawing, but it is not very clear; instead, I explain it with a folding paper activity. (T8)

> For fractions and decimal notations, I draw 2D objects to show whole and part relationships, such as squares, rectangles, and circles. (T2)

Also, teachers matched the two-dimensional diagrams they gave in their lessons with area problems.

> When calculating the areas of polygons, 2D drawings help students a lot in understand the subject. (T9)
>
> Calculating the area of a pentagon by drawing a square, what is its circumference, I showed them mainly on the figure. (T11)

Half of the teachers reported that they shared the drawings and images of three-dimensional objects with their students and that these drawings were under the title of solid objects. Only one teacher's statement is included here despite the agreement of many teachers on this notion.

> I describe three-dimensional objects, solid objects in three dimensions. I can show images and videos about the experiences gained here in the course. But I have never used any technological program in my classroom lessons. I generally use it to show the open state of solid objects, in short distances, from which direction the ant will walk, and from which surface you walk. I also make use of lots of drawings on space geometry subjects. (T1)

4.1.2.2 Through Digital Materials

The placement of diagrams in teaching resources has another sub-theme that focuses on the concepts instructors conveyed "through digital diagrams," including mathematical software and packages.

Initially, some mathematics teachers shared their usage of digital diagrams through mathematical packages such as VITAMIN, one of Turkey's richest online education platforms, and EBA, a social quality electronic educational content network designed and governed by the Ministry of National Education of the Republic of Turkey. They generally choose these packages since they are unfamiliar with using other software due to their lack of free time to learn them.

> I usually use the dynamic activities in EBA (Education Information Network for schools in Turkey), but I do not know much about designing activities in GeoGebra. However, I prefer ready-made materials on solid objects, space, and volume. (T2)

> While working at a private school, I used the VITAMIN (Online Educational Platform by SEBIT) package in-class lessons. I mostly used videos. When I switched to online education, I had to use technology a little more. But I still can't find much time to design something in these programs. I want to learn GeoGebra and add something to myself, but it's not because of a lack of time. Everything was already ready in VITAMIN through membership. When we gave a student homework and did an exam, all the results were already in front of us. Where is it missing etc.? From there, we could use it when we wanted to enrich it with visuals related to the subjects or in three dimensions. (T4)

> I describe three-dimensional objects, solid objects in three dimensions. I can show images and videos about the experiences gained here in the course. But I have never used any technological program in my classroom lessons. I generally use it to show the open state of solid objects, in short distances, from which direction the ant will walk, and from which surface you walk. I also make use of lots of drawings on space geometry subjects. (T1)

On the other hand, other teachers knew how to use digital software such as Tinkercad and GeoGebra related to geometry and designed their materials accordingly for teaching geometry.

> I used intelligent board applications for drawing. In addition, I showed programs related to geometry in the three-dimensional design section in the field of informatics, a program called Tinkercad, for example, how a cube is drawn there, how its details are, how pentagons, hexagons, and ellipses are drawn, and I showed different versions of it. By the way, I ultimately learned it myself. In other words, I studied informatics and did not receive such education; I learned it from YouTube videos and applied it myself. I even improvised some of them instantly (T3)

4.2 Awareness of Mathematics Teachers to Understand Students' Difficulties, Misconceptions, and Improvements

The primary goal of the second research question of this study is to determine whether participants are aware of potential misunderstandings their students might experience when using 3D geometry in 2D settings. The question "How aware are the mathematics teachers of the misconceptions and difficulties that may occur in students while teaching 3D geometry in 2D environments?" Through content analysis, opinions from the semi-structured, open-ended, and task-based interviews of participant mathematics instructors were examined.

Table 4.4 Themes and sub-topics for the mathematics teachers' awareness regarding difficulties and misconceptions

Research Question 2	**Themes & Sub-themes**
2. Recognizing difficulties, misconceptions, and improvements	2.1. Detecting misconceptions and improvements in Task 1 2.1.1. Paper-based version of Task 1
	2.1.2. Improvements for Task 1 2.1.3. Digital version of Task 1
	2.2. Detecting misconceptions and improvements in Task 2 2.2.1. Paper-based version of Task 2 2.2.2. Improvements for Task 2 2.2.3. Digital version of Task 2
	2.3. Detecting misconceptions and improvements in Task 3 2.3.1. Paper-based version of Task 3 2.3.2. Improvements for Task 3 2.3.3. Digital version of Task 3

The teacher's answers in this section will be reported following Task 1, Task 2, and Task 3, respectively. Additionally, their answers will be indicated to the questions in the paper-based and digital versions under each task. The topics of the second study question, shown in The primary goal of the second research question of this study is to determine whether participants are aware of potential misunderstandings their students might experience when using 3D geometry in 2D settings. The question "How aware are the mathematics teachers of the misconceptions and difficulties that may occur in students while teaching 3D geometry in 2D environments?" Through content analysis, opinions from the semi-structured, open-ended, and task-based interviews of participant mathematics instructors were examined.

Table 4.4 highlight the difficulties, misconceptions, and improvements. The following parts will go into further detail on each of these subjects.

4.2.1 Detecting Difficulties, Misconceptions, and Improvements in Task 1

The first theme is "detecting difficulties, misconceptions, and improvements in Task 1," which deals with the difficulties and misconceptions that instructors mention and specific details on how they address them. This theme is related to findings addressing the study's second research question.

The data revealed three sub-themes detection of difficulties, misconceptions, and improvements in Task 1: paper-based version of Task 1, improvements for Task 1, and digital version of Task 1.

Table 4.5 The sub-themes related to "detection of difficulties, misconceptions, and improvements in Task 1"

Theme	**Sub-themes**
1. Detection of difficulties, misconceptions, and improvements in Task 1	1.1. Paper-based version of Task 1 1.2. Improvements for Task 1 1.3. Digital Version of Task 1

Table 4.5 shows three sub-themes in identifying difficulties, misconceptions, and improvements in Task 1: the paper-based version of Task 1, improvements for Task 1, and the digital version of Task 1.

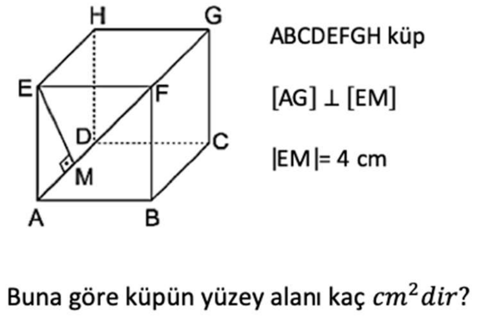
Table 4.6 The mathematics teachers' awareness regarding difficulties and misconceptions' frequency table for Task 1

Themes	Sub-themes	Codes	Number of Participants	ID
Recognizing Difficulties, Misconceptions, and Improvements for Task 1	A paper-based version of Task 1	Incomprehensible	7	T4,T7,T2,T5 T6,T10,T12
		Unsuitable	3	T1, T3, T6
		Diagonal is unclear	6	T5,T12,T2 T4, T7, T9
	Improvements for Task 1	Give extra info.	6	T6, T7, T4 T8,T10,T11
		Changing the perspective view	3	T4, T6, T10
		Coloring shape	2	T5, T11
	Digital version of Task 1	Easy to explain	5	T7, T3, T8 T9, T12
		It makes the task enjoyable	4	T5, T2 T10, T6
		Chance to see different views	7	T4, T11, T1 T2,T5,T7,T9

4.2.1.1 Paper-based Version of Task 1

According to the study, teachers found Task 1, given below in Figure 4.1, unsuitable and incomprehensible, and the drawing of diagonal objects unclear. Also, the development proposal for the task will be given subsequently.

Figure 4.1. A paper-based question that is asked in Task 1



Most teachers (seven) thought that the drawing in the task was incomprehensible.

> They say this is a drawing, and it's wrong, sir. Firstly. It's like students say AG is perpendicular to AF, not G. That's why it will not be straightforward for the student. (T4)

> Points A, F, and G appear to be on the same line. It is challenging for the student to perceive this. (T7)

> I think it would be better if I could interpret daily life in a way that would stay in the child's mind. But if I give a visual like this, the child cannot imagine. (T2)

> If even my eyes have difficulty perceiving and I can't think of a solution, it is more difficult for the student. (T5)

Three mathematics teachers stated that this task presentation is neither proper nor transparent for students.

> In the exercises, the student cannot see that line is not like that. Still, after solving it after a while, s/he can show that the passing line connects the two furthest points of the cube, which we later call the object diagonal, and that it is the object diagonal. Still, in the first question, we do not think it is correct in the exercise part. (T1)

Together, six teachers could not understand the actual situation of the drawing's points and lines.

> Here, I perceive the line as the surface diagonal. If that's how I understand it, I think it's challenging for children to figure out the actual situation. (T5)
>
> Later, when I read the question a little carefully, I saw that it went from A to G. But does it go from A to G? As if there was a way from M to G too. (T12)
>
> Even when I looked at it, I just perceived it now. I thought it was going from A to F, but it was going from A to G. (T2)

4.2.1.2 Improvements for Task 1

When the teachers were asked for their proposals for the improvements to make Task 1 more understandable for the students, the answers retrieved were grouped as giving extra information, changing the perspective view, and coloring the pictures.

Giving extra information was the most proposed (six participants) option to improve Task 1.

> It would be clearer if it were stated first that G is the diagonal of the object. Later, information could be given that AG and EM are perpendicular. (T6)
> For example, I am writing all kinds of details; you will not take into account the thickness of the object will not be taken into account, or whether AFG is linear or should be specified. (T7)

Coloring the picture was the least recommended improvement proposal mentioned by only two mathematics teachers.

> Three of them had to be different colors; that's what I would do. One color from AM, another color from MF. That would add dimension, I think, a color from ED. (T5)
>
> There would be color differences, and perspectives would be different. That way, I would like a new shape. (T11)

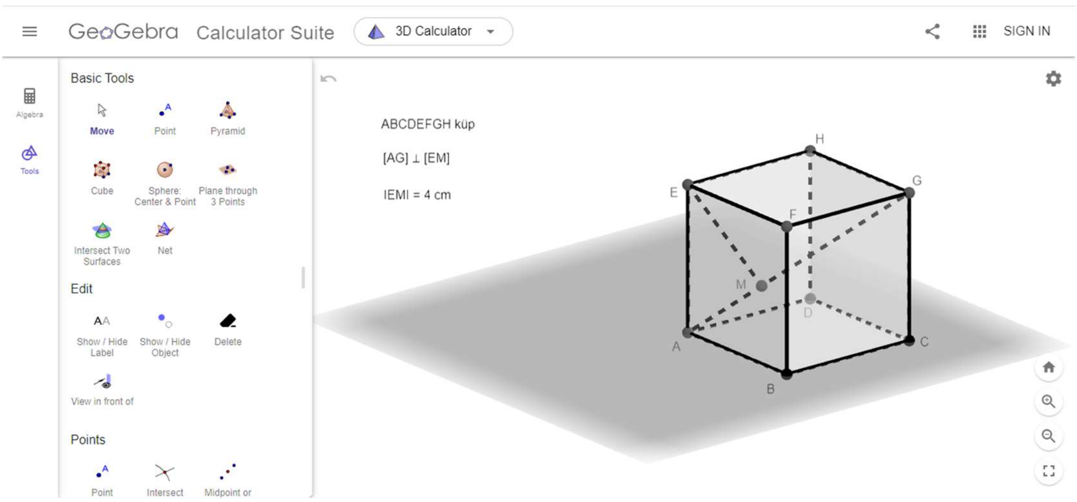
Changing the view of perspective was the second most recommended and possibly the one matching the digital environments by three teachers. However, these teachers advise working on the two-dimensional view of the object.

> If we change this after changing the perspective, it will be a bit more liberating since it is always difficult to fit these three-dimensional shapes perfectly into two dimensions. (T4)
>
> The shape could be turned slightly to the right as it is, or it could be stated more clearly that AG is the diagonal of the object. This is a cube. When we turn the sudden shape slightly to the right, it seems like we can see it more easily that it does not pass through that F. (T6)
>
> As a teacher of ten years, for example, I use the drawing of the cube facing a little more to the right rather than from the left. For example, we could draw the view of the right isometric here. (T10)

4.2.1.3 Digital Version of Task 1

For the digital version of tasks given in *Figure 4.2*, the ideas of the mathematics teachers are classified as easy to explain; the digital environment makes the tasks enjoyable and allows students to see different perspectives according to their answers in the interview.

Figure 4.2. Digital version of Task 1 on GeoGebra



First of all, five mathematics teachers stated that explaining details that belong to diagrams in the digital version is more straightforward.

> If the child has the right to translate it this way, it will not be forced. But even if it is like this, even if AG and EM are perpendicular, it has to be stated that AMG is still linear. The thing is that F seems to be on it, too, due to perspective. Yeah. The problem with F is already gone. As soon as the child sees it a little differently, we can explain it to the child much more quickly if we have this material. (T7)
>
> We can quickly explain this to children with the material we currently have. You know, is that linear thing in the head? Oh, it's not. It looks different from there. Because the child cannot see it with their eyes, we say that it is a mistake from the eyes of the teacher, but we can only save the child with explanations. But no matter how much you explain, he cannot understand it without seeing it. I think there is no problem. Explaining it this way solves the problem. (T3)

Secondly, four teachers stated that this version makes the task more enjoyable for students and teachers and makes it more straightforward for understanding.

> If we show it like this in the lesson, there is nothing beyond perfect perception and optical illusion, and everything will look fun. But it is not fun in two dimensions; if the student sees this shape in two dimensions, he will not have fun for a long time, and he will not be able to perceive it anyway, so there will be a problem for the exam again. Do this both in this way and in two dimensions in the exam; this was the first link. (T5)
>
> Children see it from several different points of view, not from one point of view, so I think they will interpret it more clearly. (T2)
>
> Here's what I'm saying if we can do the question in three dimensions, here are the exams, but it will get more enjoyable when the child turns to two-dimensional after seeing this. (T10)

Finally, according to seven teachers, it allows students to see the actual shape from different perspectives.

> Not being able to translate the book now is being able to translate it this way. It also attracts the attention of the student. I can see every aspect of it, and I'm

> the judge. Look, it looks different from the top. I mean, you know. It works both at the point of participation. The student likes it, for example. (T4)
>
> It can be seen in a much more logical, instructive way, so such programs contribute to learning and give the student the ability to look at all aspects from different angles. (T11)
>
> Now, for example, I see linearity from A to G; for example, I can see it more clearly when I rotate it like this, or I can even leave it like this when I rotate it like this, for example, when I look from there, I see linearity from A to G and linearity descending from H to M. I see it this way. (T1)

4.2.2 Detecting Difficulties, Misconceptions, and Improvements in Task 2

The second sub-theme is "detecting difficulties, misconceptions, and improvements in Task 2," based on the data related to the study's second research question. The sub-themes of identifying difficulties, misconceptions, and improving Task 2 include its paper-based form, enhancements, and digital equivalents, as shown in Table 4.7 below. The data revealed three sub-themes as detection of difficulties, misconceptions, and improvements in Task 2: paper-based version of Task 2, improvements for Task 2, and digital version of Task 2.

Table 4.7 The sub-themes related to "detection of difficulties, misconceptions, and improvements in Task 2"

Theme	**Sub-themes**
2. Detection of difficulties, misconceptions, and improvements in Task 2	2.1. Paper-based version of Task 2 2.2. Improvements for Task 2 2.3. Digital Version of Task 2

The Task 2 paper-based version, Task 2 improvements, and Task 2 digital version are the three sub-themes that make up the detection of misunderstandings and improvements shown in Table 4.7.

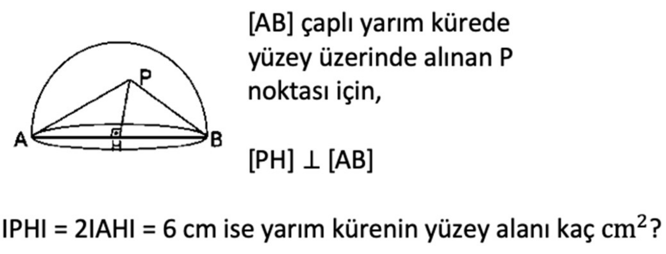
Table 4.8 Mathematics teachers' awareness regarding difficulties and misconceptions' frequency table for Task 2

Themes	Sub-themes	Codes	Number of Participants	ID
Recognizing Difficulties, Misconceptions, and Improvements for Task 2	A paper-based version of Task 2	Incomprehensible	5	T2, T7, T6, T4, T9
		Misleading	6	T11,T8,T5, T2,T10,T11
	Improvements for Task 2	Giving extra info	2	T3, T7
		Using concrete material	3	T8, T6, T5
		Coloring	4	T8, T10, T3, T12
	Digital version of Task 2	Easy to imagine	5	T6, T11, T2, T5, T7
		Misconception remover	1	T1
		Beneficial	4	T3, T7, T8, T10

4.2.2.1 Paper-based Version of Task 2

First, the study results showed that mathematics teachers believed that the paper-based version of Task 2, which is given in *Figure 4.3*, was incomprehensible and misleading to both students and teachers. To clarify and exemplify these results, the following examples from mathematics teachers are given concerning these subjects.

Figure 4.3. A paper-based question that is asked in Task 2



Six mathematics teachers stated that the questions' drawings and statements might cause students not fully to comprehend the issue.

> After saying that point P is on the surface, we have to think on the surface there now. But as a teacher, I cannot understand precisely what is shown in the figure. (T11)
>
> The way we give it to children is very misleading. On the surface, I think of him as on the surface, but of course, the student can think of him inside. (T8)

On the other hand, incomprehensibility is another property that five mathematics teachers determined this task as.

> In the question, this is written on the surface. I mean, it's in the air. When I first looked, it looked like it was in the air. I saw it as if it was going to the middle of the sphere but not reaching the top of the sphere. Just now. I felt like there was a small triangle inside, but it didn't reach the top of the sphere; in the question, it says it's taken on the surface. It means it was taken from one side of the surface. I made it something. So, point P is touching the surface. (T2)
>
> The child can think of it as planar if it does not indicate that point P is any point on the surface. Most students can't think in three dimensions anyway. Intermediate and higher students may think it is three-dimensional. (T7)
>
> As if there were no spheres at the moment, if we had eliminated arcs A and B, it would be as if there was a direct triangle. (T6)

4.2.2.2 Improvements for Task 2

For Task 2, teachers proposed improvements mainly on the development of the missing parts, especially in drawings. Creating it in a dynamic environment, giving extra information, using concrete materials simultaneously and coloring were in the majority as well, and they are explained further below through examples.
Some teachers reported that they want to create their activities in geometrical digital environments, but they are not familiar with the necessary tools to do so as they want.

> We can examine the state of that triangle by placing it in a three-dimensional program or by shifting the p point on the object (T1).
>
> Of course, if possible, I would love to prepare three-dimensional questions for such children. That would be nice. Because, after all, it is perceived as very three-dimensional for the brain (T2).

Again, for Task 2, two teachers thought that providing more information about the questions or the task would benefit students' understanding.

> So, as a start, maybe I could have completed the whole sphere, I could have it completed in a giant sphere so that it would be different and more settled in your eyes, but I couldn't draw it right now. The location of the P point is not specific, but I can say something like this; at least, I can show that children can see this shape as a giant sphere in terms of integration and think a little differently from a perspective. (T3)
>
> We will fill in our shortcomings by writing. This might be a little more understandable. But children with a high level of knowledge can perceive it. It seems like a good question, but he made a statement because it's there. (T7)

In addition, three teachers argued that using concrete materials simultaneously will make the concepts more transparent for students.

> I would design a material for them to imagine inside the surface to figure out where this P point is. (T8)

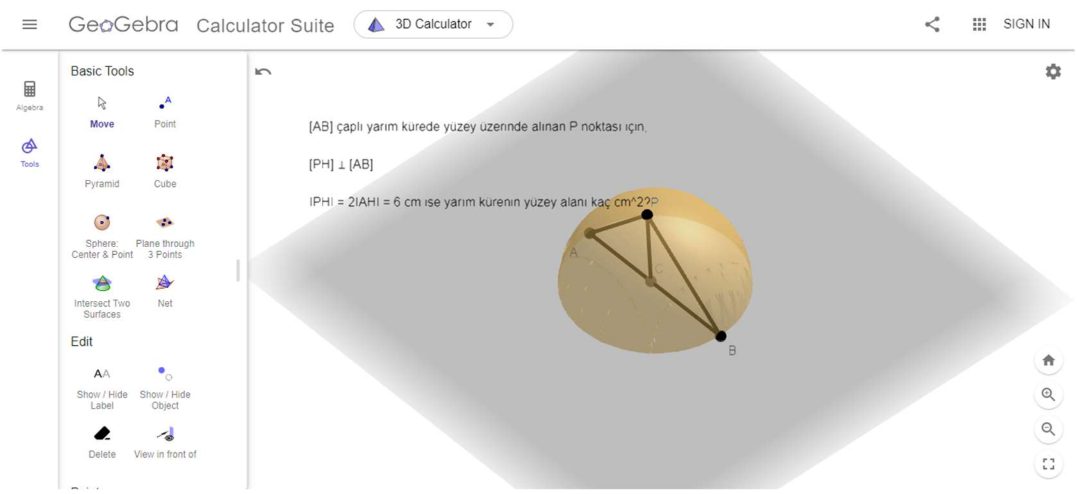
Finally, four teachers stated that coloring the shape will enforce the effect of a three-dimensional view of perspective.

> This image is given in one color if it is drawn with different colors so that children can understand it better. Also, this way, the concept of three dimensions is better integrated. (T8)

4.2.2.3 Digital Version of Task 2

For the digital version of Task 2, given in *Figure 4.4*, teachers made organized comments such as easy to imagine, misconception remover, and beneficial for students. Some teachers think giving extra information is better than showing the diagrams digitally.

Figure 4.4. Digital version of Task 2 on GeoGebra



Five mathematics teachers stated that the digital diagram makes this task's concepts more manageable and understandable.

> My teacher, when the figure is shown visually in a three-dimensional environment through a program like this, things that do not come to life in my mind come to life. It's alive right now. When we looked at the question a moment ago, we saw something completely different, but now we see

> something different. In other words, it was as if we were looking at it as if we saw a semi-circle, not just a hemisphere. But now, we can see the hemisphere itself. (T6)

Also, one teacher saw this version of the task as a misconception remover.

> The student can use it very quickly, and many think that it can solve the perception of being on the surface and that our misconceptions can be eliminated. (T1)

Four teachers also stated that the digital diagram version of this task benefits students in their understanding process.

> When we look at it in terms of solution, there is no extreme change; it just helps the child to analyze that point better with questions such as whether it stays at the top or in the corner; I think this shape is to play it, let me put it that way. (T3)

> Moving it like this also changes the perception in the child's head. When we bring it a little further down here. I can explain this to the child in the lesson so that you can think of it like a tent, as if you are spinning in the sky, but not every child can visualize it in his mind. Now we can explain this very well. (T7)

Unlike the rest, one teacher argued that students might not deal with technology in this case; giving extra information is a better option for the task.

> Suppose the student was asked like this, in two dimensions and really in two dimensions. In that case, this is 6 cm on the surface, here is 3 cm AH 3 cm, if it were like this and such, the student would not be confused at all, and there was no need, you know, is not on the surface, this is the surface, but we gave this and that, in that question. Giving it became very difficult for 3D; it became easier for 3D, and we made it harder for 2D; if it gave it, I would say, why didn't it give it? (T6)

4.2.3 Detecting Difficulties, Misconceptions, and Improvements in Task 3

The third and final sub-theme is titled "detecting difficulties, misconceptions and improvements in Task 3," and it is based on the data related to the study's second research question. The sub-themes of identifying difficulties, misconceptions, and improving Task 3 include its paper-based form, developments, and digital types, as shown in Table 4.9 below.

Table 4.9 The sub-themes related to "detection of difficulties, misconceptions, and improvements in Task 3"

Theme	**Sub-themes**
3. Detection of difficulties, misconceptions, and improvements in Task 3	3.1. Paper-based version of Task 3 3.2. Improvements for Task 3 3.3. Digital Version of Task 3

The paper-based version of Task 3, improvements for Task 3, and the digital version of Task 3 are the three sub-themes that make up the detection of misunderstandings and improvements shown in Table 4.9.

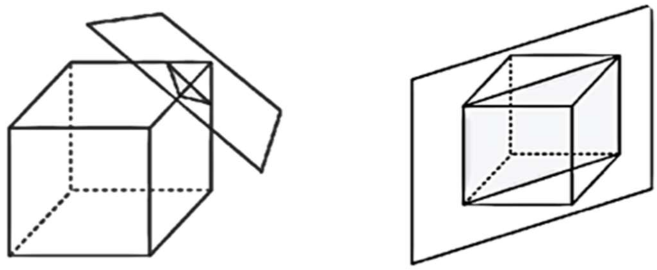
Table 4.10 Mathematics teachers' awareness regarding difficulties and misconceptions' frequency table for Task 3

Themes	Sub-themes	Codes	Number of Participants	ID
Recognizing Difficulties, Misconceptions, and Improvements for Task 3	A paper-based version of Task 3	Not seeing extra polygons	7	T3,T8,T5,T10, T2,T4,T6
		Explaining it by drawing	2	T2, T3
	Improvements for Task 3	Creating it in a dynamic environment	2	T6, T4
		Using concrete materials	4	T3,T10,T7,T9
	Digital version of Task 3	Students may have difficulties	7	T4, T3, T7, T8,T10,T11
		Teachers may not handle technology.	5	T10, T5, T6,T11,T12
		Improve the different thinking skills of students	1	T12

4.2.3.1 Paper-based Version of Task 3

The study findings showed that mathematics teachers shared their ideas on the paper-based version of Task 2, which is given in *Figure 4.5*, and could not find any other polygon other than students and using concrete materials in activities. Teachers' sentences exemplifying this finding are given below.

Figure 4.5. A paper-based question that is asked in Task 3



The majority of the teachers (seven teachers) could not see extra polygons except the polygons which are given in the question.

> When I look now, nothing more comes out as far as I can see, but I may be looking at the wrong thing. Let's see, I may not be able to see it as a pentagon with 3 and 4 sides, I don't know, but when I look at it now, I don't see much so that it may be my lack of geometry. (T3)
>
> For example, as I said, when they cut vertically, a square can form. Again, when they cut horizontally, I expect them to realize that a square is formed. Apart from that, I couldn't think of any other polygon. (T8)

For this task, four teachers shared that using concrete materials simultaneously as diagrams can solve the confusion in students' minds.

> If there is a wrong interpretation, for example, at that moment, I will find an example that the child will live with his eyes and five organs and try to find the right one. If he gave the wrong answer. (T3)
>
> If there is an everyday cube-shaped object, I can pick it up and show it to the children so they can see it more realistically; I would like more details; only triangles, quadrilaterals, and polygons are formed. (T10)
>
> I say let's take two dots from the ceiling. From here, I say, let's connect them with ropes. I mean, how are the ropes holding up? See here visualization. It is even more challenging to animate before him, but I can sometimes animate it in his environment. I use such materials. But it's three-dimensional, huh? I sometimes make the books fold differently with our concrete materials

because we don't have the material—in high school. But digital would be much more effective if it was a three-dimensional material. (T7)

> Although I need to take a cubed object for this, it would be wrong to bring something sharp with a cube-shaped object, but let's say it's a ruler. I think they can observe that a square is formed when I cut that ruler by holding it vertically; they will understand this. In other words, I would resort to concrete material if they could not grasp it through visuals. Or I would take one myself, cut myself and show it to the children. They probably would. (T9)

4.2.3.2 Improvements for the Task 3

For Task 3, mathematics teachers did not give more improvements than they did for other tasks. Creating it in a dynamic environment and explaining it by drawing are two sub-topics of this section.

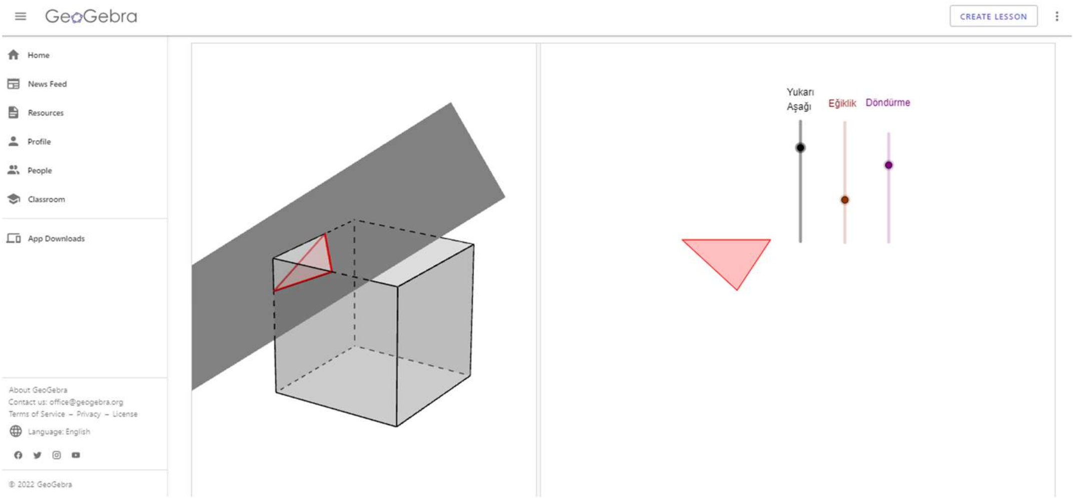
> It is easier to benefit from programs that contain mathematical expressions, as I said, to use those programs. (T6)

> But I showed it by drawing the thing you know on the board with a pencil. Three, four, five, six. So that's it. For example, we lower the cube from the corner to the bottom to make it a pentagon. For example, if we straightened it a little bit from the corner, it was a K on the upper face. (T4)

4.2.3.3 Digital Version of Task 3

The digital version of Task 3, given in *Figure 4.6*, was shown to the teachers. Only one teacher among the twelve could see the point as an intersection; almost all teachers could see three and four-sided polygons to consist of student answers. For the other-sided polygons, no answer was provided by the teachers.

Figure 4.6. Digital version of Task 3 on GeoGebra



Five mathematics teachers said they believe many teachers may be unable to handle technology. Congruently, it was stated by seven teachers that the students also might have difficulties using slides that are not close to digital diagrams for the task.

> Which is fine; I mean, it's an area where teachers should improve themselves. Now, you know, there is no problem with a new graduate, so for example, I had many friends at the beginning of the pandemic that could not turn on or off Zoom. For example, I have some shortcomings with me in these points. (T4)
>
> When we rotate it now, for example, we saw the quadrilateral; we started to rotate it more, I don't know, we turned it into a pentagon. Unfortunately, no Turkish student can think like this if we talk about Turkish students. Unfortunately, it is infrequent, that is, the 1% and the 2% are that much. They may be able to think in detail; you know, I can say that they are above a particular class in terms of intelligence level. Still, of course, it is questioning; on the other hand, this questioning, that is, the student enters the given thing. (T3)
>
> Could it be a difficulty, you know, it could be a problem in terms of use for children. There could be problems such as turning in a different direction, freezing as we just experienced, or forgetting from the other side while trying on one side. (T10)

On the contrary, one teacher stated that the digital version of this task would improve the different thinking styles of the students.

> They will observe that it is formed, now I see that I see that different polygons are formed in it; for example, the child will try this and say that yes, different shapes can be formed, they may think that we can create different shapes when we cut different cubes. I think this is more beneficial, and I think it develops children's different thinking skills. (T12)

Also, practicing themselves is essential for students' permanent knowledge construction, so students have a large percentage of information from the digital version of a diagram.

> I wanted you to realize themselves, into their transformations in moving forward. For example, it is necessary to prepare something like this at work or to find ready-made materials and have students do it. So what happens when we pull it up and down? Let's look at a change. It may be difficult to revive in the center, but when the student sees it this way, I do this on the board. It happened to me in the exam. Here you can comment that it was like this more easily. (T6)

4.3 Advice from Teachers to Prevent the Students' Difficulties and Misconceptions which Derive from Diagrams

From the viewpoint of the mathematics teachers, the third research question of the current study is to determine what might be done to minimize the misunderstandings deriving from diagrams. The final question, "How can students' difficulties and misconceptions be reduced from the perspectives of mathematics teachers?" was asked in this context. Through content analysis, the opinions of the participating mathematics teachers were examined.

Table 4.11 Reducing misconceptions from the perspectives of mathematics teachers' themes and sub-topics

Themes	Sub-themes	Codes	Number of Participants	ID
Principle of clarity	Course materials	Bringing concrete materials to class	2	T3, T12
		Giving detailed information	3	T2, T7, T8
	Exam materials	Integrating each question with a QR code	4	T8, T6 T10, T12
		Consistency of drawings and information	2	T2, T3
		Exporting multiple images	5	T7, T2, T4 T11, T9
		Coloring questions	2	T2, T10
Proposals for teacher education		Digital courses for preservice teachers	6	T8, T11, T1 T3, T5, T6
		The aesthetic aspect of teachers should be improved	2	T5, T1
		Preparing digital workshops for teachers	6	T5,T6,T4,T7 T11, T12
		Integrating geometry and math as a discipline	1	T1
		Creating math meetings for materials	2	T9, T4

Topics of the third study question, shown in Table 4.11, are the principle of clarity and proposals for teacher education. These topics will be covered in further detail in the following sections.

4.3.1 Principle of Clarity

The first theme for the recommendations of mathematics teachers is the “principle of clarity," which consist of course and exam materials based on the research results of the last research question of the study shown in Table 4.12 alongside the sub-themes about the ideas of mathematics teachers on the principle of clarity.

Table 4.12 The sub-themes relating to the notions of mathematics teachers on the principle of clarity

Theme	**Sub-themes**
1. Principle of Clarity	1.1. Course materials 1.2. Exam materials

Three sub-themes of the theme titled ″principle of clarity″ are shown in Table 4.12. The names of these sub-themes are course materials and exam materials.

4.3.1.1 Course Materials

Previously, it was mentioned that ″course materials″ was the name of the first sub-theme of the principle of clarity. In this study section, mathematics teachers’ answers were categorized for course materials as bringing concrete materials to the classroom and giving detailed information.

As highlighted before, mathematics teachers thought that bringing concrete materials for the activity and explaining the details of this object would be beneficial for the students’ understanding.

> In the exam, it is not possible to give it concretely anyway, according to Turkish laws. At most, I can do this; I throw it, rectangular prism, square prism at work. I put it somewhere in the classroom; I give an example, and it stays there, so if the child cannot imagine it in the exams, I can show it. (T3)

> For example, when I take him to class, he takes the cube out of the corner. He sees it; he touches it. Neither surface was formed. The child perceives things by seeing in his best hand, but this. (T12)

Also, they argued that giving more detailed information is essential for the course materials to prevent difficulties and misconceptions about diagrams.

> If the extra was like this, I don't have a suggestion for combining; again, I recommend that the figures be given on detailed information and given correctly. (T2)

4.3.1.2 Exam Materials

Secondly, ″exam materials″ was the name of the second sub-theme of the principle of clarity. According to this part of the study, mathematics teachers' answers were categorized for exam materials as integrating each question with a QR code, consistency of drawings and information, exporting multiple images, and coloring questions.
Integrating each question with a QR code was the first advice of mathematics teachers to be able to change the manipulation of questions in the exams.

> Instead, there may be a site where we can go to a three-dimensional shape when we have a QR code reader. Let him direct us there a cube that will rotate this image with a child's phone or on the intelligent board used in this classroom; I think it is much better to transfer such a program to books with a data matrix, just as we rotate it in GeoGebra, and also for questions required in question banks. (T8)

> For example, when we come to three-dimensional objects, I think it may even be on other subjects, but there are QR codes on the edges of the question banks, but this solution is telling. (T6)

Also, mathematics teachers thought that the drawings and information given below the questions should be more transparent and consistent.

> For example, if I give a square shape, the side length of the square is given as 3 cm, while a 5 or 10 cm shape is given next to it, but it is given the same square length; there are two squares, the side lengths of both are perceived to look like thresholds, but 3 cm in one and 10 cm in the other. There is inconsistency in the information given with the shapes. (T2)
>
> I give it to the child there; I do not leave a comment, anyway, I will give everything in detail, you know, let me give everything clearly without the need for the child to make up or derive something additional so that this information is sufficient for the solution. (T3)

One teacher added that exporting multiple drawings could be a good solution, and students will have the advantage of seeing the object from multiple perspectives.

> Then it can be viewed from several directions, you know, instead of one shape, three shapes can be put and viewed from different angles. But this, of course, requires a separate effort. But why not? (T7)

Finally, two mathematics teachers advised that using more colors in the questions may prevent confusion.

> The purpose of coloring, you know, is to make it easier for children to imagine the three-dimensional and to make it easier for them to see by adding a little more dimension by coloring. (T2)
>
> Every book is printed in color, and OSYM's questions are prepared in color. National education exams are colorful, and maybe this can be solved with this coloring. (T10)

4.3.2 Proposals for Teacher Education

The proposals for teacher education is the second theme for the recommendations of mathematics teachers and has no sub-themes, which is why this part does not include any tables for themes and sub-themes. This section categorized teacher advice as

digital courses for preservice teachers. Teachers' aesthetic aspect should be improved, preparing digital workshops for teachers, integrating geometry and mathematics as a discipline, and creating mathematics meetings for materials.
Firstly, teachers advise universities to prepare and open digital lessons related to each pre-service teacher's discipline.

> If we have a lesson that includes direct technology, it would be much better if the teachers showed us the technological sites more, if we had the chance to apply them at that moment. (T8)
>
> When the ball is thrown at us, we have a little bit of everything, so some learn, some do not; I think it would be much better to teach it as a lesson instead; I think integrating the technological lesson is missing in universities. (T11)

Also, some teachers suggest that the aesthetic aspect of mathematics teachers should be improved.

> It's not a table; it's a picture we draw; it's not the harmonious glow of colors; mathematics is an aesthetic thing. (T5)
>
> The same thing exists in 3D diagrams; the more aesthetically pleasing we make, the more attractive the student will come to us, and the more accessible learning will be. I always think that a mathematician should have a very high aesthetic sense. (T1)

Additionally, preparing digital workshops for teachers was another proposal mostly applicable to in-service teachers.

> The fact that he can use digital in three-dimensional shapes is more enjoyable for students because children see three-dimensionality. Still, they have a hard time putting it on paper at first, and after they don't sit down, it's a very big geometric, a part of mathematical geometry, or that's missing, later with a lack of mathematics. Mathematics is becoming a situation that continues with hatred, so teachers should prepare it; national education should give a great incentive for this; national education should send it to teachers, especially if it can't prepare its materials; what else can it do? You just showed me. (T5)

> It cannot be said that our fifty-one teachers use technology and the old teaching. So, maybe we need to be in a way that can transfer experience to us or teach them technology. (T6)

> If there are many different perspectives as concise shapes, if we support it with digital, it will be an excellent thing with Z book applications, and teachers need to be trained on this subject. (T7)

> Most teachers know how to use the Z book, but how they use it, my students also know from my daughter, for example, they open the book, you write the book, the children use it as a summary or something. However, the purpose of the Z book is to create a different perception for the child. Teachers do not use the Z book correctly; they do not use it to make their work easier. I think the goal should be to facilitate the child's understanding, but as I said, teachers should be given separate training on this subject. (T4)

One teacher added by stating that integrating geometry and mathematics as a discipline is important since some teachers are separated depending on geometry or mathematics.

> But you know, as I said geometry, I understood, with the perception that I will be able to explain, that teachers already refer geometry lessons to each other. If this is not the biggest problem we usually face, it would be better if we had to explain it. I cut you off from my point of view; if I were to evaluate it from my perspective, I did not want to teach geometry in my first years, it was a lesson that I liked more than mathematics, but we can't manage to explain it, now I think that I started to make up for the deficiencies in myself in the institutions we work as I had to, our more experienced teachers know this better than we do, so we can't explain it. As we go on and say that we can solve the problem, but I can't, that problem continues. (T1)

Finally, mathematics teachers stated that they are hungry for new meetings in which they can talk about their class activities, and they suggest that creating mathematics meetings would be beneficial for their schools.

> Not every teacher is so into digital, it's a deficiency, either the National Education teachers will be able to open a computer that will be competent with the training, or the program or the video or the video, you know, the video should be eliminated for clear things, because I think at least one video should be used in three dimensions. (T9)

Here are these things, namely GeoGebra. I don't know if there is any other that I know. I don't know if such programs are being taught; I don't know right now, but I know this; for example, during the seminar period in the state, there are helpful topics, but they are all shown to be done, so nothing is done. (T4)

CHAPTER 5

DISCUSSION AND IMPLICATIONS

This chapter presents the discussion and implications of this study's findings. For each study topic, a brief review of the findings is given first, and the findings are then examined in light of the relevant studies. Then, prospects for future practice and research are provided.

5.1 Discussion of Findings

Since they cannot describe Euclidean 3D diagrams, middle and high school mathematics teachers lack particular knowledge of 3D diagrams in teaching geometry. How teachers should assist students in comprehending and misreading diagrams is controversial. The suggestions for reducing misconceptions and difficulties deriving from diagrams were also to be taken from the opinions of the middle and high school mathematics teachers. This section discusses the knowledge of specialized content on 3D diagrams for mathematics teachers, the awareness of mathematics teachers to understand student misconceptions and difficulties, and how to prevent misconceptions and difficulties by benefitting from teacher suggestions.

5.1.1 Specialized Content Knowledge of Mathematics Teachers on 3D Diagrams

Particularly in the past three decades, research on mathematics teacher expertise has significantly changed (Bastian et al., 2022). The number of studies in this area has substantially risen, the nature and extent of the research have broadened, and there are now a wide variety of frameworks being utilized to investigate mathematics teacher knowledge. The expanding variety of teacher knowledge frameworks attests to the study field's richness and multiple perspectives (Scheiner et al., 2019). In this section, addressing the first research question, learning the opinions of middle and high school mathematics teachers about specialized content knowledge on 3D diagrams was attempted.

Table 5.1 Summary of Specialized Content Knowledge of Mathematics Teachers on 3D Diagrams

Role of Diagrams in Geometry		Diagrams' Place in Teaching Resources	
STUDY	LITERATURE	STUDY	LITERATURE
Teachers thought that diagrams significantly affect teaching and learning geometry	The representational mode of diagrams (Herbst, 2004) Diagram was very effective for quickly introducing students to 3D (Accessina & Rogora, 2006)	• Teachers draw a lot on paper and on board • They are aware of the digital materials • They do not have enough capacity and information to use them	The inadequate teaching of 3D geometry results from the absence of such visual skills (Accassina and Rogora, 2006).
Diagrams ; • Assistants for understanding geometry • Makes the subject attractive and enjoyable • Increasing the memorability of concepts • Provide concretization and help with imagination	Children may comprehend geometric concepts more readily when connected to **real- world** situations because they find these connections more intriguing, familiar, and reasonable (Duatepe Paksu, 2009).		

As a result, the researcher discusses the role of diagrams in geometry and their place in teaching resources through the sub-titles. Understanding the world through mathematics helps us develop mental discipline. Mathematics fosters logical reasoning, critical analysis, inventiveness, abstract or spatial thinking, problem-solving aptitude, and good communication skills. A crucial element of arithmetic is geometry. When the middle and secondary school curricula are reviewed, geometry can be seen as an essential academic topic. Students regard it as one of the most challenging areas of mathematics. Numerous studies carried out in the past decades have documented the challenges that students face while studying geometry (Nader, 2022). Since Euclid's Elements were first used to teach geometry to high school students over a century ago, a lot has changed in how geometry is taught and learned (Sinclair, 2008). Geometry started being taught explicitly in primary schools in the 1960s. Given the cumulation of teachers like Froebel, Montessori, Pestalozzi, Steiner, Boole, and Somervell, who created curricula featuring spatial awareness of concrete objects, it may be due to geometry in primary school only recently emerging that there was so little research to report on for authors of the Handbook, Clements, and Battista (1992) and then Battista (2007).

According to the findings of the present study, middle and high school mathematics teachers think diagrams significantly affect teaching and learning geometry. General ideas were that diagrams are assistants for understanding geometry since they make the subject attractive and enjoyable, alongside increasing the memorability of concepts. To support this with literature, the demonstrative geometry that students are reportedly studying in school geometry, where the objects of study are marketed as mathematical ideas defined axiomatically and whose qualities are stated to be proven deductively, is supported by the representational mode of diagrams (Herbst, 2004). Participants in the present study also thought that diagrams provide concretization and help with imagination. According to previous research, children may comprehend geometric concepts more readily when connected to real-world situations because they find these connections more intriguing, familiar, and reasonable (Duatepe Paksu, 2009). In other words, it can be stated hereby that the literature supports the comments of the teachers.

All answers from participants in the current study were grouped in terms of whether they used paper-based or digital materials for the place of diagrams in their respective teaching resources. Mathematics teachers mentioned that they draw a lot on paper and the board in their lessons and are aware of the digital materials. Yet, they do not have enough capacity and information to use them, and therefore, they cannot prefer using them. However, Accassina and Rogora (2006) stated that if students should understand the characteristics and connections of spatial arrangements; they must acquire visual thinking skills. The inadequate teaching of 3D geometry results from the absence of such visual skills. Thus, the researcher is interested in learning how Cabri3D might help students enhance their visual thinking skills because standard ways of doing so are ineffective or need extensive training to be effective.

5.1.2 Teachers' Awareness of Students' Misconceptions and Difficulties on 3D Diagrams in Teaching Geometry

The discussion in this section will align with paper-based and digital tasks for all tasks in parallel with the findings section for the second research question. In Turkey, students of all age groups are subject to a national examination system, and they train to demonstrate what they have learned in the lessons in this system cumulatively. So, how much do we, as teachers, know about how students perceive the diagrams we use in exams and lessons? Modern mathematics avoided using diagrams and focused primarily on the formal aspect of geometry. It was stated that diagrams made geometry particularly challenging for students since they confounded them with their empirical facts and the professors' insistence on employing deductive reasoning only (Laborde et al., 2006). Here, the teacher's duty is essential to understand students' difficulties. To continue with the importance of misconceptions and detecting them for both the learning and teaching process, a person's assessment of a subject that accords with their logic but conflicts with the conceptual understanding of domain experts are referred to as having a misconception (Baki, 2015). Additionally, it is essential to address misconceptions outside of student failure and mistakes since they are expressed with wrong meanings based on misunderstandings and misinterpretations (Ojose, 2016).

If this is the case, at this point, it was necessary to investigate the expertise of the teachers.

Table 5.2 Summary of the Teachers’ Awareness of Students’ Misconceptions and Difficulties on 3D Diagrams in Teaching Geometry

Paper-based Tasks		**Digital Tasks**	
STUDY	LITERATURE	STUDY	LITERATURE
In all questions the mathematics teachers had difficulty solving the all problems. • Insufficient geometric knowledge • Obstacles to understanding drawings.	Thom and McGarvey (2015) shared the significance of diagrams in children's geometric thinking.	• Teachers try to work on their own • They stated they had difficulty in using tech. • More transparent and responsive task.	Moving in digital allows students to generate hypotheses and develop a plan for completing the geometric assignment (Komatsu & Jones, 2020).
Diagrams did not transmit notions of 3D since many teachers in the study thought 3D drawings were 2D when they were on a sheet of paper.	Creating solid conceptual pictures of those items can benefit from manipulatives, diagrams, and digital diagrams (Accescina & Rogora, 2006)	Ideas; • Students can rotate the shapes as they wish, • Contribution to the development of the student's imagination.	

The present study aimed to measure how well the teachers know about using diagrams and how good they are at noticing their deficiencies and efficiencies. In all the questions asked on paper, the mathematics teachers had difficulty solving the problems themselves, let alone seeing the difficulties that the students might experience and understanding the details of the problem. This may be due to two reasons: they may have insufficient geometric knowledge or there are severe obstacles to understanding the drawings. In this context, Thom and McGarvey (2015), investigated whether drawing serves as a mechanism by which children become aware of geometric concepts and relationships that offer fresh ways of thinking about the significance of diagrams in children's geometric thinking. They emphasize how crucial it is for instructors and academics to examine how diagrams are generated in relation to spoken language, gestures, and context. On the other hand, assuming that teachers have a low ability to detect misconceptions in geometry, there is a long process in the background that includes compulsory education, university education, and teacher education, all of which need to be examined in detail. For example, these teachers state that they did not go through adequate training after they started practicing their profession. Teacher views on this part will be discussed in the next part.

Secondly, when teachers were provided with the digital versions of the questions asked on paper and required to work on them on their own, they stated that while they had difficulty in using technology; the diagrams in two dimensions were not more responsive and more transparent, but this was much less in the three-dimensional environment. Scientists have shown that students struggle to match archetypal visuals with verbal or written descriptions and definitions while recognizing forms, even at the elementary school level (Clements & Battista 1992). Following this statement, their teachers should not have any problems using three-dimensional tasks for students to achieve these skills. Moreover, a DGE's dragging capability gives users access to various diagrams while maintaining the limitations of a geometrical task. The learner may observe the constructed figure from various perspectives by moving it. This can reduce the likelihood that they would base their conclusions on data that may be inaccurate, such as the self-attributes of the particular, static given graphic (Widder et al., 2014). This allows students to generate hypotheses and develop a plan for completing the geometric assignment (Komatsu & Jones, 2020). In addition, the mathematics teachers argued that in the digital versions of the tasks, as in their own experience, the students can rotate the shapes as they wish, which can positively affect the learning process and even contribute to the development of the student's imagination. Concerning this statement, Kaur (2015) focuses on how youngsters could stretch the traditional limits of their nearly exclusive exposure to regular triangles by using inclusive relations to recognize other triangles using dynamic geometry programs.

On the other hand, it is stated that diagrams did not transmit notions of three-dimensional since many mathematics teachers in the present study thought three-dimensional drawings were two-dimensional when they were on a sheet of paper. Concretization is an essential concept in mathematics education, but could these concretizations cause knowledge loss when made through visualization? Could it be the basis for the student's utterly different perception of a concept? According to a study, examining the relationships between geometric objects and creating solid conceptual pictures of those items can benefit from manipulatives, diagrams, and digital diagrams (Accescina & Rogora, 2006).

5.1.3 Advice from Teachers on How to Prevent Diagram Derived Difficulties and Misconceptions

The discussion of this part will align with the third research question, ways to reduce the difficulties and misconceptions arising from diagrams and teacher suggestions regarding this phenomenon, as well as their recommendations for teacher education. For the principle of clarity, according to the answers of the mathematics teachers in the sample, the teachers predicted that coloring the questions and using concrete materials to demonstrate the visuals used in the lessons as course materials would save the students from ambiguity. One of the recommendations was for teachers to add drawings that see the object from multiple angles rather than a singular one. According to Benning et al. (2018), teachers mentioned that students could better visualize the orientation of a 3D geometric object when it is rotated clockwise or anti-clockwise about the origin by utilizing GeoGebra tools. Although this suggestion seems compatible with the field research, it will be limited to comments as there are primarily evidence-based studies related to 3D diagrams. Also, students who are given proof problems may have trouble answering ones that rely on the personal qualities of a single drawing, which are unique to one illustration of an idea but are not crucial features of the concept. Instead, diagrams should address the universal attributes that are employed in the demonstrating and calculating procedures (Haj-Yahya, 2020).

In the perspective of the teachers who participated in the present study, it is necessary to provide students with access to multiple images for one object in both course and exam materials, and this will increase their understanding. Giving students a proof problem and more than one diagram, with a shape provided in various places in each picture, influences their geometrical constructions (Widder et al., 2014). The participants in the present study also stated that this is not possible with the current exam system in Turkey. Nonetheless, if it can be done, using QR codes for each question and an activity in the exam materials will somewhat eliminate the problems arising from the diagrams. Furthermore, they also stated that they wanted to see the exams at least in the three-dimensional related part of the digital environment. At this point, it is explained that the geometric actions of the students become more explicitly conscious while using digital methods to alter forms (Clements & Sarama, 2007). With this consciousness, they may further their mathematical movements.

Finally, in this part, proposals for teacher education gathered from the twelve participants in the present study are placed and discussed. Although the maths teachers were unaware of the problems their students might experience with the diagrams, they noted their own deficiencies. Assuming that teachers have low spatial abilities, there is a long process in the background that includes compulsory education, university education, and teacher education, all of which need to be examined in detail. These teachers also stated that they did not go through adequate training after they started practicing their profession. According to Kapur (2018), teachers play a crucial role in advancing societal welfare and helping educational institutions achieve their targeted goals and objectives. They must overcome various obstacles while carrying out their job responsibilities. Also, teachers have a crucial role in ensuring the successful development and growth of students. In this regard, according to empirical data, geometry is not being taught to students in a way that helps them comprehend the relationships between ideas (Crompton et al.,2018). That is why mathematics teachers should improve themselves through their education and working years regarding their academic background, content knowledge of all types, and pedagogical knowledge.

5.1.4 Implications for Future Research

The current study examines how mathematics instructors see the use of three-dimensional diagrams and how to use teacher views to help students understand these diagrams more clearly. In this study, only the opinions of mathematics teachers are considered. For further study, it is advised that three-dimensional diagrams be investigated by looking at the opinions and experiences of mathematics teacher educators and mathematic teachers throughout geometry teaching. As Patton (2002) indicates, gathering information from several sources can assist the information to be more complete. Also, in line with this research, teacher perspectives can be examined in detail by associating them with middle and high school curriculums for further studies.

The data of this study were gathered through open-ended, semi-structured, and task-based interviews with mathematics teachers. Various (two) researchers were used to assess and evaluate the data to enhance knowledge of the problem under investigation by presenting different points of view. This is believed to provide the researchers with a more holistic view of the diagrams and their effects on teaching geometry. This is thought to give academics a more comprehensive perspective on the diagrams and how they affect geometry teaching. As a result, it can also be thought of as gathering data from multiple sources for further research into the use of three-dimensional diagrams. Finally, based on the level of use of 3D diagrams by teachers in online environments, it can be examined in a larger-scale study whether it differentiates according to levels

5.1.5 Limitations of the Study

Due to nature of interview, math teachers may not have been sufficiently willing to answer or could not focus on the questions due to the length of the process. This might apply to any study that uses interviews to gather data. If the complete anonymity of the participants cannot be ensured, they would not share their best knowledge to provide a deeper understanding of the subject. This may be a condition that affects the interviewee psychologically. Additionally, maintaining participant identity throughout interviews may be challenging in the context of this study. As a result, they would have presented a restriction for this study by preventing participants from sharing as much as they might have, and to provide a more a thorough grasp of the setting considered in this study. For this study, interviews were conducted online. This circumstance might have affected the trust of the participants in confidentiality issues while sharing their identities on a recording because the researcher could not see the participants in their natural environment. For example, if a participant does not want to share their image, we cannot guess under what circumstances they participated in the interview. On this point, the researchers have to trust the participant, which in turn effects the reliability of the information obtained.

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APPENDICES

A. APPENDIX A: INTERVIEW PROTOCOL

Veri Toplama Aracı
Görüşme Protokolü

Değerli öğretmenimiz,

Bu araştırma, Orta Doğu Teknik Üniversitesi, Matematik Eğitimi Yüksek Lisans programı, yüksek lisans tezi kapsamında ortaokul ve lise matematik öğretmenlerinin üç boyutlu diyagramlar üzerine görüşleri hakkında veri toplama amacı ile hazırlanmıştır. Araştırmaya katılmak tamamen **gönüllülük** esasına dayalıdır. Bilgileriniz araştırma kapsamı dışında hiçbir kişi ya da kuruluşla paylaşılmayacaktır. Araştırmaya katılmak istediğiniz takdirde, lütfen tüm soruları eksiksiz tamamlayınız. Katılımınız için teşekkür ederim.

İlkay YILDIZEL SAYGILI
Orta Doğu Teknik Üniversitesi, Matematik Eğitimi Yüksek Lisans Öğrencisi

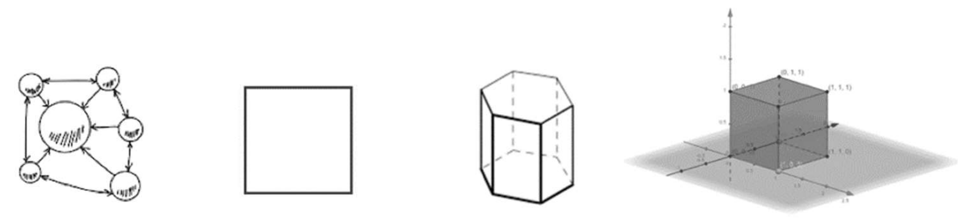
Kişisel Bilgiler

1. Cinsiyetiniz?
   - o Kadın
   - o Erkek
2. Kaç yıldır öğretmenlik yapmaktasınız?
   - o 0-5
   - o 6-10
   - o 11-15
   - o 16 ve üstü

3. Öğretmenlik deneyiminiz hakkında kısaca bilgi verebilir misiniz?
4. Geometri/matematik konularının öğretiminde kullandığınız dinamik programlar varsa nelerdir ve ne şekilde kullandığınızdan bahsedebilir misiniz?
   Teknolojiyi hangi konuları öğretirken kullanırsınız?
   Ya da hangi konularda neden kullanmıyorsunuz?
   Bir ders, kurs dahilinde mi yoksa kendi çabanızla mı öğrendiniz?

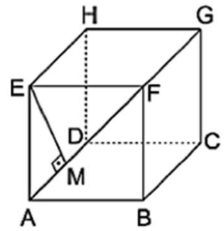
Görüşme Soruları

Diyagramlar; eskiz diyagramları, Öklid 2 Boyutlu ve 3 Boyutlu diyagramlar, dijital diyagramlar alt kategorileri ile geometrik nesneleri ve ilişkilerini temsil eden çizimlerdir.



Eskiz Diyagram 2 Boyutlu Diyagram 3 Boyutlu Diyagram Dijital Diyagram

1. Diyagramların geometri öğretimindeki rolü hakkında ne düşünüyorsunuz?
2. Üç boyutlu geometri konularını/derslerini planlarken hangi tür diyagramlara ne kadar yer verirsiniz ve bunları ne amaçla kullanıyorsunuz?
   Üç boyutlu programlardan bahsetmiştiniz, bunları nasıl değerlendiriyorsunuz?
3. Aşağıda MEB 12.sınıf ders kitabından alınan bir örnek verilmiştir.
   Sizce bu soruda öğrenciden ne yapması bekleniyor?
   Bu beklenti için bu tarz bir çizim uygun mu?
   Siz olsaydınız, böyle bir beceriyi öğrenciye kazandırmak için bu soruyu nasıl sorardınız?



ABCDEFGH küp

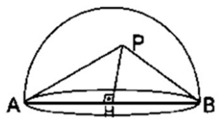
[AG] ⊥ [EM]

|EM|= 4 cm

Buna göre küpün yüzey alanı kaç $cm^2 dir$?

4. Daha önce yapılan bir çalışmada, anket sorusu olarak öğrencilere A, M, F ve G noktalarının doğrusal olup olmadığı sorulmuş ve öğrencilerin doğrusal olduğunu düşündükleri ortaya çıkmıştır. Öğrencilerin bu kanısını aşmak için ne önerebilirsiniz?
5. Aşağıda 12.sınıf geometri soru bankasından alınan bir soruya yer verilmiştir. Bu soruyu öğrencilerden nasıl çözmelerini beklersiniz? Sizce, öğrencilerin p noktasının yerini nasıl algıladıkları onların çözüm stratejilerini nasıl değiştirir?

   Bu tarz bir soruyu öğrenciler için daha anlaşılır kılmak için neler önerirsiniz?

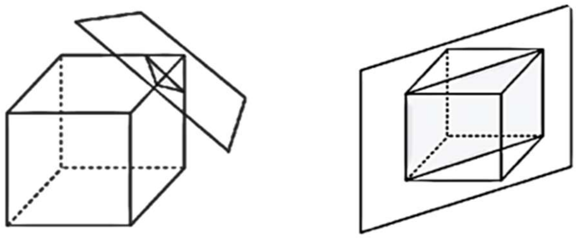


[AB] çaplı yarım kürede yüzey üzerinde alınan P noktası için,

[PH] ⊥ [AB]

IPHI = 2IAHI = 6 cm ise yarım kürenin yüzey alanı kaç cm^2?

6. Öğrencilerinize bir küp ile bir düzlem kesiştiğinde oluşabilecek olası geometrik şekilleri sorduğunuzu varsayalım. Öğrencilerden, aşağıda örnekleri gösterildiği gibi sadece 3 ve 4 kenarlı çokgenler şeklinde cevaplar gelseydi, olası çözüm yollarınız neler olur ve karşılaşabilecekleri zorluklar hakkında nasıl bir yol izlersiniz?



7. Yukarıda verilen soruları 3 boyutlu dinamik ortamda yorumlamak isteseniz, nasıl cevaplar verirsiniz?
   https://www.geogebra.org/calculator/smqwtvbn
   https://www.geogebra.org/calculator/hecsmmzc
   https://www.geogebra.org/m/kvtjst5w
8. Bir öğretmen olarak öğrencilerinizin 3 boyutlu geometrik objelerin 2 boyuttaki gösterimlerine ait diyagramlar ile ilgili kavram yanılgılarının üstesinden nasıl gelirsiniz?
   Diyagramların hangi özellikleri içermesine özen gösterirdiniz?
   Yani diyagram çizimlerini siz yapacak olsanız neyi nasıl gösterirdiniz?
9. Öğretim materyalleri hazırlamadaki açıklık ilkesi; öğrencilere kazandırılacak özelliklerin ve bu amaçla düzenlenecek etkinliklerin, öğrenci açısından net, kolay ve anlaşılır bir dil ve düzende gerçekleştirilmesidir.
   Üç boyutlu geometri konularını içeren sınav ve ders materyallerine bu ilkenin nasıl aktarılması gerektiğini düşünüyorsunuz?
   Bunun ile ilgili öğretmen eğitimine yönelik önerileriniz varsa nelerdir?

Görüşmemiz bitmiştir, katkılarınız için teşekkür ederim.

B. APPENDIX B: THE APPROVAL OF THE METU HUMAN SUBJECT ETHICS COMMITTEE

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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www.ueam.metu.edu.tr

20 HAZİRAN 2022

Konu: Değerlendirme Sonucu

Gönderen: ODTÜ İnsan Araştırmaları Etik Kurulu (İAEK)

İlgi: İnsan Araştırmaları Etik Kurulu Başvurusu

Sayın Ayhan Kürşat ERBAŞ

Danışmanlığını yürüttüğünüz İlkay Yıldızel Saygılı'nın "Yüzyüze ve Online Platformlarda Çalışan Ortaokul ve Lise Matematik Öğretmenlerinin Geometri Öğretiminde 3 Boyutlu Diyagramların Kullanımına İlişkin Perspektifleri" başlıklı araştırması İnsan Araştırmaları Etik Kurulu tarafından uygun görülerek gerekli onay **0382-ODTUİAEK-2022** protokol numarası ile onaylanmıştır.

Bilgilerinize saygılarımla sunarım.

Prof. Dr. Mine MISIRLISOY
Başkan

Doç. Dr. İ.Semih AKÇOMAK
Üye

Dr. Öğretim Üyesi Müge GÜNDÜZ
Üye

Dr. Öğretim Üyesi Şerife SEVİNÇ
Üye

Dr. Öğretim Üyesi Murat Perit ÇAKIR
Üye

Dr. Öğretim Üyesi Süreyya ÖZCAN KABASAKAL
Üye

Dr. Öğretim Üyesi A. Emre TURGUT
Üye

C. APPENDIX C: INFORMED CONSENT FORM

Ek2. Gönüllü Katılım Formu Haziran 2022

ARAŞTIRMAYA GÖNÜLLÜ KATILIM FORMU

Bu araştırma, ODTÜ Matematik ve Fen Bilimleri Eğitimi Bölümü Yüksek Lisans öğrencisi İlkay Yıldızel Saygılı tarafından Prof. Dr. Ayhan Kürşat Erbaş danışmanlığındaki yüksek lisans tezi kapsamında yürütülmektedir. Bu form sizi araştırma koşulları hakkında bilgilendirmek için hazırlanmıştır.

Çalışmanın Amacı Nedir?

Araştırmanın amacı, üç boyutlu geometri öğretiminde kullanılan diyagramlar hakkında öğretmenlerin görüşlerini öğrenmektir.

Bize Nasıl Yardımcı Olmanızı İsteyeceğiz?

Araştırmaya katılmayı kabul ederseniz, sizden bir görüşme protokolüne katılmanız beklenmektedir. Yaklaşık olarak 45- 60 dakika sürmesi beklenen bu çalıştayda sizlere bir dizi açık uçlu soru yöneltilecek ve bu sorulara neden belirli bir cevap verdiğiniz sorulacaktır. Daha sonra içerik analizi ile değerlendirilmek üzere cevaplarınızın ses kaydı alınacaktır.

Sizden Topladığımız Bilgileri Nasıl Kullanacağız?

Araştırmaya katılımınız tamamen gönüllülük temelinde olmalıdır. Çalışmada sizden kimlik veya kurum belirleyici hiçbir bilgi istenmemektedir. Cevaplarınız tamamıyla gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Katılımcılardan elde edilecek bilgiler toplu halde değerlendirilecek ve bilimsel yayımlarda kullanılacaktır.

Katılımınızla ilgili bilmeniz gerekenler:

Çalıştay, genel olarak kişisel rahatsızlık verecek sorular veya uygulamalar içermemektedir. Ancak, katılım sırasında sorulardan ya da herhangi başka bir nedenden ötürü kendinizi rahatsız hissederseniz çalıştayı yarıda bırakıp çıkmakta serbestsiniz. Böyle bir durumda çalışmayı uygulayan kişiye çalışmadan çıkmak istediğinizi söylemek yeterli olacaktır.

Araştırmayla ilgili daha fazla bilgi almak isterseniz:

Çalıştay sonunda, bu çalışmayla ilgili sorularınız cevaplanacaktır. Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz. Çalışma hakkında daha fazla bilgi almak için Matematik ve Fen Bilimleri Eğitimi Bölümü öğretim üyelerinden Prof. Dr. Ayhan Kürşat Erbaş (Tel: 0 312 210 3652 ;E-posta: erbas@metu.edu.tr) ya da yüksek lisans öğrencisi İlkay Yıldızel Saygılı (Tel: 05544802219 ;E-posta: e202282@metu.edu.tr) ile iletişim kurabilirsiniz.

Yukarıdaki bilgileri okudum ve bu çalışmaya tamamen gönüllü olarak katılıyorum.
(Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim Soyad Tarih İmza
---/----/-----

D. CALL FOR INVITATION

Tez Çalışmama Katılmak İster misiniz?

Merhabalar!

Ben matematik öğretmeni İlkay Yıldızel Saygılı, Prof. Dr. Ayhan Kürşat Erbaş süpervizörlüğünde tez çalışması yürütmekteyim. Araştırmanın amacı ,üç boyutlu geometri öğretiminde kullanılan diyagramlar hakkında öğretmenlerin görüşlerini öğrenmektir.

Araştırmaya katkı sağlamak ve detaylı biigi almak için:

ilkayyildizelsaygili@gmail.com

05544802219